Exercise 8.6 Page No: 8.41

1. Determine the nature of the roots of the following quadratic equations:

Important Notes:

- A quadratic equation is in the form $ax^2 + bx + c = 0$
- To find the nature of roots, first find determinant "D"
- $D = b^2 – 4ac$
- If $D > 0$, equation has real and distinct roots
- If $D < 0$, equation has no real roots
- If $D = 0$, equation has 1 root

(i) $2x^2 - 3x + 5 = 0$

Solution:

Here, a= 2, b= -3, c= 5

$D = b^2 – 4ac$

$= (-3)^2 - 4(2)(5)$

$= 9 – 40$

$= -31 < 0$

It's seen that D<0 and hence, the given equation does not have any real roots.

(ii) $2x^2 - 6x + 3 = 0$

Solution:

Here, a= 2, b= -6, c= 3

$D = (-6)^2 - 4(2)(3)$

$= 36 – 24$

$= 12 > 0$

It's seen that D>0 and hence, the given equation have real and distinct roots.

(iii) $(3/5)x^2 – (2/3) + 1 = 0$

Solution:

Here, a= 3/5, b= -2/3, c= 1

$D = (-2/3)^2 - 4(3/5)(1)$

$= 4/9 – 12/5$

$= -88/45 < 0$

It's seen that D<0 and hence, the given equation does not have any real roots.

(iv) $3x^2 - 4\sqrt{3}x + 4 = 0$

Solution:

Here, a= 3, b= $-4\sqrt{3}$, c= 4

$D = (-4\sqrt{3})^2 - 4(3)(4)$

$= 48 – 48$

= 0

It's seen that D = 0 and hence, the given equation has only 1 real and equal root.

(v) $3x^2 - 2\sqrt{6}x + 2 = 0$

Solution:

Here, a= 3, b= - 2√6, c= 2

$D = (-2\sqrt{6})^2 - 4(3)(2)$

= 24 – 24

= 0

It's seen that D = 0 and hence, the given equation has only 1 real and equal root.

2. Find the values of k for which the roots are real and equal in each of the following equations:

(i) $kx^2 + 4x + 1 = 0$

Solution:

The given equation $kx^2 + 4x + 1 = 0$ is in the form of $ax^2 + bx + c = 0$

Where a = k, b = 4, c = 1

For the equation to have real and equal roots, the condition is

$D = b^2 - 4ac = 0$

⇒ $4^2 - 4(k)(1) = 0$

⇒ 16 – 4k = 0

⇒ k = 4

The value of k is 4.

(ii) $kx^2 - 2\sqrt{5}x + 4 = 0$

Solution:

The given equation $kx^2 - 2\sqrt{5}x + 4 = 0$ is in the form of $ax^2 + bx + c = 0$

Where a = k, b = - 2√5, c = 4

For the equation to have real and equal roots, the condition is

$D = b^2 - 4ac = 0$

⇒ $(-2\sqrt{5})^2 - 4(k)(4) = 0$

⇒ 20 –16k = 0

⇒ k = 5/4

The value of k is 5/4.

(iii) $3x^2 - 5x + 2k = 0$

Solution:

The given equation $3x^2 – 5x + 2k = 0$ is in the form of $ax^2 + bx + c = 0$

Where a = 3, b = -5, c = 2k

For the equation to have real and equal roots, the condition is

$D = b^2 - 4ac = 0$

⇒ $(-5)^2 - 4(3)(2k) = 0$

⇒ $25 - 24k = 0$
⇒ $k = 25/24$
The value of k is 25/24.

(iv) $4x^2 + kx + 9 = 0$
Solution:

The given equation $4x^2 + kx + 9 = 0$ is in the form of $ax^2 + bx + c = 0$
Where a = 4, b = k, c = 9
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
⇒ $k^2 - 4(4)(9) = 0$
⇒ $k^2 - 144 = 0$
⇒ $k = \pm 12$
The value of k is 12 or -12.

(v) $2kx^2 - 40x + 25 = 0$
Solution:

The given equation $2kx^2 - 40x + 25 = 0$ is in the form of $ax^2 + bx + c = 0$
Where a = 2k, b = -40, c = 25
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
⇒ $(-40)^2 - 4(2k)(25) = 0$
⇒ $1600 - 200k = 0$
⇒ $k = 8$
The value of k is 8.

(vi) $9x^2 - 24x + k = 0$
Solution:

The given equation $9x^2 - 24x + k = 0$ is in the form of $ax^2 + bx + c = 0$
Where a = 9, b = -24, c = k
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
⇒ $(-24)^2 - 4(9)(k) = 0$
⇒ $576 - 36k = 0$
⇒ $k = 16$
The value of k is 16.

(vii) $4x^2 - 3kx + 1 = 0$
Solution:

The given equation $4x^2 - 3kx + 1 = 0$ is in the form of $ax^2 + bx + c = 0$
Where a = 4, b = -3k, c = 1
For the equation to have real and equal roots, the condition is

$D = b^2 - 4ac = 0$
⇒ $(-3k)^2 - 4(4)(1) = 0$
⇒ $9k^2 – 16 = 0$
⇒ $k = \pm 4/3$
The value of k is ± 4/3.

(viii) $x^2 - 2(5 + 2k)x + 3(7 + 10k) = 0$
Solution:

The given equation $x^2 – 2(5 + 2k)x + 3(7 + 10k) = 0$ is in the form of $ax^2 + bx + c = 0$
Where a = 1, b = -2(5 + 2k), c = 3(7 + 10k)
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
⇒ $(-2(5 + 2k))^2 - 4(1)(3(7 + 10k)) = 0$
⇒ $4(5 + 2k)^2 - 12(7 + 10k) = 0$
⇒ $25 + 4k^2 + 20k - 21 - 30k = 0$
⇒ $4k^2 - 10k + 4 = 0$
⇒ $2k^2 - 5k + 2 = 0$ [dividing by 2]
Now, solving for k by factorization we have
⇒ $2k^2 - 4k - k + 2 = 0$
⇒ $2k(k - 2) - 1(k - 2) = 0$
⇒ $(k - 2)(2k - 1) = 0$,
k = 2 and k = 1/2,
So, the value of k can either be 2 or 1/2

(ix) $(3k +1)x^2+ 2(k +1)x + k = 0$
Solution:

The given equation $(3k +1)x^2 + 2(k +1)x + k = 0$ is in the form of $ax^2 + bx + c = 0$
Where a = (3k +1), b = 2(k + 1), c = k
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
⇒ $(2(k + 1))^2 - 4(3k +1)(k) = 0$
⇒ $4(k +1)^2 - 4(3k^2 + k) = 0$
⇒ $(k + 1)^2 - k(3k + 1) = 0$
⇒ $2k^2 – k – 1 = 0$
Now, solving for k by factorization we have
⇒ $2k^2 - 2k + k – 1 = 0$
⇒ $2k(k - 1) + 1(k - 1) = 0$
⇒ $(k - 1)(2k + 1) = 0$,
k = 1 and k = -1/2,
So, the value of k can either be 1 or -1/2

(x) $kx^2 + kx + 1 = – 4x^2 – x$
Solution:

The given equation $kx^2 + kx + 1 = -4x^2 – x$
This can be rewritten as,
$(k + 4)x^2 + (k + 1)x + 1 = 0$
Now, this in the form of $ax^2 + bx + c = 0$
Where $a = (k +4)$, $b = (k + 1)$, $c = 1$
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
$\Rightarrow \quad (k + 1)^2 - 4(k +4)(1) = 0$
$\Rightarrow \quad (k +1)^2 - 4k – 16 = 0$
$\Rightarrow \quad k^2 + 2k + 1 – 4k - 16 = 0$
$\Rightarrow \quad k^2 – 2k – 15 = 0$
Now, solving for k by factorization we have
$\Rightarrow k^2 - 5k + 3k – 15 = 0$
$\Rightarrow k(k - 5) + 3(k - 5) = 0$
$\Rightarrow (k + 3)(k - 5) = 0$,
$k = -3$ and $k = 5$,
So, the value of k can either be -3 or 5.

(xi) $(k + 1)x^2 + 2(k + 3)x + (k + 8) = 0$
Solution:

The given equation $(k +1)x^2 + 2(k +3)x + (k +8) = 0$ is in the form of $ax^2 + bx + c = 0$
Where $a = (k +1)$, $b = 2(k + 3)$, $c = (k + 8)$
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
$\Rightarrow \quad (2(k + 3))^2 - 4(k +1)(k + 8) = 0$
$\Rightarrow \quad 4(k +3)^2 - 4(k^2 + 9k + 8) = 0$
$\Rightarrow \quad (k + 3)^2 - (k^2 + 9k + 8) = 0$
$\Rightarrow \quad k^2 +6k + 9 – k^2 – 9k – 8 = 0$
$\Rightarrow \quad -3k + 1 = 0$
$\Rightarrow \quad k = 1/3$
So, the value of k is 1/3.

(xii) $x^2 - 2kx + 7k - 12 = 0$
Solution:

The given equation $x^2 - 2kx + 7k - 12 = 0$ is in the form of $ax^2 + bx + c = 0$
Where $a = 1$, $b = -2k$, $c = 7k – 12$
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
$\Rightarrow \quad (-2k)^2 - 4(1)(7k - 12) = 0$
$\Rightarrow \quad 4k^2 - 4(7k – 12) = 0$
$\Rightarrow \quad k^2 - 7k + 12 = 0$
Now, solving for k by factorization we have
$\Rightarrow k^2 - 4k - 3k + 12 = 0$

$\Rightarrow$ (k - 4)(k - 3) = 0,
k = 4 and k = 3,
So, the value of k can either be 4 or 3.

(xiii) $(k + 1)x^2 - 2(3k + 1)x + 8k + 1 = 0$
Solution:

The given equation $(k +1)x^2 - 2(3k +1)x + 8k + 1 = 0$ is in the form of $ax^2 + bx + c = 0$
Where a = (k +1), b = -2(3k + 1), c = 8k + 1
For the equation to have real and equal roots, the condition is

$D = b^2 - 4ac = 0$

$\Rightarrow$ $(-2(3k + 1))^2 - 4(k +1)(8k + 1) = 0$

$\Rightarrow$ $4(3k +1)^2 - 4(k + 1)(8k + 1) = 0$

$\Rightarrow$ $(3k + 1)^2 - (k + 1)(8k + 1) = 0$

$\Rightarrow$ $9k^2 + 6k + 1 – (8k^2 + 9k + 1) = 0$

$\Rightarrow$ $9k^2 + 6k + 1 – 8k^2 - 9k - 1 = 0$

$\Rightarrow$ $k^2 - 3k = 0$

$\Rightarrow$ $k(k - 3) = 0$

Either k = 0 Or, k - 3 = 0 $\Rightarrow$ k = 3,
So, the value of k can either be 0 or 3

(xiv) $5x^2 - 4x + 2 + k(4x^2 - 2x + 1) = 0$
Solution:

The given equation $5x^2 - 4x + 2 + k(4x^2 - 2x + 1) = 0$
This can be rewritten as,
$x^2(5 + 4k) - x(4 + 2k) + 2 - k = 0$
Now, this in the form of $ax^2 + bx + c = 0$
Where a = (4k +5), b = -(2k + 4), c = 2 - k
For the equation to have real and equal roots, the condition is

$D = b^2 - 4ac = 0$

$\Rightarrow$ $(-(2k + 4))^2 - 4(4k +5)(2 - k) = 0$

$\Rightarrow$ $(2k +4)^2 - 4(4k +5)(2 - k) = 0$

$\Rightarrow$ $16 + 4k^2 + 16k - 4(10 - 5k + 8k - 4k^2) = 0$

$\Rightarrow$ $16 + 4k^2 + 16k - 40 + 20k - 32k + 16k^2 = 0$

$\Rightarrow$ $20k^2 + 4k - 24 = 0$

$\Rightarrow$ $5k^2 + k – 6 = 0$

Now, solving for k by factorization we have
$\Rightarrow 5k^2 + 6k - 5k – 6 = 0$
$\Rightarrow$ 5k(k - 1) + 6(k - 1) = 0
$\Rightarrow$ (k - 1)(5k + 6) = 0,
k = 1 and k = -6/5,
So, the value of k can either be -3 or 5.

(xv) $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$
Solution:

The given equation $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$ is in the form of $ax^2 + bx + c = 0$
Where $a = (4 - k)$, $b = (2k + 4)$, $c = (8k + 1)$
For the equation to have real and equal roots, the condition is

$D = b^2 - 4ac = 0$
$\Rightarrow (2k + 4)^2 - 4(4 - k)(8k + 1) = 0$
$\Rightarrow 4k^2 + 16k + 16 - 4(-8k^2 + 32k + 4 - k) = 0$
$\Rightarrow 4k^2 + 16k + 16 + 32k^2 - 124k - 16 = 0$
$\Rightarrow 36k^2 - 108k = 0$

Taking common,
$\Rightarrow 9k(k -3) = 0$
Now, either $9k = 0 \Rightarrow k = 0$ or $k – 3 = 0 \Rightarrow k = 3$,
So, the value of k can either be 0 or 3.

(xvi) $(2k + 1)x^2 + 2(k + 3)x + (k +5) = 0$
Solution:

The given equation $(2k +1)x^2 + 2(k +3)x + (k + 5) = 0$ is in the form of $ax^2 + bx + c = 0$
Where $a = (2k +1)$, $b = 2(k + 3)$, $c = (k + 5)$
For the equation to have real and equal roots, the condition is

$D = b^2 - 4ac = 0$
$\Rightarrow (2(k + 3))^2 - 4(2k +1)(k + 5) = 0$
$\Rightarrow 4(k +3)^2 - 4(2k^2 + 11k + 5) = 0$
$\Rightarrow (k + 3)^2 - (2k^2 + 11k + 5) = 0$ [dividing by 4 both sides]
$\Rightarrow k^2 + 5k - 4 = 0$

Now, solving for k by completing the square we have
$\Rightarrow k^2 + 2 \times (5/2) \times k + (5/2)^2 = 4 + (5/2)^2$
$\Rightarrow (k + 5/2)^2 = 4 + 25/4 = \sqrt{41}/4$
$\Rightarrow k + (5/2) = \pm \sqrt{41}/2$
$\Rightarrow k = (\sqrt{41} – 5)/2$ or $–(\sqrt{41} + 5)/2$
So, the value of k can either be $(\sqrt{41} – 5)/2$ or $–(\sqrt{41} + 5)/2$

(xvii) $4x^2 - 2(k + 1)x + (k + 4) = 0$
Solution:

The given equation $4x^2 - 2(k +1)x + (k + 4)= 0$ is in the form of $ax^2 + bx + c = 0$
Where $a = 4$, $b = -2(k + 1)$, $c = (k + 4)$
For the equation to have real and equal roots, the condition is

$D = b^2 - 4ac = 0$
$\Rightarrow (-2(k + 1))^2 - 4(4)(k + 4) = 0$
$\Rightarrow 4(k +1)^2 - 16(k + 4) = 0$
$\Rightarrow (k + 1)^2 - 4(k + 4) = 0$
$\Rightarrow k^2 – 2k – 15 = 0$

Now, solving for k by factorization we have
$\Rightarrow k^2 - 5k + 3k – 15 = 0$

⇒ $k(k - 5) + 3(k - 5) = 0$
⇒ $(k - 5)(k + 3) = 0$,
$k = 5$ and $k = -3$,
So, the value of k can either be 5 or -3.

3. In the following, determine the set of values of k for which the given quadratic equation has real roots:
(i) $2x^2 + 3x + k = 0$
Solution:

Given,
$2x^2 + 3x + k = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, $a = 2, b = 3, c = k$
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = 9 - 4(2)(k) \geq 0$
⇒ $9 - 8k \geq 0$
⇒ $k \leq 9/8$
The value of k should not exceed 9/8 to have real roots.

(ii) $2x^2 + x + k = 0$
Solution:

Given,
$2x^2 + x + k = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, $a = 2, b = 1, c = k$
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = 1^2 - 4(2)(k) \geq 0$
⇒ $1 – 8k \geq 0$
⇒ $k \leq 1/8$
The value of k should not exceed 1/8 to have real roots.

(iii) $2x^2 - 5x - k = 0$
Solution:

Given,
$2x^2 – 5x – k = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, $a = 2, b = -5, c = -k$
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = (-5)^2 - 4(2)(-k) \geq 0$
⇒ $25 + 8k \geq 0$
⇒ $k \geq -25/8$
The value of k should be lesser than -25/8 to have real roots.

(iv) $kx^2 + 6x + 1 = 0$
Solution:

Given,
$2x^2 + x + k = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, a = 2, b = 1, c = k
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = 1^2 - 4(2)(k) \geq 0$
$\Rightarrow 1 – 8k \geq 0$
$\Rightarrow k \leq 1/8$
The value of k should not exceed 1/8 to have real roots.

(v) $3x^2 + 2x + k = 0$
Solution:

Given,
$3x^2 + 2x + k = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, a =3, b = 2, c = k
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = (2)^2 - 4(3)(k) \geq 0$
$\Rightarrow 4 - 12k \geq 0$
$\Rightarrow 4 \geq 12k$
$\Rightarrow k \leq 1/3$
The value of k should not exceed 1/3 to have real roots.

4. Find the values of k for which the following equations have real and equal roots
(i) $x^2 – 2(k + 1)x + k^2 = 0$
Solution:

Given,
$x^2 – 2(k + 1)x + k^2 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, a =1, b = -2(k + 1), c = k^2
For the given quadratic equation to have real roots $D = b^2 - 4ac = 0$
$D = (-2(k + 1))^2 - 4(1)(k^2) = 0$
$\Rightarrow 4k^2 + 8k + 4 - 4k^2 = 0$
$\Rightarrow 8k + 4 = 0$
$\Rightarrow k = -4/8$
$\Rightarrow k = -1/2$
The value of k should -1/2 to have real and equal roots.

(ii)$k^2x^2 – 2 (2k - 1)x + 4 = 0$
Solution:

Given,
$k^2x^2 – 2 (2k - 1)x + 4 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, $a = k^2$, $b = –2 (2k - 1)$, $c = 4$
For the given quadratic equation to have real roots $D = b^2 - 4ac = 0$
$D = (-2(2k - 1))^2 - 4(4)(k^2) = 0$
$\Rightarrow 4k^2 - 4k + 1 - 4k^2 = 0$ [dividing by 4 both sides]
$\Rightarrow -4k + 1 = 0$
$\Rightarrow k = 1/4$
The value of k should 1/4 to have real and equal roots.

(iii) $(k + 1)x^2 – 2(k - 1)x + 1 = 0$
Solution:

Given,
$(k + 1)x^2 – 2(k - 1)x + 1 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, $a = (k + 1)$, $b = -2(k - 1)$, $c = 1$
For the given quadratic equation to have real roots $D = b^2 - 4ac = 0$
$D = (-2(k - 1))^2 - 4(1)(k + 1) = 0$
$\Rightarrow 4k^2 – 2k + 1 – k - 1 = 0$ [dividing by 4 both sides]
$\Rightarrow k^2 – 3k = 0$
$\Rightarrow k(k - 3) = 0$
$\Rightarrow k = 0$ or $k = 3$
The value of k can be 0 or 3 to have real and equal roots.

5. Find the values of k for which the following equations have real roots
(i) $2x^2 + kx + 3 = 0$
Solution:

Given,
$2x^2 + kx + 3 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, $a = 2$, $b = k$, $c = 3$
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = (k)^2 - 4(3)(2) \geq 0$
$\Rightarrow k^2 - 24 \geq 0$
$\Rightarrow k^2 \geq 24$
$\Rightarrow k \geq 2\sqrt{6}$ and $k \leq -2\sqrt{6}$ [After taking square root on both sides]
The value of k can be represented as $(\infty, 2\sqrt{6}]$ U $[-2\sqrt{6}, -\infty)$

(ii) $kx(x - 2) + 6 = 0$
Solution:

Given,
$kx(x - 2) + 6 = 0$

It can be rewritten as,
$kx^2 – 2kx + 6 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, a =k, b = -2k, c = 6
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = (-2k)^2 - 4(k)(6) \geq 0$
$\Rightarrow 4k^2 - 24k \geq 0$
$\Rightarrow 4k(k - 6) \geq 0$
$\Rightarrow k \geq 0$ and $k \geq 6$
$\Rightarrow k \geq 6$
The value of k should be greater than or equal to 6 to have real roots.

(iii) $x^2 – 4kx + k = 0$
Solution:

Given,
$x^2 – 4kx + k = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, a =1, b = -4k, c = k
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = (-4k)^2 - 4(1)(k) \geq 0$
$\Rightarrow 16k^2 - 4k \geq 0$
$\Rightarrow 4k(4k - 1) \geq 0$
$\Rightarrow k \geq 0$ and $k \geq 1/4$
$\Rightarrow k \geq 1/4$
The value of k should be greater than or equal to 1/4 to have real roots.

(iv) $kx(x - 2\sqrt{5}) + 10 = 0$
Solution:

Given,
$kx(x - 2\sqrt{5}) + 10 = 0$
It can be rewritten as,
$kx^2 – 2\sqrt{5}kx + 10 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, a =k, b = $-2\sqrt{5}k$, c = 10
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = (-2\sqrt{5}k)^2 - 4(k)(10) \geq 0$
$\Rightarrow 20k^2 - 40k \geq 0$
$\Rightarrow 20k(k - 2) \geq 0$
$\Rightarrow k \geq 0$ and $k \geq 2$
$\Rightarrow k \geq 2$
The value of k should be greater than or equal to 2 to have real roots.

(v) $kx(x - 3) + 9 = 0$
Solution:

Given,
$kx(x - 3) + 9 = 0$
It can be rewritten as,
$kx^2 – 3kx + 9 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, $a = k$, $b = -3k$, $c = 9$
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = (-3k)^2 - 4(k)(9) \geq 0$
$\Rightarrow 9k^2 - 36k \geq 0$
$\Rightarrow 9k(k - 4) \geq 0$
$\Rightarrow k \geq 0$ and $k \geq 4$
$\Rightarrow k \geq 4$
The value of k should be greater than or equal to 4 to have real roots.

(vi) $4x^2 + kx + 3 = 0$
Solution:

Given,
$4x^2 + kx + 3 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, $a = 4$, $b = k$, $c = 3$
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = (k)^2 - 4(4)(3) \geq 0$
$\Rightarrow k^2 - 48 \geq 0$
$\Rightarrow k^2 \geq 48$
$\Rightarrow k \geq 4\sqrt{3}$ and $k \leq -4\sqrt{3}$ [After taking square root on both sides]
The value of k can be represented as $(\infty, 4\sqrt{3}]$ U $[-4\sqrt{3}, -\infty)$

6. Find the values of k for which the given quadratic equation has real and distinct roots.
(i) $kx^2 + 2x + 1 = 0$
Solution:

Given,
$kx^2 + 2x + 1 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, $a =k$, $b = 2$, $c = 1$
For the given quadratic equation to have real roots $D = b^2 - 4ac > 0$
$D = (2)^2 - 4(1)(k) > 0$
$\Rightarrow 4 - 4k > 0$
$\Rightarrow 4k < 4$
$\Rightarrow k < 1$
The value of k should be lesser than 1 to have real and distinct roots.

(ii) $kx^2 + 6x + 1 = 0$

Solution:

Given,
$kx^2 + 6x + 1 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, a =k, b = 6, c = 1
For the given quadratic equation to have real roots $D = b^2 - 4ac > 0$
$D = (6)^2 - 4(1)(k) > 0$
$\Rightarrow 36 - 4k > 0$
$\Rightarrow 4k < 36$
$\Rightarrow k < 9$
The value of k should be lesser than 9 to have real and distinct roots.

7. For what value of k, $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$, is a perfect square.
Solution:

Given,
$(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$
It is in the form of $ax^2 + bx + c = 0$
Where, a = 4 - k, b = 2k + 4, c = 8k + 1
Calculating the discriminant, $D = b^2 - 4ac$
$= (2k + 4)^2 - 4(4 - k)(8k + 1)$
$= 4k^2 + 16 + 4k - 4(32 + 4 - 8k^2 - k)$
$= 4(k^2 + 4 + k - 32 - 4 + 8k^2 + k)$
$= 4(9k^2 - 27k)$
As the given equation is a perfect square, then D = 0
$\Rightarrow \quad 4(9k^2 - 27k) = 0$
$\Rightarrow \quad (9k^2 - 27k) = 0$
$\Rightarrow \quad 3k(k - 3) = 0$
Thus, 3k = 0 $\Rightarrow$ k = 0 Or, k-3 = 0 $\Rightarrow$k = 3
Hence, the value of k should be 0 or 3 for the given to be perfect square.

8. Find the least positive value of k for which the equation $x^2 + kx + 4 = 0$ has real roots.
Solution:

Given,
$x^2 + kx + 4 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, a =1, b = k, c = 4
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = (k)^2 - 4(1)(4) \geq 0$
$\Rightarrow k^2 - 16 \geq 0$
$\Rightarrow k \geq 4$ and $k \leq -4$
Considering the least positive value, we have
$\Rightarrow k = 4$
Thus, the least value of k is 4 for the given equation to have real roots.

9. Find the values of k for which the quadratic equation $(3k + 1)x^2 + 2(k + 1)x + 1 = 0$ has equal roots. Also, find the roots.
Solution:

The given equation $(3k +1)x^2 + 2(k +1)x + 1 = 0$ is in the form of $ax^2 + bx + c = 0$
Where $a = (3k +1)$, $b = 2(k + 1)$, $c = 1$
For the equation to have real and equal roots, the condition is

$D = b^2 - 4ac = 0$
$\Rightarrow \quad (2(k + 1))^2 - 4(3k +1)(1) = 0$
$\Rightarrow \quad (k +1)^2 - (3k + 1) = 0$ [After dividing by 4 both sides]
$\Rightarrow \quad k^2 + 2k + 1 - 3k – 1 = 0$
$\Rightarrow \quad k^2 - k = 0$
$\Rightarrow \quad k(k - 1) = 0$

Either $k = 0$ Or, $k - 3 = 0 \Rightarrow k = 1$,
So, the value of k can either be 0 or 1

Now, using $k = 0$ in the given quadratic equation we get

$(3(0) + 1)x^2 + 2(0 + 1)x + 1 = 0$
$x^2 + 2x + 1 = 0$
$\Rightarrow (x + 1)^2 = 0$

Thus, $x = -1$ is the root of the given quadratic equation.

Next, on using $k = 1$ in the given quadratic equation we get

$(3(1) + 1)x^2 + 2(1 + 1)x + 1 = 0$
$4x^2 + 4x + 1 = 0$
$\Rightarrow (2x + 1)^2 = 0$

Thus, $2x = -1 \Rightarrow x = -1/2$ is the root of the given quadratic equation.

10. Find the values of p for which the quadratic equation $(2p + 1)x^2 – (7p + 2)x + (7p - 3) = 0$ has equal roots. Also, find the roots.
Solution:

The given equation $(2p + 1)x^2 – (7p + 2)x + (7p - 3) = 0$ is in the form of $ax^2 + bx + c = 0$
Where $a = (2p +1)$, $b = -(7p + 2)$, $c = (7p - 3)$
For the equation to have real and equal roots, the condition is

$D = b^2 - 4ac = 0$
$\Rightarrow \quad (-(7p + 2))^2 - 4(2p +1)(7p - 3) = 0$
$\Rightarrow \quad (7p + 2)^2 - 4(14p^2 + p - 3) = 0$
$\Rightarrow \quad 49p^2 + 28p + 4 – 56p^2 – 4p + 12 = 0$
$\Rightarrow \quad -7p^2 + 24p + 16 = 0$

Solving for p by factorization,

$\Rightarrow \quad -7p^2 + 28p - 4p + 16 = 0$
$\Rightarrow \quad -7p(p - 4) -4(p - 4) = 0$
$\Rightarrow \quad (p - 4) (-7p - 4) = 0$

Either $p - 4 = 0 \Rightarrow p = 4$ Or, $7p + 4 = 0 \Rightarrow p = -4/7$,
So, the value of k can either be 4 or -4/7

Now, using k = 4 in the given quadratic equation we get

$(2(4) + 1)x^2 – (7(4) + 2)x + (7(4) - 3) = 0$

$9x^2 - 30x + 25 = 0$

$\Rightarrow (3x - 5)^2 = 0$

Thus, x = 5/3 is the root of the given quadratic equation.

Next, on using k = 1 in the given quadratic equation we get

$(2(-4/7) + 1)x^2 – (7(-4/7) + 2)x + (7(-4/7) - 3) = 0$

$x^2 - 14x + 49 = 0$

$\Rightarrow (x - 7)^2 = 0$

Thus, x – 7 = 0 ⇒ x = 7 is the root of the given quadratic equation.

11. If -5 is a root of the quadratic equation $2x^2 + px -15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.

Solution:

Given,

-5 is as root of $2x^2 + px - 15 = 0$

So, on substituting x = -5 the LHS will become zero and satisfy the equation.

⇒ $2(-5)^2 + p(-5) - 15 = 0$

⇒ $50 – 5p – 15 = 0$

⇒ $35 = 5p$

⇒ $p = 7$

Now, substituting the value of p in the second equation we have

$(7)(x^2 + x) + k = 0$

⇒ $7x^2 + 7x + k = 0$

It's given that the above equation has equal roots.

Thus the discriminant, D = 0

The equation $7x^2 + 7x + k = 0$ is in the form of $ax^2 + bx + c = 0$

Where a = 7, b = 7, c = k

$D = b^2 – 4ac$

⇒ $7^2 – 4(7)(k) = 0$

⇒ $49 – 28k = 0$

⇒ $k = 49/28 = 7/4$

Therefore, the value of k is 7/4.

12. If 2 is a root of the quadratic equation $3x^2 + px -8 = 0$ and the quadratic equation $4x^2 – 2px + k = 0$ has equal roots, find the value of k.

Solution:

Given,

2 is as root of $3x^2 + px -8 = 0$

So, on substituting x = 2 the LHS will become zero and satisfy the equation.

⇒ $3(2)^2 + p(2) - 8 = 0$

⇒ $12 + 2p - 8 = 0$

⇒ $4 + 2p = 0$

⇒ $p = -2$

Now, substituting the value of p in the second equation we have

$4x^2 - 2(-2)x + k = 0$

⇒ $4x^2 + 4x + k = 0$

It's given that the above equation has equal roots.

Thus the discriminant, D = 0

The equation $4x^2 + 4x + k = 0$ is in the form of $ax^2 + bx + c = 0$

Where a = 4, b = 4, c = k

$D = b^2 - 4ac$

⇒ $4^2 - 4(4)(k) = 0$

⇒ $16 - 16k = 0$ [dividing by 16 both sides]

⇒ $k = 1$

Therefore, the value of k is 1.