Exercise 8.4 Page No: 8.26

Find the roots of the following quadratic equations (if they exist) by the method of completing the square.

1. $x^2 - 4\sqrt{2}x + 6 = 0$

Solution:

Given equation,

$$x^2 - 4\sqrt{2x} + 6 = 0$$

$$x^2 - 2 \times x \times 2\sqrt{2} + (2\sqrt{2})^2 - (2\sqrt{2})^2 + 6 = 0$$

$$(x - 2\sqrt{2})^2 = (2\sqrt{2})^2 - 6$$

$$(x - 2\sqrt{2})^2 = (4 \times 2) - 6 \quad = 8 - 6$$

$$(x - 2\sqrt{2})^2 = 2$$

$$(x - 2\sqrt{2}) = \pm\sqrt{2}$$

$$(x - 2\sqrt{2}) = \sqrt{2} \text{ or } (x - 2\sqrt{2}) = -\sqrt{2}$$

$$x = \sqrt{2} + 2\sqrt{2} \text{ or } x = -\sqrt{2} + 2\sqrt{2}$$

$$\Rightarrow x = 3\sqrt{2} \text{ or } x = \sqrt{2}$$

Thus, the roots of the given quadratic equation are $x = 3\sqrt{2}$ and $x = \sqrt{2}$.

2. $2x^2 - 7x + 3 = 0$

Solution:

Given equation,

$$2x^2 - 7x + 3 = 0$$

$$2\left(x^2 - \frac{7x}{2} + \frac{3}{2}\right) = 0$$

$$x^2 - 2 \times \frac{7}{2} \times \frac{1}{2} \times x + \frac{3}{2} = 0$$

$$x^2 - 2 \times \frac{7}{4} \times x + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$$

$$x^2 - 2 \times \frac{7}{4} \times x + \left(\frac{7}{4}\right)^2 - \left(\frac{49}{16}\right) + \frac{3}{2} = 0$$

$$\left(x-\frac{7}{4}\right)^2-\frac{49}{16}+\frac{3}{2}=0$$

$$\left(x-\frac{7}{4}\right)^2=\frac{49}{16}-\frac{3}{2}$$

$$\left(x-\frac{7}{4}\right)^2=\frac{49-26}{16}$$

$$\left(x-\frac{7}{4}\right)^2=\frac{25}{16}$$

$$\left(x-\frac{7}{4}\right)^2=\left(\frac{5}{4}\right)^2$$

$$x-\frac{7}{4}=\pm\frac{5}{4}$$

$$x-\frac{7}{4}=\frac{5}{4} \text{ or } x-\frac{7}{4}=-\frac{5}{4}$$

$$x=\frac{7}{4}+\frac{5}{4} \text{ or } x=\frac{7}{4}-\frac{5}{4}$$

⇒ x = 12/4 = 3 or x = 2/4 = 1/2

Thus, the roots of the given quadratic equation are x = 3 and x = 1/2.

3. $3x^2 + 11x + 10 = 0$

Solution:

Given equation,

$$x^2+\frac{11x}{3}+\frac{10}{3}=0$$

$$x^2+2\times\frac{1}{2}\times\frac{11x}{3}+\frac{10}{3}=0$$

$$x^2+2\times\frac{11x}{6}+\left(\frac{11}{6}\right)^2-\left(\frac{11}{6}\right)^2+\frac{10}{3}=0$$

$$\left(x+\frac{11}{6}\right)^2=\left(\frac{11}{6}\right)^2-\frac{10}{3}$$

$$\left(x+\frac{11}{6}\right)^2=\frac{121}{36}-\frac{10}{3}$$

$$\left(x+\frac{11}{6}\right)^2=\frac{121-120}{36}$$

$$\left(x+\frac{11}{6}\right)^2=\frac{1}{36}$$

$$\left(x+\frac{11}{6}\right)^2=\left(\frac{1}{6}\right)^2$$

$$x+\frac{11}{6}=\pm\frac{1}{6}$$

$$x+\frac{11}{6}=\frac{1}{6}\text{ or }x+\frac{11}{6}=\frac{-1}{6}$$

$$x=\frac{1}{6}-\frac{11}{6}\text{ or }x=\frac{-1}{6}-\frac{11}{6}$$

$$x=\frac{-10}{6}\text{ or }x=\frac{-12}{6}=-2$$

⇒ x = -5/3 = 3 or x = -2

Thus, the roots of the given quadratic equation are x = -5/3 and x = -2.

4. $2x^2 + x – 4 = 0$

Solution:

Given equation,

$2x^2 + x – 4 = 0$

$$2\left(x^2+\frac{x}{2}-\frac{4}{2}\right)=0$$

$$x^2+2\times\frac{1}{2}\times\frac{1}{2}\times x-2=0$$

$$x^2+2\times\frac{1}{4}\times x+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2-2=0$$

$$\left(x+\frac{1}{4}\right)^2=\left(\frac{1}{4}\right)^2+2$$

$$\left(x+\frac{1}{4}\right)^2=\frac{1}{16}+2$$

$$\left(x+\frac{1}{4}\right)^2=\frac{1+2\times16}{16}$$

$$\left(x+\frac{1}{4}\right)^2=\frac{1+32}{16}$$

$$\left(x+\frac{1}{4}\right)^2=\frac{33}{16}$$

$$\left(x+\frac{1}{4}\right)=\pm\sqrt{\frac{33}{16}}$$

$$\left(x+\frac{1}{4}\right)=\sqrt{\frac{33}{16}}$$

$$\text{or } \left(x+\frac{1}{4}\right)=-\sqrt{\frac{33}{16}}$$

$$x=\frac{\sqrt{33}}{4}-\frac{1}{4} \text{ or } x=-\frac{\sqrt{33}}{4}-\frac{1}{4}$$

$$x=\frac{\sqrt{33}-1}{4} \text{ or } x=-\frac{\sqrt{33}-1}{4}$$

Thus, the roots of the given quadratic equation are $x=\frac{\sqrt{33}-1}{4}$ or $x=\frac{-\sqrt{33}-1}{4}$

5. $2x^2 + x + 4 = 0$
Solution:

Given equation,

$2x^2 + x + 4 = 0$

$x^2 + x/2 + 2 = 0$

$$x^2+2\times\frac{1}{2}\times\frac{1}{2}\times x+2=0$$

$$x^2+2\times\frac{1}{4}\times x+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2+2=0$$

$$x^2+2\times\frac{1}{4}\times x+\left(\frac{1}{4}\right)^2=\left(\frac{1}{4}\right)^2-2$$

$$\left(x+\frac{1}{4}\right)^2=\frac{1-32}{16}$$

$$\left(x+\frac{1}{4}\right)^2=\frac{-31}{16}$$

$$\left(x+\frac{1}{4}\right)=\pm\sqrt{-\frac{31}{16}}$$

$$\left(x+\frac{1}{4}\right)=\frac{\sqrt{-31}}{4} \text{ or } \left(x+\frac{1}{4}\right)=\frac{-\sqrt{-31}}{4}$$

$$x=\frac{\sqrt{-31}-1}{4} \text{ or } x=\frac{-\sqrt{-31}-1}{4}$$

Thus, the above are the two roots of the given quadratic equation.