Exercise 8.1

1. Which of the following are quadratic equations?

(i) $x^2 + 6x – 4 = 0$

Solution:

Let $p(x) = x^2 + 6x – 4$,

It's clearly seen that $p(x) = x^2 + 6x – 4$ is a quadratic polynomial. Thus, the given equation is a quadratic equation.

(ii) $\sqrt{3}x^2 – 2x + 1/2 = 0$

Solution:

Let $p(x) = \sqrt{3}x^2 – 2x + 1/2$,

It's clearly seen that $p(x) = \sqrt{3}x^2 – 2x + 1/2$ having real coefficients is a quadratic polynomial. Thus, the given equation is a quadratic equation.

(iii) $x^2 + 1/x^2 = 5$

Solution:

Given,

$x^2 + 1/x^2 = 5$

On multiplying by x^2 on both sides we have,

$x^4 + 1 = 5x^2$

$\Rightarrow \quad x^4 – 5x^2 + 1 = 0$

It's clearly seen that $x^4 – 5x^2 + 1$ is not a quadratic polynomial as its degree is 4. Thus, the given equation is not a quadratic equation.

(iv) $x – 3/x = x^2$

Solution:

Given,

$x – 3/x = x^2$

On multiplying by x on both sides we have,

$x^2 – 3 = x^3$

$\Rightarrow \quad x^3 – x^2 + 3 = 0$

It's clearly seen that $x^3 – x^2 + 3$ is not a quadratic polynomial as its degree is 3. Thus, the given equation is not a quadratic equation.

(v) $2x^2 - \sqrt{(3x)} + 9 = 0$

Solution:

It's clearly seen that $2x^2 - \sqrt{(3x)} + 9$ is not a polynomial because it contains a term involving $x^{1/2}$, where 1/2 is not an integer. Thus, the given equation is not a quadratic equation.

(vi) $x^2 - 2x - \sqrt{x} - 5 = 0$
Solution:

It's clearly seen that $x^2 - 2x - \sqrt{x} - 5$ is not a polynomial because it contains a term involving $x^{1/2}$, where 1/2 is not an integer. Thus, the given equation is not a quadratic equation.

(vii) $3x^2 - 5x + 9 = x^2 - 7x + 3$
Solution:

Given,
$3x^2 - 5x + 9 = x^2 - 7x + 3$
On simplifying the equation, we have
$2x^2 + 2x + 6 = 0$
$\Rightarrow$ $x^2 + x + 3 = 0$ (dividing by 2 on both sides)
Now, it's clearly seen that $x^2 + x + 3$ is a quadratic polynomial. Thus, the given equation is a quadratic equation.

(viii) $x + 1/x = 1$
Solution:

Given,
$x + 1/x = 1$
On multiplying by x on both sides we have,
$x^2 + 1 = x$
$\Rightarrow$ $x^2 - x + 1 = 0$
It's clearly seen that $x^2 - x + 1$ is a quadratic polynomial. Thus, the given equation is a quadratic equation.

(ix) $x^2 - 3x = 0$
Solution:

Let $p(x) = x^2 - 3x$,
It's clearly seen that $p(x) = x^2 - 3x$ is a quadratic polynomial. Thus, the given equation is a quadratic equation.

(x) $(x + 1/x)^2 = 3(x + 1/x) + 4$
Solution:

Given,
$(x + 1/x)^2 = 3(x + 1/x) + 4$
$\Rightarrow$ $x^2 + 1/x^2 + 2 = 3x + 3/x + 4$
$\Rightarrow$ $x^4 + 1 + 2x^2 = 3x^3 + 3x + 4x^2$
$\Rightarrow$ $x^4 - 3x^3 - 2x^2 - 3x + 1 = 0$
Now, it's clearly seen that $x^4 - 3x^3 - 2x^2 - 3x + 1$ is not a quadratic polynomial since its degree is 4. Thus, the given equation is not a quadratic equation.

(xi) $(2x + 1)(3x + 2) = 6(x – 1)(x – 2)$
Solution:

Given,
$(2x + 1)(3x + 2) = 6(x – 1)(x – 2)$
⇒ $6x^2 + 4x + 3x + 2 = 6x^2 -12x - 6x + 12$
⇒ $7x + 2 = -18x + 12$
⇒ $25x – 10 = 0$
Now, it's clearly seen that $25x – 10$ is not a quadratic polynomial since its degree is 1. Thus, the given equation is not a quadratic equation.

(xii) $x + 1/x = x^2, x \neq 0$
Solution:

Given,
$x + 1/x = x^2$
On multiplying by x on both sides we have,
$x^2 + 1 = x^3$
⇒ $x^3 – x^2 – 1 = 0$
Now, it's clearly seen that $x^3 – x^2 – 1$ is not a quadratic polynomial since its degree is 3. Thus, the given equation is not a quadratic equation.

(xiii) $16x^2 – 3 = (2x + 5)(5x - 3)$
Solution:

Given,
$16x^2 – 3 = (2x + 5)(5x - 3)$
$16x^2 – 3 = 10x^2 – 6x + 25x – 15$
⇒ $6x^2 – 19x + 12 = 0$
Now, it's clearly seen that $6x^2 – 19x + 12$ is a quadratic polynomial. Thus, the given equation is a quadratic equation.

(xiv) $(x + 2)^3 = x^3 – 4$
Solution:

Given,
$(x + 2)^3 = x^3 – 4$
On expanding, we get
⇒ $x^3 + 6x^2 + 8x + 8 = x^3 – 4$
⇒ $6x^2 + 8x + 12 = 0$
Now, it's clearly seen that $6x^2 + 8x + 12$ is a quadratic polynomial. Thus, the given equation is a quadratic equation.

(xv) $x(x + 1) + 8 = (x + 2)(x - 2)$
Solution:

Given,

$x(x + 1) + 8 = (x + 2)(x - 2)$

$x^2 + x + 8 = x^2 – 4$

⇒ $x + 12 = 0$

Now, it's clearly seen that $x + 12$ is a not quadratic polynomial since its degree is 1. Thus, the given equation is not a quadratic equation.

2. In each of the following, determine whether the given values are solutions of the given equation or not:

(i) $x^2 - 3x + 2 = 0$, $x = 2$, $x = - 1$

Solution:

Here we have,

LHS = $x^2 - 3x + 2$

Substituting $x = 2$ in LHS, we get

$(2)^2 – 3(2) + 2$

⇒ $4 – 6 + 2 = 0$ = RHS

⇒ LHS = RHS

Thus, $x = 2$ is a solution of the given equation.

Similarly,

Substituting $x = -1$ in LHS, we get

$(-1)^2 – 3(-1) + 2$

⇒ $1 + 3 + 2 = 6 \neq$ RHS

⇒ LHS ≠ RHS

Thus, $x = -1$ is not a solution of the given equation.

(ii) $x^2 + x + 1 = 0$, $x = 0$, $x = 1$

Solution:

Here we have,

LHS = $x^2 + x + 1$

Substituting $x = 0$ in LHS, we get

$(0)^2 + 0 + 1$

⇒ $1 \neq$ RHS

⇒ LHS ≠ RHS

Thus, $x = 0$ is not a solution of the given equation.

Similarly,

Substituting $x = 1$ in LHS, we get

$(1)^2 + 1 + 1$

⇒ $3 \neq$ RHS

⇒ LHS ≠ RHS

Thus, $x = 1$ is not a solution of the given equation.

(iii) $x^2 - 3\sqrt{3}x + 6 = 0$, $x = \sqrt{3}$ and $x = -2\sqrt{3}$
Solution:

Here we have,
LHS = $x^2 - 3\sqrt{3}x + 6$
Substituting $x = \sqrt{3}$ in LHS, we get
$(\sqrt{3})^2 - 3\sqrt{3}(\sqrt{3}) + 6$
⇒ $3 - 9 + 6 = 0$ = RHS
⇒ LHS = RHS
Thus, $x = \sqrt{3}$ is a solution of the given equation.

Similarly,
Substituting $x = -2\sqrt{3}$ in LHS, we get
$(-2\sqrt{3})^2 - 3\sqrt{3}(-2\sqrt{3}) + 6$
⇒ $12 + 18 + 6 = 36 \neq$ RHS
⇒ LHS ≠ RHS
Thus, $x = -2\sqrt{3}$ is not a solution of the given equation.

(iv) $x + 1/x = 13/6$, $x = 5/6$, $x = 4/3$
Solution:

Here we have,
LHS = $x + 1/x$
Substituting $x = 5/6$ in LHS, we get
$(5/6) + 1/(5/6) = 5/6 + 6/5$
⇒ $61/30 \neq$ RHS
⇒ LHS ≠ RHS
Thus, $x = 5/6$ is not a solution of the given equation.

Similarly,
Substituting $x = 4/3$ in LHS, we get
$(4/3) + 1/(4/3) = 4/3 + 3/4$
⇒ $25/12 \neq$ RHS
⇒ LHS ≠ RHS
Thus, $x = 3/4$ is not a solution of the given equation.

(v) $2x^2 - x + 9 = x^2 + 4x + 3$, $x = 2$, $x = 3$
Solution:

Here we have,
$2x^2 - x + 9 = x^2 + 4x + 3$
⇒ $x^2 - 5x + 6 = 0$

LHS = $x^2 - 5x + 6$
Substituting $x = 2$ in LHS, we get
$(2)^2 - 5(2) + 6$

⇒ $4 - 10 + 6 = 0$ = RHS

⇒ LHS = RHS

Thus, $x = 2$ is a solution of the given equation.

Similarly,

Substituting $x = 3$ in LHS, we get

$(3)^2 - 5(3) + 6$

⇒ $9 - 15 + 6 = 0$ = RHS

⇒ LHS = RHS

Thus, $x = 3$ is a solution of the given equation.

(vi) $x^2 - \sqrt{2}x - 4 = 0, x = -\sqrt{2}, x = -2\sqrt{2}$

Solution:

Here we have,

LHS = $x^2 - \sqrt{2}x - 4$

Substituting $x = -\sqrt{2}$ in LHS, we get

$(-\sqrt{2})^2 - \sqrt{2}(-\sqrt{2}) - 4$

⇒ $4 + 2 - 4 = 2 \neq$ RHS

⇒ LHS ≠ RHS

Thus, $x = -\sqrt{2}$ is a solution of the given equation.

Similarly,

Substituting $x = -2\sqrt{2}$ in LHS, we get

$(-2\sqrt{2})^2 - \sqrt{2}(-2\sqrt{2}) - 4$

⇒ $8 + 4 - 4 = 8 \neq$ RHS

⇒ LHS ≠ RHS

Thus, $x = -2\sqrt{2}$ is not a solution of the given equation.

(vii) $a^2x^2 - 3abx + 2b^2 = 0, x = a/b, x = b/a$

Solution:

We have,

LHS = $a^2x^2 - 3abx + 2b^2$ and RHS = 0

Substituting the $x = \frac{a}{b}$ and $x = \frac{b}{a}$ in

$$\text{LHS} = a^2\left(\frac{a}{b}\right)^2 - 3ab\left(\frac{a}{b}\right) + 2b^2$$

$$= \frac{a^4}{b^2} - 3a^2 + 2b^2$$

≠ RHS

And, for x = b/a

$$\text{LHS} = a^2\left(\frac{b}{a}\right)^2 - 3ab\left(\frac{b}{a}\right) + 2b^2$$

$$= b^2 - 3b^2 + 2b^2 = 0$$

= RHS

Therefore, x = b/a is a solution of the given equation.

Exercise 8.2 Page No: 8.8

1. The product of two consecutive positive integers is 306. Form the quadratic equation to find the integers, if x denotes the smaller integer.

Solution:

Let the two integers be x and x+1, x taken as the smaller integer.
From the question, the product of these two integers is 306
So,

$x(x + 1) = 306$

$\Rightarrow \quad x^2 + x - 306 = 0$

Thus, the required quadratic equation is $x^2 + x - 306 = 0$

2. John and Jivani together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 128. Form the quadratic equation to find how many marbles they to start with, if John had x marbles.

Solution:

Given,
John and Jilani together have a total of 45 marbles.
Let John have x marbles.
So, Jivani will be having (45 - x) marbles.

Number of marbles John had after losing 5 marbles = x - 5
Number of marbles Jivani had after losing 5 marbles = (45 - x) - 5 = 40 – x

Now, according to the question the product of the marbles that they are having now is 128
So,

$(x - 5)(40 - x) = 128$

$\Rightarrow \quad 40x - x^2 - 200 = 128$

$\Rightarrow \quad x^2 - 45x + 128 + 200 = 0$

$\Rightarrow \quad x^2 - 45x + 328 = 0$

Thus the required quadratic equation is $x^2 - 45x + 328 = 0$.

3. A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of articles produced in a day. On a particular day, the total cost of production was Rs. 750. If x denotes the number of toys produced that day, form the quadratic equation to find x.

Solution:

Given that x denotes the number of toys produced in a day.
So, the cost of production of each toy = (55 - x)
And, the total cost of production is the product of number of toys produced in a day and cost of production of each toy i.e, x (55 - x)
From the question, it's given that
The total cost of production on that particular day is Rs.750

So,

⇒ $x(55 - x) = 750$

⇒ $55x - x^2 = 750$

⇒ $x^2 - 55x + 750 = 0$

Thus, the required quadratic equation is $x^2 - 55x + 750 = 0$.

Exercise 8.3

Solve the following quadratic equation by factorization:

1. (x – 4)(x + 2) = 0

Solution:

Given,

(x - 4) (x + 2) = 0

So, either x - 4 = 0 ⇒ x = 4

Or, x + 2 = 0, ⇒ x = - 2

Thus, the roots of the given quadratic equation are 4 and -2 respectively.

2. (2x + 3) (3x - 7) = 0

Solution:

Given,

(2x + 3) (3x - 7) = 0.

So, either 2x + 3 = 0, ⇒ x = - 3/2

Or, 3x -7 = 0, ⇒ x = 7/3

Thus, the roots of the given quadratic equation are x = -3/2 and x = 7/3 respectively.

3. $3x^2$ - 14x - 5 = 0

Solution:

Given.

$3x^2 - 14x - 5 = 0$

⇒ $3x^2 - 14x - 5 = 0$

⇒ $3x^2 - 15x + x - 5 = 0$

⇒ $3x(x - 5) + 1(x - 5) = 0$

⇒ $(3x + 1)(x - 5) = 0$

Now, either 3x + 1 = 0 ⇒ x = -1/3

Or, x - 5 = 0 ⇒ x = 5

Thus, the roots of the given quadratic equation are 5 and x = - 1/3 respectively.

4. Find the roots of the equation $9x^2$ - 3x - 2 = 0.

Solution:

Given,

$9x^2 - 3x - 2 = 0.$

⇒ $9x^2 - 3x - 2 = 0.$

⇒ $9x^2 - 6x + 3x - 2 = 0$

⇒ $3x (3x - 2) + 1(3x - 2) = 0$

⇒ $(3x - 2)(3x + 1) = 0$

Now, either 3x - 2 = 0 ⇒ x = 2/3

Or, $3x + 1 = 0$ $\Rightarrow x = -1/3$
Thus, the roots of the given quadratic equation are $x = 2/3$ and $x = -1/3$ respectively.

5. $\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}$

Solution:

Given,

$$\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}$$

$$\frac{x+5-x+1}{(x-1)(x+5)} = \frac{6}{7}$$

$$\frac{6}{x^2+4x-5} = \frac{6}{7}$$

Dividing by 6 both the sides and cross-multiplying we get
$x^2 + 4x - 12 = 0$
$\Rightarrow$ $x^2 + 6x - 2x - 12 = 0$
$\Rightarrow$ $x(x + 6) - 2(x - 6) = 0$
$\Rightarrow$ $(x + 6)(x - 2) = 0$
Now, either $x + 6 = 0 \Rightarrow x = -6$
Or, $x - 2 = 0 \Rightarrow x = 2$
Thus, the roots of the given quadratic equation are 2 and – 6 respectively.

6. $6x^2 + 11x + 3 = 0$
Solution:

Given equation is $6x^2 + 11x + 3 = 0$.
$\Rightarrow$ $6x^2 + 9x + 2x + 3 = 0$
$\Rightarrow$ $3x(2x + 3) + 1(2x + 3) = 0$
$\Rightarrow$ $(2x + 3)(3x + 1) = 0$
Now, either $2x + 3 = 0$ $\Rightarrow x = -3/2$
Or, $3x + 1 = 0$ $\Rightarrow x = -1/3$
Thus, the roots of the given quadratic equation are $x = -3/2$ and $x = -1/3$ respectively.

7. $5x^2 - 3x - 2 = 0$
Solution:

Given equation is $5x^2 - 3x - 2 = 0$.
$\Rightarrow$ $5x^2 - 3x - 2 = 0$.
$\Rightarrow$ $5x^2 - 5x + 2x - 2 = 0$

⇒ $5x(x - 1) + 2(x - 1) = 0$
⇒ $(5x + 2)(x - 1) = 0$
Now, either $5x + 2 = 0$ ⇒ $x = -2/5$
Or, $x - 1 = 0$ ⇒ $x = 1$
Thus, the roots of the given quadratic equation are 1 and $x = -2/5$ respectively.

8. $48x^2 - 13x - 1 = 0$
Solution:

Given equation is $48x^2 - 13x - 1 = 0$.
⇒ $48x^2 - 13x - 1 = 0$.
⇒ $48x^2 - 16x + 3x - 1 = 0$.
⇒ $16x(3x - 1) + 1(3x - 1) = 0$
⇒ $(16x + 1)(3x - 1) = 0$
Either $16x + 1 = 0$ ⇒ $x = -1/16$
Or, $3x – 1 = 0$ ⇒ $x = 1/3$
Thus, the roots of the given quadratic equation are $x = -1/16$ and $x = 1/3$ respectively.

9. $3x^2 = -11x - 10$
Solution:

Given equation is $3x^2 = -11x - 10$
⇒ $3x^2 + 11x + 10 = 0$
⇒ $3x^2 + 6x + 5x + 10 = 0$
⇒ $3x(x + 2) + 5(x + 2) = 0$
⇒ $(3x + 2)(x + 2) = 0$
Now, either $3x + 2 = 0$ ⇒ $x = -2/3$
Or, $x + 2 = 0$ ⇒ $x = -2$
Thus, the roots of the given quadratic equation are $x = -2/3$ and -2 respectively.

10. $25x(x + 1) = - 4$
Solution:

Given equation is $25x(x + 1) = -4$
$25x(x + 1) = -4$
⇒ $25x^2 + 25x + 4 = 0$
⇒ $25x^2 + 20x + 5x + 4 = 0$
⇒ $5x (5x + 4) + 1(5x + 4) = 0$
⇒ $(5x + 4)(5x + 1) = 0$
Now, either $5x + 4 = 0$ therefore $x = - 4/5$
Or, $5x + 1 = 0$ therefore $x = -1 /5$
Thus, the roots of the given quadratic equation are $x = - 4/5$ and $x = -1/5$ respectively.

11. 16x – 10/x = 27
Solution:

Given,
16x – 10/x = 27
On multiplying x on both the sides we have,
⇒ $16x^2 - 10 = 27x$
⇒ $16x^2 - 27x - 10 = 0$
⇒ $16x^2 - 32x + 5x - 10 = 0$
⇒ 16x(x - 2) +5(x – 2) = 0
⇒ (16x + 5) (x - 2) = 0
Now, either 16x + 5 = 0 ⇒ x = -5/16
Or, x – 2 = 0 ⇒ x = 2
Thus, the roots of the given quadratic equation are x = - 5/16 and x = 2 respectively.

12. $\frac{1}{x} - \frac{1}{x-2} = 3$

Solution:

Given equation is,

$$\frac{1}{x} - \frac{1}{x-2} = 3$$

$$\frac{x-2-x}{x(x-2)} = 3$$

$$\frac{2}{x(x-2)} = 3$$

On cross multiplying on both the sides we get,
2 = 3x(x - 2)
$2 = 3x^2 - 6x$
$3x^2 - 6x - 2 = 0$
⇒ $3x^2 - 3x - 3x - 2 = 0$

$$3x^2 - (3+\sqrt{3})x - (3-\sqrt{3})x + [(\sqrt{3}^2) - 1^2] = 0$$

$$3x^2 - (3+\sqrt{3})x - (3-\sqrt{3})x + [(\sqrt{3}^2) - 1^2][(\sqrt{3}^2) - 1^2] = 0$$

$$\sqrt{3}^2x^2 - \sqrt{3}(\sqrt{3}+1)x - \sqrt{3}(\sqrt{3}-1)x + (\sqrt{3}+1)(\sqrt{3}-1) = 0$$

$$\sqrt{3}x(\sqrt{3}+1)x - (\sqrt{3}x - (\sqrt{3}+1))(\sqrt{3}-1) = 0$$

$$(\sqrt{3}x - \sqrt{3} - 1)(\sqrt{3}x - \sqrt{3} + 1)(\sqrt{3} - 1) = 0$$

Now, either

$$(\sqrt{3}x - \sqrt{3} - 1) = 0 \quad \text{or} \quad (\sqrt{3}x - \sqrt{3} + 1)(\sqrt{3} - 1) = 0$$

Thus,

$$x = \frac{\sqrt{3}+1}{\sqrt{3}} \quad \text{or} \quad x = \frac{\sqrt{3}-1}{\sqrt{3}}$$

are the solutions of the given quadratic equations.

13. x – 1/x = 3, x ≠ 0
Solution:

Given,
x – 1/x = 3
On multiplying x on both the sides we have,
⇒ $x^2 - 1 = 3x$
⇒ $x^2 - 3x - 1 = 0$

$$x^2 - (\frac{3}{2} + \frac{3}{2})x - 1 = 0$$

$$x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x - 1 = 0$$

$$x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x - \frac{-4}{4} = 0$$

$$x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x - \frac{9-13}{4} = 0$$

$$x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x - \frac{(3)^2-(\sqrt{13}^2)}{(2)^2} = 0$$

$$x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x + (\frac{3+\sqrt{13}}{2})(\frac{3-\sqrt{13}}{2}) = 0$$

$$(x - \frac{3+\sqrt{13}}{2})(x - \frac{3-\sqrt{13}}{2}) = 0$$

Either, $(x - \frac{3+\sqrt{13}}{2}) = 0$; $\Rightarrow x = \frac{3+\sqrt{13}}{2}$

Or, $(x - \frac{3-\sqrt{13}}{2}) = 0$; $\Rightarrow x = \frac{3-\sqrt{13}}{2}$

Therefore, the roots of the given quadratic equation are $\frac{3+\sqrt{13}}{2}$ and $\frac{3-\sqrt{13}}{2}$ respectively.

14. $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

Solution:

Given,

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$$

$$\frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$$

$$\frac{-11}{(x+4)(x-7)} = \frac{11}{30}$$

Dividing by 11 both the sides and cross-multiplying we get,

⇒ $x^2 - 3x - 28 = -30$

⇒ $x^2 - 3x - 2 = 0$

⇒ $x^2 - 2x - x - 2 = 0$

⇒ $x(x - 2) - 1(x - 2) = 0$

⇒ $(x - 2)(x - 1) = 0$

Now, either $x – 2 = 0$ ⇒ $x = 2$

Or, $x – 1 = 0$ ⇒ $x = 1$

Thus, the roots of the given quadratic equation are 1 and 2 respectively.

15. $\frac{1}{x-3} + \frac{2}{x-2} = \frac{8}{x}$

Solution:

Given,

$$\frac{1}{x-3} + \frac{2}{x-2} = \frac{8}{x}$$

$$\frac{x-2+2(x-3)}{(x-3)(x-2)} = \frac{8}{x}$$

$$\frac{3x-8}{(x-3)(x-2)} = \frac{8}{x}$$

On cross multiplying we get,

⇒ $x(3x – 8) = 8(x – 3)(x – 2)$

⇒ $3x^2 – 8x = 8(x^2 – 5x + 6)$

⇒ $8x^2 – 40x + 48 – (3x^2 – 8x) = 0$

⇒ $5x^2 – 32x + 48 = 0$

⇒ $5x^2 – 20x – 12x + 48 = 0$

⇒ $5x(x – 4) – 12(x – 4) = 0$

⇒ $(x – 4)(5x - 12) = 0$

Now, either $x - 4 = 0 \Rightarrow x = 4$
Or, $5x - 12 = 0 \Rightarrow x = 5/12$
Thus, the roots of the given quadratic equation are 1 and 2 respectively.

16. $a^2x^2 - 3abx + 2b^2 = 0$
Solution:

Given equation is $a^2x^2 - 3abx + 2b^2 = 0$
$\Rightarrow a^2x^2 - abx - 2abx + 2b^2 = 0$
$\Rightarrow ax(ax - b) - 2b(ax - b) = 0$
$\Rightarrow (ax - b)(ax - 2b) = 0$
Now, either $ax - b = 0 \Rightarrow x = b/a$
Or, $ax - 2b = 0 \Rightarrow x = 2b/a$
Thus, the roots of the quadratic equation are $x = 2b/a$ and $x = b/a$ respectively.

17. $9x^2 – 6b^2x – (a^4 – b^4) = 0$
Solution:

Given,
$$9x^2 - 6b^2x - (a^4 - b^4) = 0$$
$$\Rightarrow 9x^2 - 6b^2x - (a^2 - b^2)(a^2 + b^2) = 0$$
$$\Rightarrow 9x^2 + 3(a^2 - b^2)] - 3(a^2 + b^2)x - (a^2 - b^2)(a^2 + b^2) = 0$$
$$\Rightarrow 3x[3x + a^2 + b^2)] - (a^2 + b^2)[3x + (a^2 - b^2)] = 0$$
$$\Rightarrow [3x - (a^2 + b^2)][3x + (a^2 - b^2)] = 0$$
$$\Rightarrow 3x - (a^2 + b^2) = 0 \; or \; 3x + (a^2 - b^2) = 0$$
$$\Rightarrow x = \frac{a^2+b^2}{3} \; or \; x = \frac{b^2-a^2}{3}$$
Thus, the roots of the quadratic equation are $x = (b^2 – a^2)/3$ and $x = (a^2 + b^2)/3$ respectively.

18. $4x^2 + 4bx - (a^2 - b^2) = 0$
Solution:

Given,
$4x^2 + 4bx - (a^2 - b^2) = 0$
For factorizing,
$4(a^2 - b^2) = -4(a - b)(a + b) = [-2(a-b)][2(a + b)]$
$\Rightarrow 2(b - a)*2(b + a)$
$\Rightarrow 4x^2+ (2(b - a) + 2(b + a)) - (a - b)(a + b) = 0$
So, now
$4x^2 + 2(b - a)x++ 2(b + a)x + (b - a)(a + b) = 0$
$\Rightarrow 2x(2x + (b - a)) +(a + b)(2x + (b - a)) = 0$
$\Rightarrow (2x + (b - a))(2x + b + a) = 0$

Now, either $(2x + (b - a)) = 0$ $\Rightarrow x = (a - b)/2$
Or, $(2x + b + a) = 0$ $\Rightarrow x = -(a + b)/2$
Thus, the roots of the given quadratic equation are $x = -(a + b)/2$ and $x = (a - b)/2$ respectively.

19. $ax^2 + (4a^2 - 3b)x - 12ab = 0$
Solution:

Given equation is $ax^2 + (4a^2 - 3b)x - 12ab = 0$
$\Rightarrow$ $ax^2 + 4a^2x - 3bx - 12ab = 0$
$\Rightarrow$ $ax(x + 4a) - 3b(x + 4a) = 0$
$\Rightarrow$ $(x + 4a)(ax - 3b) = 0$
Now, either $x + 4a = 0$ $\Rightarrow x = -4a$
Or, $ax - 3b = 0$ $\Rightarrow x = 3b/a$
Thus, the roots of the given quadratic equation are $x = 3b/a$ and $-4a$ respectively.

20. $2x^2 + ax – a^2 = 0$
Solution:

Given,

$$2x^2 + ax - a^2 = 0$$
$$\Rightarrow 2x^2 + 2ax - ax - a^2 = 0$$
$$\Rightarrow 2x(x + a) - a(x + a) = 0$$
$$\Rightarrow (2x - a)(x + a) = 0$$
$$\Rightarrow 2x - a = 0 \ or \ x + a = 0$$
$$\Rightarrow x = \frac{a}{2} \ or \ x = -a$$

Thus, the roots of the given quadratic equation are $x = a/2$ and $-a$ respectively.

21. $16/x – 1 = 15/(x + 1), x \neq 0, -1$
Solution:

Given,

$$\frac{16}{x} - 1 = \frac{15}{x+1}$$
$$\Rightarrow \frac{16-x}{x} = \frac{15}{x+1}$$
$$\Rightarrow (16 - x)(x + 1) = 15x$$
$$\Rightarrow -x^2 + 16 + 15x = 15x$$
$$\Rightarrow -x^2 + 16 = 0$$
$$\Rightarrow -x^2 - 16 = 0$$

$\Rightarrow (x-4)(x+4)=0$

$\Rightarrow x-4=0\ x+4=0$

$\Rightarrow x=4\ or\ x=-4$

Thus, the roots of the given quadratic equation are x = 4 and -4 respectively.

22. $\frac{x+3}{x+2}=\frac{3x-7}{2x-3}$ **, x ≠ -2, 3/2**

Solution:

Given,

$$\frac{x+3}{x+2}=\frac{3x-7}{2x-3}$$

On cross-multiplying we get,

$(x + 3)(2x - 3) = (x + 2)(3x - 7)$

⇒ $2x^2 - 3x + 6x - 9 = 3x^2 - x - 14$

⇒ $2x^2 + 3x - 9 = 3x^2 - x - 14$

⇒ $x^2 - 3x - x - 14 + 9 = 0$

⇒ $x^2 - 5x + x - 5 = 0$

⇒ $x(x - 5) + 1(x - 5) = 0$

⇒ $(x - 5)(x + l) - 0$

Now, either x - 5 = 0 or x + 1 = 0

⇒ x = 5 and x = -1

Thus, the roots of the given quadratic equation are 5 and -1 respectively.

23. $\frac{2x}{x-4}+\frac{2x-5}{x-3}=\frac{25}{3}$ **, x ≠ 3, 4**

Solution:

The given equation is

$$\frac{2x}{x-4}+\frac{2x-5}{x-3}=\frac{25}{3}$$

$$\frac{2x(x-3)+(2x-5)(x-4)}{(x-4)(x-3)}=\frac{25}{3}$$

$$\frac{2x^2-6x+2x^2-5x-8x+20}{x^2-4x-3x+12}$$

$$\frac{4x^2-19x+20}{x^2-7x+12}=\frac{25}{3}$$

On cross multiplying, we have

$3(4x^2 - 19x + 20) = 25(x^2 - 7x + 12)$

$\Rightarrow 12x^2 - 57x + 60 = 25x^2 - 175x + 300$

$\Rightarrow 13x^2 - 78x - 40x + 240 = 0$

$\Rightarrow 13x^2 - 118x + 240 = 0$

$\Rightarrow 13x^2 - 78x - 40x + 240 = 0$

$\Rightarrow 13x(x - 6) - 40(x - 6) = 0$

$\Rightarrow (x - 6)(13x - 40) = 0$

Now, either $x - 6 = 0 \Rightarrow x = 6$

Or, $13x - 40 = 0 \Rightarrow x = 40/13$

Thus, the roots of the given quadratic equation are 6 and 40/13 respectively.

24. $\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$ **$x \neq 0, 2$**

Solution:

Given equation is,

$$\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$$

$$\frac{x(x+3) - (x-2)(1-x)}{x(x-2)} = \frac{17}{4}$$

$$\frac{x^2 + 3x - x + x^2 + 2 - 2x}{x^2 - 2x} = \frac{17}{4}$$

$$\frac{2x^2 + 2}{x^2 - 2x} = \frac{17}{4}$$

On cross multiplying, we get

$4(2x^2 + 2) = 17(x^2 - 2x)$

$\Rightarrow$ $8x^2 + 8 = 17x^2 - 34x$

$\Rightarrow$ $9x^2 - 34x - 8 = 0$

$\Rightarrow$ $9x^2 - 36x + 2x - 8 = 0$

$\Rightarrow$ $9x(x - 4) + 2(x - 4) = 0$

$\Rightarrow$ $9x + 2)(x - 4) = 0$

Now, either $9x + 2 = 0 \Rightarrow x = -2/9$

Or, $x - 4 = 0 \Rightarrow x = 4$

Thus, the roots of the given quadratic equation are $x = -2/9$ and 4 respectively.

25. $\frac{x-3}{x+3} - \frac{x+3}{x-3} = \frac{48}{7}, x \neq 3, x \neq -3$

Solution:

Given equation is,

$$\frac{x-3}{x+3} - \frac{x+3}{x-3} = \frac{48}{7}$$

$$\Rightarrow \frac{(x-3)^2 - (x+3)^2}{(x+3)(x-3)} = \frac{48}{7}$$

$$\Rightarrow \frac{(x^2-6x+9)-(x^2+6x+9)}{x^2-9} = \frac{48}{7}$$

$$\Rightarrow \frac{x^2-6x+9-x^2-6x-9}{x^2-9} = \frac{48}{7}$$

$$\Rightarrow \frac{-12x}{x^2-9} = \frac{48}{7}$$

On cross-multiplying, we get

$7(-12x) = 48(x^2 - 9)$

$\Rightarrow$ $-84x = 48x^2 – 432$

$\Rightarrow$ $48x^2 + 84x – 432 = 0$

$\Rightarrow$ $4x^2 + 7x – 36 = 0$ dividing by 12]

$\Rightarrow$ $4x^2 + 16x – 9x – 36 = 0$

$\Rightarrow$ $4x(x + 4) – 9(x – 4) = 0$

$\Rightarrow$ $(4x – 9)(x + 4) = 0$

Now, either $4x – 9 = 0$ $\Rightarrow x = 9/4$

Or, $x + 4 = 0$ $\Rightarrow x = -4$

Thus, the roots of the given quadratic equation are x = 9/4 and -4 respectively.

26. $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$, $x \neq$ **0**

Solution:

Given equation is,

$$\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$$

$$\frac{(x-1)+2(x-2)}{(x-2)(x-1)} = \frac{6}{x}$$

$$\frac{(x-1)+2x-4}{(x^2-2x-x+2)}=\frac{6}{x}$$

$$\frac{3x-5}{(x^2-3x+2)}=\frac{6}{x}$$

On cross multiplying, we have

$x(3x - 5) = 6(x^2 - 3x + 2)$

⇒ $3x^2 - 5x = 6x^2 - 18x + 12$

⇒ $3x^2 - 13x + 12 = 0$

⇒ $3x^2 - 9x - 4x + 12 = 0$

⇒ $3x(x - 3) - 4(x - 3) = 0$

⇒ $(x - 3)(3x - 4) = 0$

Now, either $x - 3 = 0 \Rightarrow x = 3$

Or, $3x - 4 = 0 \Rightarrow 4/3$.

Thus, the roots of the given quadratic equation are 3 and 4/3 respectively.

27. $\frac{x+1}{x-1}-\frac{x-1}{x+1}=\frac{5}{6}$ **, $x \neq 1, -1$**

Solution:

The given equation is,

$$\frac{x+1}{x-1}-\frac{x-1}{x+1}=\frac{5}{6}$$

$$\frac{(x+1)^2-(x-1)^2}{x^2-1}=\frac{5}{6}$$

$$\frac{4x}{x^2-1}=\frac{5}{6}$$

On cross – multiplying we have,

⇒ $6(4x) = 5(x^2 - 1) = 24x$

⇒ $5x^2 - 5 = 5x^2 - 24x - 5 =0$

⇒ $5x^2 - 25x + x - 5 = 0$

⇒ $5x(x - 5) + 1(x - 5) = 0$

⇒ $(5x + 1)(x - 5) = 0$

Now, either $x - 5 = 0 \Rightarrow x = 5$

Or, $5x + 1 = 0 \Rightarrow x = -1/5$

Thus, the roots of the given quadratic equation are $x = -1/5$ and 5 respectively.

28. $\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=\frac{5}{2}$ **, $x \neq 1, -1/2$**

Solution:

The given equation is,

$$\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=\frac{5}{2}$$

$$\frac{(x-1)^2+(2x+1)^2}{2x^2-2x+x-1}=\frac{5}{2}$$

$$\frac{x^2-2x+1+4x^2+4x+1}{2x^2-x-1}=\frac{5}{2}$$

$$\frac{5x^2+2x+2}{2x^2-x-1}=\frac{5}{2}$$

On cross – multiplying we have,

⇒ $2(5x^2 + 2x + 2) = 5(2x^2 - x - 1)$

⇒ $10x^2 + 4x + 4 = 10x^2 - 5x - 5$

⇒ $4x + 5x + 4 + 5 = 0$

⇒ $9x + 9 = 0$

⇒ $9x = -9$

Thus, x = -1 is the only root of the given equation.

29. $\frac{4}{x}-3=\frac{5}{2x+3}, x \neq 0, -\frac{3}{2}$

Solution:

Given equation is,

$$\Rightarrow \frac{4-3x}{x}=\frac{5}{(2x+3)}$$

$\Rightarrow 5x = (2x + 3)(4 - 3x)$

$\Rightarrow 5x = 8x - 6x^2 + 12 - 9x$

$\Rightarrow 5x - 8x + 6x^2 - 12 + 9x = 0$

$\Rightarrow 6x^2 + 6x - 12 = 0$

$\Rightarrow x^2 + x - 2 = 0$ (Dividing by 6)

$\Rightarrow x^2 + 2x - x - 2 = 0$

$\Rightarrow x(x + 2) - 1(x + 2) = 0$

$\Rightarrow (x - 1)(x + 2) = 0$

$\Rightarrow x - 1 = 0$ or $x + 2 = 0$

$\therefore x = 1$ or $x = -2$

Thus, the roots of the given quadratic equation are x = 1 and x = -2 respectively.

30.
Solution:

Given equation is,

$$\frac{x-4}{x-5}+\frac{x-6}{x-7}=\frac{10}{3}$$

$$\frac{(x-4)(x-7)+(x-5)(x-6)}{(x-5)(x-7)}=\frac{10}{3}$$

$$\frac{x^2-7x-4x+28+x^2-6x-5x+30}{x^2-7x-5x+35}$$

$$\frac{2x^2-22x+58}{x^2-12x+35}=\frac{10}{3}$$

On cross-multiplying we have,

$3(2x^2 - 22x + 58) = 10(x^2 - 12x + 35)$

⇒ $6x^2 - 66x + 174 = 10x^2 - 120x + 350$

⇒ $4x^2 - 54x + 176 = 0$

⇒ $2x^2 - 27x + 88 = 0$

⇒ $2x^2 - 16x - 11x + 88 = 0$

⇒ $2x(x - 8) - 11(x + 8) = 0$

⇒ $(x - 8)(2x - 11) = 0$

Now, either x - 8 = 0 ⇒ x = 8

Or, 2x – 11 = 0 ⇒ x = 11/2

Thus, the roots of the given quadratic equation are x = 11/2 and 8 respectively.

Exercise 8.4 Page No: 8.26

Find the roots of the following quadratic equations (if they exist) by the method of completing the square.

1. $x^2 - 4\sqrt{2}x + 6 = 0$

Solution:

Given equation,

$$x^2 - 4\sqrt{2x} + 6 = 0$$

$$x^2 - 2 \times x \times 2\sqrt{2} + (2\sqrt{2})^2 - (2\sqrt{2})^2 + 6 = 0$$

$$(x - 2\sqrt{2})^2 = (2\sqrt{2})^2 - 6$$

$$(x - 2\sqrt{2})^2 = (4 \times 2) - 6 \quad = 8 - 6$$

$$(x - 2\sqrt{2})^2 = 2$$

$$(x - 2\sqrt{2}) = \pm\sqrt{2}$$

$$(x - 2\sqrt{2}) = \sqrt{2} \text{ or } (x - 2\sqrt{2}) = -\sqrt{2}$$

$$x = \sqrt{2} + 2\sqrt{2} \text{ or } x = -\sqrt{2} + 2\sqrt{2}$$

$$\Rightarrow x = 3\sqrt{2} \text{ or } x = \sqrt{2}$$

Thus, the roots of the given quadratic equation are $x = 3\sqrt{2}$ and $x = \sqrt{2}$.

2. $2x^2 - 7x + 3 = 0$

Solution:

Given equation,

$2x^2 - 7x + 3 = 0$

$$2\left(x^2 - \frac{7x}{2} + \frac{3}{2}\right) = 0$$

$$x^2 - 2 \times \frac{7}{2} \times \frac{1}{2} \times x + \frac{3}{2} = 0$$

$$x^2 - 2 \times \frac{7}{4} \times x + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$$

$$x^2 - 2 \times \frac{7}{4} \times x + \left(\frac{7}{4}\right)^2 - \left(\frac{49}{16}\right) + \frac{3}{2} = 0$$

$$\left(x-\frac{7}{4}\right)^2-\frac{49}{16}+\frac{3}{2}=0$$

$$\left(x-\frac{7}{4}\right)^2=\frac{49}{16}-\frac{3}{2}$$

$$\left(x-\frac{7}{4}\right)^2=\frac{49-26}{16}$$

$$\left(x-\frac{7}{4}\right)^2=\frac{25}{16}$$

$$\left(x-\frac{7}{4}\right)^2=\left(\frac{5}{4}\right)^2$$

$$x-\frac{7}{4}=\pm\frac{5}{4}$$

$$x-\frac{7}{4}=\frac{5}{4} \text{ or } x-\frac{7}{4}=-\frac{5}{4}$$

$$x=\frac{7}{4}+\frac{5}{4} \text{ or } x=\frac{7}{4}-\frac{5}{4}$$

⇒ x = 12/4 = 3 or x = 2/4 = 1/2

Thus, the roots of the given quadratic equation are x = 3 and x = 1/2.

3. $3x^2 + 11x + 10 = 0$

Solution:

Given equation,

$$x^2+\frac{11x}{3}+\frac{10}{3}=0$$

$$x^2+2\times\frac{1}{2}\times\frac{11x}{3}+\frac{10}{3}=0$$

$$x^2+2\times\frac{11x}{6}+\left(\frac{11}{6}\right)^2-\left(\frac{11}{6}\right)^2+\frac{10}{3}=0$$

$$\left(x+\frac{11}{6}\right)^2=\left(\frac{11}{6}\right)^2-\frac{10}{3}$$

$$\left(x+\frac{11}{6}\right)^2 = \frac{121}{36} - \frac{10}{3}$$

$$\left(x+\frac{11}{6}\right)^2 = \frac{121-120}{36}$$

$$\left(x+\frac{11}{6}\right)^2 = \frac{1}{36}$$

$$\left(x+\frac{11}{6}\right)^2 = \left(\frac{1}{6}\right)^2$$

$$x+\frac{11}{6} = \pm\frac{1}{6}$$

$$x+\frac{11}{6} = \frac{1}{6} \text{ or } x+\frac{11}{6} = \frac{-1}{6}$$

$$x = \frac{1}{6} - \frac{11}{6} \text{ or } x = \frac{-1}{6} - \frac{11}{6}$$

$$x = \frac{-10}{6} \text{ or } x = \frac{-12}{6} = -2$$

⇒ x = -5/3 = 3 or x = -2

Thus, the roots of the given quadratic equation are x = -5/3 and x = -2.

4. $2x^2 + x - 4 = 0$

Solution:

Given equation,

$2x^2 + x - 4 = 0$

$$2\left(x^2 + \frac{x}{2} - \frac{4}{2}\right) = 0$$

$$x^2 + 2 \times \frac{1}{2} \times \frac{1}{2} \times x - 2 = 0$$

$$x^2 + 2 \times \frac{1}{4} \times x + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 2 = 0$$

$$\left(x+\frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 + 2$$

$$\left(x+\frac{1}{4}\right)^2 = \frac{1}{16} + 2$$

$$\left(x+\frac{1}{4}\right)^2=\frac{1+2\times16}{16}$$

$$\left(x+\frac{1}{4}\right)^2=\frac{1+32}{16}$$

$$\left(x+\frac{1}{4}\right)^2=\frac{33}{16}$$

$$\left(x+\frac{1}{4}\right)=\pm\sqrt{\frac{33}{16}}$$

$$\left(x+\frac{1}{4}\right)=\sqrt{\frac{33}{16}}$$

$$\text{or }\left(x+\frac{1}{4}\right)=-\sqrt{\frac{33}{16}}$$

$$x=\frac{\sqrt{33}}{4}-\frac{1}{4}\text{ or }x=-\frac{\sqrt{33}}{4}-\frac{1}{4}$$

$$x=\frac{\sqrt{33}-1}{4}\text{ or }x=-\frac{\sqrt{33}-1}{4}$$

Thus, the roots of the given quadratic equation are $x=\frac{\sqrt{33}-1}{4}$ or $x=\frac{-\sqrt{33}-1}{4}$

5. $2x^2 + x + 4 = 0$

Solution:

Given equation,

$2x^2 + x + 4 = 0$

$x^2 + x/2 + 2 = 0$

$$x^2+2\times\frac{1}{2}\times\frac{1}{2}\times x+2=0$$

$$x^2+2\times\frac{1}{4}\times x+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2+2=0$$

$$x^2+2\times\frac{1}{4}\times x+\left(\frac{1}{4}\right)^2=\left(\frac{1}{4}\right)^2-2$$

$$\left(x+\frac{1}{4}\right)^2=\frac{1-32}{16}$$

$$\left(x+\frac{1}{4}\right)^2=\frac{-31}{16}$$

$$\left(x+\frac{1}{4}\right)=\pm\sqrt{-\frac{31}{16}}$$

$$\left(x+\frac{1}{4}\right)=\frac{\sqrt{-31}}{4} \text{ or } \left(x+\frac{1}{4}\right)=\frac{-\sqrt{-31}}{4}$$

$$x=\frac{\sqrt{-31}-1}{4} \text{ or } x=\frac{-\sqrt{-31}-1}{4}$$

Thus, the above are the two roots of the given quadratic equation.

Exercise 8.5

1. Write the discriminant of the following quadratic equations:

(i) $2x^2 - 5x + 3 = 0$

Solution:

Given equation,
$2x^2 - 5x + 3 = 0$
It is in the form of $ax^2 + bx + c = 0$
Where, a = 2, b = -5 and c = 3
So, the discriminant is given by $D = b^2 - 4ac$
$D = (-5)^2 - 4 \times 2 \times 3$
$D = 25 - 24 = 1$
Hence, the discriminant of the given quadratic equation is 1.

(ii) $x^2 + 2x + 4 = 0$

Solution:

Given equation,
$x^2 + 2x + 4 = 0$
It is in the form of $ax^2 + bx + c = 0$
Where, a = 1, b = 2 and c = 4
So, the discriminant is given by $D = b^2 - 4ac$
$D = (2)^2 - 4 \times 1 \times 4$
$D = 4 - 16 = -12$
Hence, the discriminant of the given quadratic equation is - 12.

(iii) $(x - 1)(2x - 1) = 0$

Solution:

Given equation,
$(x - 1)(2x - 1) = 0$
On expanding it, we get
$2x^2 - 3x + 1 = 0$
It is in the form of $ax^2 + bx + c = 0$
Where, a = 2, b = -3, c = 1
So, the discriminant is given by $D = b^2 - 4ac$
$D = (-3)^2 - 4 \times 2 \times 1$
$D = 9 - 8 = 1$
Hence, the discriminant of the given quadratic equation is 1.

(iv) $x^2 - 2x + k = 0, k \in R$

Solution:

Given equation,
$x^2 - 2x + k = 0$

It is in the form of $ax^2 + bx + c = 0$
Where, a = 1, b = -2, and c = k
So, the discriminant is given by $D = b^2 - 4ac$
$D = (-2)^2 - 4(1)(k)$
$= 4 - 4k$
Hence, the discriminant of the given equation is (4 - 4k).

(v) $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$

Solution:

Given equation,
$\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$
It is in the form of $ax^2 + bx + c = 0$
Here $a = \sqrt{3}, b = 2\sqrt{2}x$ and $c = -2\sqrt{3}$
So, the discriminant is given by $D = b^2 - 4ac$
$= (2\sqrt{2})^2 - (4 \times \sqrt{3} \times -2\sqrt{3})$
D = 8 + 24 = 32
Thus, the discriminant of the given equation is 32.

(vi) $x^2 - x + 1 = 0$
Solution:

Given equation,
$x^2 - x + 1 = 0$ It is in the form of $ax^2 + bx + c = 0$
Where, a = 1, b = -1 and c = 1
So, the discriminant is given by $D = b^2 - 4ac$
$D = (-1)^2 - 4 \times 1 \times 1$
D = 1 - 4 = - 3
Thus, the discriminant of the given equation is -3.

Exercise 8.6 Page No: 8.41

1. Determine the nature of the roots of the following quadratic equations:

Important Notes:

- A quadratic equation is in the form $ax^2 + bx + c = 0$
- To find the nature of roots, first find determinant "D"
- $D = b^2 - 4ac$
- If $D > 0$, equation has real and distinct roots
- If $D < 0$, equation has no real roots
- If $D = 0$, equation has 1 root

(i) $2x^2 - 3x + 5 = 0$

Solution:

Here, a= 2, b= -3, c= 5

$D = b^2 - 4ac$

$= (-3)^2 - 4(2)(5)$

$= 9 - 40$

$= -31 < 0$

It's seen that D<0 and hence, the given equation does not have any real roots.

(ii) $2x^2 - 6x + 3 = 0$

Solution:

Here, a= 2, b= -6, c= 3

$D = (-6)^2 - 4(2)(3)$

$= 36 - 24$

$= 12 > 0$

It's seen that D>0 and hence, the given equation have real and distinct roots.

(iii) $(3/5)x^2 - (2/3) + 1 = 0$

Solution:

Here, a= 3/5, b= -2/3, c= 1

$D = (-2/3)^2 - 4(3/5)(1)$

$= 4/9 - 12/5$

$= -88/45 < 0$

It's seen that D<0 and hence, the given equation does not have any real roots.

(iv) $3x^2 - 4\sqrt{3}x + 4 = 0$

Solution:

Here, a= 3, b= $-4\sqrt{3}$, c= 4

$D = (-4\sqrt{3})^2 - 4(3)(4)$

$= 48 - 48$

= 0
It's seen that D = 0 and hence, the given equation has only 1 real and equal root.

(v) $3x^2 - 2\sqrt{6}x + 2 = 0$
Solution:

Here, a= 3, b= $-2\sqrt{6}$, c= 2
$D = (-2\sqrt{6})^2 - 4(3)(2)$
$= 24 – 24$
$= 0$
It's seen that D = 0 and hence, the given equation has only 1 real and equal root.

2. Find the values of k for which the roots are real and equal in each of the following equations:
(i) $kx^2 + 4x + 1 = 0$
Solution:

The given equation $kx^2 + 4x + 1 = 0$ is in the form of $ax^2 + bx + c = 0$
Where a = k, b = 4, c = 1
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
⇒ $4^2 - 4(k)(1) = 0$
⇒ $16 – 4k = 0$
⇒ $k = 4$
The value of k is 4.

(ii) $kx^2 - 2\sqrt{5}x + 4 = 0$
Solution:

The given equation $kx^2 - 2\sqrt{5}x + 4 = 0$ is in the form of $ax^2 + bx + c = 0$
Where a = k, b = $-2\sqrt{5}$, c = 4
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
⇒ $(-2\sqrt{5})^2 - 4(k)(4) = 0$
⇒ $20 – 16k = 0$
⇒ $k = 5/4$
The value of k is 5/4.

(iii) $3x^2 - 5x + 2k = 0$
Solution:

The given equation $3x^2 – 5x + 2k = 0$ is in the form of $ax^2 + bx + c = 0$
Where a = 3, b = -5, c = 2k
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
⇒ $(-5)^2 - 4(3)(2k) = 0$

⇒ $25 - 24k = 0$
⇒ $k = 25/24$
The value of k is 25/24.

(iv) $4x^2 + kx + 9 = 0$
Solution:

The given equation $4x^2 + kx + 9 = 0$ is in the form of $ax^2 + bx + c = 0$
Where a = 4, b = k, c = 9
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
⇒ $k^2 - 4(4)(9) = 0$
⇒ $k^2 - 144 = 0$
⇒ $k = \pm 12$
The value of k is 12 or -12.

(v) $2kx^2 - 40x + 25 = 0$
Solution:

The given equation $2kx^2 - 40x + 25 = 0$ is in the form of $ax^2 + bx + c = 0$
Where a = 2k, b = -40, c = 25
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
⇒ $(-40)^2 - 4(2k)(25) = 0$
⇒ $1600 - 200k = 0$
⇒ $k = 8$
The value of k is 8.

(vi) $9x^2 - 24x + k = 0$
Solution:

The given equation $9x^2 - 24x + k = 0$ is in the form of $ax^2 + bx + c = 0$
Where a = 9, b = -24, c = k
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
⇒ $(-24)^2 - 4(9)(k) = 0$
⇒ $576 - 36k = 0$
⇒ $k = 16$
The value of k is 16.

(vii) $4x^2 - 3kx + 1 = 0$
Solution:

The given equation $4x^2 - 3kx + 1 = 0$ is in the form of $ax^2 + bx + c = 0$
Where a = 4, b = -3k, c = 1
For the equation to have real and equal roots, the condition is

$D = b^2 - 4ac = 0$

⇒ $(-3k)^2 - 4(4)(1) = 0$

⇒ $9k^2 – 16 = 0$

⇒ $k = \pm 4/3$

The value of k is ± 4/3.

(viii) $x^2 - 2(5 + 2k)x + 3(7 + 10k) = 0$

Solution:

The given equation $x^2 – 2(5 + 2k)x + 3(7 + 10k) = 0$ is in the form of $ax^2 + bx + c = 0$

Where a = 1, b = -2(5 + 2k), c = 3(7 + 10k)

For the equation to have real and equal roots, the condition is

$D = b^2 - 4ac = 0$

⇒ $(-2(5 + 2k))^2 - 4(1)(3(7 + 10k)) = 0$

⇒ $4(5 + 2k)^2 - 12(7 + 10k) = 0$

⇒ $25 + 4k^2 + 20k - 21 - 30k = 0$

⇒ $4k^2 - 10k + 4 = 0$

⇒ $2k^2 - 5k + 2 = 0$ [dividing by 2]

Now, solving for k by factorization we have

⇒ $2k^2 - 4k - k + 2 = 0$

⇒ $2k(k - 2) - 1(k - 2) = 0$

⇒ $(k - 2)(2k - 1) = 0$,

k = 2 and k = 1/2,

So, the value of k can either be 2 or 1/2

(ix) $(3k +1)x^2+ 2(k +1)x + k = 0$

Solution:

The given equation $(3k +1)x^2 + 2(k +1)x + k = 0$ is in the form of $ax^2 + bx + c = 0$

Where a = (3k +1), b = 2(k + 1), c = k

For the equation to have real and equal roots, the condition is

$D = b^2 - 4ac = 0$

⇒ $(2(k + 1))^2 - 4(3k +1)(k) = 0$

⇒ $4(k +1)^2 - 4(3k^2 + k) = 0$

⇒ $(k + 1)^2 - k(3k + 1) = 0$

⇒ $2k^2 – k – 1 = 0$

Now, solving for k by factorization we have

⇒ $2k^2 - 2k + k – 1 = 0$

⇒ $2k(k - 1) + 1(k - 1) = 0$

⇒ $(k - 1)(2k + 1) = 0$,

k = 1 and k = -1/2,

So, the value of k can either be 1 or -1/2

(x) $kx^2 + kx + 1 = – 4x^2 – x$

Solution:

The given equation $kx^2 + kx + 1 = -4x^2 - x$
This can be rewritten as,
$(k + 4)x^2 + (k + 1)x + 1 = 0$
Now, this in the form of $ax^2 + bx + c = 0$
Where $a = (k + 4)$, $b = (k + 1)$, $c = 1$
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
$\Rightarrow (k + 1)^2 - 4(k + 4)(1) = 0$
$\Rightarrow (k + 1)^2 - 4k - 16 = 0$
$\Rightarrow k^2 + 2k + 1 - 4k - 16 = 0$
$\Rightarrow k^2 - 2k - 15 = 0$
Now, solving for k by factorization we have
$\Rightarrow k^2 - 5k + 3k - 15 = 0$
$\Rightarrow k(k - 5) + 3(k - 5) = 0$
$\Rightarrow (k + 3)(k - 5) = 0$,
$k = -3$ and $k = 5$,
So, the value of k can either be -3 or 5.

(xi) $(k + 1)x^2 + 2(k + 3)x + (k + 8) = 0$
Solution:

The given equation $(k + 1)x^2 + 2(k + 3)x + (k + 8) = 0$ is in the form of $ax^2 + bx + c = 0$
Where $a = (k + 1)$, $b = 2(k + 3)$, $c = (k + 8)$
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
$\Rightarrow (2(k + 3))^2 - 4(k + 1)(k + 8) = 0$
$\Rightarrow 4(k + 3)^2 - 4(k^2 + 9k + 8) = 0$
$\Rightarrow (k + 3)^2 - (k^2 + 9k + 8) = 0$
$\Rightarrow k^2 + 6k + 9 - k^2 - 9k - 8 = 0$
$\Rightarrow -3k + 1 = 0$
$\Rightarrow k = 1/3$
So, the value of k is 1/3.

(xii) $x^2 - 2kx + 7k - 12 = 0$
Solution:

The given equation $x^2 - 2kx + 7k - 12 = 0$ is in the form of $ax^2 + bx + c = 0$
Where $a = 1$, $b = -2k$, $c = 7k - 12$
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
$\Rightarrow (-2k)^2 - 4(1)(7k - 12) = 0$
$\Rightarrow 4k^2 - 4(7k - 12) = 0$
$\Rightarrow k^2 - 7k + 12 = 0$
Now, solving for k by factorization we have
$\Rightarrow k^2 - 4k - 3k + 12 = 0$

$\Rightarrow$ (k - 4)(k - 3) = 0,
k = 4 and k = 3,
So, the value of k can either be 4 or 3.

(xiii) $(k + 1)x^2 - 2(3k + 1)x + 8k + 1 = 0$
Solution:

The given equation $(k + 1)x^2 - 2(3k + 1)x + 8k + 1 = 0$ is in the form of $ax^2 + bx + c = 0$
Where a = (k +1), b = -2(3k + 1), c = 8k + 1
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
$\Rightarrow (-2(3k + 1))^2 - 4(k + 1)(8k + 1) = 0$
$\Rightarrow 4(3k + 1)^2 - 4(k + 1)(8k + 1) = 0$
$\Rightarrow (3k + 1)^2 - (k + 1)(8k + 1) = 0$
$\Rightarrow 9k^2 + 6k + 1 – (8k^2 + 9k + 1) = 0$
$\Rightarrow 9k^2 + 6k + 1 – 8k^2 - 9k - 1 = 0$
$\Rightarrow k^2 - 3k = 0$
$\Rightarrow k(k - 3) = 0$
Either k = 0 Or, k - 3 = 0 $\Rightarrow$ k = 3,
So, the value of k can either be 0 or 3

(xiv) $5x^2 - 4x + 2 + k(4x^2 - 2x + 1) = 0$
Solution:

The given equation $5x^2 - 4x + 2 + k(4x^2 - 2x + 1) = 0$
This can be rewritten as,
$x^2(5 + 4k) - x(4 + 2k) + 2 - k = 0$
Now, this in the form of $ax^2 + bx + c = 0$
Where a = (4k +5), b = -(2k + 4), c = 2 - k
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
$\Rightarrow (-(2k + 4))^2 - 4(4k + 5)(2 - k) = 0$
$\Rightarrow (2k + 4)^2 - 4(4k + 5)(2 - k) = 0$
$\Rightarrow 16 + 4k^2 + 16k - 4(10 - 5k + 8k - 4k^2) = 0$
$\Rightarrow 16 + 4k^2 + 16k - 40 + 20k - 32k + 16k^2 = 0$
$\Rightarrow 20k^2 + 4k - 24 = 0$
$\Rightarrow 5k^2 + k – 6 = 0$
Now, solving for k by factorization we have
$\Rightarrow 5k^2 + 6k - 5k – 6 = 0$
$\Rightarrow$ 5k(k - 1) + 6(k - 1) = 0
$\Rightarrow$ (k - 1)(5k + 6) = 0,
k = 1 and k = -6/5,
So, the value of k can either be -3 or 5.

(xv) $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$
Solution:

The given equation $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$ is in the form of $ax^2 + bx + c = 0$
Where $a = (4 - k)$, $b = (2k + 4)$, $c = (8k + 1)$
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
$\Rightarrow (2k + 4)^2 - 4(4 - k)(8k + 1) = 0$
$\Rightarrow 4k^2 + 16k + 16 - 4(-8k^2 + 32k + 4 - k) = 0$
$\Rightarrow 4k^2 + 16k + 16 + 32k^2 - 124k - 16 = 0$
$\Rightarrow 36k^2 - 108k = 0$
Taking common,
$\Rightarrow 9k(k - 3) = 0$
Now, either $9k = 0 \Rightarrow k = 0$ or $k – 3 = 0 \Rightarrow k = 3$,
So, the value of k can either be 0 or 3.

(xvi) $(2k + 1)x^2 + 2(k + 3)x + (k + 5) = 0$
Solution:

The given equation $(2k + 1)x^2 + 2(k + 3)x + (k + 5) = 0$ is in the form of $ax^2 + bx + c = 0$
Where $a = (2k + 1)$, $b = 2(k + 3)$, $c = (k + 5)$
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
$\Rightarrow (2(k + 3))^2 - 4(2k + 1)(k + 5) = 0$
$\Rightarrow 4(k + 3)^2 - 4(2k^2 + 11k + 5) = 0$
$\Rightarrow (k + 3)^2 - (2k^2 + 11k + 5) = 0$ [dividing by 4 both sides]
$\Rightarrow k^2 + 5k - 4 = 0$
Now, solving for k by completing the square we have
$\Rightarrow k^2 + 2 \times (5/2) \times k + (5/2)^2 = 4 + (5/2)^2$
$\Rightarrow (k + 5/2)^2 = 4 + 25/4 = \sqrt{41}/4$
$\Rightarrow k + (5/2) = \pm \sqrt{41}/2$
$\Rightarrow k = (\sqrt{41} – 5)/2$ or $–(\sqrt{41} + 5)/2$
So, the value of k can either be $(\sqrt{41} – 5)/2$ or $–(\sqrt{41} + 5)/2$

(xvii) $4x^2 - 2(k + 1)x + (k + 4) = 0$
Solution:

The given equation $4x^2 - 2(k + 1)x + (k + 4) = 0$ is in the form of $ax^2 + bx + c = 0$
Where $a = 4$, $b = -2(k + 1)$, $c = (k + 4)$
For the equation to have real and equal roots, the condition is
$D = b^2 - 4ac = 0$
$\Rightarrow (-2(k + 1))^2 - 4(4)(k + 4) = 0$
$\Rightarrow 4(k + 1)^2 - 16(k + 4) = 0$
$\Rightarrow (k + 1)^2 - 4(k + 4) = 0$
$\Rightarrow k^2 – 2k – 15 = 0$
Now, solving for k by factorization we have
$\Rightarrow k^2 - 5k + 3k – 15 = 0$

⇒ $k(k - 5) + 3(k - 5) = 0$
⇒ $(k - 5)(k + 3) = 0$,
$k = 5$ and $k = -3$,
So, the value of k can either be 5 or -3.

3. In the following, determine the set of values of k for which the given quadratic equation has real roots:
(i) $2x^2 + 3x + k = 0$
Solution:

Given,
$2x^2 + 3x + k = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, $a = 2$, $b = 3$, $c = k$
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = 9 - 4(2)(k) \geq 0$
⇒ $9 - 8k \geq 0$
⇒ $k \leq 9/8$
The value of k should not exceed 9/8 to have real roots.

(ii) $2x^2 + x + k = 0$
Solution:

Given,
$2x^2 + x + k = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, $a = 2$, $b = 1$, $c = k$
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = 1^2 - 4(2)(k) \geq 0$
⇒ $1 - 8k \geq 0$
⇒ $k \leq 1/8$
The value of k should not exceed 1/8 to have real roots.

(iii) $2x^2 - 5x - k = 0$
Solution:

Given,
$2x^2 - 5x - k = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, $a = 2$, $b = -5$, $c = -k$
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = (-5)^2 - 4(2)(-k) \geq 0$
⇒ $25 + 8k \geq 0$
⇒ $k \geq -25/8$
The value of k should be lesser than -25/8 to have real roots.

(iv) $kx^2 + 6x + 1 = 0$
Solution:

Given,
$2x^2 + x + k = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, a = 2, b = 1, c = k
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = 1^2 - 4(2)(k) \geq 0$
$\Rightarrow 1 - 8k \geq 0$
$\Rightarrow k \leq 1/8$
The value of k should not exceed 1/8 to have real roots.

(v) $3x^2 + 2x + k = 0$
Solution:

Given,
$3x^2 + 2x + k = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, a =3, b = 2, c = k
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = (2)^2 - 4(3)(k) \geq 0$
$\Rightarrow 4 - 12k \geq 0$
$\Rightarrow 4 \geq 12k$
$\Rightarrow k \leq 1/3$
The value of k should not exceed 1/3 to have real roots.

4. Find the values of k for which the following equations have real and equal roots
(i) $x^2 - 2(k + 1)x + k^2 = 0$
Solution:

Given,
$x^2 - 2(k + 1)x + k^2 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, a =1, b = -2(k + 1), c = k^2
For the given quadratic equation to have real roots $D = b^2 - 4ac = 0$
$D = (-2(k + 1))^2 - 4(1)(k^2) = 0$
$\Rightarrow 4k^2 + 8k + 4 - 4k^2 = 0$
$\Rightarrow 8k + 4 = 0$
$\Rightarrow k = -4/8$
$\Rightarrow k = -1/2$
The value of k should -1/2 to have real and equal roots.

(ii) $k^2x^2 - 2(2k - 1)x + 4 = 0$
Solution:

Given,
$k^2x^2 – 2 (2k - 1)x + 4 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, $a = k^2$, $b = –2 (2k - 1)$, $c = 4$
For the given quadratic equation to have real roots $D = b^2 - 4ac = 0$
$D = (-2(2k - 1))^2 - 4(4)(k^2) = 0$
$\Rightarrow 4k^2 - 4k + 1 - 4k^2 = 0$ [dividing by 4 both sides]
$\Rightarrow -4k + 1 = 0$
$\Rightarrow k = 1/4$
The value of k should 1/4 to have real and equal roots.

(iii) $(k + 1)x^2 – 2(k - 1)x + 1 = 0$
Solution:

Given,
$(k + 1)x^2 – 2(k - 1)x + 1 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, $a = (k + 1)$, $b = -2(k - 1)$, $c = 1$
For the given quadratic equation to have real roots $D = b^2 - 4ac = 0$
$D = (-2(k - 1))^2 - 4(1)(k + 1) = 0$
$\Rightarrow 4k^2 – 2k + 1 – k - 1 = 0$ [dividing by 4 both sides]
$\Rightarrow k^2 – 3k = 0$
$\Rightarrow k(k - 3) = 0$
$\Rightarrow k = 0$ or $k = 3$
The value of k can be 0 or 3 to have real and equal roots.

5. Find the values of k for which the following equations have real roots
(i) $2x^2 + kx + 3 = 0$
Solution:

Given,
$2x^2 + kx + 3 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, $a = 2$, $b = k$, $c = 3$
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = (k)^2 - 4(3)(2) \geq 0$
$\Rightarrow k^2 - 24 \geq 0$
$\Rightarrow k^2 \geq 24$
$\Rightarrow k \geq 2\sqrt{6}$ and $k \leq -2\sqrt{6}$ [After taking square root on both sides]
The value of k can be represented as $(\infty, 2\sqrt{6}]$ U $[-2\sqrt{6}, -\infty)$

(ii) $kx(x - 2) + 6 = 0$
Solution:

Given,
$kx(x - 2) + 6 = 0$

It can be rewritten as,
$kx^2 - 2kx + 6 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, a =k, b = -2k, c = 6
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = (-2k)^2 - 4(k)(6) \geq 0$
$\Rightarrow 4k^2 - 24k \geq 0$
$\Rightarrow 4k(k - 6) \geq 0$
$\Rightarrow k \geq 0$ and $k \geq 6$
$\Rightarrow k \geq 6$
The value of k should be greater than or equal to 6 to have real roots.

(iii) $x^2 - 4kx + k = 0$
Solution:

Given,
$x^2 - 4kx + k = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, a =1, b = -4k, c = k
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = (-4k)^2 - 4(1)(k) \geq 0$
$\Rightarrow 16k^2 - 4k \geq 0$
$\Rightarrow 4k(4k - 1) \geq 0$
$\Rightarrow k \geq 0$ and $k \geq 1/4$
$\Rightarrow k \geq 1/4$
The value of k should be greater than or equal to 1/4 to have real roots.

(iv) $kx(x - 2\sqrt{5}) + 10 = 0$
Solution:

Given,
$kx(x - 2\sqrt{5}) + 10 = 0$
It can be rewritten as,
$kx^2 - 2\sqrt{5}kx + 10 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, a =k, b = $-2\sqrt{5}k$, c = 10
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = (-2\sqrt{5}k)^2 - 4(k)(10) \geq 0$
$\Rightarrow 20k^2 - 40k \geq 0$
$\Rightarrow 20k(k - 2) \geq 0$
$\Rightarrow k \geq 0$ and $k \geq 2$
$\Rightarrow k \geq 2$
The value of k should be greater than or equal to 2 to have real roots.

(v) $kx(x - 3) + 9 = 0$
Solution:

Given,
$kx(x - 3) + 9 = 0$
It can be rewritten as,
$kx^2 – 3kx + 9 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, $a = k$, $b = -3k$, $c = 9$
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = (-3k)^2 - 4(k)(9) \geq 0$
$\Rightarrow 9k^2 - 36k \geq 0$
$\Rightarrow 9k(k - 4) \geq 0$
$\Rightarrow k \geq 0$ and $k \geq 4$
$\Rightarrow k \geq 4$
The value of k should be greater than or equal to 4 to have real roots.

(vi) $4x^2 + kx + 3 = 0$
Solution:

Given,
$4x^2 + kx + 3 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, $a = 4$, $b = k$, $c = 3$
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = (k)^2 - 4(4)(3) \geq 0$
$\Rightarrow k^2 - 48 \geq 0$
$\Rightarrow k^2 \geq 48$
$\Rightarrow k \geq 4\sqrt{3}$ and $k \leq -4\sqrt{3}$ [After taking square root on both sides]
The value of k can be represented as $(\infty, 4\sqrt{3}] \cup [-4\sqrt{3}, -\infty)$

6. Find the values of k for which the given quadratic equation has real and distinct roots.
(i) $kx^2 + 2x + 1 = 0$
Solution:

Given,
$kx^2 + 2x + 1 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, $a = k$, $b = 2$, $c = 1$
For the given quadratic equation to have real roots $D = b^2 - 4ac > 0$
$D = (2)^2 - 4(1)(k) > 0$
$\Rightarrow 4 - 4k > 0$
$\Rightarrow 4k < 4$
$\Rightarrow k < 1$
The value of k should be lesser than 1 to have real and distinct roots.

(ii) $kx^2 + 6x + 1 = 0$

Solution:

Given,
$kx^2 + 6x + 1 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, a =k, b = 6, c = 1
For the given quadratic equation to have real roots $D = b^2 - 4ac > 0$
$D = (6)^2 - 4(1)(k) > 0$
$\Rightarrow 36 - 4k > 0$
$\Rightarrow 4k < 36$
$\Rightarrow k < 9$
The value of k should be lesser than 9 to have real and distinct roots.

7. For what value of k, $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$, is a perfect square.
Solution:

Given,
$(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$
It is in the form of $ax^2 + bx + c = 0$
Where, a = 4 - k, b = 2k + 4, c = 8k + 1
Calculating the discriminant, $D = b^2 - 4ac$
$= (2k + 4)^2 - 4(4 - k)(8k + 1)$
$= 4k^2 + 16 + 4k - 4(32 + 4 - 8k^2 - k)$
$= 4(k^2 + 4 + k - 32 - 4 + 8k^2 + k)$
$= 4(9k^2 - 27k)$
As the given equation is a perfect square, then D = 0
$\Rightarrow \quad 4(9k^2 - 27k) = 0$
$\Rightarrow \quad (9k^2 - 27k) = 0$
$\Rightarrow \quad 3k(k - 3) = 0$
Thus, 3k = 0 $\Rightarrow$ k = 0 Or, k-3 = 0 $\Rightarrow$k = 3
Hence, the value of k should be 0 or 3 for the given to be perfect square.

8. Find the least positive value of k for which the equation $x^2 + kx + 4 = 0$ has real roots.
Solution:

Given,
$x^2 + kx + 4 = 0$
It's of the form of $ax^2 + bx + c = 0$
Where, a =1, b = k, c = 4
For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$
$D = (k)^2 - 4(1)(4) \geq 0$
$\Rightarrow k^2 - 16 \geq 0$
$\Rightarrow k \geq 4$ and $k \leq -4$
Considering the least positive value, we have
$\Rightarrow k = 4$
Thus, the least value of k is 4 for the given equation to have real roots.

9. Find the values of k for which the quadratic equation $(3k + 1)x^2 + 2(k + 1)x + 1 = 0$ has equal roots. Also, find the roots.
Solution:

The given equation $(3k +1)x^2 + 2(k +1)x + 1 = 0$ is in the form of $ax^2 + bx + c = 0$
Where $a = (3k +1)$, $b = 2(k + 1)$, $c = 1$
For the equation to have real and equal roots, the condition is

$D = b^2 - 4ac = 0$

$\Rightarrow \quad (2(k + 1))^2 - 4(3k +1)(1) = 0$

$\Rightarrow \quad (k +1)^2 - (3k + 1) = 0$ [After dividing by 4 both sides]

$\Rightarrow \quad k^2 + 2k + 1 - 3k - 1 = 0$

$\Rightarrow \quad k^2 - k = 0$

$\Rightarrow \quad k(k - 1) = 0$

Either $k = 0$ Or, $k - 3 = 0 \Rightarrow k = 1$,
So, the value of k can either be 0 or 1

Now, using k = 0 in the given quadratic equation we get

$(3(0) + 1)x^2 + 2(0 + 1)x + 1 = 0$

$x^2 + 2x + 1 = 0$

$\Rightarrow (x + 1)^2 = 0$

Thus, x = -1 is the root of the given quadratic equation.

Next, on using k = 1 in the given quadratic equation we get

$(3(1) + 1)x^2 + 2(1 + 1)x + 1 = 0$

$4x^2 + 4x + 1 = 0$

$\Rightarrow (2x + 1)^2 = 0$

Thus, $2x = -1 \Rightarrow x = -1/2$ is the root of the given quadratic equation.

10. Find the values of p for which the quadratic equation $(2p + 1)x^2 – (7p + 2)x + (7p - 3) = 0$ has equal roots. Also, find the roots.
Solution:

The given equation $(2p + 1)x^2 – (7p + 2)x + (7p - 3) = 0$ is in the form of $ax^2 + bx + c = 0$
Where $a = (2p +1)$, $b = -(7p + 2)$, $c = (7p - 3)$
For the equation to have real and equal roots, the condition is

$D = b^2 - 4ac = 0$

$\Rightarrow \quad (-(7p + 2))^2 - 4(2p +1)(7p - 3) = 0$

$\Rightarrow \quad (7p + 2)^2 - 4(14p^2 + p - 3) = 0$

$\Rightarrow \quad 49p^2 + 28p + 4 – 56p^2 – 4p + 12 = 0$

$\Rightarrow \quad -7p^2 + 24p + 16 = 0$

Solving for p by factorization,

$\Rightarrow \quad -7p^2 + 28p - 4p + 16 = 0$

$\Rightarrow \quad -7p(p - 4) -4(p - 4) = 0$

$\Rightarrow \quad (p - 4) (-7p - 4) = 0$

Either $p - 4 = 0 \Rightarrow p = 4$ Or, $7p + 4 = 0 \Rightarrow p = -4/7$,
So, the value of k can either be 4 or -4/7

Now, using k = 4 in the given quadratic equation we get

$(2(4) + 1)x^2 – (7(4) + 2)x + (7(4) - 3) = 0$

$9x^2 - 30x + 25 = 0$

$\Rightarrow (3x - 5)^2 = 0$

Thus, x = 5/3 is the root of the given quadratic equation.

Next, on using k = 1 in the given quadratic equation we get

$(2(-4/7) + 1)x^2 – (7(-4/7) + 2)x + (7(-4/7) - 3) = 0$

$x^2 - 14x + 49 = 0$

$\Rightarrow (x - 7)^2 = 0$

Thus, x – 7 = 0 $\Rightarrow$ x = 7 is the root of the given quadratic equation.

11. If -5 is a root of the quadratic equation $2x^2 + px -15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.

Solution:

Given,

-5 is as root of $2x^2 + px - 15 = 0$

So, on substituting x = -5 the LHS will become zero and satisfy the equation.

$\Rightarrow$ $2(-5)^2 + p(-5) - 15 = 0$

$\Rightarrow$ $50 – 5p – 15 = 0$

$\Rightarrow$ $35 = 5p$

$\Rightarrow$ $p = 7$

Now, substituting the value of p in the second equation we have

$(7)(x^2 + x) + k = 0$

$\Rightarrow$ $7x^2 + 7x + k = 0$

It's given that the above equation has equal roots.

Thus the discriminant, D = 0

The equation $7x^2 + 7x + k = 0$ is in the form of $ax^2 + bx + c = 0$

Where a = 7, b = 7, c = k

$D = b^2 – 4ac$

$\Rightarrow$ $7^2 – 4(7)(k) = 0$

$\Rightarrow$ $49 – 28k = 0$

$\Rightarrow$ $k = 49/28 = 7/4$

Therefore, the value of k is 7/4.

12. If 2 is a root of the quadratic equation $3x^2 + px -8 = 0$ and the quadratic equation $4x^2 – 2px + k = 0$ has equal roots, find the value of k.

Solution:

Given,

2 is as root of $3x^2 + px -8 = 0$

So, on substituting x = 2 the LHS will become zero and satisfy the equation.

$\Rightarrow$ $3(2)^2 + p(2) - 8 = 0$

⇒ $12 + 2p - 8 = 0$
⇒ $4 + 2p = 0$
⇒ $p = -2$

Now, substituting the value of p in the second equation we have
$4x^2 - 2(-2)x + k = 0$
⇒ $4x^2 + 4x + k = 0$
It's given that the above equation has equal roots.
Thus the discriminant, D = 0
The equation $4x^2 + 4x + k = 0$ is in the form of $ax^2 + bx + c = 0$
Where a = 4, b = 4, c = k
$D = b^2 - 4ac$
⇒ $4^2 - 4(4)(k) = 0$
⇒ $16 - 16k = 0$ [dividing by 16 both sides]
⇒ $k = 1$
Therefore, the value of k is 1.

Exercise 8.7

1. Find two consecutive numbers whose squares have the sum of 85.
Solution:

Let the two consecutive be considered as (x) and (x +1) respectively.
Given that,
The sum of their squares is 85.
Expressing the same by equation we have,
$x^2 + (x + 1)^2 = 85$
⇒ $x^2 + x^2 + 2x + 1 = 85$
⇒ $2x^2 + 2x + 1 - 85 = 0$
⇒ $2x^2 + 2x - 84 = 0$
⇒ $2(x^2 + x - 42) = 0$
Solving for x by factorization method, we get
$x^2 + 7x - 6x - 42 = 0$
⇒ $x(x + 7) - 6(x + 7) = 0$
⇒ $(x - 6)(x + 7) = 0$
Now, either, $x - 6 = 0$ ⇒ $x = 6$
Or, $x + 7 = 0$ ⇒ $x = -7$
Thus, the consecutive numbers whose sum of squares can be (6, 7) or (-7, -6).

2. Divide 29 into two parts so that the sum of the squares of the parts is 425.
Solution:

Let's assume that one part is (x), so the other part will be (29 - x).
From the question, the sum of the squares of these two parts is 425.
Expressing the same by equation we have,
$x^2 + (29 - x)^2 = 425$
⇒ $x^2 + x^2 + 841 + -58x = 425$
⇒ $2x^2 - 58x + 841 - 425 = 0$
⇒ $2x^2 - 58x + 416 = 0$
⇒ $x^2 - 29x + 208 = 0$
Solving for x by factorization method, we get
$x^2 - 13x - 16x + 208 = 0$
⇒ $x(x - 13) - 16(x - 13) = 0$
⇒ $(x - 13)(x - 16) = 0$
Now, either $x - 13 = 0$ ⇒ $x = 13$
Or, $x - 16 = 0$ ⇒ $x = 16$
Thus, the two parts whose sum of the squares is 425 are 13 and 16 respectively.

3. Two squares have sides x cm and (x + 4) cm. The sum of their areas is 656 cm^2. Find the sides of the squares.
Solution:

Given,

The sides of the two squares are = x cm and (x + 4) cm respectively.
The sum of the areas of these squares = 656 cm^2
We know that,
Area of the square = side * side
So, the area of the squares are x^2 and $(x + 4)^2$.
From the given condition,
$x^2 + (x + 4)^2 = 656$
⇒ $x^2 + x^2 + 8x + 16 = 656$
⇒ $2x^2 + 8x – 640 = 0$
⇒ $x^2 + 4x – 320 = 0$ [dividing by 2 both sides]
Solving for x by factorization method,
$x^2 + 20x – 16x – 320 = 0$
⇒ $x(x + 20) – 16(x + 20) = 0$
⇒ $(x + 20)(x –16) = 0$
Now, either x + 20 = 0 ⇒ x = – 20
Or, x - 16 = 0 ⇒ x = 16
No negative value is considered as the value of the side of the square can never be negative.
Thus, the side of the square is 16.
And, x + 4 = 16 + 4 = 20 cm
Therefore, the side of the other square is 20 cm.

4. The sum of two numbers is 48 and their product is 432. Find the numbers.
Solution:

Given that the sum of two numbers is 48.
So, assuming one number to be x, then the other number will be 48 - x
Also given that their product is 432.
Which means, x(48 - x) = 432
⇒ $48x - x^2 = 432$
⇒ $x^2 - 48x + 432 = 0$
Solving for x by factorization method,
⇒ $x^2 - 36x - 12x + 432 = 0$
⇒ $x(x - 36) - 12(x - 36) = 0$
⇒ $(x - 36)(x - 12) = 0$
Now, either x - 36 = 0 ⇒ x = 36
Or, x - 12 = 0 ⇒ x = 12
Therefore, the two numbers are 12 and 36 respectively.

5. If an integer is added to its square, the sum is 90. Find the integer with the help of quadratic equation.
Solution:

Assume the integer be x. Then its square will be x^2.
And given, their sum is 90
⇒ $x + x^2 = 90$
⇒ $x^2 + x - 90 = 0$

Solving for x by factorization method, we have

$x^2 + 10x - 9x - 90 = 0$

$\Rightarrow$ $x(x + 10) - 9(x + 10) = 0$

$\Rightarrow$ $(x + 10)(x - 9) = 0$

Now, either $x + 10 = 0$ $\Rightarrow x = -10$

Or, $x - 9 = 0$ $\Rightarrow x = 9$

Thus, the values of the integer are 9 and -10 respectively.

6. Find the whole number which when decreased by 20 is equal to 69 times the reciprocal of the number.

Solution:

Let the whole number be x.

When it is decreased by 20 $\Rightarrow (x - 20)$

And, the reciprocal of the whole number is 1/x

From the given condition, we have

$(x - 20) = 69 \times (1/x)$

$\Rightarrow$ $x(x - 20) = 69$

$\Rightarrow$ $x^2 - 20x - 69 = 0$

Solving for x by factorization method, we have

$\Rightarrow$ $x^2 - 23x + 3x - 69 = 0$

$\Rightarrow$ $x(x - 23) + 3(x - 23) = 0$

$\Rightarrow$ $(x - 23)(x + 3) = 0$

Thus, x is either 23 Or -3

As we that a whole number is always positive, $x = -3$ is not considered. Therefore, the whole number is 23.

7. Find two consecutive natural numbers whose product is 20.

Solution:

Let the two consecutive natural numbers be x and x + 1 respectively.

Given that their product is 20.

Which means, $x(x + 1) = 20$

$\Rightarrow$ $x^2 + x - 20 = 0$

Solving for x by factorization method, we have

$\Rightarrow$ $x^2 + 5x - 4x - 20 = 0$

$\Rightarrow$ $x(x + 5) - 4(x + 5) = 0$

$\Rightarrow$ $(x + 5)(x - 4) = 0$

Now, either $x + 5 = 0 \Rightarrow x = -5$

Or, $x - 4 = 0 \Rightarrow x = 4$

Considering only the positive value of x since it a natural number. i.e, x = 4

Thus, the two consecutive natural numbers are 4 and 5 respectively.

8. The sum of the squares of two consecutive odd positive integers is 394. Find them.

Solution:

Let's assume the consecutive odd positive integer to be 2x - 1 and 2x + 1 respectively. [Keeping the common difference as 2]
Now, it's given that the sum of their squares is 394.
Which means,

$(2x - 1)^2 + (2x + 1)^2 = 394$

$4x^2 + 1 - 4x + 4x^2 + 1 + 4x = 394$

By cancelling out the equal and opposite terms, we get

$8x^2 + 2 = 394$

$8x^2 = 392$

$x^2 = 49$

$x = 7$ and -7

Since we need only consecutive odd positive integers, we only consider x = 7.
Now,

$2x - 1 = 14 - 1 = 13$

$2x + 1 = 14 + 1 = 15$

Thus, the two consecutive odd positive numbers are 13 and 15 respectively.

9. The sum of two numbers is 8 and 15 times the sum of the reciprocal is also 8. Find the numbers.
Solution:

Let tone of the number be x so, the other number will be (8 – x) as given their sum is 8.
Also given, 15 times the sum of their reciprocals is 8.
Which means,

$$=15\left(\frac{1}{x}+\frac{1}{8-x}\right)=8$$

$$=15\frac{8-x+x}{x(8-x)}=8$$

$$=15\times\frac{8}{8x-x^2}=8$$

⇒ $120 = 8(8x - x^2)$

⇒ $120 = 64x - 8x^2$

⇒ $8x^2 - 64x + 120 = 0$

⇒ $8(x^2 - 8x + 15) = 0$

⇒ $x^2 - 8x + 15 = 0$

Solving for x by factorization method, we have

⇒ $x^2 - 5x - 3x + 15 = 0$

⇒ $x(x - 5) - 3(x - 5) = 0$

⇒ $(x - 5)(x - 3) = 0$

Now, either $x - 5 = 0$ ⇒ $x = 5$

Or, $x - 3 = 0$ ⇒ $x = 3$

Thus, the two numbers are 5 and 3 respectively.

10. The sum of a number and its positive square root is 6/25. Find the numbers.
Solution:

Let the number be x.
According to the question, we have

$$x + \sqrt{x} = \frac{6}{25}$$

Let us assume that $x = y^2$,
So, we get

$y^2 + y = 6/25$

⇒ $25y^2 + 25y - 6 = 0$

⇒ $25y^2 + 30y – 5y – 6 = 0$

⇒ $5y(5y + 6) – 1(5y + 6) = 0$

⇒ $(5y + 6) (5y – 1) = 0$

Now, either $5y + 6 = 0$ ⇒ $y = -6/5$
Or, $5y – 1 = 0$ ⇒ $y = 1/5$
Since, it's given that only positive square root to be considered we take y = 1/5 only
Thus, $x = (1/5)^2 = 1/25$
Hence, the number is 1/25.

10. The sum of a number and its square is 63/4, Find the numbers.
Solution:

Let the number be x.
So, its square will be x^2.
From the question, it's given that sum of the number and its square is 63/4
Which means,

$x + x^2 = 63/4$

⇒ $4x + 4x^2 = 63$

⇒ $4x^2 + 4x – 63 = 0$

Solving for x by factorization method, we have

⇒ $4x^2 + 18x - 14x – 63 = 0$

⇒ $2x(2x + 9) – 7(2x - 9) = 0$

⇒ $(2x - 7)(2x + 9) = 0$

Now, either $2x -7 = 0$ ⇒ $x = 7/2$
Or, $2x + 9 = 0$ ⇒ $x = -9/2$
Thus, the numbers are 7/2 and -9/2.

12. There are three consecutive integers such that the square of the first increased by the product of the other two gives 154. What are the integers?
Solution:

Let's consider the three consecutive numbers to be x, x + 1, x + 2 respectively. And x being the

first integer of the sequence.
From the question, it's understood that
$x^2 + (x + 1)(x + 2) = 154$
$\Rightarrow \quad x^2 + x^2 + 3x + 2 = 154$
$\Rightarrow \quad 2x^2 + 3x - 152 = 0$
Solving for x by factorization method, we have
$\Rightarrow \quad 2x^2 + 19x – 16x - 152 = 0$
$\Rightarrow \quad x(2x + 19) – 8(2x - 19) = 0$
$\Rightarrow \quad (2x - 19)(x - 8) = 0$
Now, either $2x – 19 = 0 \quad \Rightarrow \quad x = 19/2$ (which is not an integer)
Or, $x – 8 = 0 \Rightarrow \quad x = 8$
Hence, considering x = 8 the three consecutive integers are 8, 9 and 10.

13. The product of two successive integral multiples of 5 is 300. Determine the multiples.
Solution:

Given that the product of two successive integral multiples of 5 is 300
Let's assume the integers be 5x and 5(x+1), where x and x+1 are two consecutive multiples
Then, according to the question, we have
$5x[5(x + 1)] = 300$
$\Rightarrow \quad 25x(x + 1) = 300$
$\Rightarrow \quad x^2 + x = 12$
$\Rightarrow \quad x^2 + x - 12 = 0$
Solving for x by factorization method, we have
$\Rightarrow \quad x^2 + 4x - 3x - 12 = 0$
$\Rightarrow \quad x(x + 4) - 3(x + 4) = 0$
$\Rightarrow \quad (x + 4)(x - 3) = 0$
Now, either $x + 4 = 0 \quad \Rightarrow x = -4$
Or, $x - 3 = 0 \quad \Rightarrow x = 3$
For, $x = – 4$
$5x = – 20$ and $5(x + 1) = -15$
And, for $x = 3$
$5x = 15$ and $5(x + 1) = 20$
Thus, the two successive integral multiples can be 15, 20 or -15 and -20 respectively.

14. The sum of the squares of two numbers is 233 and one of the numbers is 3 less than twice the other number. Find the numbers.
Solution:

Let one of the number be x. Then the other number will be 2x – 3.
From the question:
$x^2 + (2x - 3)^2 = 233$
$\Rightarrow \quad x^2 + 4x^2 + 9 - 12x = 233$
$\Rightarrow \quad 5x^2 - 12x - 224 = 0$
$\Rightarrow \quad 5x^2 - 40x + 28x - 224 = 0$
$\Rightarrow \quad 5x(x – 8) + 28(x – 8) = 0$

⇒ $(5x + 28)(x – 8) = 0$

Now, $5x + 28$ cannot be 0

so, $x - 8 = 0$ ⇒ $x = 8$

Considering the value of $x = 8$, we have

$2x – 3 = 15$

Thus, the two numbers are 8 and 15 respectively.

15. Find the consecutive even integers whose squares have the sum 340.

Solution:

Let's consider the three consecutive even numbers to be 2x and 2x + 2respectively.

From the question, it's given that sum of the squares of these integers is 340.

Which means,

$(2x)^2 + (2x + 2)^2 = 340$

⇒ $4x^2 + 4x^2 + 8x + 4 = 340$

⇒ $8x^2 + 8x – 336 = 0$

⇒ $8(x^2 + x - 42) = 0$

⇒ $x^2 + x – 42 = 0$

Solving for x by factorization method, we have

⇒ $x^2 + 7x – 6x – 42 = 0$

⇒ $(x + 7)(x - 6) = 0$

Thus, x can be either -7 or 6.

If x = -7, the number are -14 (2x(-7)) and -12 (2x(-7) + 2)

Similarly, if x = 6, the numbers are 12 and 14.

Therefore, the consecutive integers are either -14, -12 or 12, 14.

16. The difference of two number is 4. If the difference of their reciprocals is 4/21, find the numbers.

Solution:

Let the two numbers be x and x - 4 respectively.

Since, given that the difference of two numbers is 4.

Now, from the question, we have

$$\frac{1}{x-4} - \frac{1}{x} = \frac{4}{21}$$

$$\frac{x-x+4}{x(x-4)} = \frac{4}{21}$$

⇒ $84 = 4x(x - 4)$

⇒ $x^2 - 4x - 21 = 0$

Solving for x by factorization method, we have

⇒ $x^2 - 7x + 3x - 21 = 0$

⇒ $(x - 7)(x + 3) = 0$

Now, either $x - 7 = 0$ ⇒ $x = 7$

Or, $x + 3 = 0 \Rightarrow \quad x = -3$
Thus, the required numbers are – 3 and 7 respectively.

17. Find two natural numbers which differ by 3 and whose squares have the sum 117.
Solution:

Let the numbers be x and x – 3, as its given the number differ by 3.
From the question, it’s given that sum of squares of these numbers is 117.

$x^2 + (x - 3)^2 = 117$
$\Rightarrow \quad x^2 + x^2 + 9 - 6x - 117 = 0$
$\Rightarrow \quad 2x^2 - 6x - 108 = 0$
$\Rightarrow \quad x^2 - 3x - 54 = 0$

Solving for x by factorization method, we have

$\Rightarrow \quad x^2 - 9x + 6x - 54 = 0$
$\Rightarrow \quad x(x - 9) + 6(x - 9) = 0$
$\Rightarrow \quad (x - 9)(x + 6) = 0$

Now, either $x - 9 = 0 \Rightarrow \quad x = 9$
Or, $x + 6 = 0 \Rightarrow \quad x = -6$
Considering only the positive value of x as natural numbers are always positive i.e, x = 9.
So, x - 3 = 6.
Thus, the two numbers are 6 and 9 respectively.

18. The sum of the squares of three consecutive natural numbers is 149. Find the numbers.
Solution:

Let the three consecutive natural numbers be x, x + 1, and x + 2 respectively.
From the question, we have

$x^2 + (x +1)^2 + (x + 2)^2 = 149$
$\Rightarrow \quad x^2 + x^2 + x^2 + 1 + 2x + 4 + 4x = 149$
$\Rightarrow \quad 3x^2 + 6x - 144 = 0$
$\Rightarrow \quad x^2 + 2x - 48 = 0$ [dividing by 3 both sides]

Solving for x by factorization method, we have

$x^2 + 8x - 6x - 48 = 0$
$\Rightarrow \quad x(x + 8) - 6(x + 8) = 0$
$\Rightarrow \quad (x + 8)(x - 6) = 0$

Now, either $x + 8 = 0 \Rightarrow \quad x = -8$
Or, $x - 6 = 0 \Rightarrow \quad x = 6$
Considering only the positive value of x that is 6 and discarding the negative value as the numbers considered are natural numbers.
So, x = 6, x + 1 = 7 and x + 2 = 8.
Thus, the three consecutive numbers are 6, 7, and 8 respectively.

19. The sum of two numbers is 16. The sum of their reciprocals is 1/3. Find the numbers.
Solution:

Let’s consider one of the two natural numbers as x then the other number will be 16 – x, as its

given their sum is 16.
Now, form the question we can form the below equation

$$\frac{1}{x} + \frac{1}{16-x} = \frac{1}{3}$$

$$\frac{16-x+x}{x(16-x)} = \frac{1}{3}$$

$$\frac{16}{x(16-x)} = \frac{1}{3}$$

⇒ $16x - x^2 = 48$
⇒ $-16x + x^2 + 48 = 0$
⇒ $x^2 - 16x + 48 = 0$
Solving for x by factorization method, we have
⇒ $x^2 - 12x - 4x + 48 = 0$
⇒ $x(x - 12) - 4(x - 12) = 0$
⇒ $(x - 12)(x - 4) = 0$
So, either $x - 12 = 0$ ⇒ $x = 12$
Or, $x - 4 = 0$ ⇒ $x = 4$
Thus, the two numbers are 4 and 12 respectively.

20. Determine two consecutive multiples of 3 whose product is 270.
Solution:

Let the two consecutive multiples of 3 be 3x and 3x + 3
From the question, it's given that
$3x*(3x + 3) = 270$
⇒ $x(3x + 3) = 90$ [Dividing by 3 both sides]
⇒ $3x^2 + 3x = 90$
⇒ $3x^2 + 3x - 90 = 0$
⇒ $x^2 + x - 30 = 0$
Solving for x by factorization method, we have
$x^2 + 6x - 5x - 30 = 0$
⇒ $x(x + 6) - 5(x + 6) = 0$
⇒ $(x + 6)(x - 5) = 0$
Now, either $x + 6 = 0$ ⇒ $x = -6$
Or, $x - 5 = 0$ ⇒ $x = 5$
Considering the positive value of x, we have only
x = 5, so 3x = 15 and 3x + 3 = 18.
Thus, the two consecutive multiples of 3 are 15 and 18 respectively.

21. The sum of a number and its reciprocal is 17/4. Find the number.
Solution:

Let the number be x.
Then from the question, we have

$x + 1/x = 17/4$

$$\frac{x^2+1}{x} = \frac{17}{4}$$

⇒ $4(x^2+1) = 17x$
⇒ $4x^2 + 4 - 17x = 0$
⇒ $4x^2 + 4 - 16x - x = 0$
⇒ $4x(x - 4) - 1(x - 4) = 0$
⇒ $(4x - 1)(x - 4) = 0$

Now, either $x - 4 = 0$ ⇒ $x = 4$
Or, $4x - 1 = 0$ ⇒ $x = 1/4$
Thus, the value of x is 4.

Exercise 8.8 Page No: 8.58

1. The speed of a boat in still water is 8km/hr. It can go 15 km upstream and 22 km downstream in 5 hours. Find the speed of the stream.
Solution:

Let the speed of stream be x km/hr
Given, speed of boat in still water is 8km/hr.
So, speed of downstream = (8 + x) km/hr
And, speed of upstream = (8 - x) km/hr
Using, speed = distance/ time
Time taken by the boat to go 15 km upstream = 15/(8 - x)hr
And, time taken by the boat to return 22 km downstream = 22/(8 + x)hr
From the question, the boat returns to the same point in 5 hr.

$$\text{so, } \frac{15}{(8-x)} + \frac{22}{(8+x)} = 5$$

$$\frac{15(8+x) + 22(8-x)}{(8-x)(8+x)} = 5$$

$$\frac{120 + 15x + 176 - 22x}{64 - x^2} = 5$$

$$\frac{296 - 7x}{64 - x^2} = 5$$

$5x^2 - 7x + 296 - 320 = 0$
$5x^2 - 7x - 24 = 0$
$5x^2 - 15x + 8x - 24 = 0$ [by factorisation method]
$5x(x - 3) + 8(x - 3) = 0$
$(x - 3)(5x + 8) = 0$
$\therefore x = 3, x = -8/5$
As the speed of the stream can never be negative, only the positive solution is considered.
Therefore, the speed of the stream is 3 km/hr.

2. A train, traveling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/hr more. Find the original speed of the train.
Solution:

Let the original speed of train be x km/hr
When increased by 5, speed of the train = (x + 5) km/hr
Using, speed = distance/ time
Time taken by the train for original uniform speed to cover 360 km = 360/x hr.
And, time taken by the train for increased speed to cover 360 km = 360/(x + 5) hr.
Given, that the difference in the times is 48 mins. ⇒ 48/60 hour

This can be expressed as below:

$$\frac{360}{x} - \frac{360}{(x+5)} = \frac{48}{60}$$

$$\frac{360(x+5) - 360x}{x(x+5)} = \frac{4}{5}$$

$$\frac{360x + 1800 - 360x}{x^2 + 5x} = \frac{4}{5}$$

$1800(5) = 4(x^2 + 5x)$
$9000 = 4x^2 + 20x$
$4x^2 + 20x - 9000 = 0$
$x^2 + 5x - 2250 = 0$
$x^2 + 50x - 45x - 2250 = 0$ [by factorisation method]
$x(x + 50) - 45(x + 50) = 0$
$(x + 50)(x - 45) = 0$
$\therefore x = -50$ or $x = 45$
Since, the speed of the train can never be negative x = -50 is not considered.
Therefore, the original speed of train is 45 km/hr.

3. A fast train takes one hour less than a slow train for a journey of 200 km. If the speed of the slow train is 10 km/hr less than that of the fast train, find the speed of the two trains.
Solution:

Let's consider the speed of the fast train as x km/hr
Then, the speed of the slow train will be = (x -10) km/hr
Using, speed = distance/ time
Time taken by the fast train to cover 200 km = 200/x hr
And, time taken by the slow train to cover 200 km = 200/(x - 10) hr
Given, that the difference in the times is 1 hour.
This can be expressed as below:

$$\frac{200}{x} - \frac{200}{(x-10)} = 1$$

$$\frac{(200(x-10) - 200x)}{x(x-10)} = 1$$

$$\frac{200x - 2000 - 200x}{x^2 - 10x} = 1$$

$x^2 - 10x = -2000$
$x^2 - 10x + 2000 = 0$

$x^2 - 50x + 40x + 2000 = 0$ [by factorisation method]
$x(x - 50) + 40(x - 50) = 0$
$(x - 50)(x + 40) = 0$
$x = 50$ or $x = -40$
As, the speed of train can never be negative we neglect $x = -40$
Thus, speed of the fast train is 50 km/hr
And the speed of slow train $(50 - 10) = 40$ km/hr

4. A passenger train takes one hour less for a journey of 150 km if its speed is increased 5 km/hr from its usual speed. Find the usual speed of the train.
Solution:

Let's assume the usual speed of train as x km/hr
Then, the increased speed of the train = (x + 5) km/hr
Using, speed = distance/ time
Time taken by the train under usual speed to cover 150 km = 150/x hr
Time taken by the train under increased speed to cover 150 km = 150(x + 5)hr
Given, that the difference in the times is 1 hour.
This can be expressed as below:

$$\text{So, } \frac{150}{x} - \frac{150}{(x+5)} = 1$$

$$\frac{150(x+5) - 150x}{x(x+5)} = 1$$

$$\frac{150x + 750 - 150x}{x^2 + 5x} = 1$$

$750 = x^2 + 5x$
$x^2 + 5x - 750 = 0$
$x^2 - 25x + 30x - 750 = 0$ [by factorisation method]
$x(x - 25) + 30(x - 25) = 0$
$(x - 25)(x + 30) = 0$
$x = 25$ or $x = -30$ (neglected as the speed of the train can never be negative)
Hence, the usual speed of the train is $x = 25$ km/hr

5. The time taken by a person to cover 150 km was 2.5 hrs more than the time taken in the return journey. If he returned at the speed of 10 km/hr more than the speed of going, what was the speed per hour in each direction?
Solution:

Let the ongoing speed of person be x km/hr,
Then, the returning speed of the person is = (x + 10) km/hr (from the question)
Using, speed = distance/ time

Time taken by the person in going direction to cover 150 km = 150/x hr
And, time taken by the person in returning direction to cover 150 km = 150/(x + 10)hr
Given, that the difference in the times is 2.5 hour ⇒ 5/2 hours
This can be expressed as below:

$$\frac{150}{x} - \frac{150}{(x+10)} = \frac{5}{2}$$

$$\frac{150(x+10) - 150x}{x(x+10)} = \frac{5}{2}$$

$$\frac{150x + 1500 - 150x}{x^2 + 10x} = \frac{5}{2}$$

$$\frac{1500}{x^2 + 10x} = \frac{5}{2}$$

$3000 = 5x^2 + 50x$
$5x^2 + 50x - 3000 = 0$
$5(x^2 + 10x - 600) = 0$
$x^2 + 10x - 600 = 0$
$x^2 - 20x + 30x - 600 = 0$ [by factorisation method]
$x(x - 20) + 30(x - 20) = 0$
$(x - 20)(x + 30) = 0$
x = 20 or x = -30(neglected) As the speed of train can never be negative.
Thus, x = 20 Then, (x + 10) (20 + 10) = 30
Therefore, the ongoing speed of person is 20km/hr.
And the returning speed of the person is 30 km/hr.

6. A plane left 40 minutes late due to bad weather and in order to reach the destination, 1600 km away in time, it had to increase its speed by 400 km/hr from its usual speed. Find the usual speed of the plane.
Solution:

Let's assume the usual speed of the plane to be x km/hr,
Then the increased speed of the plane is = (x + 4000) km/hr
Using, speed = distance/ time
Time taken by the plane under usual speed to cover 1600 km = 1600/x hr
Time taken by the plane under increased speed to cover 1600 km = 1600/(x + 400) hr
Given, that the difference in the times is 40mins ⇒ 40/60 hours
This can be expressed as below:

$$\frac{1600}{x} - \frac{1600}{(x+400)} = \frac{40}{60}$$

$$\frac{1600(x+400) - 1600x}{x(x+400)} = \frac{2}{3}$$

$$\frac{1600x + 640000 - 1600x}{x^2 + 400x} = \frac{2}{3}$$

$1920000 = 2x^2 + 800x$
$2x^2 + 800x - 1920000 = 0$
$2(x^2 + 400x - 960000) = 0$
$x^2 + 400x - 960000 = 0$
$x^2 - 800x + 1200x - 960000 = 0$ [by factorisation method]
$x(x - 800) + 1200(x - 800) = 0$
$(x - 800)(x + 1200) = 0$
$x = 800$ or $x = -1200$ (neglected)
As the speed of the train can never be negative.
Thus, the usual speed of the train is 800 km/hr.

7. An aero plane takes 1 hour less for a journey of 1200 km if its speed is increased by 100 km/hr from its usual speed of the plane. Find its usual speed.
Solution:

Let's consider the usual speed of plane as x km/hr,
Then, the increased speed of the plane is = (x + 100) km/hr
Using, speed = distance/ time
Time taken by the plane under usual speed to cover 1200 km = 1200/x hr
Time taken by the plane under increased speed to cover 1200 km = 1200/(x + 100)hr
Given, that the difference in the times is 1 hour.
So, this can be expressed as below:

$$\frac{1200}{x} - \frac{1200}{(x+100)} = 1$$

$$\frac{1200(x+100) - 1200x}{x(x+100)} = 1$$

$$\frac{1200x + 120000 - 1200x}{x^2 + 100x} = 1$$

$120000 = x^2 + 100x$
$x^2 + 100x - 120000 = 0$
$x^2 - 300x + 400x - 120000 = 0$ [by factorisation method]
$x(x - 300) + 400(x - 300) = 0$

x = 300 or x = - 400 neglected as the speed of the aero plane can never be negative.
Therefore, the usual speed of train is 300 km/hr.

Exercise 8.9 Page No: 8.61

1. Ashu is x years old while his mother Mrs. Veena is x^2 years old. Five years hence Mrs. Veena will be three times old as Ashu. Find their present ages.

Solution:

Given, Ashu's present age is x years and his mother Mrs. Veena is x^2 years.
After 5 years, Ashu age will be $(x + 5)$ years
And his mother Mrs. Veena age will be $(x^2 + 5)$ years
Given relationship between their ages can be expressed as:
$x^2 + 5 = 3(x + 5)$
$x^2 + 5 = 3x + 15$ $x^2 + 5 - 3x - 15 = 0$
$x^2 - 5x + 2x + 10 = 0$
$x(x - 5) + 2(x - 5) = 0$
$(x - 5)(x + 2) = 0$
$x = 5$ or $x = -2$(neglected) since, the age can never be negative
Hence, Ashu's present age is 5 years and his mother's age is 25 years.

2. The sum of the ages of a man and his son is 45 years. Five years ago, the product of their ages was four times the man's age at the time. Find their present ages.

Solution:

Let the present age of the man be x years
Then, the present age of his son will be $= (45 - x)$ years
Five years ago, man's age $= (x - 5)$ years
And, his son's age $= (45 - x - 5) = (40 - x)$ years
Given relationship between their ages can be expressed as:
$(x - 5)(40 - x) = 4(x - 5)$
$40x - x^2 + 5x - 200 = 4x - 20$
$-x^2 + 45x - 200 = 4x - 20$
$-x^2 + 45x - 200 - 4x + 20 = 0$
$-x^2 + 41x - 180 = 0$
$x^2 - 36x - 5x + 180 = 0$ [By factorisation method]
$x(x - 36) - 5(x - 36) = 0$
$(x - 36)(x - 5) = 0$
$x = 36$ or $x = 5$,
But, the father's age can never be 5 years
Thus, when $x = 36$, $45 - x = 45 - 36 = 9$
Therefore, the man's present age is 36 years and his son's age is 9 years.

3. The product of Shikha's age five years ago and her age 8 years later is 30, her age at both times being given in years. Find her present age.

Solution:

Let's assume the present age of Shikha be x years
So, 8 years later, age of her $= (x + 8)$ years

Five years ago, her age = (x - 5) years
Given relationship between the ages can be expressed as:
$(x - 5)(x + 8) = 30$
$x^2 + 8x - 5x - 40 = 30$
$x^2 + 3x - 40 - 30 = 0$
$x^2 + 3x - 70 = 0$ [By factorisation method]
$x(x - 7) + 10(x - 7) = 0$
$(x - 7)(x + 10) = 0$
x = 7 or x = -10 (neglected)
Since, the age can never be negative.
Therefore, the present age of Shikha is 7 years.

4. The product of Ramu's age (in years) five years ago and his age (in years) nine years later is 15. Determine Ramu's present age.
Solution:

Let the present age of Ramu be x years
So, 9 years later, the age of him = (x + 9) years
And, five years ago, his age = (x - 5) years
Given relationship between the ages can be expressed as:
$(x - 5)(x + 5) = 15$
$x^2 + 9x - 5x - 45 = 15$
$x^2 + 4x - 45 - 15 = 0$
$x^2 + 4x - 60 = 0$
$x^2 - 6x + 10x - 60 = 0$ [By factorisation method]
$x(x - 6) + 10(x - 6) = 0$
$(x - 6)(x + 10) = 0$
x = 6 or x = - 10(neglected) as the age can be never be negative.
Hence, the present age of Ramu is 6 years.

Exercise 8.10 Page No: 8.64

1. The hypotenuse of a right triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 5 cm. Find the lengths of these sides.
Solution:

Let the length of one side of the right triangle be x cm
So, the other side will be = (x + 5) cm [as they differ by 5cm]
And given that hypotenuse = 25 cm
On applying Pythagoras Theorem, we have
$x^2 + (x + 5)^2 = 25^2$
$x^2 + x^2 + 10x + 25 = 625$
$2x^2 + 10x + 25 - 625 = 0$
$2x^2 + 10x - 600 = 0$
$x^2 + 5x - 300 = 0$
$x^2 - 15x + 20x - 300 = 0$ [By factorisation method]
$x(x - 15) + 20(x - 15) = 0$
$(x - 15)(x + 20) = 0$
x = 15 or x = - 20 (neglected) As the side of triangle can never be negative.
Thus, when x = 15 ⇒ x + 5 = 15 + 5 = 20
Hence, the length of side of right triangle is 15 cm and other side is 20 cm

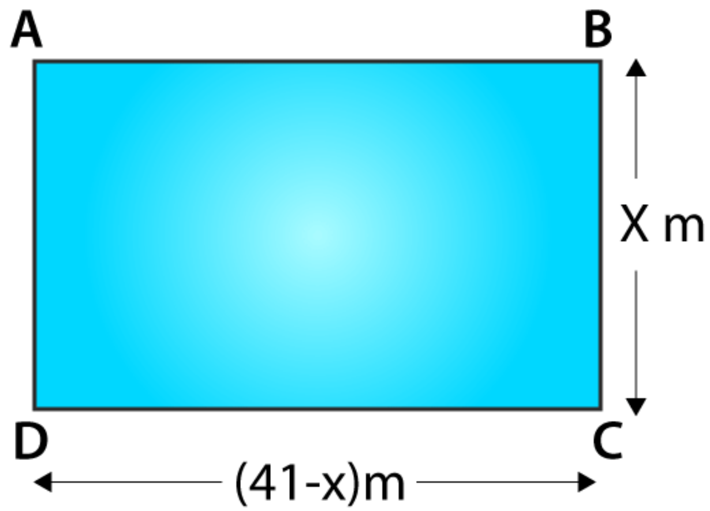
2. The diagonal of a rectangular field is 60 meters more than the shorter side. If the longer side is 30 meters more than the shorter side, find the sides of the field.
Solution:

Let's consider the length of smaller side of rectangle as x metres
Then, the larger side will be (x + 30) metres and diagonal will be = (x + 60) metre [From given relation]
Now, by using Pythagoras theorem we have,
$x^2 + (x + 30)^2 = (x + 60)^2$
$x^2 + x^2 + 60x + 900 = x^2 + 120x + 3600$
$2x^2 + 60x + 900 - x^2 - 120x - 3600 = 0$
$x^2 - 60x - 2700 = 0$
$x^2 - 90x + 30x - 2700 = 0$ [By factorisation method]
$x(x - 90) + 30(x - 90) = 0$
$(x - 90)(x + 30) = 0$
x = 90 or x = -30 (this is neglected as the side of a rectangle can never be negative)
Therefore, we only take x = 90,
⇒ x + 30 = 90 + 30 = 120
Thus, the length of smaller side of rectangle is 90 metres and the larger side is 120 metres.

Exercise 8.11 Page No: 8.70

1. The perimeter of the rectangular field is 82 m and its area is 400 m^2. Find the breadth of the rectangle?

Solution:



Given,

Perimeter = 82 m and its area = 400 m^2

Let the breadth of the rectangle be considered as x m.

We know that,

Perimeter of a rectangle = 2(length + breadth)

82 = 2(length + x)

41 = (length + x)

⇒Length = (41 - x)m

We also know that,

Area of the rectangle = length * breadth

400 = (41 - x)(x)

$400 = 41x - x^2$

$x^2 - 41x + 400 = 0$

$x^2 - 25x - 16x + 400 = 0$

x(x - 25) - 16(x - 25) = 0

(x - 16)(x - 25) = 0

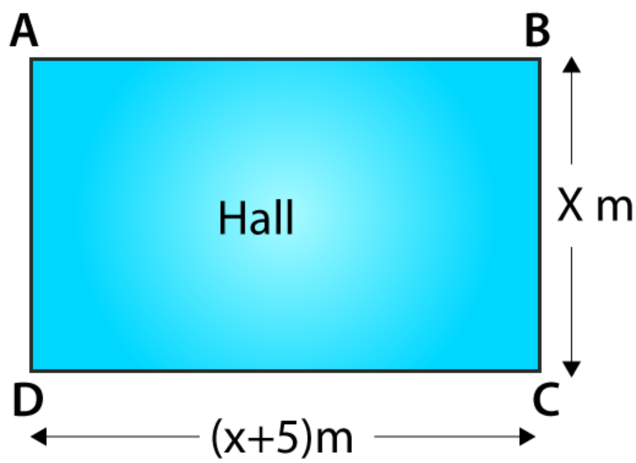
Now, either x - 16 = 0 ⇒ x = 16

Or, x - 25 = 0 ⇒ x = 25

Therefore, the breadth of the rectangle can be either 16 m or 25 m respectively.

2. The length of the hall is 5 m more than its breadth. If the area of the floor of the hall is 84 m^2, what are the length and breadth of the hall?

Solution:



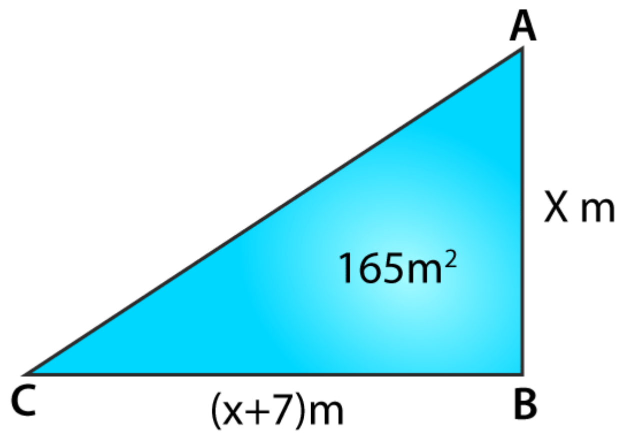
Considering the breadth of the rectangle be x m
Then, the length of the hall is 5 m more than its breadth i.e, = (x + 5) m
Given, area of the hall is = 84 m^2
As the shape of the hall is rectangular, its area is given by
Area of the rectangular hall = length * breadth
$84 = x(x + 5)$
$x^2 + 5x - 84 = 0$
$x^2 + 12x - 7x - 84 = 0$
$x(x + 12) - 7(x + 12) = 0$
$(x + 12)(x - 7) = 0$
Now, either $x + 12 = 0 \Rightarrow x = -12$ (neglected since the side of a rectangle can never be negative)
Or, $x - 7 = 0 \quad \Rightarrow x = 7$
So, only x = 7 is considered.
$\Rightarrow \quad x + 5 = 12$
Thus, the length and breadth of the rectangle is 7 and 12 respectively.

3. Two squares have sides x and (x + 4) cm. The sum of their area is 656 cm^2. Find the sides of the square.
Solution:

Let S_1 and S_2 be the two squares.
And, let x cm be the side square S_1 and (x + 4) cm be the side of the square S_2.
So,
Area of the square $S_1 = x^2$ cm^2
Area of the square $S_2 = (x + 4)^2$ cm^2
From the question, we have
Area of the square S_1 + Area of the square S_2 = 656 cm^2
$\Rightarrow x^2\ cm^2 + (x + 4)^2\ cm^2 = 656\ cm^2$
$x^2 + x^2 + 16 + 8x - 656 = 0$
$2x^2 + 16 + 8x - 656 = 0$
$2(x^2 + 4x - 320) = 0$
$x^2 + 4x - 320 = 0$

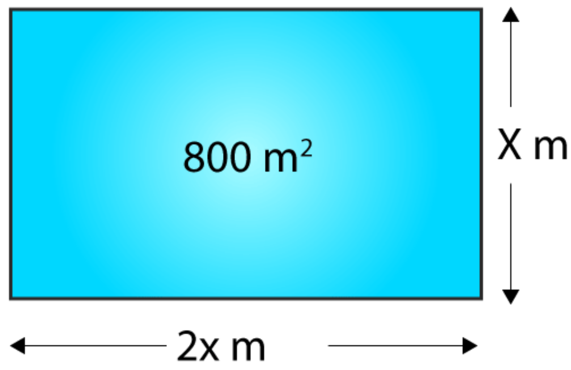
$x^2 + 20x - 16x - 320 = 0$
$x(x + 20) - 16(x + 20) = 0$
$(x + 20)(x - 16) = 0$
Now, either $x + 20 = 0 \Rightarrow x = -20$
Or, $x - 16 = 0 \Rightarrow x = 16$
As the value of x cannot be negative, we choose the value of $x = 16 \Rightarrow x + 4 = 20$
Therefore,
The side of the square $S_1 = 16$ cm
The side of the square $S_2 = 20$ cm

4. The area of a right-angled triangle is 165 cm^2. Determine its base and altitude if the latter exceeds the former by 7m.
Solution:



Let the altitude of the right angle triangle be considered as x m
So given that, the altitude exceeds the base by 7m ⇒ altitude = (x – 7)m
We know that,
Area of the triangle = 1/2 × base × altitude
⇒ $165 = 1/2 \times (x - 7) \times x$
$x(x - 7) = 330$
$x^2 - 7x - 330 = 0$
$x^2 - 22x + 15x - 330 = 0$
$x(x - 22) + 15(x - 22) = 0$
$(x - 22)(x + 15) = 0$
Now, either $x - 22 = 0 \Rightarrow x = 22$
Or, $x + 15 = 0 \Rightarrow x = -15$ (neglected)
Since the value of x cannot be negative, so the value of x = 22 is only considered
⇒ $x - 7 = 15$
Therefore the base and altitude of the right angled triangle are 15 cm and 22 cm respectively.

5. Is it possible to design a rectangular mango grove whose length is twice its breadth and the area is 800 m^2? If so, find its length and breadth.
Solution:



Let the breadth of the rectangular mango grove be x m
Given that, the length of rectangle is twice of its breadth.
So, length = 2x
Area of the grove = 800 m^2 (given)
We know that,
Area of the rectangle = length * breadth
800 = x(2x)
$2x^2 - 800 = 0$
$x^2 - 400 = 0$
⇒ x = √400 = 20 (neglecting the negative sq. root as side can never be negative)
Therefore,
The breadth of the rectangular groove is 20 m.
And, the length of the rectangular groove is 40 m.
Yes, it is possible to design a rectangular groove whose length is twice of its breadth.

Exercise 8.12 Page No: 8.73

1. A takes 10 days less than the time taken by B to finish a piece of work. If both A and B together can finish the work in 12 days, find the time taken by B to finish the work.

Solution:

Let's consider that B takes x days to complete the piece of work.
So, B's 1 day work = 1/x
Now, A takes 10 days less than that of B to finish the same piece of work so, that is (x –10) days
⇒ A's 1 day work = 1/(x - 10)
Same work both working together for 12 days, then
(A and B)'s 1 day's work = 1/12
From the question, it's understood that

$$\text{A's 1 day work} + \text{B's 1 day work} = \frac{1}{x-10} + \frac{1}{x}$$

$$= \frac{1}{x} + \frac{1}{x-10} = \frac{1}{12}$$

$$= \frac{x-10+x}{x(x-10)} = \frac{1}{12}$$

⇒ 12(2x - 10) = x(x - 10)
⇒ 24x - 120 = x^2 - 10x
⇒ x^2 - 10x - 24x + 120 = 0
⇒ x^2 - 34x + 120 = 0
⇒ x^2 - 30x - 4x + 120 = 0
⇒ x(x - 30) - 4(x - 30) = 0
⇒ (x - 30)(x - 4) = 0
Now, either x - 30 = 0 ⇒ x = 30
Or, x - 4 = 0 ⇒ x = 4
It's clear that the value of x cannot be less than 10, so the value of x = 30 is chosen.
Therefore, the time taken by B to finish the piece of work is 30 days.

2. If two pipes function simultaneously, a reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours will the second pipe take to fill the reservoir?

Solution:

Let's consider that the faster pipe takes x hours to fill the reservoir.
Then, the portion of reservoir filled by faster pipe in one hour = 1/x
Given that, the slower pipe takes 10 hours more than that of faster pipe to fill the reservoir that is (x + 10) hours
Portion of reservoir filled by slower pipe = 1/(x + 10)

Now, it's given that if both the pipes function simultaneously, the same reservoir can be filled in

12 hours
Thus, we know that the portion of the reservoir filled by both pipes in one hour = 1/12
Now,
Portion of reservoir filled by slower pipe in one hour + Portion of reservoir filled by faster pipe in one hour = 1/x + 1/(x + 10)

So, the portion of reservoir filled by both pipes = 1/12, hence

$$= \frac{1}{x} + \frac{1}{x+10} = \frac{1}{12}$$

⇒ 12(2x + 10) = x(x + 10)
⇒ x^2 - 14x - 120 = 0
⇒ x^2 - 20x + 6x - 120 = 0
⇒ x(x - 20) + 6(x - 20) = 0
⇒ (x - 20)(x + 6) = 0
Now, either x - 20 = 0 ⇒ x = 20
Or, x + 6 = 0 ⇒ x = – 6 (can be neglected)
Since the value of time cannot be negative.
Thus, the value of x is taken as 20 hours.
Therefore the time taken by the slower pipe to fill the reservoir = x + 10 = 30 hours.

Exercise 8.13 Page No: 8.80

1. A piece of cloth costs Rs. 35. If the piece were 4 m longer and each metre costs Rs. 1 less, the cost would remain unchanged. How long is the piece?
Solution:

Let's assume the length of the cloth to be 'a' meters.
Given, piece of cloth costs Rs. 35 and if the piece were 4 m longer and each metre costs Rs. 1 less, the cost remains unchanged.
From the above condition, we have
Cost of 1m of cloth = 35/a

$\Rightarrow (a + 4) \times (\frac{35}{a} - 1) = 35$

$\Rightarrow (a + 4)(35 – a) = 35a$
$\Rightarrow 35a + 140 – a^2 – 4a = 35a$
$\Rightarrow a^2 + 4a – 140 = 0$
Solving for 'a' by factorization method,
$\Rightarrow a^2 + 14a – 10a – 140 = 0$
$\Rightarrow a(a + 14) – 10(a + 14) = 0$
$\Rightarrow (a – 10)(a + 14) = 0$
$\Rightarrow a = 10$ m or $a = -14$ (neglected)
Since, the length of a cloth can never be negative.
So, a = 10
Hence, the piece of cloth was 10 m long.

2. Some students planned a picnic. The budget for food was Rs. 480. But eight of these failed to go and thus the cost of food for each member increased by Rs. 10. How many students attended the picnic?
Solution:

Let the number of students who planned the picnic be 'x'.
And given, budget for the food was Rs. 480
So, cost of food for each member = 480/x
Also given, eight of these failed to go and hence the cost of food for each member increased by Rs. 10
This can be expressed as below:

$\Rightarrow (x - 8) \times (\frac{480}{x} + 10) = 480$

$\Rightarrow (x – 8)(480 + 10x) = 480x$
$\Rightarrow 480x + 10x^2 – 3840 – 80x = 480x$
$\Rightarrow x^2 – 8x – 384 = 0$
$\Rightarrow x^2 – 24x + 16x – 384 = 0$
$\Rightarrow x(x – 24) + 16(x – 24) = 0$
$\Rightarrow (x + 16)(x – 24) = 0$
$\Rightarrow x = 24$ or

x = -16 (this solution is neglected since the number of students can never be negative.)

Hence, the number of students who planned the picnic was 24.
And, the number of students who attended the picnic = 24 – 8 = 16

3. A dealer sells an article for Rs. 24 and gains as much percent as the cost price of the article. Find the cost price of the article.
Solution:

Let the cost price be assumed as Rs x.
Given, the dealer sells an article for Rs. 24 and gains as much percent as the cost price of the article.
It's given that he gains as much as the cost price of the article, thus, Gain% = x%

Gain% = $\frac{SP - CP}{CP} \times 100$

$\Rightarrow \mathbf{x} = \frac{24 - \mathbf{x}}{\mathbf{x}} \times 100$
$\Rightarrow x^2 = (24-x) \times 100$
$\Rightarrow x^2 + 100x - 2400 = 0$
$\Rightarrow x^2 + 120x - 20x - 2400 = 0$
$\Rightarrow (x + 120)(x - 20) = 0$
$\Rightarrow x = 20$ or -120
Since money cannot be negative, we call neglect -120
$\Rightarrow x = 20$
Therefore, the cost price of the article is Rs 20

4. Out of a group of swans, 7/2 times the square root of the total number are playing on the share of a pond. The two remaining ones are swinging in water. Find the total number of swans.
Solution:

Let's assume the number of swans in the pond be 'a'.
Given that, out of a group of swans, 7/2 times the square root of the total number are playing on the share of a pond. The two remaining ones are swinging in water.
So, expressing the total number of swans in a equation, we have
$\Rightarrow \frac{7}{2}\sqrt{a} + 2 = a$
$\Rightarrow 7\sqrt{a} = 2a - 4$
On squaring both sides,
$\Rightarrow 49a = 4a^2 + 16 - 16a$
$\Rightarrow 4a^2 - 65a + 16 = 0$
$\Rightarrow 4a^2 - 64a - a + 16 = 0$ [By factorisation method]
$\Rightarrow 4a(a - 16) - (a - 16) = 0$
$\Rightarrow (4a - 1)(a - 16) = 0$

$\Rightarrow$ a =1/4 or a = 16
Since, number of swans can only be a natural number we can neglect the solution of a = 1/4
Hence, the total number of swans is 16.

5. If the list price of a toy is reduced by Rs. 2, a person can buy 2 toys more for Rs. 360. Find the original price of the toy.
Solution:

Let the original price of the toy be 'x'.
Given that, when the list price of a toy is reduced by Rs. 2, the person can buy 2 toys more for Rs. 360.
The number of toys he can buy at the original price for Rs. 360 = 360/x
According to the question,

$$\Rightarrow \frac{360}{x-2} = \frac{360}{x} + 2$$

$\Rightarrow 360x = (x - 2)(360 + 2x)$
$\Rightarrow 360x = 360x + 2x^2 - 720 - 4x$
$\Rightarrow x^2 - 2x - 360 = 0$
$\Rightarrow x^2 - 20x + 18x - 360 = 0$ [By factorisation method]
$\Rightarrow x(x - 20) + 18(x - 20) = 0$
$\Rightarrow (x + 18)(x - 20) = 0$
$\Rightarrow x + 18 = 0$ or $x - 20 = 0$
$\Rightarrow x = -18$ or $x = 20$
As, the price can't be negative, a = -18 is neglected.
Thus, the original price of the toy is Rs 20.

6. Rs. 9000 were divided equally among a certain number of persons. Had there been 20 more persons, each would have got Rs. 160 less. Find the original number of persons.
Solution:

Let's consider the original number of people as 'a'.
Given that, Rs. 9000 were divided equally among a certain number of persons. Had there been 20 more persons, each would have got Rs. 160 less.
Amount which each receives when a persons are present = 9000/a
Expressing the given condition we have,

$$\Rightarrow \frac{9000}{a+20} = \frac{9000}{a} - 160$$

$\Rightarrow 9000a = (9000 - 160a)(a + 20)$
$\Rightarrow 9000a = 9000a + 180000 - 160a^2 - 3200a$
$\Rightarrow a^2 + 20a - 1125 = 0$
$\Rightarrow a^2 + 45a - 25a - 1125 = 0$
$\Rightarrow a(a + 45) - 25(a + 45) = 0$

$\Rightarrow (a - 25)(a + 45) = 0$
$\Rightarrow a = 25$ or $a = -45$ (neglected as the number of people can never be negative)
Therefore, the original number of people = 25.