Exercise 9.1 Page No: 9.5

1. Write the first terms of each of the following sequences whose n^{th} term are:
(i) $a_n = 3n + 2$
(ii) $a_n = (n – 2)/3$
(iii) $a_n = 3^n$
(iv) $a_n = (3n – 2)/ 5$
(v) $a_n = (-1)^n . 2^n$
(vi) $a_n = n(n – 2)/2$
(vii) $a_n = n^2 - n + 1$
(viii) $a_n = n^2 - n + 1$
(ix) $a_n = (2n – 3)/ 6$

Solutions:

(i) $a_n = 3n + 2$

Given sequence whose $a_n = 3n + 2$
To get the first five terms of given sequence, put n = 1, 2, 3, 4, 5 and we get
$a_1 = (3 \times 1) + 2 = 3 + 2 = 5$
$a_2 = (3 \times 2) + 2 = 6 + 2 = 8$
$a_3 = (3 \times 3) + 2 = 9 + 2 = 11$
$a_4 = (3 \times 4) + 2 = 12 + 2 = 14$
$a_5 = (3 \times 5) + 2 = 15 + 2 = 17$
∴ the required first five terms of the sequence whose n^{th} term, $a_n = 3n + 2$ are 5, 8, 11, 14, 17.

(ii) $a_n = (n – 2)/3$

Given sequence whose $a_n = \frac{n-2}{3}$
On putting n = 1, 2, 3, 4, 5 then can get the first five terms

$$a_1 = \frac{1-2}{3} = \frac{-1}{3}; a_2 = \frac{2-2}{3} = 0$$

$$a_3 = \frac{3-2}{3} = \frac{1}{3}; a_4 = \frac{4-2}{3} = \frac{2}{3}$$

$$a_5 = \frac{5-2}{3} = 1$$

∴ the required first five terms of the sequence whose n^{th} term, $a_n = \frac{n-2}{3}$ are $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, 1.$

(iii) $a_n = 3^n$

Given sequence whose $a_n = 3^n$
To get the first five terms of given sequence, put n = 1, 2, 3, 4, 5 in the above
$a_1 = 3^1 = 3;$
$a_2 = 3^2 = 9;$
$a_3 = 27;$
$a_4 = 3^4 = 81;$

$a_5 = 3^5 = 243$.
∴ the required first five terms of the sequence whose n^{th} term, $a_n = 3^n$ are 3, 9, 27, 81, 243.

(iv) $a_n = (3n – 2)/ 5$

Given sequence whose $a_n = \frac{3n-2}{5}$

To get the first five terms of the sequence, put n = 1, 2, 3, 4, 5 in the above
And, we get

$$a_1 = \frac{3\times 1-2}{5} = \frac{3-2}{5} = \frac{1}{5}$$

$$a_2 = \frac{3\times 2-2}{5} = \frac{6-2}{5} = \frac{4}{5}$$

$$a_3 = \frac{3\times 3-2}{5} = \frac{9-2}{5} = \frac{7}{5}$$

$$a_4 = \frac{3\times 4-2}{5} = \frac{12-2}{5} = \frac{10}{5}$$

$$a_5 = \frac{3\times 5-2}{5} = \frac{15-2}{5} = \frac{13}{5}$$

∴ the required first five terms of the sequence are 1/5, 4/5, 7/5, 10/5, 13/5

(v) $a_n = (-1)^n 2^n$

Given sequence whose $a_n = (-1)^n 2^n$
To get first five terms of the sequence, put n = 1, 2, 3, 4, 5 in the above.
$a_1 = (-1)^1.2^1 = (-1).2 = -2$
$a_2 = (-1)^2.2^2 = (-1).4 = 4$
$a_3 = (-1)^3.2^3 = (-1).8 = -8$
$a_4 = (-1)^4.2^4 = (-1).16 = 16$
$a_5 = (-1)^5.2^5 = (-1).32 = -32$
∴ the first five terms of the sequence are – 2, 4, - 8, 16, - 32.

(vi) $a_n = n(n – 2)/2$

The given sequence is, $a_n = \frac{n(n-2)}{2}$
To get the first five terms of the sequence, put n = 1, 2, 3, 4, 5.
And, we get

$$a_1 = \frac{1(1-2)}{2} = \frac{1-1}{2} = \frac{-1}{2}$$

$$a_2 = \frac{2(2-2)}{2} = \frac{2.0}{2} = 0$$

$$a_3 = \frac{3(3-2)}{2} = \frac{3.1}{2} = \frac{3}{2}$$

$$a_4 = \frac{4(4-2)}{2} = \frac{4.2}{2} = 4$$

$$a_5 = \frac{5(5-2)}{2} = \frac{5.3}{2} = \frac{15}{2}$$

∴ the required first five terms are -1/2, 0, 3/2, 4, 15/2

(vii) $a_n = n^2 - n + 1$

The given sequence whose, $a_n = n^2 - n + 1$

To get the first five terms of given sequence, put n = 1, 2, 3, 4, 5.

And, we get

$a_1 = 1^2 - 1 + 1 = 1$

$a_2 = 2^2 - 2 + 1 = 3$

$a_3 = 3^2 - 3 + 1 = 7$

$a_4 = 4^2 - 4 + 1 = 13$

$a_5 = 5^2 - 5 + 1 = 21$

∴ the required first five terms of the sequence are 1, 3, 7, 13, 21.

(viii) $a_n = 2n^2 - 3n + 1$

The given sequence whose $a_n = 2n^2 - 3n + 1$

To get the first five terms of the sequence, put n = 1, 2, 3, 4, 5.

And, we get

$a_1 = 2.1^2 - 3.1 + 1 = 2 - 3 + 1 = 0$

$a_2 = 2.2^2 - 3.2 + 1 = 8 - 6 + 1 = 3$

$a_3 = 2.3^2 - 3.3 + 1 = 18 - 9 + 1 = 10$

$a_4 = 2.4^2 - 3.4 + 1 = 32 - 12 + 1 = 21$

$a_5 = 2.5^2 - 3.5 + 1 = 50 - 15 + 1 = 36$

∴ the required first five terms of the sequence are 0, 3, 10, 21, 36.

(ix) $a_n = (2n – 3)/ 6$

Given sequence whose, $a_n = \frac{2n-3}{6}$

To get the first five terms of the sequence we put n = 1, 2, 3, 4, 5.

And, we get

$$a_1 = \frac{2.1 - 3}{6} = \frac{2 - 3}{6} = \frac{-1}{6}$$

$$a_2 = \frac{2.2 - 3}{6} = \frac{4 - 3}{6} = \frac{1}{6}$$

$$a_3 = \frac{2.3 - 3}{6} = \frac{6 - 3}{6} = \frac{3}{6} = 1/2$$

$$a_4 = \frac{2.4 - 3}{6} = \frac{8 - 3}{6} = \frac{5}{6}$$

$$a_5 = \frac{2.5 - 3}{6} = \frac{10 - 3}{6} = \frac{7}{6}$$

∴ the required first five terms of the sequence are -1/6, 1/6, 1/2, 5/6 and 7/6

Exercise 9.2 Page No: 9.8

1. Show that the sequence defined by $a_n = 5n – 7$ is an A.P., find its common difference.
Solution:

Given, $a_n = 5n – 7$
Now putting n = 1, 2, 3, 4 we get,
$a_1 = 5(1) – 7 = 5 – 7 = -2$
$a_2 = 5(2) – 7 = 10 – 7 = 3$
$a_3 = 5(3) – 7 = 15 – 7 = 8$
$a_4 = 5(4) – 7 = 20 – 7 = 13$
We can see that,
$a_2 – a_1 = 3 – (-2) = 5$
$a_3 – a_2 = 8 – (3) = 5$
$a_4 – a_3 = 13 – (8) = 5$
Since the difference between the terms is common, we can conclude that the given sequence defined by $a_n = 5n – 7$ is an A.P with common difference 5.

2. Show that the sequence defined by $a_n = 3n^2 – 5$ is not an A.P.
Solution:

Given, $a_n = 3n^2 – 5$
Now putting n = 1, 2, 3, 4 we get,
$a_1 = 3(1)^2 – 5= 3 – 5 = -2$
$a_2 = 3(2)^2 – 5 = 12 – 5 = 7$
$a_3 = 3(3)^2 – 5 = 27 – 5 = 22$
$a_4 = 3(4)^2 – 5 = 48 – 5 = 43$
We can see that,
$a_2 – a_1 = 7 – (-2) = 9$
$a_3 – a_2 = 22 – 7 = 15$
$a_4 – a_3 = 43 – 22 = 21$
Since the difference between the terms is not common and varying, we can conclude that the given sequence defined by $a_n = 3n^2 – 5$ is not an A.P.

3. The general term of a sequence is given by $a_n = -4n + 15$. Is the sequence an A.P.? If so, find its 15^{th} term and the common difference.
Solution:

Given, $a_n = -4n + 15$
Now putting n = 1, 2, 3, 4 we get,
$a_1 = -4(1) + 15 = -4 + 15 = 11$
$a_2 = -4(2) + 15 = -8 + 15 = 7$
$a_3 = -4(3) + 15 = -12 + 15 = 3$
$a_4 = -4(4) + 15 = -16 + 15 = -1$
We can see that,

$a_2 - a_1 = 7 - (11) = -4$
$a_3 - a_2 = 3 - 7 = -4$
$a_4 - a_3 = -1 - 3 = -4$
Since the difference between the terms is common, we can conclude that the given sequence defined by $a_n = -4n + 15$ is an A.P with common difference of -4.

Hence, the 15th term will be
$a_{15} = -4(15) + 15 = -60 + 15 = -45$

Exercise 9.3 Page No: 9.11

1. For the following arithmetic progressions write the first term a and the common difference d:
(i) - 5, -1, 3, 7,…
(ii) 1/5, 3/5, 5/5, 7/5,…
(iii) 0.3, 0.55, 0.80, 1.05,…
(iv) -1.1, – 3.1, – 5.1, – 7.1,…
Solution:

We know that if a is the first term and d is the common difference, the arithmetic progression is a, a + d, a + 2d + a + 3d,….

(i) – 5, –1, 3, 7,…
Given arithmetic series is – 5, –1, 3, 7,...
c a, a + d, a + 2d + a + 3d,….
Thus, by comparing these two we get, a = – 5, a + d = 1, a + 2d = 3, a + 3d = 7
First term (a) = – 5
By subtracting second and first term, we get
(a + d) - (a) = d
-1 - (- 5) = d
4 = d
⇒ Common difference (d) = 4.

(ii) 1/5, 3/5, 5/5, 7/5, ...
Given arithmetic series is 1/5, 3/5, 5/5, 7/5, ...
It is seen that, it's of the form of 1/5, 2/5, 5/5, 7/5, ... a, a + d, a + 2d, a + 3d,
Thus, by comparing these two, we get
a = 1/5, a + d = 3/5, a + 2d = 5/5, a + 3d = 7/5
First term (a) = 1/5
By subtracting first term from second term, we get
d = (a + d)-(a)
d = 3/5 - 1/5
d = 2/5
⇒ common difference (d) = 2/5

(iii) 0.3, 0.55, 0.80, 1.05, ...
Given arithmetic series 0.3, 0.55, 0.80, 1.05, ...
It is seen that, it's of the form of a, a + d, a + 2d, a + 3d,
Thus, by comparing we get,
a = 0.3, a + d = 0.55, a + 2d = 0.80, a + 3d = 1.05
First term (a) = 0.3.
By subtracting first term from second term. We get
d = (a + d) - (a)
d = 0.55 - 0.3
d = 0.25
⇒ Common difference (d) = 0.25

(iv) –1.1, – 3.1, – 5.1, –7.1, ...
General series is –1.1, – 3.1, – 5.1, –7.1, ...
It is seen that, it's of the form of a, a + d, a + 2d, a + 3d, ...
Thus, by comparing these two, we get
a = –1.1, a + d = –3.1, a + 2d = –5.1, a + 3d = –71
First term (a) = –1.1
Common difference (d) = (a + d) - (a)
= -3.1 – (– 1.1)
⇒ Common difference (d) = – 2

2. Write the arithmetic progression when first term a and common difference d are as follows:
(i) a = 4, d = – 3
(ii) a = –1, d = 1/2
(iii) a = –1.5, d = – 0.5
Solution:

We know that, if first term (a) = a and common difference = d, then the arithmetic series is: a, a + d, a + 2d, a + 3d,

(i) a = 4, d = -3
Given, first term (a) = 4
Common difference (d) = -3
Then arithmetic progression is: a, a + d, a + 2d, a + 3d, ...
⇒ 4, 4 - 3, a + 2(-3), 4 + 3(-3), ...
⇒ 4, 1, – 2, – 5, – 8 ....

(ii) a = -1, d = 1/2
Given, first term (a) = -1
Common difference (d) = 1/2
Then arithmetic progression is: a, a + d, a + 2d, a + 3d,
⇒ -1, -1 + 1/2, -1, 2½, -1 + 3½, ...
⇒ -1, -1/2, 0, 1/2

(iii) a = –1.5, d = – 0.5
Given First term (a) = –1.5
Common difference (d) = – 0 5
Then arithmetic progression is; a, a + d, a + 2d, a + 3d, ...
⇒ -1.5, -1.5, -0.5, –1.5 + 2(– 0.5), –1.5 + 3(– 0.5)
⇒ – 1.5, – 2, – 2.5, – 3, ...

3. In which of the following situations, the sequence of numbers formed will form an A.P.?
(i) The cost of digging a well for the first metre is Rs 150 and rises by Rs 20 for each succeeding metre.
(ii) The amount of air present in the cylinder when a vacuum pump removes each time 1/4 of their remaining in the cylinder.
(iii) Divya deposited Rs 1000 at compound interest at the rate of 10% per annum. The amount at

the end of first year, second year, third year, …, and so on.
Solution:

(i) Given,
Cost of digging a well for the first meter (c_1) = Rs.150.
And, the cost rises by Rs.20 for each succeeding meter
Then,
Cost of digging for the second meter (c_2) = Rs.150 + Rs 20 = Rs 170
Cost of digging for the third meter (c_3) = Rs.170 + Rs 20 = Rs 210
Hence, its clearly seen that the costs of digging a well for different lengths are 150, 170, 190, 210, ...
Evidently, this series is in A·P.
With first term (a) = 150, common difference (d) = 20

(ii) Given,
Let the initial volume of air in a cylinder be V liters each time $3^{th}/4$ of air in a remaining i.e 1 -1/4
First time, the air in cylinder is V.
Second time, the air in cylinder is 3/4 V.
Third time, the air in cylinder is $(3/4)^2$ V.
Thus, series is V, 3/4 V, $(3/4)^2$ V,$(3/4)^3$ V, ...
Hence, the above series is not a A.P.

(iii) Given,
Divya deposited Rs 1000 at compound interest of 10% p.a
So, the amount at the end of first year is = 1000 + 0.1(1000) = Rs 1100
And, the amount at the end of second year is = 1100 + 0.1(1100) = Rs 1210
And, the amount at the end of third year is = 1210 + 0.1(1210) = Rs 1331
Cleary, these amounts 1100, 1210 and 1331 are not in an A.P since the difference between them is not the same.

Exercise 9.4

1. Find:

(i) 10^{th} tent of the AP 1, 4, 7, 10....
(ii) 18^{th} term of the AP √2, 3√2, 5√2, …….
(iii) n^{th} term of the AP 13, 8, 3, -2, ...
(iv) 10^{th} term of the AP -40, -15, 10, 35, ...
(v) 8^{th} term of the AP 11, 104, 91, 78, ...
(vi) 11^{th} tenor of the AP 10.0, 10.5, 11.0, 11.2, ...
(vii) 9^{th} term of the AP 3/4, 5/4, 7/4 + 9/4, ...

Solution:

(i) Given A.P. is 1, 4, 7, 10, ...
First term (a) = 1
Common difference (d) = Second term - First term
= 4 - 1 = 3.
We know that, n^{th} term in an A.P = a + (n - 1)d
Then, 10^{th} term in the A.P is 1 + (10 - 1)3
= 1 + 9x3
= 1 + 27
= 28
∴ 10^{th} term of A. P. is 28

(ii) Given A.P. is √2, 3√2, 5√2, …….
First term (a) = √2
Common difference = Second term – First term
= 3√2 - √2
⇒ d = 2√2
We know that, n^{th} term in an A. P. = a + (n - 1)d
Then, 18^{th} term of A. P. = √2 + (18 - 1)2√2
= √2 + 17.2√2
= √2 (1+34)
= 35√2
∴ 18^{th} term of A. P. is 35√2

(iii) Given A. P. is 13, 8, 3, - 2, ...
First term (a) = 13
Common difference (d) = Second term first term
= 8 - 13 = – 5
We know that, n^{th} term of an A.P. a_n = a +(n - 1)d
= 13 + (n - 1) - 5
= 13 - 5n + 5
∴ n^{th} term of the A.P is a_n = 18 - 5n

(iv) Given A. P. is - 40, -15, 10, 35, ...
First term (a) = -40

Common difference (d) = Second term - fast term
= -15 - (- 40)
= 40 - 15
= 25
We know that, n^{th} term of an A.P. $a_n = a + (n - 1)d$
Then, 10^{th} term of A. P. $a_{10} = -40 + (10 - 1)25$
= – 40 + 9.25
= – 40 + 225
= 185
∴ 10^{th} term of the A. P. is 185

(v) Given sequence is 117, 104, 91, 78, ...
First term (a) = 117
Common difference (d) = Second term - first term
= 104 - 117
= – 13
We know that, n^{th} term = a + (n - 1)d
Then, 8^{th} term = a + (8 - 1)d
= 117 + 7(-13)
= 117 - 91
= 26
∴ 8^{th} term of the A. P. is 26

(vi) Given A. P is 10.0, 10.5, 11.0, 11.5,
First term (a) = 10.0
Common difference (d) = Second term - first term
= 10.5 - 10.0 = 0.5
We know that, n^{th} term $a_n = a + (n - 1)d$
Then, 11^{th} term $a_{11} = 10.0 + (11 - 1)0.5$
= 10.0 + 10 x 0.5
= 10.0 + 5
=15.0
∴ 11^{th} term of the A. P. is 15.0

(vii) Given A. P is 3/4, 5/4, 7/4, 9/4, ...
First term (a) = 3/4
Common difference (d) = Second term - first term
= 5/4 - 3/4
= 2/4
We know that, n^{th} term $a_n = a + (n - 1)d$
Then, 9^{th} term $a_9 = a + (9 - 1)d$

$$= \frac{3}{4} + 8.\frac{2}{4}$$

$$= \frac{3}{4} + \frac{16}{4}$$

$$= \frac{19}{4}$$

∴ 9^{th} term of the A. P. is 19/4.

2.(i) Which term of the AP 3, 8, 13, ... is 248?
(ii) Which term of the AP 84, 80, 76, ... is 0?
(iii) Which term of the AP 4. 9, 14, ... is 254?
(iv) Which term of the AP 21. 42, 63, 84, ... is 420?
(v) Which term of the AP 121, 117. 113, ... is its first negative term?
Solution:

(i) Given A.P. is 3, 8, 13, ...
First term (a) = 3
Common difference (d) = Second term - first term
= 8 - 3
= 5
We know that, n^{th} term (a_n) = a + (n - 1)d
And, given n^{th} term a_n = 248
248 = 3+(n - 1)5
248 = -2 + 5n
5n = 250
n =250/5 = 50
∴ 50^{th} term in the A.P is 248.

(ii) Given A. P is 84, 80, 76, ...
First term (a) = 84
Common difference (d) = a_2 - a
= 80 - 84
= – 4
We know that, n^{th} term (a_n) = a +(n - 1)d
And, given nth term is 0
0 = 84 + (n - 1) - 4
84 = +4(n - 1)
n - 1 = 84/4 = 21
n = 21 + 1 = 22
∴ 22^{nd} term in the A.P is 0.

(iii) Given A. P 4, 9, 14, ...

First term (a) = 4
Common difference (d) = $a_2 - a_1$
= 9 - 4
= 5
We know that, n^{th} term (a_n) = a + (n - 1)d
And, given n^{th} term is 254
4 + (n - 1)5 = 254
(n - 1)·5 = 250
n - 1 = 250/5 = 50
n = 51
∴ 51^{th} term in the A.P is 254.

(iv) Given A. P 21, 42, 63, 84, ...
a = 21, d = $a_2 - a_1$
= 42 - 21
= 21
We know that, n^{th} term (a_n) = a +(n - 1)d
And, given n^{th} term = 420
21 + (n - 1)21 = 420
(n - 1)21 = 399
n - 1 = 399/21 = 19
n = 20
∴ 20^{th} term is 420.

(v) Given A.P is 121, 117, 113, ...
Fiat term (a) = 121
Common difference (d) = 117 - 121
= - 4
We know that, n^{th} term a_n = a + (n - 1)d
And, for some n^{th} term is negative i.e., $a_n < 0$
121 + (n - 1) - 4 < 0
121 + 4 - 4n < 0
125 - 4n < 0
4n > 125
n > 125/4
n > 31.25
The integer which comes after 31.25 is 32.
∴ 32^{nd} term in the A.P will be the first negative term.

3.(i) Is 68 a term of the A.P. 7, 10, 13,… ?
(ii) Is 302 a term of the A.P. 3, 8, 13, …. ?
(iii) Is -150 a term of the A.P. 11, 8, 5, 2, … ?
Solutions:

(i) Given, A.P. 7, 10, 13,…
Here, a = 7 and d = $a_2 - a_1$ = 10 – 7 = 3

We know that, n^{th} term $a_n = a + (n - 1)d$
Required to check n^{th} term $a_n = 68$
$a + (n - 1)d = 68$
$7 + (n - 1)3 = 68$
$7 + 3n – 3 = 68$
$3n + 4 = 68$
$3n = 64$
$\Rightarrow n = 64/3$, which is not a whole number.
Therefore, 68 is not a term in the A.P.

(ii) Given, A.P. 3, 8, 13,…
Here, $a = 3$ and $d = a_2 – a_1 = 8 – 3 = 5$
We know that, n^{th} term $a_n = a + (n - 1)d$
Required to check n^{th} term $a_n = 302$
$a + (n - 1)d = 302$
$3 + (n - 1)5 = 302$
$3 + 5n – 5 = 302$
$5n - 2 = 302$
$5n = 304$
$\Rightarrow n = 304/5$, which is not a whole number.
Therefore, 302 is not a term in the A.P.

(iii) Given, A.P. 11, 8, 5, 2, …
Here, $a = 11$ and $d = a_2 – a_1 = 8 – 11 = -3$
We know that, n^{th} term $a_n = a + (n - 1)d$
Required to check n^{th} term $a_n = -150$
$a + (n - 1)d = -150$
$11 + (n - 1)(-3) = -150$
$11 – 3n + 3 = -150$
$3n = 150 + 14$
$3n = 164$
$\Rightarrow n = 164/3$, which is not a whole number.
Therefore, -150 is not a term in the A.P.

4. How many terms are there in the A.P.?
(i) 7, 10, 13, ….., 43
(ii) -1, -5/6, -2/3, -1/2, … , 10/3
(iii) 7, 13, 19, …, 205
(iv) 18, 15½, 13, …., -47
Solution:

(i) Given, A.P. 7, 10, 13, ….., 43
Here, $a = 7$ and $d = a_2 – a_1 = 10 – 7 = 3$
We know that, n^{th} term $a_n = a + (n - 1)d$
And, given n^{th} term $a_n = 43$

$a + (n - 1)d = 43$
$7 + (n - 1)(3) = 43$
$7 + 3n – 3 = 43$
$3n = 43 – 4$
$3n = 39$
$\Rightarrow n = 13$
Therefore, there are 13 terms in the given A.P.

(ii) Given, A.P. -1, -5/6, -2/3, -1/2, … , 10/3
Here, $a = -1$ and $d = a_2 – a_1 = -5/6 – (-1) = 1/6$
We know that, n^{th} term $a_n = a + (n - 1)d$
And, given n^{th} term $a_n = 10/3$
$a + (n - 1)d = 10/3$
$-1 + (n - 1)(1/6) = 10/3$
$-1 + n/6 – 1/6 = 10/3$
$n/6 = 10/3 + 1 + 1/6$
$n/6 = (20 + 6 + 1)/6$
$n = (20 + 6 + 1)$
$\Rightarrow n = 27$
Therefore, there are 27 terms in the given A.P.

(iii) Given, A.P. 7, 13, 19, …, 205
Here, $a = 7$ and $d = a_2 – a_1 = 13 – 7 = 6$
We know that, n^{th} term $a_n = a + (n - 1)d$
And, given n^{th} term $a_n = 205$
$a + (n - 1)d = 205$
$7 + (n - 1)(6) = 205$
$7 + 6n – 6 = 205$
$6n = 205 – 1$
$n = 204/6$
$\Rightarrow n = 34$
Therefore, there are 34 terms in the given A.P.

(iv) Given, A.P. 18, 15½, 13, …., -47
Here, $a = 7$ and $d = a_2 – a_1 = 15½ – 18 = 5/2$
We know that, n^{th} term $a_n = a + (n - 1)d$
And, given n^{th} term $a_n = -47$
$a + (n - 1)d = 43$
$18 + (n - 1)(-5/2) = -47$
$18 – 5n/2 + 5/2 = -47$
$36 – 5n + 5 = -94$
$5n = 94 + 36 + 5$
$5n = 135$
$\Rightarrow n = 27$
Therefore, there are 27 terms in the given A.P.

5. The first term of an A.P. is 5, the common difference is 3 and the last term is 80; find the number of terms.
Solution:

Given,
$a = 5$ and $d = 3$
We know that, n^{th} term $a_n = a + (n - 1)d$
So, for the given A.P. $a_n = 5 + (n - 1)3 = 3n + 2$
Also given, last term = 80
$\Rightarrow 3n + 2 = 80$
$3n = 78$
$n = 78/3 = 26$
Therefore, there are 26 terms in the A.P.

6. The 6^{th} and 17^{th} terms of an A.P. are 19 and 41 respectively, find the 40^{th} term.
Solution:

Given,
$a_6 = 19$ and $a_{17} = 41$
We know that, n^{th} term $a_n = a + (n - 1)d$
So,
$a_6 = a + (6-1)d$
$\Rightarrow a + 5d = 19$ …… (i)
Similarity,
$a_{17} = a + (17 - 1)d$
$\Rightarrow a + 16d = 41$ …… (ii)
Solving (i) and (ii),
(ii) – (i) $\Rightarrow$
$a + 16d – (a + 5d) = 41 – 19$
$11d = 22$
$\Rightarrow d = 2$
Using d in (i), we get
$a + 5(2) = 19$
$a = 19 – 10 = 9$

Now, the 40^{th} term is given by $a_{40} = 9 + (40 - 1)2 = 9 + 78 = 87$
Therefore the 40^{th} term is 87.

7. If 9^{th} term of an A.P. is zero, prove its 29^{th} term is double the 19^{th} term.
Solution:

Given,
$a_9 = 0$
We know that, n^{th} term $a_n = a + (n - 1)d$
So, $a + (9 - 1)d = 0 \Rightarrow a + 8d = 0$ ……(i)
Now,

29^{th} term is given by $a_{29} = a + (29 - 1)d$
$\Rightarrow a_{29} = a + 28d$
And, $a_{29} = (a + 8d) + 20d$ [using (i)]
$\Rightarrow a_{29} = 20d$ ….. (ii)

Similarly, 19^{th} term is given by $a_{19} = a + (19 - 1)d$
$\Rightarrow a_{19} = a + 18d$
And, $a_{19} = (a + 8d) + 10d$ [using (i)]
$\Rightarrow a_{19} = 10d$ …..(iii)

On comparing (ii) and (iii), it's clearly seen that
$a_{29} = 2(a_{19})$

Therefore, 29^{th} term is double the 19^{th} term.

8. If 10 times the 10^{th} term of an A.P. is equal to 15 times the 15^{th} term, show that 25^{th} term of the A.P. is zero.
Solution:

Given,
10 times the 10^{th} term of an A.P. is equal to 15 times the 15^{th} term.
We know that, n^{th} term $a_n = a + (n - 1)d$
$\Rightarrow 10(a_{10}) = 15(a_{15})$
$10(a + (10 - 1)d) = 15(a + (15 - 1)d)$
$10(a + 9d) = 15(a + 14d)$
$10a + 90d = 15a + 210d$
$5a + 120d = 0$
$5(a + 24d) = 0$
$a + 24d = 0$
$a + (25 – 1)d = 0$
$\Rightarrow a_{25} = 0$
Therefore, the 25^{th} term of the A.P. is zero.

9. The 10^{th} and 18^{th} terms of an A.P. are 41 and 73 respectively. Find 26^{th} term.
Solution:

Given,
$A_{10} = 41$ and $a_{18} = 73$
We know that, n^{th} term $a_n = a + (n - 1)d$
So,
$a_{10} = a + (10 - 1)d$
$\Rightarrow a + 9d = 41$ …… (i)
Similarity,
$a_{18} = a + (18 - 1)d$
$\Rightarrow a + 17d = 73$ …… (ii)
Solving (i) and (ii),

(ii) – (i) ⇒
a + 17d – (a + 9d) = 73 – 41
8d = 32
⇒ d = 4
Using d in (i), we get
a + 9(4) = 41
a = 41 – 36 = 5

Now, the 26th term is given by a_{26} = 5 + (26 - 1)4 = 5 + 100 = 105
Therefore the 26th term is 105.

10. In a certain A.P. the 24th term is twice the 10th term. Prove that the 72nd term is twice the 34th term.
Solution:

Given,
24th term is twice the 10th term.
We know that, nth term $a_n = a + (n - 1)d$
⇒ $a_{24} = 2(a_{10})$
a + (24 - 1)d = 2(a + (10 - 1)d)
a + 23d = 2(a + 9d)
a + 23d = 2a + 18d
a = 5d …. (1)

Now, the 72nd term can be expressed as
a_{72} = a + (72 - 1)d
= a + 71d
= a + 5d + 66d
= a + a + 66d [using (1)]
= 2(a + 33d)
= 2(a + (34 - 1)d)
= 2(a_{34})
⇒ $a_{72} = 2(a_{34})$
Hence, the 72nd term is twice the 34th term of the given A.P.

11. The 26th, 11th and the last term of an A.P. are 0, 3 and -1/5, respectively. Find the common difference and the number of terms.
Solution:

Given,
a_{26} = 0 , a_{11} = 3 and a_n (last term) = -1/5 of an A.P.
We know that, nth term $a_n = a + (n - 1)d$
Then,
a_{26} = a + (26 - 1)d
⇒ a + 25d = 0 …..(1)

And,
$a_{11} = a + (11 - 1)d$
$\Rightarrow a + 10d = 3$ …… (2)

Solving (1) and (2),
(1) – (2) $\Rightarrow$
$a + 25d – (a + 10d) = 0 – 3$
$15d = -3$
$\Rightarrow d = -1/5$
Using d in (1), we get
$a + 25(-1/5) = 0$
$a = 5$

Now, given that the last term $a_n = -1/5$
$\Rightarrow 5 + (n - 1)(-1/5) = -1/5$
$5 + -n/5 + 1/5 = -1/5$
$25 – n + 1 = -1$
$n = 27$
Therefore, the A.P has 27 terms and its common difference is -1/5.

12. If the n^{th} term of the A.P. 9, 7, 5, …. is same as the n^{th} term of the A.P. 15, 12, 9, … find n.
Solution:

Given,
$A.P_1$ = 9, 7, 5, …. and $A.P_2$ = 15, 12, 9, …
And, we know that, n^{th} term $a_n = a + (n - 1)d$

For $A.P_1$,
a = 9, d = Second term – first term = 9 – 7 = -2
And, its n^{th} term $a_n = 9 + (n - 1)(-2) = 9 – 2n + 2$
$a_n = 11 – 2n$ …..(i)
Similarly, for $A.P_2$
a = 15, d = Second term – first term = 12 – 15 = -3
And, its n^{th} term $a_n = 15 + (n - 1)(-3) = 15 – 3n + 3$
$a_n = 18 - 3n$ …..(ii)

According to the question, its given that
n^{th} term of the $A.P_1$ = n^{th} term of the $A.P_2$
$\Rightarrow 11 – 2n = 18 - 3n$
$n = 7$
Therefore, the 7^{th} term of the both the A.Ps are equal.

13. Find the 12^{th} term from the end of the following arithmetic progressions:
(i) 3, 5, 7, 9, …. 201
(ii) 3,8,13, … ,253
(iii) 1, 4, 7, 10, … ,88

Solution:

In order the find the 12^{th} term for the end of an A.P. which has n terms, its done by simply finding the $((n - 12) + 1)^{th}$ of the A.P
And we know, n^{th} term $a_n = a + (n - 1)d$

(i) Given A.P = 3, 5, 7, 9, …. 201
Here, a = 3 and d = (5 - 3) = 2
Now, find the number of terms when the last term is known i.e, 201
$a_n = 3 + (n - 1)2 = 201$
$3 + 2n – 2 = 201$
$2n = 200$
$n = 100$
Hence, the A.P has 100 terms.

So, the 12^{th} term from the end is same as $(100 – 12 + 1)^{th}$ of the A.P which is the 89^{th} term.
$\Rightarrow a_{89} = 3 + (89 - 1)2$
$= 3 + 88(2)$
$= 3 + 176$
$= 179$
Therefore, the 12^{th} term from the end of the A.P is 179.

(ii) Given A.P = 3,8,13, … ,253
Here, a = 3 and d = (8 - 3) = 5
Now, find the number of terms when the last term is known i.e, 253
$a_n = 3 + (n - 1)5 = 253$
$3 + 5n – 5 = 253$
$5n = 253 + 2 = 255$
$n = 255/5$
$n = 51$
Hence, the A.P has 51 terms.

So, the 12^{th} term from the end is same as $(51 – 12 + 1)^{th}$ of the A.P which is the 40^{th} term.
$\Rightarrow a_{40} = 3 + (40 - 1)5$
$= 3 + 39(5)$
$= 3 + 195$
$= 198$
Therefore, the 12^{th} term from the end of the A.P is 198.

(iii) Given A.P = 1, 4, 7, 10, … ,88
Here, a = 1 and d = (4 - 1) = 3
Now, find the number of terms when the last term is known i.e, 88
$a_n = 1 + (n - 1)3 = 88$
$1 + 3n – 3 = 88$
$3n = 90$
$n = 30$

Hence, the A.P has 30 terms.

So, the 12th term from the end is same as $(30 – 12 + 1)^{th}$ of the A.P which is the 19th term.
$\Rightarrow a_{89} = 1 + (19 - 1)3$
$= 1 + 18(3)$
$= 1 + 54$
$= 55$
Therefore, the 12th term from the end of the A.P is 55.

14. The 4th term of an A.P. is three times the first and the 7th term exceeds twice the third term by 1. Find the first term and the common difference.
Solution:

Let's consider the first term and the common difference of the A.P to be a and d respectively.
Then, we know that $a_n = a + (n - 1)d$
Given conditions,
4th term of an A.P. is three times the first
Expressing this by equation we have,
$\Rightarrow a_4 = 3(a)$
$a + (4 - 1)d = 3a$
$3d = 2a \Rightarrow a = 3d/2$…….(i)
And,
7th term exceeds twice the third term by 1
$\Rightarrow a_7 = 2(a_3) + 1$
$a + (7 – 1)d = 2(a + (3–1)d) + 1$
$a + 6d = 2a + 4d + 1$
$a – 2d +1 = 0$ ….. (ii)

Using (i) in (ii), we have
$3d/2 – 2d + 1 = 0$
$3d – 4d + 2 = 0$
$d = 2$
So, putting d = 2 in (i), we get a
$\Rightarrow a = 3$

Therefore, the first term is 3 and the common difference is 2.

15. Find the second term and the nth term of an A.P. whose 6th term is 12 and the 8th term is 22.
Solution:

Given, in an A.P
$a_6 = 12$ and $a_8 = 22$
We know that $a_n = a + (n - 1)d$
So,
$a_6 = a + (6-1)d = a + 5d = 12$ …. (i)
And,

$a_8 = a + (8-1)d = a + 7d = 22$ ……. (ii)

Solving (i) and (ii), we have
(ii) - (i) ⇒
$a + 7d – (a + 5d) = 22 – 12$
$2d = 10$
$d = 5$
Putting d in (i) we get,
$a + 5(5) = 12$
$a = 12 – 25$
$a = -13$
Thus, for the A.P: a = -13 and d = 5
So, the n^{th} term is given by $a_n = a + (n-1)d$
$a_n = -13 + (n-1)5 = -13 + 5n – 5$
⇒ $a_n = 5n – 18$
Hence, the second term is given by $a_2 = 5(2) – 18 = 10 – 18 = -8$

16. How many numbers of two digit are divisible by 3?
Solution:

The first 2 digit number divisible by 3 is 12. And, the last 2 digit number divisible by 3 is 99.
So, this forms an A.P.
12, 15, 18, 21, …. , 99
Where, a = 12 and d = 3
Finding the number of terms in this A.P
⇒ $99 = 12 + (n-1)3$
$99 = 12 + 3n – 3$
$90 = 3n$
$n = 90/3 = 30$
Therefore, there are 30 two digit numbers divisible by 3.

17. An A.P. consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find 32^{nd} term.
Solution:

Given, an A.P of 60 terms
And, a = 7 and $a_{60} = 125$
We know that $a_n = a + (n - 1)d$
⇒ $a_{60} = 7 + (60 - 1)d = 125$
$7 + 59d = 125$
$59d = 118$
$d = 2$

So, the 32^{nd} term is given by
$a_{32} = 7 + (32 -1)2 = 7 + 62 = 69$

$\Rightarrow a_{32} = 69$

18. The sum of 4^{th} and 8^{th} terms of an A.P. is 24 and the sum of the 6^{th} and 10^{th} terms is 34. Find the first term and the common difference of the A.P.
Solution:

Given, in an A.P
The sum of 4^{th} and 8^{th} terms of an A.P. is 24
$\Rightarrow a_4 + a_8 = 24$
And, we know that $a_n = a + (n - 1)d$
$[a + (4-1)d] + [a + (8-1)d] = 24$
$2a + 10d = 24$
$a + 5d = 12$ …. (i)

Also given that,
the sum of the 6^{th} and 10^{th} terms is 34
$\Rightarrow a_6 + a_{10} = 34$
$[a + 5d] + [a + 9d] = 34$
$2a + 14d = 34$
$a + 7d = 17$ …… (ii)

Subtracting (i) form (ii), we have
$a + 7d – (a + 5d) = 17 – 12$
$2d = 5$
$d = 5/2$
Using d in (i) we get,
$a + 5(5/2) = 12$
$a = 12 – 25/2$
$a = -1/2$

Therefore, the first term is -1/2 and the common difference is 5/2.

19. The first term of an A.P. is 5 and its 100^{th} term is -292. Find the 50^{th} term of this A.P.
Solution:

Given, an A.P whose
$a = 5$ and $a_{100} = -292$
We know that $a_n = a + (n - 1)d$
$a100 = 5 + 99d = -292$
$99d = -297$
$d = -3$

Hence, the 50^{th} term is
$a_{50} = a + 49d = 5 + 49(-3) = 5 – 147 = -142$

20. Find $a_{30} – a_{20}$ for the A.P.
(i) -9, -14, -19, -24 **(ii) a, a+d, a+2d, a+3d, ……**
Solution:

We know that $a_n = a + (n - 1)d$
So, $a_{30} – a_{20} = (a + 29d) – (a + 19) =10d$

(i) Given A.P. -9, -14, -19, -24
Here, a = -9 and d = -14 – (-9) = = -14 + 9 = -5
So, $a_{30} – a_{20} = 10d$
$= 10(-5)$
$= -50$

(ii) Given A.P. a, a+d, a+2d, a+3d, ……
So, $a_{30} – a_{20} = (a + 29d) – (a + 19d$
$=10d$

21. Write the expression $a_n – a_k$ for the A.P. a, a+d, a+2d, …..
Hence, find the common difference of the A.P. for which
(i) 11th term is 5 and 13th term is 79.
(ii) $a_{10} – a_5 = 200$
(iii) 20th term is 10 more than the 18th term.
Solution:

Given A.P. a, a+d, a+2d, …..
So, $a_n = a + (n-1)d = a + nd – d$
And, $a_k = a + (k-1)d = a + kd – d$
$a_n - a_k = (a + nd – d) – (a + kd – d)$
$= (n – k)d$

(i) Given 11th term is 5 and 13th term is 79,
Here n = 13 and k = 11,
$a_{13} – a_{11} = (13 – 11)d = 2d$
$\Rightarrow 79 – 5 = 2d$
$d = 74/2 = 37$

(ii) Given, $a_{10} – a_5 = 200$
$\Rightarrow (10 - 5)d = 200$
$5d = 200$
$d = 40$

(iii) Given, 20th term is 10 more than the 18th term.
$\Rightarrow a_{20} – a_{18} = 10$
$(20 - 18)d = 10$
$2d = 10$
$d = 5$

22. Find n if the given value of x is the n^{th} term of the given A.P.
(i) 25, 50, 75, 100, ; x = 1000 **(ii) -1, -3, -5, -7, …; x = -151**
(iii) 5½, 11, 16½, 22, ….; x = 550 **(iv) 1, 21/11, 31/11, 41/11, …; x = 171/11**
Solution:

(i) Given A.P. 25, 50, 75, 100, …… ,1000
Here, a = 25 d = 50 – 25 = 25
Last term (n^{th} term) = 1000
We know that $a_n = a + (n - 1)d$
⇒ 1000 = 25 + (n-1)25
1000 = 25 + 25n – 25
n = 1000/25
n = 40

(ii) Given A.P. -1, -3, -5, -7, …., -151
Here, a = -1 d = -3 – (-1) = -2
Last term (n^{th} term) = -151
We know that $a_n = a + (n - 1)d$
⇒ -151 = -1 + (n-1)(-2)
-151 = -1 - 2n + 2
n = 152/2
n = 76

(iii) Given A.P. 5½, 11, 16½, 22, … , 550
Here, a = 5½ d = 11 – (5½) = 5½ = 11/2
Last term (n^{th} term) = 550
We know that $a_n = a + (n - 1)d$
⇒ 550 = 5½ + (n-1)(11/2)
550 x 2 = 11+ 11n – 11
1100 = 11n
n = 100

(iv) Given A.P. 1, 21/11, 31/11, 41/11, 171/11
Here, a = 1 d = 21/11 – 1 = 10/11
Last term (n^{th} term) = 171/11
We know that $a_n = a + (n - 1)d$
⇒ 171/11 = 1 + (n-1)10/11
171 = 11 + 10n – 10
n = 170/10
n = 17

23. The eighth term of an A.P is half of its second term and the eleventh term exceeds one third of its fourth term by 1. Find the 15^{th} term.
Solution:

Given, an A.P in which,

$a_8 = 1/2(a_2)$
$a_{11} = 1/3(a_4) + 1$
We know that $a_n = a + (n - 1)d$
$\Rightarrow a_8 = 1/2(a_2)$
$a + 7d = 1/2(a + d)$
$2a + 14d = a + d$
$a + 13d = 0$ …… (i)

And, $a_{11} = 1/3(a_4) + 1$
$a + 10d = 1/3(a + 3d) + 1$
$3a + 30d = a + 3d + 3$
$2a + 27d = 3$ …… (ii)

Solving (i) and (ii), by (ii) – 2x(i) ⇒
$2a + 27d – 2(a + 13d) = 3 - 0$
$d = 3$
Putting d in (i) we get,
$a + 13(3) = 0$
$a = -39$

Thus, the 15th term $a_{15} = -39 + 14(3) = -39 + 42 = 3$

24. Find the arithmetic progression whose third term is 16 and the seventh term exceeds its fifth term by 12.
Solution:

Given, in an A.P
$a_3 = 16$ and $a_7 = a_5 + 12$
We know that $a_n = a + (n - 1)d$
$\Rightarrow a + 2d = 16$…… (i)
And,
$a + 6d = a + 4d + 12$
$2d = 12$
$\Rightarrow d = 6$

Using d in (i), we have
$a + 2(6) = 16$
$a = 16 – 12 = 4$

Hence, the A.P is 4, 10, 16, 22, …….

25. The 7th term of an A.P. is 32 and its 13th term is 62. Find the A.P.
Solution:

Given,
$a_7 = 32$ and $a_{13} = 62$

From $a_n - a_k = (a + nd – d) – (a + kd – d)$
$= (n – k)d$

$a_{13} – a_7 = (13 - 7)d = 62 – 32 = 30$
$6d = 30$
$d = 5$

Now,
$a_7 = a + (7 - 1)5 = 32$
$a + 30 = 32$
$a = 2$

Hence, the A.P is 2, 7, 12, 17, ……

26. Which term of the A.P. 3, 10, 17, …. will be 84 more than its 13th term ?
Solution:

Given, A.P. 3, 10, 17, ….
Here, $a = 3$ and $d = 10 – 3 = 7$
According the question,
$a_n = a_{13} + 84$
Using $a_n = a + (n - 1)d$,
$3 + (n - 1)7 = 3 + (13 - 1)7 + 84$
$3 + 7n – 7 = 3 + 84 + 84$
$7n = 168 + 7$
$n = 175/7$
$n = 25$

Therefore, it the 25th term which is 84 more than its 13th term.

27. Two arithmetic progressions have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?
Solution:

Let the two A.Ps be $A.P_1$ and $A.P_2$
For $A.P_1$ the first term = a and the common difference = d
And for $A.P_2$ the first term = b and the common difference = d
So, from the question we have
$a_{100} – b_{100} = 100$
$(a + 99d) – (b + 99d) = 100$
$a - b = 100$

Now, the difference between their 1000th terms is,
$(a + 999d) – (b + 999d) = a – b = 100$

Therefore, the difference between their 1000th terms is also 100.

Exercise 9.5

1. Find the value of x for which (8x + 4), (6x – 2) and (2x + 7) are in A.P.
Solution:

Given,
(8x + 4), (6x – 2) and (2x + 7) are in A.P.
So, the common difference between the consecutive terms should be the same.
$(6x - 2) - (8x + 4) = (2x + 7) - (6x - 2)$
$\Rightarrow 6x - 2 - 8x - 4 = 2x + 7 - 6x + 2$
$\Rightarrow -2x - 6 = -4x + 9$
$\Rightarrow -2x + 4x = 9 + 6$
$\Rightarrow 2x = 15$
Therefore, x = 15/2

2. If x + 1, 3x and 4x + 2 are in A.P., find the value of x.
Solution:

Given,
x + 1, 3x and 4x + 2 are in A.P.
So, the common difference between the consecutive terms should be the same.
$3x - x - 1 = 4x + 2 - 3x$
$\Rightarrow 2x - 1 = x + 2$
$\Rightarrow 2x - x = 2 + 1$
$\Rightarrow x = 3$
Therefore, x = 3

3. Show that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in A.P.
Solution:

If $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ have to be in A.P. then,
It should satisfy the condition,
2b = a + c [for a, b, c are in A.P]
Thus,
$2(a^2 + b^2) = (a - b)^2 + (a + b)^2$
$2(a^2 + b^2) = a^2 + b^2 - 2ab + a^2 + b^2 + 2ab$
$2(a^2 + b^2) = 2a^2 + 2b^2 = 2(a^2 + b^2)$
LHS = RHS
Hence proved.

4. The sum of three terms of an A.P. is 21 and the product of the first and the third terms exceeds the second term by 6, find three terms.
Solution:

Let's consider the three terms of the A.P. to be a – d, a, a + d
so, the sum of three terms = 21

$\Rightarrow a - d + a + a + d = 21$
$\Rightarrow 3a = 21$
$\Rightarrow a = 7$

And, product of the first and 3rd = 2nd term + 6
$\Rightarrow (a - d)(a + d) = a + 6$
$a^2 - d^2 = a + 6$
$\Rightarrow (7)^2 - d^2 = 7 + 6$
$\Rightarrow 49 - d^2 = 13$
$\Rightarrow d^2 = 49 - 13 = 36$
$\Rightarrow d^2 = (6)^2$
$\Rightarrow d = 6$

Hence, the terms are $7 - 6, 7, 7 + 6 \Rightarrow 1, 7, 13$

5. Three numbers are in A.P. If the sum of these numbers be 27 and the product 648, find the numbers.
Solution:

Let the three numbers of the A.P. be $a - d, a, a + d$
From the question,
Sum of these numbers = 27
$a - d + a + a + d = 27$
$\Rightarrow 3a = 27$
$a = 27/3 = 9$

Now, product of these numbers = 648
$(a - d)(a)(a + d) = 648$
$a(a^2 - d^2) = 648$
$a^2 - 648/a = d^2$
$9^2 - (648/9) = d^2$
$9^3 - 648 = 9d^2$
$729 - 648 = 9d^2$
$81 = 9d^2$
$d^2 = 9$
$d = 3$ or -3

Hence, the terms are 9-3, 9 and 9+3 $\Rightarrow$ 6, 9, 12 or 12, 9, 6 (for $d = -3$)

6. Find the four numbers in A.P., whose sum is 50 and in which the greatest number is 4 times the least.
Solution:

Let's consider the four terms of the A.P. to be $(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$.
From the question,
Sum of these terms = 50

$\Rightarrow$ (a – 3d) + (a – d) + (a + d) + (a + 3d) = 50
$\Rightarrow$ a – 3d + a – d + a + d + a – 3d= 50
$\Rightarrow$ 4a = 50
$\Rightarrow$ a = 50/4 = 25/2
And, also given that the greatest number = 4 x least number
$\Rightarrow$ a + 3d = 4 (a – 3d)
$\Rightarrow$ a + 3d = 4a – 12d
$\Rightarrow$ 4a – a = 3d + 12d
$\Rightarrow$3a = 15d
$\Rightarrow$a = 5d
Using the value of a in the above equation, we have
$\Rightarrow$25/2 = 5d
$\Rightarrow$ d = 5/2

So, the terms will be:
(a – 3d) = (25/2 – 3(5/2)), (a – d) = (25/2 – 5/2), (25/2 + 5/2) and (25/2 + 3(5/2)).
$\Rightarrow$ 5, 10, 15, 20

Exercise 9.6

1. Find the sum of the following arithmetic progressions:
(i) 50, 46, 42, ... to 10 terms
(ii) 1, 3, 5, 7, ... to 12 terms
(iii) 3, 9/2, 6, 15/2, ... to 25 terms
(iv) 41, 36, 31, ... to 12 terms
(v) a + b, a - b, a - 3b, ... to 22 terms
(vi) $(x - y)^2$, $(x^2 + y^2)$, $(x + y)^2$, to 22 tams

(vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}$, to n terms

(viii) – 26, – 24, – 22, ... to 36 terms
Solution:

In an A.P if the first term = a, common difference = d, and if there are n terms.
Then, sum of n terms is given by:

$$S_n = \frac{n}{2}\{2a + (n - 1)d\}$$

(i) Given A.P.is 50, 46, 42 to 10 term.
First term (a) = 50
Common difference (d) = 46 - 50 = – 4
n^{th} term (n) = 10

$$\text{Then } S_{10} = \frac{10}{2}\{2.50 + (10 - 1) - 4\}$$

= 5{100 - 9.4}
= 5{100 - 36}
= S × 64
∴ S_{10} = 320

(ii) Given A.P is, 1, 3, 5, 7, ...to 12 terms.
First term (a) = 1
Common difference (d) = 3 - 1 = 2
n^{th} term (n) = 12

$$\text{Sum of } n^{th} \text{ terms } S_{12} = \frac{12}{2} \times \{2.1 + (12 - 1).2\}$$

= 6 × {2 + 22} = 6.24
∴ S_{12} = 144

(iii) Given A.P. is 3, 9/2, 6, 15/2, ... to 25 terms
First term (a) = 3
Common difference (d) = 9/2 - 3 = 3/2
Sum of n terms S_n, given n = 25

$$S_{25} = \frac{n}{2}(2a + (n-1)d)$$

$$S_{25} = \frac{25}{2}(2.3 + 25 - 1) \times \frac{3}{2}$$

$$= \frac{25}{2}\left(6 + 24.\frac{3}{2}\right)$$

$$= \frac{25}{2}\ (6 + 36)$$

$$= \frac{25}{2}\ (42)$$

$\therefore S_{25} = 525$

(iv) Given expression is 41, 36, 31, ….. to 12 terms.
First term (a) = 41
Common difference (d) = 36 - 41 = -5
Sum of n terms S_n, given n = 12

$$S_{12} = \frac{n}{2}(2a + (n-1)d)$$

$$S_{12} = \frac{12}{6}(2.41 + 12 - 1) \times -5$$

$$= 6(82 + 11 \times (-5))$$

$$= 6 \times 27$$

$$= 162$$

$\therefore\ S_{12} = 162.$

(v) a + b, a – b, a - 3b, ….. to 22 terms
First term (a) = a + b
Common difference (d) = a - b - a - b = -2b
Sum of n terms S_n = n/2{2a(n - 1). d}
Here n = 22
S_{22} = 22/2{2.(a + b) + (22 - 1). -2b}
= 11{2(a + b) - 22b)
= 11{2a - 20b}

= 22a - 440b

$\therefore S_{22}$ = 22a - 440b

(vi) $(x - y)^2, (x^2 + y^2), (x + y)^2, \ldots$ to n terms

First term (a) = $(x - y)^2$

Common difference (d) = $x^2 + y^2 - (x - y)^2$

= $x^2 + y^2 - (x^2 + y^2 - 2xy)$

= $x^2 + y^2 - x^2 + y^2 + 2xy$

= 2xy

Sum of n^{th} terms $S_n = n/2\{2a(n - 1).\ d\}$

= $n/2\{2(x - y)^2 + (n - 1).\ 2xy\}$

= $n\{(x - y)^2 + (n - 1)xy\}$

$\therefore S_n = n\{(x — y)^2 + (n — 1).\ xy)$

(vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \ldots\ldots$ to n terms

First term (a) $= \frac{x-y}{x+y}$

Common difference (d) $= \frac{3x-2y}{x+y} - \frac{x-y}{x+y}$

$= \frac{3x-2y-x-y}{x+y}$

$= \frac{2x-y}{x+y}$

Sum of n terms $S_n = \frac{n}{2}(2a + (n-1).\ d)$

$= \frac{n}{2}\left[2.\frac{x-y}{x+y} = \ + (n-1).\ 2 - \frac{2x-y}{x+y}\right\}$

$$= \frac{n}{2(x+y)}\{2(x-y)+(n-1)(2x-y)\}$$

$$= \frac{n}{2(x+y)},\{2x-2y+2nx-ny-2x+y\}$$

$$= \frac{n}{2(x+y)}\{n(2x - y) - y\}$$

$$\therefore S_n = \frac{n}{2(x+y)}\{n(2x - y) - y\}$$

(viii) Given expression -26, - 24. -22, to 36 terms
First term (a) = -26
Common difference (d) = -24 - (-26)
= -24 + 26 = 2
Sum of n terms, S_n = n/2{2a + (n - 1)d) for n = 36
S_n = 36/2{2(-26) + (36 - 1)2}
= 18[-52 + 70]
= 18x18
= 324
∴ S_n = 324

2. Find the sum to n terms of the A.P. 5, 2, –1, – 4, –7, ...
Solution:

Given AP is 5, 2, -1, -4, -7, ...
Here, a = 5, d = 2 - 5 = -3
We know that,
S_n = n/2{2a + (n - 1)d}
= n/2{2.5 + (n - 1) - 3}
= n/2{10 - 3(n - 1)}
= n/2{13 - 3n)
∴ S_n = n/2(13 - 3n)

3. Find the sum of n terms of an A.P. whose the terms is given by a_n = 5 - 6n.
Solution:

Given nth term of the A.P as a_n = 5 - 6n
Put n = 1, we get
a_1 = 5 - 6.1 = -1
So, first term (a) = -1
Last term (a_n) = 5 - 6n = 1
Then, S_n = n/2(-1 + 5 - 6n)

= n/2(4 - 6n) = n(2 - 3n)

4. Find the sum of last ten terms of the A.P. : 8, 10, 12, 14, .. , 126
Solution:

Given A.P. 8, 10, 12, 14, .. , 126
Here, a = 8 , d = 10 – 8 = 2
We know that, $a_n = a + (n - 1)d$
So, to find the number of terms
126 = 8 + (n - 1)2
126 = 8 + 2n - 2
2n = 120
n = 60

Next, let's find the 51^{st} term
$a_{51} = 8 + 50(2) = 108$

So, the sum of last ten terms is the sum of $a_{51} + a_{52} + a_{53} + \ldots\ldots + a_{60}$
Here, n = 10, a = 108 and l = 126
S = 10/2 [108 + 126]
= 5(234)
= 1170
Hence, the sum of last ten terms of the A.P is 1170.

5. Find the sum of first 15 terms of each of the following sequences having n^{th} term as:
(i) $a_n = 3 + 4n$
(ii) $b_n = 5 + 2n$
(iii) $x_n = 6 - n$
(iv) $y_n = 9 – 5n$
Solution:

(i) Given an A.P. whose n^{th} term is given by $a_n = 3 + 4n$
To find the sum of the n terms of the given A.P., using the formula,
$S_n = n(a + l)/ 2$
Where, a = the first term l = the last term.
Putting n = 1 in the given a_n, we get
a = 3 + 4(1) = 3 + 4 = 7
For the last term (l), here n = 15
$a_{15} = 3 + 4(15) = 63$

So, $S_n = 15(7 + 63)/2$
= 15 x 35
= 525
Therefore, the sum of the 15 terms of the given A.P. is $S_{15} = 525$

(ii) Given an A.P. whose n^{th} term is given by $b_n = 5 + 2n$

To find the sum of the n terms of the given A.P., using the formula,
$S_n = n(a + l)/ 2$
Where, a = the first term l = the last term.
Putting n = 1 in the given b_n, we get
$a = 5 + 2(1) = 5 + 2 = 7$
For the last term (l), here n = 15
$a_{15} = 5 + 2(15) = 35$

So, $S_n = 15(7 + 35)/2$
$= 15 \times 21$
$= 315$
Therefore, the sum of the 15 terms of the given A.P. is $S_{15} = 315$

(iii) Given an A.P. whose n^{th} term is given by $x_n = 6 - n$
To find the sum of the n terms of the given A.P., using the formula
$S_n = n(a + l)/ 2$
Where, a = the first term l = the last term.
Putting n = 1 in the given x_n, we get
$a = 6 – 1 = 5$
For the last term (l), here n = 15
$a_{15} = 6 – 15 = -9$

So, $S_n = 15(5 – 9)/2$
$= 15 \times (-2)$
$= -30$
Therefore, the sum of the 15 terms of the given A.P. is $S_{15} = -30$

(iv) Given an A.P. whose n^{th} term is given by $y_n = 9 – 5n$
To find the sum of the n terms of the given A.P., using the formula,
$S_n = n(a + l)/ 2$
Where, a = the first term l = the last term.
Putting n = 1 in the given y_n, we get
$a = 9 - 5(1) = 9 – 5 = 4$
For the last term (l), here n = 15
$a_{15} = 9 - 5(15) = -66$

So, $S_n = 15(4 - 66)/2$
$= 15 \times (-31)$
$= -465$
Therefore, the sum of the 15 terms of the given A.P. is $S_{15} = -465$

6. Find the sum of first 20 terms the sequence whose n^{th} term is $a_n = An + B$.
Solution:

Given an A.P. whose nth term is given by, $a_n = An + B$
We need to find the sum of first 20 terms.

To find the sum of the n terms of the given A.P., we use the formula,
$S_n = n(a + l)/ 2$
Where, a = the first term l = the last term,
Putting n = 1 in the given a_n, we get
a = A(1) + B = A + B
For the last term (l), here n = 20
$A_{20} = A(20) + B = 20A + B$
$S_{20} = 20/2((A + B) + 20A + B)$
= 10[21A + 2B]
= 210A + 20B
Therefore, the sum of the first 20 terms of the given A.P. is 210 A + 20B

7. Find the sum of first 25 terms of an A.P whose n^{th} term is given by $a_n = 2 - 3n$.
Solution:

Given an A.P. whose n^{th} term is given by $a_n = 2 – 3n$
To find the sum of the n terms of the given A.P., we use the formula,
$S_n = n(a + l)/ 2$
Where, a = the first term l = the last term.
Putting n = 1 in the given a_n, we get
a = 2 - 3(1) = -1
For the last term (l), here n = 25
$a_{25} = 2 - 3(25) = -73$

So, $S_n = 25(-1 - 73)/2$
= 25 x (-37)
= -925
Therefore, the sum of the 25 terms of the given A.P. is $S_{25} = -465$

8. Find the sum of first 25 terms of an A.P whose n^{th} term is given by $a_n = 7 - 3n$.
Solution:

Given an A.P. whose n^{th} term is given by $a_n = 7 – 3n$
To find the sum of the n terms of the given A.P., we use the formula,
$S_n = n(a + l)/ 2$
Where, a = the first term l = the last term.
Putting n = 1 in the given a_n, we get
a = 7 - 3(1) = 7 – 3 = 4
For the last term (l), here n = 25
$a_{15} = 7 - 3(25) = -68$

So, $S_n = 25(4 - 68)/2$
= 25 x (-32)
= -800
Therefore, the sum of the 15 terms of the given A.P. is $S_{25} = -80$

9. If the sum of a certain number of terms starting from first term of an A.P. is 25, 22, 19, . . ., is 116. Find the last term.
Solution:

Given the sum of the certain number of terms of an A.P. = 116
We know that, $S_n = n/2[2a + (n - 1)d]$
Where; a = first term for the given A.P.
d = common difference of the given A.P.
n = number of terms So for the given A.P.(25, 22, 19,...)
Here we have, the first term (a) = 25
The sum of n terms $S_n = 116$
Common difference of the A.P. (d) = $a_2 - a_1 = 22 – 25 = -3$
Now, substituting values in S_n
$\Longrightarrow 116 = n/2[2(25) + (n - 1)(-3)]$
$\Longrightarrow (n/2)[50 + (-3n + 3)] = 116$
$\Longrightarrow (n/2)[53 - 3n] = 116$
$\Longrightarrow 53n - 3n^2 = 116 \times 2$
Thus, we get the following quadratic equation,
$3n^2 - 53n + 232 = 0$
By factorization method of solving, we have
$\Longrightarrow 3n^2 - 24n - 29n + 232 = 0$
$\Longrightarrow 3n(n - 8) - 29 (n - 8) = 0$
$\Longrightarrow (3n - 29)(n - 8) = 0$
So, $3n - 29 = 0$
$\Longrightarrow n = 29/3$
Also, $n - 8 = 0$
$\Longrightarrow n = 8$
Since, n cannot be a fraction, so the number of terms is taken as 8.
So, the term is:
$a_8 = a_1 + 7d = 25 + 7(-3) = 25 - 21 = 4$
Hence, the last term of the given A.P. such that the sum of the terms is 116 is 4.

10. (i) How many terms of the sequence 18, 16, 14.... should be taken so that their sum is zero.
(ii) How many terms are there in the A.P. whose first and fifth terms are -14 and 2 respectively and the sum of the terms is 40?
(iii) How many terms of the A.P. 9, 17, 25, . . . must be taken so that their sum is 636?
(iv) How many terms of the A.P. 63, 60, 57, . . . must be taken so that their sum is 693?
(v) How many terms of the A.P. is 27, 24, 21. . . should be taken that their sum is zero?
Solution:

(i) Given AP. is 18, 16, 14, ...
We know that,
$S_n = n/2[2a + (n - 1)d]$
Here,
The first term (a) = 18
The sum of n terms (S_n) = 0 (given)

Common difference of the A.P.
(d) = a_2 - a_1 = 16 - 18 = – 2
So, on substituting the values in S_n
⟹ 0 = n/2[2(18) + (n − 1)(−2)]
⟹ 0 = n/2[36 + (−2n + 2)]
⟹ 0 = n/2[38 − 2n] Further, n/2
⟹ n = 0 Or, 38 - 2n = 0
⟹ 2n = 38
⟹ n = 19
Since, the number of terms cannot be zer0, hence the number of terms (n) should be 19.

(ii) Given, the first term (a) = -14, Filth term (a_5) = 2, Sum of terms (S_n) = 40 of the A.P.
If the common difference is taken as d.
Then, a_5 = a + 4d
⟹ 2 = -14 + 4d
⟹ 2 + 14 = 4d
⟹ 4d = 16
⟹ d = 4

Next, we know that S_n = n/2[2a + (n − 1)d]
Where; a = first term for the given A.P.
d = common difference of the given A.P.
n = number of terms

Now, on substituting the values in S_n
⟹ 40 = n/2[2(−14) + (n − 1)(4)]
⟹ 40 = n/2[−28 + (4n − 4)]
⟹ 40 = n/2[−32 + 4n]
⟹ 40(2) = - 32n + $4n^2$
So, we get the following quadratic equation,
$4n^2$ - 32n - 80 = 0
⟹ n^2 - 8n + 20 = 0
On solving by factorization method, we get
$4n^2$ - 10n + 2n + 20 = 0
⟹ n(n - 10) + 2(n - 10) = 0
⟹ (n + 2)(n - 10) = 0
Either, n + 2 = 0
⟹ n = -2
Or, n - 10 = 0
⟹ n = 10
Since the number of terms cannot be negative**.**
Therefore, the number of terms (n) is 10.

(iii) Given AP is 9, 17, 25,...
We know that,
S_n = n/2[2a + (n − 1)d]

Here we have,
The first term (a) = 9 and the sum of n terms (S_n) = 636
Common difference of the A.P. (d) = $a_2 - a_1$ = 17 - 9 = 8
Substituting the values in S_n, we get
$\Rightarrow 636 = n/2[2(9) + (n - 1)(8)]$
$\Rightarrow 636 = n/2[18 + (8n - 8)]$
$\Rightarrow 636(2) = (n)[10 + 8n]$
$\Rightarrow 1271 = 10n + 8n^2$
Now, we get the following quadratic equation,
$\Rightarrow 8n^2 + 10n - 1272 = 0$
$\Rightarrow 4n^2 + 5n - 636 = 0$
On solving by factorisation method, we have
$\Rightarrow 4n^2 - 48n + 53n - 636 = 0$
$\Rightarrow 4n(n - 12) - 53(n - 12) = 0$
$\Rightarrow (4n - 53)(n - 12) = 0$
Either $4n - 53 = 0 \Rightarrow n = 534$
Or, $n - 12 = 0 \Rightarrow n = 12$
Since, the number of terms cannot be a fraction.
Therefore, the number of terms (n) is 12.

(iv) Given A.P. is 63, 60, 57,...
We know that,
$S_n = n/2[2a + (n - 1)d]$
Here we have,
the first term (a) = 63
The sum of n terms (S_n) = 693
Common difference of the A.P. (d) = $a_2 - a_1$ = 60 - 63 = –3
On substituting the values in S_n we get
$\Rightarrow 693 = n/2[2(63) + (n - 1)(-3)]$
$\Rightarrow 693 = n/2[163+(-3n + 3)]$
$\Rightarrow 693 = n/2[129 - 3n]$
$\Rightarrow 693(2) = 129n - 3n^2$
Now, we get the following quadratic equation.
$\Rightarrow 3n^2 - 129n + 1386 = 0$
$\Rightarrow n^2 - 43n + 462$
Solving by factorisation method, we have
$\Rightarrow n^2 - 22n - 21n + 462 = 0$
$\Rightarrow n(n - 22) -21(n - 22) = 0$
$\Rightarrow (n - 22) (n - 21) = 0$
Either, $n - 22 = 0 \Rightarrow n = 22$
Or, $n - 21 = 0 \Rightarrow n = 21$
Now, the 22nd term will be $a_{22} = a_1 + 21d = 63 + 21(-3) = 63 - 63 = 0$
So, the sum of 22 as well as 21 terms is 693.
Therefore, the number of terms (n) is 21 or 22.

(v) Given A.P. is 27, 24, 21. . .
We know that,
$S_n = n/2[2a + (n - 1)d]$
Here we have, the first term (a) = 27
The sum of n terms (S_n) = 0
Common difference of the A.P. (d) = $a_2 - a_1 = 24 - 27 = -3$
On substituting the values in S_n, we get
$\Rightarrow 0 = n/2[2(27) + (n - 1)(-3)]$
$\Rightarrow 0 = (n)[54 + (n - 1)(-3)]$
$\Rightarrow 0 = (n)[54 - 3n + 3]$
$\Rightarrow 0 = n[57 - 3n]$ Further we have, n = 0 Or, 57 - 3n = 0
$\Rightarrow 3n = 57$
$\Rightarrow n = 19$
The number of terms cannot be zero,
Hence, the numbers of terms (n) is 19.

11. Find the sum of the first
(i) 11 terms of the A.P. : 2, 6, 10, 14, . . .
(ii) 13 terms of the A.P. : -6, 0, 6, 12, . . .
(iii) 51 terms of the A.P. : whose second term is 2 and fourth term is 8.
Solution:

We know that the sum of terms for different arithmetic progressions is given by
$S_n = n/2[2a + (n - 1)d]$
Where; a = first term for the given A.P. d = common difference of the given A.P. n = number of terms

(i) Given A.P 2, 6, 10, 14,... to 11 terms.
Common difference (d) = $a_2 - a_1 = 10 - 6 = 4$
Number of terms (n) = 11
First term for the given A.P. (a) = 2
So,
$S_{11} = 11/2[2(2) + (11 - 1)4]$
$= 11/2[2(2) + (10)4]$
$= 11/2[4 + 40]$
$= 11 \times 22$
$= 242$
Hence, the sum of first 11 terms for the given A.P. is 242

(ii) Given A.P. – 6, 0, 6, 12, ... to 13 terms.
Common difference (d) = $a_2 - a_1 = 6 - 0 = 6$
Number of terms (n) = 13
First term (a) = -6
So,
$S_{13} = 13/2[2(-6) + (13 - 1)6]$
$= 13/2[(-12) + (12)6]$
$= 13/2[60] = 390$

Hence, the sum of first 13 terms for the given A.P. is 390

(iii) 51 terms of an AP whose $a_2 = 2$ and $a_4 = 8$
We know that, $a_2 = a + d$
$2 = a + d$ ...(2)
Also, $a_4 = a + 3d$
$8 = a + 3d$ ... (2)
Subtracting (1) from (2), we have
$2d = 6$
$d = 3$
Substituting d = 3 in (1), we get
$2 = a + 3$
$\Longrightarrow a = -1$

Given that the number of terms (n) = 51
First term (a) = -1
So,
$S_n = 51/2[2(-1) + (51 - 1)(3)]$
$= 51/2[-2 + 150]$
$= 51/2[158]$
$= 3774$
Hence, the sum of first 51 terms for the A.P. is 3774.

12. Find the sum of
(i) the first 15 multiples of 8
(ii) the first 40 positive integers divisible by (a) 3 (b) 5 (c) 6.
(iii) all 3 - digit natural numbers which are divisible by 13.
(iv) all 3 - digit natural numbers which are multiples of 11.

Solution:
We know that the sum of terms for an A.P is given by
$S_n = n/2[2a + (n - 1)d]$
Where; a = first term for the given A.P. d = common difference of the given A.P. n = number of terms

(i) Given, first 15 multiples of 8.
These multiples form an A.P: 8, 16, 24, …… , 120
Here, a = 8 , d = 61 – 8 = 8 and the number of terms(n) = 15
Now, finding the sum of 15 terms, we have

$$S_n = \frac{15}{2}[2(8) + (15 - 1)8]$$

$$= \frac{15}{2}[16 + (14)8]$$

$$= \frac{15}{2}[16 + 12]$$

$$= \frac{15}{2}[128]$$

$$= 960$$ \

Hence, the sum of the first 15 multiples of 8 is 960

(ii)(a) First 40 positive integers divisible by 3.
Hence, the first multiple is 3 and the 40^{th} multiple is 120.
And, these terms will form an A.P. with the common difference of 3.
Here, First term (a) = 3
Number of terms (n) = 40
Common difference (d) = 3
So, the sum of 40 terms
$S_{40} = 40/2[2(3) + (40 - 1)3]$
$= 20[6 + (39)3]$
$= 20(6 + 117)$
$= 20(123) = 2460$
Thus, the sum of first 40 multiples of 3 is 2460.

(b) First 40 positive integers divisible by 5
Hence, the first multiple is 5 and the 40^{th} multiple is 200.
And, these terms will form an A.P. with the common difference of 5.
Here, First term (a) = 5
Number of terms (n) = 40
Common difference (d) = 5
So, the sum of 40 terms
$S_{40} = 40/2[2(5) + (40 - 1)5]$
$= 20[10 + (39)5]$
$= 20 (10 + 195)$
$= 20 (205) = 4100$
Hence, the sum of first 40 multiples of 5 is 4100.

(c) First 40 positive integers divisible by 6

Hence, the first multiple is 6 and the 40th multiple is 240.
And, these terms will form an A.P. with the common difference of 6.
Here, First term (a) = 6
Number of terms (n) = 40
Common difference (d) = 6
So, the sum of 40 terms

S_{40} = 40/2[2(6) + (40 − 1)6]
= 20[12 + (39)6]
=20(12 + 234)
= 20(246) = 4920

Hence, the sum of first 40 multiples of 6 is 4920.

(iii) All 3 digit natural number which are divisible by 13.
So, we know that the first 3 digit multiple of 13 is 104 and the last 3 digit multiple of 13 is 988.
And, these terms form an A.P. with the common difference of 13.
Here, first term (a) = 104 and the last term (l) = 988
Common difference (d) = 13
Finding the number of terms in the A.P. by, $a_n = a + (n - 1)d$
We have,
988 = 104 + (n - 1)13
⟹ 988 = 104 + 13n -13
⟹ 988 = 91 + 13n
⟹ 13n = 897
⟹ n = 69
Now, using the formula for the sum of n terms, we get

S_{69} = 69/2[2(104) + (69 − 1)13]
= 69/2[208 + 884]
= 69/2[1092]
= 69(546)
= 37674

Hence, the sum of all 3 digit multiples of 13 is 37674.

(iv) All 3 digit natural number which are multiples of 11.
So, we know that the first 3 digit multiple of 11 is 110 and the last 3 digit multiple of 13 is 990.
And, these terms form an A.P. with the common difference of 11.
Here, first term (a) = 110 and the last term (l) = 990
Common difference (d) = 11
Finding the number of terms in the A.P. by, $a_n = a + (n - 1)d$
We get,
990 = 110 + (n - 1)11
⟹ 990 = 110 + 11n -11
⟹ 990 = 99 + 11n
⟹ 11n = 891
⟹ n = 81
Now, using the formula for the sum of n terms, we get

S_{81} = 81/2[2(110) + (81 − 1)11]

= 81/2[220 + 880]
= 81/2[1100]
= 81(550)
= 44550

Hence, the sum of all 3 digit multiples of 11 is 44550.

12. Find the sum:
(i) 2 + 4 + 6 + . . . + 200
(ii) 3 + 11 + 19 + . . . + 803
(iii) (-5) + (-8) + (-11) + . . . + (- 230)
(iv) 1 + 3 + 5 + 7 + . . . + 199

(v) $7 + 10\frac{1}{2} + 14 + \cdots + 84$

(vi) 34 + 32 + 30 + . . . + 10
(vii) 25 + 28 + 31 + . . . + 100
Solution:

We know that the sum of terms for an A.P is given by
$S_n = n/2[2a + (n - 1)d]$
Where; a = first term for the given A.P. d = common difference of the given A.P. n = number of terms
Or $S_n = n/2[a + l]$
Where; a = first term for the given A.P. ;l = last term for the given A.P

(i) Given series. 2 + 4 + 6 + . . . + 200 which is an A.P
Where, a = 2 ,d = 4 – 2 = 2 and last term ($a_n = l$) = 200
We know that, $a_n = a + (n - 1)d$
So,
200 = 2 + (n - 1)2
200 = 2 + 2n – 2
n = 200/2 = 100

Now, for the sum of these 100 terms
S_{100} = 100/2 [2 + 200]
= 50(202)
= 10100

Hence, the sum of terms of the given series is 10100.

(ii) Given series. 3 + 11 + 19 + . . . + 803 which is an A.P
Where, a = 3 ,d = 11 – 3 = 8 and last term ($a_n = l$) = 803
We know that, $a_n = a + (n - 1)d$
So,
803 = 3 + (n - 1)8
803 = 3 + 8n – 8
n = 808/8 = 101

Now, for the sum of these 101 terms
S_{101} = 101/2 [3 + 803]
= 101(806)/2
= 101 x 403
= 40703
Hence, the sum of terms of the given series is 40703.

(iii) Given series (-5) + (-8) + (-11) + . . . + (- 230) which is an A.P
Where, a = -5 ,d = -8 – (-5) = -3 and last term (a_n = l) = -230
We know that, a_n = a + (n - 1)d
So,
-230 = -5 + (n - 1)(-3)
-230 = -5 - 3n + 3
3n = -2 + 230
n = 228/3 = 76

Now, for the sum of these 76 terms
S_{76} = 76/2 [-5 + (-230)]
= 38 x (-235)
= -8930
Hence, the sum of terms of the given series is -8930.

(iv) Given series. 1 + 3 + 5 + 7 + . . . + 199 which is an A.P
Where, a = 1 ,d = 3 – 1 = 2 and last term (a_n = l) = 199
We know that, a_n = a + (n - 1)d
So,
199 = 1 + (n - 1)2
199 = 1 + 2n – 2
n = 200/2 = 100

Now, for the sum of these 100 terms
S_{100} = 100/2 [1 + 199]
= 50(200)
= 10000
Hence, the sum of terms of the given series is 10000.

(v) Given series $7 + 10\frac{1}{2} + 14 + \cdots + 84$ which is an A.P

Where, a = 7, d = 10 ½ - 7 = (21 – 14)/2 = 7/2 and last term (a_n = l) = 84
We know that, a_n = a + (n - 1)d
So,
84 = 7 + (n - 1)(7/2)
168 = 14 + 7n – 7
n = (168 – 7)/7 = 161/7 = 23

Now, for the sum of these 23 terms

S_{23} = 23/2 [7 + 84]

= 23(91)/2

= 2093/2

Hence, the sum of terms of the given series is 2093/2.

(vi) Given series, 34 + 32 + 30 + . . . + 10 which is an A.P

Where, a = 34 ,d = 32 – 34 = -2 and last term (a_n = l) = 10

We know that, a_n = a + (n - 1)d

So,

10 = 34 + (n - 1)(-2)

10 = 34 - 2n + 2

n = (36 – 10)/2 = 13

Now, for the sum of these 13 terms

S_{13} = 13/2 [34 + 10]

= 13(44)/2

= 13 x 22

= 286

Hence, the sum of terms of the given series is 286.

(vii) Given series, 25 + 28 + 31 + . . . + 100 which is an A.P

Where, a = 25 ,d = 28 – 25 = 3 and last term (a_n = l) = 100

We know that, a_n = a + (n - 1)d

So,

100 = 25 + (n - 1)(3)

100 = 25 + 3n - 3

n = (100 – 22)/3 = 26

Now, for the sum of these 26 terms

S_{100} = 26/2 [24 + 100]

= 13(124)

= 1625

Hence, the sum of terms of the given series is 1625.

14. The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution:

Given, the first term of the A.P (a) = 17

The last term of the A.P (l) = 350

The common difference (d) of the A.P. = 9

Let the number of terms be n. And, we know that; l = a + (n - 1)d

So, 350 = 17 + (n- 1) 9

⟹ 350 = 17 + 9n - 9

⟹ 350 = 8 + 9n

$\Longrightarrow$ 350 - 8 = 9n
Thus we get, n = 38
Now, finding the sum of terms
S_n = n/2[a + l]
= 38/2(17 + 350)
= 19 × 367
= 6973
Hence, the number of terms is of the A.P is 38 and their sum is 6973.

15. The third term of an A.P. is 7 and the seventh term exceeds three times the third term by 2. Find the first term, the common difference and the sum of first 20 terms.
Solution:

Let's consider the first term as a and the common difference as d.
Given,
a_3 = 7 .... (1) and,
$a_7 = 3a_3 + 2$ .... (2)
So, using (1) in (2), we get,
a_7 = 3(7) + 2 = 21 + 2 = 23 .... (3)
Also, we know that
a_n = a +(n - 1)d
So, the 3th term (for n = 3),
a_3 = a + (3 - 1)d
$\Longrightarrow$ 7 = a + 2d (Using 1)
$\Longrightarrow$ a = 7 - 2d .... (4)
Similarly, for the 7th term (n = 7),
a_7 = a + (7 - 1) d 24 = a + 6d = 23 (Using 3)
a = 23 - 6d .... (5)
Subtracting (4) from (5), we get,
a - a = (23 - 6d) - (7 - 2d)
$\Longrightarrow$ 0 = 23 - 6d - 7 + 2d
$\Longrightarrow$ 0 = 16 - 4d
$\Longrightarrow$ 4d = 16
$\Longrightarrow$ d = 4
Now, to find a, we substitute the value of d in (4), a =7 - 2(4)
$\Longrightarrow$ a = 7 - 8
a = -1
Hence, for the A.P. a = -1 and d = 4
For finding the sum, we know that
$S_n = n/2[2a + (n - 1)d]$ and n = 20 (given0
S_{20} = 20/2[2(−1) + (20 − 1)(4)]
= (10)[-2 + (19)(4)]
= (10)[-2 + 76]
= (10)[74]
= 740
Hence, the sum of first 20 terms for the given A.P. is 740

16. The first term of an A.P. is 2 and the last term is 50. The sum of all these terms is 442. Find the common difference.

Solution:

Given,
The first term of the A.P (a) = 2
The last term of the A.P (l) = 50
Sum of all the terms $S_n = 442$
So, let the common difference of the A.P. be taken as d.
The sum of all the terms is given as,
$442 = (n/2)(2 + 50)$
$\Longrightarrow 442 = (n/2)(52)$
$\Longrightarrow 26n = 442$
$\Longrightarrow n = 17$
Now, the last term is expressed as
$50 = 2 + (17 - 1)d$
$\Longrightarrow 50 = 2 + 16d$
$\Longrightarrow 16d = 48$
$\Longrightarrow d = 3$
Thus, the common difference of the A.P. is d = 3.

17. If 12th term of an A.P. is -13 and the sum of the first four terms is 24, what is the sum of first 10 terms?

Solution:

Let us take the first term as a and the common difference as d.
Given,
$a_{12} = -13$ $S_4 = 24$
Also, we know that $a_n = a + (n - 1)d$
So, for the 12th term
$a_{12} = a + (12 - 1)d = -13$
$\Longrightarrow a + 11d = -13$
$a = -13 - 11d$ .... (1)
And, we that for sum of terms
$S_n = n/2[2a + (n - 1)d]$
Here, n = 4
$S_4 = 4/2[2(a) + (4 - 1)d]$
$\Longrightarrow 24 = (2)[2a + (3)(d)]$
$\Longrightarrow 24 = 4a + 6d$
$\Longrightarrow 4a = 24 - 6d$

$$\Longrightarrow a = 6 - \frac{6}{4}d \; \ldots. (2)$$

Subtracting (1) from (2), we have

$$\Rightarrow a - a = (6 - \frac{6}{4}d) - (-13 - 11d)$$

$$\Rightarrow 0 = 6 - \frac{6}{4}d + 13d + 11d$$

$$\Rightarrow 0 = 19 + \frac{44d - 6d}{4s}$$

Further simplifying for d, we get,

$$\Rightarrow 0 = 19 + \frac{38}{4}d$$

$$\Rightarrow -19 = \frac{19}{2}d$$

$\Rightarrow$ -19 × 2 = 19d
$\Rightarrow$ d = – 2
On substituting the value of d in (1), we find a
a = -13 - 11(-2)
a = -13 + 22
a = 9
Next, the sum of 10 term is given by
S_{10} = 10/2[2(9) + (10 − 1)(−2)]
= (5)[19 + (9)(-2)]
= (5)(18 - 18) = 0
Thus, the sum of first 10 terms for the given A.P. is S_{10} = 0.

18. Find the sum of first 22 terms of an A.P. in which d = 22 and a_{22} = 149.
Solution:

Let the first term be taken as a.
Given,
a_{22} = 149 and the common difference d = 22
Also, we know that
a_n = a + (n - 1) d
So, the 22nd term is given by
a_{22} = a + (22 - 1)d
149 = a + (21) (22)
a = 149 - 462
a = - 313
Now, for the sum of term
S_n = n/2[2a + (n − 1)d]
Here, n = 22
S_{22} = 22/2[2(−313) + (22 − 1)(22)]

= (11)[– 626 + 462]
= (11)[–164]
= – 1804
Hence, the sum of first 22 terms for the given A.P. is S_{22} = -1804

19. In an A.P., if the first term is 22, the common difference is – 4 and the sum to n terms is 64, find n.
Solution:

Given that,
a = 22, d = – 4 and S_n = 64
Let us consider the number of terms as n.
For sum of terms in an A.P, we know that
$S_n = n/2[2a + (n - 1)d]$
Where; a = first term for the given A.P. d = common difference of the given A.P. n = number of terms

So,
$\Rightarrow S_n = n/2[2(22) + (n - 1)(-4)]$
$\Rightarrow 64 = n/2[2(22) + (n - 1)(-4)]$
$\Rightarrow 64(2) = n(48 - 4n)$
$\Rightarrow 128 = 48n - 4n^2$
After rearranging the terms, we have a quadratic equation
$4n^2 - 48n + 128 = 0$,
$n^2 - 12n + 32 = 0$ [dividing by 4 on both sides]
$n^2 - 12n + 32 = 0$
Solving by factorisation method,
$n^2 - 8n - 4n + 32 = 0$
$n (n - 8) - 4 (n - 8) = 0$
$(n - 8) (n - 4) = 0$
So, we get $n - 8 = 0 \Rightarrow n = 8$
Or, $n - 4 = 0 \Rightarrow n = 4$
Hence, the number of terms can be either n = 4 or 8.

20. In an A.P., if the 5th and 12th terms are 30 and 65 respectively, what is the sum of first 20 terms?
Solution:

Let's take the first term as a and the common difference to be d
Given that,
$a_5 = 30$ and $a_{12} = 65$
And, we know that $a_n = a + (n - 1)d$
So,
$a_5 = a + (5 - 1)d$
$30 = a + 4d$
$a = 30 - 4d$ .... (i)

Similarly, $a_{12} = a + (12 - 1) d$
$65 = a + 11d$
$a = 65 - 11d$ .... (ii)

Subtracting (i) from (ii), we have
$a - a = (65 - 11d) - (30 - 4d)$
$0 = 65 - 11d - 30 + 4d$
$0 = 35 - 7d$
$7d = 35$
$d = 5$
Putting d in (i), we get
$a = 30 - 4(5)$
$a = 30 - 20$
$a = 10$
Thus for the A.P; d = 5 and a = 10
Next, to find the sum of first 20 terms of this A.P., we use the following formula for the sum of n terms of an A.P.,
$S_n = n/2[2a + (n - 1)d]$
Where;
a = first term of the given A.P.
d = common difference of the given A.P.
n = number of terms
Here n = 20, so we have
$S_{20} = 20/2[2(10) + (20 - 1)(5)]$
$= (10)[20 + (19)(5)]$
$= (10)[20 + 95]$
$= (10)[115]$
$= 1150$
Hence, the sum of first 20 terms for the given A.P. is 1150

21. Find the sum of first 51 terms of an A.P. whose second and third terms are 14 and 18 respectively.
Solution:

Let's take the first term as a and the common difference as d.
Given that,
$a_2 = 14$ and $a_3 = 18$
And, we know that $a_n = a + (n - 1)d$
So,
$a_2 = a + (2 - 1)d$
$\Longrightarrow 14 = a + d$
$\Longrightarrow a = 14 - d$ .... (i)
Similarly,
$a_3 = a + (3 - 1)d$
$\Longrightarrow 18 = a + 2d$
$\Longrightarrow a = 18 - 2d$ .... (ii)

Subtracting (i) from (ii), we have
a - a = (18 - 2d) - (14 - d)
0 = 18 - 2d - 14 + d
0 = 4 - d
d = 4
Putting d in (i), to find a
a = 14 - 4
a = 10
Thus, for the A.P. d = 4 and a = 10
Now, to find sum of terms
$S_n = n/2(2a + (n - 1)d)$
Where,
a = the first term of the A.P.
d = common difference of the A.P.
n = number of terms So, using the formula for
n = 51,
$\Rightarrow S_{51} = 51/2[2(10) + (51 - 1)(4)]$
$= 51/2[20 + (40)4]$
$= 51/2[220]$
$= 51(110)$
$= 5610$
Hence, the sum of the first 51 terms of the given A.P. is 5610

22. If the sum of 7 terms of an A.P. is 49 and that of 17 terms is 289, find the sum of n terms.
Solution:

Given,
Sum of 7 terms of an A.P. is 49
$\Rightarrow S_7 = 49$
And, sum of 17 terms of an A.P. is 289
$\Rightarrow S_{17} = 289$
Let the first term of the A.P be a and common difference as d.
And, we know that the sum of n terms of an A.P is
$S_n = n/2[2a + (n - 1)d]$
So,
$S_7 = 49 = 7/2[2a + (7 - 1)d]$
$= 7/2 [2a + 6d]$
$= 7[a + 3d]$
$\Rightarrow 7a + 21d = 49$
a + 3d = 7 ….. (i)

Similarly,
$S_{17} = 17/2[2a + (17 - 1)d]$
$= 17/2 [2a + 16d]$
$= 17[a + 8d]$
$\Rightarrow 17[a + 8d] = 289$

$a + 8d = 17$ ….. (ii)
Now, subtracting (i) from (ii), we have
$a + 8d – (a + 3d) = 17 – 7$
$5d = 10$
$d = 2$

Putting d in (i), we find a
$a + 3(2) = 7$
$a = 7 – 6 = 1$
So, for the A.P: a = 1 and d = 2

For the sum of n terms is given by,
$S_n = n/2[2(1) + (n − 1)(2)]$
$= n/2[2 + 2n - 2]$
$= n/2[2n]$
$= n^2$
Therefore, the sum of n terms of the A.P is given by n^2.

23. The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.
Solution:

Sum of first n terms of an A.P is given by $S_n = n/2(2a + (n − 1)d)$
Given,
First term (a) = 5, last term (a_n) = 45 and sum of n terms (S_n) = 400
Now, we know that
$a_n = a + (n - 1)d$
$\Longrightarrow 45 = 5 + (n - 1)d$
$\Longrightarrow 40 = nd - d$
$\Longrightarrow nd - d = 40$ .... (1)
Also,
$S_n = n/2(2(a) + (n − 1)d)$
$400 = n/2(2(5) + (n − 1)d)$
$800 = n (10 + nd - d)$
$800 = n (10 + 40)$ [using (1)]
$\Longrightarrow n = 16$

Putting n in (1), we find d
$nd - d = 40$
$16d - d = 40$
$15d = 40$
$d = 8/3$
Therefore, the common difference of the given A.P. is 8/3.

24. In an A.P. the first term is 8, n^{th} term is 33 and the sum of first n term is 123. Find n and the d, the common difference.

Solution:

Given,
The first term of the A.P (a) = 8
The nth term of the A.P (l) = 33
And, the sum of all the terms $S_n = 123$
Let the common difference of the A.P. be d.
So, find the number of terms by

$123 = (n/2)(8 + 33)$
$123 = (n/2)(41)$
$n = (123 \times 2)/ 41$
$n = 246/41$
$n = 6$

Next, to find the common difference of the A.P. we know that

$l = a + (n - 1)d$
$33 = 8 + (6 - 1)d$
$33 = 8 + 5d$
$5d = 25$
$d = 5$

Thus, the number of terms is n = 6 and the common difference of the A.P. is d = 5.

25. In an A.P. the first term is 22, n^{th} term is -11 and the sum of first n term is 66. Find n and the d, the common difference.
Solution:

Given,
The first term of the A.P (a) = 22
The nth term of the A.P (l) = -11
And, sum of all the terms $S_n = 66$
Let the common difference of the A.P. be d.
So, finding the number of terms by
$66 = (n/2)[22 + (-11)]$
$66 = (n/2)[22 - 11]$
$(66)(2) = n(11)$
$6 \times 2 = n$
$n = 12$
Now, for finding d
We know that, $l = a + (n - 1)d$
$- 11 = 22 + (12 - 1)d$
$-11 = 22 + 11d$
$11d = -33$
$d = -3$
Hence, the number of terms is n = 12 and the common difference d = -3

26. The first and the last terms of an A.P. are 7 and 49 respectively. If sum of all its terms is 420, find the common difference.

Solution:

Given,
First term (a) = 7, last term (a_n) = 49 and sum of n terms (S_n) = 420
Now, we know that
$a_n = a + (n - 1)d$
$\Rightarrow 49 = 7 + (n - 1)d$
$\Rightarrow 43 = nd - d$
$\Rightarrow nd - d = 42$ ..... (1)
Next,
$S_n = n/2(2(7) + (n - 1)d)$
$\Rightarrow 840 = n[14 + nd - d]$
$\Rightarrow 840 = n[14 + 42]$ [using (1)]
$\Rightarrow 840 = 54n$
$\Rightarrow n = 15$ .... (2)
So, by substituting (2) in (1), we have
$nd - d = 42$
$\Rightarrow 15d - d = 42$
$\Rightarrow 14d = 42$
$\Rightarrow d = 3$
Therefore, the common difference of the given A.P. is 3.

27. The first and the last terms of an A.P are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.
Solution:

Given,
First term (a) = 5 and the last term (l) = 45
Also, $S_n = 400$
We know that,
$a_n = a + (n - 1)d$
$\Rightarrow 45 = 5 + (n - 1)d$
$\Rightarrow 40 = nd - d$
$\Rightarrow nd - d = 40$ ..... (1)
Next,
$S_n = n/2(2(5) + (n - 1)d)$
$\Rightarrow 400 = n[10 + nd - d]$
$\Rightarrow 800 = n[10 + 40]$ [using (1)]
$\Rightarrow 800 = 50n$
$\Rightarrow n = 16$ .... (2)
So, by substituting (2) in (1), we have
$nd - d = 40$
$\Rightarrow 16d - d = 40$
$\Rightarrow 15d = 40$
$\Rightarrow d = 8/3$

Therefore, the common difference of the given A.P. is 8/3.

28. The sum of first q terms of an A.P. is 162. The ratio of its 6th term to its 13th term is 1: 2. Find the first and 15th term of the A.P.
Solution:

Let a be the first term and d be the common difference.
And we know that, sum of first n terms is:
$S_n = n/2(2a + (n - 1)d)$
Also, nth term is given by $a_n = a + (n - 1)d$

From the question, we have
$S_q = 162$ and $a_6 : a_{13} = 1 : 2$
So,
$2a_6 = a_{13}$
$\Rightarrow 2\,[a + (6 - 1d)] = a + (13 - 1)d$
$\Rightarrow 2a + 10d = a + 12d$
$\Rightarrow a = 2d$ .... (1)
And, $S_9 = 162$
$\Rightarrow S_9 = 9/2(2a + (9 - 1)d)$
$\Rightarrow 162 = 9/2(2a + 8d)$
$\Rightarrow 162 \times 2 = 9[4d + 8d]$ [from (1)]
$\Rightarrow 324 = 9 \times 12d$
$\Rightarrow d = 3$
$\Rightarrow a = 2(3)$ [from (1)]
$\Rightarrow a = 6$
Hence, the first term of the A.P. is 6
For the 15th term, $a_{15} = a + 14d = 6 + 14 \times 3 = 6 + 42$
Therefore, $a_{15} = 48$

29. If the 10th term of an A.P. is 21 and the sum of its first 10 terms is 120, find its nth term.
Solution:

Let's consider a to be the first term and d be the common difference.
And we know that, sum of first n terms is:
$S_n = n/2(2a + (n - 1)d)$ and nth term is given by: $a_n = a + (n - 1)d$

Now, from the question we have
$S_{10} = 120$
$\Rightarrow 120 = 10/2(2a + (10 - 1)d)$
$\Rightarrow 120 = 5(2a + 9d)$
$\Rightarrow 24 = 2a + 9d$ .... (1)
Also given that, $a_{10} = 21$
$\Rightarrow 21 = a + (10 - 1)d$
$\Rightarrow 21 = a + 9d$ .... (2)
Subtracting (2) from (1), we get

$24 - 21 = 2a + 9d - a - 9d$
$\Rightarrow a = 3$
Now, on putting a = 3 in equation (2), we get
$3 + 9d = 21$
$9d = 18$
$d = 2$
Thus, we have the first term(a) = 3 and the common difference(d) = 2
Therefore, the n^{th} term is given by
$a_n = a + (n - 1)d = 3 + (n - 1)2$
$= 3 + 2n - 2$
$= 2n + 1$
Hence, the n^{th} term of the A.P is $(a_n) = 2n + 1$.

30. The sum of first 7 terms of an A.P. is 63 and the sum of its next 7 terms is 161. Find the 28^{th} term of this A.P.
Solution:

Let's take a to be the first term and d to be the common difference.
And we know that, sum of first n terms
$S_n = n/2(2a + (n - 1)d)$
Given that sum of the first 7 terms of an A.P. is 63.
$S_7 = 63$
And sum of next 7 terms is 161.
So, the sum of first 14 terms = Sum of first 7 terms + sum of next 7 terms
$S_{14} = 63 + 161 = 224$
Now, having
$S_7 = 7/2(2a + (7 - 1)d)$
$\Rightarrow 63(2) = 7(2a + 6d)$
$\Rightarrow 9 \times 2 = 2a + 6d$
$\Rightarrow 2a + 6d = 18 \ldots. (1)$
And,
$S_{14} = 14/2(2a + (14 - 1)d)$
$\Rightarrow 224 = 7(2a + 13d)$
$\Rightarrow 32 = 2a + 13d \ldots. (2)$
Now, subtracting (1) from (2), we get
$\Rightarrow 13d - 6d = 32 - 18$
$\Rightarrow 7d = 14$
$\Rightarrow d = 2$
Using d in (1), we have
$2a + 6(2) = 18$
$2a = 18 - 12$
$a = 3$
Thus, from n^{th} term
$\Rightarrow a_{28} = a + (28 - 1)d$
$= 3 + 27 (2)$
$= 3 + 54 = 57$

Therefore, the 28^{th} term is 57.

31. The sum of first seven terms of an A.P. is 182. If its 4^{th} and 17^{th} terms are in ratio 1: 5, find the A.P.
Solution:

Given that,
$S_{17} = 182$
And, we know that the sum of first n term is:
$S_n = n/2(2a + (n - 1)d)$
So,
$S_7 = 7/2(2a + (7 - 1)d)$
$182 \times 2 = 7(2a + 6d)$
$364 = 14a + 42d$
$26 = a + 3d$
$a = 26 - 3d$ ... (1)
Also, it's given that 4^{th} term and 17^{th} term are in a ratio of 1: 5. So, we have
$\Longrightarrow 5(a_4) = 1(a_{17})$
$\Longrightarrow 5 (a + 3d) = 1 (a + 16d)$
$\Longrightarrow 5a + 15d = a + 16d$
$\Longrightarrow 4a = d$ .... (2)
Now, substituting (2) in (1), we get
$\Longrightarrow 4 (26 - 3d) = d$
$\Longrightarrow 104 - 12d = d$
$\Longrightarrow 104 = 13d$
$\Longrightarrow d = 8$
Putting d in (2), we get
$\Longrightarrow 4a = d$
$\Longrightarrow 4a = 8$
$\Longrightarrow a = 2$
Therefore, the first term is 2 and the common difference is 8. So, the A.P. is 2, 10, 18, 26, ...

32. The n^{th} term of an A.P is given by (-4n + 15). Find the sum of first 20 terms of this A.P.
Solution:

Given,
The n^{th} term of the A.P = (-4n + 15)
So, by putting n = 1 and n = 20 we can find the first ans 20^{th} term of the A.P
$a = (-4(1) + 15) = 11$
And,
$a_{20} = (-4(20) + 15) = -65$
Now, for find the sum of 20 terms of this A.P we have the first and last term.
So, using the formula
$S_n = n/2(a + l)$
$S_{20} = 20/2(11 + (-65))$
$= 10(-54)$

= -540

Therefore, the sum of first 20 terms of this A.P. is -540.

33. In an A.P. the sum of first ten terms is -150 and the sum of its next 10 term is -550. Find the A.P.

Solution:

Let's take a to be the first term and d to be the common difference.

And we know that, sum of first n terms

$S_n = n/2(2a + (n - 1)d)$

Given that sum of the first 10 terms of an A.P. is -150.

$S_{10} = -150$

And the sum of next 10 terms is -550.

So, the sum of first 20 terms = Sum of first 10 terms + sum of next 10 terms

$S_{20} = -150 + -550 = -700$

Now, having

$S_{10} = 10/2(2a + (10 - 1)d)$

$\Longrightarrow -150 = 5(2a + 9d)$

$\Longrightarrow -30 = 2a + 9d$

$\Longrightarrow 2a + 9d = -30 \ldots. (1)$

And,

$S_{20} = 20/2(2a + (20 - 1)d)$

$\Longrightarrow -700 = 10(2a + 19d)$

$\Longrightarrow -70 = 2a + 19d \ldots. (2)$

Now, subtracting (1) from (2), we get

$\Longrightarrow 19d - 9d = -70 - (-30)$

$\Longrightarrow 10d = -40$

$\Longrightarrow d = -4$

Using d in (1), we have

$2a + 9(-4) = -30$

$2a = -30 + 36$

$a = 6/2 = 3$

Hence, we have a = 3 and d = -4

So, the A.P is 3, -1, -5, -9, -13,…..

34. Sum of the first 14 terms of an A.P. is 1505 and its first term is 10. Find its 25^{th} term.

Solution

Given,

First term of the A.P is 1505 and

$S_{14} = 1505$

We know that, the sum of first n terms is

$S_n = n/2(2a + (n - 1)d)$

So,

$S_{14} = 14/2(2(10) + (14 - 1)d) = 1505$

$7(20 + 13d) = 1505$

$20 + 13d = 215$
$13d = 215 – 20$
$d = 195/13$
$d = 15$

Thus, the 25th term is given by
$a_{25} = 10 + (25 - 1)15$
$= 10 + (24)15$
$= 10 + 360$
$= 370$
Therefore, the 25th term of the A.P is 370

35. In an A.P. , the first term is 2, the last term is 29 and the sum of the terms is 155. Find the common difference of the A.P.
Solution:

Given,
The first term of the A.P. (a) = 2
The last term of the A.P. (l) = 29
And, sum of all the terms (S_n) = 155
Let the common difference of the A.P. be d.
So, find the number of terms by sum of terms formula
$S_n = n/2\ (a + l)$
$155 = n/2(2 + 29)$
$155(2) = n(31)$
$31n = 310$
$n = 10$
Using n for the last term, we have
$l = a + (n - 1)d$
$29 = 2 + (10 - 1)d$
$29 = 2 + (9)d$
$29 - 2 = 9d$
$9d = 27$
$d = 3$
Hence, the common difference of the A.P. is d = 3

36. The first and the last term of an A.P are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?
Solution:

Given,
In an A.P first term (a) = 17 and the last term (l) = 350
And, the common difference (d) = 9
We know that,
$a_n = a + (n - 1)d$
so,

$a_n = l = 17 + (n - 1)9 = 350$
$17 + 9n – 9 = 350$
$9n = 350 – 8$
$n = 342/9$
$n = 38$

So, the sum of all the term of the A.P is given by
S_n = n/2 (a + l)
= 38/2(17 + 350)
= 19(367)
= 6973
Therefore, the sum of terms of the A.P is 6973.

37. Find the number of terms of the A.P. –12, –9, –6, . . . , 21. If 1 is added to each term of this A.P., then find the sum of all terms of the A.P. thus obtained.
Solution:

Given,
First term, a = -12
Common difference, $d = a_2 - a_1 = – 9 – (- 12)$
$d = - 9 + 12 = 3$
And, we know that n^{th} term $= a_n = a + (n - 1)d$
$\Longrightarrow 21 = -12 + (n - 1)3$
$\Longrightarrow 21 = -12 + 3n - 3$
$\Longrightarrow 21 = 3n - 15$
$\Longrightarrow 36 = 3n$
$\Longrightarrow n = 12$
Thus, the number of terms is 12.
Now, if 1 is added to each of the 12 terms, the sum will increase by 12.
Hence, the sum of all the terms of the A.P. so obtained is
$\Longrightarrow S_{12} + 12 = 12/2[a + l] + 12$
$= 6[-12 + 21] + 12$
$= 6 \times 9 + 12$
$= 66$
Therefore, the sum after adding 1 to each of the terms in the A.P is 66.