

Exercise 8.3

Page No: 8.19

Solve the following quadratic equation by factorization:

1. $(x - 4)(x + 2) = 0$

Solution:

Given,

$$(x - 4)(x + 2) = 0$$

$$\text{So, either } x - 4 = 0 \Rightarrow x = 4$$

$$\text{Or, } x + 2 = 0, \Rightarrow x = -2$$

Thus, the roots of the given quadratic equation are 4 and -2 respectively.

2. $(2x + 3)(3x - 7) = 0$

Solution:

Given,

$$(2x + 3)(3x - 7) = 0.$$

$$\text{So, either } 2x + 3 = 0, \Rightarrow x = -3/2$$

$$\text{Or, } 3x - 7 = 0, \Rightarrow x = 7/3$$

Thus, the roots of the given quadratic equation are $x = -3/2$ and $x = 7/3$ respectively.

3. $3x^2 - 14x - 5 = 0$

Solution:

Given.

$$3x^2 - 14x - 5 = 0$$

$$\Rightarrow 3x^2 - 14x - 5 = 0$$

$$\Rightarrow 3x^2 - 15x + x - 5 = 0$$

$$\Rightarrow 3x(x - 5) + 1(x - 5) = 0$$

$$\Rightarrow (3x + 1)(x - 5) = 0$$

$$\text{Now, either } 3x + 1 = 0 \Rightarrow x = -1/3$$

$$\text{Or, } x - 5 = 0 \Rightarrow x = 5$$

Thus, the roots of the given quadratic equation are 5 and $x = -1/3$ respectively.

4. Find the roots of the equation $9x^2 - 3x - 2 = 0$.

Solution:

Given,

$$9x^2 - 3x - 2 = 0.$$

$$\Rightarrow 9x^2 - 3x - 2 = 0.$$

$$\Rightarrow 9x^2 - 6x + 3x - 2 = 0$$

$$\Rightarrow 3x(3x - 2) + 1(3x - 2) = 0$$

$$\Rightarrow (3x - 2)(3x + 1) = 0$$

$$\text{Now, either } 3x - 2 = 0 \Rightarrow x = 2/3$$

Or, $3x + 1 = 0 \Rightarrow x = -1/3$

Thus, the roots of the given quadratic equation are $x = 2/3$ and $x = -1/3$ respectively.

5.
$$\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}$$

Solution:

Given,

$$\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}$$

$$\frac{x+5-x+1}{(x-1)(x+5)} = \frac{6}{7}$$

$$\frac{6}{x^2+4x-5} = \frac{6}{7}$$

Dividing by 6 both the sides and cross-multiplying we get

$$x^2 + 4x - 12 = 0$$

$$\Rightarrow x^2 + 6x - 2x - 12 = 0$$

$$\Rightarrow x(x+6) - 2(x-6) = 0$$

$$\Rightarrow (x+6)(x-2) = 0$$

Now, either $x + 6 = 0 \Rightarrow x = -6$

Or, $x - 2 = 0 \Rightarrow x = 2$

Thus, the roots of the given quadratic equation are 2 and -6 respectively.

6. $6x^2 + 11x + 3 = 0$

Solution:

Given equation is $6x^2 + 11x + 3 = 0$.

$$\Rightarrow 6x^2 + 9x + 2x + 3 = 0$$

$$\Rightarrow 3x(2x+3) + 1(2x+3) = 0$$

$$\Rightarrow (2x+3)(3x+1) = 0$$

Now, either $2x + 3 = 0 \Rightarrow x = -3/2$

Or, $3x + 1 = 0 \Rightarrow x = -1/3$

Thus, the roots of the given quadratic equation are $x = -3/2$ and $x = -1/3$ respectively.

7. $5x^2 - 3x - 2 = 0$

Solution:

Given equation is $5x^2 - 3x - 2 = 0$.

$$\Rightarrow 5x^2 - 3x - 2 = 0.$$

$$\Rightarrow 5x^2 - 5x + 2x - 2 = 0$$

$$\Rightarrow 5x(x - 1) + 2(x - 1) = 0$$

$$\Rightarrow (5x + 2)(x - 1) = 0$$

Now, either $5x + 2 = 0 \Rightarrow x = -2/5$

Or, $x - 1 = 0 \Rightarrow x = 1$

Thus, the roots of the given quadratic equation are 1 and $x = -2/5$ respectively.

8. $48x^2 - 13x - 1 = 0$

Solution:

Given equation is $48x^2 - 13x - 1 = 0$.

$$\Rightarrow 48x^2 - 13x - 1 = 0.$$

$$\Rightarrow 48x^2 - 16x + 3x - 1 = 0.$$

$$\Rightarrow 16x(3x - 1) + 1(3x - 1) = 0$$

$$\Rightarrow (16x + 1)(3x - 1) = 0$$

Either $16x + 1 = 0 \Rightarrow x = -1/16$

Or, $3x - 1 = 0 \Rightarrow x = 1/3$

Thus, the roots of the given quadratic equation are $x = -1/16$ and $x = 1/3$ respectively.

9. $3x^2 = -11x - 10$

Solution:

Given equation is $3x^2 = -11x - 10$

$$\Rightarrow 3x^2 + 11x + 10 = 0$$

$$\Rightarrow 3x^2 + 6x + 5x + 10 = 0$$

$$\Rightarrow 3x(x + 2) + 5(x + 2) = 0$$

$$\Rightarrow (3x + 2)(x + 2) = 0$$

Now, either $3x + 2 = 0 \Rightarrow x = -2/3$

Or, $x + 2 = 0 \Rightarrow x = -2$

Thus, the roots of the given quadratic equation are $x = -2/3$ and -2 respectively.

10. $25x(x + 1) = -4$

Solution:

Given equation is $25x(x + 1) = -4$

$$25x(x + 1) = -4$$

$$\Rightarrow 25x^2 + 25x + 4 = 0$$

$$\Rightarrow 25x^2 + 20x + 5x + 4 = 0$$

$$\Rightarrow 5x(5x + 4) + 1(5x + 4) = 0$$

$$\Rightarrow (5x + 4)(5x + 1) = 0$$

Now, either $5x + 4 = 0$ therefore $x = -4/5$

Or, $5x + 1 = 0$ therefore $x = -1/5$

Thus, the roots of the given quadratic equation are $x = -4/5$ and $x = -1/5$ respectively.

11. $16x - 10/x = 27$

Solution:

Given,

$$16x - 10/x = 27$$

On multiplying x on both the sides we have,

$$\Rightarrow 16x^2 - 10 = 27x$$

$$\Rightarrow 16x^2 - 27x - 10 = 0$$

$$\Rightarrow 16x^2 - 32x + 5x - 10 = 0$$

$$\Rightarrow 16x(x - 2) + 5(x - 2) = 0$$

$$\Rightarrow (16x + 5)(x - 2) = 0$$

$$\text{Now, either } 16x + 5 = 0 \quad \Rightarrow x = -5/16$$

$$\text{Or, } x - 2 = 0 \quad \Rightarrow x = 2$$

Thus, the roots of the given quadratic equation are $x = -5/16$ and $x = 2$ respectively.

12. $\frac{1}{x} - \frac{1}{x-2} = 3$

Solution:

Given equation is,

$$\frac{1}{x} - \frac{1}{x-2} = 3$$

$$\frac{x-2-x}{x(x-2)} = 3$$

$$\frac{2}{x(x-2)} = 3$$

On cross multiplying on both the sides we get,

$$2 = 3x(x - 2)$$

$$2 = 3x^2 - 6x$$

$$3x^2 - 6x - 2 = 0$$

$$\Rightarrow 3x^2 - 3x - 3x - 2 = 0$$

$$3x^2 - (3 + \sqrt{3})x - (3 - \sqrt{3})x + [(\sqrt{3}^2) - 1^2] = 0$$

$$3x^2 - (3 + \sqrt{3})x - (3 - \sqrt{3})x + [(\sqrt{3}^2) - 1^2][(\sqrt{3}^2) - 1^2] = 0$$

$$\sqrt{3}^2 x^2 - \sqrt{3}(\sqrt{3} + 1)x - \sqrt{3}(\sqrt{3} - 1)x + (\sqrt{3} + 1)(\sqrt{3} - 1) = 0$$

$$\sqrt{3}x(\sqrt{3} + 1)x - (\sqrt{3}x - (\sqrt{3} + 1))(\sqrt{3} - 1) = 0$$

$$(\sqrt{3}x - \sqrt{3} - 1)(\sqrt{3}x - \sqrt{3} + 1)(\sqrt{3} - 1) = 0$$

Now, either

$$(\sqrt{3}x - \sqrt{3} - 1) = 0 \quad \text{or} \quad (\sqrt{3}x - \sqrt{3} + 1)(\sqrt{3} - 1) = 0$$

Thus,

$$x = \frac{\sqrt{3}+1}{\sqrt{3}} \quad \text{or} \quad x = \frac{\sqrt{3}-1}{\sqrt{3}}$$

are the solutions of the given quadratic

equations.

13. $x - 1/x = 3, x \neq 0$

Solution:

Given,

$$x - 1/x = 3$$

On multiplying x on both the sides we have,

$$\Rightarrow x^2 - 1 = 3x$$

$$\Rightarrow x^2 - 3x - 1 = 0$$

$$x^2 - \left(\frac{3}{2} + \frac{3}{2}\right)x - 1 = 0$$

$$x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x - 1 = 0$$

$$x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x - \frac{-4}{4} = 0$$

$$x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x - \frac{9-13}{4} = 0$$

$$x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x - \frac{(3)^2 - (\sqrt{13})^2}{(2)^2} = 0$$

$$x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x + \left(\frac{3+\sqrt{13}}{2}\right)\left(\frac{3-\sqrt{13}}{2}\right) = 0$$

$$\left(x - \frac{3+\sqrt{13}}{2}\right)\left(x - \frac{3-\sqrt{13}}{2}\right) = 0$$

$$\text{Either, } \left(x - \frac{3+\sqrt{13}}{2}\right) = 0; \Rightarrow x = \frac{3+\sqrt{13}}{2}$$

$$\text{Or, } \left(x - \frac{3-\sqrt{13}}{2}\right) = 0; \Rightarrow x = \frac{3-\sqrt{13}}{2}$$

Therefore, the roots of the given quadratic equation are $\frac{3+\sqrt{13}}{2}$ and $\frac{3-\sqrt{13}}{2}$ respectively.

14. $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

Solution:

Given,

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$$

$$\frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$$

$$\frac{-11}{(x+4)(x-7)} = \frac{11}{30}$$

Dividing by 11 both the sides and cross-multiplying we get,

$$\Rightarrow x^2 - 3x - 28 = -30$$

$$\Rightarrow x^2 - 3x - 2 = 0$$

$$\Rightarrow x^2 - 2x - x - 2 = 0$$

$$\Rightarrow x(x-2) - 1(x-2) = 0$$

$$\Rightarrow (x-2)(x-1) = 0$$

Now, either $x-2=0 \Rightarrow x=2$

Or, $x-1=0 \Rightarrow x=1$

Thus, the roots of the given quadratic equation are 1 and 2 respectively.

15.
$$\frac{1}{x-3} + \frac{2}{x-2} = \frac{8}{x}$$

Solution:

Given,

$$\frac{1}{x-3} + \frac{2}{x-2} = \frac{8}{x}$$

$$\frac{x-2+2(x-3)}{(x-3)(x-2)} = \frac{8}{x}$$

$$\frac{3x-8}{(x-3)(x-2)} = \frac{8}{x}$$

On cross multiplying we get,

$$\Rightarrow x(3x-8) = 8(x-3)(x-2)$$

$$\Rightarrow 3x^2 - 8x = 8(x^2 - 5x + 6)$$

$$\Rightarrow 8x^2 - 40x + 48 - (3x^2 - 8x) = 0$$

$$\Rightarrow 5x^2 - 32x + 48 = 0$$

$$\Rightarrow 5x^2 - 20x - 12x + 48 = 0$$

$$\Rightarrow 5x(x-4) - 12(x-4) = 0$$

$$\Rightarrow (x-4)(5x-12) = 0$$

Now, either $x - 4 = 0 \Rightarrow x = 4$

Or, $5x - 12 = 0 \Rightarrow x = 5/12$

Thus, the roots of the given quadratic equation are 1 and 2 respectively.

16. $a^2x^2 - 3abx + 2b^2 = 0$

Solution:

Given equation is $a^2x^2 - 3abx + 2b^2 = 0$

$$\Rightarrow a^2x^2 - abx - 2abx + 2b^2 = 0$$

$$\Rightarrow ax(ax - b) - 2b(ax - b) = 0$$

$$\Rightarrow (ax - b)(ax - 2b) = 0$$

Now, either $ax - b = 0 \Rightarrow x = b/a$

Or, $ax - 2b = 0 \Rightarrow x = 2b/a$

Thus, the roots of the quadratic equation are $x = 2b/a$ and $x = b/a$ respectively.

17. $9x^2 - 6b^2x - (a^4 - b^4) = 0$

Solution:

Given,

$$9x^2 - 6b^2x - (a^4 - b^4) = 0$$

$$\Rightarrow 9x^2 - 6b^2x - (a^2 - b^2)(a^2 + b^2) = 0$$

$$\Rightarrow 9x^2 + 3(a^2 - b^2) - 3(a^2 + b^2)x - (a^2 - b^2)(a^2 + b^2) = 0$$

$$\Rightarrow 3x[3x + a^2 + b^2] - (a^2 + b^2)[3x + (a^2 - b^2)] = 0$$

$$\Rightarrow [3x - (a^2 + b^2)][3x + (a^2 - b^2)] = 0$$

$$\Rightarrow 3x - (a^2 + b^2) = 0 \text{ or } 3x + (a^2 - b^2) = 0$$

$$\Rightarrow x = \frac{a^2 + b^2}{3} \text{ or } x = \frac{b^2 - a^2}{3}$$

Thus, the roots of the quadratic equation are $x = (b^2 - a^2)/3$ and $x = (a^2 + b^2)/3$ respectively.

18. $4x^2 + 4bx - (a^2 - b^2) = 0$

Solution:

Given,

$$4x^2 + 4bx - (a^2 - b^2) = 0$$

For factorizing,

$$4(a^2 - b^2) = -4(a - b)(a + b) = [-2(a - b)][2(a + b)]$$

$$\Rightarrow 2(b - a) \cdot 2(b + a)$$

$$\Rightarrow 4x^2 + (2(b - a) + 2(b + a))x - (a - b)(a + b) = 0$$

So, now

$$4x^2 + 2(b - a)x + 2(b + a)x + (b - a)(a + b) = 0$$

$$\Rightarrow 2x(2x + (b - a)) + (a + b)(2x + (b - a)) = 0$$

$$\Rightarrow (2x + (b - a))(2x + b + a) = 0$$

Now, either $(2x + (b - a)) = 0 \Rightarrow x = (a - b)/2$

Or, $(2x + b + a) = 0 \Rightarrow x = -(a + b)/2$

Thus, the roots of the given quadratic equation are $x = -(a + b)/2$ and $x = (a - b)/2$ respectively.

19. $ax^2 + (4a^2 - 3b)x - 12ab = 0$

Solution:

Given equation is $ax^2 + (4a^2 - 3b)x - 12ab = 0$

$$\Rightarrow ax^2 + 4a^2x - 3bx - 12ab = 0$$

$$\Rightarrow ax(x + 4a) - 3b(x + 4a) = 0$$

$$\Rightarrow (x + 4a)(ax - 3b) = 0$$

Now, either $x + 4a = 0 \Rightarrow x = -4a$

Or, $ax - 3b = 0 \Rightarrow x = 3b/a$

Thus, the roots of the given quadratic equation are $x = 3b/a$ and $-4a$ respectively.

20. $2x^2 + ax - a^2 = 0$

Solution:

Given,

$$2x^2 + ax - a^2 = 0$$

$$\Rightarrow 2x^2 + 2ax - ax - a^2 = 0$$

$$\Rightarrow 2x(x + a) - a(x + a) = 0$$

$$\Rightarrow (2x - a)(x + a) = 0$$

$$\Rightarrow 2x - a = 0 \text{ or } x + a = 0$$

$$\Rightarrow x = \frac{a}{2} \text{ or } x = -a$$

Thus, the roots of the given quadratic equation are $x = a/2$ and $-a$ respectively.

21. $16/x - 1 = 15/(x + 1), x \neq 0, -1$

Solution:

Given,

$$\frac{16}{x} - 1 = \frac{15}{x+1}$$

$$\Rightarrow \frac{16-x}{x} = \frac{15}{x+1}$$

$$\Rightarrow (16 - x)(x + 1) = 15x$$

$$\Rightarrow -x^2 + 16 + 15x = 15x$$

$$\Rightarrow -x^2 + 16 = 0$$

$$\Rightarrow -x^2 - 16 = 0$$

$$\Rightarrow (x - 4)(x + 4) = 0$$

$$\Rightarrow x - 4 = 0 \quad x + 4 = 0$$

$$\Rightarrow x = 4 \text{ or } x = -4$$

Thus, the roots of the given quadratic equation are $x = 4$ and -4 respectively.

22. $\frac{x + 3}{x + 2} = \frac{3x - 7}{2x - 3}$, $x \neq -2, 3/2$

Solution:

Given,

$$\frac{x + 3}{x + 2} = \frac{3x - 7}{2x - 3}$$

On cross-multiplying we get,

$$(x + 3)(2x - 3) = (x + 2)(3x - 7)$$

$$\Rightarrow 2x^2 - 3x + 6x - 9 = 3x^2 - x - 14$$

$$\Rightarrow 2x^2 + 3x - 9 = 3x^2 - x - 14$$

$$\Rightarrow x^2 - 3x - x - 14 + 9 = 0$$

$$\Rightarrow x^2 - 5x + x - 5 = 0$$

$$\Rightarrow x(x - 5) + 1(x - 5) = 0$$

$$\Rightarrow (x - 5)(x + 1) - 0$$

Now, either $x - 5 = 0$ or $x + 1 = 0$

$$\Rightarrow x = 5 \text{ and } x = -1$$

Thus, the roots of the given quadratic equation are 5 and -1 respectively.

23. $\frac{2x}{x - 4} + \frac{2x - 5}{x - 3} = \frac{25}{3}$, $x \neq 3, 4$

Solution:

The given equation is

$$\frac{2x}{x - 4} + \frac{2x - 5}{x - 3} = \frac{25}{3}$$

$$\frac{2x(x - 3) + (2x - 5)(x - 4)}{(x - 4)(x - 3)} = \frac{25}{3}$$

$$\frac{2x^2 - 6x + 2x^2 - 5x - 8x + 20}{x^2 - 4x - 3x + 12}$$

$$\frac{4x^2 - 19x + 20}{x^2 - 7x + 12} = \frac{25}{3}$$

On cross multiplying, we have

$$\begin{aligned} 3(4x^2 - 19x + 20) &= 25(x^2 - 7x + 12) \\ \Rightarrow 12x^2 - 57x + 60 &= 25x^2 - 175x + 300 \\ \Rightarrow 13x^2 - 78x - 40x + 240 &= 0 \\ \Rightarrow 13x^2 - 118x + 240 &= 0 \\ \Rightarrow 13x^2 - 78x - 40x + 240 &= 0 \\ \Rightarrow 13x(x - 6) - 40(x - 6) &= 0 \\ \Rightarrow (x - 6)(13x - 40) &= 0 \end{aligned}$$

Now, either $x - 6 = 0 \Rightarrow x = 6$

Or, $13x - 40 = 0 \Rightarrow x = 40/13$

Thus, the roots of the given quadratic equation are 6 and 40/13 respectively.

24. $\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}, x \neq 0, 2$

Solution:

Given equation is,

$$\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$$

$$\frac{x(x+3) - (x-2)(1-x)}{x(x-2)} = \frac{17}{4}$$

$$\frac{x^2 + 3x - x + x^2 + 2 - 2x}{x^2 - 2x} = \frac{17}{4}$$

$$\frac{2x^2 + 2}{x^2 - 2x} = \frac{17}{4}$$

On cross multiplying, we get

$$4(2x^2 + 2) = 17(x^2 - 2x)$$

$$\Rightarrow 8x^2 + 8 = 17x^2 - 34x$$

$$\Rightarrow 9x^2 - 34x - 8 = 0$$

$$\Rightarrow 9x^2 - 36x + 2x - 8 = 0$$

$$\Rightarrow 9x(x - 4) + 2(x - 4) = 0$$

$$\Rightarrow (9x + 2)(x - 4) = 0$$

Now, either $9x + 2 = 0 \Rightarrow x = -2/9$

Or, $x - 4 = 0 \Rightarrow x = 4$

Thus, the roots of the given quadratic equation are $x = -2/9$ and 4 respectively.

25. $\frac{x-3}{x+3} - \frac{x+3}{x-3} = \frac{48}{7}, x \neq 3, x \neq -3$

Solution:

Given equation is,

$$\frac{x-3}{x+3} - \frac{x+3}{x-3} = \frac{48}{7}$$

$$\Rightarrow \frac{(x-3)^2 - (x+3)^2}{(x+3)(x-3)} = \frac{48}{7}$$

$$\Rightarrow \frac{(x^2 - 6x + 9) - (x^2 + 6x + 9)}{x^2 - 9} = \frac{48}{7}$$

$$\Rightarrow \frac{x^2 - 6x + 9 - x^2 - 6x - 9}{x^2 - 9} = \frac{48}{7}$$

$$\Rightarrow \frac{-12x}{x^2 - 9} = \frac{48}{7}$$

On cross-multiplying, we get

$$7(-12x) = 48(x^2 - 9)$$

$$\Rightarrow -84x = 48x^2 - 432$$

$$\Rightarrow 48x^2 + 84x - 432 = 0$$

$$\Rightarrow 4x^2 + 7x - 36 = 0 \quad \text{[dividing by 12]}$$

$$\Rightarrow 4x^2 + 16x - 9x - 36 = 0$$

$$\Rightarrow 4x(x + 4) - 9(x - 4) = 0$$

$$\Rightarrow (4x - 9)(x + 4) = 0$$

Now, either $4x - 9 = 0 \Rightarrow x = 9/4$

Or, $x + 4 = 0 \Rightarrow x = -4$

Thus, the roots of the given quadratic equation are $x = 9/4$ and -4 respectively.

26. $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}, x \neq 0$

Solution:

Given equation is,

$$\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$$

$$\frac{(x-1) + 2(x-2)}{(x-2)(x-1)} = \frac{6}{x}$$

$$\frac{(x - 1) + 2x - 4}{(x^2 - 2x - x + 2)} = \frac{6}{x}$$

$$\frac{3x - 5}{(x^2 - 3x + 2)} = \frac{6}{x}$$

On cross multiplying, we have

$$x(3x - 5) = 6(x^2 - 3x + 2)$$

$$\Rightarrow 3x^2 - 5x = 6x^2 - 18x + 12$$

$$\Rightarrow 3x^2 - 13x + 12 = 0$$

$$\Rightarrow 3x^2 - 9x - 4x + 12 = 0$$

$$\Rightarrow 3x(x - 3) - 4(x - 3) = 0$$

$$\Rightarrow (x - 3)(3x - 4) = 0$$

$$\text{Now, either } x - 3 = 0 \Rightarrow x = 3$$

$$\text{Or, } 3x - 4 = 0 \Rightarrow 4/3.$$

Thus, the roots of the given quadratic equation are 3 and 4/3 respectively.

27. $\frac{x + 1}{x - 1} - \frac{x - 1}{x + 1} = \frac{5}{6}, x \neq 1, -1$

Solution:

The given equation is,

$$\frac{x + 1}{x - 1} - \frac{x - 1}{x + 1} = \frac{5}{6}$$

$$\frac{(x + 1)^2 - (x - 1)^2}{x^2 - 1} = \frac{5}{6}$$

$$\frac{4x}{x^2 - 1} = \frac{5}{6}$$

On cross - multiplying we have,

$$\Rightarrow 6(4x) = 5(x^2 - 1) = 24x$$

$$\Rightarrow 5x^2 - 5 = 5x^2 - 24x - 5 = 0$$

$$\Rightarrow 5x^2 - 25x + x - 5 = 0$$

$$\Rightarrow 5x(x - 5) + 1(x - 5) = 0$$

$$\Rightarrow (5x + 1)(x - 5) = 0$$

$$\text{Now, either } x - 5 = 0 \Rightarrow x = 5$$

$$\text{Or, } 5x + 1 = 0 \Rightarrow x = -1/5$$

Thus, the roots of the given quadratic equation are $x = -1/5$ and 5 respectively.

28. $\frac{x - 1}{2x + 1} + \frac{2x + 1}{x - 1} = \frac{5}{2}, x \neq 1, -1/2$

Solution:

The given equation is,

$$\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = \frac{5}{2}$$

$$\frac{(x-1)^2 + (2x+1)^2}{2x^2 - 2x + x - 1} = \frac{5}{2}$$

$$\frac{x^2 - 2x + 1 + 4x^2 + 4x + 1}{2x^2 - x - 1} = \frac{5}{2}$$

$$\frac{5x^2 + 2x + 2}{2x^2 - x - 1} = \frac{5}{2}$$

On cross – multiplying we have,

$$\Rightarrow 2(5x^2 + 2x + 2) = 5(2x^2 - x - 1)$$

$$\Rightarrow 10x^2 + 4x + 4 = 10x^2 - 5x - 5$$

$$\Rightarrow 4x + 5x + 4 + 5 = 0$$

$$\Rightarrow 9x + 9 = 0$$

$$\Rightarrow 9x = -9$$

Thus, $x = -1$ is the only root of the given equation.

29. $\frac{4}{x} - 3 = \frac{5}{2x+3}, x \neq 0, -\frac{3}{2}$

Solution:

Given equation is,

$$\Rightarrow \frac{4-3x}{x} = \frac{5}{2x+3}$$

$$\Rightarrow 5x = (2x+3)(4-3x)$$

$$\Rightarrow 5x = 8x - 6x^2 + 12 - 9x$$

$$\Rightarrow 5x - 8x + 6x^2 - 12 + 9x = 0$$

$$\Rightarrow 6x^2 + 6x - 12 = 0$$

$$\Rightarrow x^2 + x - 2 = 0 \quad \text{(Dividing by 6)}$$

$$\Rightarrow x^2 + 2x - x - 2 = 0$$

$$\Rightarrow x(x+2) - 1(x+2) = 0$$

$$\Rightarrow (x-1)(x+2) = 0$$

$$\Rightarrow x-1 = 0 \text{ or } x+2 = 0$$

$$\therefore x = 1 \text{ or } x = -2$$

Thus, the roots of the given quadratic equation are $x = 1$ and $x = -2$ respectively.

30.

Solution:

Given equation is,

$$\frac{x-4}{x-5} + \frac{x-6}{x-7} = \frac{10}{3}$$

$$\frac{(x-4)(x-7) + (x-5)(x-6)}{(x-5)(x-7)} = \frac{10}{3}$$

$$\frac{x^2 - 7x - 4x + 28 + x^2 - 6x - 5x + 30}{x^2 - 7x - 5x + 35}$$

$$\frac{2x^2 - 22x + 58}{x^2 - 12x + 35} = \frac{10}{3}$$

On cross-multiplying we have,

$$3(2x^2 - 22x + 58) = 10(x^2 - 12x + 35)$$

$$\Rightarrow 6x^2 - 66x + 174 = 10x^2 - 120x + 350$$

$$\Rightarrow 4x^2 - 54x + 176 = 0$$

$$\Rightarrow 2x^2 - 27x + 88 = 0$$

$$\Rightarrow 2x^2 - 16x - 11x + 88 = 0$$

$$\Rightarrow 2x(x - 8) - 11(x + 8) = 0$$

$$\Rightarrow (x - 8)(2x - 11) = 0$$

Now, either $x - 8 = 0 \Rightarrow x = 8$

Or, $2x - 11 = 0 \Rightarrow x = 11/2$

Thus, the roots of the given quadratic equation are $x = 11/2$ and 8 respectively.