EXERCISE 16

1. State the correspondence between the vertices, sides and angles of the following pairs of congruent triangles.
   (i). ΔABC ≅ ΔEFD
   Solution:-
   Two triangles are congruent if pairs of corresponding sides and corresponding angles are equal. If we write ΔABC ≅ ΔEFD, it would mean that,
   Correspondence between vertices:
   A ↔ E, B ↔ F, C ↔ D
   Correspondence between sides:
   AB = EF, BC = FD, CA = DE
   Correspondence between angles:
   ∠A = ∠E, ∠B = ∠F, ∠C = ∠D

   (ii). ΔCAB ≅ ΔQRP
   Solution:-
   Two triangles are congruent if pairs of corresponding sides and corresponding angles are equal. If we write ΔCAB ≅ ΔQRP, it would mean that,
   Correspondence between vertices:
   C ↔ Q, A ↔ R, B ↔ P
   Correspondence between sides:
   CA = QR, AB = RP, BC = PQ
   Correspondence between angles:
   ∠C = ∠Q, ∠A = ∠R, ∠B = ∠P

   (iii). ΔXZY ≅ ΔQPR
   Solution:-
   Two triangles are congruent if pairs of corresponding sides and corresponding angles are equal. If we write ΔXZY ≅ ΔQPR, it would mean that,
   Correspondence between vertices:
   X ↔ Q, Z ↔ P, Y ↔ R
   Correspondence between sides:
   XZ = QP, ZY = PR, XY = QR
   Correspondence between angles:
   ∠X = ∠Q, ∠Z = ∠P, ∠Y = ∠R

   (iv). ΔMPN ≅ ΔSQR
   Solution:-
   Two triangles are congruent if pairs of corresponding sides and corresponding angles are equal. If we write ΔMPN ≅ ΔSQR, it would mean that,
   Correspondence between vertices:
   M ↔ S, P ↔ Q, N ↔ R
Correspondence between sides:
  MP = SQ, PN = QR, MN = SR
Correspondence between angles:
  \( \angle M = \angle S, \angle P = \angle Q, \angle N = \angle R \)

2. Given below are pairs of congruent triangles. State the property of congruence and name the congruent triangles in each case.

   (i) [Diagram of congruent triangles]

   Solution:
   SAS congruence property:- Two triangles are congruent if the two sides and the included angle of one are respectively equal to the two sides and the included angle of the other.
ΔACB \cong \Delta DEF

(ii). Solution:-
RHS congruence property: Two right triangles are congruent if the hypotenuse and one side of the first triangle are respectively equal to the hypotenuse and one side of the second.
ΔRPQ \cong \Delta LNM

(iii). Solution:-
SSS congruence property: Two triangles are congruent if the three sides of one triangle are respectively equal to the three sides of the other triangle.
ΔYXZ \cong \Delta TRS

(iv). Solution:-
ASA congruence property: Two triangles are congruent if the two angles and the included side of one are respectively equal to the two angles and the included side of the other.
ΔDEF \cong \Delta PNM

(v). Solution:-
ASA congruence property: Two triangles are congruent if the two angles and the included side of one are respectively equal to the two angles and the included side of the other.
ΔACB \cong \Delta ACD

3. In Fig. (i), PL \perp OA and PM \perp OB such that PL = PM. Is ΔPLO \cong ΔPMO? Give reasons in support of your answer.

Solution:-
From the question:-
Is given that $PL \perp OA$, $PM \perp OB$ and $PL = PM$  
To prove: 
\[ \Delta PLO \cong \Delta PMO \]
Proof: 
From the fig, 
In $\triangle PLO$ and $\triangle PMO$,  
\[ \angle PLO = \angle PMO = 90^\circ \]
\[ PO = PO \] (common side)  
\[ PL = PM \] (given)  
\[ \therefore \Delta PLO \cong \Delta PMO \]
Yes. $\Delta PLO \cong \Delta PMO$ by the RHS congruence property

4. In fig. (ii), $AD = BC$ and $AD \parallel BC$. Is $AB = DC$? Give reasons in support of your answer.

Solution:
From the question,  
Is given that $AD = BC$ and $AD \parallel BC$  
To prove:  
$AB = DC$  
Proof:  
In $\triangle ABC$ and $\triangle CDA$,  
\[ BC = DA \] (given)  
\[ AD \parallel BC \] (given)  
\[ \angle BCA = \angle DAC \] (alternate angles)  
\[ AC = AC \] (common)  
\[ \therefore \Delta ABC \cong \Delta CDA \]
$AB = CD$  
Yes. $AB = CD$ by the SAS congruence property.

5. In the adjoining figure, $AB = AC$ and $BD = DC$. Prove that $\Delta ADB \cong \Delta ADC$ and hence show that  
(i) $\angle ADB = \angle ADC = 90^\circ$, (ii) $\angle BAD = \angle CAD$. 

Solution:--
From the question,  
Is given that $AD = BC$ and $AD \parallel BC$  
To prove:  
$AB = DC$  
Proof:  
In $\triangle ABC$ and $\triangle CDA$,  
\[ BC = DA \] (given)  
\[ AD \parallel BC \] (given)  
\[ \angle BCA = \angle DAC \] (alternate angles)  
\[ AC = AC \] (common)  
\[ \therefore \Delta ABC \cong \Delta CDA \]
$AB = CD$  
Yes. $AB = CD$ by the SAS congruence property.
Solution:

Given,

AB = AC and BD = DC

To prove,

ΔADB ≅ ΔADC

Proof,

In the right triangles ADB and ADC, we have:

Hypotenuse AB = Hypotenuse AC  \(\text{given}\)
BD = DC  \(\text{given}\)
AD = AD  \(\text{common}\)

∴ ΔADB ≅ ΔADC

By SSS congruence property:

∠ADB = ∠ADC  \(\text{corresponding parts of the congruent triangles}\) ... (1)

∠ADB and ∠ADC are on the straight line.
∴ ∠ADB + ∠ADC = 180°
∠ADB + ∠ADB = 180°
2∠ADB = 180°
∠ADB = 180°/2
∠ADB = 90°

From (1):

∠ADB = ∠ADC = 90°

(ii) ∠BAD = ∠CAD  \(\therefore \text{corresponding parts of the congruent triangles}\)

6. In the adjoining figure, ABC is a triangle in which AD is the bisector of ∠A. If AD ⊥ BC, show that ΔABC is isosceles.

Solution:-
Given:

- AD is the bisector of \(\angle A\)
- AD \(\perp BC\)

So we have,

\[\angle DAB = \angle DAC\]  ... (1)

To prove,

\(\Delta ABC\) is isosceles.

Proof,

In \(\Delta DAB\) and \(\Delta DAC\),

\[\angle BDA = \angle CDA = 90°\]
\[DA = DA\] (common)
\[\angle DAB = \angle DAC\] (from 1)

By ASA congruence property,

\(\Delta DAB \cong \Delta DAC\)

\(AB = AC\)

Hence, \(\Delta ABC\) is isosceles.

7. In the adjoining figure, \(AB = AD\) and \(CB = CD\). Prove that \(\Delta ABC \cong \Delta ADC\).

Solution:

Given,

\(AB = AD\) and \(CB = CD\)

To prove,

\(\Delta ABC \cong \Delta ADC\).
Proof,
In \( \triangle ABC \) and \( \triangle ADC \)
\[
\begin{align*}
AB &= AD \quad \text{(given)} \\
CB &= CD \quad \text{(given)} \\
AC &= AC \quad \text{(common)}
\end{align*}
\]
\( \therefore \triangle ABC \cong \triangle ADC. \)
(By SSS congruence property)

8. In the given figure, \( PA \perp AB, QB \perp AB \) and \( PA = QB \). Prove that \( \triangle OAP = \triangle OBQ \). Is \( OA = OB? \)
Solution:

Given,
\( PA \perp AB, QB \perp AB \) and \( PA = QB \)
To prove,
\( \triangle OAP = \triangle OBQ \)
Is \( OA = OB? \)
Proof,
In \( \triangle OAP \) and \( \triangle OBQ \)
\[
\begin{align*}
PA &= QB \quad \text{(given)} \\
\angle POA &= \angle QOB \quad \text{(vertically opposite angles)} \\
\angle OAP &= \angle OBQ = 90^\circ
\end{align*}
\]
From AAS congruence property,
\( \triangle OAP \cong \triangle OBQ \)
Then,
\( OA = OB \quad \text{(corresponding parts of the congruent triangles)} \)

9. In the given figure, triangles \( ABC \) and \( DCB \) are right-angled at \( A \) and \( D \) respectively and \( AC = DB \). Prove that \( \triangle ABC = \triangle DCB \).
Solution:-
Given,
Triangles ABC and DCB are right-angled at A and D respectively.
AC = DB
To prove,
\( \triangle ABC = \triangle DCB \)
Proof,
In \( \triangle ABC \) and \( \triangle DCB \):
AC = DB \hspace{1cm} \text{(given)}
BC = BC \hspace{1cm} \text{(common)}
\( \angle CAB = \angle BDC = 90^\circ \)
From the RHS congruence property,
\( \triangle ABC \cong \triangle DCB \).