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1. The speed of a boat in still water is 8km/hr. It can go 15 km upstream and 22 km downstream in 5 hours. Find the speed of the stream.
Solution:

Let the speed of stream be x km/hr
Given, speed of boat in still water is 8km/hr.
So, speed of downstream = (8 + x) km/hr
And, speed of upstream = (8 - x) km/hr
Using, speed = distance/ time
Time taken by the boat to go 15 km upstream = 15/(8 - x)hr
And, time taken by the boat to return 22 km downstream = 22/(8 + x)hr
From the question, the boat returns to the same point in 5 hr.

$$\text{so, } \frac{15}{(8-x)} + \frac{22}{(8+x)} = 5$$

$$\frac{15(8+x) + 22(8-x)}{(8-x)(8+x)} = 5$$

$$\frac{120 + 15x + 176 - 22x}{64 - x^2} = 5$$

$$\frac{296 - 7x}{64 - x^2} = 5$$

$5x^2 - 7x + 296 - 320 = 0$
$5x^2 - 7x - 24 = 0$
$5x^2 - 15x + 8x - 24 = 0$ [by factorisation method]
$5x(x - 3) + 8(x - 3) = 0$
$(x - 3)(5x + 8) = 0$
$\therefore x = 3, x = - 8/5$
As the speed of the stream can never be negative, only the positive solution is considered.
Therefore, the speed of the stream is 3 km/hr.

2. A train, traveling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/hr more. Find the original speed of the train.
Solution:

Let the original speed of train be x km/hr
When increased by 5, speed of the train = (x + 5) km/hr
Using, speed = distance/ time
Time taken by the train for original uniform speed to cover 360 km = 360/x hr.
And, time taken by the train for increased speed to cover 360 km = 360/(x + 5) hr.
Given, that the difference in the times is 48 mins. ⇒ 48/60 hour

This can be expressed as below:

$$\frac{360}{x} - \frac{360}{(x+5)} = \frac{48}{60}$$

$$\frac{360(x+5) - 360x}{x(x+5)} = \frac{4}{5}$$

$$\frac{360x + 1800 - 360x}{x^2 + 5x} = \frac{4}{5}$$

$1800(5) = 4(x^2 + 5x)$
$9000 = 4x^2 + 20x$
$4x^2 + 20x - 9000 = 0$
$x^2 + 5x - 2250 = 0$
$x^2 + 50x - 45x - 2250 = 0$ [by factorisation method]
$x(x + 50) - 45(x + 50) = 0$
$(x + 50)(x - 45) = 0$
$\therefore x = -50$ or $x = 45$
Since, the speed of the train can never be negative x = -50 is not considered.
Therefore, the original speed of train is 45 km/hr.

3. A fast train takes one hour less than a slow train for a journey of 200 km. If the speed of the slow train is 10 km/hr less than that of the fast train, find the speed of the two trains.
Solution:

Let's consider the speed of the fast train as x km/hr
Then, the speed of the slow train will be = (x -10) km/hr
Using, speed = distance/ time
Time taken by the fast train to cover 200 km = 200/x hr
And, time taken by the slow train to cover 200 km = 200/(x - 10) hr
Given, that the difference in the times is 1 hour.
This can be expressed as below:

$$\frac{200}{x} - \frac{200}{(x-10)} = 1$$

$$\frac{(200(x-10) - 200x)}{x(x-10)} = 1$$

$$\frac{200x - 2000 - 200x}{x^2 - 10x} = 1$$

$x^2 - 10x = -2000$
$x^2 - 10x + 2000 = 0$

$x^2 - 50x + 40x + 2000 = 0$ [by factorisation method]
$x(x - 50) + 40(x - 50) = 0$
$(x - 50)(x + 40) = 0$
$x = 50$ or $x = - 40$
As, the speed of train can never be negative we neglect $x = -40$
Thus, speed of the fast train is 50 km/hr
And the speed of slow train $(50 - 10) = 40$ km/hr

4. A passenger train takes one hour less for a journey of 150 km if its speed is increased 5 km/hr from its usual speed. Find the usual speed of the train.
Solution:

Let's assume the usual speed of train as x km/hr
Then, the increased speed of the train = (x + 5) km/hr
Using, speed = distance/ time
Time taken by the train under usual speed to cover 150 km = 150/x hr
Time taken by the train under increased speed to cover 150 km = 150(x + 5)hr
Given, that the difference in the times is 1 hour.
This can be expressed as below:

$$\text{So, } \frac{150}{x} - \frac{150}{(x+5)} = 1$$

$$\frac{150(x+5) - 150x}{x(x+5)} = 1$$

$$\frac{150x + 750 - 150x}{x^2 + 5x} = 1$$

$750 = x^2 + 5x$
$x^2 + 5x - 750 = 0$
$x^2 - 25x + 30x -750 = 0$ [by factorisation method]
$x(x - 25) + 30(x - 25) = 0$
$(x - 25) (x + 30) = 0$
$x = 25$ or $x = -30$ (neglected as the speed of the train can never be negative)
Hence, the usual speed of the train is $x = 25$ km/hr

5. The time taken by a person to cover 150 km was 2.5 hrs more than the time taken in the return journey. If he returned at the speed of 10 km/hr more than the speed of going, what was the speed per hour in each direction?
Solution:

Let the ongoing speed of person be x km/hr,
Then, the returning speed of the person is = (x + 10) km/hr (from the question)
Using, speed = distance/ time

Time taken by the person in going direction to cover 150 km = 150/x hr
And, time taken by the person in returning direction to cover 150 km = 150/(x + 10)hr
Given, that the difference in the times is 2.5 hour ⇒ 5/2 hours
This can be expressed as below:

$$\frac{150}{x} - \frac{150}{(x+10)} = \frac{5}{2}$$

$$\frac{150(x+10) - 150x}{x(x+10)} = \frac{5}{2}$$

$$\frac{150x + 1500 - 150x}{x^2 + 10x} = \frac{5}{2}$$

$$\frac{1500}{x^2 + 10x} = \frac{5}{2}$$

$3000 = 5x^2 + 50x$
$5x^2 + 50x - 3000 = 0$
$5(x^2 + 10x - 600) = 0$
$x^2 + 10x - 600 = 0$
$x^2 - 20x + 30x - 600 = 0$ [by factorisation method]
$x(x - 20) + 30(x - 20) = 0$
$(x - 20)(x + 30) = 0$
x = 20 or x = -30(neglected) As the speed of train can never be negative.
Thus, x = 20 Then, (x + 10) (20 + 10) = 30
Therefore, the ongoing speed of person is 20km/hr.
And the returning speed of the person is 30 km/hr.

6. A plane left 40 minutes late due to bad weather and in order to reach the destination, 1600 km away in time, it had to increase its speed by 400 km/hr from its usual speed. Find the usual speed of the plane.
Solution:

Let's assume the usual speed of the plane to be x km/hr,
Then the increased speed of the plane is = (x + 4000) km/hr
Using, speed = distance/ time
Time taken by the plane under usual speed to cover 1600 km = 1600/x hr
Time taken by the plane under increased speed to cover 1600 km = 1600/(x + 400) hr
Given, that the difference in the times is 40mins ⇒ 40/60 hours
This can be expressed as below:

$$\frac{1600}{x} - \frac{1600}{(x+400)} = \frac{40}{60}$$

$$\frac{1600(x+400) - 1600x}{x(x+400)} = \frac{2}{3}$$

$$\frac{1600x + 640000 - 1600x}{x^2 + 400x} = \frac{2}{3}$$

$1920000 = 2x^2 + 800x$
$2x^2 + 800x - 1920000 = 0$
$2(x^2 + 400x - 960000) = 0$
$x^2 + 400x - 960000 = 0$
$x^2 - 800x + 1200x - 960000 = 0$ [by factorisation method]
$x(x - 800) + 1200(x - 800) = 0$
$(x - 800)(x + 1200) = 0$
$x = 800$ or $x = -1200$ (neglected)
As the speed of the train can never be negative.
Thus, the usual speed of the train is 800 km/hr.

7. An aero plane takes 1 hour less for a journey of 1200 km if its speed is increased by 100 km/hr from its usual speed of the plane. Find its usual speed.
Solution:

Let's consider the usual speed of plane as x km/hr,
Then, the increased speed of the plane is = (x + 100) km/hr
Using, speed = distance/ time
Time taken by the plane under usual speed to cover 1200 km = 1200/x hr
Time taken by the plane under increased speed to cover 1200 km = 1200/(x + 100)hr
Given, that the difference in the times is 1 hour.
So, this can be expressed as below:

$$\frac{1200}{x} - \frac{1200}{(x+100)} = 1$$

$$\frac{1200(x+100) - 1200x}{x(x+100)} = 1$$

$$\frac{1200x + 120000 - 1200x}{x^2 + 100x} = 1$$

$120000 = x^2 + 100x$
$x^2 + 100x - 120000 = 0$
$x^2 - 300x + 400x - 120000 = 0$ [by factorisation method]
$x(x - 300) + 400(x - 300) = 0$

x = 300 or x = - 400 neglected as the speed of the aero plane can never be negative.
Therefore, the usual speed of train is 300 km/hr.