Exercise 9.4

1. Find:
(i) 10^{th} tent of the AP 1, 4, 7, 10....
(ii) 18^{th} term of the AP √2, 3√2, 5√2, …….
(iii) n^{th} term of the AP 13, 8, 3, -2, ...
(iv) 10^{th} term of the AP -40, -15, 10, 35, ...
(v) 8^{th} term of the AP 11, 104, 91, 78, ...
(vi) 11^{th} tenor of the AP 10.0, 10.5, 11.0, 11.2, ...
(vii) 9^{th} term of the AP 3/4, 5/4, 7/4 + 9/4, ...
Solution:

(i) Given A.P. is 1, 4, 7, 10, ...
First term (a) = 1
Common difference (d) = Second term - First term
= 4 - 1 = 3.
We know that, n^{th} term in an A.P = a + (n - 1)d
Then, 10^{th} term in the A.P is 1 + (10 - 1)3
= 1 + 9x3
= 1 + 27
= 28
∴ 10^{th} term of A. P. is 28

(ii) Given A.P. is √2, 3√2, 5√2, …….
First term (a) = √2
Common difference = Second term – First term
= 3√2 - √2
⇒ d = 2√2
We know that, n^{th} term in an A. P. = a + (n - 1)d
Then, 18^{th} term of A. P. = √2 + (18 - 1)2√2
= √2 + 17.2√2
= √2 (1+34)
= 35√2
∴ 18^{th} term of A. P. is 35√2

(iii) Given A. P. is 13, 8, 3, - 2, ...
First term (a) = 13
Common difference (d) = Second term first term
= 8 - 13 = – 5
We know that, n^{th} term of an A.P. a_n = a +(n - 1)d
= 13 + (n - 1) - 5
= 13 - 5n + 5
∴ n^{th} term of the A.P is a_n = 18 - 5n

(iv) Given A. P. is - 40, -15, 10, 35, ...
First term (a) = -40

Common difference (d) = Second term - fast term
= -15 - (- 40)
= 40 - 15
= 25
We know that, n^{th} term of an A.P. $a_n = a + (n - 1)d$
Then, 10^{th} term of A. P. $a_{10} = -40 + (10 - 1)25$
= – 40 + 9.25
= – 40 + 225
= 185
∴ 10^{th} term of the A. P. is 185

(v) Given sequence is 117, 104, 91, 78, ...
First term (a) = 117
Common difference (d) = Second term - first term
= 104 - 117
= – 13
We know that, n^{th} term = a + (n - 1)d
Then, 8^{th} term = a + (8 - 1)d
= 117 + 7(-13)
= 117 - 91
= 26
∴ 8^{th} term of the A. P. is 26

(vi) Given A. P is 10.0, 10.5, 11.0, 11.5,
First term (a) = 10.0
Common difference (d) = Second term - first term
= 10.5 - 10.0 = 0.5
We know that, n^{th} term $a_n = a + (n - 1)d$
Then, 11^{th} term $a_{11} = 10.0 + (11 - 1)0.5$
= 10.0 + 10 x 0.5
= 10.0 + 5
=15.0
∴ 11^{th} term of the A. P. is 15.0

(vii) Given A. P is 3/4, 5/4, 7/4, 9/4, ...
First term (a) = 3/4
Common difference (d) = Second term - first term
= 5/4 - 3/4
= 2/4
We know that, n^{th} term $a_n = a + (n - 1)d$
Then, 9^{th} term $a_9 = a + (9 - 1)d$

$$= \frac{3}{4} + 8.\frac{2}{4}$$

$$= \frac{3}{4} + \frac{16}{4}$$

$$= \frac{19}{4}$$

∴ 9^{th} term of the A. P. is 19/4.

2.(i) Which term of the AP 3, 8, 13, ... is 248?
(ii) Which term of the AP 84, 80, 76, ... is 0?
(iii) Which term of the AP 4. 9, 14, ... is 254?
(iv) Which term of the AP 21. 42, 63, 84, ... is 420?
(v) Which term of the AP 121, 117. 113, ... is its first negative term?
Solution:

(i) Given A.P. is 3, 8, 13, ...
First term (a) = 3
Common difference (d) = Second term - first term
= 8 - 3
= 5
We know that, n^{th} term (a_n) = a + (n - 1)d
And, given n^{th} term a_n = 248
248 = 3+(n - 1)5
248 = -2 + 5n
5n = 250
n =250/5 = 50
∴ 50^{th} term in the A.P is 248.

(ii) Given A. P is 84, 80, 76, ...
First term (a) = 84
Common difference (d) = a_2 - a
= 80 - 84
= – 4
We know that, n^{th} term (a_n) = a +(n - 1)d
And, given nth term is 0
0 = 84 + (n - 1) - 4
84 = +4(n - 1)
n - 1 = 84/4 = 21
n = 21 + 1 = 22
∴ 22^{nd} term in the A.P is 0.

(iii) Given A. P 4, 9, 14, ...

First term (a) = 4
Common difference (d) = $a_2 - a_1$
= 9 - 4
= 5
We know that, n^{th} term (a_n) = a + (n - 1)d
And, given n^{th} term is 254
4 + (n - 1)5 = 254
(n - 1)·5 = 250
n - 1 = 250/5 = 50
n = 51
∴ 51^{th} term in the A.P is 254.

(iv) Given A. P 21, 42, 63, 84, ...
a = 21, d = $a_2 - a_1$
= 42 - 21
= 21
We know that, n^{th} term (a_n) = a +(n - 1)d
And, given n^{th} term = 420
21 + (n - 1)21 = 420
(n - 1)21 = 399
n - 1 = 399/21 = 19
n = 20
∴ 20^{th} term is 420.

(v) Given A.P is 121, 117, 113, ...
Fiat term (a) = 121
Common difference (d) = 117 - 121
= - 4
We know that, n^{th} term a_n = a + (n - 1)d
And, for some n^{th} term is negative i.e., $a_n < 0$
121 + (n - 1) - 4 < 0
121 + 4 - 4n < 0
125 - 4n < 0
4n > 125
n > 125/4
n > 31.25
The integer which comes after 31.25 is 32.
∴ 32^{nd} term in the A.P will be the first negative term.

3.(i) Is 68 a term of the A.P. 7, 10, 13,… ?
(ii) Is 302 a term of the A.P. 3, 8, 13, …. ?
(iii) Is -150 a term of the A.P. 11, 8, 5, 2, … ?
Solutions:

(i) Given, A.P. 7, 10, 13,…
Here, a = 7 and d = $a_2 - a_1$ = 10 – 7 = 3

We know that, n^{th} term $a_n = a + (n - 1)d$
Required to check n^{th} term $a_n = 68$
$a + (n - 1)d = 68$
$7 + (n - 1)3 = 68$
$7 + 3n – 3 = 68$
$3n + 4 = 68$
$3n = 64$
$\Rightarrow n = 64/3$, which is not a whole number.
Therefore, 68 is not a term in the A.P.

(ii) Given, A.P. 3, 8, 13,…
Here, $a = 3$ and $d = a_2 – a_1 = 8 – 3 = 5$
We know that, n^{th} term $a_n = a + (n - 1)d$
Required to check n^{th} term $a_n = 302$
$a + (n - 1)d = 302$
$3 + (n - 1)5 = 302$
$3 + 5n – 5 = 302$
$5n - 2 = 302$
$5n = 304$
$\Rightarrow n = 304/5$, which is not a whole number.
Therefore, 302 is not a term in the A.P.

(iii) Given, A.P. 11, 8, 5, 2, …
Here, $a = 11$ and $d = a_2 – a_1 = 8 – 11 = -3$
We know that, n^{th} term $a_n = a + (n - 1)d$
Required to check n^{th} term $a_n = -150$
$a + (n - 1)d = -150$
$11 + (n - 1)(-3) = -150$
$11 – 3n + 3 = -150$
$3n = 150 + 14$
$3n = 164$
$\Rightarrow n = 164/3$, which is not a whole number.
Therefore, -150 is not a term in the A.P.

4. How many terms are there in the A.P.?
(i) 7, 10, 13, ….., 43
(ii) -1, -5/6, -2/3, -1/2, … , 10/3
(iii) 7, 13, 19, …, 205
(iv) 18, 15½, 13, …., -47
Solution:

(i) Given, A.P. 7, 10, 13, ….., 43
Here, $a = 7$ and $d = a_2 – a_1 = 10 – 7 = 3$
We know that, n^{th} term $a_n = a + (n - 1)d$
And, given n^{th} term $a_n = 43$

$a + (n - 1)d = 43$
$7 + (n - 1)(3) = 43$
$7 + 3n – 3 = 43$
$3n = 43 – 4$
$3n = 39$
$\Rightarrow n = 13$
Therefore, there are 13 terms in the given A.P.

(ii) Given, A.P. -1, -5/6, -2/3, -1/2, … , 10/3
Here, $a = -1$ and $d = a_2 – a_1 = -5/6 – (-1) = 1/6$
We know that, n^{th} term $a_n = a + (n - 1)d$
And, given n^{th} term $a_n = 10/3$
$a + (n - 1)d = 10/3$
$-1 + (n - 1)(1/6) = 10/3$
$-1 + n/6 – 1/6 = 10/3$
$n/6 = 10/3 + 1 + 1/6$
$n/6 = (20 + 6 + 1)/6$
$n = (20 + 6 + 1)$
$\Rightarrow n = 27$
Therefore, there are 27 terms in the given A.P.

(iii) Given, A.P. 7, 13, 19, …, 205
Here, $a = 7$ and $d = a_2 – a_1 = 13 – 7 = 6$
We know that, n^{th} term $a_n = a + (n - 1)d$
And, given n^{th} term $a_n = 205$
$a + (n - 1)d = 205$
$7 + (n - 1)(6) = 205$
$7 + 6n – 6 = 205$
$6n = 205 – 1$
$n = 204/6$
$\Rightarrow n = 34$
Therefore, there are 34 terms in the given A.P.

(iv) Given, A.P. 18, 15½, 13, …., -47
Here, $a = 7$ and $d = a_2 – a_1 = 15½ – 18 = 5/2$
We know that, n^{th} term $a_n = a + (n - 1)d$
And, given n^{th} term $a_n = -47$
$a + (n - 1)d = 43$
$18 + (n - 1)(-5/2) = -47$
$18 – 5n/2 + 5/2 = -47$
$36 – 5n + 5 = -94$
$5n = 94 + 36 + 5$
$5n = 135$
$\Rightarrow n = 27$
Therefore, there are 27 terms in the given A.P.

5. The first term of an A.P. is 5, the common difference is 3 and the last term is 80; find the number of terms.
Solution:

Given,
a = 5 and d = 3
We know that, n^{th} term $a_n = a + (n - 1)d$
So, for the given A.P. $a_n = 5 + (n - 1)3 = 3n + 2$
Also given, last term = 80
$\Rightarrow 3n + 2 = 80$
$3n = 78$
$n = 78/3 = 26$
Therefore, there are 26 terms in the A.P.

6. The 6^{th} and 17^{th} terms of an A.P. are 19 and 41 respectively, find the 40^{th} term.
Solution:

Given,
$a_6 = 19$ and $a_{17} = 41$
We know that, n^{th} term $a_n = a + (n - 1)d$
So,
$a_6 = a + (6-1)d$
$\Rightarrow a + 5d = 19$ …… (i)
Similarity,
$a_{17} = a + (17 - 1)d$
$\Rightarrow a + 16d = 41$ …… (ii)
Solving (i) and (ii),
(ii) – (i) $\Rightarrow$
$a + 16d – (a + 5d) = 41 – 19$
$11d = 22$
$\Rightarrow d = 2$
Using d in (i), we get
$a + 5(2) = 19$
$a = 19 – 10 = 9$

Now, the 40^{th} term is given by $a_{40} = 9 + (40 - 1)2 = 9 + 78 = 87$
Therefore the 40^{th} term is 87.

7. If 9^{th} term of an A.P. is zero, prove its 29^{th} term is double the 19^{th} term.
Solution:

Given,
$a_9 = 0$
We know that, n^{th} term $a_n = a + (n - 1)d$
So, $a + (9 - 1)d = 0 \Rightarrow a + 8d = 0$ ……(i)
Now,

29^{th} term is given by $a_{29} = a + (29 - 1)d$
$\Rightarrow a_{29} = a + 28d$
And, $a_{29} = (a + 8d) + 20d$ [using (i)]
$\Rightarrow a_{29} = 20d$ ….. (ii)

Similarly, 19^{th} term is given by $a_{19} = a + (19 - 1)d$
$\Rightarrow a_{19} = a + 18d$
And, $a_{19} = (a + 8d) + 10d$ [using (i)]
$\Rightarrow a_{19} = 10d$ …..(iii)

On comparing (ii) and (iii), it's clearly seen that
$a_{29} = 2(a_{19})$

Therefore, 29^{th} term is double the 19^{th} term.

8. If 10 times the 10^{th} term of an A.P. is equal to 15 times the 15^{th} term, show that 25^{th} term of the A.P. is zero.
Solution:

Given,
10 times the 10^{th} term of an A.P. is equal to 15 times the 15^{th} term.
We know that, n^{th} term $a_n = a + (n - 1)d$
$\Rightarrow 10(a_{10}) = 15(a_{15})$
$10(a + (10 - 1)d) = 15(a + (15 - 1)d)$
$10(a + 9d) = 15(a + 14d)$
$10a + 90d = 15a + 210d$
$5a + 120d = 0$
$5(a + 24d) = 0$
$a + 24d = 0$
$a + (25 – 1)d = 0$
$\Rightarrow a_{25} = 0$
Therefore, the 25^{th} term of the A.P. is zero.

9. The 10^{th} and 18^{th} terms of an A.P. are 41 and 73 respectively. Find 26^{th} term.
Solution:

Given,
$A_{10} = 41$ and $a_{18} = 73$
We know that, n^{th} term $a_n = a + (n - 1)d$
So,
$a_{10} = a + (10 - 1)d$
$\Rightarrow a + 9d = 41$ …… (i)
Similarity,
$a_{18} = a + (18 - 1)d$
$\Rightarrow a + 17d = 73$ …… (ii)
Solving (i) and (ii),

(ii) – (i) ⇒
$a + 17d – (a + 9d) = 73 – 41$
$8d = 32$
$\Rightarrow d = 4$
Using d in (i), we get
$a + 9(4) = 41$
$a = 41 – 36 = 5$

Now, the 26th term is given by $a_{26} = 5 + (26 - 1)4 = 5 + 100 = 105$
Therefore the 26th term is 105.

10. In a certain A.P. the 24th term is twice the 10th term. Prove that the 72nd term is twice the 34th term.
Solution:

Given,
24th term is twice the 10th term.
We know that, nth term $a_n = a + (n - 1)d$
$\Rightarrow a_{24} = 2(a_{10})$
$a + (24 - 1)d = 2(a + (10 - 1)d)$
$a + 23d = 2(a + 9d)$
$a + 23d = 2a + 18d$
$a = 5d$ …. (1)

Now, the 72nd term can be expressed as
$a_{72} = a + (72 - 1)d$
$= a + 71d$
$= a + 5d + 66d$
$= a + a + 66d$ [using (1)]
$= 2(a + 33d)$
$= 2(a + (34 - 1)d)$
$= 2(a_{34})$
$\Rightarrow a_{72} = 2(a_{34})$
Hence, the 72nd term is twice the 34th term of the given A.P.

11. The 26th, 11th and the last term of an A.P. are 0, 3 and -1/5, respectively. Find the common difference and the number of terms.
Solution:

Given,
$a_{26} = 0$, $a_{11} = 3$ and a_n (last term) = -1/5 of an A.P.
We know that, nth term $a_n = a + (n - 1)d$
Then,
$a_{26} = a + (26 - 1)d$
$\Rightarrow a + 25d = 0$ …..(1)

And,
$a_{11} = a + (11 - 1)d$
$\Rightarrow a + 10d = 3$ …… (2)

Solving (1) and (2),
(1) – (2) ⇒
$a + 25d - (a + 10d) = 0 - 3$
$15d = -3$
$\Rightarrow d = -1/5$
Using d in (1), we get
$a + 25(-1/5) = 0$
$a = 5$

Now, given that the last term $a_n = -1/5$
$\Rightarrow 5 + (n - 1)(-1/5) = -1/5$
$5 + -n/5 + 1/5 = -1/5$
$25 - n + 1 = -1$
$n = 27$
Therefore, the A.P has 27 terms and its common difference is -1/5.

12. If the n^{th} term of the A.P. 9, 7, 5, …. is same as the n^{th} term of the A.P. 15, 12, 9, … find n.
Solution:

Given,
$A.P_1 = 9, 7, 5, \ldots$ and $A.P_2 = 15, 12, 9, \ldots$
And, we know that, n^{th} term $a_n = a + (n - 1)d$

For $A.P_1$,
a = 9, d = Second term – first term = 9 – 7 = -2
And, its n^{th} term $a_n = 9 + (n - 1)(-2) = 9 - 2n + 2$
$a_n = 11 - 2n$ …..(i)
Similarly, for $A.P_2$
a = 15, d = Second term – first term = 12 – 15 = -3
And, its n^{th} term $a_n = 15 + (n - 1)(-3) = 15 - 3n + 3$
$a_n = 18 - 3n$ …..(ii)

According to the question, its given that
n^{th} term of the $A.P_1$ = n^{th} term of the $A.P_2$
$\Rightarrow 11 - 2n = 18 - 3n$
$n = 7$
Therefore, the 7^{th} term of the both the A.Ps are equal.

13. Find the 12^{th} term from the end of the following arithmetic progressions:
(i) 3, 5, 7, 9, …. 201
(ii) 3,8,13, … ,253
(iii) 1, 4, 7, 10, … ,88

Solution:

In order the find the 12th term for the end of an A.P. which has n terms, its done by simply finding the ((n -12) + 1)th of the A.P
And we know, nth term $a_n = a + (n - 1)d$

(i) Given A.P = 3, 5, 7, 9, …. 201
Here, a = 3 and d = (5 - 3) = 2
Now, find the number of terms when the last term is known i.e, 201
$a_n = 3 + (n - 1)2 = 201$
$3 + 2n – 2 = 201$
$2n = 200$
$n = 100$
Hence, the A.P has 100 terms.

So, the 12th term from the end is same as (100 – 12 + 1)th of the A.P which is the 89th term.
$\Rightarrow a_{89} = 3 + (89 - 1)2$
$= 3 + 88(2)$
$= 3 + 176$
$= 179$
Therefore, the 12th term from the end of the A.P is 179.

(ii) Given A.P = 3,8,13, … ,253
Here, a = 3 and d = (8 - 3) = 5
Now, find the number of terms when the last term is known i.e, 253
$a_n = 3 + (n - 1)5 = 253$
$3 + 5n – 5 = 253$
$5n = 253 + 2 = 255$
$n = 255/5$
$n = 51$
Hence, the A.P has 51 terms.

So, the 12th term from the end is same as (51 – 12 + 1)th of the A.P which is the 40th term.
$\Rightarrow a_{40} = 3 + (40 - 1)5$
$= 3 + 39(5)$
$= 3 + 195$
$= 198$
Therefore, the 12th term from the end of the A.P is 198.

(iii) Given A.P = 1, 4, 7, 10, … ,88
Here, a = 1 and d = (4 - 1) = 3
Now, find the number of terms when the last term is known i.e, 88
$a_n = 1 + (n - 1)3 = 88$
$1 + 3n – 3 = 88$
$3n = 90$
$n = 30$

Hence, the A.P has 30 terms.

So, the 12^{th} term from the end is same as $(30 – 12 + 1)^{th}$ of the A.P which is the 19^{th} term.
$\Rightarrow a_{89} = 1 + (19 - 1)3$
$= 1 + 18(3)$
$= 1 + 54$
$= 55$
Therefore, the 12^{th} term from the end of the A.P is 55.

14. The 4^{th} term of an A.P. is three times the first and the 7^{th} term exceeds twice the third term by 1. Find the first term and the common difference.
Solution:

Let's consider the first term and the common difference of the A.P to be a and d respectively.
Then, we know that $a_n = a + (n - 1)d$
Given conditions,
4^{th} term of an A.P. is three times the first
Expressing this by equation we have,
$\Rightarrow a_4 = 3(a)$
$a + (4 - 1)d = 3a$
$3d = 2a \Rightarrow a = 3d/2$…….(i)
And,
7^{th} term exceeds twice the third term by 1
$\Rightarrow a_7 = 2(a_3) + 1$
$a + (7 – 1)d = 2(a + (3–1)d) + 1$
$a + 6d = 2a + 4d + 1$
$a – 2d +1 = 0$ ….. (ii)

Using (i) in (ii), we have
$3d/2 – 2d + 1 = 0$
$3d – 4d + 2 = 0$
$d = 2$
So, putting d = 2 in (i), we get a
$\Rightarrow a = 3$

Therefore, the first term is 3 and the common difference is 2.

15. Find the second term and the n^{th} term of an A.P. whose 6^{th} term is 12 and the 8^{th} term is 22.
Solution:

Given, in an A.P
$a_6 = 12$ and $a_8 = 22$
We know that $a_n = a + (n - 1)d$
So,
$a_6 = a + (6-1)d = a + 5d = 12$ …. (i)
And,

$a_8 = a + (8-1)d = a + 7d = 22$ ……. (ii)

Solving (i) and (ii), we have
(ii) - (i) ⇒
$a + 7d – (a + 5d) = 22 – 12$
$2d = 10$
$d = 5$
Putting d in (i) we get,
$a + 5(5) = 12$
$a = 12 – 25$
$a = -13$
Thus, for the A.P: a = -13 and d = 5
So, the n^{th} term is given by $a_n = a + (n-1)d$
$a_n = -13 + (n-1)5 = -13 + 5n – 5$
⇒ $a_n = 5n – 18$
Hence, the second term is given by $a_2 = 5(2) – 18 = 10 – 18 = -8$

16. How many numbers of two digit are divisible by 3?
Solution:

The first 2 digit number divisible by 3 is 12. And, the last 2 digit number divisible by 3 is 99.
So, this forms an A.P.
12, 15, 18, 21, …. , 99
Where, a = 12 and d = 3
Finding the number of terms in this A.P
⇒ $99 = 12 + (n-1)3$
$99 = 12 + 3n – 3$
$90 = 3n$
$n = 90/3 = 30$
Therefore, there are 30 two digit numbers divisible by 3.

17. An A.P. consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find 32^{nd} term.
Solution:

Given, an A.P of 60 terms
And, a = 7 and $a_{60} = 125$
We know that $a_n = a + (n - 1)d$
⇒ $a_{60} = 7 + (60 - 1)d = 125$
$7 + 59d = 125$
$59d = 118$
$d = 2$

So, the 32^{nd} term is given by
$a_{32} = 7 + (32 -1)2 = 7 + 62 = 69$

⇒ $a_{32} = 69$

18. The sum of 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 34. Find the first term and the common difference of the A.P.
Solution:

Given, in an A.P
The sum of 4th and 8th terms of an A.P. is 24
⇒ $a_4 + a_8 = 24$
And, we know that $a_n = a + (n - 1)d$
$[a + (4-1)d] + [a + (8-1)d] = 24$
$2a + 10d = 24$
$a + 5d = 12$ …. (i)

Also given that,
the sum of the 6th and 10th terms is 34
⇒ $a_6 + a_{10} = 34$
$[a + 5d] + [a + 9d] = 34$
$2a + 14d = 34$
$a + 7d = 17$ …… (ii)

Subtracting (i) form (ii), we have
$a + 7d – (a + 5d) = 17 – 12$
$2d = 5$
$d = 5/2$
Using d in (i) we get,
$a + 5(5/2) = 12$
$a = 12 – 25/2$
$a = -1/2$

Therefore, the first term is -1/2 and the common difference is 5/2.

19. The first term of an A.P. is 5 and its 100th term is -292. Find the 50th term of this A.P.
Solution:

Given, an A.P whose
$a = 5$ and $a_{100} = -292$
We know that $a_n = a + (n - 1)d$
$a100 = 5 + 99d = -292$
$99d = -297$
$d = -3$

Hence, the 50th term is
$a_{50} = a + 49d = 5 + 49(-3) = 5 – 147 = -142$

20. Find $a_{30} – a_{20}$ for the A.P.
(i) -9, -14, -19, -24 **(ii) a, a+d, a+2d, a+3d, ……**
Solution:

We know that $a_n = a + (n - 1)d$
So, $a_{30} – a_{20} = (a + 29d) – (a + 19) = 10d$

(i) Given A.P. -9, -14, -19, -24
Here, a = -9 and d = -14 – (-9) = = -14 + 9 = -5
So, $a_{30} – a_{20} = 10d$
$= 10(-5)$
$= -50$

(ii) Given A.P. a, a+d, a+2d, a+3d, ……
So, $a_{30} – a_{20} = (a + 29d) – (a + 19d$
$=10d$

21. Write the expression $a_n – a_k$ for the A.P. a, a+d, a+2d, …..
Hence, find the common difference of the A.P. for which
(i) 11th term is 5 and 13th term is 79.
(ii) $a_{10} – a_5 = 200$
(iii) 20th term is 10 more than the 18th term.
Solution:

Given A.P. a, a+d, a+2d, …..
So, $a_n = a + (n-1)d = a + nd – d$
And, $a_k = a + (k-1)d = a + kd – d$
$a_n - a_k = (a + nd – d) – (a + kd – d)$
$= (n – k)d$

(i) Given 11th term is 5 and 13th term is 79,
Here n = 13 and k = 11,
$a_{13} – a_{11} = (13 – 11)d = 2d$
$\Rightarrow 79 – 5 = 2d$
$d = 74/2 = 37$

(ii) Given, $a_{10} – a_5 = 200$
$\Rightarrow (10 - 5)d = 200$
$5d = 200$
$d = 40$

(iii) Given, 20th term is 10 more than the 18th term.
$\Rightarrow a_{20} – a_{18} = 10$
$(20 - 18)d = 10$
$2d = 10$
$d = 5$

22. Find n if the given value of x is the n^{th} term of the given A.P.
(i) 25, 50, 75, 100, ; x = 1000 **(ii) -1, -3, -5, -7, …; x = -151**
(iii) 5½, 11, 16½, 22, ….; x = 550 **(iv) 1, 21/11, 31/11, 41/11, …; x = 171/11**
Solution:

(i) Given A.P. 25, 50, 75, 100, …… ,1000
Here, a = 25 d = 50 – 25 = 25
Last term (n^{th} term) = 1000
We know that $a_n = a + (n - 1)d$
⇒ 1000 = 25 + (n-1)25
1000 = 25 + 25n – 25
n = 1000/25
n = 40

(ii) Given A.P. -1, -3, -5, -7, …., -151
Here, a = -1 d = -3 – (-1) = -2
Last term (n^{th} term) = -151
We know that $a_n = a + (n - 1)d$
⇒ -151 = -1 + (n-1)(-2)
-151 = -1 - 2n + 2
n = 152/2
n = 76

(iii) Given A.P. 5½, 11, 16½, 22, … , 550
Here, a = 5½ d = 11 – (5½) = 5½ = 11/2
Last term (n^{th} term) = 550
We know that $a_n = a + (n - 1)d$
⇒ 550 = 5½ + (n-1)(11/2)
550 x 2 = 11+ 11n – 11
1100 = 11n
n = 100

(iv) Given A.P. 1, 21/11, 31/11, 41/11, 171/11
Here, a = 1 d = 21/11 – 1 = 10/11
Last term (n^{th} term) = 171/11
We know that $a_n = a + (n - 1)d$
⇒ 171/11 = 1 + (n-1)10/11
171 = 11 + 10n – 10
n = 170/10
n = 17

23. The eighth term of an A.P is half of its second term and the eleventh term exceeds one third of its fourth term by 1. Find the 15^{th} term.
Solution:

Given, an A.P in which,

$a_8 = 1/2(a_2)$
$a_{11} = 1/3(a_4) + 1$
We know that $a_n = a + (n - 1)d$
$\Rightarrow a_8 = 1/2(a_2)$
$a + 7d = 1/2(a + d)$
$2a + 14d = a + d$
$a + 13d = 0$ …… (i)

And, $a_{11} = 1/3(a_4) + 1$
$a + 10d = 1/3(a + 3d) + 1$
$3a + 30d = a + 3d + 3$
$2a + 27d = 3$ …… (ii)

Solving (i) and (ii), by (ii) – 2x(i) ⇒
$2a + 27d – 2(a + 13d) = 3 - 0$
$d = 3$
Putting d in (i) we get,
$a + 13(3) = 0$
$a = -39$

Thus, the 15^{th} term $a_{15} = -39 + 14(3) = -39 + 42 = 3$

24. Find the arithmetic progression whose third term is 16 and the seventh term exceeds its fifth term by 12.
Solution:

Given, in an A.P
$a_3 = 16$ and $a_7 = a_5 + 12$
We know that $a_n = a + (n - 1)d$
$\Rightarrow a + 2d = 16$…… (i)
And,
$a + 6d = a + 4d + 12$
$2d = 12$
$\Rightarrow d = 6$

Using d in (i), we have
$a + 2(6) = 16$
$a = 16 – 12 = 4$

Hence, the A.P is 4, 10, 16, 22, …….

25. The 7^{th} term of an A.P. is 32 and its 13^{th} term is 62. Find the A.P.
Solution:

Given,
$a_7 = 32$ and $a_{13} = 62$

From $a_n - a_k = (a + nd – d) – (a + kd – d)$
$= (n – k)d$

$a_{13} – a_7 = (13 - 7)d = 62 – 32 = 30$
$6d = 30$
$d = 5$

Now,
$a_7 = a + (7 - 1)5 = 32$
$a + 30 = 32$
$a = 2$

Hence, the A.P is 2, 7, 12, 17, ……

26. Which term of the A.P. 3, 10, 17, …. will be 84 more than its 13th term ?
Solution:

Given, A.P. 3, 10, 17, ….
Here, $a = 3$ and $d = 10 – 3 = 7$
According the question,
$a_n = a_{13} + 84$
Using $a_n = a + (n - 1)d$,
$3 + (n - 1)7 = 3 + (13 - 1)7 + 84$
$3 + 7n – 7 = 3 + 84 + 84$
$7n = 168 + 7$
$n = 175/7$
$n = 25$

Therefore, it the 25th term which is 84 more than its 13th term.

27. Two arithmetic progressions have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?
Solution:

Let the two A.Ps be $A.P_1$ and $A.P_2$
For $A.P_1$ the first term = a and the common difference = d
And for $A.P_2$ the first term = b and the common difference = d
So, from the question we have
$a_{100} – b_{100} = 100$
$(a + 99d) – (b + 99d) = 100$
$a - b = 100$

Now, the difference between their 1000th terms is,
$(a + 999d) – (b + 999d) = a – b = 100$

Therefore, the difference between their 1000th terms is also 100.