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STANDARD NINE

MATHEMATICS

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Department Of School Education

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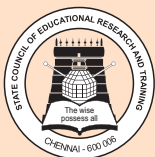
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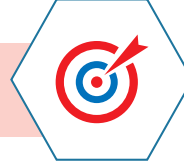
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Numbers and letters, they are known as
eyes to humans, they are. Kural 392

Learning Outcomes

To transform the classroom processes into
learning centric with a set of bench marks



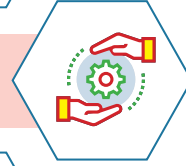
Note

To provide additional inputs for students in the
content



Activity / Project

To encourage students to involve in activities to
learn mathematics



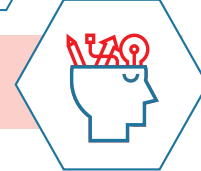
ICT Corner

To encourage learner's understanding of content through
application of technology



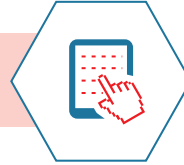
Thinking Corner

To kindle the inquisitiveness of students in learning mathematics. To
make the students to have a diverse thinking



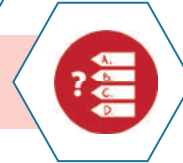
Points to Remember

To recall the points learnt in the topic



Multiple Choice Questions

To provide additional assessment items on the content



Progress Check

Self evaluation of the learner's progress



Exercise

To evaluate the learners' in
understanding the content



*“The essence of mathematics is not to make simple things complicated
but to make complicated things simple” -S. Gudder*



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- Once the camera detects the QR code, a url appears in the screen. Click the url and go to the content page.



SYMBOLS

$=$	equal to	\sim	similarly
\neq	not equal to	Δ	symmetric difference
$<$	less than	\mathbb{N}	natural numbers
\leq	less than or equal to	\mathbb{W}	whole numbers
$>$	greater than	\mathbb{Z}	integers
\geq	greater than or equal to	\mathbb{R}	real numbers
\approx	equivalent to	\triangle	triangle
\cup	union	\angle	angle
\cap	intersection	\perp	perpendicular to
\mathbb{U}	universal Set	\parallel	parallel to
\in	belongs to	\Rightarrow	implies
\notin	does not belong to	\therefore	therefore
\subset	proper subset of	\because	since (or) because
\subseteq	subset of or is contained in	$ $	absolute value
$\not\subset$	not a proper subset of	\simeq	approximately equal to
$\not\subseteq$	not a subset of or is not contained in	$ (\text{or}) $	such that
$A' \text{ (or) } A^c$	complement of A	$\equiv \text{ (or) } \cong$	congruent
$\emptyset \text{ (or) } \{ \}$	empty set or null set or void set	\equiv	identically equal to
$n(A)$	number of elements in the set A	π	pi
$P(A)$	power set of A	\pm	plus or minus
Σ	summation	$P(E)$	probability of an event E



E-book



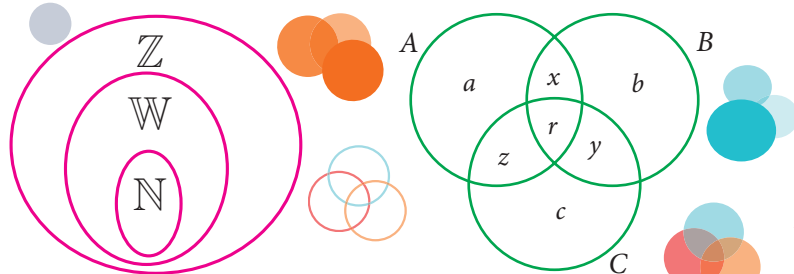
Evaluation



DIGI Links



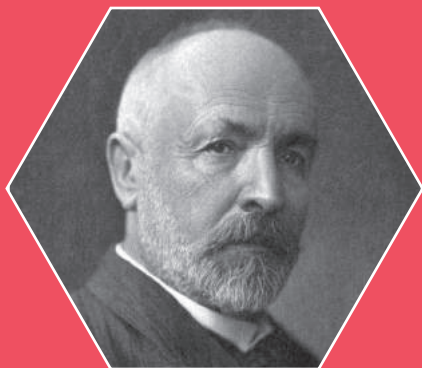
1



SET LANGUAGE

A set is a many that allows itself to thought of as a one

-Georg Cantor



Georg Cantor
(AD (CE) 1845 - 1918)

The theory of sets was developed by German mathematician Georg Cantor. Today it is used in almost every branch of Mathematics. In Mathematics, sets are convenient because all mathematical structures can be regarded as sets.



Learning Outcomes



- To describe and represent a set in different forms.
- To identify different types of sets.
- To understand and perform set operations and apply this in Venn diagram.
- To know the commutative, associative and distributive properties among sets.
- To understand and verify De Morgan's laws.
- To use set language in solving life oriented word problems.

1.1 Introduction

In our daily life, we often deal with collection of objects like books, stamps, coins, etc. Set language is a mathematical way of representing a collection of objects.

Let us look at the following pictures. What do they represent?

Here, Fig.1.1 represents a collection of fruits and Fig. 1.2 represents a collection of house- hold items.

We observe in these cases, our attention turns from one individual object to a collection of objects based on their characteristics. Any such collection is called a set.

1.2 Set

A set is a well-defined collection of objects.



Fig. 1.1



Fig. 1.2

Here “well-defined collection of objects” means that given a specific object it must be possible for us to decide whether the object is an element of the given collection or not.

The objects of a set are called its members or elements.

For example,

1. The collection of all books in a District Central Library.
2. The collection of all colours in a rainbow.
3. The collection of prime numbers.

We see that in the adjacent box, statements (1), (2), and (4) are well defined and therefore they are sets. Whereas (3) and (5) are not well defined because the words good and beautiful are difficult to agree on. I might consider a student to be good and you may not. I might consider the Jasmine is the beautiful flower but you may not. So we will consider only those collections to be sets where there is no such ambiguity.

Which of the following are sets ?

1. Collection of Natural numbers.
2. Collection of English alphabets.
3. Collection of good students in a class.
4. Collection of States in our country.
5. Collection of beautiful flowers in a garden.

Therefore (3) and (5) are not sets.



Activity - 1

Discuss and give as many examples of collections from your daily life situations, which are sets and which are not sets.

Note



- Elements of a set are listed only once.
- The order of listing the elements of the set does not change the set.

For example, the collection 1,2,3,4,5,6,7,8, ... as well as the collection 1, 3, 2, 4, 5, 7, 6, 8, ... are the same though listed in different order. Since it is necessary to know whether an object is an element in the set or not, we do not want to list that element many times.



Notation

A set is usually denoted by capital letters of the English Alphabets A, B, P, Q, X, Y , etc.

The elements of a set is written within curly brackets “{ }”

If x is an element of a set A or x belongs to A , we write $x \in A$.

If x is not an element of a set A or x does not belongs to A , we write $x \notin A$.

For example,

Consider the set $A = \{2, 3, 5, 7\}$ then

2 is an element of A ; we write $2 \in A$

5 is an element of A ; we write $5 \in A$

6 is not an element of A ; we write $6 \notin A$

Example 1.1

Consider the set $A = \{\text{Ashwin, Murali Vijay, Vijay Shankar, Badrinath}\}$.

Fill in the blanks with the appropriate symbol \in or \notin .

(i) Murali Vijay _____ A . (ii) Ashwin _____ A . (iii) Badrinath _____ A .

(iv) Ganguly _____ A . (v) Tendulkar _____ A

Solution

(i) Murali Vijay $\in A$. (ii) Ashwin $\in A$ (iii) Badrinath $\in A$

(iv) Ganguly $\notin A$. (v) Tendulkar $\notin A$.

1.3 Representation of a Set

The collection of odd numbers can be described in many ways:

(1) “The set of odd numbers” is a fine description, we understand it well.

(2) It can be written as $\{1, 3, 5, \dots\}$

(3) Also, it can be said as the collection of all numbers x where x is an odd number.

All of them are equivalent and useful. For instance, the two descriptions “The collection of all solutions to the equation $x-5 = 3$ ” and $\{8\}$ refer to the same set.

A set can be represented in any one of the following three ways or forms:

1.3.1 Descriptive Form

In descriptive form, a set is described in words.

For example,

(i) The set of all vowels in English alphabets.

(ii) The set of whole numbers.

1.3.2 Set Builder Form or Rule Form

In set builder form, all the elements are described by a rule.

For example,

- (i) $A = \{x : x \text{ is a vowel in English alphabets}\}$
- (ii) $B = \{x | x \text{ is a whole number}\}$

Note



The symbol ':' or '|' stands for "such that".

1.3.3 Roster Form or Tabular Form

A set can be described by listing all the elements of the set.

For example,

- (i) $A = \{a, e, i, o, u\}$
- (ii) $B = \{0, 1, 2, 3, \dots\}$

Note



Three dots (...) in the example (ii) is called ellipsis. It indicates that the pattern of the listed elements continues in the same manner.

Can this form of representation be possible always?



Activity-2

Write the following sets in respective forms.

S.No.	Descriptive Form	Set Builder Form	Roster Form
1	The set of all natural numbers less than 10		
2		$\{x : x \text{ is a multiple of 3, } x \in \mathbb{N}\}$	
3			$\{2, 4, 6, 8, 10\}$
4	The set of all days in a week.		
5			$\{\dots -3, -2, -1, 0, 1, 2, 3 \dots\}$

Example 1.2

Write the set of letters of the following words in Roster form

- (i) ASSESSMENT
- (ii) PRINCIPAL

Solution

- (i) ASSESSMENT

- (ii) PRINCIPAL

$$X = \{A, S, E, M, N, T\}$$

$$Y = \{P, R, I, N, C, A, L\}$$



Exercise 1.1

1. Which of the following are sets?
 - (i) The collection of prime numbers upto 100.
 - (ii) The collection of rich people in India.
 - (iii) The collection of all rivers in India.
 - (iv) The collection of good Hockey players.
2. List the set of letters of the following words in Roster form.
 - (i) INDIA
 - (ii) PARALLELOGRAM
 - (iii) MISSISSIPPI
 - (iv) CZECHOSLOVAKIA
3. Consider the following sets $A = \{0, 3, 5, 8\}$, $B = \{2, 4, 6, 10\}$ and $C = \{12, 14, 18, 20\}$.
 - (a) State whether True or False:
 - (i) $18 \in C$
 - (ii) $6 \notin A$
 - (iii) $14 \notin C$
 - (iv) $10 \in B$
 - (v) $5 \in B$
 - (vi) $0 \in B$
 - (b) Fill in the blanks:
 - (i) $3 \in \underline{\hspace{1cm}}$
 - (ii) $14 \in \underline{\hspace{1cm}}$
 - (iii) $18 \underline{\hspace{1cm}} B$
 - (iv) $4 \underline{\hspace{1cm}} B$
4. Represent the following sets in Roster form.
 - (i) $A =$ The set of all even natural numbers less than 20.
 - (ii) $B = \{y : y = \frac{1}{2n}, n \in \mathbb{N}, n \leq 5\}$
 - (iii) $C = \{x : x \text{ is perfect cube, } 27 < x < 216\}$
 - (iv) $D = \{x : x \in \mathbb{Z}, -5 < x \leq 2\}$
5. Represent the following sets in set builder form.
 - (i) $B =$ The set of all Cricket players in India who scored double centuries in One Day Internationals.
 - (ii) $C = \{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots\}$
 - (iii) $D =$ The set of all tamil months in a year.
 - (iv) $E =$ The set of odd Whole numbers less than 9.

6. Represent the following sets in descriptive form.

(i) $P = \{\text{January, June, July}\}$

(ii) $Q = \{7, 11, 13, 17, 19, 23, 29\}$

(iii) $R = \{x : x \in \mathbb{N}, x < 5\}$

(iv) $S = \{x : x \text{ is a consonant in English alphabets}\}$

1.4 Types of Sets

There is a very special set of great interest: the empty collection ! Why should one care about the empty collection? Consider the set of solutions to the equation $x^2 + 1 = 0$. It has no elements at all in the set of Real Numbers. Also consider all rectangles with one angle greater than 90 degrees. There is no such rectangle and hence this describes an empty set.

So, the empty set is important, interesting and deserves a special symbol too.

1.4.1 Empty Set or Null Set

A set consisting of no element is called the *empty set* or *null set* or *void set*.

It is denoted by \emptyset or $\{ \}$.

For example,

(i) $A = \{x : x \text{ is an odd integer and divisible by } 2\}$

$\therefore A = \{ \}$ or \emptyset

(ii) The set of all integers between 1 and 2.

1.4.2. Singleton Set

A set which has only one element is called a *singleton set*.

For example,

(i) $A = \{x : 3 < x < 5, x \in \mathbb{N}\}$

(ii) The set of all even prime numbers.

1.4.3 Finite Set

A set with finite number of elements is called a *finite set*.

Thinking Corner

Are the sets $\{0\}$ and $\{\emptyset\}$ empty sets?

Note

An empty set has no elements, so \emptyset is a finite set.

For example,

1. The set of family members.
2. The set of indoor/outdoor games you play.
3. The set of curricular subjects you learn in school.
4. $A = \{x : x \text{ is a factor of } 36\}$

1.4.4 Infinite Set

A set which is not finite is called an *infinite set*.

For example,

- (i) $\{5, 10, 15, \dots\}$ (ii) The set of all points on a line.

To discuss further about the types of sets, we need to know the cardinality of sets.

Cardinal number of a set : When a set is finite, it is very useful to know how many elements it has. The number of elements in a set is called the Cardinal number of the set.

The cardinal number of a set A is denoted by $n(A)$

Example 1.3

If $A = \{1, 2, 3, 4, 5, 7, 9, 11\}$, find $n(A)$.

Solution

$$A = \{1, 2, 3, 4, 5, 7, 9, 11\}$$

Since set A contains 8 elements, $n(A) = 8$.

Thinking Corner

Is the set of natural numbers a finite set?

Thinking Corner

If $A = \{1, b, b, \{4, 2\}, \{x, y, z\}, d, \{d\}\}$, then $n(A)$ is _____

1.4.5 Equivalent Sets

Two finite sets A and B are said to be equivalent if they contain the same number of elements. It is written as $A \approx B$.

If A and B are equivalent sets, then $n(A) = n(B)$

For example,

Consider $A = \{\text{ball, bat}\}$ and

$B = \{\text{history, geography}\}$.

Here A is equivalent to B because $n(A) = n(B) = 2$.

Thinking Corner

Let $A = \{x : x \text{ is a colour in national flag of India}\}$ and $B = \{\text{Red, Blue, Green}\}$. Are these two sets equivalent?

Example 1.4

Are $P = \{x : -3 \leq x \leq 0, x \in \mathbb{Z}\}$ and $Q = \text{The set of all prime factors of } 210$, equivalent sets?

Solution

$P = \{-3, -2, -1, 0\}$, The prime factors of 210 are 2, 3, 5, and 7 and so, $Q = \{2, 3, 5, 7\}$

$n(P) = 4$ and $n(Q) = 4$. Therefore P and Q are equivalent sets.

1.4.6 Equal Sets

Two sets are said to be **equal** if they contain exactly the same elements, otherwise they are said to be unequal.

In other words, two sets A and B are said to be equal, if

- (i) every element of A is also an element of B
- (ii) every element of B is also an element of A

For example,

Consider the sets $A = \{1, 2, 3, 4\}$ and $B = \{4, 2, 3, 1\}$

Since A and B contain exactly the same elements, A and B are equal sets.

Note



- If A and B are equal sets, we write $A = B$.
- If A and B are unequal sets, we write $A \neq B$.

A set does not change, if one or more elements of the set are repeated.

For example, if we are given

$A = \{a, b, c\}$ and $B = \{a, a, b, b, b, c\}$ then, we write $B = \{a, b, c\}$. Since, every element of A is also an element of B and every element of B is also an element of A , the sets A and B are equal.

Example 1.5

Are $A = \{x : x \in \mathbb{N}, 4 \leq x \leq 8\}$ and
 $B = \{4, 5, 6, 7, 8\}$ equal sets?

Solution

$A = \{4, 5, 6, 7, 8\}$, $B = \{4, 5, 6, 7, 8\}$
 A and B are equal sets.

Note



Equal sets are equivalent sets but equivalent sets need not be equal sets. **For example,** if $A = \{p, q, r, s, t\}$ and $B = \{4, 5, 6, 7, 8\}$. Here $n(A) = n(B)$, so A and B are equivalent but not equal.

1.4.7 Universal Set

A Universal set is a set which contains all the elements of all the sets under consideration and is usually denoted by U .

For example,

- (i) If we discuss about elements in Natural numbers, then the universal set U is the set of all Natural numbers. $U = \{x : x \in \mathbb{N}\}$.
- (ii) If $A = \{\text{earth, mars, jupiter}\}$, then the universal set U is the planets of solar system.

1.4.8 Subset

Let A and B be two sets. If every element of A is also an element of B , then A is called a subset of B . We write $A \subseteq B$.

$A \subseteq B$ is read as “ A is a subset of B ”

Thus $A \subseteq B$, if $a \in A$ implies $a \in B$.

If A is not a subset of B , we write $A \not\subseteq B$

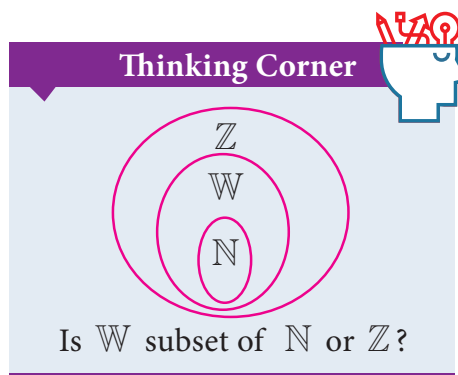
Clearly, if A is a subset of B , then $n(A) \leq n(B)$.

Since every element of A is also an element of B , the set B must have at least as many elements as A , thus $n(A) \leq n(B)$.

The other way is also true. Suppose that $n(A) > n(B)$, then A has more elements than B , and hence there is at least one element in A that cannot be in B , so A is not a subset of B .

For example,

$$(i) \{1\} \subseteq \{1, 2, 3\} \quad (ii) \{2, 4\} \not\subseteq \{1, 2, 3\}$$



Example 1.6

Insert the appropriate symbol \subseteq or $\not\subseteq$ in each blank to make a true statement. (i) $\{10, 20, 30\}$ ____ $\{10, 20, 30, 40\}$ (ii) $\{p, q, r\}$ ____ $\{w, x, y, z\}$

Solution

$$(i) \{10, 20, 30\} \subseteq \{10, 20, 30, 40\}$$

Since every element of $\{10, 20, 30\}$ is also an element of $\{10, 20, 30, 40\}$, we get $\{10, 20, 30\} \subseteq \{10, 20, 30, 40\}$.

$$(ii) \{p, q, r\} \not\subseteq \{w, x, y, z\}$$

Since the element p belongs to $\{p, q, r\}$ but does not belong to $\{w, x, y, z\}$, shows that $\{p, q, r\} \not\subseteq \{w, x, y, z\}$.



Activity-3

Discuss with your friends and give examples of subsets of sets from your daily life situation.

Example 1.7

Write all the subsets of $A = \{a, b\}$.

Solution

$$A = \{a, b\}$$

Subsets of A are $\emptyset, \{a\}, \{b\}$ and $\{a, b\}$.

Note



- If $A \subseteq B$ and $B \subseteq A$, then $A = B$.
In fact this is how we defined equality of sets.
- Empty set is a subset of every set.
This is not easy to see ! Let A be any set. The only way for the empty set to be not a subset of A would be to have an element x in it but with x not in A . But how can x be in the empty set ? That is impossible. So this only way being impossible, the empty set must be a subset of A . (Is your head spinning? Think calmly, explain it to a friend, and you will agree it is alright !)
- Every set is a subset of itself. (Try and argue why?)

1.4.9. Proper Subset

Let A and B be two sets. If A is a subset of B and $A \neq B$, then A is called a proper subset of B and we write $A \subset B$.

For example,

If $A = \{1, 2, 5\}$ and $B = \{1, 2, 3, 4, 5\}$ then A is a proper subset of B ie. $A \subset B$.

1.4.10 Disjoint Sets

Two sets A and B are said to be disjoint if they do not have common elements.

In other words, if $A \cap B = \emptyset$, then A and B are said to be disjoint sets.

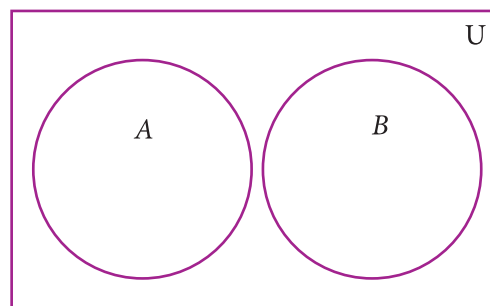


Fig. 1.3

Example 1.8

Verify whether $A = \{20, 22, 23, 24\}$ and $B = \{25, 30, 40, 45\}$ are disjoint sets.

Solution

$$A = \{20, 22, 23, 24\}, B = \{25, 30, 40, 45\}$$

$$\begin{aligned} A \cap B &= \{20, 22, 23, 24\} \cap \{25, 30, 40, 45\} \\ &= \{ \} \end{aligned}$$

Since $A \cap B = \emptyset$, A and B are disjoint sets.

Note



If $A \cap B \neq \emptyset$, then A and B are said to be overlapping sets. Thus if two sets have at least one common element, they are called overlapping sets.

1.4.11 Power Set

The set of all subsets of a set A is called the **power set of 'A'**. It is denoted by $P(A)$.

For example,

- (i) If $A = \{2, 3\}$, then find the power set of A .

The subsets of A are $\emptyset, \{2\}, \{3\}, \{2, 3\}$.

The power set of A ,

$$P(A) = \{\emptyset, \{2\}, \{3\}, \{2, 3\}\}$$

- (ii) If $A = \{\emptyset, \{\emptyset\}\}$, then the power set of A is $\{\emptyset, \{\emptyset, \{\emptyset\}\}, \{\emptyset\}, \{\{\emptyset\}\}\}$.

An important property.

We already noted that $n(A) \leq n[P(A)]$. But how big is $P(A)$? Think about this a bit, and see whether you come to the following conclusion:

- (i) If $n(A) = m$, then $n[P(A)] = 2^m$
(ii) The number of proper subsets of a set A is $n[P(A)] - 1 = 2^m - 1$.

Example 1.9

Find the number of subsets and the number of proper subsets of a set $X = \{a, b, c, x, y, z\}$.

Solution Given $X = \{a, b, c, x, y, z\}$. Then, $n(X) = 6$

$$\text{The number of subsets} = n[P(X)] = 2^6 = 64$$

$$\begin{aligned} \text{The number of proper subsets} &= n[P(X)] - 1 = 2^6 - 1 \\ &= 64 - 1 = 63 \end{aligned}$$

Thinking Corner

Every set has only one improper subset. Verify this fact using any set.



Exercise 1.2

- Find the cardinal number of the following sets.
 - $M = \{p, q, r, s, t, u\}$
 - $P = \{x : x = 3n + 2, n \in \mathbb{W} \text{ and } x < 15\}$
 - $Q = \{y : y = \frac{4}{3n}, n \in \mathbb{N} \text{ and } 2 < n \leq 5\}$
 - $R = \{x : x \text{ is an integer, } x \in \mathbb{Z} \text{ and } -5 \leq x < 5\}$
 - $S = \text{The set of all leap years between 1882 and 1906.}$
- Identify the following sets as finite or infinite.
 - $X = \text{The set of all districts in Tamilnadu.}$
 - $Y = \text{The set of all straight lines passing through a point.}$
 - $A = \{x : x \in \mathbb{Z} \text{ and } x < 5\}$
 - $B = \{x : x^2 - 5x + 6 = 0, x \in \mathbb{N}\}$



3. Which of the following sets are equivalent or unequal or equal sets?
- (i) A = The set of vowels in the English alphabets.
 B = The set of all letters in the word "VOWEL"
- (ii) $C = \{2, 3, 4, 5\}$ $D = \{x : x \in \mathbb{W}, 1 < x < 5\}$
- (iii) $X = \{x : x \text{ is a letter in the word "LIFE"}\}$ $Y = \{F, I, L, E\}$
- (iv) $G = \{x : x \text{ is a prime number and } 3 < x < 23\}$ $H = \{x : x \text{ is a divisor of } 18\}$
4. Identify the following sets as null set or singleton set.
- (i) $A = \{x : x \in \mathbb{N}, 1 < x < 2\}$
- (ii) B = The set of all even natural numbers which are not divisible by 2
- (iii) $C = \{0\}$.
- (iv) D = The set of all triangles having four sides.
5. State which pairs of sets are disjoint or overlapping?
- (i) $A = \{f, i, a, s\}$ and $B = \{a, n, f, h, s\}$
- (ii) $C = \{x : x \text{ is a prime number, } x > 2\}$ and $D = \{x : x \text{ is an even prime number}\}$
- (iii) $E = \{x : x \text{ is a factor of } 24\}$ and $F = \{x : x \text{ is a multiple of } 3, x < 30\}$
6. If $S = \{\text{square, rectangle, circle, rhombus, triangle}\}$, list the elements of the following subset of S .
- (i) The set of shapes which have 4 equal sides.
- (ii) The set of shapes which have radius.
- (iii) The set of shapes in which the sum of all interior angles is 180° .
- (iv) The set of shapes which have 5 sides.
7. If $A = \{a, \{a, b\}\}$, write all the subsets of A .
8. Write down the power set of the following sets:
- (i) $A = \{a, b\}$ (ii) $B = \{1, 2, 3\}$ (iii) $D = \{p, q, r, s\}$ (iv) $E = \emptyset$
9. Find the number of subsets and the number of proper subsets of the following sets.
- (i) $W = \{\text{red, blue, yellow}\}$ (ii) $X = \{x^2 : x \in \mathbb{N}, x^2 \leq 100\}$.
10. (i) If $n(A) = 4$, find $n[P(A)]$. (ii) If $n(A) = 0$, find $n[P(A)]$.
(iii) If $n[P(A)] = 256$, find $n(A)$.

1.5 Set Operations

We started with numbers and very soon we learned arithmetical operations on them. In algebra we learnt expressions and soon started adding and multiplying them as well, writing $(x^2+2)(x-3)$ etc. Now that we know sets, the natural question is, what can we do with sets, what are natural operations on them?



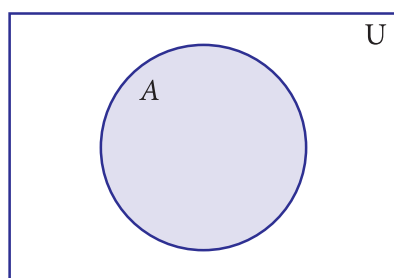
When two or more sets combine together to form one set under the given conditions, then operations on sets can be carried out. We can visualize the relationship between sets and set operations using Venn diagram.

John Venn was an English mathematician. He invented Venn diagrams which pictorially represent the relations between sets. Venn diagrams are used in the field of Set Theory, Probability, Statistics, Logic and Computer Science.

1.5.1 Complement of a Set

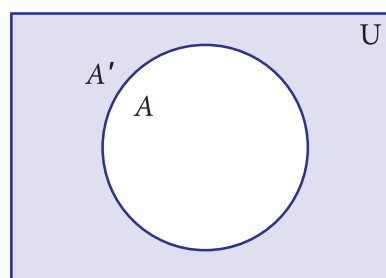
The Complement of a set A is the set of all elements of U (the universal set) that are not in A . It is denoted by A' or A^c . In symbols $A' = \{x : x \in U, x \notin A\}$

Venn diagram for complement of a set



A (shaded region)

Fig. 1.4



A' (shaded region)

Fig. 1.5

For example,

If $U = \{\text{all boys in a class}\}$ and $A = \{\text{boys who play Cricket}\}$, then complement of the set A is $A' = \{\text{boys who do not play Cricket}\}$.

Example 1.10

If $U = \{c, d, e, f, g, h, i, j\}$ and $A = \{c, d, g, j\}$, find A' .

Solution

$U = \{c, d, e, f, g, h, i, j\}$, $A = \{c, d, g, j\}$

$A' = \{e, f, h, i\}$

Note

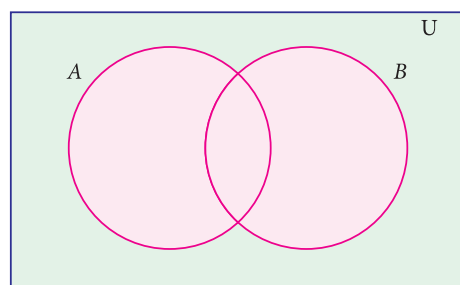
- $(A')' = A$
- $U' = \emptyset$
- $\emptyset' = U$

1.5.2 Union of Two Sets

The union of two sets A and B is the set of all elements which are either in A or in B or in both. It is denoted by $A \cup B$ and read as A union B .

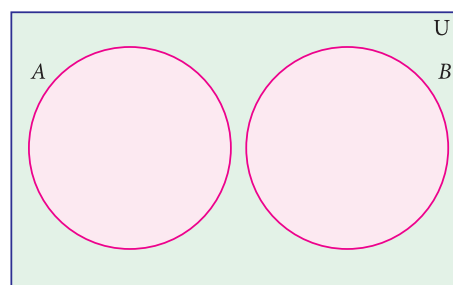
In symbol, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

The union of two sets can be represented by Venn diagram as given below



Sets A and B have common elements

Fig. 1.6



Sets A and B are disjoint

Fig. 1.7

For example,

If $P = \{\text{Asia, Africa, Antarctica, Australia}\}$ and $Q = \{\text{Europe, North America, South America}\}$, then the union set of P and Q is $P \cup Q = \{\text{Asia, Africa, Antarctica, Australia, Europe, North America, South America}\}$.

Note

- $A \cup A = A$
- $A \cup \emptyset = A$
- $A \cup U = U$ where A is any subset of universal set U
- $A \subseteq A \cup B$ and $B \subseteq A \cup B$
- $A \cup B = B \cup A$ (union of two sets is commutative)

Example 1.11

If $P = \{m, n\}$ and $Q = \{m, i, j\}$, then, represent P and Q in Venn diagram and hence find $P \cup Q$.

Solution

Given $P = \{m, n\}$ and $Q = \{m, i, j\}$

From the venn diagram, $P \cup Q = \{n, m, i, j\}$.

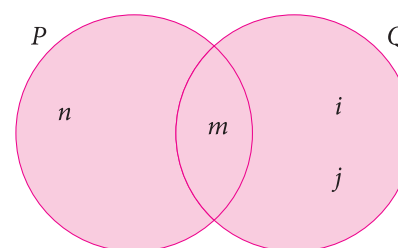


Fig. 1.8

1.5.3 Intersection of Two Sets

The intersection of two sets A and B is the set of all elements common to both A and B. It is denoted by $A \cap B$ and read as A intersection B.

In symbol, $A \cap B = \{x : x \in A \text{ and } x \in B\}$

Intersection of two sets can be represented by a Venn diagram as given below

For example,

If $A = \{1, 2, 6\}$; $B = \{2, 3, 4\}$, then $A \cap B = \{2\}$ because 2 is common element of the sets A and B.

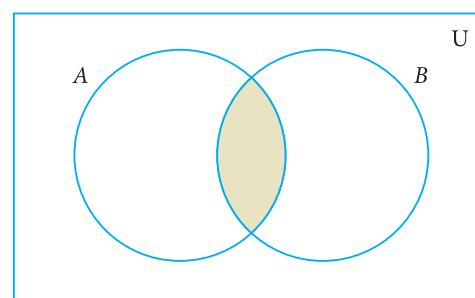


Fig. 1.9

Example 1.12

Let $A = \{x : x \text{ is an even natural number and } 1 < x \leq 12\}$ and $B = \{x : x \text{ is a multiple of 3, } x \in \mathbb{N} \text{ and } x \leq 12\}$ be two sets. Find $A \cap B$.

Solution

Here $A = \{2, 4, 6, 8, 10, 12\}$ and $B = \{3, 6, 9, 12\}$

$$A \cap B = \{6, 12\}$$

Example 1.13

If $A = \{2, 3\}$ and $C = \{\}$, find $A \cap C$.

Solution

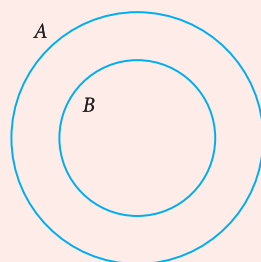
There is no common element and hence $A \cap C = \{\}$

Note

- $A \cap A = A$ (ii) $A \cap \emptyset = \emptyset$
- $A \cap U = A$ where A is any subset of universal set U
- $A \cap B \subseteq A$ and $A \cap B \subseteq B$
- $A \cap B = B \cap A$ (Intersection of two sets is commutative)

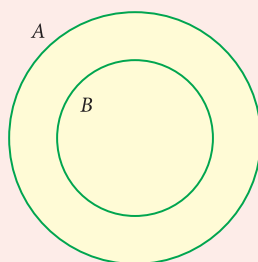
Note

- When $B \subset A$, the union and intersection of A and B are represented in Venn diagram as follows



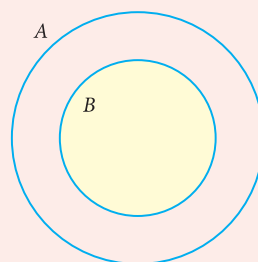
$$B \subset A$$

Fig. 1.10



$$\text{shaded region is } A \cup B = A$$

Fig. 1.11



$$\text{shaded region is } A \cap B = B$$

Fig. 1.12

- If A and B are any two non empty sets such that $A \cup B = A \cap B$, then $A = B$
- Let $n(A) = p$ and $n(B) = q$ then
 - (a) Minimum of $n(A \cup B) = \max\{p, q\}$
 - (b) Maximum of $n(A \cup B) = p + q$
 - (c) Minimum of $n(A \cap B) = 0$
 - (d) Maximum of $n(A \cap B) = \min\{p, q\}$

1.5.4 Difference of Two Sets

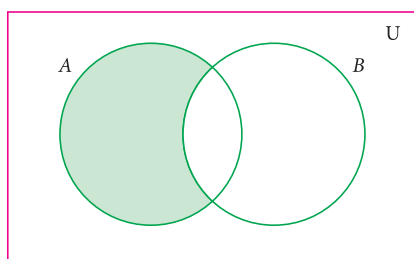
Let A and B be two sets, the difference of sets A and B is the set of all elements which are in A , but not in B . It is denoted by $A-B$ or $A \setminus B$ and read as A difference B .

In symbol, $A-B = \{x : x \in A \text{ and } x \notin B\}$

$B-A = \{y : y \in B \text{ and } y \notin A\}$.

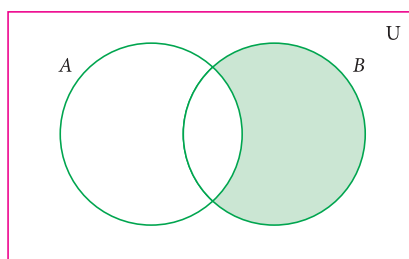


Venn diagram for set difference



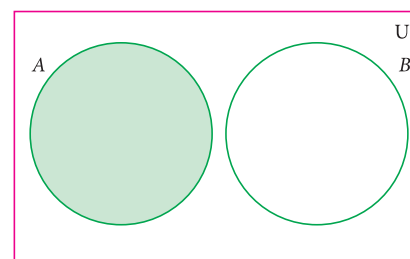
$A-B$

Fig. 1.13



$B-A$

Fig. 1.14



$A-B$

Fig. 1.15

Example 1.14

If $A = \{-3, -2, 1, 4\}$ and $B = \{0, 1, 2, 4\}$, find (i) $A-B$ (ii) $B-A$.

Solution

$$A-B = \{-3, -2, 1, 4\} - \{0, 1, 2, 4\} = \{-3, -2\}$$

$$B-A = \{0, 1, 2, 4\} - \{-3, -2, 1, 4\} = \{0, 2\}$$

1.5.5 Symmetric Difference of Sets

The symmetric difference of two sets A and B is the set $(A-B) \cup (B-A)$. It is denoted by $A \Delta B$.

$$A \Delta B = \{x : x \in A-B \text{ or } x \in B-A\}$$

Example 1.15

If $A = \{6, 7, 8, 9\}$ and $B = \{8, 10, 12\}$, find $A \Delta B$.

Solution

$$A-B = \{6, 7, 9\}$$

$$B-A = \{10, 12\}$$

$$A \Delta B = (A-B) \cup (B-A) = \{6, 7, 9\} \cup \{10, 12\}$$

$$A \Delta B = \{6, 7, 9, 10, 12\}.$$

Note

- $A' = U - A$
- $A-B = A \cap B'$
- $A-A = \emptyset$
- $A-\emptyset = A$
- $A-B = B-A \Leftrightarrow A=B$
- $A-B = A$ if $A \cap B = \emptyset$

Thinking Corner

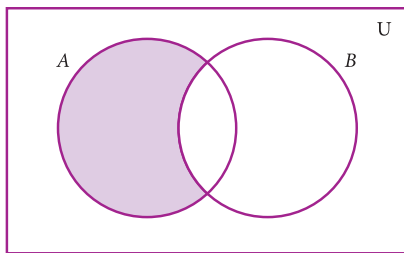
What is $(A-B) \cap (B-A)$?

Example 1.16

Represent $A \Delta B$ through Venn diagram.

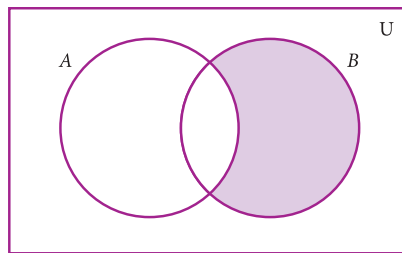
Solution

$$A \Delta B = (A - B) \cup (B - A)$$



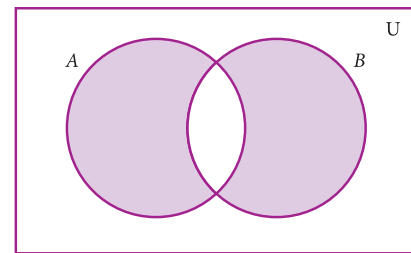
$A - B$

Fig. 1.16



$B - A$

Fig. 1.17



$(A - B) \cup (B - A)$

Fig. 1.18

Note

- $A \Delta A = \emptyset$
- $A \Delta B = B \Delta A$
- $A \Delta B = \{x : x \in A \cup B \text{ and } x \notin A \cap B\}$
- $A \Delta B = (A \cup B) - (A \cap B)$

Example 1.17

From the given Venn diagram, write the elements of

- (i) A (ii) B (iii) $A - B$ (iv) $B - A$
 (v) A' (vi) B' (vii) U

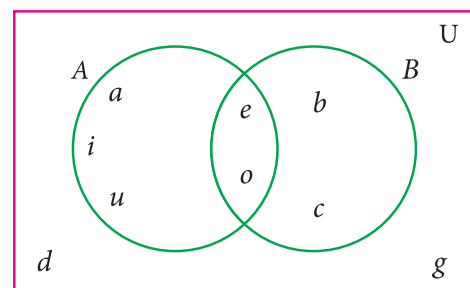


Fig. 1.19

Solution

- (i) $A = \{a, e, i, o, u\}$ (ii) $B = \{b, c, e, o\}$
 (iii) $A - B = \{a, i, u\}$ (iv) $B - A = \{b, c\}$
 (v) $A' = \{b, c, d, g\}$ (vi) $B' = \{a, d, g, i, u\}$
 (vii) $U = \{a, b, c, d, e, g, i, o, u\}$

Example 1.18

Draw Venn diagram and shade the region representing the following sets

- (i) A' (ii) $(A - B)'$ (iii) $(A \cup B)'$

Solution

(i) A'

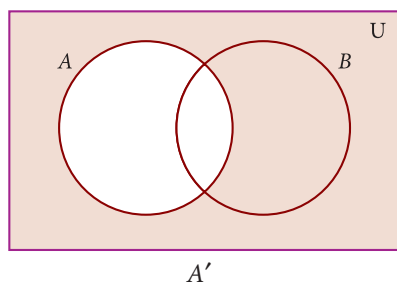


Fig. 1.20

(ii) $(A-B)'$

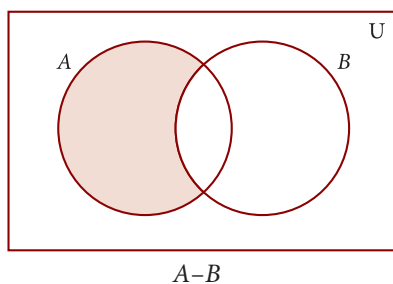


Fig. 1.21

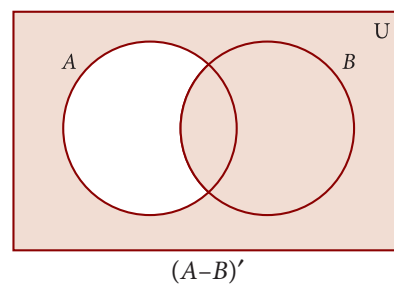


Fig. 1.22

(iii) $(A \cup B)'$

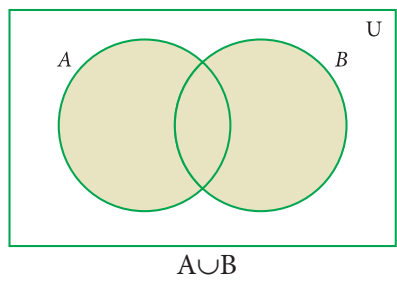


Fig. 1.23

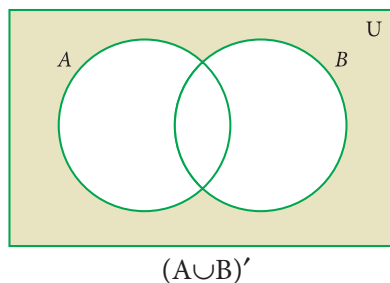


Fig. 1.24



Exercise 1.3

- Using the given Venn diagram, write the elements of
 - A
 - B
 - $A \cup B$
 - $A \cap B$
 - $A - B$
 - $B - A$
 - A'
 - B'
 - U

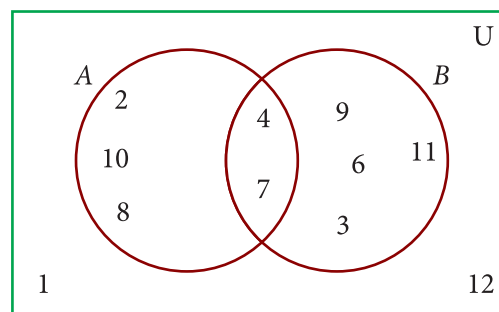


Fig. 1.25

- Find $A \cup B$, $A \cap B$, $A - B$ and $B - A$ for the following sets.
 - $A = \{2, 6, 10, 14\}$ and $B = \{2, 5, 14, 16\}$
 - $A = \{a, b, c, e, u\}$ and $B = \{a, e, i, o, u\}$
 - $A = \{x : x \in N, x \leq 10\}$ and $B = \{x : x \in W, x < 6\}$
 - $A =$ Set of all letters in the word “mathematics” and
 $B =$ Set of all letters in the word “geometry”



3. If $U=\{a, b, c, d, e, f, g, h\}$, $A=\{b, d, f, h\}$ and $B=\{a, d, e, h\}$, find the following sets.
- (i) A' (ii) B' (iii) $A' \cup B'$ (iv) $A' \cap B'$ (v) $(A \cup B)'$
- (vi) $(A \cap B)'$ (vii) $(A')'$ (viii) $(B')'$
4. Let $U=\{0, 1, 2, 3, 4, 5, 6, 7\}$, $A=\{1, 3, 5, 7\}$ and $B=\{0, 2, 3, 5, 7\}$, find the following sets.
- (i) A' (ii) B' (iii) $A' \cup B'$ (iv) $A' \cap B'$ (v) $(A \cup B)'$
- (vi) $(A \cap B)'$ (vii) $(A')'$ (viii) $(B')'$
5. Find the symmetric difference between the following sets.
- (i) $P = \{2, 3, 5, 7, 11\}$ and $Q = \{1, 3, 5, 11\}$
- (ii) $R = \{l, m, n, o, p\}$ and $S = \{j, l, n, q\}$
- (iii) $X = \{5, 6, 7\}$ and $Y = \{5, 7, 9, 10\}$
6. Using the set symbols, write down the expressions for the shaded region in the following

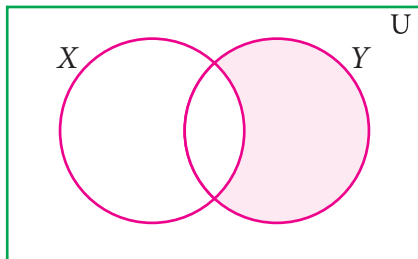


Fig. 1.26

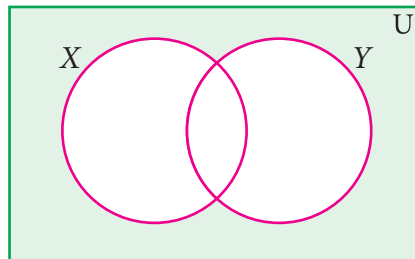


Fig. 1.27

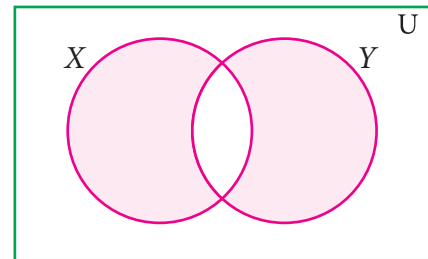


Fig. 1.28

7. Let A and B be two overlapping sets and the universal set be U . Draw appropriate Venn diagram for each of the following,
- (i) $A \cup B$ (ii) $A \cap B$ (iii) $(A \cap B)'$ (iv) $(B - A)'$ (v) $A' \cup B'$ (vi) $A' \cap B'$
- (vii) What do you observe from the Venn diagram (iii) and (v)?

1.6 Properties of Set Operations

It is an interesting investigation to find out if operations among sets (like union, intersection, etc) follow mathematical properties such as Commutativity, Associativity, etc., We have seen numbers having many of these properties; whether sets also possess these, is to be explored.

We first take up the properties of set operations on union and intersection.

1.6.1 Commutative Property

In set language, commutative situations can be seen when we perform operations. **For example**, we can look into the Union (and Intersection) of sets to find out if the operation is commutative.

Let $A = \{2, 3, 8, 10\}$ and $B = \{1, 3, 10, 13\}$ be two sets.

Then, $A \cup B = \{1, 2, 3, 8, 10, 13\}$ and

$$B \cup A = \{1, 2, 3, 8, 10, 13\}$$

From the above, we see that $A \cup B = B \cup A$.

This is called **Commutative property of union of sets**.

Now, $A \cap B = \{3, 10\}$ and $B \cap A = \{3, 10\}$. Then, we see that $A \cap B = B \cap A$.

This is called **Commutative property of intersection of sets**.

Commutative property: For any two sets A and B

$$(i) A \cup B = B \cup A \quad (ii) A \cap B = B \cap A$$

Example 1.19

If $A = \{b, e, f, g\}$ and $B = \{c, e, g, h\}$, then verify the commutative property of (i) union of sets (ii) intersection of sets.

Solution

Given, $A = \{b, e, f, g\}$ and $B = \{c, e, g, h\}$

$$(i) A \cup B = \{b, c, e, f, g, h\} \quad \dots (1)$$

$$B \cup A = \{b, c, e, f, g, h\} \quad \dots (2)$$

From (1) and (2) we have $A \cup B = B \cup A$

It is verified that union of sets is commutative.

$$(ii) A \cap B = \{e, g\} \quad \dots (3)$$

$$B \cap A = \{e, g\} \quad \dots (4)$$

From (3) and (4) we get, $A \cap B = B \cap A$

It is verified that intersection of sets is commutative.

Note

For any set A ,

• $A \cup A = A$ and $A \cap A = A$
[Idempotent Laws].

• $A \cup \phi = A$ and $A \cap U = A$
[Identity Laws].

Thinking Corner

Given, $P = \{l, n, p\}$ and
 $Q = \{j, l, m, n, o, p\}$. If P and Q
are disjoint sets, then what will be
 Q and $P \cap Q$?

Note



Recall that subtraction on numbers is not commutative. Is set difference commutative? We expect that the set difference is not commutative as well. For instance, consider $A = \{a, b, c\}$, $B = \{b, c, d\}$. $A - B = \{a\}$,

$$B - A = \{d\}; \text{ we see that } A - B \neq B - A.$$

1.6.2 Associative Property

Now, we perform operations on union and intersection for three sets.

Let $A = \{-1, 0, 1, 2\}$, $B = \{-3, 0, 2, 3\}$ and $C = \{0, 1, 3, 4\}$ be three sets.

$$\text{Now, } B \cup C = \{-3, 0, 1, 2, 3, 4\}$$

$$\begin{aligned} A \cup (B \cup C) &= \{-1, 0, 1, 2\} \cup \{-3, 0, 1, 2, 3, 4\} \\ &= \{-3, -1, 0, 1, 2, 3, 4\} \quad \dots (1) \end{aligned}$$

$$\text{Then, } A \cup B = \{-3, -1, 0, 1, 2, 3\}$$

$$\begin{aligned} (A \cup B) \cup C &= \{-3, -1, 0, 1, 2, 3\} \cup \{0, 1, 3, 4\} \\ &= \{-3, -1, 0, 1, 2, 3, 4\} \quad \dots (2) \end{aligned}$$

From (1) and (2), $A \cup (B \cup C) = (A \cup B) \cup C$.

This is associative property of union among sets A , B , and C .

$$\text{Now, } B \cap C = \{0, 3\}$$

$$\begin{aligned} A \cap (B \cap C) &= \{-1, 0, 1, 2\} \cap \{0, 3\} \\ &= \{0\} \quad \dots (3) \end{aligned}$$

$$\text{Then, } A \cap B = \{0, 2\}$$

$$\begin{aligned} (A \cap B) \cap C &= \{0, 2\} \cap \{0, 1, 3, 4\} \\ &= \{0\} \quad \dots (4) \end{aligned}$$

From (3) and (4), $A \cap (B \cap C) = (A \cap B) \cap C$.

This is associative property of intersection among sets A , B and C .

Associative property: For any three sets A , B and C

$$(i) \quad A \cup (B \cap C) = (A \cup B) \cap C \quad (ii) \quad A \cap (B \cup C) = (A \cap B) \cup C$$

Example 1.20

$$\text{If } A = \left\{-\frac{1}{2}, 0, \frac{1}{4}, \frac{3}{4}, 2\right\}, B = \left\{0, \frac{1}{4}, \frac{3}{4}, 2, \frac{5}{2}\right\} \text{ and } C = \left\{-\frac{1}{2}, \frac{1}{4}, 1, 2, \frac{5}{2}\right\},$$

then verify $A \cap (B \cap C) = (A \cap B) \cap C$.

Solution

$$\text{Now, } (B \cap C) = \left\{ \frac{1}{4}, 2, \frac{5}{2} \right\}$$

$$A \cap (B \cap C) = \left\{ \frac{1}{4}, 2 \right\} \quad \dots (1)$$

$$\text{Then, } A \cap B = \left\{ 0, \frac{1}{4}, \frac{3}{4}, 2 \right\}$$

$$(A \cap B) \cap C = \left\{ \frac{1}{4}, 2 \right\} \quad \dots (2)$$

From (1) and (2), it is verified that

$$(A \cap B) \cap C = A \cap (B \cap C)$$

Note



The set difference in general is not associative

that is, $(A - B) - C \neq A - (B - C)$.

But, if the sets A , B and C are mutually disjoint then the set difference is associative

that is, $(A - B) - C = A - (B - C)$.



Exercise 1.4

1. If $P = \{1, 2, 5, 7, 9\}$, $Q = \{2, 3, 5, 9, 11\}$, $R = \{3, 4, 5, 7, 9\}$ and $S = \{2, 3, 4, 5, 8\}$, then find

(i) $(P \cup Q) \cup R$ (ii) $(P \cap Q) \cap S$ (iii) $(Q \cap S) \cap R$

2. Test for the commutative property of union and intersection of the sets

$P = \{x : x \text{ is a real number between 2 and 7}\}$ and

$Q = \{x : x \text{ is an irrational number between 2 and 7}\}.$

3. If $A = \{p, q, r, s\}$, $B = \{m, n, q, s, t\}$ and $C = \{m, n, p, q, s\}$, then verify the associative property of union of sets.

4. Verify the associative property of intersection of sets for $A = \{-11, \sqrt{2}, \sqrt{5}, 7\}$,
 $B = \{\sqrt{3}, \sqrt{5}, 6, 13\}$ and $C = \{\sqrt{2}, \sqrt{3}, \sqrt{5}, 9\}.$

5. If $A = \{x : x = 2^n, n \in W \text{ and } n < 4\}$, $B = \{x : x = 2n, n \in \mathbb{N} \text{ and } n \leq 4\}$ and

$C = \{0, 1, 2, 5, 6\}$, then verify the associative property of intersection of sets.

1.6.3 Distributive Property

In lower classes, we have studied distributive property of multiplication over addition on numbers. That is, $a \times (b + c) = (a \times b) + (a \times c)$. In the same way we can define distributive properties on sets.

Distributive property: For any three sets A , B and C

- (i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [Intersection over union]
- (ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ [Union over intersection]

Example 1.21

If $A = \{0, 2, 4, 6, 8\}$, $B = \{x : x \text{ is a prime number and } x < 11\}$ and $C = \{x : x \in N \text{ and } 5 \leq x < 9\}$ then verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Solution

Given $A = \{0, 2, 4, 6, 8\}$, $B = \{2, 3, 5, 7\}$ and $C = \{5, 6, 7, 8\}$

First, we find $B \cap C = \{5, 7\}$, $A \cup (B \cap C) = \{0, 2, 4, 5, 6, 7, 8\}$... (1)

Next, $A \cup B = \{0, 2, 3, 4, 5, 6, 7, 8\}$, $A \cup C = \{0, 2, 4, 5, 6, 7, 8\}$

Then, $(A \cup B) \cap (A \cup C) = \{0, 2, 4, 5, 6, 7, 8\}$... (2)

From (1) and (2), it is verified that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Example 1.22

Verify $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ using Venn diagrams.

Solution

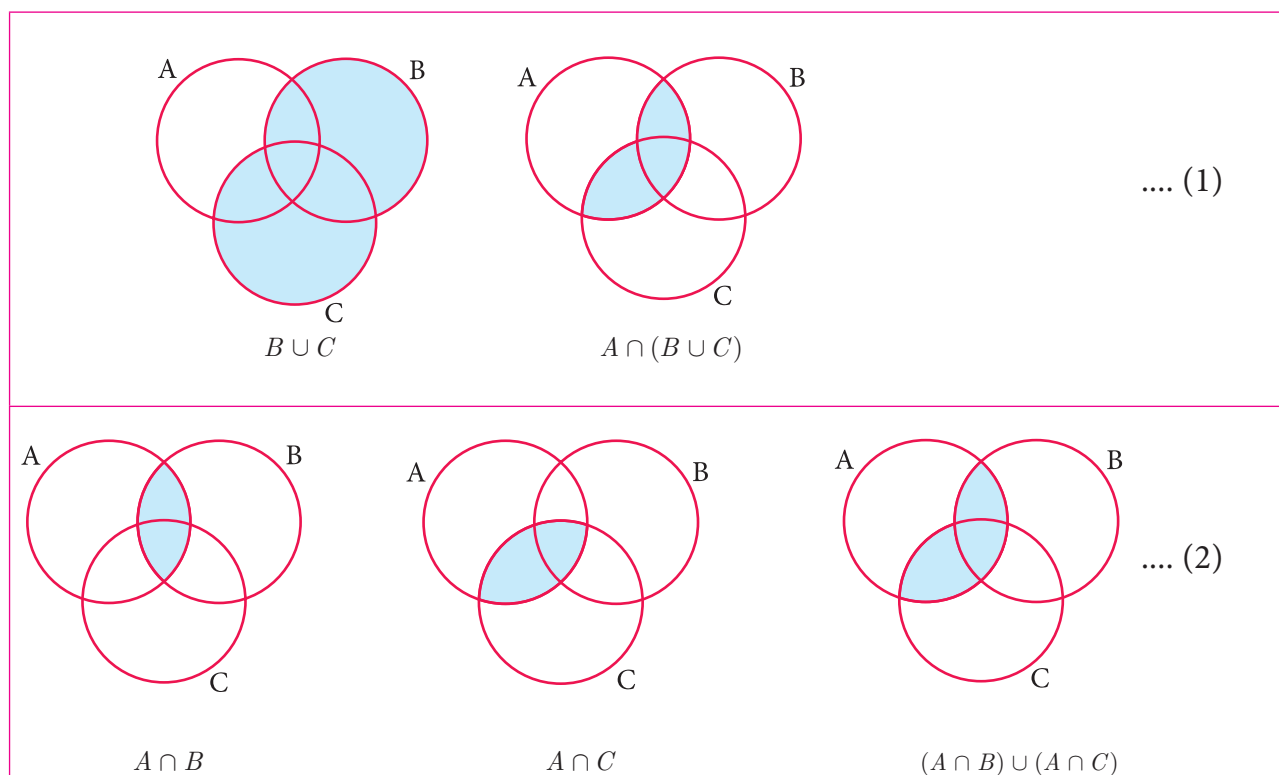


Fig.1.29

From (1) and (2), $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ is verified.

1.7 De Morgan's Laws

Augustus De Morgan (1806 – 1871) was a British mathematician. He was born on 27th June 1806 in Madurai, Tamilnadu, India. His father was posted in India by the East India Company. When he was seven months old, his family moved back to England. De Morgan was educated at Trinity College, Cambridge, London. He formulated laws for set difference and complementation. These are called De Morgan's laws.



1.7.1 De Morgan's Laws for Set Difference

These laws relate the set operations union, intersection and set difference.

Let us consider three sets A, B and C as $A = \{-5, -2, 1, 3\}$, $B = \{-3, -2, 0, 3, 5\}$ and $C = \{-2, -1, 0, 4, 5\}$.

Now, $B \cup C = \{-3, -2, -1, 0, 3, 4, 5\}$

$$A - (B \cup C) = \{-5, 1\} \quad \dots (1)$$

Then, $A - B = \{-5, 1\}$ and $A - C = \{-5, 1, 3\}$

$$(A - B) \cup (A - C) = \{-5, 1, 3\} \quad \dots (2)$$

$$(A - B) \cap (A - C) = \{-5, 1\} \quad \dots (3)$$

From (1) and (2), we see that

$$A - (B \cup C) \neq (A - B) \cup (A - C)$$

But note that from (1) and (3), we see that

$$A - (B \cup C) = (A - B) \cap (A - C)$$

Now, $B \cap C = \{-2, 0, 5\}$

$$A - (B \cap C) = \{-5, 1, 3\} \quad \dots (4)$$

From (3) and (4) we see that

$$A - (B \cap C) \neq (A - B) \cap (A - C)$$

But note that from (2) and (4), we get $A - (B \cap C) = (A - B) \cup (A - C)$

Thinking Corner



$$(A - B) \cup (A - C) \cup (A \cap B) = \underline{\hspace{2cm}}$$

De Morgan's laws for set difference : For any three sets A, B and C

(i) $A - (B \cup C) = (A - B) \cap (A - C)$ (ii) $A - (B \cap C) = (A - B) \cup (A - C)$

Example 1.23

Verify $A - (B \cup C) = (A - B) \cap (A - C)$ using Venn diagrams.

Solution

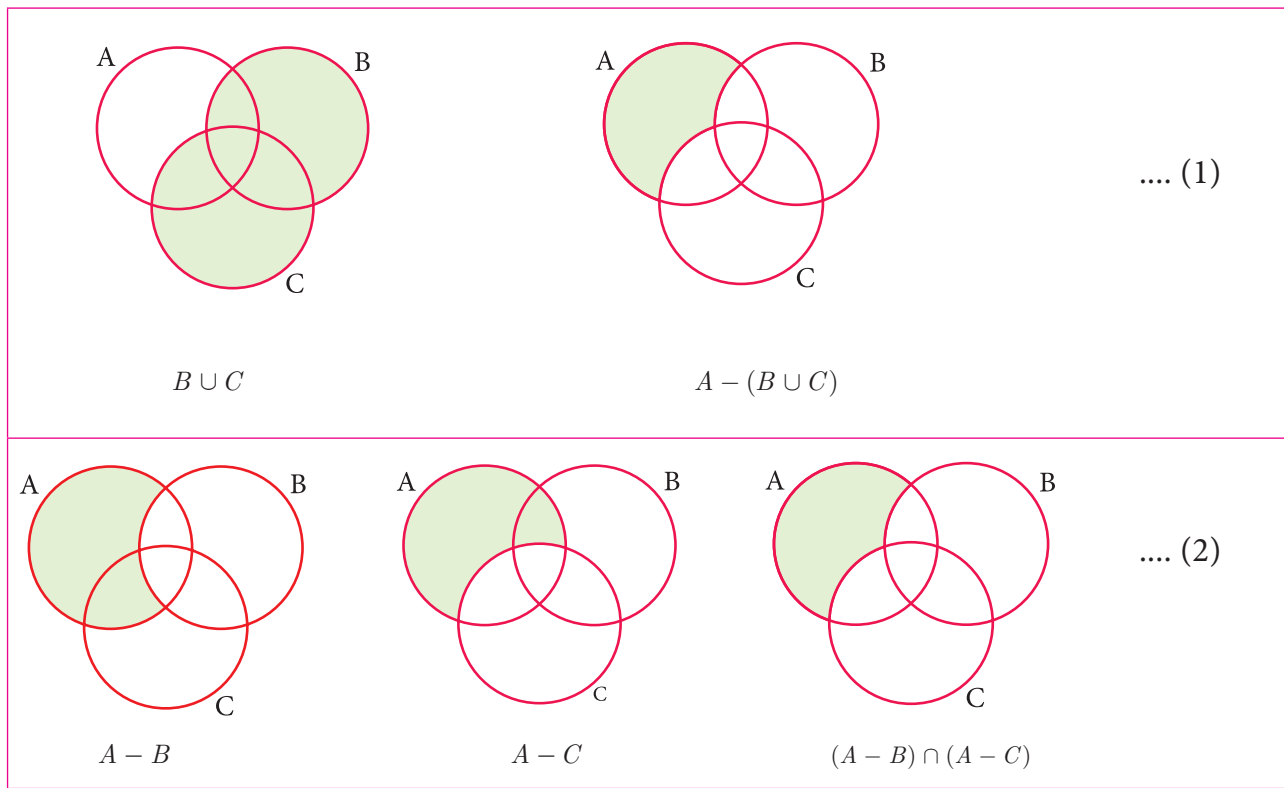


Fig.1.30

From (1) and (2), we get $A - (B \cup C) = (A - B) \cap (A - C)$. Hence it is verified.

Example 1.24

If $P = \{x : x \in \mathbb{W} \text{ and } 0 < x < 10\}$, $Q = \{x : x = 2n+1, n \in \mathbb{W} \text{ and } n < 5\}$ and $R = \{2, 3, 5, 7, 11, 13\}$, then verify $P - (Q \cap R) = (P - Q) \cup (P - R)$

Solution The roster form of sets P , Q and R are

$$P = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}, \quad Q = \{1, 3, 5, 7, 9\}$$

$$\text{and } R = \{2, 3, 5, 7, 11, 13\}$$

$$\text{First, we find } (Q \cap R) = \{3, 5, 7\}$$

$$\text{Then, } P - (Q \cap R) = \{1, 2, 4, 6, 8, 9\} \dots (1)$$

$$\text{Next, } P - Q = \{2, 4, 6, 8\} \text{ and}$$

$$P - R = \{1, 4, 6, 8, 9\}$$

$$\text{and so, } (P - Q) \cup (P - R) = \{1, 2, 4, 6, 8, 9\} \dots (2)$$

Hence from (1) and (2), it is verified that $P - (Q \cap R) = (P - Q) \cup (P - R)$.

Finding the elements of set Q

Given, $x = 2n + 1$

$$n = 0 \rightarrow x = 2(0) + 1 = 0 + 1 = 1$$

$$n = 1 \rightarrow x = 2(1) + 1 = 2 + 1 = 3$$

$$n = 2 \rightarrow x = 2(2) + 1 = 4 + 1 = 5$$

$$n = 3 \rightarrow x = 2(3) + 1 = 6 + 1 = 7$$

$$n = 4 \rightarrow x = 2(4) + 1 = 8 + 1 = 9$$

Therefore, x takes values such as 1, 3, 5, 7 and 9.

1.7.2 De Morgan's Laws for Complementation

These laws relate the set operations on union, intersection and complementation.

Let us consider universal set $U = \{0, 1, 2, 3, 4, 5, 6\}$, $A = \{1, 3, 5\}$ and $B = \{0, 3, 4, 5\}$.

Now, $A \cup B = \{0, 1, 3, 4, 5\}$

Then, $(A \cup B)' = \{2, 6\}$ (1)

Next, $A' = \{0, 2, 4, 6\}$ and $B' = \{1, 2, 6\}$

Then, $A' \cap B' = \{2, 6\}$ (2)

From (1) and (2), we get $(A \cup B)' = A' \cap B'$

Also, $A \cap B = \{3, 5\}$,

$(A \cap B)' = \{0, 1, 2, 4, 6\}$ (3)

$A' = \{0, 2, 4, 6\}$ and $B' = \{1, 2, 6\}$

$A' \cup B' = \{0, 1, 2, 4, 6\}$ (4)

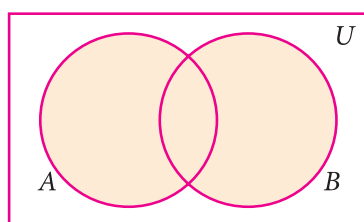
From (3) and (4), we get $(A \cap B)' = A' \cup B'$

De Morgan's laws for complementation : Let ' U ' be the universal set containing finite sets A and B. Then (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

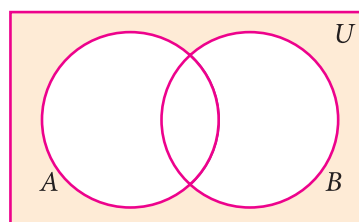
Example 1.25

Verify $(A \cup B)' = A' \cap B'$ using Venn diagrams.

Solution



$A \cup B$



$(A \cup B)'$

.... (1)

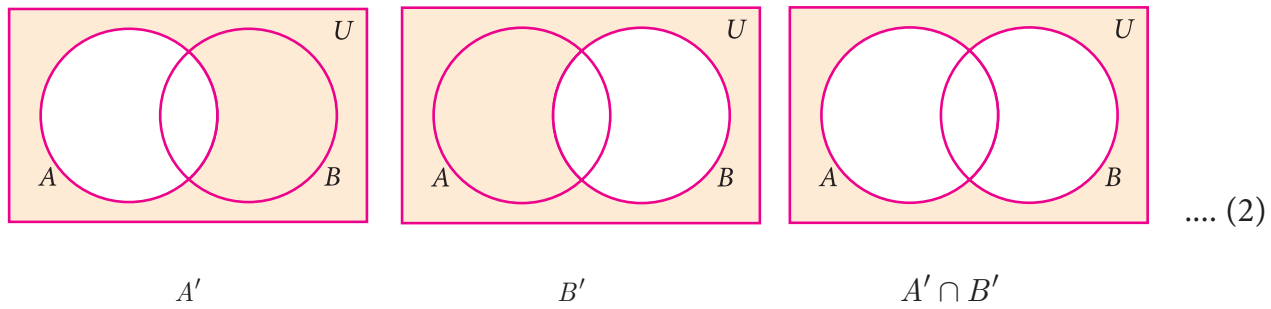


Fig.1.31

From (1) and (2), it is verified that $(A \cup B)' = A' \cap B'$

Example 1.26

If $U = \{x : x \in \mathbb{Z}, -2 \leq x \leq 10\}$,
 $A = \{x : x = 2p + 1, p \in \mathbb{Z}, -1 \leq p \leq 4\}$, $B = \{x : x = 3q + 1, q \in \mathbb{Z}, -1 \leq q < 4\}$,
 verify De Morgan's laws for complementation.

Solution

Given $U = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,
 $A = \{-1, 1, 3, 5, 7, 9\}$ and $B = \{-2, 1, 4, 7, 10\}$

Law (i) $(A \cup B)' = A' \cap B'$

$$\begin{aligned} \text{Now, } A \cup B &= \{-2, -1, 1, 3, 4, 5, 7, 9, 10\} \\ (A \cup B)' &= \{0, 2, 6, 8\} \end{aligned} \quad \dots (1)$$

$$\begin{aligned} \text{Then, } A' &= \{-2, 0, 2, 4, 6, 8, 10\} \text{ and } B' = \{-1, 0, 2, 3, 5, 6, 8, 9\} \\ A' \cap B' &= \{0, 2, 6, 8\} \end{aligned} \quad \dots (2)$$

From (1) and (2), it is verified that $(A \cup B)' = A' \cap B'$

Law (ii) $(A \cap B)' = A' \cup B'$

$$\begin{aligned} \text{Now, } A \cap B &= \{1, 7\} \\ (A \cap B)' &= \{-2, -1, 0, 2, 3, 4, 5, 6, 8, 9, 10\} \end{aligned} \quad \dots (3)$$

$$\text{Then, } A' \cup B' = \{-2, -1, 0, 2, 3, 4, 5, 6, 8, 9, 10\} \quad \dots (4)$$

From (3) and (4), it is verified that $(A \cap B)' = A' \cup B'$

Thinking Corner



$$A \cap (A \cup B)' = \underline{\hspace{2cm}}$$

Thinking Corner



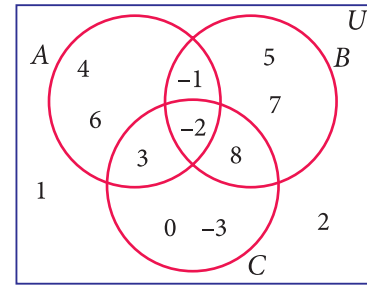
$$(A \cup B)' \cup (A' \cap B) = \underline{\hspace{2cm}}$$



Exercise 1.5

1. Using the adjacent Venn diagram, find the following sets:

- (i) $A - B$ (ii) $B - C$ (iii) $A' \cup B'$
(iv) $A' \cap B'$ (v) $(B \cup C)'$
(vi) $A - (B \cup C)$ (vii) $A - (B \cap C)$



2. If $K = \{a, b, d, e, f\}$, $L = \{b, c, d, g\}$ and $M = \{a, b, c, d, h\}$, then find the following:

- (i) $K \cup (L \cap M)$ (ii) $K \cap (L \cup M)$
(iii) $(K \cup L) \cap (K \cup M)$ (iv) $(K \cap L) \cup (K \cap M)$

and verify distributive laws.

3. If $A = \{x : x \in \mathbb{Z}, -2 < x \leq 4\}$, $B = \{x : x \in \mathbb{W}, x \leq 5\}$, $C = \{-4, -1, 0, 2, 3, 4\}$, then

verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

4. Verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ using Venn diagrams.

5. If $A = \{b, c, e, g, h\}$, $B = \{a, c, d, g, i\}$ and $C = \{a, d, e, g, h\}$, then show that

$$A - (B \cap C) = (A - B) \cup (A - C).$$

6. If $A = \{x : x = 6n, n \in \mathbb{W} \text{ and } n < 6\}$, $B = \{x : x = 2n, n \in \mathbb{N} \text{ and } 2 < n \leq 9\}$ and $C = \{x : x = 3n, n \in \mathbb{N} \text{ and } 4 \leq n < 10\}$, then show that $A - (B \cap C) = (A - B) \cup (A - C)$

7. If $A = \{-2, 0, 1, 3, 5\}$, $B = \{-1, 0, 2, 5, 6\}$ and $C = \{-1, 2, 5, 6, 7\}$, then show that $A - (B \cup C) = (A - B) \cap (A - C)$.

8. If $A = \{y : y = \frac{a+1}{2}, a \in \mathbb{W} \text{ and } a \leq 5\}$, $B = \{y : y = \frac{2n-1}{2}, n \in \mathbb{W} \text{ and } n < 5\}$ and

$$C = \left\{-1, -\frac{1}{2}, 1, \frac{3}{2}, 2\right\}, \text{ then show that } A - (B \cup C) = (A - B) \cap (A - C).$$

9. Verify $A - (B \cap C) = (A - B) \cup (A - C)$ using Venn diagrams.

10. If $U = \{4, 7, 8, 10, 11, 12, 15, 16\}$, $A = \{7, 8, 11, 12\}$ and $B = \{4, 8, 12, 15\}$, then verify De Morgan's Laws for complementation.

11. Verify $(A \cap B)' = A' \cup B'$ using Venn diagrams.

1.8 Application on Cardinality of Sets:

We have learnt about the union, intersection, complement and difference of sets.

Now we will go through some practical problems on sets related to everyday life.

Results :

If A and B are two finite sets, then

$$(i) \quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$(ii) \quad n(A - B) = n(A) - n(A \cap B)$$

$$(iii) \quad n(B - A) = n(B) - n(A \cap B)$$

$$(iv) \quad n(A') = n(U) - n(A)$$

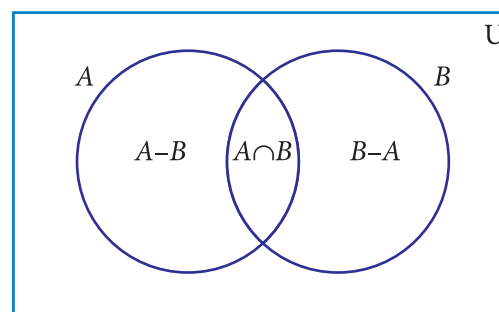


Fig. 1.32

Note

From the above results we may get,

$$\bullet \quad n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$\bullet \quad n(U) = n(A) + n(A')$$

$$\bullet \quad \text{If } A \text{ and } B \text{ are disjoint sets then, } n(A \cup B) = n(A) + n(B).$$

Example 1.27

From the Venn diagram, verify that

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Solution From the venn diagram,

$$A = \{5, 10, 15, 20\}$$

$$B = \{10, 20, 30, 40, 50\}$$

$$\text{Then } A \cup B = \{5, 10, 15, 20, 30, 40, 50\}$$

$$A \cap B = \{10, 20\}$$

$$n(A) = 4, \quad n(B) = 5, \quad n(A \cup B) = 7, \quad n(A \cap B) = 2$$

$$n(A \cup B) = 7$$

→ (1)

$$n(A) + n(B) - n(A \cap B) = 4 + 5 - 2$$

$$= 7$$

→ (2)

$$\text{From (1) and (2), } n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

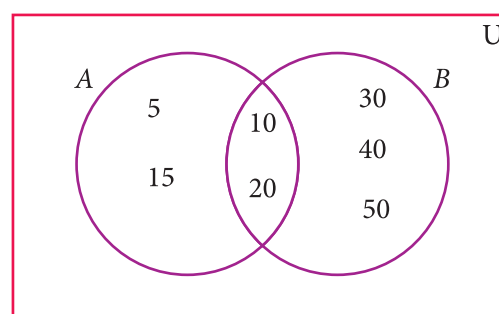


Fig. 1.33

Example 1.28

If $n(A) = 36$, $n(B) = 10$, $n(A \cup B) = 40$, and $n(A') = 27$ find $n(U)$ and $n(A \cap B)$.

Solution $n(A) = 36$, $n(B) = 10$, $n(A \cup B) = 40$, $n(A') = 27$

$$(i) \quad n(U) = n(A) + n(A') = 36 + 27 = 63$$

$$(ii) \quad n(A \cap B) = n(A) + n(B) - n(A \cup B) = 36 + 10 - 40 = 46 - 40 = 6$$



Activity-4

Fill in the blanks with appropriate cardinal numbers.						
S.No.	$n(A)$	$n(B)$	$n(A \cup B)$	$n(A \cap B)$	$n(A - B)$	$n(B - A)$
1	30	45	65			
2	20		55	10		
3	50	65		25		
4	30	43	70			

Example 1.29

Let $A = \{b, d, e, g, h\}$ and $B = \{a, e, c, h\}$. Verify that $n(A - B) = n(A) - n(A \cap B)$.

Solution

$$A = \{b, d, e, g, h\}, B = \{a, e, c, h\}$$

$$A - B = \{b, d, g\}$$

$$n(A - B) = 3 \quad \dots (1)$$

$$A \cap B = \{e, h\}$$

$$n(A \cap B) = 2, \quad n(A) = 5$$

$$n(A) - n(A \cap B) = 5 - 2$$

$$= 3 \quad \dots (2)$$

Form (1) and (2) we get $n(A - B) = n(A) - n(A \cap B)$.

Example 1.30

In a school, all students play either Hockey or Cricket or both. 300 play Hockey, 250 play Cricket and 110 play both games. Find

- the number of students who play only Hockey.
- the number of students who play only Cricket.
- the total number of students in the School.

Solution:

Let H be the set of all students who play Hockey and C be the set of all students who play Cricket.

$$\text{Then } n(H) = 300, n(C) = 250 \text{ and } n(H \cap C) = 110$$

Using Venn diagram,

From the Venn diagram,

- The number of students who play only Hockey = 190

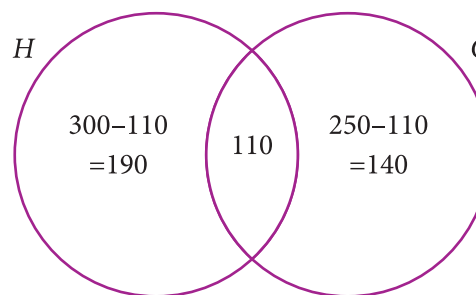


Fig. 1.34



- (ii) The number of students who play only Cricket = 140
(iii) The total number of students in the school = $190 + 110 + 140 = 440$

Aliter

- (i) The number of students who play only Hockey

$$\begin{aligned}n(H-C) &= n(H) - n(H \cap C) \\&= 300 - 110 = 190\end{aligned}$$

- (ii) The number of students who play only Cricket

$$\begin{aligned}n(C-H) &= n(C) - n(H \cap C) \\&= 250 - 110 = 140\end{aligned}$$

- (iii) The total number of students in the school

$$\begin{aligned}n(H \cup C) &= n(H) + n(C) - n(H \cap C) \\&= 300 + 250 - 110 = 440\end{aligned}$$



Example 1.31

In a party of 60 people, 35 had Vanilla ice cream, 30 had Chocolate ice cream. All the people had at least one ice cream. Then how many of them had,

- (i) both Vanilla and Chocolate ice cream.
(ii) only Vanilla ice cream.
(iii) only Chocolate ice cream.

Solution :

Let V be the set of people who had Vanilla ice cream and C be the set of people who had Chocolate ice cream.

Then $n(V) = 35$, $n(C) = 30$, $n(V \cup C) = 60$,

Let x be the number of people who had both ice creams.

From the Venn diagram

$$35 - x + x + 30 - x = 60$$

$$65 - x = 60$$

$$x = 5$$

Hence 5 people had both ice creams.

- (i) Number of people who had only Vanilla ice cream = $35 - x$

$$= 35 - 5 = 30$$

- (ii) Number of people who had only Chocolate ice cream = $30 - x$

$$= 30 - 5 = 25$$

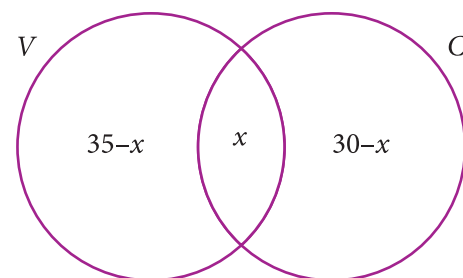


Fig. 1.35

We have learnt to solve problems involving two sets using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. Suppose we have three sets, we can apply this formula to get a similar formula for three sets.

For any three finite sets A , B and C

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

Note



Let us consider the following results which will be useful in solving problems using Venn diagram. Let three sets A , B and C represent the students. From the Venn diagram,

Number of students in only set $A = a$, only set $B = b$, only set $C = c$.

- Total number of students in only one set $= (a + b + c)$
- Total number of students in only two sets $= (x + y + z)$
- Number of students exactly in three sets $= r$
- Total number of students in atleast two sets (two or more sets) $= x + y + z + r$
- Total number of students in 3 sets $= (a + b + c + x + y + z + r)$

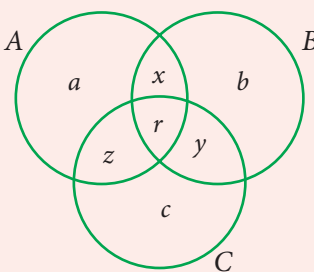


Fig.1.36

Example 1.32

In a college, 240 students play cricket, 180 students play football, 164 students play hockey, 42 play both cricket and football, 38 play both football and hockey, 40 play both cricket and hockey and 16 play all the three games. If each student participate in atleast one game, then find (i) the number of students in the college (ii) the number of students who play only one game.

Solution Let C , F and H represent sets of students who play Cricket, Football and Hockey respectively.

Then, $n(C) = 240$, $n(F) = 180$, $n(H) = 164$, $n(C \cap F) = 42$, $n(F \cap H) = 38$, $n(C \cap H) = 40$, $n(C \cap F \cap H) = 16$.

Let us represent the given data in a Venn diagram.

(i) The number of students in the college

$$= 174 + 26 + 116 + 22 + 102 + 24 + 16 = 480$$

(ii) The number of students who play only one game

$$= 174 + 116 + 102 = 392$$

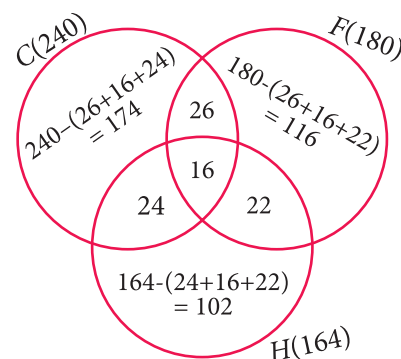


Fig.1.37

Example 1.33

In a residential area with 600 families $\frac{3}{5}$ owned scooter, $\frac{1}{3}$ owned car, $\frac{1}{4}$ owned bicycle, 120 families owned scooter and car, 86 owned car and bicycle while 90 families owned scooter and bicycle. If $\frac{2}{15}$ of families owned all the three types of vehicles, then find (i) the number of families owned atleast two types of vehicle. (ii) the number of families owned no vehicle.

Solution Let S , C and B represent sets of families who owned Scooter, Car and Bicycle respectively.

$$\text{Given, } n(U) = 600 \quad n(S) = \frac{3}{5} \times 600 = 360$$

$$n(C) = \frac{1}{3} \times 600 = 200 \quad n(B) = \frac{1}{4} \times 600 = 150$$

$$n(S \cap C \cap B) = \frac{2}{15} \times 600 = 80$$

From Venn diagram,

(i) The number of families owned atleast two types of vehicles = $40 + 6 + 10 + 80 = 136$

(ii) The number of families owned no vehicle
 $= 600 - (\text{owned atleast one vehicle})$
 $= 600 - (230 + 40 + 74 + 6 + 54 + 10 + 80)$
 $= 600 - 494 = 106$

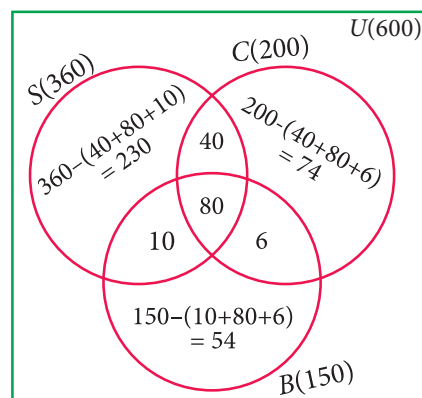


Fig.1.38

Example 1.34

In a group of 100 students, 85 students speak Tamil, 40 students speak English, 20 students speak French, 32 speak Tamil and English, 13 speak English and French and 10 speak Tamil and French. If each student knows atleast any one of these languages, then find the number of students who speak all these three languages.

Solution Let A , B and C represent sets of students who speak Tamil, English and French respectively.

$$\text{Given, } n(A \cup B \cup C) = 100, n(A) = 85, n(B) = 40, n(C) = 20, \\ n(A \cap B) = 32, n(B \cap C) = 13, n(A \cap C) = 10.$$

We know that,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$100 = 85 + 40 + 20 - 32 - 13 - 10 + n(A \cap B \cap C)$$

$$\text{Then, } n(A \cap B \cap C) = 100 - 90 = 10$$

Therefore, 10 students speak all the three languages.

Example 1.35

A survey was conducted among 200 magazine subscribers of three different magazines A , B and C . It was found that 75 members do not subscribe magazine A , 100 members do not subscribe magazine B , 50 members do not subscribe magazine C and 125 subscribe atleast two of the three magazines. Find

- Number of members who subscribe exactly two magazines.
- Number of members who subscribe only one magazine.

Solution

Total number of subscribers = 200

Magazine	Do not subscribe	Subscribe
A	75	125
B	100	100
C	50	150

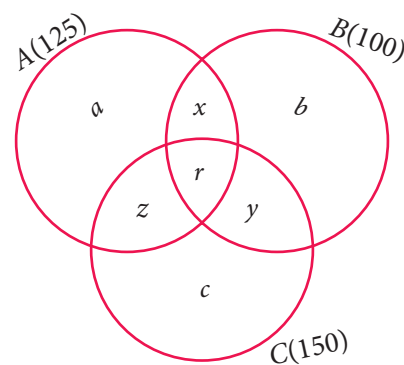


Fig.1.39

From the Venn diagram,

Number of members who subscribe only one magazine = $a + b + c$

Number of members who subscribe exactly two magazines = $x + y + z$

and 125 members subscribe atleast two magazines.

That is, $x + y + z + r = 125$... (1)

Now, $n(A \cup B \cup C) = 200$, $n(A) = 125$, $n(B) = 100$, $n(C) = 150$, $n(A \cap B) = x + r$

$n(B \cap C) = y + r$, $n(A \cap C) = z + r$, $n(A \cap B \cap C) = r$

We know that,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$200 = 125 + 100 + 150 - x - r - y - r - z - r + r$$

$$= 375 - (x + y + z + r) - r$$

$$= 375 - 125 - r \quad [\because x + y + z + r = 125]$$

$$200 = 250 - r \quad \Rightarrow r = 50$$

From (1) $x + y + z + 50 = 125$

We get, $x + y + z = 75$

Therefore, number of members who subscribe exactly two magazines = 75.

From Venn diagram,

$$(a + b + c) + (x + y + z + r) = 200 \quad \dots (2)$$

substitute (1) in (2),

$$a + b + c + 125 = 200$$

$$a + b + c = 75$$

Therefore, number of members who subscribe only one magazine = 75.



Exercise 1.6

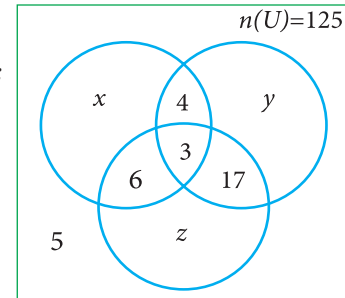
1. (i) If $n(A) = 25$, $n(B) = 40$, $n(A \cup B) = 50$ and $n(B') = 25$, find $n(A \cap B)$ and $n(U)$.
(ii) If $n(A) = 300$, $n(A \cup B) = 500$, $n(A \cap B) = 50$ and $n(B') = 350$, find $n(B)$ and $n(U)$.
2. If $U = \{x : x \in \mathbb{N}, x \leq 10\}$, $A = \{2, 3, 4, 8, 10\}$ and $B = \{1, 2, 5, 8, 10\}$, then verify that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
3. Verify $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$ for the following sets.
 - (i) $A = \{a, c, e, f, h\}$, $B = \{c, d, e, f\}$ and $C = \{a, b, c, f\}$
 - (ii) $A = \{1, 3, 5\}$, $B = \{2, 3, 5, 6\}$ and $C = \{1, 5, 6, 7\}$
4. In a class, all students take part in either music or drama or both. 25 students take part in music, 30 students take part in drama and 8 students take part in both music and drama. Find
 - (i) The number of students who take part in only music.
 - (ii) The number of students who take part in only drama.
 - (iii) The total number of students in the class.
5. In a party of 45 people, each one likes tea or coffee or both. 35 people like tea and 20 people like coffee. Find the number of people who
 - (i) like both tea and coffee.
 - (ii) do not like Tea.
 - (iii) do not like coffee.
6. In an examination 50% of the students passed in Mathematics and 70% of students passed in Science while 10% students failed in both subjects. 300 students passed in both the subjects. Find the total number of students who appeared in the examination, if they took examination in only two subjects.
7. A and B are two sets such that $n(A - B) = 32 + x$, $n(B - A) = 5x$ and $n(A \cap B) = x$. Illustrate the information by means of a Venn diagram. Given that $n(A) = n(B)$, calculate the value of x .
8. Out of 500 car owners investigated, 400 owned car A and 200 owned car B , 50 owned both A and B cars. Is this data correct?
9. In a colony, 275 families buy Tamil newspaper, 150 families buy English newspaper, 45 families buy Hindi newspaper, 125 families buy Tamil and English newspapers, 17 families buy English and Hindi newspapers, 5 families buy Tamil and Hindi newspapers and 3 families buy all the three newspapers. If each family buy atleast one of these newspapers then find



- (i) Number of families buy only one newspaper
- (ii) Number of families buy atleast two newspapers
- (iii) Total number of families in the colony.

10. A survey of 1000 farmers found that 600 grew paddy, 350 grew ragi, 280 grew corn, 120 grew paddy and ragi, 100 grew ragi and corn, 80 grew paddy and corn. If each farmer grew atleast any one of the above three, then find the number of farmers who grew all the three.

11. In the adjacent diagram, if $n(U) = 125$, y is two times of x and z is 10 more than x , then find the value of x, y and z .



12. Each student in a class of 35 plays atleast one game among chess, carrom and table tennis. 22 play chess, 21 play carrom, 15 play table tennis, 10 play chess and table tennis, 8 play carrom and table tennis and 6 play all the three games. Find the number of students who play (i) chess and carrom but not table tennis (ii) only chess (iii) only carrom (**Hint:** Use Venn diagram)

13. In a class of 50 students, each one come to school by bus or by bicycle or on foot. 25 by bus, 20 by bicycle, 30 on foot and 10 students by all the three. Now how many students come to school exactly by two modes of transport?



Exercise 1.7



Multiple Choice Questions



1. Which of the following is correct?
(1) $\{7\} \in \{1,2,3,4,5,6,7,8,9,10\}$ (2) $7 \in \{1,2,3,4,5,6,7,8,9,10\}$
(3) $7 \notin \{1,2,3,4,5,6,7,8,9,10\}$ (4) $\{7\} \not\subseteq \{1,2,3,4,5,6,7,8,9,10\}$
2. The set $P = \{x \mid x \in \mathbb{Z}, -1 < x < 1\}$ is a
(1) Singleton set (2) Power set (3) Null set (4) Subset
3. If $U = \{x \mid x \in \mathbb{N}, x < 10\}$ and $A = \{x \mid x \in \mathbb{N}, 2 \leq x < 6\}$ then $(A')'$ is
(1) $\{1, 6, 7, 8, 9\}$ (2) $\{1, 2, 3, 4\}$ (3) $\{2, 3, 4, 5\}$ (4) $\{ \}$
4. If $B \subseteq A$ then $n(A \cap B)$ is
(1) $n(A - B)$ (2) $n(B)$ (3) $n(B - A)$ (4) $n(A)$
5. If $A = \{x, y, z\}$ then the number of non- empty subsets of A is
(1) 8 (2) 5 (3) 6 (4) 7





6. Which of the following is correct?

- (1) $\emptyset \subseteq \{a, b\}$ (2) $\emptyset \in \{a, b\}$ (3) $\{a\} \in \{a, b\}$ (4) $a \subseteq \{a, b\}$

7. If $A \cup B = A \cap B$, then

- (1) $A \neq B$ (2) $A = B$ (3) $A \subset B$ (4) $B \subset A$

8. If $B - A$ is B , then $A \cap B$ is

- (1) A (2) B (3) U (4) \emptyset

9. From the adjacent diagram $n[P(A \Delta B)]$ is

- (1) 8 (2) 16
(3) 32 (4) 64

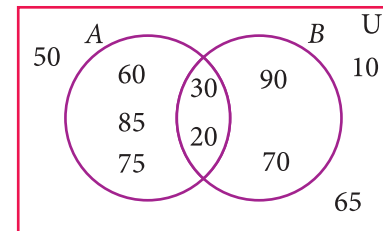


Fig. 1.40

10. If $n(A) = 10$ and $n(B) = 15$, then the minimum and maximum number of elements in $A \cap B$ is

- (1) 10,15 (2) 15,10 (3) 10,0 (4) 0,10

11. Let $A = \{\emptyset\}$ and $B = P(A)$, then $A \cap B$ is

- (1) $\{\emptyset, \{\emptyset\}\}$ (2) $\{\emptyset\}$ (3) \emptyset (4) $\{0\}$

12. In a class of 50 boys, 35 boys play Carrom and 20 boys play Chess then the number of boys play both games is

- (1) 5 (2) 30 (3) 15 (4) 10.

13. If $U = \{x : x \in \mathbb{N} \text{ and } x < 10\}$, $A = \{1, 2, 3, 5, 8\}$ and $B = \{2, 5, 6, 7, 9\}$, then $n[(A \cup B)']$ is

- (1) 1 (2) 2 (3) 4 (4) 8

14. For any three sets P, Q and R, $P - (Q \cap R)$ is

- (1) $P - (Q \cup R)$ (2) $(P \cap Q) - R$
(3) $(P - Q) \cup (P - R)$ (4) $(P - Q) \cap (P - R)$

15. Which of the following is true?

- (1) $A - B = A \cap B$ (2) $A - B = B - A$
(3) $(A \cup B)' = A' \cup B'$ (4) $(A \cap B)' = A' \cup B'$

16. If $n(A \cup B \cup C) = 100$, $n(A) = 4x$, $n(B) = 6x$, $n(C) = 5x$, $n(A \cap B) = 20$, $n(B \cap C) = 15$, $n(A \cap C) = 25$ and $n(A \cap B \cap C) = 10$, then the value of x is

- (1) 10 (2) 15 (3) 25 (4) 30

17. For any three sets A, B and C, $(A - B) \cap (B - C)$ is equal to

- (1) A only (2) B only (3) C only (4) ϕ



18. If J = Set of three sided shapes, K = Set of shapes with two equal sides and L = Set of shapes with right angle, then $J \cap K \cap L$ is
- (1) Set of isosceles triangles (2) Set of equilateral triangles
(3) Set of isosceles right triangles (4) Set of right angled triangles
19. The shaded region in the Venn diagram is
- (1) $Z - (X \cup Y)$ (2) $(X \cup Y) \cap Z$
(3) $Z - (X \cap Y)$ (4) $Z \cup (X \cap Y)$
20. In a city, 40% people like only one fruit, 35% people like only two fruits, 20% people like all the three fruits. How many percentage of people do not like any one of the above three fruits?
- (1) 5 (2) 8 (3) 10 (4) 15

Points to Remember



- A set is a well defined collection of objects.
- Sets are represented in three forms (i) Descriptive form (ii) Set – builder form (iii) Roster form.
- If every element of A is also an element of B , then A is called a subset of B .
- If $A \subseteq B$ and $A \neq B$, then A is a proper subset of B .
- The power set of the set A is the set of all the subsets of A and it is denoted by $P(A)$.
- The number of subsets of a set with m elements is 2^m .
- The number of proper subsets of a set with m elements is $2^m - 1$.
- If $A \cap B = \emptyset$ then A and B are disjoint sets. If $A \cap B \neq \emptyset$ then A and B are overlapping.
- The difference of two sets A and B is the set of all elements in A but not in B .
- The symmetric difference of two sets A and B is the union of $A - B$ and $B - A$.
- **Commutative Property**

For any two sets A and B ,

$$A \cup B = B \cup A ; \quad A \cap B = B \cap A$$





■ Associative Property

For any three sets A , B and C

$$A \cup (B \cap C) = (A \cup B) \cap C \quad ; \quad A \cap (B \cup C) = (A \cap B) \cup C$$

■ Distributive Property

For any three sets A , B and C

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \quad \left[\text{Intersection over union} \right]$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \quad \left[\text{Union over intersection} \right]$$

■ De Morgan's Laws for Set Difference

For any three sets A , B and C

$$A - (B \cup C) = (A - B) \cap (A - C)$$

$$A - (B \cap C) = (A - B) \cup (A - C)$$

■ De Morgan's Laws for Complementation

Consider an Universal set and A , B are two subsets, then

$$(A \cup B)' = A' \cap B' \quad ; \quad (A \cap B)' = A' \cup B'$$

■ Cardinality of Sets

If A and B are any two sets, then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

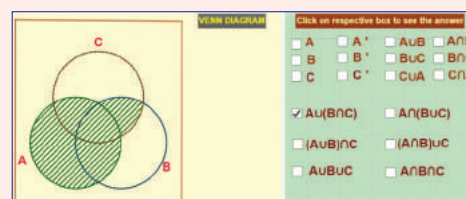
If A , B and C are three sets, then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$



ICT Corner 1

Expected Result is shown in this picture



Step - 1 : Open the Browser, type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named “Set Language” will open. In the work sheet there are two activities. 1. Venn Diagram for two sets and 2. Venn Diagram for three sets. In the first activity Click on the boxes on the right side to see the respective shading and analyse.

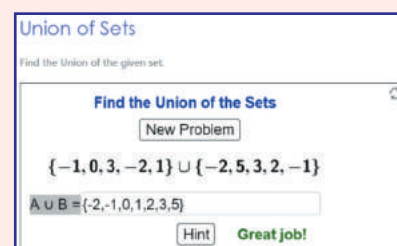
Step - 2 : Do the same for the second activity for three sets.

Scan the QR Code.



ICT Corner 2

Expected Result is shown in this picture



Step - 1

Open the Browser and copy and paste the Link given below (or) by typing the URL given (or) Scan the QR Code.

Step - 2

GeoGebra worksheet “Union of Sets” will appear. You can create new problems by clicking on the box “NEW PROBLEM”

Step-3

Enter your answer by typing the correct numbers in the Question Box and then hit enter. If you have any doubt, you can hit the “HINT” button

Step-4

If your answer is correct “GREAT JOB” menu will appear. And if your answer is Wrong “Try Again!” menu will appear.

Keep on working new problems until you get 5 consecutive trials as correct.

