Exercise 9.6

1. Find the sum of the following arithmetic progressions:
(i) 50, 46, 42, ... to 10 terms
(ii) 1, 3, 5, 7, ... to 12 terms
(iii) 3, 9/2, 6, 15/2, ... to 25 terms
(iv) 41, 36, 31, ... to 12 terms
(v) a + b, a - b, a - 3b, ... to 22 terms
(vi) $(x - y)^2$, $(x^2 + y^2)$, $(x + y)^2$, to 22 tams

(vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}$, to n terms

(viii) – 26, – 24, – 22, ... to 36 terms
Solution:

In an A.P if the first term = a, common difference = d, and if there are n terms.
Then, sum of n terms is given by:

$$S_n = \frac{n}{2}\{2a + (n - 1)d\}$$

(i) Given A.P.is 50, 46, 42 to 10 term.
First term (a) = 50
Common difference (d) = 46 - 50 = – 4
n^{th} term (n) = 10

$$\text{Then } S_{10} = \frac{10}{2}\{2.50 + (10 - 1) - 4\}$$

= 5{100 - 9.4}
= 5{100 - 36}
= S × 64
∴ S_{10} = 320

(ii) Given A.P is, 1, 3, 5, 7, ...to 12 terms.
First term (a) = 1
Common difference (d) = 3 - 1 = 2
n^{th} term (n) = 12

$$\text{Sum of } n^{th} \text{ terms } S_{12} = \frac{12}{2} \times \{2.1 + (12 - 1).2\}$$

= 6 × {2 + 22} = 6.24
∴ S_{12} = 144

(iii) Given A.P. is 3, 9/2, 6, 15/2, ... to 25 terms
First term (a) = 3
Common difference (d) = 9/2 - 3 = 3/2
Sum of n terms S_n, given n = 25

$$S_{25} = \frac{n}{2}(2a + (n-1)d)$$

$$S_{25} = \frac{25}{2}(2.3 + 25 - 1) \times \frac{3}{2}$$

$$= \frac{25}{2}\left(6 + 24.\frac{3}{2}\right)$$

$$= \frac{25}{2}\ (6 + 36)$$

$$= \frac{25}{2}\ (42)$$

$\therefore S_{25} = 525$

(iv) Given expression is 41, 36, 31, ….. to 12 terms.
First term (a) = 41
Common difference (d) = 36 - 41 = -5
Sum of n terms S_n, given n = 12

$$S_{12} = \frac{n}{2}(2a + (n-1)d)$$

$$S_{12} = \frac{12}{6}(2.41 + 12 - 1) \times -5$$

$$= 6\big(82 + 11 \times (-5)\big)$$

$$= 6 \times 27$$

$$= 162$$

$\therefore\ \ S_{12} = 162.$

(v) a + b, a – b, a - 3b, ….. to 22 terms
First term (a) = a + b
Common difference (d) = a - b - a - b = -2b
Sum of n terms S_n = n/2{2a(n - 1). d}
Here n = 22
S_{22} = 22/2{2.(a + b) + (22 - 1). -2b}
= 11{2(a + b) - 22b)
= 11{2a - 20b}

$= 22a - 440b$
$\therefore S_{22} = 22a - 440b$

(vi) $(x - y)^2, (x^2 + y^2), (x + y)^2, \ldots$ to n terms
First term (a) = $(x - y)^2$
Common difference (d) = $x^2 + y^2 - (x - y)^2$
$= x^2 + y^2 - (x^2 + y^2 - 2xy)$
$= x^2 + y^2 - x^2 + y^2 + 2xy$
$= 2xy$
Sum of n^{th} terms $S_n = n/2\{2a(n - 1). d\}$
$= n/2\{2(x - y)^2 + (n - 1). 2xy\}$
$= n\{(x - y)^2 + (n - 1)xy\}$
$\therefore S_n = n\{(x — y)^2 + (n — 1). xy)$

(vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \ldots\ldots$ to n terms

First term (a) $= \frac{x-y}{x+y}$

Common difference (d) $= \frac{3x-2y}{x+y} - \frac{x-y}{x+y}$

$= \frac{3x-2y-x-y}{x+y}$

$= \frac{2x-y}{x+y}$

Sum of n terms $S_n = \frac{n}{2}(2a + (n-1).d)$

$= \frac{n}{2}\left[2.\frac{x-y}{x+y} = + (n-1).2 - \frac{2x-y}{x+y}\right\}$

$$= \frac{n}{2(x+y)}\{2(x-y)+(n-1)(2x-y)\}$$

$$= \frac{n}{2(x+y)}, \{2x-2y+2nx-ny-2x+y\}$$

$$= \frac{n}{2(x+y)}\{n(2x - y) - y\}$$

$$\therefore S_n = \frac{n}{2(x+y)}\{n(2x - y) - y\}$$

(viii) Given expression -26, - 24. -22, to 36 terms
First term (a) = -26
Common difference (d) = -24 - (-26)
= -24 + 26 = 2
Sum of n terms, S_n = n/2{2a + (n - 1)d) for n = 36
S_n = 36/2{2(-26) + (36 - 1)2}
= 18[-52 + 70]
= 18x18
= 324
∴ S_n = 324

2. Find the sum to n terms of the A.P. 5, 2, –1, – 4, –7, ...
Solution:

Given AP is 5, 2, -1, -4, -7, ...
Here, a = 5, d = 2 - 5 = -3
We know that,
S_n = n/2{2a + (n - 1)d}
= n/2{2.5 + (n - 1) - 3}
= n/2{10 - 3(n - 1)}
= n/2{13 - 3n)
∴ S_n = n/2(13 - 3n)

3. Find the sum of n terms of an A.P. whose the terms is given by a_n = 5 - 6n.
Solution:

Given nth term of the A.P as a_n = 5 - 6n
Put n = 1, we get
a_1 = 5 - 6.1 = -1
So, first term (a) = -1
Last term (a_n) = 5 - 6n = 1
Then, S_n = n/2(-1 + 5 - 6n)

$= n/2(4 - 6n) = n(2 - 3n)$

4. Find the sum of last ten terms of the A.P. : 8, 10, 12, 14, .. , 126
Solution:

Given A.P. 8, 10, 12, 14, .. , 126
Here, $a = 8$, $d = 10 – 8 = 2$
We know that, $a_n = a + (n - 1)d$
So, to find the number of terms
$126 = 8 + (n - 1)2$
$126 = 8 + 2n - 2$
$2n = 120$
$n = 60$

Next, let's find the 51^{st} term
$a_{51} = 8 + 50(2) = 108$

So, the sum of last ten terms is the sum of $a_{51} + a_{52} + a_{53} + \ldots\ldots + a_{60}$
Here, $n = 10$, $a = 108$ and $l = 126$
$S = 10/2 [108 + 126]$
$= 5(234)$
$= 1170$
Hence, the sum of last ten terms of the A.P is 1170.

5. Find the sum of first 15 terms of each of the following sequences having n^{th} term as:
(i) $a_n = 3 + 4n$
(ii) $b_n = 5 + 2n$
(iii) $x_n = 6 - n$
(iv) $y_n = 9 – 5n$
Solution:

(i) Given an A.P. whose n^{th} term is given by $a_n = 3 + 4n$
To find the sum of the n terms of the given A.P., using the formula,
$S_n = n(a + l)/ 2$
Where, a = the first term l = the last term.
Putting $n = 1$ in the given a_n, we get
$a = 3 + 4(1) = 3 + 4 = 7$
For the last term (l), here $n = 15$
$a_{15} = 3 + 4(15) = 63$

So, $S_n = 15(7 + 63)/2$
$= 15 \times 35$
$= 525$
Therefore, the sum of the 15 terms of the given A.P. is $S_{15} = 525$

(ii) Given an A.P. whose n^{th} term is given by $b_n = 5 + 2n$

To find the sum of the n terms of the given A.P., using the formula,
$S_n = n(a + l)/ 2$
Where, a = the first term l = the last term.
Putting n = 1 in the given b_n, we get
$a = 5 + 2(1) = 5 + 2 = 7$
For the last term (l), here n = 15
$a_{15} = 5 + 2(15) = 35$

So, $S_n = 15(7 + 35)/2$
$= 15 \times 21$
$= 315$
Therefore, the sum of the 15 terms of the given A.P. is $S_{15} = 315$

(iii) Given an A.P. whose n^{th} term is given by $x_n = 6 - n$
To find the sum of the n terms of the given A.P., using the formula
$S_n = n(a + l)/ 2$
Where, a = the first term l = the last term.
Putting n = 1 in the given x_n, we get
$a = 6 – 1 = 5$
For the last term (l), here n = 15
$a_{15} = 6 – 15 = -9$

So, $S_n = 15(5 – 9)/2$
$= 15 \times (-2)$
$= -30$
Therefore, the sum of the 15 terms of the given A.P. is $S_{15} = -30$

(iv) Given an A.P. whose n^{th} term is given by $y_n = 9 – 5n$
To find the sum of the n terms of the given A.P., using the formula,
$S_n = n(a + l)/ 2$
Where, a = the first term l = the last term.
Putting n = 1 in the given y_n, we get
$a = 9 - 5(1) = 9 – 5 = 4$
For the last term (l), here n = 15
$a_{15} = 9 - 5(15) = -66$

So, $S_n = 15(4 - 66)/2$
$= 15 \times (-31)$
$= -465$
Therefore, the sum of the 15 terms of the given A.P. is $S_{15} = -465$

6. Find the sum of first 20 terms the sequence whose n^{th} term is $a_n = An + B$.
Solution:

Given an A.P. whose nth term is given by, $a_n = An + B$
We need to find the sum of first 20 terms.

To find the sum of the n terms of the given A.P., we use the formula,
$S_n = n(a + l)/ 2$
Where, a = the first term l = the last term,
Putting n = 1 in the given a_n, we get
$a = A(1) + B = A + B$
For the last term (l), here n = 20
$A_{20} = A(20) + B = 20A + B$
$S_{20} = 20/2((A + B) + 20A + B)$
$= 10[21A + 2B]$
$= 210A + 20B$
Therefore, the sum of the first 20 terms of the given A.P. is 210 A + 20B

7. Find the sum of first 25 terms of an A.P whose n^{th} term is given by $a_n = 2 - 3n$.
Solution:

Given an A.P. whose n^{th} term is given by $a_n = 2 – 3n$
To find the sum of the n terms of the given A.P., we use the formula,
$S_n = n(a + l)/ 2$
Where, a = the first term l = the last term.
Putting n = 1 in the given a_n, we get
$a = 2 - 3(1) = -1$
For the last term (l), here n = 25
$a_{25} = 2 - 3(25) = -73$

So, $S_n = 25(-1 - 73)/2$
$= 25 \times (-37)$
$= -925$
Therefore, the sum of the 25 terms of the given A.P. is $S_{25} = -465$

8. Find the sum of first 25 terms of an A.P whose n^{th} term is given by $a_n = 7 - 3n$.
Solution:

Given an A.P. whose n^{th} term is given by $a_n = 7 – 3n$
To find the sum of the n terms of the given A.P., we use the formula,
$S_n = n(a + l)/ 2$
Where, a = the first term l = the last term.
Putting n = 1 in the given a_n, we get
$a = 7 - 3(1) = 7 – 3 = 4$
For the last term (l), here n = 25
$a_{15} = 7 - 3(25) = -68$

So, $S_n = 25(4 - 68)/2$
$= 25 \times (-32)$
$= -800$
Therefore, the sum of the 15 terms of the given A.P. is $S_{25} = -80$

9. If the sum of a certain number of terms starting from first term of an A.P. is 25, 22, 19, . . ., is 116. Find the last term.
Solution:

Given the sum of the certain number of terms of an A.P. = 116
We know that, $S_n = n/2[2a + (n - 1)d]$
Where; a = first term for the given A.P.
d = common difference of the given A.P.
n = number of terms So for the given A.P.(25, 22, 19,...)
Here we have, the first term (a) = 25
The sum of n terms $S_n = 116$
Common difference of the A.P. (d) = $a_2 - a_1 = 22 – 25 = -3$
Now, substituting values in S_n
$\Rightarrow 116 = n/2[2(25) + (n - 1)(-3)]$
$\Rightarrow (n/2)[50 + (-3n + 3)] = 116$
$\Rightarrow (n/2)[53 - 3n] = 116$
$\Rightarrow 53n - 3n^2 = 116 \times 2$
Thus, we get the following quadratic equation,
$3n^2 - 53n + 232 = 0$
By factorization method of solving, we have
$\Rightarrow 3n^2 - 24n - 29n + 232 = 0$
$\Rightarrow 3n(n - 8) - 29 (n - 8) = 0$
$\Rightarrow (3n - 29)(n - 8) = 0$
So, $3n - 29 = 0$
$\Rightarrow n = 29/3$
Also, $n - 8 = 0$
$\Rightarrow n = 8$
Since, n cannot be a fraction, so the number of terms is taken as 8.
So, the term is:
$a_8 = a_1 + 7d = 25 + 7(-3) = 25 - 21 = 4$
Hence, the last term of the given A.P. such that the sum of the terms is 116 is 4.

10. (i) How many terms of the sequence 18, 16, 14.... should be taken so that their sum is zero.
(ii) How many terms are there in the A.P. whose first and fifth terms are -14 and 2 respectively and the sum of the terms is 40?
(iii) How many terms of the A.P. 9, 17, 25, . . . must be taken so that their sum is 636?
(iv) How many terms of the A.P. 63, 60, 57, . . . must be taken so that their sum is 693?
(v) How many terms of the A.P. is 27, 24, 21. . . should be taken that their sum is zero?
Solution:

(i) Given AP. is 18, 16, 14, ...
We know that,
$S_n = n/2[2a + (n - 1)d]$
Here,
The first term (a) = 18
The sum of n terms (S_n) = 0 (given)

Common difference of the A.P.
(d) = a_2 - a_1 = 16 - 18 = – 2
So, on substituting the values in S_n
⟹ 0 = n/2[2(18) + (n − 1)(−2)]
⟹ 0 = n/2[36 + (−2n + 2)]
⟹ 0 = n/2[38 − 2n] Further, n/2
⟹ n = 0 Or, 38 - 2n = 0
⟹ 2n = 38
⟹ n = 19
Since, the number of terms cannot be zer0, hence the number of terms (n) should be 19.

(ii) Given, the first term (a) = -14, Filth term (a_5) = 2, Sum of terms (S_n) = 40 of the A.P.
If the common difference is taken as d.
Then, a_5 = a + 4d
⟹ 2 = -14 + 4d
⟹ 2 + 14 = 4d
⟹ 4d = 16
⟹ d = 4

Next, we know that S_n = n/2[2a + (n − 1)d]
Where; a = first term for the given A.P.
d = common difference of the given A.P.
n = number of terms

Now, on substituting the values in S_n
⟹ 40 = n/2[2(−14) + (n − 1)(4)]
⟹ 40 = n/2[−28 + (4n − 4)]
⟹ 40 = n/2[−32 + 4n]
⟹ 40(2) = - 32n + $4n^2$
So, we get the following quadratic equation,
$4n^2$ - 32n - 80 = 0
⟹ n^2 - 8n + 20 = 0
On solving by factorization method, we get
$4n^2$ - 10n + 2n + 20 = 0
⟹ n(n - 10) + 2(n - 10) = 0
⟹ (n + 2)(n - 10) = 0
Either, n + 2 = 0
⟹ n = -2
Or, n - 10 = 0
⟹ n = 10
Since the number of terms cannot be negative**.**
Therefore, the number of terms (n) is 10.

(iii) Given AP is 9, 17, 25,...
We know that,
S_n = n/2[2a + (n − 1)d]

Here we have,
The first term (a) = 9 and the sum of n terms (S_n) = 636
Common difference of the A.P. (d) = a_2 - a_1 = 17 - 9 = 8
Substituting the values in S_n, we get
$\Longrightarrow 636 = n/2[2(9) + (n - 1)(8)]$
$\Longrightarrow 636 = n/2[18 + (8n - 8)]$
$\Longrightarrow 636(2) = (n)[10 + 8n]$
$\Longrightarrow 1271 = 10n + 8n^2$
Now, we get the following quadratic equation,
$\Longrightarrow 8n^2 + 10n - 1272 = 0$
$\Longrightarrow 4n^2 + 5n - 636 = 0$
On solving by factorisation method, we have
$\Longrightarrow 4n^2 - 48n + 53n - 636 = 0$
$\Longrightarrow 4n(n - 12) - 53(n - 12) = 0$
$\Longrightarrow (4n - 53)(n - 12) = 0$
Either $4n - 53 = 0 \Longrightarrow n = 534$
Or, $n - 12 = 0 \Longrightarrow n = 12$
Since, the number of terms cannot be a fraction.
Therefore, the number of terms (n) is 12.

(iv) Given A.P. is 63, 60, 57,...
We know that,
$S_n = n/2[2a + (n - 1)d]$
Here we have,
the first term (a) = 63
The sum of n terms (S_n) = 693
Common difference of the A.P. (d) = a_2 - a_1 = 60 - 63 = –3
On substituting the values in S_n we get
$\Longrightarrow 693 = n/2[2(63) + (n - 1)(-3)]$
$\Longrightarrow 693 = n/2[163+(-3n + 3)]$
$\Longrightarrow 693 = n/2[129 - 3n]$
$\Longrightarrow 693(2) = 129n - 3n^2$
Now, we get the following quadratic equation.
$\Longrightarrow 3n^2 - 129n + 1386 = 0$
$\Longrightarrow n^2 - 43n + 462$
Solving by factorisation method, we have
$\Longrightarrow n^2 - 22n - 21n + 462 = 0$
$\Longrightarrow n(n - 22) -21(n - 22) = 0$
$\Longrightarrow (n - 22) (n - 21) = 0$
Either, $n - 22 = 0 \Longrightarrow n = 22$
Or, $n - 21 = 0 \Longrightarrow n = 21$
Now, the 22^{nd} term will be $a_{22} = a_1 + 21d = 63 + 21(-3) = 63 - 63 = 0$
So, the sum of 22 as well as 21 terms is 693.
Therefore, the number of terms (n) is 21 or 22.

(v) Given A.P. is 27, 24, 21. . .
We know that,
$S_n = n/2[2a + (n - 1)d]$
Here we have, the first term (a) = 27
The sum of n terms (S_n) = 0
Common difference of the A.P. (d) = $a_2 - a_1 = 24 - 27 = -3$
On substituting the values in S_n, we get
$\Rightarrow 0 = n/2[2(27) + (n - 1)(-3)]$
$\Rightarrow 0 = (n)[54 + (n - 1)(-3)]$
$\Rightarrow 0 = (n)[54 - 3n + 3]$
$\Rightarrow 0 = n [57 - 3n]$ Further we have, n = 0 Or, 57 - 3n = 0
$\Rightarrow 3n = 57$
$\Rightarrow n = 19$
The number of terms cannot be zero,
Hence, the numbers of terms (n) is 19.

11. Find the sum of the first
(i) 11 terms of the A.P. : 2, 6, 10, 14, . . .
(ii) 13 terms of the A.P. : -6, 0, 6, 12, . . .
(iii) 51 terms of the A.P. : whose second term is 2 and fourth term is 8.
Solution:

We know that the sum of terms for different arithmetic progressions is given by
$S_n = n/2[2a + (n - 1)d]$
Where; a = first term for the given A.P. d = common difference of the given A.P. n = number of terms

(i) Given A.P 2, 6, 10, 14,... to 11 terms.
Common difference (d) = $a_2 - a_1 = 10 - 6 = 4$
Number of terms (n) = 11
First term for the given A.P. (a) = 2
So,
$S_{11} = 11/2[2(2) + (11 - 1)4]$
$= 11/2[2(2) + (10)4]$
$= 11/2[4 + 40]$
$= 11 \times 22$
$= 242$
Hence, the sum of first 11 terms for the given A.P. is 242

(ii) Given A.P. – 6, 0, 6, 12, ... to 13 terms.
Common difference (d) = $a_2 - a_1 = 6 - 0 = 6$
Number of terms (n) = 13
First term (a) = -6
So,
$S_{13} = 13/2[2(-6) + (13 - 1)6]$
$= 13/2[(-12) + (12)6]$
$= 13/2[60] = 390$

Hence, the sum of first 13 terms for the given A.P. is 390

(iii) 51 terms of an AP whose $a_2 = 2$ and $a_4 = 8$
We know that, $a_2 = a + d$
$2 = a + d$ ...(2)
Also, $a_4 = a + 3d$
$8 = a + 3d$ ... (2)
Subtracting (1) from (2), we have
$2d = 6$
$d = 3$
Substituting d = 3 in (1), we get
$2 = a + 3$
$\Longrightarrow a = -1$

Given that the number of terms (n) = 51
First term (a) = -1
So,
$S_n = 51/2[2(-1) + (51 - 1)(3)]$
$= 51/2[-2 + 150]$
$= 51/2[158]$
$= 3774$
Hence, the sum of first 51 terms for the A.P. is 3774.

12. Find the sum of
(i) the first 15 multiples of 8
(ii) the first 40 positive integers divisible by (a) 3 (b) 5 (c) 6.
(iii) all 3 - digit natural numbers which are divisible by 13.
(iv) all 3 - digit natural numbers which are multiples of 11.

Solution:
We know that the sum of terms for an A.P is given by
$S_n = n/2[2a + (n - 1)d]$
Where; a = first term for the given A.P. d = common difference of the given A.P. n = number of terms

(i) Given, first 15 multiples of 8.
These multiples form an A.P: 8, 16, 24, …… , 120
Here, a = 8 , d = 61 – 8 = 8 and the number of terms(n) = 15
Now, finding the sum of 15 terms, we have

$$S_n = \frac{15}{2}[2(8) + (15-1)8]$$

$$= \frac{15}{2}[16 + (14)8]$$

$$= \frac{15}{2}[16 + 12]$$

$$= \frac{15}{2}[128]$$

$$= 960$$ \

Hence, the sum of the first 15 multiples of 8 is 960

(ii)(a) First 40 positive integers divisible by 3.
Hence, the first multiple is 3 and the 40^{th} multiple is 120.
And, these terms will form an A.P. with the common difference of 3.
Here, First term (a) = 3
Number of terms (n) = 40
Common difference (d) = 3
So, the sum of 40 terms
$S_{40} = 40/2[2(3) + (40 - 1)3]$
$= 20[6 + (39)3]$
$= 20(6 + 117)$
$= 20(123) = 2460$
Thus, the sum of first 40 multiples of 3 is 2460.

(b) First 40 positive integers divisible by 5
Hence, the first multiple is 5 and the 40^{th} multiple is 200.
And, these terms will form an A.P. with the common difference of 5.
Here, First term (a) = 5
Number of terms (n) = 40
Common difference (d) = 5
So, the sum of 40 terms
$S_{40} = 40/2[2(5) + (40 - 1)5]$
$= 20[10 + (39)5]$
$= 20 (10 + 195)$
$= 20 (205) = 4100$
Hence, the sum of first 40 multiples of 5 is 4100.

(c) First 40 positive integers divisible by 6

Hence, the first multiple is 6 and the 40th multiple is 240.
And, these terms will form an A.P. with the common difference of 6.
Here, First term (a) = 6
Number of terms (n) = 40
Common difference (d) = 6
So, the sum of 40 terms
S_{40} = 40/2[2(6) + (40 − 1)6]
= 20[12 + (39)6]
=20(12 + 234)
= 20(246) = 4920
Hence, the sum of first 40 multiples of 6 is 4920.

(iii) All 3 digit natural number which are divisible by 13.
So, we know that the first 3 digit multiple of 13 is 104 and the last 3 digit multiple of 13 is 988.
And, these terms form an A.P. with the common difference of 13.
Here, first term (a) = 104 and the last term (l) = 988
Common difference (d) = 13
Finding the number of terms in the A.P. by, $a_n = a + (n − 1)d$
We have,
988 = 104 + (n - 1)13
⟹ 988 = 104 + 13n -13
⟹ 988 = 91 + 13n
⟹ 13n = 897
⟹ n = 69
Now, using the formula for the sum of n terms, we get
S_{69} = 69/2[2(104) + (69 − 1)13]
= 69/2[208 + 884]
= 69/2[1092]
= 69(546)
= 37674
Hence, the sum of all 3 digit multiples of 13 is 37674.

(iv) All 3 digit natural number which are multiples of 11.
So, we know that the first 3 digit multiple of 11 is 110 and the last 3 digit multiple of 13 is 990.
And, these terms form an A.P. with the common difference of 11.
Here, first term (a) = 110 and the last term (l) = 990
Common difference (d) = 11
Finding the number of terms in the A.P. by, $a_n = a + (n − 1)d$
We get,
990 = 110 + (n - 1)11
⟹ 990 = 110 + 11n -11
⟹ 990 = 99 + 11n
⟹ 11n = 891
⟹ n = 81
Now, using the formula for the sum of n terms, we get
S_{81} = 81/2[2(110) + (81 − 1)11]

= 81/2[220 + 880]
= 81/2[1100]
= 81(550)
= 44550
Hence, the sum of all 3 digit multiples of 11 is 44550.

12. Find the sum:
(i) 2 + 4 + 6 + . . . + 200
(ii) 3 + 11 + 19 + . . . + 803
(iii) (-5) + (-8) + (-11) + . . . + (- 230)
(iv) 1 + 3 + 5 + 7 + . . . + 199
(v) $7 + 10\frac{1}{2} + 14 + \cdots + 84$
(vi) 34 + 32 + 30 + . . . + 10
(vii) 25 + 28 + 31 + . . . + 100
Solution:

We know that the sum of terms for an A.P is given by
$S_n = n/2[2a + (n - 1)d]$
Where; a = first term for the given A.P. d = common difference of the given A.P. n = number of terms
Or $S_n = n/2[a + l]$
Where; a = first term for the given A.P. ;l = last term for the given A.P

(i) Given series. 2 + 4 + 6 + . . . + 200 which is an A.P
Where, a = 2 ,d = 4 – 2 = 2 and last term (a_n = l) = 200
We know that, $a_n = a + (n - 1)d$
So,
200 = 2 + (n - 1)2
200 = 2 + 2n – 2
n = 200/2 = 100

Now, for the sum of these 100 terms
S_{100} = 100/2 [2 + 200]
= 50(202)
= 10100
Hence, the sum of terms of the given series is 10100.

(ii) Given series. 3 + 11 + 19 + . . . + 803 which is an A.P
Where, a = 3 ,d = 11 – 3 = 8 and last term (a_n = l) = 803
We know that, $a_n = a + (n - 1)d$
So,
803 = 3 + (n - 1)8
803 = 3 + 8n – 8
n = 808/8 = 101

Now, for the sum of these 101 terms
S_{101} = 101/2 [3 + 803]
= 101(806)/2
= 101 x 403
= 40703
Hence, the sum of terms of the given series is 40703.

(iii) Given series (-5) + (-8) + (-11) + . . . + (- 230) which is an A.P
Where, a = -5 ,d = -8 – (-5) = -3 and last term (a_n = l) = -230
We know that, a_n = a + (n - 1)d
So,
-230 = -5 + (n - 1)(-3)
-230 = -5 - 3n + 3
3n = -2 + 230
n = 228/3 = 76

Now, for the sum of these 76 terms
S_{76} = 76/2 [-5 + (-230)]
= 38 x (-235)
= -8930
Hence, the sum of terms of the given series is -8930.

(iv) Given series. 1 + 3 + 5 + 7 + . . . + 199 which is an A.P
Where, a = 1 ,d = 3 – 1 = 2 and last term (a_n = l) = 199
We know that, a_n = a + (n - 1)d
So,
199 = 1 + (n - 1)2
199 = 1 + 2n – 2
n = 200/2 = 100

Now, for the sum of these 100 terms
S_{100} = 100/2 [1 + 199]
= 50(200)
= 10000
Hence, the sum of terms of the given series is 10000.

(v) Given series $7 + 10\frac{1}{2} + 14 + \cdots + 84$ which is an A.P

Where, a = 7, d = 10 ½ - 7 = (21 – 14)/2 = 7/2 and last term (a_n = l) = 84
We know that, a_n = a + (n - 1)d
So,
84 = 7 + (n - 1)(7/2)
168 = 14 + 7n – 7
n = (168 – 7)/7 = 161/7 = 23

Now, for the sum of these 23 terms
S_{23} = 23/2 [7 + 84]
= 23(91)/2
= 2093/2
Hence, the sum of terms of the given series is 2093/2.

(vi) Given series, 34 + 32 + 30 + . . . + 10 which is an A.P
Where, a = 34 ,d = 32 – 34 = -2 and last term (a_n = l) = 10
We know that, $a_n = a + (n - 1)d$
So,
10 = 34 + (n - 1)(-2)
10 = 34 - 2n + 2
n = (36 – 10)/2 = 13

Now, for the sum of these 13 terms
S_{13} = 13/2 [34 + 10]
= 13(44)/2
= 13 x 22
= 286
Hence, the sum of terms of the given series is 286.

(vii) Given series, 25 + 28 + 31 + . . . + 100 which is an A.P
Where, a = 25 ,d = 28 – 25 = 3 and last term (a_n = l) = 100
We know that, $a_n = a + (n - 1)d$
So,
100 = 25 + (n - 1)(3)
100 = 25 + 3n - 3
n = (100 – 22)/3 = 26

Now, for the sum of these 26 terms
S_{100} = 26/2 [24 + 100]
= 13(124)
= 1625
Hence, the sum of terms of the given series is 1625.

14. The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?
Solution:

Given, the first term of the A.P (a) = 17
The last term of the A.P (l) = 350
The common difference (d) of the A.P. = 9
Let the number of terms be n. And, we know that; l = a + (n - 1)d
So, 350 = 17 + (n- 1) 9
⟹ 350 = 17 + 9n - 9
⟹ 350 = 8 + 9n

$\Longrightarrow$ 350 - 8 = 9n
Thus we get, n = 38
Now, finding the sum of terms
S_n = n/2[a + l]
= 38/2(17 + 350)
= 19 × 367
= 6973
Hence, the number of terms is of the A.P is 38 and their sum is 6973.

15. The third term of an A.P. is 7 and the seventh term exceeds three times the third term by 2. Find the first term, the common difference and the sum of first 20 terms.
Solution:

Let's consider the first term as a and the common difference as d.
Given,
a_3 = 7 .... (1) and,
$a_7 = 3a_3 + 2$ .... (2)
So, using (1) in (2), we get,
a_7 = 3(7) + 2 = 21 + 2 = 23 .... (3)
Also, we know that
a_n = a +(n - 1)d
So, the 3th term (for n = 3),
a_3 = a + (3 - 1)d
$\Longrightarrow$ 7 = a + 2d (Using 1)
$\Longrightarrow$ a = 7 - 2d .... (4)
Similarly, for the 7th term (n = 7),
a_7 = a + (7 - 1) d 24 = a + 6d = 23 (Using 3)
a = 23 - 6d .... (5)
Subtracting (4) from (5), we get,
a - a = (23 - 6d) - (7 - 2d)
$\Longrightarrow$ 0 = 23 - 6d - 7 + 2d
$\Longrightarrow$ 0 = 16 - 4d
$\Longrightarrow$ 4d = 16
$\Longrightarrow$ d = 4
Now, to find a, we substitute the value of d in (4), a =7 - 2(4)
$\Longrightarrow$ a = 7 - 8
a = -1
Hence, for the A.P. a = -1 and d = 4
For finding the sum, we know that
S_n = n/2[2a + (n − 1)d] and n = 20 (given0
S_{20} = 20/2[2(−1) + (20 − 1)(4)]
= (10)[-2 + (19)(4)]
= (10)[-2 + 76]
= (10)[74]
= 740
Hence, the sum of first 20 terms for the given A.P. is 740

16. The first term of an A.P. is 2 and the last term is 50. The sum of all these terms is 442. Find the common difference.

Solution:

Given,
The first term of the A.P (a) = 2
The last term of the A.P (l) = 50
Sum of all the terms $S_n = 442$
So, let the common difference of the A.P. be taken as d.
The sum of all the terms is given as,
442 = (n/2)(2 + 50)
⟹ 442 = (n/2)(52)
⟹ 26n = 442
⟹ n = 17
Now, the last term is expressed as
50 = 2 + (17 - 1)d
⟹ 50 = 2 + 16d
⟹ 16d = 48
⟹ d = 3
Thus, the common difference of the A.P. is d = 3.

17. If 12th term of an A.P. is -13 and the sum of the first four terms is 24, what is the sum of first 10 terms?

Solution:

Let us take the first term as a and the common difference as d.
Given,
$a_{12} = -13$ $S_4 = 24$
Also, we know that $a_n = a + (n - 1)d$
So, for the 12th term
$a_{12} = a + (12 - 1)d = -13$
⟹ a + 11d = -13
a = -13 - 11d .... (1)
And, we that for sum of terms
$S_n = n/2[2a + (n - 1)d]$
Here, n = 4
$S_4 = 4/2[2(a) + (4 - 1)d]$
⟹ 24 = (2)[2a + (3)(d)]
⟹ 24 = 4a + 6d
⟹ 4a = 24 - 6d

$$\Longrightarrow a = 6 - \frac{6}{4}d \; \ldots.(2)$$

Subtracting (1) from (2), we have

$$\Rightarrow a - a = (6 - \frac{6}{4}d) - (-13 - 11d)$$

$$\Rightarrow 0 = 6 - \frac{6}{4}d + 13d + 11d$$

$$\Rightarrow 0 = 19 + \frac{44d - 6d}{4s}$$

Further simplifying for d, we get,

$$\Rightarrow 0 = 19 + \frac{38}{4}d$$

$$\Rightarrow -19 = \frac{19}{2}d$$

$\Rightarrow$ -19 × 2 = 19d

$\Rightarrow$ d = – 2

On substituting the value of d in (1), we find a

a = -13 - 11(-2)

a = -13 + 22

a = 9

Next, the sum of 10 term is given by

S_{10} = 10/2[2(9) + (10 − 1)(−2)]

= (5)[19 + (9)(-2)]

= (5)(18 - 18) = 0

Thus, the sum of first 10 terms for the given A.P. is S_{10} = 0.

18. Find the sum of first 22 terms of an A.P. in which d = 22 and a_{22} = 149.

Solution:

Let the first term be taken as a.

Given,

a_{22} = 149 and the common difference d = 22

Also, we know that

a_n = a + (n - 1) d

So, the 22nd term is given by

a_{22} = a + (22 - 1)d

149 = a + (21) (22)

a = 149 - 462

a = - 313

Now, for the sum of term

S_n = n/2[2a + (n − 1)d]

Here, n = 22

S_{22} = 22/2[2(−313) + (22 − 1)(22)]

$= (11)[-626 + 462]$
$= (11)[-164]$
$= -1804$
Hence, the sum of first 22 terms for the given A.P. is $S_{22} = -1804$

19. In an A.P., if the first term is 22, the common difference is – 4 and the sum to n terms is 64, find n.
Solution:

Given that,
$a = 22$, $d = -4$ and $S_n = 64$
Let us consider the number of terms as n.
For sum of terms in an A.P, we know that
$S_n = n/2[2a + (n - 1)d]$
Where; a = first term for the given A.P. d = common difference of the given A.P. n = number of terms

So,
$\Rightarrow S_n = n/2[2(22) + (n - 1)(-4)]$
$\Rightarrow 64 = n/2[2(22) + (n - 1)(-4)]$
$\Rightarrow 64(2) = n(48 - 4n)$
$\Rightarrow 128 = 48n - 4n^2$
After rearranging the terms, we have a quadratic equation
$4n^2 - 48n + 128 = 0$,
$n^2 - 12n + 32 = 0$ [dividing by 4 on both sides]
$n^2 - 12n + 32 = 0$
Solving by factorisation method,
$n^2 - 8n - 4n + 32 = 0$
$n (n - 8) - 4 (n - 8) = 0$
$(n - 8) (n - 4) = 0$
So, we get $n - 8 = 0 \Rightarrow n = 8$
Or, $n - 4 = 0 \Rightarrow n = 4$
Hence, the number of terms can be either n = 4 or 8.

20. In an A.P., if the 5th and 12th terms are 30 and 65 respectively, what is the sum of first 20 terms?
Solution:

Let's take the first term as a and the common difference to be d
Given that,
$a_5 = 30$ and $a_{12} = 65$
And, we know that $a_n = a + (n - 1)d$
So,
$a_5 = a + (5 - 1)d$
$30 = a + 4d$
$a = 30 - 4d$ .... (i)

Similarly, $a_{12} = a + (12 - 1) d$
$65 = a + 11d$
$a = 65 - 11d$ .... (ii)

Subtracting (i) from (ii), we have
$a - a = (65 - 11d) - (30 - 4d)$
$0 = 65 - 11d - 30 + 4d$
$0 = 35 - 7d$
$7d = 35$
$d = 5$
Putting d in (i), we get
$a = 30 - 4(5)$
$a = 30 - 20$
$a = 10$
Thus for the A.P; d = 5 and a = 10
Next, to find the sum of first 20 terms of this A.P., we use the following formula for the sum of n terms of an A.P.,
$S_n = n/2[2a + (n - 1)d]$
Where;
a = first term of the given A.P.
d = common difference of the given A.P.
n = number of terms
Here n = 20, so we have
$S_{20} = 20/2[2(10) + (20 - 1)(5)]$
$= (10)[20 + (19)(5)]$
$= (10)[20 + 95]$
$= (10)[115]$
$= 1150$
Hence, the sum of first 20 terms for the given A.P. is 1150

21. Find the sum of first 51 terms of an A.P. whose second and third terms are 14 and 18 respectively.
Solution:

Let's take the first term as a and the common difference as d.
Given that,
$a_2 = 14$ and $a_3 = 18$
And, we know that $a_n = a + (n - 1)d$
So,
$a_2 = a + (2 - 1)d$
$\Rightarrow 14 = a + d$
$\Rightarrow a = 14 - d$ .... (i)
Similarly,
$a_3 = a + (3 - 1)d$
$\Rightarrow 18 = a + 2d$
$\Rightarrow a = 18 - 2d$ .... (ii)

Subtracting (i) from (ii), we have
a - a = (18 - 2d) - (14 - d)
0 = 18 - 2d - 14 + d
0 = 4 - d
d = 4
Putting d in (i), to find a
a = 14 - 4
a = 10
Thus, for the A.P. d = 4 and a = 10
Now, to find sum of terms
$S_n = n/2(2a + (n - 1)d)$
Where,
a = the first term of the A.P.
d = common difference of the A.P.
n = number of terms So, using the formula for
n = 51,
$\Longrightarrow S_{51} = 51/2[2(10) + (51 - 1)(4)]$
$= 51/2[20 + (40)4]$
$= 51/2[220]$
$= 51(110)$
$= 5610$
Hence, the sum of the first 51 terms of the given A.P. is 5610

22. If the sum of 7 terms of an A.P. is 49 and that of 17 terms is 289, find the sum of n terms.
Solution:

Given,
Sum of 7 terms of an A.P. is 49
$\Longrightarrow S_7 = 49$
And, sum of 17 terms of an A.P. is 289
$\Longrightarrow S_{17} = 289$
Let the first term of the A.P be a and common difference as d.
And, we know that the sum of n terms of an A.P is
$S_n = n/2[2a + (n - 1)d]$
So,
$S_7 = 49 = 7/2[2a + (7 - 1)d]$
$= 7/2 [2a + 6d]$
$= 7[a + 3d]$
$\Longrightarrow 7a + 21d = 49$
a + 3d = 7 ….. (i)

Similarly,
$S_{17} = 17/2[2a + (17 - 1)d]$
$= 17/2 [2a + 16d]$
$= 17[a + 8d]$
$\Longrightarrow 17[a + 8d] = 289$

$a + 8d = 17$ ….. (ii)
Now, subtracting (i) from (ii), we have
$a + 8d - (a + 3d) = 17 - 7$
$5d = 10$
$d = 2$

Putting d in (i), we find a
$a + 3(2) = 7$
$a = 7 - 6 = 1$
So, for the A.P: a = 1 and d = 2

For the sum of n terms is given by,
$S_n = n/2[2(1) + (n - 1)(2)]$
$= n/2[2 + 2n - 2]$
$= n/2[2n]$
$= n^2$
Therefore, the sum of n terms of the A.P is given by n^2.

23. The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.
Solution:

Sum of first n terms of an A.P is given by $S_n = n/2(2a + (n - 1)d)$
Given,
First term (a) = 5, last term (a_n) = 45 and sum of n terms (S_n) = 400
Now, we know that
$a_n = a + (n - 1)d$
$\Longrightarrow 45 = 5 + (n - 1)d$
$\Longrightarrow 40 = nd - d$
$\Longrightarrow nd - d = 40$ .... (1)
Also,
$S_n = n/2(2(a) + (n - 1)d)$
$400 = n/2(2(5) + (n - 1)d)$
$800 = n (10 + nd - d)$
$800 = n (10 + 40)$ [using (1)]
$\Longrightarrow n = 16$

Putting n in (1), we find d
$nd - d = 40$
$16d - d = 40$
$15d = 40$
$d = 8/3$
Therefore, the common difference of the given A.P. is 8/3.

24. In an A.P. the first term is 8, n^{th} term is 33 and the sum of first n term is 123. Find n and the d, the common difference.

Solution:

Given,
The first term of the A.P (a) = 8
The nth term of the A.P (l) = 33
And, the sum of all the terms S_n = 123
Let the common difference of the A.P. be d.
So, find the number of terms by
$123 = (n/2)(8 + 33)$
$123 = (n/2)(41)$
$n = (123 \times 2)/ 41$
$n = 246/41$
$n = 6$
Next, to find the common difference of the A.P. we know that
$l = a + (n - 1)d$
$33 = 8 + (6 - 1)d$
$33 = 8 + 5d$
$5d = 25$
$d = 5$
Thus, the number of terms is n = 6 and the common difference of the A.P. is d = 5.

25. In an A.P. the first term is 22, n^{th} term is -11 and the sum of first n term is 66. Find n and the d, the common difference.
Solution:

Given,
The first term of the A.P (a) = 22
The nth term of the A.P (l) = -11
And, sum of all the terms S_n = 66
Let the common difference of the A.P. be d.
So, finding the number of terms by
$66 = (n/2)[22 + (-11)]$
$66 = (n/2)[22 - 11]$
$(66)(2) = n(11)$
$6 \times 2 = n$
$n = 12$
Now, for finding d
We know that, $l = a + (n - 1)d$
$- 11 = 22 + (12 - 1)d$
$-11 = 22 + 11d$
$11d = -33$
$d = -3$
Hence, the number of terms is n = 12 and the common difference d = -3

26. The first and the last terms of an A.P. are 7 and 49 respectively. If sum of all its terms is 420, find the common difference.

Solution:

Given,
First term (a) = 7, last term (a_n) = 49 and sum of n terms (S_n) = 420
Now, we know that
$a_n = a + (n - 1)d$
$\Rightarrow 49 = 7 + (n - 1)d$
$\Rightarrow 43 = nd - d$
$\Rightarrow nd - d = 42$ ..... (1)
Next,
$S_n = n/2(2(7) + (n - 1)d)$
$\Rightarrow 840 = n[14 + nd - d]$
$\Rightarrow 840 = n[14 + 42]$ [using (1)]
$\Rightarrow 840 = 54n$
$\Rightarrow n = 15$ .... (2)
So, by substituting (2) in (1), we have
$nd - d = 42$
$\Rightarrow 15d - d = 42$
$\Rightarrow 14d = 42$
$\Rightarrow d = 3$
Therefore, the common difference of the given A.P. is 3.

27. The first and the last terms of an A.P are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.
Solution:

Given,
First term (a) = 5 and the last term (l) = 45
Also, $S_n = 400$
We know that,
$a_n = a + (n - 1)d$
$\Rightarrow 45 = 5 + (n - 1)d$
$\Rightarrow 40 = nd - d$
$\Rightarrow nd - d = 40$ ..... (1)
Next,
$S_n = n/2(2(5) + (n - 1)d)$
$\Rightarrow 400 = n[10 + nd - d]$
$\Rightarrow 800 = n[10 + 40]$ [using (1)]
$\Rightarrow 800 = 50n$
$\Rightarrow n = 16$ .... (2)
So, by substituting (2) in (1), we have
$nd - d = 40$
$\Rightarrow 16d - d = 40$
$\Rightarrow 15d = 40$
$\Rightarrow d = 8/3$

Therefore, the common difference of the given A.P. is 8/3.

28. The sum of first q terms of an A.P. is 162. The ratio of its 6^{th} term to its 13^{th} term is 1: 2. Find the first and 15^{th} term of the A.P.
Solution:

Let a be the first term and d be the common difference.
And we know that, sum of first n terms is:
$S_n = n/2(2a + (n - 1)d)$
Also, nth term is given by $a_n = a + (n - 1)d$

From the question, we have
$S_q = 162$ and $a_6 : a_{13} = 1 : 2$
So,
$2a_6 = a_{13}$
$\Rightarrow 2 [a + (6 - 1d)] = a + (13 - 1)d$
$\Rightarrow 2a + 10d = a + 12d$
$\Rightarrow a = 2d$ .... (1)
And, $S_9 = 162$
$\Rightarrow S_9 = 9/2(2a + (9 - 1)d)$
$\Rightarrow 162 = 9/2(2a + 8d)$
$\Rightarrow 162 \times 2 = 9[4d + 8d]$ [from (1)]
$\Rightarrow 324 = 9 \times 12d$
$\Rightarrow d = 3$
$\Rightarrow a = 2(3)$ [from (1)]
$\Rightarrow a = 6$
Hence, the first term of the A.P. is 6
For the 15^{th} term, $a_{15} = a + 14d = 6 + 14 \times 3 = 6 + 42$
Therefore, $a_{15} = 48$

29. If the 10^{th} term of an A.P. is 21 and the sum of its first 10 terms is 120, find its n^{th} term.
Solution:

Let's consider a to be the first term and d be the common difference.
And we know that, sum of first n terms is:
$S_n = n/2(2a + (n - 1)d)$ and n^{th} term is given by: $a_n = a + (n - 1)d$

Now, from the question we have
$S_{10} = 120$
$\Rightarrow 120 = 10/2(2a + (10 - 1)d)$
$\Rightarrow 120 = 5(2a + 9d)$
$\Rightarrow 24 = 2a + 9d$ .... (1)
Also given that, $a_{10} = 21$
$\Rightarrow 21 = a + (10 - 1)d$
$\Rightarrow 21 = a + 9d$ .... (2)
Subtracting (2) from (1), we get

$24 - 21 = 2a + 9d - a - 9d$
$\Longrightarrow a = 3$
Now, on putting a = 3 in equation (2), we get
$3 + 9d = 21$
$9d = 18$
$d = 2$
Thus, we have the first term(a) = 3 and the common difference(d) = 2
Therefore, the n^{th} term is given by
$a_n = a + (n - 1)d = 3 + (n - 1)2$
$= 3 + 2n - 2$
$= 2n + 1$
Hence, the n^{th} term of the A.P is $(a_n) = 2n + 1$.

30. The sum of first 7 terms of an A.P. is 63 and the sum of its next 7 terms is 161. Find the 28^{th} term of this A.P.
Solution:

Let's take a to be the first term and d to be the common difference.
And we know that, sum of first n terms
$S_n = n/2(2a + (n - 1)d)$
Given that sum of the first 7 terms of an A.P. is 63.
$S_7 = 63$
And sum of next 7 terms is 161.
So, the sum of first 14 terms = Sum of first 7 terms + sum of next 7 terms
$S_{14} = 63 + 161 = 224$
Now, having
$S_7 = 7/2(2a + (7 - 1)d)$
$\Longrightarrow 63(2) = 7(2a + 6d)$
$\Longrightarrow 9 \times 2 = 2a + 6d$
$\Longrightarrow 2a + 6d = 18 \ldots. (1)$
And,
$S_{14} = 14/2(2a + (14 - 1)d)$
$\Longrightarrow 224 = 7(2a + 13d)$
$\Longrightarrow 32 = 2a + 13d \ldots. (2)$
Now, subtracting (1) from (2), we get
$\Longrightarrow 13d - 6d = 32 - 18$
$\Longrightarrow 7d = 14$
$\Longrightarrow d = 2$
Using d in (1), we have
$2a + 6(2) = 18$
$2a = 18 - 12$
$a = 3$
Thus, from n^{th} term
$\Longrightarrow a_{28} = a + (28 - 1)d$
$= 3 + 27 (2)$
$= 3 + 54 = 57$

Therefore, the 28th term is 57.

31. The sum of first seven terms of an A.P. is 182. If its 4th and 17th terms are in ratio 1: 5, find the A.P.
Solution:

Given that,
$S_{17} = 182$
And, we know that the sum of first n term is:
$S_n = n/2(2a + (n - 1)d)$
So,
$S_7 = 7/2(2a + (7 - 1)d)$
$182 \times 2 = 7(2a + 6d)$
$364 = 14a + 42d$
$26 = a + 3d$
$a = 26 - 3d$ ... (1)
Also, it's given that 4th term and 17th term are in a ratio of 1: 5. So, we have
$\Longrightarrow 5(a_4) = 1(a_{17})$
$\Longrightarrow 5 (a + 3d) = 1 (a + 16d)$
$\Longrightarrow 5a + 15d = a + 16d$
$\Longrightarrow 4a = d$ .... (2)
Now, substituting (2) in (1), we get
$\Longrightarrow 4 (26 - 3d) = d$
$\Longrightarrow 104 - 12d = d$
$\Longrightarrow 104 = 13d$
$\Longrightarrow d = 8$
Putting d in (2), we get
$\Longrightarrow 4a = d$
$\Longrightarrow 4a = 8$
$\Longrightarrow a = 2$
Therefore, the first term is 2 and the common difference is 8. So, the A.P. is 2, 10, 18, 26, ...

32. The nth term of an A.P is given by (-4n + 15). Find the sum of first 20 terms of this A.P.
Solution:

Given,
The nth term of the A.P = (-4n + 15)
So, by putting n = 1 and n = 20 we can find the first ans 20th term of the A.P
$a = (-4(1) + 15) = 11$
And,
$a_{20} = (-4(20) + 15) = -65$
Now, for find the sum of 20 terms of this A.P we have the first and last term.
So, using the formula
$S_n = n/2(a + l)$
$S_{20} = 20/2(11 + (-65))$
$= 10(-54)$

= -540

Therefore, the sum of first 20 terms of this A.P. is -540.

33. In an A.P. the sum of first ten terms is -150 and the sum of its next 10 term is -550. Find the A.P.

Solution:

Let's take a to be the first term and d to be the common difference.

And we know that, sum of first n terms

$S_n = n/2(2a + (n - 1)d)$

Given that sum of the first 10 terms of an A.P. is -150.

$S_{10} = -150$

And the sum of next 10 terms is -550.

So, the sum of first 20 terms = Sum of first 10 terms + sum of next 10 terms

$S_{20} = -150 + -550 = -700$

Now, having

$S_{10} = 10/2(2a + (10 - 1)d)$

$\Longrightarrow -150 = 5(2a + 9d)$

$\Longrightarrow -30 = 2a + 9d$

$\Longrightarrow 2a + 9d = -30$ . . . . (1)

And,

$S_{20} = 20/2(2a + (20 - 1)d)$

$\Longrightarrow -700 = 10(2a + 19d)$

$\Longrightarrow -70 = 2a + 19d$ .... (2)

Now, subtracting (1) from (2), we get

$\Longrightarrow 19d - 9d = -70 - (-30)$

$\Longrightarrow 10d = -40$

$\Longrightarrow d = -4$

Using d in (1), we have

$2a + 9(-4) = -30$

$2a = -30 + 36$

$a = 6/2 = 3$

Hence, we have a = 3 and d = -4

So, the A.P is 3, -1, -5, -9, -13,…..

34. Sum of the first 14 terms of an A.P. is 1505 and its first term is 10. Find its 25^{th} term.

Solution

Given,

First term of the A.P is 1505 and

$S_{14} = 1505$

We know that, the sum of first n terms is

$S_n = n/2(2a + (n - 1)d)$

So,

$S_{14} = 14/2(2(10) + (14 - 1)d) = 1505$

$7(20 + 13d) = 1505$

$20 + 13d = 215$
$13d = 215 – 20$
$d = 195/13$
$d = 15$

Thus, the 25th term is given by
$a_{25} = 10 + (25 - 1)15$
$= 10 + (24)15$
$= 10 + 360$
$= 370$
Therefore, the 25th term of the A.P is 370

35. In an A.P. , the first term is 2, the last term is 29 and the sum of the terms is 155. Find the common difference of the A.P.
Solution:

Given,
The first term of the A.P. (a) = 2
The last term of the A.P. (l) = 29
And, sum of all the terms (S_n) = 155
Let the common difference of the A.P. be d.
So, find the number of terms by sum of terms formula
$S_n = n/2 (a + l)$
$155 = n/2(2 + 29)$
$155(2) = n(31)$
$31n = 310$
$n = 10$
Using n for the last term, we have
$l = a + (n - 1)d$
$29 = 2 + (10 - 1)d$
$29 = 2 + (9)d$
$29 - 2 = 9d$
$9d = 27$
$d = 3$
Hence, the common difference of the A.P. is d = 3

36. The first and the last term of an A.P are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?
Solution:

Given,
In an A.P first term (a) = 17 and the last term (l) = 350
And, the common difference (d) = 9
We know that,
$a_n = a + (n - 1)d$
so,

$a_n = l = 17 + (n - 1)9 = 350$
$17 + 9n – 9 = 350$
$9n = 350 – 8$
$n = 342/9$
$n = 38$

So, the sum of all the term of the A.P is given by
$S_n = n/2\ (a + l)$
$= 38/2(17 + 350)$
$= 19(367)$
$= 6973$
Therefore, the sum of terms of the A.P is 6973.

37. Find the number of terms of the A.P. –12, –9, –6, . . . , 21. If 1 is added to each term of this A.P., then find the sum of all terms of the A.P. thus obtained.
Solution:

Given,
First term, $a = -12$
Common difference, $d = a_2 - a_1 = – 9 – (- 12)$
$d = - 9 + 12 = 3$
And, we know that n^{th} term $= a_n = a + (n - 1)d$
$\Longrightarrow 21 = -12 + (n - 1)3$
$\Longrightarrow 21 = -12 + 3n - 3$
$\Longrightarrow 21 = 3n - 15$
$\Longrightarrow 36 = 3n$
$\Longrightarrow n = 12$
Thus, the number of terms is 12.
Now, if 1 is added to each of the 12 terms, the sum will increase by 12.
Hence, the sum of all the terms of the A.P. so obtained is
$\Longrightarrow S_{12} + 12 = 12/2[a + l] + 12$
$= 6[-12 + 21] + 12$
$= 6 \times 9 + 12$
$= 66$
Therefore, the sum after adding 1 to each of the terms in the A.P is 66.