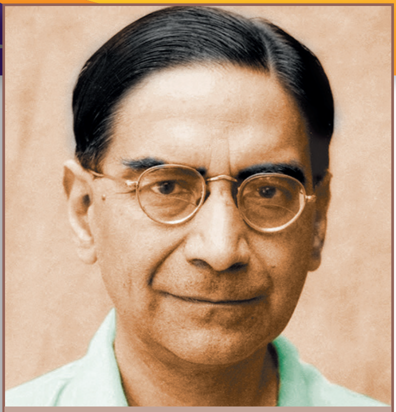
8 STATISTICS AND PROBABILITY

"Life is a School of Probability" - Walter Bagehot


Prasanta Chandra Mahalanobis

Prasanta Chandra Mahalanobis, born at Kolkata, was an Indian statistician who devised a measure of comparison between two data sets. He introduced innovative techniques for conducting large-scale sample surveys and calculated acreages and crop yields by using the method of random sampling. For his pioneering work, he was awarded the Padma Vibhushan, one of India's highest honours, by the Indian government in 1968 and he is hailed as "Father of Indian Statistics". The Government of India has designated 29th June every year, coinciding with his birth anniversary, as "National Statistics Day".

Learning Outcomes

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- To recall the measures of central tendency.
- To recall mean for ungrouped and grouped data.
- To understand the concept of dispersion.
- To understand and compute range, standard deviation, variance and coefficient of variation.
- To understand random experiments, sample space and use of a tree diagram.
- To define and describe events-mutually exclusive, complementary, certain and impossible events.
- To understand addition theorem on probability and apply it in solving some simple problems.

8.1 Introduction

'STATISTICS' is derived from the Latin word 'status' which means a political state. Today, statistics has become an integral part of everyone's life, unavoidable whether making a plan for our future, doing a business, a marketing research or preparing economic reports. It is also extensively used in opinion polls, doing advanced research. The study of statistics is concerned with scientific methods for collecting, organising, summarising, presenting, analysing data and making meaningful decisions. In earlier classes we have studied about collection of data, presenting the data in tabular form, graphical form and calculating the Measures of Central Tendency. Now, in this class, let us study about the Measures of Dispersion.

Recall

Measures of Central Tendency

It is often convenient to have one number that represent the whole data. Such a number is called a Measures of Central Tendency.

The Measures of Central Tendency usually will be near to the middle value of the data. For a given data there exist several types of central tendencies.

The most common among them are

- Arithmetic Mean • Median • Mode

Thinking Corner

(i) Does the mean, median and mode are same for a given data?

(ii) What is the difference between the arithmetic mean and average?

Note

Data : The numerical representation of facts is called data.

Observation : Each entry in the data is called an observation.

Variable : The quantities which are being considered in a survey are called variables. Variables are generally denoted by x_i, $i=1,2,3,\ldots,n$.

Frequencies : The number of times, a variable occurs in a given data is called the frequency of that variable. Frequencies are generally denoted as f_i, $i=1,2,3,\ldots,n$.

In this class we have to recall the Arithmetic Mean.

Arithmetic Mean

The Arithmetic Mean or Mean of the given values is sum of all the observations divided by the total number of observations. It is denoted by $\overline{x}$ (pronounced as x bar)

$$\overline{x} = \frac{\text{Sum of all the observations}}{\text{Number of observations}}$$

Thinking Corner

The mean of n observation is $\overline{x}$, if first term is increased by 1 second term is increased by 2 and so on. What will be the new mean?

Methods of finding Mean

- Ungrouped data
  - Direct Method: $\overline{X} = \frac{\sum_{i=1}^{n} x_i}{n}$
- Grouped data
  - Direct Method: $\overline{X} = \frac{\sum_{i=1}^{n} x_i f_i}{\sum_{i=1}^{n} f_i}$
  - Assumed Mean Method: $\overline{X} = A + \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$ where $d_i = x_i - A$
  - Step Deviation Method: $\overline{X} = A + C \times \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$ where $d_i = \frac{x_i - A}{c}$

We apply the respective formulae depending upon the provided information in the problem.

Progress Check

1. The sum of all the observations divided by number of observations is ________.
2. If the sum of 10 data values is 265 then their mean is ________.
3. If the sum and mean of a data are 407 and 11 respectively, then the number of observations in the data are ________.

8.2 Measures of Dispersion

The following data provide the runs scored by two batsmen in the last 10 matches.

Batsman A: 25, 20, 45, 93, 8, 14, 32, 87, 72, 4

Batsman B: 33, 50, 47, 38, 45, 40, 36, 48, 37, 26

$$\text{Mean of Batsman A} = \frac{25+20+45+93+8+14+32+87+72+4}{10} = 40$$

$$\text{Mean of Batsman B} = \frac{33+50+47+38+45+40+36+48+37+26}{10} = 40$$

The mean of both the data are same (40), but they differ significantly.

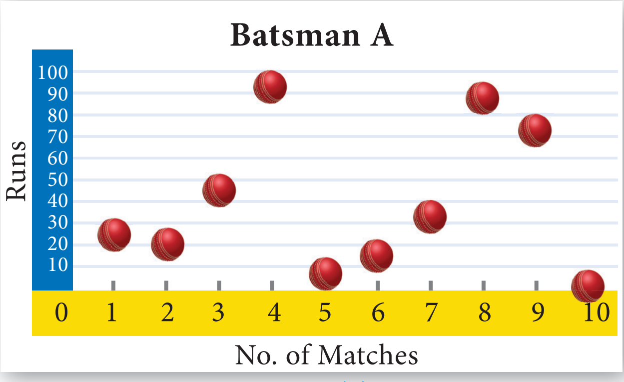


Fig. 8.1(a)

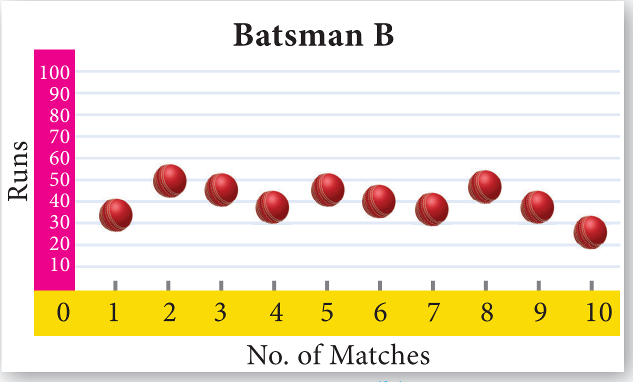


Fig. 8.1(b)

From the above diagram, runs of batsman B are grouped around the mean. But the runs of batsman A are scattered from 0 to 100.

Thus, some additional statistical information may be required to determine how the values are spread in data. For this, we shall discuss Measures of Dispersion.

Dispersion is a measure which gives an idea about the scatteredness of the values.

Measures of Variation (or) Dispersion of a data provide an idea of how observations spread out (or) scattered throughout the data.

Different Measures of Dispersion are

1. Range
2. Mean deviation
3. Quartile deviation
4. Standard deviation
5. Variance
6. Coefficient of Variation

8.2.1 Range

The difference between the largest value and the smallest value is called Range.

$$\text{Range } R = L - S$$

$$\text{Coefficient of range} = \frac{L-S}{L+S}$$

where L - Largest value; S - Smallest value

Progress Check

The range of first 10 prime numbers is ________

Example 8.1 Find the range and coefficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.

Solution Largest value $L = 67$; Smallest value $S = 18$

$$\text{Range R} = L - S = 67 - 18 = 49$$

$$\text{Coefficient of range} = \frac{L-S}{L+S}$$

$$\text{Coefficient of range} = \frac{67-18}{67+18} = \frac{49}{85} = 0.576$$

Example 8.2 Find the range of the following distribution.

Age (in years)	16-18	18-20	20-22	22-24	24-26	26-28
Number of students	0	4	6	8	2	2

Solution Here Largest value $L = 28$

Smallest value $S = 18$

Range $R = L - S$

$R = 28 - 18 = 10$ Years

Note

If the frequency of initial class is zero, then the next class will be considered for the calculation of range.

Example 8.3 The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value.

Solution Range $R = 13.67$

Largest value $L = 70.08$

Range $R = L - S$

$13.67 = 70.08 - S$

$S = 70.08 - 13.67 = 56.41$

Therefore, the smallest value is 56.41.

Note

The range of a set of data does not give the clear idea about the dispersion of the data from measures of Central Tendency. For this, we need a measure which depend upon the deviation from the Central Tendency.

8.2.2 Deviations from the mean

For a given data with n observations $x_1, x_2, \ldots x_n$, the deviations from the mean $\bar{x}$ are $x_1 - \bar{x},\ x_2 - \bar{x},\ \ldots,\ x_n - \bar{x}$.

8.2.3 Squares of deviations from the mean

The squares of deviations from the mean $\bar{x}$ of the observations $x_1, x_2, \ldots, x_n$ are $(x_1 - \bar{x})^2, (x_2 - \bar{x})^2, \ldots, (x_n - \bar{x})^2$ or $\sum_{i=1}^{n}(x_i - \bar{x})^2$

Note

We note that $\left(x_i - \overline{x}\right)^2 \geq 0$ for all observations x_i, $i = 1,2,3,\ldots,n$. If the deviations from the mean $\left(x_i - \overline{x}\right)$ are small, then the squares of the deviations will be very small.

8.2.4 Variance

The mean of the squares of the deviations from the mean is called Variance. It is denoted by σ^2 (read as sigma square).

$$\text{Variance} = \text{Mean of squares of deviations}$$

$$= \frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \ldots + (x_n - \overline{x})^2}{n}$$

$$\text{Variance } \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n}$$

Thinking Corner

Can variance be negative?

8.2.5 Standard Deviation

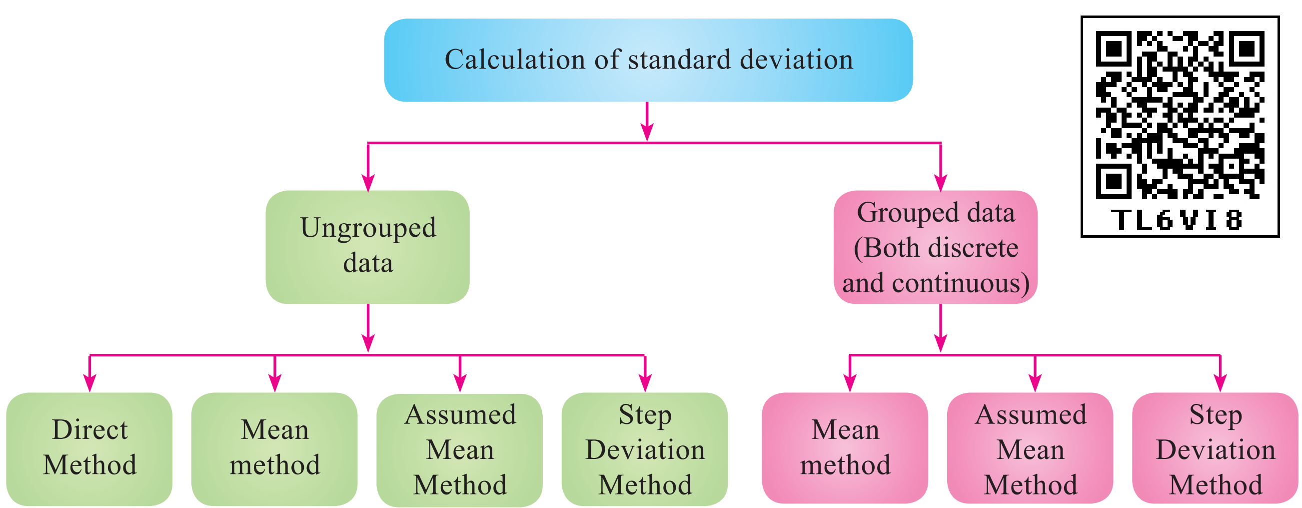
The positive square root of Variance is called Standard deviation. That is, Standard deviation is the positive square root of the mean of the squares of deviations of the given values from their mean.

Standard deviation gives a clear idea about how far the values are spreading or deviating from the mean.

DO YOU KNOW?

Karl Pearson was the first person to use the word standard deviation. German mathematician Gauss used the word Mean error.

$$\text{Standard deviation } \sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n}}$$



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Calculation of Standard Deviation for ungrouped data

(i) Direct Method

$$\text{Standard deviation } \sigma = \sqrt{\frac{\Sigma(x_i - \overline{x})^2}{n}}$$

$$= \sqrt{\frac{\Sigma(x_i^2 - 2x_i\overline{x} + \overline{x}^2)}{n}}$$

Note

The standard deviation and mean have same units in which the data are given.

$$= \sqrt{\frac{\Sigma x_i^2}{n} - 2\overline{x}\frac{\Sigma x_i}{n} + \frac{\overline{x}^2}{n} \times \left(1+1+\dots \ to \ \text{n times}\right)}$$

$$= \sqrt{\frac{\Sigma x_i^2}{n} - 2\overline{x} \times \overline{x} + \frac{\overline{x}^2}{n} \times n} = \sqrt{\frac{\Sigma x_i^2}{n} - 2\overline{x}^2 + \overline{x}^2} = \sqrt{\frac{\Sigma x_i^2}{n} - \overline{x}^2}$$

Standard deviation, $\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$

Note

- While computing standard deviation, arranging data in ascending order is not mandatory.
- If the data values are given directly then to find standard deviation we can use the formula $\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$
- If the data values are not given directly but the squares of the deviations from the mean of each observation is given then to find standard deviation we can use the formula $\sigma = \sqrt{\frac{\Sigma \left(x_i - \overline{x}\right)^2}{n}}$.

Example 8.4 The number of televisions sold in each day of a week are 13, 8, 4, 9, 7, 12, 10. Find its standard deviation.

Solution

x_i	x_i^2
13	169
8	64
4	16
9	81
7	49
12	144
10	100
$\Sigma x_i = 63$	$\Sigma x_i^2 = 623$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$= \sqrt{\frac{623}{7} - \left(\frac{63}{7}\right)^2}$$

$$= \sqrt{89 - 81} = \sqrt{8}$$

gives, $\sigma \simeq 2.83$

Thinking Corner

Can the standard deviation be more than the variance?

Progress Check

If the variance is 0.49 then the standard deviation is ______.

(ii) Mean method

Another convenient way of finding standard deviation is to use the following formula.

Standard deviation (by mean method) $\sigma = \sqrt{\frac{\Sigma(x_i - \overline{x})^2}{n}}$

If $d_i = x_i - \overline{x}$ are the deviations, then $\sigma = \sqrt{\frac{\Sigma d_i^2}{n}}$

Example 8.5 The amount of rainfall in a particular season for 6 days are given as 17.8 cm, 19.2 cm, 16.3 cm, 12.5 cm, 12.8 cm and 11.4 cm. Find its standard deviation.

Solution Arranging the numbers in ascending order we get, 11.4, 12.5, 12.8, 16.3, 17.8, 19.2. Number of observations $n = 6$

$$\text{Mean} = \frac{11.4 + 12.5 + 12.8 + 16.3 + 17.8 + 19.2}{6} = \frac{90}{6} = 15$$

x_i	$d_i = x_i - \overline{x} = x - 15$	d_i^2
11.4	–3.6	12.96
12.5	–2.5	6.25
12.8	–2.2	4.84
16.3	1.3	1.69
17.8	2.8	7.84
19.2	4.2	17.64
		$\Sigma d_i^2 = 51.22$

Standard deviation $\sigma = \sqrt{\frac{\Sigma d_i^2}{n}}$

$= \sqrt{\frac{51.22}{6}} = \sqrt{8.53}$

Hence, $\sigma \simeq 2.9$

(iii) Assumed Mean method

When the mean value is not an integer (since calculations are very tedious in decimal form) then it is better to use the assumed mean method to find the standard deviation.

Let $x_1, x_2, x_3, \ldots, x_n$ be the given data values and let $\overline{x}$ be their mean.

Let d_i be the deviation of x_i from the assumed mean A, which is the middle most value of the given data.

$$d_i = x_i - A \text{ gives, } x_i = d_i + A$$

$$\Sigma d_i = \Sigma(x_i - A)$$

$$= \Sigma x_i - (A + A + A + \ldots \textit{ to } \; n \text{ times})$$

$$\Sigma d_i = \Sigma x_i - A \times n$$

$$\frac{\Sigma d_i}{n} = \frac{\Sigma x_i}{n} - A$$

$$\overline{d} = \overline{x} - A \text{ (or) } \quad \overline{x} = \overline{d} + A$$

Now, Standard deviation

$$\sigma = \sqrt{\frac{\Sigma(x_i - \overline{x})^2}{n}} = \sqrt{\frac{\Sigma(d_i + A - \overline{d} - A)^2}{n}}$$

$$= \sqrt{\frac{\Sigma(d_i - \overline{d})^2}{n}} = \sqrt{\frac{\Sigma(d_i^2 - 2d_i \times \overline{d} + \overline{d}^2)}{n}}$$

$$= \sqrt{\frac{\Sigma d_i^2}{n} - 2\overline{d}\frac{\Sigma d_i}{n} + \frac{\overline{d}^2}{n}(1 + 1 + 1 + \ldots \text{ to } \; n \text{ times})}$$

$$= \sqrt{\frac{\Sigma d_i^2}{n} - 2\overline{d} \times \overline{d} + \frac{\overline{d}^2}{n} \times n} \qquad \text{(since } \overline{d} \text{ is a constant)}$$

$$= \sqrt{\frac{\Sigma d_i^2}{n} - \overline{d}^2}$$

Standard deviation $\sigma = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$

Thinking Corner

For any collection of n values, can you find the value of

(i) $\Sigma(x_i - \overline{x})$ (ii) $(\Sigma x_i) - \overline{x}$

Example 8.6 The marks scored by 10 students in a class test are 25, 29, 30, 33, 35, 37, 38, 40, 44, 48. Find the standard deviation.

Solution The mean of marks is 35.9 which is not an integer. Hence we take assumed mean, $A = 35$, $n = 10$.

x_i	$d_i = x_i - A$ $d_i = x_i - 35$	d_i^2
25	−10	100
29	−6	36
30	−5	25
33	−2	4
35	0	0
37	2	4
38	3	9
40	5	25
44	9	81
48	13	169
	$\Sigma d_i = 9$	$\Sigma d_i^2 = 453$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$$

$$= \sqrt{\frac{453}{10} - \left(\frac{9}{10}\right)^2}$$

$$= \sqrt{45.3 - 0.81}$$

$$= \sqrt{44.49}$$

$$\sigma \simeq 6.67$$

(ii) Step deviation method

Let $x_1, x_2, x_3, \ldots x_n$ be the given data. Let A be the assumed mean.

Let c be the common divisor of $x_i - A$.

Let $\quad d_i = \frac{x_i - A}{c}$

Then $\quad x_i = d_i c + A \qquad \ldots(1)$

$$\Sigma x_i = \Sigma\left(d_i c + A\right) = c\Sigma d_i + A \times n$$

$$\frac{\Sigma x_i}{n} = c\frac{\Sigma d_i}{n} + A$$

$$\overline{x} = c\overline{d} + A \qquad \ldots(2)$$

$$x_i - \overline{x} = cd_i + A - c\overline{d} - A = c(d_i - \overline{d}) \quad \text{(using (1) and (2))}$$

$$\sigma = \sqrt{\frac{\Sigma(x_i - \overline{x})^2}{n}} = \sqrt{\frac{\Sigma(c(d_i - \overline{d}))^2}{n}} = \sqrt{\frac{c^2\Sigma(d_i - \overline{d})^2}{n}}$$

$$\sigma = c \times \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$$

Note

We can use any of the above methods for finding the standard deviation

Activity 1

Find the standard deviation of the marks obtained by you in all five subjects in the quarterly examination and in the midterm test separately. What do you observe from your results.

Example 8.7 The amount that the children have spent for purchasing some eatables in one day trip of a school are 5, 10, 15, 20, 25, 30, 35, 40, Using step deviation method, find the standard deviation of the amount they have spent.

Solution We note that all the observations are divisible by 5. Hence we can use the step deviation method. Let the Assumed mean $A = 20$, $n = 8$.

x_i	$d_i = x_i - A$ $d_i = x_i - 20$	$d_i = \frac{x_i - A}{c}$ $c = 5$	d_i^2
5	−15	−3	9
10	−10	−2	4
15	−5	−1	1
20	0	0	0
25	5	1	1
30	10	2	4
35	15	3	9
40	20	4	16
		$\Sigma d_i = 4$	$\Sigma d_i^2 = 44$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2} \times c$$

$$= \sqrt{\frac{44}{8} - \left(\frac{4}{8}\right)^2} \times 5 = \sqrt{\frac{11}{2} - \frac{1}{4}} \times 5$$

$$= \sqrt{5.5 - 0.25} \times 5 = 2.29 \times 5$$

$$\sigma \simeq 11.45$$

Example 8.8 Find the standard deviation of the following data 7, 4, 8, 10, 11. Add 3 to all the values then find the standard deviation for the new values.

Solution Arranging the values in ascending order we get, 4, 7, 8, 10, 11 and $n = 5$

x_i	x_i^2
4	16
7	49
8	64
10	100
11	121
$\Sigma x_i = 40$	$\Sigma x_i^2 = 350$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$= \sqrt{\frac{350}{5} - \left(\frac{40}{5}\right)^2}$$

$$\sigma = \sqrt{6} \simeq 2.45$$

When we add 3 to all the values, we get the new values as 7,10,11,13,14.

x_i	x_i^2
7	9
10	100
11	121
13	169
14	196
$\Sigma x_i = 55$	$\Sigma x_i^2 = 635$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$= \sqrt{\frac{635}{5} - \left(\frac{55}{5}\right)^2}$$

$$\sigma = \sqrt{6} \simeq 2.45$$

From the above, we see that the standard deviation will not change when we add some fixed constant to all the values.

Example 8.9 Find the standard deviation of the data 2, 3, 5, 7, 8. Multiply each data by 4. Find the standard deviation of the new values.

Solution Given, $n = 5$

x_i	x_i^2
2	49
3	9
5	25
7	49
8	64
$\Sigma x_i = 25$	$\Sigma x_i^2 = 151$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{151}{5} - \left(\frac{25}{5}\right)^2} = \sqrt{30.2 - 25} = \sqrt{5.2} \simeq 2.28$$

When we multiply each data by 4, we get the new values as 8, 12, 20, 28, 32.

x_i	x_i^2
8	64
12	144
20	400
28	784
32	1024
$\Sigma x_i = 100$	$\Sigma x_i^2 = 2416$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$= \sqrt{\frac{2416}{5} - \left(\frac{100}{5}\right)^2} = \sqrt{483.2 - 400} = \sqrt{83.2}$$

$$\sigma = \sqrt{16 \times 5.2} = 4\sqrt{5.2} \simeq 9.12$$

From the above, we see that when we multiply each data by 4 the standard deviation also get multiplied by 4.

Example 8.10 Find the mean and variance of the first n natural numbers.

Solution

$$\text{Mean } \bar{x} = \frac{\text{Sum of all the observations}}{\text{Number of observations}}$$

$$= \frac{\Sigma x_i}{n} = \frac{1+2+3+\ldots+n}{n} = \frac{n(n+1)}{2 \times n}$$

$$\text{Mean } \bar{x} = \frac{n+1}{2}$$

$$\text{Variance } \sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 \quad \left[\begin{array}{l}\Sigma x_i^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 \\ (\Sigma x_i)^2 = (1+2+3+\ldots+n)^2\end{array}\right]$$

$$= \frac{n(n+1)(2n+1)}{6 \times n} - \left[\frac{n(n+1)}{2 \times n}\right]^2$$

$$= \frac{2n^2+3n+1}{6} - \frac{n^2+2n+1}{4}$$

$$\text{Variance } \sigma^2 = \frac{4n^2+6n+2-3n^2-6n-3}{12} = \frac{n^2-1}{12}.$$

Calculation of Standard deviation for grouped data

(i) Mean method

Standard deviation $\sigma = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{N}}$

Let, $d_i = x_i - \bar{x}$

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N}}, \text{ where } N = \sum_{i=1}^{n} f_i$$

(f_i are frequency values of the corresponding data points x_i)

Example 8.11 48 students were asked to write the total number of hours per week they spent on watching television. With this information find the standard deviation of hours spent for watching television.

x	6	7	8	9	10	11	12
f	3	6	9	13	8	5	4

Solution

x_i	f_i	$x_i f_i$	$d_i = x_i - \overline{x}$	d_i^2	$f_i d_i^2$
6	3	18	–3	9	27
7	6	42	–2	4	24
8	9	72	–1	1	9
9	13	117	0	0	0
10	8	80	1	1	8
11	5	55	2	4	20
12	4	48	3	9	36
	$N = 48$	$\Sigma x_i f_i = 432$	$\Sigma d_i = 0$		$\Sigma f_i d_i^2 = 124$

Mean

$$\overline{x} = \frac{\Sigma x_i f_i}{N} = \frac{432}{48} = 9$$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N}} = \sqrt{\frac{124}{48}} = \sqrt{2.58}$$

$$\sigma \simeq 1.6$$

(ii) Assumed Mean method

Let $x_1, x_2, x_3, \ldots x_n$ be the given data with frequencies $f_1, f_2, f_3, \ldots f_n$ respectively. Let $\overline{x}$ be their mean and A be the assumed mean.

$$d_i = x_i - A$$

Standard deviation, $\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

Example 8.12 The marks scored by the students in a slip test are given below.

x	4	6	8	10	12
f	7	3	5	9	5

Find the standard deviation of their marks.

Solution Let the assumed mean, $A = 8$

x_i	f_i	$d_i = x_i - A$	$f_i d_i$	$f_i d_i^2$
4	7	–4	–28	112
6	3	–2	–6	12
8	5	0	0	0
10	9	2	18	36
12	5	4	20	80
	$N = 29$		$\Sigma f_i d_i = 4$	$\Sigma f_i d_i^2 = 240$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$$

$$= \sqrt{\frac{240}{29} - \left(\frac{4}{29}\right)^2} = \sqrt{\frac{240 \times 29 - 16}{29 \times 29}}$$

$$\sigma = \sqrt{\frac{6944}{29 \times 29}} \;; \qquad \sigma \simeq 2.87$$

Calculation of Standard deviation for continuous frequency distribution

(i) Mean method

Standard deviation $\sigma = \sqrt{\frac{\Sigma f_i \left(x_i - \overline{x}\right)^2}{N}}$ where $x_i =$ Middle value of the ith class.

$f_i =$ Frequency of the ith class.

(ii) Shortcut method (or) Step deviation method

To make the calculation simple, we provide the following formula. Let A be the assumed mean, x_i be the middle value of the ith class and c is the width of the class interval.

Let
$$d_i = \frac{x_i - A}{c}$$
$$\sigma = c \times \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$$

Example 8.13 Marks of the students in a particular subject of a class are given below.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of students	8	12	17	14	9	7	4

Find its standard deviation.

Solution Let the assumed mean, $A = 35$, $c = 10$

Marks	Midvalue (x_i)	f_i	$d_i = x_i - A$	$d_i = \frac{x_i - A}{c}$	$f_i d_i$	$f_i d_i^2$
0-10	5	8	-30	−3	−24	72
10-20	15	12	-20	−2	−24	48
20-30	25	17	-10	−1	−17	17
30-40	35	14	0	0	0	0
40-50	45	9	10	1	9	9
50-60	55	7	20	2	14	28
60-70	65	4	30	3	12	36
		$N = 71$			$\Sigma f_i d_i = -30$	$\Sigma f_i d_i^2 = 210$

Standard deviation $\sigma = c \times \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

$$\sigma = 10 \times \sqrt{\frac{210}{71} - \left(-\frac{30}{71}\right)^2} = 10 \times \sqrt{\frac{210}{71} - \frac{900}{5041}}$$
$$= 10 \times \sqrt{2.779}\ ; \quad \sigma \simeq 16.67$$

Thinking Corner

1. The standard deviation of a data is 2.8, if 5 is added to all the data values then the new standard deviation is ____.
2. If S is the standard deviation of values p, q, r then standard deviation of $p-3$, $q-3$, $r-3$ is ____.

Example 8.14 The mean and standard deviation of 15 observations are found to be 10 and 5 respectively. On rechecking it was found that one of the observation with value 8 was incorrect. Calculate the correct mean and standard deviation if the correct observation value was 23?

Solution $n = 15$, $\bar{x} = 10$, $\sigma = 5$; $\bar{x} = \frac{\Sigma x}{n}$; $\Sigma x = 15 \times 10 = 150$

Wrong observation value $= 8$, Correct observation value $= 23$.

Correct total $= 150 - 8 + 23 = 165$

$$\text{Correct mean } \overline{x} = \frac{165}{15} = 11$$

$$\text{Standard deviation } \sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$\text{Incorrect value of } \sigma = 5 = \sqrt{\frac{\Sigma x^2}{15} - \left(10\right)^2}$$

$$25 = \frac{\Sigma x^2}{15} - 100 \text{ gives, } \frac{\Sigma x^2}{15} = 125$$

$$\text{Incorrect value of } \Sigma x^2 = 1875$$

$$\text{Correct value of } \Sigma x^2 = 1875 - 8^2 + 23^2 = 2340$$

$$\text{Correct standard deviation } \sigma = \sqrt{\frac{2340}{15} - \left(11\right)^2}$$

$$\sigma = \sqrt{156 - 121} = \sqrt{35} \qquad \sigma \simeq 5.9$$

Exercise 8.1

1. Find the range and coefficient of range of the following data.
   (i) 63, 89, 98, 125, 79, 108, 117, 68
   (ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8
2. If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.
3. Calculate the range of the following data.

Income	400-450	450-500	500-550	550-600	600-650
Number of workers	8	12	30	21	6

4. A teacher asked the students to complete 60 pages of a record note book. Eight students have completed only 32, 35, 37, 30, 33, 36, 35 and 37 pages. Find the standard deviation of the pages yet to be completed by them.
5. Find the variance and standard deviation of the wages of 9 workers given below: ₹310, ₹290, ₹320, ₹280, ₹300, ₹290, ₹320, ₹310, ₹280.
6. A wall clock strikes the bell once at 1 o' clock, 2 times at 2 o' clock, 3 times at 3 o' clock and so on. How many times will it strike in a particular day. Find the standard deviation of the number of strikes the bell make a day.
7. Find the standard deviation of first 21 natural numbers.
8. If the standard deviation of a data is 4.5 and each value of the data decreased by 5, then find the new standard deviation.
9. If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation.
10. The rainfall recorded in various places of five districts in a week are given below.

Rainfall (in mm)	45	50	55	60	65	70
Number of places	5	13	4	9	5	4

Find its standard deviation.

11. In a study about viral fever, the number of people affected in a town were noted as

Age in years	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of people affected	3	5	16	18	12	7	4

Find its standard deviation.

12. The measurements of the diameters (in cms) of the plates prepared in a factory are given below. Find its standard deviation.

Diameter(cm)	21-24	25-28	29-32	33-36	37-40	41-44
Number of plates	15	18	20	16	8	7

13. The time taken by 50 students to complete a 100 meter race are given below. Find its standard deviation.

Time taken(sec)	8.5-9.5	9.5-10.5	10.5-11.5	11.5-12.5	12.5-13.5
Number of students	6	8	17	10	9

14. For a group of 100 candidates the mean and standard deviation of their marks were found to be 60 and 15 respectively. Later on it was found that the scores 45 and 72 were wrongly entered as 40 and 27. Find the correct mean and standard deviation.

15. The mean and variance of seven observations are 8 and 16 respectively. If five of these are 2, 4, 10, 12 and 14, then find the remaining two observations.

8.3 Coefficient of Variation

Comparison of two data in terms of measures of central tendencies and dispersions in some cases will not be meaningful, because the variables in the data may not have same units of measurement.

For example consider the two data

	Weight	Price
Mean	8 kg	₹ 85
Standard deviation	1.5 kg	₹ 21.60

Here we cannot compare the standard deviations 1.5kg and ₹21.60. For comparing two or more data for corresponding changes the relative measure of standard deviation, called "Coefficient of variation" is used.

Coefficient of variation of a data is obtained by dividing the standard deviation by the arithmetic mean. It is usually expressed in terms of percentage. This concept is suggested by one of the most prominent Statistician Karl Pearson.

Thus, coefficient of variation of first data $(C.V_1) = \frac{\sigma_1}{\overline{x}_1} \times 100\%$

and coefficient of variation of second data $(C.V_2) = \frac{\sigma_2}{\overline{x}_2} \times 100\%$

The data with lesser coefficient of variation is more consistent or stable than the other data. Consider the two data

A	500	900	800	900	700	400
B	300	540	480	540	420	240

	Mean	Standard deviation
A	700	191.5
B	420	114.9

If we compare the mean and standard deviation of the two data, we think that the two datas are entirely different. But mean and standard deviation of B are 60% of that of A. Because of the smaller mean the smaller standard deviation led to the misinterpretation.

To compare the dispersion of two data, coefficient of variation $= \frac{\sigma}{\bar{x}} \times 100\%$

The coefficient of variation of $A = \frac{191.5}{700} \times 100\% = 27.4\%$

The coefficient of variation of $B = \frac{114.9}{420} \times 100\% = 27.4\%$

Thus the two data have equal coefficient of variation. Since the data have equal coefficient of variation values, we can conclude that one data depends on the other. But the data values of B are exactly 60% of the corresponding data values of A. So they are very much related. Thus, we get a confusing situation.

To get clear picture of the given data, we can find their coefficient of variation. This is why we need coefficient of variation.

Progress Check

1. Coefficient of variation is a relative measure of _________.
2. When the standard deviation is divided by the mean we get________.
3. The coefficient of variation depends upon ________and ____________.
4. If the mean and standard deviation of a data are 8 and 2 respectively then the coefficient of variation is ________.
5. When comparing two data, the data with ________ coefficient of variation is inconsistent.

Example 8.15 The mean of a data is 25.6 and its coefficient of variation is 18.75. Find the standard deviation.

Solution Mean $\bar{x} = 25.6$, Coefficient of variation, C.V. = 18.75

Coefficient of variation, C.V. $= \frac{\sigma}{\bar{x}} \times 100\%$

$$18.75 = \frac{\sigma}{25.6} \times 100 \; ; \qquad \sigma = 4.8$$

Example 8.16 The following table gives the values of mean and variance of heights and weights of the 10th standard students of a school.

	Height	Weight
Mean	155 cm	46.50 kg^2
Variance	72.25 cm^2	28.09 kg^2

Which is more varying than the other?

Solution For comparing two data, first we have to find their coefficient of variations

Mean $\bar{x}_1 = 155$cm, variance $\sigma_1^2 = 72.25$ cm^2

Therefore standard deviation $\sigma_1 = 8.5$

Coefficient of variation $C.V_1 = \frac{\sigma_1}{\bar{x}_1} \times 100\%$

$$C.V_1 = \frac{8.5}{155} \times 100\% = 5.48\% \quad \text{(for heights)}$$

Mean $\bar{x}_2 = 46.50$ kg, Variance $\sigma_2^2 = 28.09$ kg^2

$$\text{Standard deviation } \sigma_2 = 5.3kg$$

$$\text{Coefficient of variation } C.V_2 = \frac{\sigma_2}{\overline{x}_2} \times 100\%$$

$$C.V_2 = \frac{5.3}{46.50} \times 100\% = 11.40\% \text{ (for weights)}$$

$C.V_1 = 5.48\%$ and $C.V_2 = 11.40\%$

Since $C.V_2 > C.V_1$, the weight of the students is more varying than the height.

Example 8.17 The consumption of number of guava and orange on a particular week by a family are given below.

Number of Guavas	3	5	6	4	3	5	4
Number of Oranges	1	3	7	9	2	6	2

Which fruit is consistently consumed by the family?

Solution First we find the coefficient of variation for guavas and oranges separately.

Number of guavas, $n = 7$

x_i	x_i^2
3	9
5	25
6	36
4	16
3	9
5	25
4	16
$\Sigma x_i = 30$	$\Sigma x_i^2 = 136$

$$\text{Mean } \overline{x}_1 = \frac{30}{7} = 4.29$$

$$\text{Standard deviation } \sigma_1 = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$\sigma_1 = \sqrt{\frac{136}{7} - \left(\frac{30}{7}\right)^2} = \sqrt{19.43 - 18.40} \simeq 1.01$$

Coefficient of variation for guavas

$$\text{C.V}_1 = \frac{\sigma_1}{\overline{x}_1} \times 100\% = \frac{1.01}{4.29} \times 100\% = 23.54\%$$

Number of oranges $n = 7$

x_i	x_i^2
1	1
3	9
7	49
9	81
2	4
6	36
2	4
$\Sigma x_i = 30$	$\Sigma x_i^2 = 184$

$$\text{Mean } \overline{x}_2 = \frac{30}{7} = 4.29$$

$$\text{Standard deviation } \sigma_2 = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$\sigma_2 = \sqrt{\frac{184}{7} - \left(\frac{30}{7}\right)^2} = \sqrt{26.29 - 18.40} = 2.81$$

Coefficient of variation for oranges

$$C.V_2 = \frac{\sigma_2}{\overline{x}_2} \times 100\% = \frac{2.81}{4.29} \times 100\% = 65.50\%$$

$C.V_1 = 23.54\%$, $C.V_2 = 65.50\%$. Since, $C.V_1 < C.V_2$, we can conclude that the consumption of guava is more consistent than orange.

Exercise 8.2

1. The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation.

2. The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.
3. If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.
4. If $n = 5$, $\overline{x} = 6$, $\Sigma x^2 = 765$, then calculate the coefficient of variation.
5. Find the coefficient of variation of 24, 26, 33, 37, 29, 31.
6. The time taken (in minutes) to complete a homework by 8 students in a day are given by 38, 40, 47, 44, 46, 43, 49, 53. Find the coefficient of variation.
7. The total marks scored by two students Sathya and Vidhya in 5 subjects are 460 and 480 with standard deviation 4.6 and 2.4 respectively. Who is more consistent in performance?
8. The mean and standard deviation of marks obtained by 40 students of a class in three subjects Mathematics, Science and Social Science are given below.

Subject	Mean	SD
Mathematics	56	12
Science	65	14
Social Science	60	10

Which of the three subjects shows highest variation and which shows lowest variation in marks?

9. The temperature of two cities A and B in a winter season are given below.

Temperature of city A (in degree Celsius)	18	20	22	24	26
Temperature of city B (in degree Celsius)	11	14	15	17	18

Find which city is more consistent in temperature changes?

8.4 Pobability


Blaise Pascal

Few centuries ago, gambling and gaming were considered to be fashionable and became widely popular among many men. As the games became more complicated, players were interested in knowing the chances of winning or losing a game from a given situation. In 1654, Chevalier de Mere, a French nobleman with a taste of gambling, wrote a letter to one of the prominent mathematician of the time, Blaise Pascal, seeking his advice about how much dividend he would get for a gambling game played by paying money. Pascal worked this problem mathematically but thought of sharing this problem and see how his good friend and mathematician Pierre de Fermat could solve. Their subsequent correspondences on the issue represented the birth of Probability Theory as a new branch of mathematics.

Random Experiment

A random experiment is an experiment in which

(i) The set of all possible outcomes are known (ii)Exact outcome is not known.

Example : 1. Tossing a coin. 2. Rolling a die. 3. Selecting a card from a pack of 52 cards.

Sample space

The set of all possible outcomes in a random experiment is called a sample space. It is generally denoted by S.

Example : When we roll a die, the possible outcomes are the face numbers 1,2,3,4,5,6 of the die. Therefore the sample space is $S = \{1,2,3,4,5,6\}$


Fig. 8.2

Sample point

Each element of a sample space is called a sample point.

8.4.1 Tree diagram

Tree diagram allow us to see visually all the possible outcomes of an experiment. Each branch in a tree diagram represent a possible outcome.

QM1NC9

Illustration

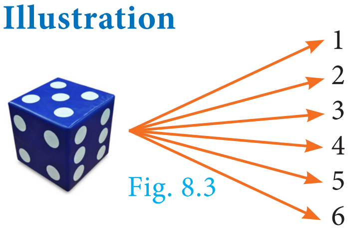


Fig. 8.3

(i) When we throw a die, then from the tree diagram (Fig.8.3), the sample space can be written as $S = \{1,2,3,4,5,6\}$

(ii) When we toss two coins, then from the tree diagram (Fig.8.4), the sample space can be written as $S=\{HH, HT, TH, TT\}$

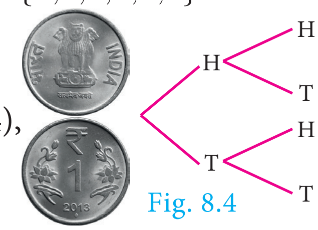


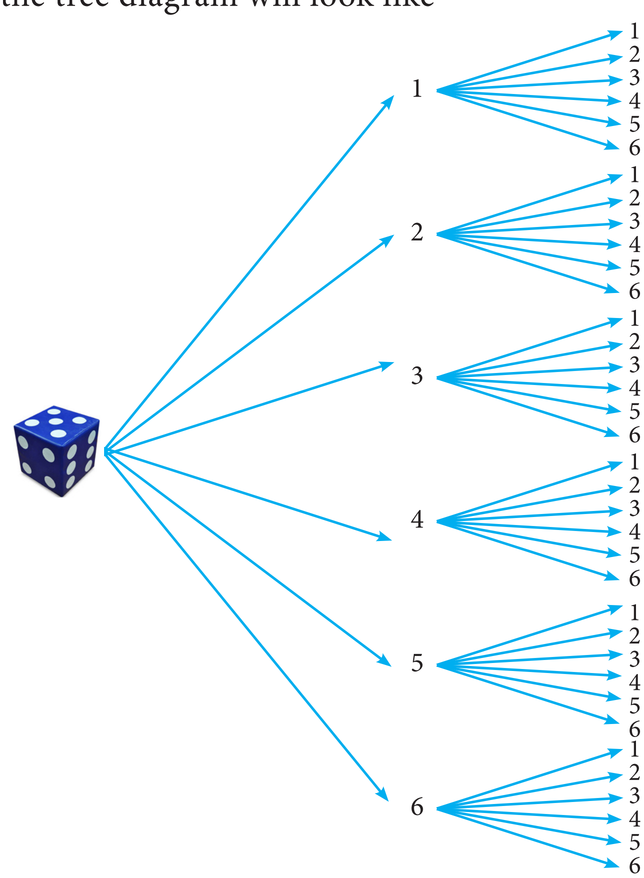
Fig. 8.4

Progress Check

1. An experiment in which a particular outcome cannot be predicted is called ________.
2. The set of all possible outcomes is called ________.

Example 8.18 Express the sample space for rolling two dice using tree diagram.

Solution When we roll two dice, since each die contain 6 faces marked with 1,2,3,4,5,6 the tree diagram will look like



Hence, the sample space can be written as

$$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$$
$$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$$
$$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$$
$$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$$
$$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$$
$$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

Event: In a random experiment, each possible outcome is called an event. Thus, an event will be a subset of the sample space.

Example : Getting two heads when we toss two coins is an event.

Trial : Performing an experiment once is called a trial.

Example : When we toss a coin thrice, then each toss of a coin is a trial.

Events	Explanation	Example
Equally likely events	Two or more events are said to be equally likely if each one of them has an equal chance of occurring.	Head and tail are equally likely events in tossing a coin.
Certain events	In an experiment, the event which surely occur is called certain event.	When we roll a die, the event of getting any natural number from one to six is a certain event.
Impossible events	In an experiment if an event has no scope to occur then it is called an impossible event.	When we toss two coins, the event of getting three heads is an impossible event.
Mutually exclusive events	Two or more events are said to be mutually exclusive if they don't have common sample points. i.e., events A, B are said to be mutually exclusive if $A \cap B = \phi$.	When we roll a die the events of getting odd numbers and even numbers are mutually exclusive events.
Exhaustive events	The collection of events whose union is the whole sample space are called exhaustive events.	When we toss a coin twice, events of getting two heads, exactly one head, no head are exhaustive events.
Complementary events	The complement of an event A is the event representing collection of sample points not in A. It is denoted A' or A^c or $\bar{A}$ The event A and its complement A' are mutually exclusive and exhaustive.	When we roll a die, the event 'rolling a 5 or 6' and the event of rolling a 1, 2, 3 or 4 are complementary events.

Note

Elementary event: If an event E consists of only one outcome then it is called an elementary event.

DO YOU KNOW?

In 1713, Bernoulli was the first to recognise the wide-range applicability of probability in fields outside gambling

8.4.2 Probability of an Event

In a random experiment, let S be the sample space and $E \subseteq S$. Then it E is an event. The probability of occurrence of E is defined as

$$P(E) = \frac{\text{Number of outcomes favourable to occurence of } E}{\text{Number of all possible outcomes}} = \frac{n(E)}{n(S)}$$

This way of defining the probability is applicable only to finite sample spaces. So in this chapter, we are dealing problems with finite sample space only.

Note

(i) $P(E) = \dfrac{n(E)}{n(S)}$

(ii) $P(S) = \dfrac{n(S)}{n(S)} = 1$. The probability of sure event is 1.

(iii) $P(\phi) = \dfrac{n(\phi)}{n(\text{s})} = \dfrac{0}{n(s)} = 0$. The probability of impossible event is 0.

(iv) Since E is a subset of S and ϕ is a subset of any set,

$$\phi \subseteq E \subseteq S$$
$$P(\phi) \le P(E) \le P(\text{S})$$
$$0 \le P(E) \le 1$$

Therefore, the probability value always lies from 0 to 1.

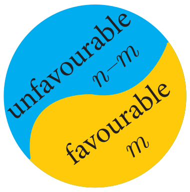
(v) The complement event of E is $\bar{E}$.

Let $P(E) = \dfrac{m}{n}$ (where m is the number of favourable outcomes of E and n is the total number of possible outcomes).

$$P(\bar{E}) = \frac{\text{Number of outcomes unfavourable to occurance of } E}{\text{Number of all possible outcomes}}$$

$$P(\bar{E}) = \frac{n-m}{n} = 1 - \frac{m}{n}$$

$$P(\bar{E}) = 1 - P(\text{E})$$



(vi) $P(E) + P(\bar{\text{E}}) = 1$

Progress Check

Which of the following values cannot be a probability of an event?

(a) -0.0001 (b) 0.5 (c) 1.001 (d) 1

(e) 20% (f) 0.253 (g) $\dfrac{1-\sqrt{5}}{2}$ (h) $\dfrac{\sqrt{3}+1}{4}$

Example 8.19 A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

Solution Total number of possible outcomes $n(S) = 5 + 4 = 9$

(i) Let A be the event of getting a blue ball.

Number of favourable outcomes for the event A. Therefore, $n(A) = 5$

Probability that the ball drawn is blue. Therefore, $P(A) = \frac{n(A)}{n(S)} = \frac{5}{9}$

(ii) $\bar{A}$ will be the event of not getting a blue ball. So $P(\bar{A}) = 1 - P(A) = 1 - \frac{5}{9} = \frac{4}{9}$

Example 8.20 Two dice are rolled. Find the probability that the sum of outcomes is (i) equal to 4 (ii) greater than 10 (iii) less than 13

Solution When we roll two dice, the sample space is given by

$$S = \{ (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$$
$$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$$
$$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$$
$$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$$
$$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$$
$$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \}; \; n(S) = 36$$

(i) Let A be the event of getting the sum of outcome values equal to 4.

Then $A = \{(1,3),(2,2),(3,1)\}; \; n(A) = 3$.

Probability of getting the sum value equal to 4 is $P(A) = \frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

(ii) Let B be the event of getting the sum of outcome values greater than 10.

$B = \{(5,6),(6,5),(6,6)\}; \; n(B) = 3$

Probability of getting the sum value greater than 10 is $P(B) = \frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

(iii) Let C be the event of getting the sum value less than 13. Here all the outcomes have the sum value less than 13. Hence $C = S$.

Therefore, $n(C) = n(S) = 36$

Probability of getting the sum value less than 13 is $P(C) = \frac{n(C)}{n(S)} = \frac{36}{36} = 1$.

Example 8.21 Two coins are tossed together. What is the probability of getting different faces on the coins?

Solution When two coins are tossed together, the sample space is

$$S = \{HH, HT, TH, TT\}; \quad n(S) = 4$$

Let A be the event of getting different faces on the coins.

$$A = \{HT, TH\}; \quad n(A) = 2$$

Probability of getting different faces on the coins is $P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

Example 8.22 From a well shuffled pack of 52 cards, one card is drawn at random. Find the probability of getting (i) red card (ii) heart card (iii) red king (iii) face card (iv) number card

Solution $n(S) = 52$

(i) Let A be the event of getting a red card.

$n(A) = 26$

Probability of getting a red card is

$$P(A) = \frac{26}{52} = \frac{1}{2}$$

(ii) Let B be the event of getting a heart card.

$n(B) = 13$

Probability of getting a heart card is

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

Suits of playing cards	Spade	Heart	Clavor	Diamond
Cards of each suit	A	A	A	A
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9
	10	10	10	10
	J	J	J	J
	Q	Q	Q	Q
	K	K	K	K
Set of playing cards in each suit	13	13	13	13

Fig 8.5

(iii) Let C be the event of getting a red king card. A red king card can be either a diamond king or a heart king.

$n(C) = 2$

Probability of getting a red king card is

$$P(C) = \frac{n(C)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

(iv) Let D be the event of getting a face card. The face cards are Jack (J), Queen (Q), and King (K).

$n(D) = 4 \times 3 = 12$

Probability of getting a face card is

$$P(D) = \frac{n(D)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

(v) Let E be the event of getting a number card. The number cards are 2, 3, 4, 5, 6, 7, 8, 9 and 10.

$n(E) = 4 \times 9 = 36$

Probability of getting a number card is

$$P(E) = \frac{n(E)}{n(S)} = \frac{36}{52} = \frac{9}{13}$$

Example 8.23 What is the probability that a leap year selected at random will contain 53 saturdays. (Hint: $366 = 52 \times 7 + 2$)

Solution A leap year has 366 days. So it has 52 full weeks and 2 days. 52 Saturdays must be in 52 full weeks.

The possible chances for the remaining two days will be the sample space.

$S = \{(\text{Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun})\}$

$n(S) = 7$

Let A be the event of getting 53[rd] Saturday.

Then $A = \{\text{Fri-Sat, Sat-Sun}\};\ n(A) = 2$

Thinking Corner

What will be the probability that a non-leap year will have 53 Saturdays?

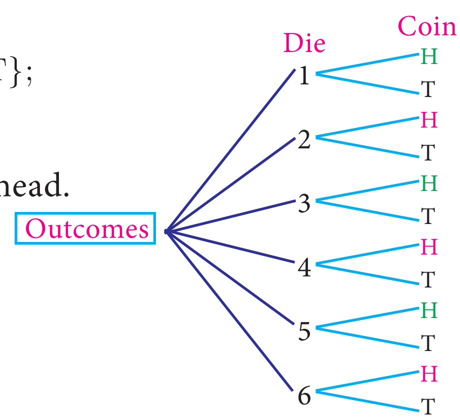
Probability of getting 53 Saturdays in a leap year is $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{2}{7}$

Example 8.24 A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.

Solution Sample space

$S = \{1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T\};$

$n(S) = 12$



Let A be the event of getting an odd number and a head.

$A = \{1H, 3H, 5H\};\ n(A) = 3$

$P(A) = \dfrac{n(A)}{n(S)} = \dfrac{3}{12} = \dfrac{1}{4}$

Activity 3	Activity 4
There are three routes R_1, R_2 and R_3 from Madhu's home to her place of work. There are four parking lots P_1, P_2, P_3, P_4 and three entrances B_1, B_2, B_3 into the office building. There are two elevators E_1 and E_2 to her floor. Using the tree diagram explain how many ways can she reach her office?	Collect the details and find the probabilities of (i) selecting a boy from your class. (ii) selecting a girl from your class. (iii) selecting a student from tenth standard. (iv) selecting a boy from tenth standard in your school. (v) selecting a girl from tenth standard in your school.

Example 8.25 A bag contains 6 green balls, some black and red balls. Number of black balls is as twice as the number of red balls. Probability of getting a green ball is thrice the probability of getting a red ball. Find (i) number of black balls (ii) total number of balls.

Solution Number of green balls is $n(G) = 6$

Let number of red balls is $n(R) = x$

Therefore, number of black balls is $n(B) = 2x$

Total number of balls $n(S) = 6 + x + 2x = 6 + 3x$

It is given that, $P(G) = 3 \times P(R)$

$$\frac{6}{6+3x} = 3 \times \frac{x}{6+3x}$$

$3x = 6$ gives, $x = 2$

(i) Number of black balls $= 2 \times 2 = 4$

(ii) Total number of balls $= 6 + (3 \times 2) = 12$

Example 8.26 A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the numbers $1,\ 2,\ 3,\ \ldots 12$. What is the probability that it will point to (i) 7 (ii) a prime number (iii) a composite number?

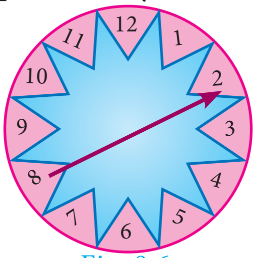


Fig. 8.6

Solution Sample space $S = \{1,2,3,4,5,6,7,8,9,10,11,12\}$; $n(S) = 12$

(i) Let A be the event of resting in 7. $n(A)=1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{12}$$

(ii) Let B be the event that the arrow will come to rest in a prime number.

$$B = \{2,3,5,7,11\};\ n(B) = 5$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{12}$$

(iii) Let C be the event that arrow will come to rest in a composite number.

$$C = \{4,6,8,9,10,12\};\ n(C)=6$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{6}{12} = \frac{1}{2}$$

Thinking Corner

What is the complement event of an impossible event?

Exercise 8.3

1. Write the sample space for tossing three coins using tree diagram.
2. Write the sample space for selecting two balls from a bag containing 6 balls numbered 1 to 6 (using tree diagram).
3. If A is an event of a random experiment such that $P(A),\ P(\bar{A}) = 17:15$ and $n(S)=640$ then find (i) $P(\bar{A})$ (ii) $n(A)$.
4. Thenmozhi throws two dice once and computes the product of the numbers appearing on the dice. Krishna throws one die and squares the number that appears on it. Who has the better chance of geting the number 36?
5. A coin is tossed thrice. What is the probability of getting two consecutive tails?
6. At a fete, cards bearing numbers 1 to 1000, one number on one card are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square number greater than 500, the player wins a prize. What is the probability that (i) the first player wins a prize (ii) the second player wins a prize, if the first has won?
7. A bag contains 12 blue balls and x red balls. If one ball is drawn at random (i) what is the probability that it will be a red ball? (ii) If 8 more red balls are put in the bag and if the probability of drawing a red ball will be twice that of the probability in (i) then find x.
8. Two unbiased dice are rolled once. Find the probability of getting
   (i) a doublet (equal numbers on both dice) (ii) the product as a prime number
   (iii) the sum as a prime number (iv) the sum as 1
9. Three fair coins are tossed together. Find the probability of getting
   (i) all heads (ii) atleast one tail
   (iii) atmost one head (iv) atmost two tails

10. Two dice are numbered 1,2,3,4,5,6 and 1,1,2,2,3,3 respectively. They are rolled and the sum of the numbers on them is noted. Find the probability of getting each sum from 2 to 9 separately.

11. A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is

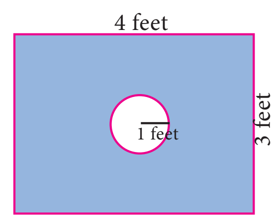
(i) white (ii) black or red

(iii) not white (iv) neither white nor black

12. In a box there are 20 non-defective and some defective bulbs. If the probability that a bulb selected at random from the box found to be defective is $\frac{3}{8}$ then, find the number of defective bulbs.

13. The king and queen of diamonds, queen and jack of hearts, jack and king of spades are removed from a deck of 52 cards and then well suffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is (i) a clavor (ii) a queen of red card (iii) a king of black card

14. Some boys are playing a game, in which the stone thrown by them landing in a circular region (given in the figure) is considered as win and landing other than the circular region is considered as loss. What is the probability to win the game?



15. Two customers Priya and Amuthan are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another day. What is the probability that both will visit the shop on

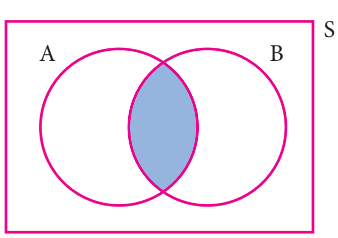
(i) the same day (ii) different days (iii) consecutive days?

16. In a game, the entry fee is ₹150. The game consists of tossing a coin 3 times. Dhana bought a ticket for entry . If one or two heads show, she gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. Find the probability that she (i) gets double entry fee (ii) just gets her entry fee (iii) loses the entry fee.

8.5 Algebra of Events

In a random experiment, let S be the sample space. Let $A \subseteq S$ and $B \subseteq S$. Then A and B are events. We say that

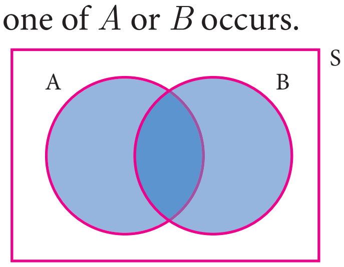
(i) $(A \cap B)$ is an event that occurs only when both A and B occurs.



$A \cap B$

Fig. 8.7(a)

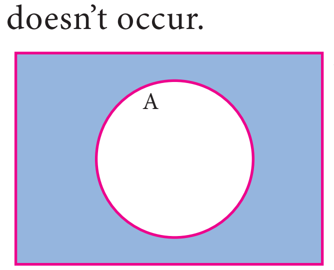
(ii) $(A \cup B)$ is an event that occurs only when at least one of A or B occurs.



$A \cup B$

Fig. 8.7(b)

(iii) $\bar{A}$ is an event that occurs only when A doesn't occur.



$\bar{A}$

Fig. 8.7(c)

Note

(i) $A \cap \bar{A} = \phi$ (ii) $A \cup \bar{A} = S$

(iii) If $A,\ B$ are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$

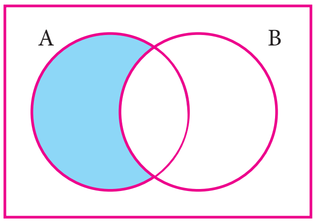
(iv) P(Union of events) $= \sum$(probability of events)

Thorem 1

If A and B are two events associated with a random experiment, then prove that

(i) $P(A \cap \bar{B}) = P(\text{only } A) = P(A) - P(A \cap B)$

(ii) $P(\bar{A} \cap B) = P(\text{only } B) = P(B) - P(A \cap B)$



$A \cap \bar{B}$

Fig. 8.8

Proof

(i) By Distributive property of sets,

1. $(A \cap B) \cup (A \cap \bar{B}) = A \cap (B \cup \bar{B}) = A \cap S = A$
2. $(A \cap B) \cap (A \cap \bar{B}) = A \cap (B \cap \bar{B}) = A \cap \phi = \phi$

Therefore, the events $A \cap B$ and $A \cap \bar{B}$ are mutually exclusive and their union is A.

Therefore, $P(A) = P\left[(A \cap B) \cup (A \cap \bar{B})\right]$

$P(A) = P(A \cap B) + P(A \cap \bar{B})$

Therefore, $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

That is, $P(A \cap \bar{B}) = P(\text{only } A) = P(A) - P(A \cap B)$

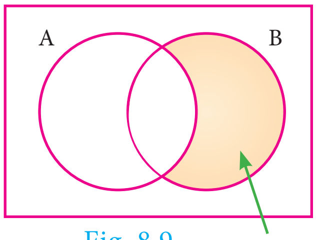


Fig. 8.9 $\bar{A} \cap B$

(ii) By Distributive property of sets,

1. $(A \cap B) \cup (\bar{A} \cap B) = (A \cup \bar{A}) \cap B = S \cap B = B$
2. $(A \cap B) \cap (\bar{A} \cap B) = (A \cap \bar{A}) \cap B = \phi \cap B = \phi$

Therefore, the events $A \cap B$ and $\bar{A} \cap B$ are mutually exclusive and their union is B.

$P(B) = P\left[(A \cap B) \cup (\bar{A} \cap B)\right]$

$P(B) = P(A \cap B) + P(\bar{A} \cap B)$

Therefore, $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

That is, $P(\bar{A} \cap B) = P(\text{only } B) = P(B) - P(A \cap B)$

Progress Check

1. $P(\text{only } A) =$ ________.
2. $P(A \cup B) + P(A \cap B) =$ ________.
3. $P(\bar{A} \cap B) =$ ________.
4. $A \cap B$ and $\bar{A} \cap B$ are ________ events.
5. $P(\bar{A} \cap \bar{B}) =$ ________.
6. If A and B are mutually exclusive events then $P(A \cap B) =$ ________.
7. If $P(A \cap B) = 0.3$, $P(\bar{A} \cap B) = 0.45$ then $P(B) =$ ________.

8.6 Addition Theorem of Probability

(i) If A and B are any two non mutually exclusive events then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(ii) If A, B and C are any three non mutually exclusive events then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

Proof

(i) Let A and B be any two events of a random experiment with sample space S.

From the Venn diagram, we have the events only A, $A \cap B$ and only B are mutually exclusive and their union is $A \cup B$

Therefore, $P(A \cup B) = P[(\text{only A}) \cup (A \cap B) \cup (\text{only } B)]$

$= P(\text{only } A) + P(A \cap B) + P(\text{only } B)$

$= [P(A) - P(A \cap B)] + P(A \cap B) + [P(B) - P(A \cap B)]$

$P(A \cup B) = [P(A) + P(B) - P(A \cap B)]$

(ii) Let A, B, C are any three events of a random experiment with sample space S.

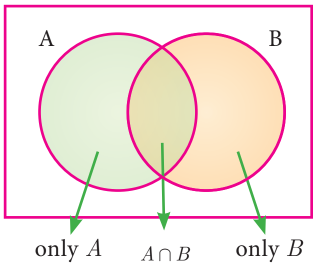


Fig. 8.10

Let $D = B \cup C$

$P(A \cup B \cup C) = P(A \cup D)$

$= P(A) + P(D) - P(A \cap D)$

$= P(A) + P(B \cup C) - P[A \cap (B \cup C)]$

$= P(A) + P(\text{B}) + P(C) - P(B \cap C) - P[(A \cap B) \cup (A \cap C)]$

$= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P[(A \cap B) \cap (A \cap C)]$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

Activity 5

The addition theorem of probability can be written easily using the following way.

$P(A \cup B) = S_1 - S_2$

$P(A \cup B \cup C) = S_1 - S_2 + S_3$

Where $S_1 \rightarrow$ Sum of probability of events taken one at a time.

$S_2 \rightarrow$ Sum of probability of events taken two at a time.

$S_3 \rightarrow$ Sum of probability of events taken three at a time.

$$P(A \cup B) = \underbrace{P(A) + P(B)}_{S_1} \underbrace{-P(A \cap B)}_{S_2}$$

$$P(A \cup B \cup C) = \underbrace{P(A) + P(B) + P(C)}_{S_1} \underbrace{-(P(A \cap B) + P(B \cap C) + P(A \cap C))}_{S_2} \underbrace{+P(A \cap B \cap C)}_{S_3}$$

Find the probability of $P(A \cup B \cup C \cup D)$ using the above way. Can you find a pattern for the number of terms in the formula?

Example 8.27 If $P(\text{A}) = 0.37$, $P(\text{B}) = 0.42$, $P(A \cap B) = 0.09$ then find $P(A \cup B)$.

Solution $P(\text{A}) = 0.37$, $P(\text{B}) = 0.42$, $P(A \cap B) = 0.09$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.37 + 0.42 - 0.09 = 0.7$$

Example 8.28 What is the probability of drawing either a king or a queen in a single draw from a well shuffled pack of 52 cards?

Solution Total number of cards $= 52$

Number of king cards $= 4$

Probability of drawing a king card $= \frac{4}{52}$

Number of queen cards $= 4$

Probability of drawing a queen card $= \frac{4}{52}$

Both the events of drawing a king and a queen are mutually exclusive

$$\Rightarrow P(A \cup B) = P(A) + P(B)$$

Therefore, probability of drawing either a king or a queen is $= \frac{4}{52} + \frac{4}{52} = \frac{2}{13}$

Example 8.29 Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

Solution When two dice are rolled together, there are $6 \times 6 = 36$ outcomes. Let S be the sample space. Then $n(S) = 36$

Let A be the event of getting a doublet and B be the event of getting face sum 4.

Then $A = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

$B = \{(1,3),(2,2),(3,1)\}$

Therefore, $A \cap B = \{(2,2)\}$

Then, $n(A) = 6$, $n(B) = 3$, $n(A \cap B) = 1$.

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

Therefore, P (getting a doublet or a total of 4) $= P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{6}{36} + \frac{3}{36} - \frac{1}{36} = \frac{8}{36} = \frac{2}{9}$$

Hence, the required probability is $\frac{2}{9}$.

Example 8.30 If A and B are two events such that $P(A)=\frac{1}{4}$, $P(B)=\frac{1}{2}$ and $P(A$ and $B)$ $\frac{1}{8}$, find (i) $P(A$ or $B)$ (ii) $P($not A and not $B)$.

Solution (i)

$$P(A \text{ or } B) = P(A\cup B)$$
$$= P(A)+P(B)-P(A\cap B)$$
$$P(A \text{ or } B) = \frac{1}{4}+\frac{1}{2}-\frac{1}{8} = \frac{5}{8}$$

(ii)

$$P(\text{not } A \text{ and not } B) = P(\bar{A}\cap\bar{B})$$
$$= P\left(\overline{A\cup B}\right)$$
$$= 1-P\left(A\cup B\right)$$
$$P(\text{not } A \text{ and not } B) = 1-\frac{5}{8} = \frac{3}{8}$$

Example 8.31 A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

Solution Total number of cards = 52; $n(S)=52$

Let A be the event of getting a king card. $n\left(A\right)=4$

$$P\left(A\right) = \frac{n(A)}{n(S)} = \frac{4}{52}$$

Let B be the event of getting a heart card. $n\left(B\right)=13$

$$P\left(B\right) = \frac{n(B)}{n(S)} = \frac{13}{52}$$

Let C be the event of getting a red card. $n\left(C\right)=26$

$$P(C) = \frac{n(C)}{n(S)} = \frac{26}{52}$$

$$P(A\cap B) = P(\text{getting heart king}) = \frac{1}{52}$$

$$P(B\cap C) = P(\text{getting red and heart})) = \frac{13}{52}$$

$$P(A\cap C) = P(\text{getting red king}) = \frac{2}{52}$$

$$P(A\cap B\cap C) = P(\text{getting heart, king which is red}) = \frac{1}{52}$$

Therefore, required probability is

$$P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$$
$$= \frac{4}{52}+\frac{13}{52}+\frac{26}{52}-\frac{1}{52}-\frac{13}{52}-\frac{2}{52}+\frac{1}{52} = \frac{28}{52} = \frac{7}{13}$$

Example 8.32 In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the students is selected at random. Find the probability that

(i) The student opted for NCC but not NSS.

(ii) The student opted for NSS but not NCC.

(iii) The student opted for exactly one of them.

Solution Total number of students $n(S) = 50$.

Let A and B be the events of students opted for NCC and NSS respectively.

$$n(A) = 28, \; n(B) = 30, \; n(A \cap B) = 18$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{28}{50}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{30}{50}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{18}{50}$$

(i) Probability of the students opted for NCC but not NSS

$$P(A \cap \overline{B}) = P(A) - P(A \cap B) = \frac{28}{50} - \frac{18}{50} = \frac{1}{5}$$

(ii) Probability of the students opted for NSS but not NCC.

$$P(A \cap \overline{B}) = P(B) - P(A \cap B) = \frac{30}{50} - \frac{18}{50} = \frac{6}{25}$$

(iii) Probability of the students opted for exactly one of them

$$= P[(A \cap \overline{B}) \cup (\overline{A} \cap B)]$$

$$= P(A \cap \overline{B}) + P(\overline{A} \cap B) = \frac{1}{5} + \frac{6}{25} = \frac{11}{25}$$

(Note that $(A \cap \overline{B}), (\overline{A} \cap B)$ are mutually exclusive events)

Example 8.33 A and B are two candidates seeking admission to IIT, the probability that A getting selected is 0.5 and the probability that both A and B getting selected is 0.3. Prove that the probability of B being selected is atmost 0.8.

Solution $P(A) = 0.5, \; P(A \cap B) = 0.3$

We have

$$P(A \cup B) \leq 1$$

$$P(\text{A}) + P(\text{B}) - P(A \cap B) \leq 1$$

$$0.5 + P(B) - 0.3 \leq 1$$

$$P(B) \leq 1 - 0.2$$

$$P(B) \leq 0.8$$

Therefore, probability of B getting selected is atmost 0.8.

Exercise 8.4

1. If $P(A)=\frac{2}{3}$, $P(B)=\frac{2}{5}$, $P(A\cup B)=\frac{1}{3}$ then find $P(A\cap B)$.
2. A and B are two events such that, $P(A)=0.42$, $P(B)=0.48$, and $P(A\cap B)=0.16$. Find (i) $P(\text{not } A)$ (ii) $P(\text{not } B)$ (iii) $P(A \text{ or } B)$
3. If A and B are two mutually exclusive events of a random experiment and $P(\text{not } A)=0.45$, $P(A\cup B)$ =0.65, then find $P(B)$.
4. The probability that atleast one of A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then find $P(\bar{A})+P(\bar{B})$.
5. The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then find the probability that neither A nor B happen.
6. Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.
7. From a well-shuffled pack of 52 cards, a card is drawn at random. Find the probability of it being either a red king or a black queen.
8. A box contains cards numbered $3, 5, 7, 9, \ldots 35, 37$. A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.
9. Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or atleast 2 heads.
10. The probability that a person will get an electrification contract is $\frac{3}{5}$ and the probability that he will not get plumbing contract is $\frac{5}{8}$. The probability of getting atleast one contract is $\frac{5}{7}$. What is the probability that he will get both?
11. In a town of 8000 people, 1300 are over 50 years and 3000 are females. It is known that 30% of the females are over 50 years. What is the probability that a chosen individual from the town is either a female or over 50 years?
12. Three coins are tossed simultaneously. Find the probability of getting exactly two heads or atleast one tail or consecutively two heads.
13. If A, B, C are any three events such that probability of B is twice as that of probability of A and probability of C is thrice as that of probability of A and if $P(A\cap B)=\frac{1}{6}$, $P(B\cap C)=\frac{1}{4}$, $P(A\cap C)=\frac{1}{8}$, $P(A\cup B\cup C)=\frac{9}{10}$ and $P(A\cap B\cap C)=\frac{1}{15}$, then find $P(A), P(B)$ and $P(C)$?
14. In a class of 35, students are numbered from 1 to 35. The ratio of boys to girls is 4:3. The roll numbers of students begin with boys and end with girls. Find the probability that a student selected is either a boy with prime roll number or a girl with composite roll number or an even roll number.

Exercise 8.5

Multiple choice questions

1. Which of the following is not a measures of dispersion?
(1) Range (2) Standard deviation
(3) Arithmetic mean (4) Variance

2. The range of the data 8, 8, 8, 8, 8. . . 8 is
(1) 0 (2) 1 (3) 8 (4) 3

3. The sum of all deviations of the data from its mean is
(1) Always positive (2) always negative (3) zero (4) non-zero integer

4. The mean of 100 observations is 40 and their standard deviation is 3. The sum of squares of all deviations is
(1) 40000 (2) 160900 (3) 160000 (4) 30000

5. Variance of first 20 natural numbers is
(1) 32.25 (2) 44.25 (3) 33.25 (4) 30

6. The standard deviation of a data is 3. If each value is multiplied by 5 then the new variance is
(1) 3 (2) 15 (3) 5 (4) 225

7. If the standard deviation of x, y, z is p then the standard deviation of $3x+5$, $3y+5$, $3z+5$ is
(1) $3p+5$ (2) $3p$ (3) $p+5$ (4) $9p+15$

8. If the mean and coefficient of variation of a data are 4 and 87.5% then the standard deviation is
(1) 3.5 (2) 3 (3) 4.5 (4) 2.5

9. Which of the following is incorrect?
(1) $P(A) > 1$ (2) $0 \le P(A) \le 1$ (3) $P(\phi) = 0$ (4) $P(A) + P(\bar{A}) = 1$

10. The probability a red marble selected at random from a jar containing p red, q blue and r green marbles is
(1) $\frac{q}{p+q+r}$ (2) $\frac{p}{p+q+r}$ (3) $\frac{p+q}{p+q+r}$ (4) $\frac{p+r}{p+q+r}$

11. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is
(1) $\frac{3}{10}$ (2) $\frac{7}{10}$ (3) $\frac{3}{9}$ (4) $\frac{7}{9}$

12. The probability of getting a job for a person is $\frac{x}{3}$. If the probability of not getting the job is $\frac{2}{3}$ then the value of x is
(1) 2 (2) 1 (3) 3 (4) 1.5

13. Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold. If the probability of Kamalam winning is $\frac{1}{9}$, then the number of tickets bought by Kamalam is

(1) 5 (2) 10 (3) 15 (4) 20

14. If a letter is chosen at random from the English alphabets $\{a, b, \ldots, z\}$, then the probability that the letter chosen precedes x

(1) $\frac{12}{13}$ (2) $\frac{1}{13}$ (3) $\frac{23}{26}$ (4) $\frac{3}{26}$

15. A purse contains 10 notes of ₹2000, 15 notes of ₹500, and 25 notes of ₹200. One note is drawn at random. What is the probability that the note is either a ₹500 note or ₹200 note?

(1) $\frac{1}{5}$ (2) $\frac{3}{10}$ (3) $\frac{2}{3}$ (4) $\frac{4}{5}$

Unit Exercise - 8

1. The mean of the following frequency distribution is 62.8 and the sum of all frequencies is 50. Compute the missing frequencies f_1 and f_2.

Class Interval	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	f_1	10	f_2	7	8

2. The diameter of circles (in mm) drawn in a design are given below.

Diameters	33-36	37-40	41-44	45-48	49-52
Number of circles	15	17	21	22	25

Calculate the standard deviation.

3. The frequency distribution is given below.

x	k	$2k$	$3k$	$4k$	$5k$	$6k$
f	2	1	1	1	1	1

In the table, k is a positive integer, has a varience of 160. Determine the value of k.

4. The standard deviation of some temperature data in degree celsius (°C) is 5. If the data were converted into degree Farenheit (°F) then what is the variance?

5. If for distribution $\sum(x-5) = 3$, $\sum(x-5)^2 = 43$, and total number of items is 18. find the mean and standard deviation.

6. Prices of peanut packets in various places of two cities are given below. In which city, prices were more stable?

Prices in city A	20	22	19	23	16
Prices in city B	10	20	18	12	15

7. If the range and coefficient of range of the data are 20 and 0.2 respectively, then find the largest and smallest values of the data.

8. If two dice are rolled, then find the probability of getting the product of face value 6 or the difference of face values 5.
9. In a two children family, find the probability that there is at least one girl in a family.
10. A bag contains 5 white and some black balls. If the probability of drawing a black ball from the bag is twice the probability of drawing a white ball then find the number of black balls.
11. The probability that a student will pass the final examination in both English and Tamil is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Tamil examination?
12. The King, Queen and Jack of the suit spade are removed from a deck of 52 cards. One card is selected from the remaining cards. Find the probability of getting (i) a diamond (ii) a queen (iii) a spade (iv) a heart card clearing the number 5.

Points to Remember

- Range $= L - S$ (L - Largest value, S - Smallest value)
- Coefficient of range $= \dfrac{L-S}{L+S}$; Variance $\sigma^2 = \dfrac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n}$
- Standard deviation $\sigma = \sqrt{\dfrac{\Sigma\left(x_i - \overline{x}\right)^2}{n}}$
- Standard deviation (ungrouped data)

  (i) Direct method $\sigma = \sqrt{\dfrac{\Sigma x_i^2}{n} - \left(\dfrac{\Sigma x_i}{n}\right)^2}$ (ii) Mean method $\sigma = \sqrt{\dfrac{\Sigma d_i^{\,2}}{n}}$

  (iii) Assumed mean method $\sigma = \sqrt{\dfrac{\Sigma d_i^2}{n} - \left(\dfrac{\Sigma d_i}{n}\right)^2}$

  (iv) Step deviation method $\sigma = c \times \sqrt{\dfrac{\Sigma d_i^2}{n} - \left(\dfrac{\Sigma d_i}{n}\right)^2}$
- Standard deviation of first n natural numbers $\sigma = \sqrt{\dfrac{n^2-1}{12}}$
- Standard deviation (grouped data)

  (i) Mean method $\sigma = \sqrt{\dfrac{\Sigma f_i d_i^2}{N}}$ (ii)Assumed mean method $\sigma = \sqrt{\dfrac{\Sigma f_i d_i^2}{N} - \left(\dfrac{\Sigma f_i d_i}{N}\right)^2}$

  (iii) Step deviation method $\sigma = C \times \sqrt{\dfrac{\Sigma f_i d_i^2}{N} - \left(\dfrac{\Sigma f_i d_i}{N}\right)^2}$ Coefficient of variation

  C.V $= \dfrac{\sigma}{\overline{x}} \times 100\%$
- If the C.V. value is less, then the observations of corresponding data are consistent. If the C.V. value is more then the observations of corresponding are inconsistent.

- In a random experiment, the set of all outcomes are known but exact outcome is not known.
- The set of all possible outcomes is called sample space.
- A, B are said to be mutually exclusive events if $A \cap B = \phi$
- Probability of event E is $P(E) = \dfrac{n(E)}{n(S)}$

  (i) The probability of sure event is 1 and the probability of impossible event is 0.

  (ii) $0 \leq P(E) \leq 1$; (iii) $P(\overline{E}) = 1 - P(E)$
- If A and B are mutually exclusive events then $P(A \cup B) = P(A) + P(B)$.
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, for any two events $A,\ B$.

  (i) $P(A \cap \overline{B}) = P(\text{only}\,A) = P(A) - P(A \cap B)$

  (ii) $P(\overline{A} \cap B) = P(\text{only}\,B) = P(B) - P(A \cap B)$

  (iii) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$
  $- P(C \cap A) + P(A \cap B \cap C)$

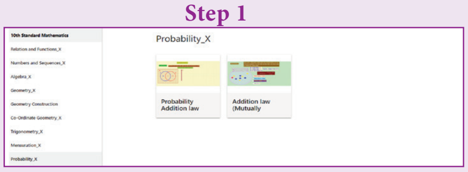
ICT CORNER

ICT 8.1

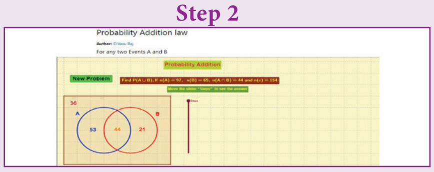
Step 1: Open the Browser type the URL Link given below (or) Scan the QR Code. Chapter named **"Probability"** will open. Select the work sheet " **Probability Addition law** "

Step 2: In the given worksheet you can change the question by clicking on "New Problem". Move the slider to see the steps.

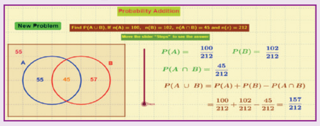
Step 1



Step 2



Expected results



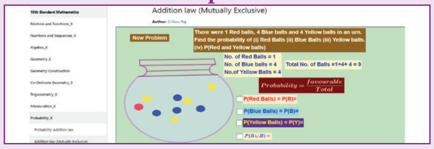
ICT 8.2

Step 1: Open the Browser type the URL Link given below (or) Scan the QR Code. Chapter named **"Probability"** will open. Select the work sheet " **Addition law Mutually Exclusive"**

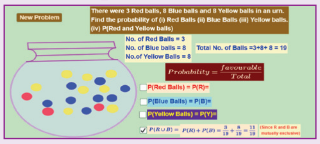
Step 2: In the given worksheet you can change the question by clicking on "New Problem". Click on the check boxes to see the respective answer.

Step 1

Step 2



Expected results



Scan the QR Code.

B374_10_MATHS_EM

ANSWERS

Exercise 1.1

1.(i) $A\times B=\{(2,1),(2,-4),(-2,1),(-2,-4),(3,1),(3,-4)\}$

$A\times A=\{(2,2),(2,-2),(2,3),(-2,2),(-2,-2),(-2,3),(3,2),(3,-2),(3,3)\}$

$B\times A=\{(1,2),(1,-2),(1,3),(-4,2),(-4,-2),(-4,3)\}$

(ii) $A\times B=\{(p,p)(p,q)(q,p)(q,q)\}$; $A\times A=\{(p,p),(p,q),(q,p),(q,q)\}$;

$B\times A=\{(p,p),(p,q),(q,p),(q,q)\}$

(iii) $A\times B=\{\ \}$; $A\times A=\{(m,m),(m,n),(n,m),(n,n)\}$; $B\times A=\{\ \}$

2. $A\times B=\{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),(2,7),(3,2),(3,3),(3,5),(3,7)\}$

$B\times A=\{(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(5,1),(5,2),(5,3),(7,1),(7,2),(7,3)\}$

3. $A=\{3,4\}$ $B=\{-2,0,3\}$

5. true

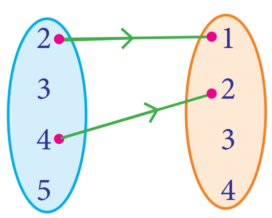
Exercise 1.2

1.(i) Not a relation (ii) Not a relation (iii) Relation (iv) Not a relation

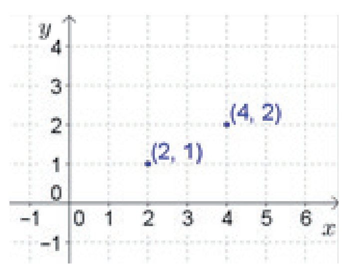
2. $\{1,2,3,4,5,6\},\{1,4,9,16,25,36\}$

3. $\{0,1,2,3,4,5\},\{3,4,5,6,7,8\}$

4. (i)(a)

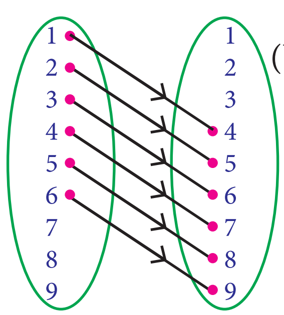


(b)

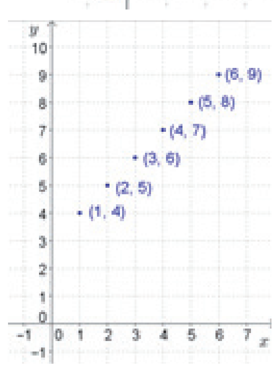


(c) $\{(2,1),(4,2)\}$

(ii)(a)

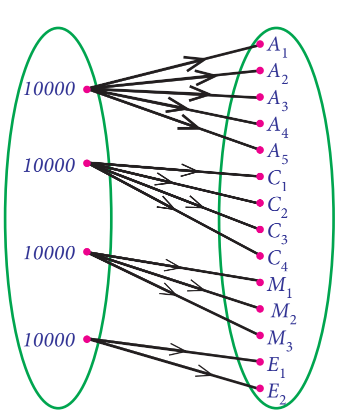


(b)



(c) $\{(1,4),(2,5),(3,6),(4,7),(5,8),(6,9)\}$

5. $\{(10000,A_1),(10000,A_2),(10000,A_3),(10000,A_4),(10000,A_5),(25000,C_1),(25000,C_2),(25000,C_3),(25000,C_4),(50000,M_1),(50000,M_2),(50000,M_3),(100000,E_1),(100000,E_2)\}$



Exercise 1.3

1. $\{1,2,3,4,...\}$, $\{1,2,3,4\}$, $\{2,4,6,8,...\}$, yes. 2. yes

3.(i) 12 (ii) $4a^2-10a+6$ (iii) 0 (iv) $x^2-7x+12$

4.(i) (a) 9 (b) 6 (c) 6 (d) 0

(ii) 9.5 (iii) (a) $\{x \,/\, 0 \le x \le 10, x \in R\}$ (b) $\{x \,/\, 0 \le x \le 9, x \in R\}$

(iv) 5 5. 2 6.(i) -2 (ii) $\frac{3}{2}$ (iii) 3 (iv) $\frac{1}{2}$

7. $4x^3-96x^2+576x$ 8. 1 9. $500t$

10.(i) Yes (ii) 1,20 (iii) 68 inches (iv) 40.54 cm

Exercise 1.4

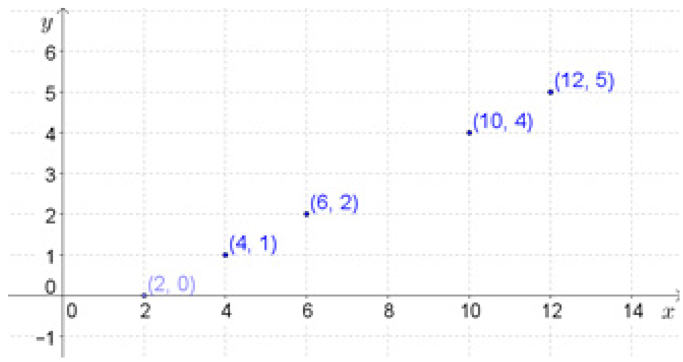
1.(i) Not a function (ii) function (iii) Not a function (iv) function

2.(i) $\{(2,0),(4,1),(6,2),(10,4),(12,5)\}$

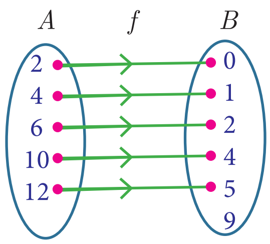
(ii)

x	2	4	6	10	12
$f(x)$	0	1	2	4	5

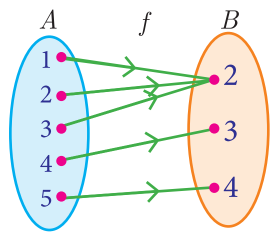
(iv)



(iii)



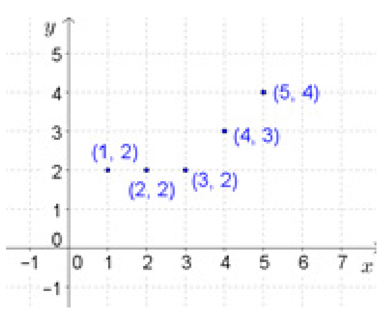
3.(i)



(ii)

x	1	2	3	4	5
$f(x)$	2	2	2	3	4

(iii)



6.(i) $\{1,8,27,64\}$ (ii) one-one and into function

7.(i) Bijective function (ii) Not bijective function 8. 1,1

9.(i) 5 (ii) 2 (iii) -2.5 (iv) 1

10.(i) 2 (ii) 10 (iii) 178 (iv) $\frac{-9}{17}$

11. Yes 12.(i) $32°F$ (ii) $82.4°F$ (iii) $24°F$

(iv) $100°C$ (v) $-40°$

Exercise 1.5

1.(i) $x^2-6,\ (x-6)^2$; not equal (ii) $\frac{2}{2x^2-1}, \frac{8}{x^2}-1$; not equal

(iii) $\frac{3-x}{3}$; not equal (iv) $x-1,\ x-1$; equal (v) $4x^2+8x+3,\ 4x^2$; not equal

2.(i) -5 (ii) $\frac{-5}{3}$ 4.(i) $a=\pm 2$ (ii) 2

5. $\left\{y \mid y = 2x^2+1,\ x \in \mathbb{N}\right\}$; $\left\{y \mid y=(2x+1)^2, x \in \mathbb{N}\right\}$ 6.(i) x^4-2x^2

(ii) $\left[x^4-2x^2\right]^2-1$ 7. f is one-one, g is one-one, $f \circ g$ is one-one 9. $-4x-1$

Exercise 1.6

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(3)	(3)	(1)	(2)	(3)	(4)	(4)	(1)	(3)	(3)	(1)	(4)	(3)	(2)	(4)

Unit exercise

1. 1,2 and $-5,1$ 2. $\{-1,0,1\}$, $\{(-1,-1),(-1,1),(0,-1),(0,0),(1,-1),(1,0),(1,1)\}$

3.(i) 4 (ii) $\sqrt{2}$ (iii) $\sqrt{a}$

4. $\{(9,3),(10,5),(11,11),(12,3),(13,13),(14,7),(15,5),(16,2),(17,17)\}$, $\{2,3,5,11,13,17\}$

5. {-1,0,1} 9.(i) $\frac{-5}{6}$ (ii) $2(x+1)$ 10.(i) $R-\{9\}$

(ii) R (iii) $[2,\infty)$ (iv) R

Exercise 2.1

1. 2, 5, 8, 11, … 2. 25, 7 6.(i) 4 (ii) 51

(iii) 144 (iv) 6 7. 174 8. $2,-1$ 9. 6

Exercise 2.2

1. Even number 2. No value 3. 10101 4. 9, 3

5. 2,3,5,7 and 3,4,2,1 6. 2040, 34 7. 999720 8. 3647 9. 2520

Exercise 2.3

1.(i) 7 (ii) 5 (iii) 2 (iv) 7 (v) 2

2. 3 3. 2,8,14,… 4. 8, 19, 30, … 5. 11 a.m

6. 8 p.m 7. Friday 9. 2 10. 6 am, Monday

Exercise 2.4

1.(i) 216,648,1944 (ii) $-7,-11,-15$ (iii) $\frac{4}{25},\frac{5}{36},\frac{6}{49}$ 2.(i) $-1,6,25,62$

(ii) $2,-6,12,-20$ (iii) -4, 2, 12, 26 3.(i) n^2+1 (ii) $\frac{n-1}{n}$

(iii) $5n-2$ 4.(i) $\frac{13}{3},\frac{15}{4}$ (ii) $-12,117$ 5. $\frac{63}{11},\frac{225}{31}$ 6. 1,1,3,7,17,41

Exercise 2.5

1.(i) A.P (ii) not an A.P (iii) A.P (iv) A.P

(v) not an A.P 2.(i) 5, 11, 17, … (ii) 7, 2, -3, … (iii) $\frac{3}{4},\frac{5}{4},\frac{7}{4},\ldots$

3.(i) -1, 2 (ii) $-3,-7$ 4. -83 5. 15

6. 93, 99 8. 4 9. 3,17,31 10. 78

11. 2,9,16 12. 5:7 13. $-3°C, 0°C, 3°C, 6°C, 9°C$ 14. 31 years

Exercise 2.6

1.(i) 3240 (ii) 999 (iii) 721 2. 20 3. 1540

5. 612.5 6. 50625 7. 168448 8.(i) ₹ 45750 (ii) ₹ 5750 9. 20 months

10.(i) 42 (ii) 2130 12. $\frac{6}{a+b}(24a-13b)$

Exercise 2.7

1.(i) G.P (ii) not a G.P (iii) G.P (iv) G.P (v) G.P

(vi) not a G.P (vii) G.P 2.(i) 6,18,54 (ii) $\sqrt{2},2,2\sqrt{2}$

(iii) 1000,400,160 3. 1 4. -18 5.(i) 12 (ii) 7 6. $5\times(3^{11})$

7. 3072 9. $\frac{9}{2}$, 3, 2 10. ₹ 76577 11. ₹ 23820, ₹ 24040

Exercise 2.8

1.(i) $\frac{25}{8}\left[1-\left(-\frac{3}{5}\right)^n\right]$ (ii) $\frac{1024}{3}\left[1-\left(\frac{1}{4}\right)^n\right]$ 2. 1820 3. 12

4.(i) $\frac{27}{2}$ (ii) 63 5. $\frac{1}{4}$ 6.(i) $\frac{4}{9}n-\frac{4\left(1-\left(\frac{1}{10}\right)^n\right)}{81}$

(ii) $\frac{10(10^n-1)}{27}-\frac{n}{3}$ 7. 3069 8. ₹ 174760 9. $\frac{41}{333}$

Exercise 2.9

1.(i) 1830 (ii) 1584 (iii) 3003 (iv) 1240 (v) 3256

(vi) 42075 (vii) 1296 2. 105625 3. 210

4. 15 5. 9 6. 4615 cm^2 7.(i) $4n^3+3n^2$ (ii) 2240

Exercise 2.10

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(3)	(1)	(2)	(3)	(4)	(1)	(4)	(3)	(1)	(3)	(3)	(4)	(2)	(2)	(3)

Unit exercise

2.(i) 35 litres (ii) 5 (iii) 3 3. 1 6. -78

8. ₹1200 9. $\sqrt{2},\ \sqrt{6},\ 3\sqrt{2},\ \ldots$ 10. ₹27636

Exercise 3.1

1.(i) 2, -1, 4 (ii) $\frac{1}{2},\frac{1}{3},\frac{1}{4}$ (iii) 35, 30, 25

2.(i) infinitely many solution (ii) no solution (iii) unique solution

3. 24 years, 51 years, 84 years 4. 137 5. 7, 3, 2

Exercise 3.2

1.(i) x^2+2x-3 (ii) x^2+1 (iii) $x(x^2+4x+4)$ (iv) $3(x^2+1)$

2.(i) $8x^3y^2$ (ii) $-36a^3b^2c$ (iii) $-48m^2n^2$ (iv) $(p-1)(p-2)(p+2)$

(v) $4(x+3)(2x+1)\ (x-3)$ (vi) $2^3x^2(2x-3y)^3(4x^2+6xy+9y^2)$

Exercise 3.3

1.(i) $105x^2y^2$, $7xy$ (ii) $(x-1)(x+1)(x^2+x+1)(x^2-x+1)$, $(x+1)$

(iii) $xy(x+y), x(x+y)$ 2.(i) $(a+6)\ (a-2)\ (a-3)$ (ii) $x(x-3a)^2(x^2+3ax+9a^2)$

3.(i) $4x^2(x-1)$ (ii) x^2-xy+y^2 4. (i) $(a+2)(a-7)$ (ii) x^2+xy+y^2

Exercise 3.4

1.(i) $\frac{x-1}{x}$ (ii) $\frac{x-9}{x-2}$ (iii) $\frac{9}{x-1}$ (iv) $\frac{p+5}{2p(p-4)}$

2.(i) -5, **5** **(ii)** **2, 3** **(iii)** **1** **(iv)** **0,** -3, **2**

Exercise 3.5

1.(i) $\frac{3x^3z}{5y^3}$ (ii) $p+4$ (iii) $\frac{3t^2}{4}$ 2.(i) $\frac{3x-4y}{2x-5}$ (ii) $\frac{x^2+xy+y^2}{3(x+2y)}$

3.(i) -5 (ii) $\frac{b-4}{b+2}$ (iii) $\frac{3y}{x-3}$ (iv) $\frac{4(2t-1)}{3}$ 4. $\frac{4}{9}$ 5. x^2+4x+4

Exercise 3.6

1.(i) $\frac{2x}{x-2}$ (ii) $\frac{2x^2+2x-7}{(x+3)(x-2)}$ (iii) x^2+xy+y^2 2.(i) $\frac{2(x-2)}{x-4}$ (ii) $\frac{1-x}{1+x}$

3. $\frac{2x^3+1}{(x^2+2)^2}$ 4. $\frac{3}{x^2-2x+4}$ 5. $\frac{(4x^2-1)}{2(4x^2+1)}$ 7. 2 hrs 24 minutes 8. 30 kg, 20 kg

Exercise 3.7

1.(i) $2\left|\frac{y^4z^6}{x^2}\right|$ (ii) $4\left|\frac{\sqrt{7}x+\sqrt{2}}{4x-1}\right|$ (iii) $\frac{11}{9}\left|\frac{(a+b)^4(x+y)^4}{(a-b)^6}\right|$ 2.(i) $|2x+5|$

(ii) $\left|1+\frac{1}{x^3}\right|$ (iii) $|3x-4y+5z|$ (iv) $|(x-2)(7x+1)(4x-1)|$ (v) $\frac{1}{6}|(4x+3)(3x+2)(x+2)|$

Exercise 3.8

1.(i) $|x^2-6x+3|$ **(ii)** $|2x^2-7x-3|$ **(iii)** $|4x^2+1|$ **(iv)** $|11x^2-9x-12|$

2. $\left|\frac{x}{y}-5+\frac{y}{x}\right|$ 3.(i) 49, -42 (ii) 144, 264 4.(i) -12, 4 (ii) 24, -32

Exercise 3.9

1.(i) $x^2+9x+20=0$ (ii) $3x^2-5x+12=0$ (iii) $2x^2+3x-2=0$

(iv) $x^2+(2-a^2)x+(a+5)^2=0$ 2.(i) -3, -28 (ii) -3, 0 (iii) $-\frac{1}{3}, -\frac{10}{3}$ (iv) $\frac{1}{3}, \frac{-4}{3}$

Exercise 3.10

1.(i) $-\frac{1}{4}, 2$ (ii) $\frac{-6}{7}$ (iii) $-2, 9$ (iv) $-\sqrt{2}, \frac{-5}{\sqrt{2}}$ (v) $\frac{1}{4}, \frac{1}{4}$ 2. 6

Exercise 3.11

1.(i) $\frac{2}{3}, \frac{2}{3}$ (ii) $-1, 3$ 2.(i) 2, $\frac{1}{2}$ (ii) $\frac{3+\sqrt{3}}{\sqrt{2}}, \frac{3-\sqrt{3}}{\sqrt{2}}$

(iii) $-1, \frac{23}{3}$ (iv) $\frac{a+b}{6}, \frac{a-b}{6}$ 3. 3.75 seconds

Exercise 3.12

1. **$5, -\frac{1}{5}$** **2.** **1.5 m** **3.** **45 km/hr** **4.** **20 years, 10 years**

5. Yes, 12 m, 16 m 6. 72 7. 28 m, 42 m 8. 2 m 9. 40, 60

Exercise 3.13

1.(i) Real and unequal (ii) Real and unequal (iii) Not real

(iv) Real and equal (v) Real and equal 2.(i) 2, 3 (ii) 1, $\frac{1}{9}$

Exercise 3.14

1.(i) $\frac{(\alpha+\beta)^2-2\alpha\beta}{3\alpha\beta}$ (ii) $\frac{\alpha+\beta}{(\alpha\beta)^2}$ (iii) $9\alpha\beta-3(\alpha+\beta)+1$

(iv) $\frac{(\alpha+\beta)^2-2\alpha\beta+3(\alpha+\beta)}{\alpha\beta}$ 2.(i) $\frac{7}{5}$ (ii) $\frac{29}{20}$ (iii) $\frac{-63}{8}$ (iv) $\frac{101}{54}$

3.(i) $x^2-44x+16=0$ (ii) $x^2-3x-1=0$ (iii) $x^2-24x-64=0$

4. $-15, 15$ 5. $-24, 24$ 6. -36

Exercise 3.15

1.(i) Real and unequal roots (ii) Real and equal roots (iii) No real roots
(iv) Real and unequal roots (v) Real and equal roots (vi) Real and unequal roots
2. –3, 4 3. No real roots 4. -1 5. $-4, 1$ 6. $-2, 7$
7. –1, 3 8. –2, 3

Exercise 3.16

1.(i) 16 (ii) 4×4 (iii) $\sqrt{7}, \frac{\sqrt{3}}{2}, 5, 0, -11, 1$

2. $1\times18, 2\times9, 3\times6, 6\times3, 9\times2, 18\times1$ and $1\times6, 2\times3, 3\times2, 6\times1$

3.(i) $\begin{pmatrix}1&3&5\\0&2&4\\1&1&6\end{pmatrix}$ (ii) $\begin{pmatrix}\frac{8}{3}&9&\frac{64}{3}\\9&\frac{64}{3}&\frac{125}{3}\\\frac{64}{3}&\frac{125}{3}&72\end{pmatrix}$ 4. $\begin{pmatrix}5&1&3\\4&-7&8\\3&9&2\end{pmatrix}$

5. $\begin{pmatrix}-\sqrt{7}&\sqrt{5}&-\sqrt{3}\\3&-2&5\end{pmatrix}$ 7.(i) 3,12,3 (ii) 4,2,0 or 2,4,0 (iii) 2,4,3

Exercise 3.17

3. $\begin{pmatrix}5&0\\\frac{3}{2}&\frac{9}{2}\end{pmatrix}, \begin{pmatrix}2&0\\\frac{3}{2}&\frac{1}{2}\end{pmatrix}$ 4.(i) $\begin{pmatrix}7&-17&-37\\-39&-11&-26\end{pmatrix}$ (ii) $\begin{pmatrix}-63&-15&-45\\15&-27&-60\end{pmatrix}$

5.(i) $4, -10, 12$ (ii) $-10, 14, 10$ 6. 4,6 7. 4 8. $-1, 5$ and $-2, 4$

Exercise 3.18

1. $P\times R$, not defined 2. 7,10 3. $3\times3, 4\times2, 4\times2, 4\times1, 1\times3$

4. $\begin{pmatrix}12&19\\10&3\end{pmatrix}, \begin{pmatrix}-10&-4\\24&23\end{pmatrix}$, $AB\neq BA$

Exercise 3.19

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
(4)	(1)	(2)	(1)	(2)	(3)	(4)	(2)	(3)	(3)	(2)	(1)	(2)	(3)	(2)	(2)	(4)	(2)	(2)	(1)

Unit exercise-3

1. 6,2,1 2. 42,78,30 3. 153 4. $(ky+x)(k^2x^2-y^2)$ 5. x^2+2x+1 6. (i) x^a-2

(ii) $-x+\frac{5}{2}$ 7. $\frac{1}{2qr}$ 8. 11 hrs, 22 hrs, 33 hrs 9. $|17x^2-18x+19|$ 10. 3,63

11. 14 km/hr 12. 120 m,40 m 13. 14 minutes 14. 25 15.(i) $x^2-6x+11=0$

(ii) $3x^2-2x+1=0$ 16. $3, \frac{9}{4}$ 17.(i) $\begin{pmatrix}750&1500&2250\\3750&4250&750\end{pmatrix}$ (ii) $\begin{pmatrix}8000&1600&24000\\40000&24000&8000\end{pmatrix}$

18. $\sin\theta$ 19. 8, 4 20. $\begin{pmatrix} 122 & 71 \\ -58 & -34 \end{pmatrix}$

Exercise 4.1

1.(i) Not similar (ii) Similar,2.5 2. 328.5 m 3. 42 m

5. $\frac{15}{13}, \frac{36}{13}$ 6. 5.6 cm, 3.25 cm 7. 2.8 cm 9. 2 m

Exercise 4.2

1.(i) 6.43 cm (ii) 1 2. 60 cm 5. 4 cm, 4 cm

8. 2.5 cm, 3.5 cm 9.(i) Not a bisector (ii) Bisector 13. 2.1 cm

Exercise 4.3

1. 30 m 2. 1 mile 3. 21.74 m 4. 12 cm, 5 cm

5. 10 m, 24 m, 26 m 6. 0.8 m

Exercise 4.4

1. 7 cm 2. 2 cm 3. 7 cm, 5 cm, 3 cm 4. 30° 5. 130°

6. $\frac{20}{3}$ cm 7. 10 cm 8. 4.8 cm 10. 2 cm 11. 2 cm 14.8.7 cm 16.4 cm

Exercise 4.5

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(3)	(2)	(4)	(1)	(4)	(1)	(2)	(3)	(1)	(4)	(2)	(2)	(2)	(4)	(1)

Unit exercise

2. $\frac{12}{5}$ cm, $\frac{10}{3}$ cm 5. $20\sqrt{13}$ km 7. 10 m 8. shadow$=\frac{4}{11}\times$(distance) 10.14 units

Exercise 5.1

1.(i) 24 sq. units (ii) 11.5 sq. units 2.(i) collinear (ii) collinear

3.(i) 44 (ii) 13 4.(i) 0 (ii) $\frac{1}{2}$ or -1 5.(i) 35 sq. units (ii) 34 sq. units

6. -5 7. $2, -1$ 8.24 sq. units, $\text{area}(\Delta ABC) = 4 \times \text{area}(\Delta PQR)$ 9. 38sq.units

10. 10 cans 11.(i) 3.75 sq. units (ii) 3 sq. units (iii) 13.88 sq. units

Exercise 5.2

1.(i) undefined (ii) 0 2.(i) 0° (ii) 45° 3.(i) $\frac{1}{\sqrt{5}}$ (ii) $-\cot\theta$

4. 3 6. 7 7. $\frac{17}{2}$ 8. 4 9.(i) yes (ii) yes 11. 5, 214. $\left(1, \frac{-3}{2}\right)$ or $\left(3, \frac{-1}{2}\right)$

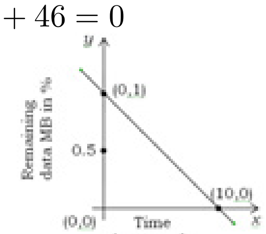
Exercise 5.3

1.(i) $2y+3=0$ (ii) $2x-5=0$ 2. $1, 45°, \frac{5}{2}$

3. $x-\sqrt{3}y-3\sqrt{3}=0$ 4. $\frac{\sqrt{3}+3}{2}, \frac{3+3\sqrt{3}}{-2}$ 5. -5 6. $x-y-16=0$

7.(i) $16x-15y-22=0$ (ii) $4x-9y+19=0$ 8. $15x-11y+46=0$

9. $x+4y-14=0$ 10. $5x+4y-3=0$ 11. (i)



(ii) 1 (iii) 7.5 seconds (iv) 10 seconds

12.(i) $3x-2y-12=0$ (ii) $3x-20y+15=0$ 13.(i) 2, -3

(ii) $-3, -4$ 14.(i) $5x+2y+3=0$ (ii) $x+y+4=0$

Exercise 5.4

1.(i) 0 (ii) undefined 2.(i) 0.7 (ii) undefined

3.(i) Parallel (ii) Perpendicular 4. 4 5. $3x + 4y + 7 = 0$

6. $2x + 5y - 2 = 0$ 7. $2x + 5y + 6 = 0$, $5x + y - 48 = 0$ 8. $5x - 3y - 8 = 0$

9. $13x + 5y - 18 = 0$ 10. $49x + 28y - 156 = 0$ 11. $31x + 15y + 30 = 0$

12. $4x + 13y - 9 = 0$

Exercise 5.5

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(2)	(1)	(2)	(3)	(3)	(4)	(2)	(2)	(1)	(3)	(3)	(1)	(2)	(1)	(2)

Unit exercise

1. Rhombus 2. $\left(\frac{7}{2}, \frac{13}{2}\right)$ 3. 0 sq.units 4. -5 6. $2x - 3y - 6 = 0$, $3x - 2y + 6 = 0$

7. 1340 litres 8. $(-1, -4)$ 9. $13x + 13y - 6 = 0$ 10. $119x + 102y - 125 = 0$

Exercise 6.2

1. 30° 2. 24 m 3. 3.66 m 4. 1.5 m 5.(i) 7 m (ii) 16.39 m

6. 10 m 7. $100\sqrt{5}$ m 8. 0.14 mile (approx)

Exercise 6.3

1. 150 m 2. 50 m 3. 32.93 m 4. 2078.4 m 6. 0.5 m / s

Exercise 6.4

1. 35.52 m 2. 69.28 m, 160 m 4. 150 m, yes

5.(i) 264 m (ii) 198 m (iii) 114.31 m 6.(i) 2.91 km (ii) 6.93 km

Exercise 6.5

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(2)	(4)	(2)	(1)	(2)	(2)	(1)	(3)	(2)	(4)	(2)	(2)	(4)	(2)	(1)

Unit exercise

5. 29.28 m/s 6. 1.97 seconds (approx) 7.(i) 24.58 km(approx)

(ii) 17.21 km (approx) (iii) 21.41 km (approx) (iv) 23.78 km (approx)

8. 200 m 9. 39.19 m

Exercise 7.1

1. 25 cm, 35 cm 2. 7 m, 35 m 3. 2992 sq.cm

4. CSA of the cone when rotated about PQ is larger. 5. 18.25 cm

6. 28 caps 7. $\sqrt{5} : 9$ 8. 56.25% 9. ₹ 302.72 10. ₹ 1357.72

Exercise 7.2

1. 4.67 m 2. 1 cm 3. 652190 cm^3 4. 63 minutes (approx)

5. 100.58 6. 5:7 7. 64:343 9. 4186.29 cm^3 10. ₹ 418.36

Exercise 7.3

1. 1642.67 cm^3 2. 66 cm^3 3. 2.46 cm^3 4. 905.14 cm^3

5. 56.51 cm^3 6. 332.5 cm^2 7.(i) $4\pi r^2$ sq. units

(ii) $4\pi r^2$ sq. units (iii) 1:1 8. 73.39 cm^2

Exercise 7.4

1. 36 cm 2. 2 hrs 3. $\frac{h}{3x^2}$ 4. 6 cm
5. 281200 cm^3 6. 1.33 cm 7. 1 cm 8. 100%

Exercise 7.5

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(4)	(1)	(1)	(2)	(3)	(2)	(2)	(3)	(3)	(1)	(4)	(1)	(1)	(2)	(4)

Unit exercise

1. 48000 words 2. 27 minutes (approx) 3. $\frac{1}{3}\pi r^3$ cu.units 4. 782.57 sq.cm
5. 450 coins 6. 4.8 cm 7. ₹ 6800 8. 2 cm 9. 17 cm 10. 2794.18 cm^3

Exercise 8.1

1.(i) 62; 0.33 (ii) 47.8; 0.64 2. 50.2 3. 250 4. 2.34
5. 222.22, 14.91 6. 6.9 7. 6.05 8. 4.5 9. 1.44, 1.2 10. 7.76 11. 14.6
12. 6 13. 1.24 14. 60.5, 14.61 15. 6 and 8

Exercise 8.2

1. 52% 2. 4.69 3. 7.2 4. 180.28% 5. 14.4%
6. 10.07% 7. Vidhya 8. Science, Social 9. City A

Exercise 8.3

1. $\{HHH, HHT, HTH, THH, THT, TTH, TTT\}$

2. $\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4)(2,4),(2,5),(2,6),$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,5),(4,6),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

3. (i) $\frac{15}{32}$ 4. Krishna 5. $\frac{3}{8}$ 6. (i) $\frac{9}{1000}$ (ii) $\frac{8}{999}$ 7. (i) $\frac{1}{4}$ (ii) $x = 4$
8.(i) $\frac{1}{6}$ (ii) $\frac{1}{6}$ (iii) $\frac{7}{36}$ 9. (i) $\frac{1}{8}$ (ii) $\frac{7}{8}$ (iii) $\frac{1}{2}$ (iv) $\frac{7}{8}$
10. $\frac{2}{36}, \frac{4}{36}, \frac{6}{36}, \frac{6}{36}, \frac{6}{36}, \frac{6}{36}, \frac{4}{36}, \frac{2}{36}$ 11.(i) $\frac{3}{13}$ (ii) $\frac{1}{2}$ (iii) $\frac{10}{13}$ (iv) $\frac{6}{13}$
12. 12 13.(i) $\frac{13}{46}$ (ii) 0 (iii) $\frac{1}{46}$ 14. $\frac{157}{600}$ 15. (i) $\frac{1}{6}$ (ii) $\frac{5}{6}$ (iii) $\frac{5}{36}$

Exercise 8.4

1. $\frac{11}{15}$ 2. (i) 0.58 (ii) 0.52 (iii) 0.74 3. 0.1 4. 1.2 5. 0.2
6. $\frac{5}{9}$ 7. $\frac{1}{13}$ 8. $\frac{13}{18}$ 9. $\frac{73}{280}$ 10. $\frac{17}{40}$ 11. 1 12. $\frac{11}{48}, \frac{11}{24}, \frac{11}{16}$

Exercise 8.5

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(3)	(1)	(3)	(2)	(3)	(4)	(2)	(1)	(1)	(2)	(2)	(2)	(3)	(3)	(4)

Unit exercise

1. 8,12 2. 5.55 3. 7 4. 81 5. 5.17, 1.53 6. City A 7. 60, 40
8. $\frac{1}{9}$ 9. $\frac{3}{4}$ 10. 10 11. $\frac{13}{20}$ 12. (i) $\frac{13}{20}$ (ii) $\frac{3}{49}$ (iii) $\frac{1}{49}$

MATHEMATICAL TERMS

Algorithm	படிமுறை
Alternate segment	ஒன்றுவிட்ட துண்டு
Altitude	குத்துயரம்
Angle bisector	கோண இருசம வெட்டி
Angle of depression	இறக்கக் கோணம்
Angle of elevation	ஏற்றக் கோணம்
Arithmetic progression	கூட்டுத்தொடர் வரிசை
Arrow diagram	அம்புக்குறி படம்
Axis	அச்சு
Axis of symmetry	சமச்சீர் அச்சு
Basic proportionality	அடிப்படை விகித சமம்
Bijection	இருபுறச் சார்பு
Cartesian product	கார்டீசியன் பெருக்கல்
Circular motion	வட்ட இயக்கம்
Clinometer	சாய்வுமாணி
Co-domain	துணை மதிப்பகம்
Coefficient of range	வீச்சுக் கெழு
Coefficient of variation	மாறுபாட்டுக் கெழு
Collinearity	நேர்க் கோட்டமைவு
Column matrix	நிரல் அணி
Combined solids	இணைந்த திண்மங்கள்
Common difference	பொது வித்தியாசம்
Common ratio	பொது விகிதம்
Completing square method	வர்க்கப் பூர்த்தி முறை
Composition of functions	சார்புகளின் இணக்கம்
Concurrency theorem	ஒருங்கிசைவுத் தேற்றம்
Concurrent	ஒருங்கிசையும்
Concyclic	ஒரேபிரதியிலுள்ள
Congruence	ஒருங்கிசைவு
Consistent	ஒருங்கமைவுடைய
Constant function	மாறிலிச் சார்பு
Coordinate axes	ஆயக்கூறு அச்சு
Counter-clock wise	வலமிருந்து இடம்
Curved surface area	வளைபரப்பு
Decompose	பிரித்தல்
Diagonal matrix	மூலைவிட்ட அணி
Dimensions	பரிமாணங்கள்
Discriminant	தன்மைக் காட்டி
Distributive property	பங்கீட்டுப் பண்பு
Domain	மதிப்பகம்
Equal matrices	சம அணிகள்
Equiangular	சமகோணம்
Event	நிகழ்ச்சி
Frustum	இடைக் கண்டம்
Functions	சார்புகள்
Geometric progression	பெருக்குத்தொடர் வரிசை
Geo-positioning system	புவி நிலைப்படுத்தல் அமைப்பு
Graphical form	வரைபடமுறை
Great circle	மீப்பெரு வட்டம்
Height and distance	உயரங்களும் தூரங்களும்
Hemisphere	அரைக் கோளம்
Hollow	உள்ளீடற்ற
Horizontal level	கிடைமட்ட வரிசை
Horizontal line test	கிடைமட்டக் கோட்டுச் சோதனை
Hyperbola	அதிபரவளையம்
Identity function	சமனிச் சார்பு
Image	நிழல் உரு
Inclination	சாய்வுக் கோணம்
Inconsistent	ஒருங்கமைவற்ற
Injection	ஒருபுறச் சார்பு
Intercept	வெட்டுத்துண்டு
Into function	உள்நோக்கிய சார்பு
Kadhams (unit of distance)	காதம்கள் (தூரத்தின் அலகு)
Latitude	அட்சரேகை
Lemma	துணைத் தேற்றம்
Line of sight	பார்வைக் கோடு
Linear equations	நேரிய சமன்பாடுகள்
Linear function	நேரிய சார்பு

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Longitude	தீர்க்கரேகை
Magnitude	அளவு
Many-one function	பலவற்றிற்கொன்றான சார்பு
Matrix	அணிகள்
Measures of central tendency	மைய நிலைப் போக்கு அளவைகள்
Measures of dispersion	சிதறல் அளவைகள்
Median	நடுக்கோடு
Modular	மட்டு
Mutually exclusive events	ஒன்றையொன்று விலக்கும் நிகழ்ச்சிகள்
Negative of a matrix	எதிர் அணி
Non-vertical lines	நேர்க் குத்தற்ற கோடுகள்
Non-zero integer	பூச்சியமற்ற முழு
Non-zero real number	பூச்சியமற்ற மெய் எண்
Null matrix / Zero matrix	பூச்சிய அணி
Null relation	சுழி தொடர்பு
Oblique cylinder	சாய்ந்த உருளை
Oblique frustum	சாய்ந்த இடைக் கண்டம்
One-one function	ஒன்றுக்கொன்றான சார்பு
Onto function	மேல் சார்பு
Ordered pair	வரிசைச் சோடிகள்
Outcomes	விளைவுகள்
Parabola	பரவளையம்
Parallel planes	இணை தளங்கள்
Perpendicular bisector	செங்குத்து சமவெட்டி
Point of contact	தொடுபுள்ளி
Point of intersection	வெட்டுப்புள்ளி
Pre-image	முன் உரு
Quadratic equation	இருபடிச் சமன்பாடுகள்
Quadratic function	இருபடிச் சார்பு
Quadratic polynomials	இருபடி பல்லுறுப்புக் கோவைகள்
Random experiment	சமவாய்ப்புச் சோதனை
Range	வீச்சகம் (அ) வீச்சு
Rational expression	விகிதமுறு கோவை
Real valued function	மெய்மதிப்புச் சார்பு
Reciprocal function	தலைகீழ்ச் சார்பு

Relations	தொடர்புகள்/ உறவுகள்
Revolutions	சுழற்சி
Right circular cone	நேர்வட்டக் கூம்பு
Right circular cylinder	நேர்வட்ட உருளை
Row matrix	நிரை அணி
Sample point	கூறுபுள்ளி
Sample space	கூறுவெளி
Scalar matrix	திசையிலி அணி
Secant	வெட்டுக்கோடு
Sequence	தொடர்வரிசை
Series	தொடர்
Similar triangle	வடிவொத்த முக்கோணம்
Simultaneous linear equations	ஒத்த நேரிய சமன்பாடுகள்
Slant height	சாயுயரம்
Slope or gradient	சாய்வு
Solid	திண்மம்
Square matrix	சதுர அணி
Standard deviation	திட்ட விலக்கம்
Surface area	புறப்பரப்பு
Table form	அட்டவணை முறை
Tangents	தொடுகோடுகள்
Theodolite	தளமட்டக் கோணமாணி
Tossed	சுண்டப்படுதல்
Total surface area	மொத்தப் பரப்பு
Transpose matrix	நிரை நிரல் மாற்று அணி
Trial	முயற்சி
Triangular matrix	முக்கோண அணி
Unbiased coins	சீரான நாணயங்கள்
Undefined	வரையறுக்கப்படாதது
Unique solution	ஒரேயொரு தீர்வு
Uniqueness	தனித் தன்மை
Unit matrix / Identity matrix	அலகு அணி
Variance	விலக்க வர்க்க சராசரி
Vertical angle	உச்சிக் கோணம்
Vertical line test	குத்துக்கோட்டுச் சோதனை
Zeros of polynomials	பல்லுறுப்புக் கோவையின் பூச்சியங்கள்

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This book has been printed on 80 GSM
Elegant Maplitho paper
Printed by offset at :