Exercise 6.1

Prove the following trigonometric identities:

1. $(1 - \cos^2 A) \text{cosec}^2 A = 1$

Solution:

Taking the L.H.S,

$(1 - \cos^2 A) \text{cosec}^2 A$

$= (\sin^2 A) \text{cosec}^2 A$ [$\because \sin^2 A + \cos^2 A = 1 \Rightarrow 1 - \sin^2 A = \cos^2 A$]

$= 1^2$

$= 1 =$ R.H.S

- Hence Proved

2. $(1 + \cot^2 A) \sin^2 A = 1$

Solution:

By using the identity,

$\text{cosec}^2 A - \cot^2 A = 1 \Rightarrow \text{cosec}^2 A = \cot^2 A + 1$

Taking,

L.H.S $= (1 + \cot^2 A) \sin^2 A$

$= \text{cosec}^2 A \sin^2 A$

$= (\text{cosec } A \sin A)^2$

$= ((1/\sin A) \times \sin A)^2$

$= (1)^2$

$= 1$

$=$ R.H.S

- Hence Proved

3. $\tan^2 \theta \cos^2 \theta = 1 - \cos^2 \theta$

Solution:

We know that,

$\sin^2 \theta + \cos^2 \theta = 1$

Taking,

L.H.S $= \tan^2 \theta \cos^2 \theta$

$= (\tan \theta \times \cos \theta)^2$

$= (\sin \theta)^2$

$= \sin^2 \theta$

$= 1 - \cos^2 \theta$

$=$ R.H.S

- Hence Proved

4. $\text{cosec } \theta \sqrt{(1 - \cos^2 \theta)} = 1$

Solution:

Using identity,

$\sin^2\theta + \cos^2\theta = 1 \quad \Rightarrow \sin^2\theta = 1 - \cos^2\theta$

Taking L.H.S,

L.H.S $= \text{cosec}\,\theta \sqrt{(1 - \cos^2\theta)}$

$= \text{cosec}\,\theta \sqrt{(\sin^2\theta)}$

$= \text{cosec}\,\theta \times \sin\theta$

$= 1$

= R.H.S

- Hence Proved

5. $(\sec^2\theta - 1)(\text{cosec}^2\theta - 1) = 1$

Solution:

Using identities,

$(\sec^2\theta - \tan^2\theta) = 1$ and $(\text{cosec}^2\theta - \cot^2\theta) = 1$

We have,

L.H.S $= (\sec^2\theta - 1)(\text{cosec}^2\theta - 1)$

$= \tan^2\theta \times \cot^2\theta$

$= (\tan\theta \times \cot\theta)^2$

$= (\tan\theta \times 1/\tan\theta)^2$

$= 1^2$

$= 1$

= R.H.S

- Hence Proved

6. $\tan\theta + 1/\tan\theta = \sec\theta\,\text{cosec}\,\theta$

Solution:

We have,

L.H.S $= \tan\theta + 1/\tan\theta$

$= (\tan^2\theta + 1)/\tan\theta$

$= \sec^2\theta / \tan\theta$ $\quad [\because \sec^2\theta - \tan^2\theta = 1]$

$= (1/\cos^2\theta) \times 1/(\sin\theta/\cos\theta)$ $\quad [\because \tan\theta = \sin\theta / \cos\theta]$

$= \cos\theta/(\sin\theta \times \cos^2\theta)$

$= 1/\cos\theta \times 1/\sin\theta$

$= \sec\theta \times \text{cosec}\,\theta$

$= \sec\theta\,\text{cosec}\,\theta$

= R.H.S

- Hence Proved

7. $\cos\theta/(1 - \sin\theta) = (1 + \sin\theta)/\cos\theta$

Solution:

We know that,

$\sin^2\theta + \cos^2\theta = 1$

So, by multiplying both the numerator and the denominator by $(1+ \sin\theta)$, we get

L.H.S = $\dfrac{\cos\theta}{1-\sin\theta}$

$$= \frac{\cos\theta(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{\cos\theta}{(1+\sin\theta)(1-\sin^2\theta)}$$

$$= \frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$$

$$= \frac{(1+\sin\theta)}{\cos\theta}$$

= R.H.S

- Hence Proved

8. **cos θ/ (1 + sin θ) = (1 - sin θ)/ cos θ**

Solution:

We know that,

$\sin^2\theta + \cos^2\theta = 1$

So, by multiplying both the numerator and the denominator by (1- sin θ), we get

L.H.S = $\frac{\cos\theta}{1+\sin\theta}$

$= \frac{\cos\theta(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$= \frac{\cos\theta(1-\sin\theta)}{(1-\sin^2\theta)}$

$= \frac{\cos\theta(1-\sin\theta)}{(\cos^2\theta)}$

$= \frac{(1-\sin\theta)}{\cos\theta}$

$= \frac{(1-\sin\theta)}{\cos\theta}$

= R.H.S

- Hence Proved

9. $\cos^2\theta + 1/(1 + \cot^2\theta) = 1$

Solution:

We already know that,

$\text{cosec}^2\theta - \cot^2\theta = 1$ and $\sin^2\theta + \cos^2\theta = 1$

Taking L.H.S,

L.H.S = $\cos^2 A + \frac{1}{1+\cot^2 A}$

$= \cos^2 A + \frac{1}{\text{cosec}^2 A}$

$= \cos^2 A + \left(\frac{1}{\text{cosec } A}\right)^2$

$= \cos^2 A + \sin^2 A$

= 1

= R.H.S

- Hence Proved

10. $\sin^2 A + 1/(1 + \tan^2 A) = 1$
Solution:

We already know that,
$\sec^2\theta - \tan^2\theta = 1$ and $\sin^2\theta + \cos^2\theta = 1$
Taking L.H.S,

$$\text{L.H.S} = \sin A^2 + \frac{1}{1+\tan^2 A}$$

$$= \sin A^2 + \frac{1}{\sec^2 A}$$

$$= \sin A^2 + \left(\frac{1}{\sec A}\right)^2$$

$= \sin^2 A + \cos^2 A$
$= 1$
= R.H.S

- Hence Proved

11. $$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \operatorname{cosec}\theta - \cot\theta$$
Solution:

We know that, $\sin^2\theta + \cos^2\theta = 1$
Taking the L.H.S,

$$\text{L.H.S} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)}{\sin\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

= R.H.S

- Hence Proved

12. 1 – cos θ/ sin θ = sin θ/ 1 + cos θ
Solution:

We know that,

$\sin^2\theta + \cos^2\theta = 1$

So, by multiplying both the numerator and the denominator by (1+ cos θ), we get

$$\text{L.H.S} = \frac{1-\cos^2\theta}{(1+\cos\theta)(\sin\theta)}$$

$$= \frac{(\sin^2\theta)}{(1+\cos\theta)(\sin\theta)}$$

$$= \frac{(\sin\theta)}{(1+\cos\theta)}$$

= R.H.S

- Hence Proved

13. sin θ/ (1 – cos θ) = cosec θ + cot θ
Solution:

Taking L.H.S,

$$\text{L.H.S} = \frac{\sin\theta}{1-\cos\theta}$$

On multiplying by its conjugates, we have

$$= \frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$$

$$= \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$$

Since, $(1 - \cos^2\theta) = \sin^2\theta$

$$= \frac{\sin\theta + (\sin\theta \times \cos\theta)}{\sin^2\theta}$$

$$= \frac{\sin\theta}{\sin^2\theta} + \frac{\sin\theta \times \cos\theta}{\sin^2\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$= \text{cosec}\ \theta + \cot\theta$

= R.H.S

- Hence Proved

14. $(1 - \sin\theta) / (1 + \sin\theta) = (\sec\theta - \tan\theta)^2$

Solution:

Taking the L.H.S,

$$\text{L. H. S} = \frac{1 - \sin\theta}{1 + \sin\theta}$$

On multiplying by its conjugate, we have

$$= \frac{1 - \sin\theta}{1 + \sin\theta} \times \frac{1 - \sin\theta}{1 - \sin\theta}$$

Since, $1 - \sin^2\theta = \cos^2\theta$

$$= \frac{(1 - \sin\theta)^2}{\cos^2\theta}$$

$$= \left(\frac{1 - \sin\theta}{\cos\theta}\right)^2$$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$= (\sec\theta - \tan\theta)^2$
= R.H.S

- Hence Proved

15. $$\frac{(1 + \cot^2\theta)\tan\theta}{\sec^2\theta} = \cot\theta$$

Solution:

Taking L.H.S,

$$\text{L. H. S} = \frac{(1 + \cot^2\theta)\tan\theta}{\sec^2\theta}$$

Here, $1 + \cot^2\theta = \text{cosec}^2\theta$

$$= \frac{\text{cosec}^2\theta \times \tan\theta}{\sec^2\theta}$$

$$= \frac{1}{\sin^2\theta} \times \frac{\cos^2\theta}{1} \times \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$= \cot\theta$
$= \text{R.H.S}$

- Hence Proved

16. $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$
Solution:

Taking L.H.S,
L.H.S = $\tan^2\theta - \sin^2\theta$

Since, $\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$

$$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$$

$$= \sin^2\theta\left[\frac{1}{\cos^2\theta} - 1\right]$$

$$= \sin^2\theta\left[\frac{1-\cos^2\theta}{\cos^2\theta}\right]$$

$$= \frac{\sin^2\theta}{\cos^2\theta} \times \sin^2\theta$$

$= \tan^2\theta\sin^2\theta$
$= \text{R.H.S}$

- Hence Proved

17. $(\operatorname{cosec}\theta + \sin\theta)(\operatorname{cosec}\theta - \sin\theta) = \cot^2\theta + \cos^2\theta$
Solution:

Taking L.H.S = $(\operatorname{cosec}\theta + \sin\theta)(\operatorname{cosec}\theta - \sin\theta)$
On multiplying we get,
$= \operatorname{cosec}^2\theta - \sin^2\theta$
$= (1 + \cot^2\theta) - (1 - \cos^2\theta)$ [Using $\operatorname{cosec}^2\theta - \cot^2\theta = 1$ and $\sin^2\theta + \cos^2\theta = 1$]
$= 1 + \cot^2\theta - 1 + \cos^2\theta$
$= \cot^2\theta + \cos^2\theta$
$= \text{R.H.S}$

- Hence Proved

18. $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \tan^2\theta + \sin^2\theta$
Solution:

Taking L.H.S = $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta)$
On multiplying we get,

$= \sec^2\theta - \sin^2\theta$
$= (1 + \tan^2\theta) - (1 - \sin^2\theta)$ [Using $\sec^2\theta - \tan^2\theta = 1$ and $\sin^2\theta + \cos^2\theta = 1$]
$= 1 + \tan^2\theta - 1 + \sin^2\theta$
$= \tan^2\theta + \sin^2\theta$
= R.H.S

- Hence Proved

19. sec A(1- sin A) (sec A + tan A) = 1
Solution:

Taking L.H.S = sec A(1 – sin A)(sec A + tan A)
Substituting sec A = 1/cos A and tan A =sin A/cos A in the above we have,

L.H.S = 1/cos A (1 – sin A)(1/cos A + sin A/cos A)
$= 1 - \sin^2 A / \cos^2 A$ [After taking L.C.M]
$= \cos^2 A / \cos^2 A$ [$\because 1 - \sin^2 A = \cos^2 A$]
= 1
= R.H.S

- Hence Proved

20. (cosec A – sin A)(sec A – cos A)(tan A + cot A) = 1
Solution:

Taking L.H.S = (cosec A – sin A)(sec A – cos A)(tan A + cot A)

Putting, $\operatorname{cosec} A = \frac{1}{\sin A}, \sec A = \frac{1}{\cos A}, \tan A = \frac{\sin A}{\cos A}, \cot A = \frac{\cos A}{\sin A}$

Subsituting the above in the L.H.S, we get

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)\left(\frac{\sin^2 A + \cos^2 A}{\sin A \ \cos A}\right)$$

$= (\cos^2 A/ \sin A)\ (\sin^2 A/ \cos A)\ (1/ \sin A \cos A)$ [$\because \sin^2\theta + \cos^2\theta = 1$]
= sin A x cos A x (1/ cos A sin A)
= R.H.S

- Hence Proved

21. $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta) = 1$
Solution:

Taking L.H.S $= (1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$
And, we know $\sin^2\theta + \cos^2\theta = 1$ and $\sec^2\theta - \tan^2\theta = 1$

So,
L.H.S $= (1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$
$= (1 + \tan^2\theta)\{(1 - \sin\theta)(1 + \sin\theta)\}$
$= (1 + \tan^2\theta)(1 - \sin^2\theta)$
$= \sec^2\theta\,(\cos^2\theta)$
$= (1/\cos^2\theta) \times \cos^2\theta$
$= 1$
= R.H.S

- Hence Proved

22. $\sin^2 A \cot^2 A + \cos^2 A \tan^2 A = 1$
Solution:

We know that,
$\cot^2 A = \cos^2 A/\sin^2 A$ and $\tan^2 A = \sin^2 A/\cos^2 A$
Substituting the above in L.H.S, we get
L.H.S $= \sin^2 A \cot^2 A + \cos^2 A \tan^2 A$
$= \{\sin^2 A (\cos^2 A/\sin^2 A)\} + \{\cos^2 A (\sin^2 A/\cos^2 A)\}$
$= \cos^2 A + \sin^2 A$
$= 1$ [$\because \sin^2\theta + \cos^2\theta = 1$]
= R.H.S

- Hence Proved

23. (i) $\cot\theta - \tan\theta = \dfrac{2\cos 2\theta - 1}{\sin\theta * \cos\theta}$

(ii) $\tan\theta - \cot\theta = \left(\dfrac{2\sin^2\theta - 1}{\sin\theta * \cos\theta}\right)$

Solution:

(i) Taking the L.H.S and using $\sin^2\theta + \cos^2\theta = 1$, we have
L.H.S $= \cot\theta - \tan\theta$

$$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta \times \cos\theta}$$

$$= \frac{\cos^2\theta - (1 - \cos^2\theta)}{\sin\theta \times \cos\theta}$$

$$= \frac{\cos^2\theta - 1 - \cos^2\theta}{\sin\theta \times \cos\theta}$$

$$= \left(\frac{2\cos^2\theta - 1}{\sin\theta \times \cos\theta}\right)$$

= R.H.S

- Hence Proved

(ii) Taking the L.H.S and using $\sin^2\theta + \cos^2\theta = 1$, we have

L.H.S $= \tan\theta - \cot\theta$

$$= \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta - (1 - \sin^2\theta)}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta - (1 + \sin^2\theta)}{\sin\theta\cos\theta}$$

$$= \left(\frac{2\sin^2\theta - 1}{\sin\theta \quad \cos\theta}\right)$$

= R.H.S

- Hence Proved

24. $(\cos^2\theta / \sin\theta) - \operatorname{cosec}\theta + \sin\theta = 0$

Solution:

Taking L.H.S and using $\sin^2\theta + \cos^2\theta = 1$, we have

$$\text{L.H.S} = \frac{\cos^2\theta}{\sin\theta} - \text{cosec}\,\theta + \sin\theta$$

$$= \left(\frac{\cos^2\theta}{\sin\theta} - \text{cosec}\,\theta\right) + \sin\theta$$

$$= \left(\frac{\cos^2\theta}{\sin\theta} - \frac{1}{\sin\theta}\right) + \sin\theta$$

$$= \left(\frac{\cos^2\theta - 1}{\sin\theta}\right) + \sin\theta$$

$$= \left(\frac{-\sin^2\theta}{\sin\theta}\right) + \sin\theta$$

$= -\sin\theta + \sin\theta$

$= 0$

= R.H.S

- Hence proved

25. $$\frac{1}{1+\sin A} + \frac{1}{1-\sin A} = 2\sec^2 A$$

Solution:

Taking L.H.S,

$$\text{LHS} = \frac{1}{1+\sin A} + \frac{1}{1-\sin A}$$

$$= \frac{(1-\sin A)+(1+\sin A)}{(1+\sin A)(1-\sin A)}$$

$$= \frac{1-\sin A+1+\sin A}{1-\sin^2 A} \quad \because (1+\sin A)(1-\sin A) = 1-\sin^2 A$$

$$= \frac{2}{1-\sin^2 A}$$

$$= \frac{2}{\cos^2 A} \quad [\because 1-\sin^2 A = \cos A]$$

$= 2\sec^2 A$

= R.H.S

- Hence proved

26. $$\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}=2\sec\theta$$

Solution:

Taking the LHS and using $\sin^2\theta + \cos^2\theta = 1$, we have

$$\text{LHS} = \frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{1+\sin^2\theta+2\sin\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$$

$= 2/\cos\theta$

$= 2\sec\theta$

= R.H.S

- Hence proved

27. $$\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta}=\frac{1+\sin^2\theta}{1-\sin^2\theta}$$

Solution:

Taking the LHS and using $\sin^2\theta + \cos^2\theta = 1$, we have

$$\text{L.H.S} = \frac{(1+\sin\theta)^2 + (1-\sin\theta)^2}{2\cos^2\theta}$$

$$= \frac{(1+2\sin\theta+\sin^2\theta)+(1-2\sin\theta+\sin^2\theta)}{2\cos^2\theta}$$

$$= \frac{1+2\sin\theta+\sin^2\theta+1-2\sin\theta+\sin^2\theta}{2\cos^2\theta}$$

$$= \frac{2+2\sin^2\theta}{2\cos^2\theta}$$

$$= \frac{2(1+\sin^2\theta)}{2(1-\sin^2\theta)}$$

$$= \frac{(1+\sin^2\theta)}{(1-\sin^2\theta)}$$

= R.H.S

- Hence proved

28. $$\frac{1+\tan^2\theta}{1+\cot^2\theta} = \left[\frac{1-\tan\theta}{\cot\theta}\right]^2 = \tan^2\theta$$

Solution:

Taking L.H.S,

$$\frac{1+\tan^2\theta}{1+\cot^2\theta}$$

Using $\sec^2\theta - \tan^2\theta = 1$ and $\text{cosec}^2\theta - \cot^2\theta = 1$

$$= \frac{1+\tan^2\theta}{1+\cot^2\theta} = \frac{\sec^2\theta}{\text{cosec}^2\theta}$$

$$= \frac{1}{\cos^2\theta \quad 1}\sin^2\theta = \tan^2\theta$$

= R.H.S

And, taking

$\left[\frac{1-\tan\theta}{\cot\theta}\right]^2 = \frac{1+tan^2\theta-2tan\theta}{1+cot^2\theta-2cot\theta}$

$= \frac{sec^2\theta-2tan\theta}{cosec^2\theta-2cot\theta}$ [Using $\sec^2\theta - \tan^2\theta = 1$ and $\mathrm{cosec}^2\theta - \cot^2\theta = 1$]

$= \frac{\frac{1}{\cos^2\theta}-\frac{2\sin\theta}{\cos\theta}}{\frac{1}{\sin^2\theta}-\frac{2\cos\theta}{\sin\theta}} = \frac{\frac{1-2\sin\theta\cos\theta}{\cos^2\theta}}{\frac{1-2\sin\theta\cos\theta}{\sin^2\theta}}$

$= \frac{sin^2\theta}{cos^2\theta}$

$= \tan^2\theta$ = R.H.S

\- Hence proved

29. $\frac{1+\sec\theta}{\sec\theta} = \frac{\sin^2\theta}{1-\cos\theta}$

Solution:

Taking L.H.S and using $\sin^2\theta + \cos^2\theta = 1$, we have

$$\text{LHS} = \frac{1+\sec\theta}{\sec\theta}$$

$$= \frac{1+\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$$= \frac{\cos\theta+1}{\cos\theta}\cdot\cos\theta$$

$$= 1+\cos\theta$$

$$= \frac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta}$$ Multiplying by (1 - cos θ) to numerator and denominator

$$= \frac{1-\cos^2\theta}{1-\cos\theta}$$

$$= \frac{\sin^2\theta}{1-\cos\theta}$$

= R.H.S

- Hence proved

30. $$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta} = 1+\tan\theta+\cot\theta$$

Solution:

Taking LHS, we have

$$\text{LHS} = \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{1}{1-\tan\theta}\left[\frac{1}{\tan\theta} - \tan^2\theta\right]$$

$$= \frac{1}{1-\tan\theta}\left[\frac{1-\tan^3\theta}{\tan\theta}\right]$$

$$= \frac{1}{1-\tan\theta}\,\frac{(1-\tan\theta)(1+\tan\theta+\tan^2\theta)}{\tan\theta} \quad \left[\text{Since, } a^3-b^3=(a-b)(a^2+ab+b^2)\right]$$

$$= \frac{1+\tan\theta+\tan^2\theta}{\tan\theta}$$

$$= \frac{1}{\tan\theta} + \frac{\tan\theta}{\tan\theta} + \frac{\tan^2\theta}{\tan\theta}$$

$= 1 + \tan\theta + \cot\theta$
= R.H.S

- Hence proved

31. $\sec^6\theta = \tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$
Solution:

From trig. Identities we have,
$\sec^2\theta - \tan^2\theta = 1$
On cubing both sides,
$(\sec^2\theta - \tan^2\theta)^3 = 1$
$\sec^6\theta - \tan^6\theta - 3\sec^2\theta\tan^2\theta(\sec^2\theta - \tan^2\theta) = 1$
[Since, $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$]
$\sec^6\theta - \tan^6\theta - 3\sec^2\theta\tan^2\theta = 1$
$\Rightarrow \sec^6\theta = \tan^6\theta + 3\sec^2\theta\tan^2\theta + 1$
Hence, L.H.S = R.H.S

- Hence proved

32. $\text{cosec}^6\theta = \cot^6\theta + 3\cot^2\theta\,\text{cosec}^2\theta + 1$
Solution:

From trig. Identities we have,

$\text{cosec}^2\theta - \cot^2\theta = 1$

On cubing both sides,

$(\text{cosec}^2\theta - \cot^2\theta)^3 = 1$

$\text{cosec}^6\theta - \cot^6\theta - 3\text{cosec}^2\theta\cot^2\theta(\text{cosec}^2\theta - \cot^2\theta) = 1$

[Since, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$]

$\text{cosec}^6\theta - \cot^6\theta - 3\text{cosec}^2\theta\cot^2\theta = 1$

$\Rightarrow \quad \text{cosec}^6\theta = \cot^6\theta + 3\text{cosec}^2\theta\cot^2\theta + 1$

Hence, L.H.S = R.H.S

- Hence proved

33. $$\frac{(1+\tan^2\theta)\cot\theta}{\text{cosec}^2\theta} = \tan\theta$$

Solution:

Taking L.H.S and using $\sec^2\theta - \tan^2\theta = 1 \Rightarrow 1 + \tan^2\theta = \sec^2\theta$

$$\text{LHS} = \frac{\sec^2\theta \cdot \cot\theta}{\text{cosec}^2\theta}$$

$$= \frac{1 \cdot \sin^2\theta}{\cos^2\theta} \cdot \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta$$

= R.H.S

- Hence proved

34. $$\frac{1+\cos A}{\sin^2 A} = \frac{1}{1-\cos A}$$

Solution:

Taking L.H.S and using the identity $\sin^2A + \cos^2A = 1$, we get

$\sin^2A = 1 - \cos^2A$

$\Rightarrow \quad \sin^2A = (1 - \cos A)(1 + \cos A)$

$$\text{LHS} = \frac{1+\cos A}{(1-\cos A)(1+\cos A)}$$

$$= \frac{1}{(1-\cos A)}$$

- Hence proved

35. $$\frac{\sec A - \tan A}{\sec A + \tan A} = \frac{\cos^2 A}{(1 + \sin A)^2}$$

Solution:

We have,

$$\text{LHS} = \frac{\sec A - \tan A}{\sec A + \tan A}$$

Rationalizing the denominator and numerator with (sec A + tan A) and using $\sec^2\theta - \tan^2\theta = 1$ we get,

$$= \frac{\sec^2 A - \tan^2 A}{(\sec A + \tan A)^2}$$

$$= \frac{1}{(\sec A + \tan A)^2}$$

$$= \frac{1}{(\sec^2 A + \tan^2 A + 2\sec A \tan A)}$$

$$= \frac{1}{\left(\frac{1}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A} + \frac{2\sin A}{\cos A}\right)}$$

$$\Rightarrow \frac{\cos^2 A}{1 + \sin^2 A + 2\sin A}$$

$$= \frac{\cos^2 A}{(1 + \sin A)^2}$$

= R.H.S

- Hence proved

36. $$1 + \frac{\cos A}{\sin A} = \frac{\sin A}{1 - \cos A}$$

Solution:

We have,

$$\text{LHS} = \frac{1 + \cos A}{\sin A}$$

On multiplying numerator and denominator by (1 – cos A), we get

$$= \frac{(1+\cos A)(1-\cos A)}{\sin A(1-\cos A)}$$

$$= \frac{1-\cos^2 A}{\sin A(1-\cos A)}$$

$$= \frac{\sin^2 A}{\sin A(1-\cos A)}$$

$$= \frac{\sin A}{1-\cos A}$$

= R.H.S

- Hence proved

37. (i) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

Solution:

Taking L.H.S and rationalizing the numerator and denominator with √(1 + sin A), we get

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A}} = \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \sqrt{\frac{(1+\sin A)}{\cos A}}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$$

= R.H.S

- Hence proved

(ii) $\sqrt{\frac{(1-\cos A)}{(1+\cos A)}} + \sqrt{\frac{(1+\cos A)}{(1-\cos A)}} = 2 \operatorname{cosec} A$

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

$$= \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}} + \sqrt{\frac{(1+\cos A)(1+\cos A)}{(1-\cos A)(1+\cos A)}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{(1-\cos^2 A)}} + \sqrt{\frac{(1+\cos A)^2}{(1-\cos^2 A)}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{(\sin^2 A)}} + \sqrt{\frac{(1+\cos A)^2}{(\sin^2 A)}}$$

$$= \frac{(1-\cos A)}{(\sin A)} + \frac{(1+\cos A)}{(\sin A)}$$

$$= \frac{(1-\cos A+1+\cos A)}{(\sin A)}$$

$$= \frac{(2)}{(\sin A)}$$

= 2 cosec A
= R.H.S

- Hence proved

38. Prove that:
(i)

$$\sqrt{\frac{(\sec\theta-1)}{(\sec\theta+1)}} + \sqrt{\frac{(\sec\theta+1)}{(\sec\theta-1)}} = 2\,\text{cosec}\,\theta$$

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

$$= \sqrt{\frac{(\sec\theta - 1)(\sec\theta - 1)}{(\sec\theta + 1)(\sec\theta - 1)}} + \sqrt{\frac{(\sec\theta + 1)(\sec\theta + 1)}{(\sec\theta - 1)(\sec\theta + 1)}}$$

$$= \sqrt{\frac{(\sec\theta - 1)^2}{(\sec^2\theta - 1)}} + \sqrt{\frac{(\sec\theta + 1)^2}{(\sec^2\theta - 1)}}$$

$$= \sqrt{\frac{(\sec\theta - 1)^2}{\tan^2\theta}} + \sqrt{\frac{(\sec\theta + 1)^2}{\tan^2\theta}}$$

$$= \frac{(\sec\theta - 1)}{\tan\theta} + \frac{(\sec\theta + 1)}{\tan\theta}$$

$$= \frac{(\sec\theta - 1 + \sec\theta + 1)}{\tan\theta}$$

$$= \frac{(2\cos\theta)}{\cos\theta\sin\theta}$$

$$= \frac{2}{\sin\theta}$$

= R.H.S

- Hence proved

(ii) $\sqrt{\frac{(1 + \sin\theta)}{(1 - \sin\theta)}} + \sqrt{\frac{(1 - \sin\theta)}{(1 + \sin\theta)}} = 2\sec\theta$

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

$$= \sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}} + \sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{(1-\sin^2\theta)}} + \sqrt{\frac{(1-\sin\theta)^2}{(1-\sin^2\theta)}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{(\cos^2\theta)}} + \sqrt{\frac{(1-\sin\theta)^2}{(\cos^2\theta)}}$$

$$= \frac{(1+\sin\theta)}{(\cos\theta)} + \frac{(1-\sin\theta)}{(\cos\theta)}$$

$$= \sqrt{\frac{(1+\sin\theta+1-\sin\theta)}{(\cos\theta)}}$$

$$= \frac{(2)}{(\cos\theta)} = 2\sec\theta$$

= R.H.S

- Hence proved

(iii) $$\sqrt{\frac{(1+\cos\theta)}{(1-\cos\theta)}} + \sqrt{\frac{(1-\cos\theta)}{(1+\cos\theta)}} = 2\,\mathrm{cosec}\,\theta$$

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

$$= \sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}} + \sqrt{\frac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}} + \sqrt{\frac{(1+\cos\theta)^2}{(1-\cos^2\theta)}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{(\sin^2\theta)}} + \sqrt{\frac{(1+\cos\theta)^2}{(\sin^2\theta)}}$$

$$= \frac{(1-\cos\theta)}{(\sin\theta)} + \frac{(1+\cos\theta)}{(\sin\theta)}$$

$$= \frac{(1-\cos\theta+1+\cos\theta)}{(\sin\theta)}$$

$$= \frac{(2)}{(\sin\theta)}$$

= 2 cosec θ
= R.H.S

- Hence proved

(iv) $\frac{\sec\theta-1}{\sec\theta+1} = \left(\frac{\sin\theta}{1+\cos\theta}\right)^2$

Solution:

Taking L.H.S, we have

$$= \frac{\sec\theta - 1}{\sec\theta + 1} = \frac{1 - \cos\theta}{1 + \cos\theta}$$

On multiplying numerator and denominator by $1 + \cos\theta$, we get

$$= \frac{(1 - \cos\theta)(1 + \cos\theta)}{(1 + \cos\theta)(1 + \cos\theta)}$$

$$= \frac{(1 - \cos^2\theta)}{(1 + \cos\theta)^2}$$

$$= \frac{\sin^2\theta}{(1 + \cos\theta)^2}$$

$$= \left(\frac{\sin\theta}{1 + \cos\theta}\right)^2$$

= R.H.S

- Hence proved

39. $(\sec A - \tan A)^2 = \frac{1 - \sin A}{1 + \sin A}$

Solution:

Taking LHS = $(\sec A – \tan A)^2$, we have

$$= \left[\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right]^2$$

$$= \frac{(1 - \sin A)^2}{\cos^2 A}$$

$$= \frac{(1 - \sin A)^2}{1 - \sin^2 A}$$

$$= \frac{(1 - \sin A)^2}{(1 + \sin A)(1 - \sin A)}$$

$$= \frac{(1 - \sin A)}{(1 + \sin A)}$$

= R.H.S

- Hence proved

40. $\frac{1-\cos A}{1+\cos A} = (\cot A - \operatorname{cosec} A)^2$

Solution:

Taking L.H.S and rationalizing the numerator and denominator with (1 – cos A), we get

$$= \frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}$$

$$= \frac{(1-\cos A)^2}{(1-\cos^2 A)}$$

$$= \frac{(1-\cos A)^2}{(\sin^2 A)}$$

$$= \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right)^2$$

= $(\operatorname{cosec} A - \cot A)^2$
= $(\cot A - \operatorname{cosec})^2$
= R.H.S

- Hence proved

41. $\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = 2 \operatorname{cosec} A \cot A$

Solution:

Considering L.H.S and taking L.C.M and on simplifying we have,

$$= \frac{\sec A + 1 + \sec A - 1}{(\sec A + 1)(\sec A - 1)}$$

$$= \frac{2 \sec A}{(\sec^2 A - 1)}$$

$$= \frac{2 \sec A}{(\tan^2 A)}$$

$$= \frac{2 \cos^2 A}{(\cos A \sin^2 A)}$$

$$= \frac{2 \cos A}{(\sin^2 A)}$$

$$= \frac{2 \cos A}{(\sin A)(\sin A)}$$

= 2 cosec A cot A = RHS

- Hence proved

42. $\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$

Solution:

Taking LHS, we have

$$= \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A}$$

$$= \frac{\cos A}{1 - \frac{\sin A}{\cos A}} + \frac{\sin A}{1 - \frac{\cos A}{\sin A}}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\cos A - \sin A}$$

= cos A + sin A

= RHS

- Hence proved

43. $$\frac{(\operatorname{cosec} A)}{(\operatorname{cosec} A - 1)} + \frac{(\operatorname{cosec} A)}{(\operatorname{cosec} A + 1)} = 2\sec^2 A$$

Solution:

Considering L.H.S and taking L.C.M and on simplifying we have,

$$= \frac{(\operatorname{cosec} A)(\operatorname{cosec} A + 1 + \operatorname{cosec} A - 1)}{(\operatorname{cosec}^2 A - 1)}$$

$$= \frac{(2\operatorname{cosec}^2 A)}{\cot^2 A}$$

$$= \frac{2\sin^2 A}{\sin^2 A.\ \cos^2 A}$$

$$= \frac{2}{\cos^2 A}$$

= $2\sec^2 A$

= RHS

- Hence proved

Exercise 6.2 Page No: 6.54

1. If $\cos\theta = 4/5$, find all other trigonometric ratios of angle θ.
Solution:

We have,
$\cos\theta = 4/5$
And we know that,
$\sin\theta = \sqrt{(1 - \cos^2\theta)}$
$\Rightarrow \sin\theta = \sqrt{(1 - (4/5)^2)}$
$= \sqrt{(1 - (16/25))}$
$= \sqrt{[(25 - 16)/25]}$
$= \sqrt{(9/25)}$
$= 3/5$
$\therefore \sin\theta = 3/5$

Since, $\text{cosec}\,\theta = 1/\sin\theta$
$= 1/(3/5)$
$\Rightarrow \text{cosec}\,\theta = 5/3$

And, $\sec\theta = 1/\cos\theta$
$= 1/(4/5)$
$\Rightarrow \text{cosec}\,\theta = 5/4$

Now,
$\tan\theta = \sin\theta/\cos\theta$
$= (3/5)/(4/5)$
$\Rightarrow \tan\theta = 3/4$

And, $\cot\theta = 1/\tan\theta$
$= 1/(3/4)$
$\Rightarrow \cot\theta = 4/3$

2. If $\sin\theta = 1/\sqrt{2}$, find all other trigonometric ratios of angle θ.
Solution:

We have,
$\sin\theta = 1/\sqrt{2}$
And we know that,
$\cos\theta = \sqrt{(1 - \sin^2\theta)}$
$\Rightarrow \cos\theta = \sqrt{(1 - (1/\sqrt{2})^2)}$
$= \sqrt{(1 - (1/2))}$
$= \sqrt{[(2 - 1)/2]}$
$= \sqrt{(1/2)}$
$= 1/\sqrt{2}$
$\therefore \cos\theta = 1/\sqrt{2}$

Since, cosec θ = 1/ sin θ
= 1/ (1/√2)
⇒ cosec θ = √2

And, sec θ = 1/ cos θ
= 1/ (1/√2)
⇒ cosec θ = √2

Now,
tan θ = sin θ/ cos θ
= (1/√2)/ (1/√2)
⇒ tan θ = 1

And, cot θ = 1/ tan θ
= 1/ (1)
⇒ cot θ = 1

3. If $\tan\theta = \frac{1}{\sqrt{2}}$, find the value of $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \cot^2\theta}$.

Solution:

Given,
tan θ = 1/√2
By using $\sec^2\theta - \tan^2\theta = 1$,

$$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+\left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{1+\frac{1}{2}} = \sqrt{\frac{3}{2}}$$

And,

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

From identity, we have

$$\text{cosec}\,\theta = \sqrt{1+\cot^2\theta} = \sqrt{1+2} = \sqrt{3}$$

Substituting the values, we get

$$\frac{\text{cosec}^2\,\theta - \sec^2\theta}{\text{cosec}^2\theta + \cot^2\theta} = \frac{(\sqrt{3})^2 - \left(\sqrt{\frac{3}{2}}\right)^2}{(\sqrt{3})^2 + (\sqrt{2})^2}$$

$$= \frac{3-\frac{3}{2}}{3+2} = \frac{\frac{3}{2}}{5} = \frac{3}{10}$$

4. If $\tan\theta = \frac{3}{4}$, find the value of $\frac{1-\cos\theta}{1+\cos\theta}$

Solution:

Given,

$\tan\theta = 3/4$

By using $\sec^2\theta - \tan^2\theta = 1$,

$$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+\left(\frac{3}{4}\right)^2} = \sqrt{1+\frac{9}{16}} = \sqrt{\frac{16+9}{16}} = \sqrt{\frac{25}{16}}$$

sec $\theta = 5/4$

Since, $\sec\theta = 1/\cos\theta$

$\Rightarrow$ $\cos\theta = 1/\sec\theta$

$= 1/(5/4)$

$= 4/5$

So, $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \cot^2\theta} = \frac{1-\frac{4}{5}}{1+\frac{4}{5}} = \frac{\frac{1}{5}}{\frac{9}{5}} = \frac{1}{9}$

5. If $\tan\theta = \frac{12}{5}$, find the value of $\frac{1+\sin\theta}{1-\sin\theta}$

Solution:

Given, $\tan\theta = 12/5$
Since, $\cot\theta = 1/\tan\theta = 1/(12/5) = 5/12$

Now, by using $\text{cosec}^2\theta - \cot^2\theta = 1$
$\text{cosec}\,\theta = \sqrt{(1 + \cot^2\theta)}$
$= \sqrt{(1 + (5/12)^2)}$
$= \sqrt{(1 + 25/144)}$
$= \sqrt{(169/25)}$
⇒ $\text{cosec}\,\theta = 13/5$
Now, we know that
$\sin\theta = 1/\text{cosec}\,\theta$
$= 1/(13/5)$
⇒ $\sin\theta = 5/13$
Putting value of $\sin\theta$ in the expression we have,

$$= \frac{1+\frac{12}{13}}{1-\frac{12}{13}} = \frac{\frac{13+12}{18}}{\frac{13-12}{18}}$$

$= 25/1$
$= 25$

6. If $\cot\theta = \frac{1}{\sqrt{3}}$, find the value of $\frac{1-\cos^2\theta}{2-\sin^2\theta}$

Solution:

Given,
$\cot\theta = 1/\sqrt{3}$
Using $\text{cosec}^2\theta - \cot^2\theta = 1$, we can find $\text{cosec}\,\theta$
$\text{cosec}\,\theta = \sqrt{(1 + \cot^2\theta)}$
$= \sqrt{(1 + (1/\sqrt{3})^2)}$
$= \sqrt{(1 + (1/3))} = \sqrt{((3 + 1)/3)}$
$= \sqrt{(4/3)}$

⇒ $\text{cosec}\ \theta = 2/\sqrt{3}$

So, $\sin\theta = 1/\text{cosec}\ \theta = 1/(2/\sqrt{3})$

⇒ $\sin\theta = \sqrt{3}/2$

And, we know that

$\cos\theta = \sqrt{(1 - \sin^2\theta)}$

$= \sqrt{(1 - (\sqrt{3}/2)^2)}$

$= \sqrt{(1 - (3/4))}$

$= \sqrt{((4 - 3)/4)}$

$= \sqrt{(1/4)}$

⇒ $\cos\theta = 1/2$

Now, using cos θ and sin θ in the expression, we have

$$= \frac{1 - \left(\frac{1}{2}\right)^2}{2 - \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1 - \frac{1}{4}}{2 - \frac{3}{4}} = \frac{\frac{3}{4}}{\frac{5}{4}}$$

$= 3/5$

7. If $\text{cosec}\ A = \sqrt{2}$, find the value of $\frac{2\sin^2 A + 3\cot^2 A}{4(\tan^2 A - \cos^2 A)}$.

Solution:

Given,

$\text{cosec}\ A = \sqrt{2}$

Using $\text{cosec}^2 A - \cot^2 A = 1$, we find cot A

$\cot A = \sqrt{\text{cosec}^2 A - 1} \quad = \sqrt{(\sqrt{2})^2 - 1} \quad = \sqrt{2 - 1} \quad = 1$

So, tan A = 1/ cot A
= 1/ 1 = 1

And, sin A = 1/ cosec A = 1/ √2

$\cos A = \sqrt{1 - \sin^2 A} \quad = \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}}$

On substituting we get,

$$= \frac{2\left[\frac{1}{\sqrt{2}}\right]^2 + 3[1]^2}{4\left[[1] - \left[\frac{1}{\sqrt{2}}\right]^2\right]}$$

$$= \frac{2 \times \frac{1}{2} + 3}{4\left[1 - \frac{1}{2}\right]} \Rightarrow \frac{1 + 3}{4.\frac{1}{2}}$$