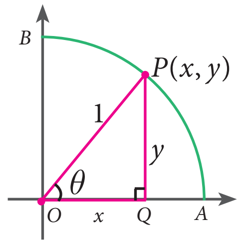
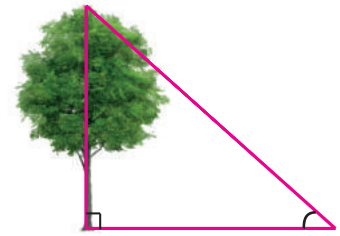
6 TRIGONOMETRY





There is perhaps nothing which so occupies the middle position of mathematics as Trigonometry.- J. F. Herbart

Leonhard Euler
(AD (CE) 1707 - 1783)

Euler, like Newton, was the greatest mathematician of his generation. He studied all areas of mathematics and continued to work hard after he had gone blind. Euler made discoveries in many areas of mathematics, especially Calculus and Trigonometry. He was the first to prove several theorems in Geometry.

FZJWSD

Learning Outcomes

- To understand the relationship among various trigonometric ratios.
- To recognize the values of trigonometric ratios and their reciprocals.
- To use the concept of complementary angles.
- To understand the usage of trigonometric tables.

6.1 Introduction

Trigonometry (which comes from Greek words trigonon means triangle and metron means measure) is the branch of mathematics that studies the relationships involving lengths of sides and measures of angles of triangles. It is a useful tool for engineers, scientists, and surveyors and is applied even in seismology and navigation.

Observe the three given right angled triangles; in particular scrutinize their measures. The corresponding angles shown in the three triangles are of the same size. Draw your attention to the lengths of "opposite" sides (meaning the side opposite to the given angle) and the "adjacent" sides (which is the side adjacent to the given angle) of the triangle.

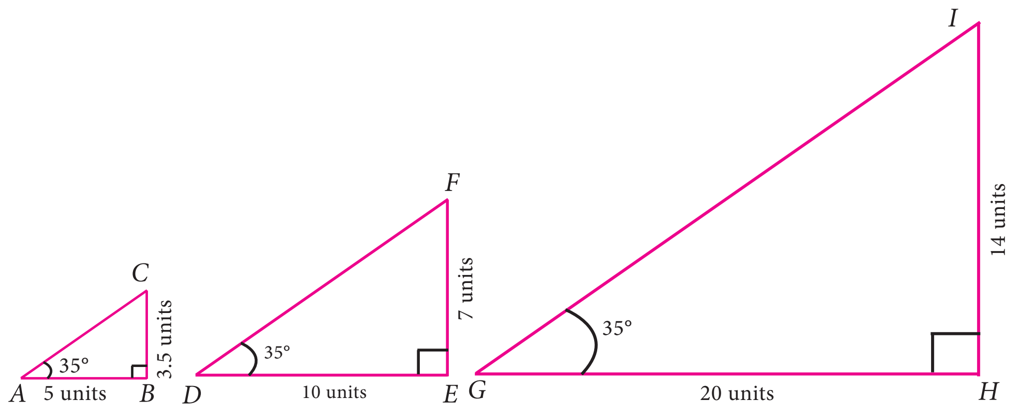


Fig. 6.1

What can you say about the ratio $\left(\frac{\textit{opposite side}}{\textit{adjacent side}}\right)$ in each case? Every right angled triangle given here has the same ratio 0.7 ; based on this finding, now what could be the length of the side marked 'x' in the Fig 6.2? Is it 15?

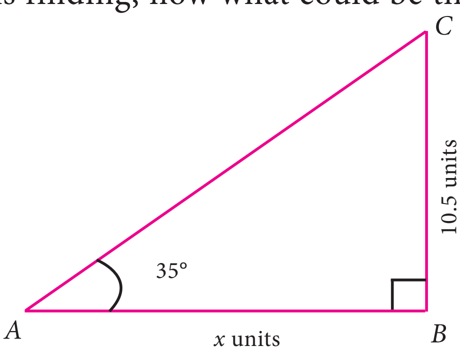
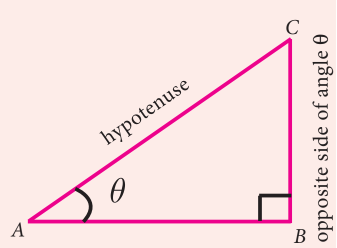
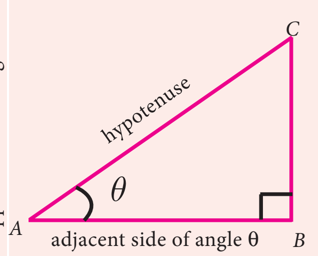
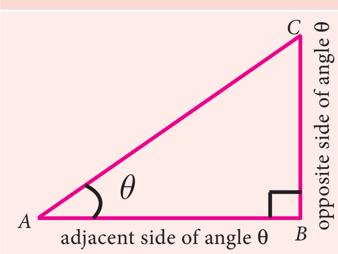


Fig. 6.2

Such remarkable ratios stunned early mathematicians and paved the way for the subject of trigonometry.

There are three basic ratios in trigonometry, each of which is one side of a right-angled triangle divided by another.

The three ratios are:

Name of the angle	sine	cosine	tangent
Short form	sin	cos	tan
Related measurements			
Relationship	$\sin\theta = \frac{\textit{opposite side}}{\textit{hypotenuse}}$	$\cos\theta = \frac{\textit{adjacent side}}{\textit{hypotenuse}}$	$\tan\theta = \frac{\textit{opposite side}}{\textit{adjacent side}}$

Example 6.1 For the measures in the figure, compute sine, cosine and tangent ratios of the angle θ.

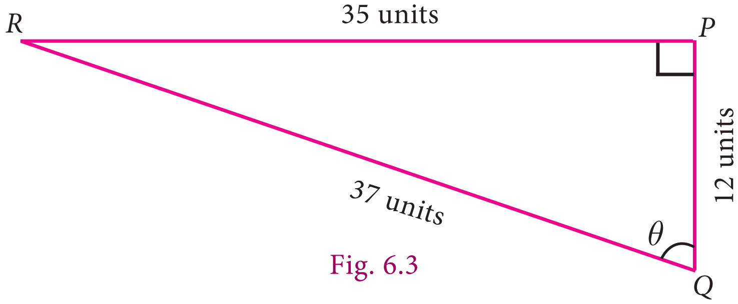


Fig. 6.3

Solution

In the given right angled triangle, note that for the given angle θ, PR is the 'opposite' side and PQ is the 'adjacent' side.

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{PR}{QR} = \frac{35}{37}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{PQ}{QR} = \frac{12}{37}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{PR}{PQ} = \frac{35}{12}$$

FZQ6G4

It is enough to leave the ratios as fractions. In case, if you want to simplify each ratio neatly in a terminating decimal form, you may opt for it, but that is not obligatory.

Note

- Since trigonometric ratios are defined in terms of ratios of sides, they are unitless numbers.
- Ratios like $\sin\theta$, $\cos\theta$, $\tan\theta$ are not to be treated like $(\sin)\times(\theta)$, $(\cos)\times(\theta)$,$(\tan)\times(\theta)$.

Thinking Corner

The given triangles ABC, DEF and GHI have measures 3-4-5, 6-8-10 and 12-16-20. Are they all right triangles? How do you know?

The angles at the vertices B, E and H are of equal size (each angle is equal to θ).

With these available details, fill up the following table and comment on the ratios that you get.

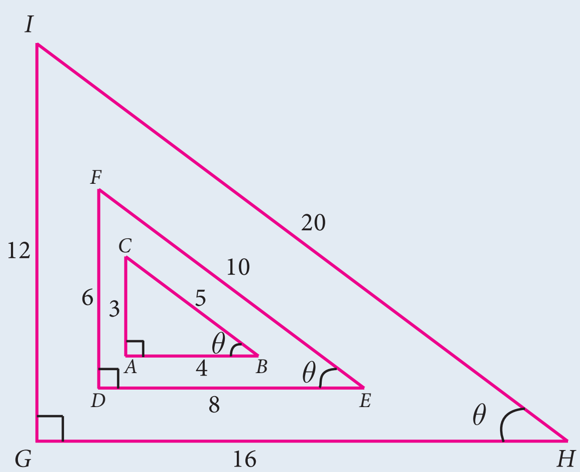


Fig. 6.4

In ΔABC	In ΔDEF	In ΔGHI
$\sin\theta = \frac{3}{5}$	$\sin\theta = \frac{6}{10} = ?$	$\sin\theta = \frac{12}{20} = ?$
$\cos\theta = ?$	$\cos\theta = ?$	$\cos\theta = ?$
$\tan\theta = \frac{3}{4}$	$\tan\theta = ?$	$\tan\theta = ?$

Reciprocal ratios

We defined three basic trigonometric ratios namely, sine, cosine and tangent. The reciprocals of these ratios are also often useful during calculations. We define them as follows:

Basic Trigonometric Ratios	Its reciprocal	Short form
$\sin\theta = \frac{\textit{opposite side}}{\textit{hypotenuse}}$	$\text{cosecant}\ \theta = \frac{\textit{hypotenuse}}{\textit{opposite side}}$	$\text{cosec}\ \theta = \frac{\textit{hypotenuse}}{\textit{opposite side}}$
$\cos\theta = \frac{\textit{adjacent side}}{\textit{hypotenuse}}$	$\text{secant}\ \theta = \frac{\textit{hypotenuse}}{\textit{adjacent side}}$	$\sec\theta = \frac{\textit{hypotenuse}}{\textit{adjacent side}}$
$\tan\theta = \frac{\textit{opposite side}}{\textit{adjacent side}}$	$\text{cotangent}\ \theta = \frac{\textit{adjacent side}}{\textit{opposite side}}$	$\cot\theta = \frac{\textit{adjacent side}}{\textit{opposite side}}$

From the above ratios we can observe easily the following relations:

$$\text{cosec}\,\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sin\theta = \frac{1}{\text{cosec}\,\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$(\sin\theta)\times(\text{cosec}\,\theta) = 1$. We usually write this as $\sin\theta\ \text{cosec}\,\theta = 1$.

$(\cos\theta)\times(\sec\theta) = 1$. We usually write this as $\cos\theta\ \sec\theta = 1$.

$(\tan\theta)\times(\cot\theta) = 1$. We usually write this as $\tan\theta\ \cot\theta = 1$.

Example 6.2 Find the six trigonometric ratios of the angle θ using the given diagram.

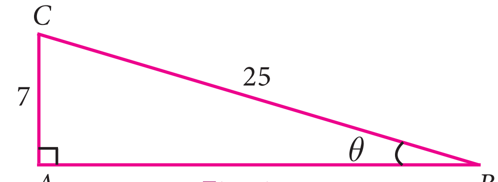


Fig. 6.5

Solution

By Pythagoras theorem,

$$AB = \sqrt{BC^2 - AC^2}$$

$$= \sqrt{(25)^2 - 7^2}$$

$$= \sqrt{625 - 49} = \sqrt{576} = 24$$

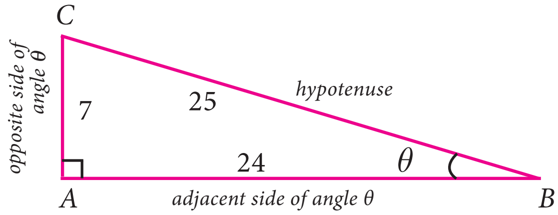


Fig. 6.6

The six trignometric ratios are

$$\sin\theta = \frac{opposite\,side}{hypotenuse} = \frac{7}{25} \qquad \cos\theta = \frac{adjacent\ side}{hypotenuse} = \frac{24}{25}$$

$$\tan\theta = \frac{opposite\,side}{adjacent\ side} = \frac{7}{24} \qquad \text{cosec}\theta = \frac{hypotenuse}{opposite\,side} = \frac{25}{7}$$

$$\sec\theta = \frac{hypotenuse}{adjacent\ side} = \frac{25}{24} \qquad \cot\theta = \frac{adjacent\ side}{opposite\,side} = \frac{24}{7}$$

Example 6.3 If $\tan A = \frac{2}{3}$, then find all the other trigonometric ratios.

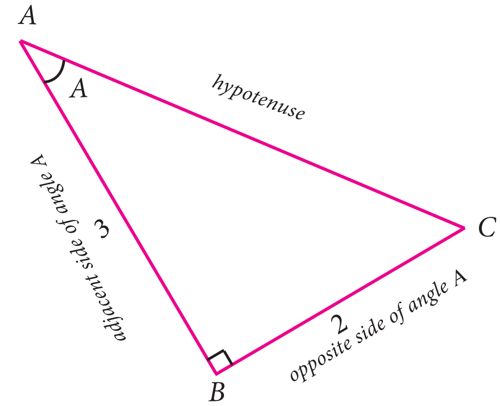


Fig. 6.7

Solution

$$\tan A = \frac{opposite\,side}{adjacent\ side} = \frac{2}{3}$$

By Pythagoras theorem,

$$AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{3^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$$

$$AC = \sqrt{13}$$

$$\sin A = \frac{opposite\,side}{hypotenuse} = \frac{2}{\sqrt{13}} \qquad \cos A = \frac{adjacent\ side}{hypotenuse} = \frac{3}{\sqrt{13}}$$

$$\text{cosec}A = \frac{hypotenuse}{opposite\,side} = \frac{\sqrt{13}}{2} \qquad \sec A = \frac{hypotenuse}{adjacent\ side} = \frac{\sqrt{13}}{3}$$

$$\cot A = \frac{adjacent\ side}{opposite\,side} = \frac{3}{2}$$

Example 6.4 If $\sec\theta = \frac{13}{5}$, then show that $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = 3$

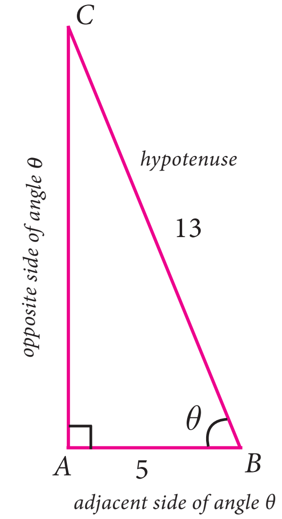


Fig. 6.8

Solution:

Let $BC = 13$ and $AB = 5$

$$\sec\theta = \frac{hypotenuse}{adjacent\ side} = \frac{BC}{AB} = \frac{13}{5}$$

By the Pythagoras theorem,

$$AC = \sqrt{BC^2 - AB^2}$$

$$= \sqrt{13^2 - 5^2}$$

$$= \sqrt{169 - 25} \quad = \quad \sqrt{144} = 12$$

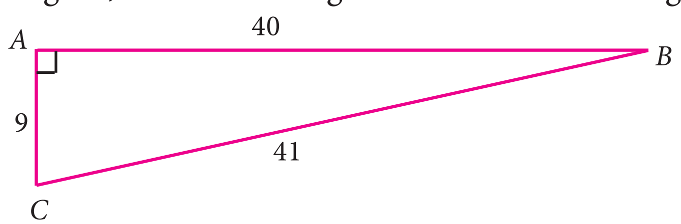
Therefore, $\sin\theta = \dfrac{AC}{BC} = \dfrac{12}{13}$; $\cos\theta = \dfrac{AB}{BC} = \dfrac{5}{13}$

$$LHS = \frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{2\times\frac{12}{13} - 3\times\frac{5}{13}}{4\times\frac{12}{13} - 9\times\frac{5}{13}} = \frac{\frac{24-15}{13}}{\frac{48-45}{13}} = \frac{9}{3} = 3 \; = RHS$$

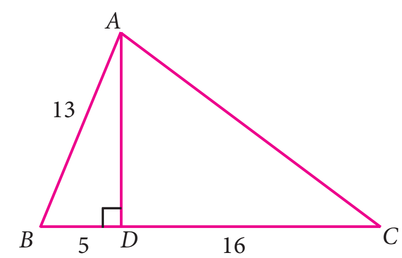
Note: We can also take the angle 'θ' at the vertex 'C' and proceed in the same way.

Exercise 6.1

1. From the given figure, find all the trigonometric ratios of angle *B*.



2. From the given figure, find the values of



   (i) $\sin B$ (ii) $\sec B$ (iii) $\cot B$

   (iv) $\cos C$ (v) $\tan C$ (vi) $\operatorname{cosec} C$

3. If $2\cos\theta = \sqrt{3}$, then find all the trigonometric ratios of angle θ .

4. If $\cos A = \dfrac{3}{5}$, then find the value of $\dfrac{\sin A - \cos A}{2\tan A}$.

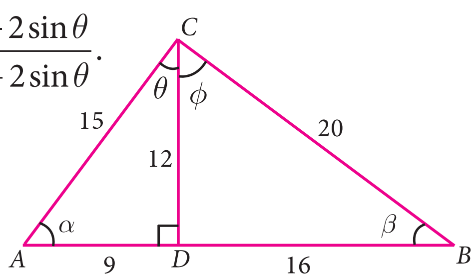
5. If $\cos A = \dfrac{2x}{1+x^2}$, then find the values of sin$A$ and tan$A$ in terms of x.

6. If $\sin\theta = \dfrac{a}{\sqrt{a^2+b^2}}$, then show that $b\sin\theta = a\cos\theta$.

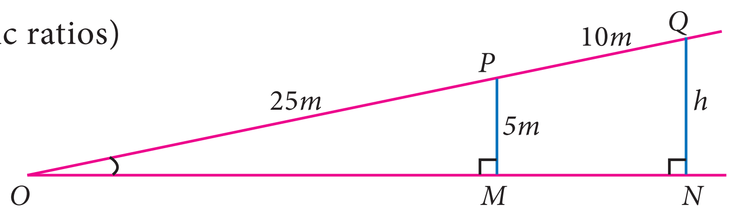
7. If $3\cot A = 2$, then find the value of $\dfrac{4\sin A - 3\cos A}{2\sin A + 3\cos A}$.

8. If $\cos\theta : \sin\theta = 1:2$, then find the value of $\dfrac{8\cos\theta - 2\sin\theta}{4\cos\theta + 2\sin\theta}$.

9. From the given figure, prove that $\theta + \phi = 90°$. Also prove that there are two other right angled triangles. Find $\sin\alpha$, $\cos\beta$ and $\tan\phi$.



10. A boy standing at a point O finds his kite flying at a point P with distance $OP=25m$. It is at a height of $5m$ from the ground. When the thread is extended by $10m$ from P, it reaches a point Q. What will be the height QN of the kite from the ground? (use trigonometric ratios)



6.2 Trigonometric Ratios of Some Special Angles

The values of trigonometric ratios of certain angles can be obtained geometrically. Two special triangles come to help here.

6.2.1 Trigonometric ratios of 45°

Consider a triangle ABC with angles 45°, 45° and 90° as shown in the figure 6.9.

It is the shape of half a square, cut along the square's diagonal. Note that it is also an isosceles triangle (both legs have the same length, a units).

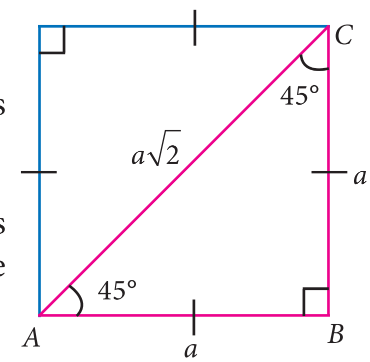


Fig. 6.9

Use Pythagoras theorem to check if the diagonal is of length $a\sqrt{2}$.

Now, from the right-angled triangle ABC,

$$\sin 45^o = \frac{opposite\ side}{hypotenuse} = \frac{BC}{AC} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\cos 45^o = \frac{adjacent\ side}{hypotenuse} = \frac{AB}{AC} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\tan 45^o = \frac{opposite\ side}{adjacent\ side} = \frac{BC}{AB} = \frac{a}{a} = 1$$

The reciprocals of these ratio can be easily foundout to be

$\operatorname{cosec} 45° = \sqrt{2}$;

$\sec 45° = \sqrt{2}$ and

$\cot 45° = 1$

6.2.2 Trigonometric Ratios of 30° and 60°

Consider an equilateral triangle PQR of side length 2 units.

Draw a bisector of $\angle P$. Let it meet QR at M.

$PQ = QR = RP = 2$ units.

$QM = MR = 1$ unit (Why?)

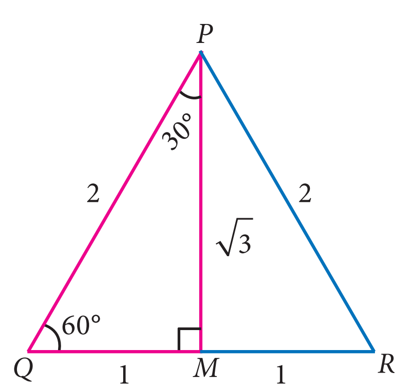


Fig. 6.10

Knowing PQ and QM, we can find PM, using Pythagoras theorem, we find that $PM = \sqrt{3}$ units.

Now, from the right-angled triangle PQM,

$$\sin 30^o = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{QM}{PQ} = \frac{1}{2}$$

$$\cos 30^o = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{PM}{PQ} = \frac{\sqrt{3}}{2}$$

$$\tan 30^o = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{QM}{PM} = \frac{1}{\sqrt{3}}$$

The reciprocals of these ratio can be easily foundout to be

$\operatorname{cosec} 30° = 2, \sec 30° = \frac{2}{\sqrt{3}}$

and $\cot 30° = \sqrt{3}$

We will use the same triangle but the other angle of measure 60° now.

$$\sin 60^o = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{PM}{PQ} = \frac{\sqrt{3}}{2}$$

$$\cos 60^o = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{QM}{PQ} = \frac{1}{2}$$

$$\tan 60^o = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{PM}{QM} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

The reciprocals of these ratio can be easily foundout to be

$\operatorname{cosec} 60° = \frac{2}{\sqrt{3}}$; $\sec 60° = 2$

and $\cot 60° = \frac{1}{\sqrt{3}}$

6.2.3 Trigonometric ratios of 0° and 90°

To find the trigonometric ratios of 0° and 90°, we take the help of what is known as a unit circle.

A unit circle is a circle of unit radius (that is of radius 1 unit), centred at the origin.

Why make a circle where the radius is 1unit?

This means that every reference triangle that we create here has a hypotenuse of 1unit, which makes it so much easier to compare angles and ratios.

We will be interested only in the positive values since we consider 'lengths' and it is hence enough to concentrate on the first quadrant.

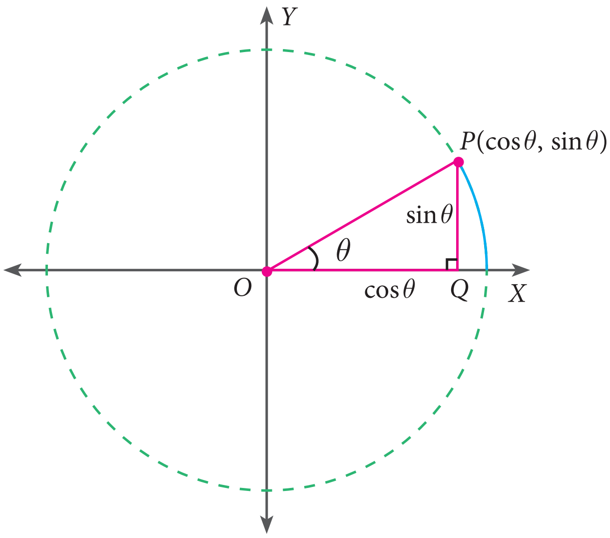


Fig. 6.11

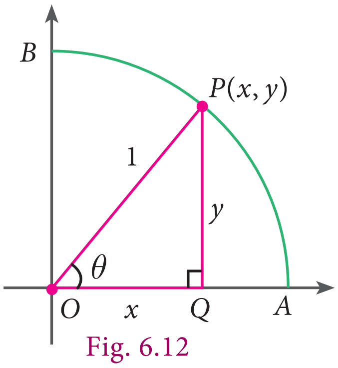


Fig. 6.12

We can see that if $P(x,y)$ be any point on the unit circle in the first quadrant and $\angle POQ = \theta$

$$\sin\theta = \frac{PQ}{OP} = \frac{y}{1} = y \ ; \quad \cos\theta = \frac{OQ}{OP} = \frac{x}{1} = x \ ; \quad \tan\theta = \frac{PQ}{OQ} = \frac{y}{x}$$

When $\theta = 0°$, OP coincides with OA, where A is (1,0) giving $x = 1, \ y = 0$.

We get thereby,

$$\sin 0° = 0 \ ; \quad \text{cosec}\, 0° = \text{not defined (why?)}$$

$$\cos 0° = 1 \ ; \quad \sec 0° = 1$$

$$\tan 0° = \frac{0}{1} = 0 \ ; \quad \cot 0° = \text{not defined (why?)}$$

When $\theta = 90°$, OP coincides with OB, where B is (0,1) giving $x = 0, \ y = 1$.

Hence,

$$\sin 90° = 1 \ ; \quad \text{cosec}\, 90° = 1$$

$$\cos 90° = 0 \ ; \quad \sec 90° = \text{not defined}$$

$$\tan 90° = \frac{1}{0} = \text{not defined} \ ; \quad \cot 90° = 0$$

Let us summarise all the results in the table given below:

Trigonometric ratio \ θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
$\text{cosec}\,\theta$	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined
$\cot\theta$	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Example 6.5

Evaluate: (i) $\sin 30^\circ + \cos 30^\circ$ (ii) $\tan 60^\circ \cot 60^\circ$

(iii) $\dfrac{\tan 45^\circ}{\tan 30^\circ + \tan 60^\circ}$

(iv) $\sin^2 45^\circ + \cos^2 45^\circ$

> **Note**
>
> (i) $(\sin\theta)^2$ is written as $\sin^2\theta = (\sin\theta)\times(\sin\theta)$
>
> (ii) $(\sin\theta)^2$ is not written as $\sin\theta^2$, because it may mean as $\sin(\theta\times\theta)$.

Solution

(i) $\sin 30^\circ + \cos 30^\circ = \dfrac{1}{2} + \dfrac{\sqrt{3}}{2} = \dfrac{1+\sqrt{3}}{2}$

(ii) $\tan 60^\circ \cot 60^\circ = \sqrt{3} \times \dfrac{1}{\sqrt{3}} = 1$

(iii) $\dfrac{\tan 45^\circ}{\tan 30^\circ + \tan 60^\circ} = \dfrac{1}{\dfrac{1}{\sqrt{3}} + \dfrac{\sqrt{3}}{1}} = \dfrac{1}{\dfrac{1+\left(\sqrt{3}\right)^2}{\sqrt{3}}} = \dfrac{1}{\dfrac{1+3}{\sqrt{3}}} = \dfrac{\sqrt{3}}{4}$

(iv) $\sin^2 45^\circ + \cos^2 45^\circ = \left(\dfrac{1}{\sqrt{2}}\right)^2 + \left(\dfrac{1}{\sqrt{2}}\right)^2 = \dfrac{1^2}{\left(\sqrt{2}\right)^2} + \dfrac{1^2}{\left(\sqrt{2}\right)^2} = \dfrac{1}{2} + \dfrac{1}{2} = 1$

Thinking Corner

The set of three numbers are called as Pythagorean triplets as they form the sides of a right angled triangle. For example,

(i) 3, 4, 5 (ii) 5, 12, 13 (iii) 7, 24, 25

Multiply each number in any of the above Pythagorean triplet by a non-zero constant. Verify whether each of the resultant set so obtained is also a Pythagorean triplet or not.

Example 6.6

Find the values of the following:

(i) $(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

(ii) $\tan^2 60^\circ - 2\tan^2 45^\circ - \cot^2 30^\circ + 2\sin^2 30^\circ + \dfrac{3}{4}\text{cosec}^2 45^\circ$

Solution

(i) $(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

$$= \left[1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right]\left[1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right]$$

$$=\left[\frac{2\sqrt{2}+2+\sqrt{2}}{2\sqrt{2}}\right]\left[\frac{2\sqrt{2}-2+\sqrt{2}}{2\sqrt{2}}\right]=\left[\frac{3\sqrt{2}+2}{2\sqrt{2}}\right]\left[\frac{3\sqrt{2}-2}{2\sqrt{2}}\right]$$

$$=\frac{18-4}{4\left(\sqrt{2}\right)^2}=\frac{14}{4\times 2}=\frac{7}{4}$$

(ii) $\tan^2 60°-2\tan^2 45°-\cot^2 30°+2\sin^2 30°+\frac{3}{4}\text{cosec}^2 45°$

$$=\left(\sqrt{3}\right)^2-2(1)^2-\left(\sqrt{3}\right)^2+2\left(\frac{1}{2}\right)^2+\frac{3}{4}\left(\sqrt{2}\right)^2$$

$$=3-2-3+\frac{1}{2}+\frac{3}{2}$$

$$=-2+\frac{4}{2}=-2+2=0$$

Note

(i) In a right angled triangle, if the angles are in the ratio $45°:45°:90°$, then the sides are in the ratio $1:1:\sqrt{2}$.

(ii) Similarly, if the angles are in the ratio $30°:60°:90°$, then the sides are in the ratio $1:\sqrt{3}:2$.

(The two set squares in your geometry box is one of the best example for the above two types of triangles).

Exercise 6.2

1. Verify the following equalities:

(i) $\sin^2 60°+\cos^2 60°=1$

(ii) $1+\tan^2 30°=\sec^2 30°$

(iii) $\cos 90°=1-2\sin^2 45°=2\cos^2 45°-1$

(iv) $\sin 30°\cos 60°+\cos 30°\sin 60°=\sin 90°$

2. Find the value of the following:

(i) $\frac{\tan 45°}{\text{cosec}30°}+\frac{\sec 60°}{\cot 45°}-\frac{5\sin 90°}{2\cos 0°}$

(ii) $(\sin 90°+\cos 60°+\cos 45°)\times(\sin 30°+\cos 0°-\cos 45°)$

(iii) $\sin^2 30°-2\cos^3 60°+3\tan^4 45°$

3. Verify $\cos 3A=4\cos^3 A-3\cos A$, when $A=30°$

4. Find the value of $8\sin 2x\ \cos 4x\ \sin 6x$, when $x=15°$

6.3 Trigonometric Ratios for Complementary Angles

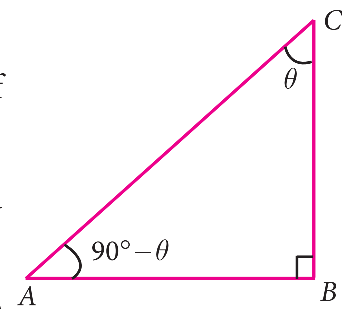


Fig. 6.13

Recall that two acute angles are said to be complementary if the sum of their measures is equal to $90°$.

What can we say about the acute angles of a right-angled triangle?

In a right angled triangle the sum of the two acute angles is equal to 90°. So, the two acute angles of a right angled triangle are always complementary to each other.

In the above figure 6.13, the triangle is right-angled at B. Therefore, if $\angle C$ is θ, then $\angle A = 90° - \theta$.

We find that

$$\left.\begin{aligned} \sin\theta &= \frac{AB}{AC} & \operatorname{cosec}\theta &= \frac{AC}{AB} \\ \cos\theta &= \frac{BC}{AC} & \sec\theta &= \frac{AC}{BC} \\ \tan\theta &= \frac{AB}{BC} & \cot\theta &= \frac{BC}{AB} \end{aligned}\right\}\ldots.(1)$$

Similarly for the angle (90° – θ), We have

$$\left.\begin{aligned} \sin(90°-\theta) &= \frac{BC}{AC} & \operatorname{cosec}(90°-\theta) &= \frac{AC}{BC} \\ \cos(90°-\theta) &= \frac{AB}{AC} & \sec(90°-\theta) &= \frac{AC}{AB} \\ \tan(90°-\theta) &= \frac{BC}{AB} & \cot(90°-\theta) &= \frac{AB}{BC} \end{aligned}\right\}\ldots..(2)$$

Comparing (1) and (2), we get

$$\begin{aligned} \sin\theta &= \cos(90°-\theta) & \operatorname{cosec}\theta &= \sec(90°-\theta) \\ \cos\theta &= \sin(90°-\theta) & \sec\theta &= \operatorname{cosec}(90°-\theta) \\ \tan\theta &= \cot(90°-\theta) & \cot\theta &= \tan(90°-\theta) \end{aligned}$$

Example 6.7

Express (i) $\sin 74°$ in terms of cosine (ii) $\tan 12°$ in terms of cotangent (iii) $\operatorname{cosec} 39°$ in terms of secant

Solution

(i) $\sin 74° = \sin(90° - 16°)$ (since, $90° - 16° = 74°$)

RHS is of the form $\sin(90° - \theta) = \cos\theta$

Therefore $\sin 74° = \cos 16°$

(ii) $\tan 12° = \tan(90° - 78°)$ (since, $12° = 90° - 78°$)

RHS is of the form $\tan(90° - \theta) = \cot\theta$

Therefore $\tan 12° = \cot 78°$

(iii) $\operatorname{cosec} 39° = \operatorname{cosec}(90° - 51°)$ (since $39° = 90° - 51°$)

RHS is of the form $\operatorname{cosec}(90° - \theta) = \sec\theta$

Therefore $\operatorname{cosec} 39° = \sec 51°$

Example 6.8 Evaluate: (i) $\frac{\sin 49°}{\cos 41°}$ (ii) $\frac{\sec 63°}{\operatorname{cosec} 27°}$

Solution

(i) $\frac{\sin 49°}{\cos 41°}$

$\sin 49° = \sin(90° - 41°) = \cos 41°$, since $49° + 41° = 90°$ (complementary),

Hence on substituting $\sin 49° = \cos 41°$ we get, $\frac{\cos 41°}{\cos 41°} = 1$

(ii) $\frac{\sec 63°}{\operatorname{cosec} 27°}$

$\sec 63° = \sec(90° - 27°) = \operatorname{cosec} 27°$, here $63°$ and $27°$ are complementary angles.

we have $\frac{\sec 63°}{\operatorname{cosec} 27°} = \frac{\operatorname{cosec} 27°}{\operatorname{cosec} 27°} = 1$

Example 6.9 Find the values of (i) $\tan 7° \tan 23° \tan 60° \tan 67° \tan 83°$

(ii) $\frac{\cos 35°}{\sin 55°} + \frac{\sin 12°}{\cos 78°} - \frac{\cos 18°}{\sin 72°}$

Solution

(i) $\tan 7° \tan 23° \tan 60° \tan 67° \tan 83°$

$= \tan 7° \tan 83° \tan 23° \tan 67° \tan 60°$ (Grouping complementary angles)

$= \tan 7° \tan(90° - 7°) \tan 23° \tan(90° - 23°) \tan 60°$

$= (\tan 7° . \cot 7°)(\tan 23° . \cot 23°) \tan 60°$

$= (1) \times (1) \times \tan 60°$

$= \tan 60° \quad = \sqrt{3}$

(ii) $\frac{\cos 35°}{\sin 55°} + \frac{\sin 12°}{\cos 78°} - \frac{\cos 18°}{\sin 72°}$

$= \frac{\cos(90° - 55°)}{\sin 55°} + \frac{\sin(90° - 78°)}{\cos 78°} - \frac{\cos(90° - 72°)}{\sin 72°}$ $\left[\begin{array}{l}\text{since}\\ \cos 35° = \cos(90° - 55°)\\ \sin 12° = \sin(90° - 78°)\\ \cos 18° = \cos(90° - 72°)\end{array}\right]$

$$=\frac{\sin 55°}{\sin 55°}+\frac{\cos 78°}{\cos 78°}-\frac{\sin 72°}{\sin 72°}$$

$$=1+1-1=1$$

Example 6.10 (i) If $\operatorname{cosec} A=\sec 34°$, then find A (ii) If $\tan B=\cot 47°$, then find B.

Solution

(i) We know that $\operatorname{cosec} A=\sec(90°-A)$

$$\sec(90°-A)=\sec(34°)$$

$$90°-A=34°$$

We get $A=90°-34°$

$$A=56°$$

(ii) We know that $\tan B=\cot(90°-B)$

$$\cot(90°-B)=\cot 47°$$

$$90°-B=47°$$

We get $B=90°-47°$

$$B=43°$$

Exercise 6.3

Find the value of the following:

(i) $\left(\frac{\cos 47°}{\sin 43°}\right)^2+\left(\frac{\sin 72°}{\cos 18°}\right)^2-2\cos^2 45°$ (ii) $\frac{\cos 70°}{\sin 20°}+\frac{\cos 59°}{\sin 31°}+\frac{\cos\theta}{\sin(90°-\theta)}-8\cos^2 60°$

(iii) $\tan 15°\ \tan 30°\ \tan 45°\ \tan 60°\ \tan 75°$

(iv) $\frac{\cot\theta}{\tan(90°-\theta)}+\frac{\cos(90°-\theta)\tan\theta\sec(90°-\theta)}{\sin(90°-\theta)\cot(90°-\theta)\operatorname{cosec}(90°-\theta)}$

Thinking Corner

(i) What are the minimum and maximum values of $\sin\theta$?

(ii) What are the minimum and maximum values of $\cos\theta$?

6.4 Method of using Trigonometric Table

We have learnt to calculate the trigonometric ratios for angles 0°, 30°, 45°, 60° and 90°. But during certain situations we need to calculate the trigonometric ratios of all the other acute angles. Hence we need to know the method of using trigonometric tables.

One degree (1°) is divided into 60 minutes ($60'$) and one minute ($1'$) is divided into 60 seconds ($60''$). Thus, $1°=60'$ and $1'=60''$.

The trigonometric tables give the values, correct to four places of decimals for the angles from 0° to 90° spaced at intervals of $60'$. A trigonometric table consists of three parts.

A column on the extreme left which contains degrees from 0° to 90°, followed by ten columns headed by $0'$, $6'$, $12'$, $18'$, $24'$, $30'$, $36'$, $42'$, $48'$ and $54'$.

Five columns under the head mean difference has values from 1,2,3,4 and 5.

For angles containing other measures of minutes (that is other than $0'$, $6'$, $12'$, $18'$, $24'$, $30'$, $36'$, $42'$, $48'$ and $54'$), the appropriate adjustment is obtained from the mean difference columns.

The mean difference is to be added in the case of sine and tangent while it is to be subtracted in the case of cosine.

Now let us understand the calculation of values of trigonometric angle from the following examples.

Example 6.11 Find the value of $\sin 64°34'$.

Solution

	$0'$	$6'$	$12'$	$18'$	$24'$	$30'$	$36'$	$42'$	$48'$	$54'$	Mean Difference				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
64°						0.9026								5	

Write $64°34' = 64°30' + 4'$

From the table we have, $\sin 64°30' = 0.9026$

Mean difference for $4' = 5$ (Mean difference to be added for sine)

$\sin 64°34' = 0.9031$

Example 6.12 Find the value of $\cos 19°59'$

Solution

	$0'$	$6'$	$12'$	$18'$	$24'$	$30'$	$36'$	$42'$	$48'$	$54'$	Mean Difference				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
19°										0.9403					5

Write $19°59' = 19°54' + 5'$

From the table we have,

$\cos 19°54' = 0.9403$

Mean difference for $5' = 5$ (Mean difference to be subtracted for cosine)

$\cos 19°59' = 0.9398$

Example 6.13 Find the value of $\tan 70°13'$

Solution

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Difference				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
70°			2.7776								26				

Write $70°13' = 70°12' + 1'$

From the table we have, $\tan 70°12' = 2.7776$

Mean difference for $1' = \quad 26$ (Mean difference to be added for tan)

$\tan 70°13' = 2.7802$

Example 6.14 Find the value of

(i) $\sin 38°36' + \tan 12°12'$ (ii) $\tan 60°25' - \cos 49°20'$

Solution

(i) $\sin 38°36' + \tan 12°12'$

$\sin 38°36' = 0.6239$

$\tan 12°12' = 0.2162$

$\sin 38°36' + \tan 12°12' = 0.8401$

(ii) $\tan 60°25' - \cos 49°20'$

$\tan 60°25' = 1.7603 + 0.0012 = 1.7615$

$\cos 49°20' = 0.6521 - 0.0004 = 0.6517$

$\tan 60°25' - \cos 49°20' = 1.1098$

Example 6.15 Find the value of θ if

(i) $\sin\theta = 0.9858$ (ii) $\cos\theta = 0.7656$

Solution

(i) $\sin\theta = 0.9858 = 0.9857 + 0.0001$

From the sine table $0.9857 = 80°18'$

Mean difference $1 = \quad 2'$

$0.9858 = 80°20'$

$\sin\theta = 0.9858 = \sin 80°20'$

$\theta = 80°20'$

(ii) $\cos\theta = 0.7656 = 0.7660 - 0.0004$

From the cosine table

$0.7660 = 40°0'$

Mean difference $4 = \quad 2'$

$0.7656 = 40°2'$

$\cos\theta = 0.7656 = \cos 40°2'$

$\theta = 40°2'$

Example 6.16

Find the area of the right angled triangle with hypotenuse 5*cm* and one of the acute angle is $48°30'$

Solution

From the figure,

$$\sin\theta = \frac{AB}{AC}$$
$$\sin 48°30' = \frac{AB}{5}$$
$$0.7490 = \frac{AB}{5}$$
$$5\times 0.7490 = AB$$
$$AB = 3.7450\ cm$$

$$\cos\theta = \frac{BC}{AC}$$
$$\cos 48°30' = \frac{BC}{5}$$
$$0.6626 = \frac{BC}{5}$$
$$0.6626\times 5 = BC$$
$$BC = 3.313\ cm$$

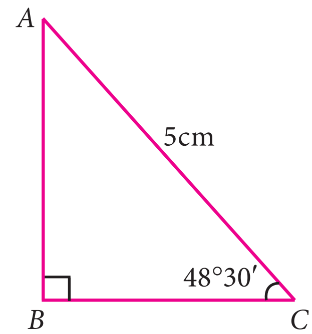
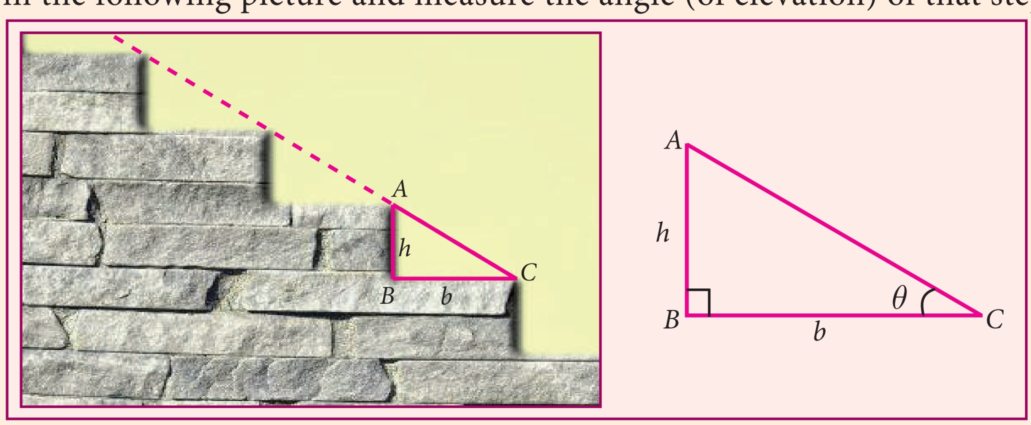


Fig. 6.14

$$\text{Area of right triangle} = \frac{1}{2}bh$$
$$= \frac{1}{2}\times BC\times AB$$
$$= \frac{1}{2}\times 3.3130\times 3.7450$$
$$= 1.6565\times 3.7450 \quad = 6.2035925\ cm^2$$

Activity

Observe the steps in your home. Measure the breadth and the height of one step. Enter it in the following picture and measure the angle (of elevation) of that step.



(i) Compare the angles (of elevation) of different steps of same height and same breadth and discuss your observation.

(ii) Sometimes few steps may not be of same height. Compare the angles (of elevation) of different steps of those different heights and same breadth and dicuss your observation.

Exercise 6.4

1. Find the value of the following:

   (i) $\sin 49°$ (ii) $\cos 74°39'$ (iii) $\tan 54°26'$ (iv) $\sin 21°21'$ (v) $\cos 33°53'$ (vi) $\tan 70°17'$

2. Find the value of θ if

   (i) $\sin\theta = 0.9975$ (ii) $\cos\theta = 0.6763$ (iii) $\tan\theta = 0.0720$

   (iv) $\cos\theta = 0.0410$ (v) $\tan\theta = 7.5958$

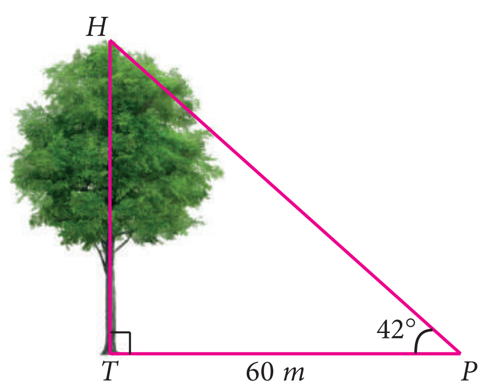
3. Find the value of the following:

   (i) $\sin 65°39' + \cos 24°57' + \tan 10°10'$ (ii) $\tan 70°58' + \cos 15°26' - \sin 84°59'$

4. Find the area of a right triangle whose hypotenuse is $10cm$ and one of the acute angle is $24°24'$

5. Find the angle made by a ladder of length $5m$ with the ground, if one of its end is $4m$ away from the wall and the other end is on the wall.

6. In the given figure, HT shows the height of a tree standing vertically. From a point P, the angle of elevation of the top of the tree (that is $\angle P$) measures $42°$ and the distance to the tree is 60 metres. Find the height of the tree.



Exercise 6.5

Multiple choice questions

1. If $\sin 30° = x$ and $\cos 60° = y$, then $x^2 + y^2$ is

   (1) $\frac{1}{2}$ (2) 0 (3) $\sin 90°$ (4) $\cos 90°$

FZTSU1

2. If $\tan\theta = \cot 37°$, then the value of θ is

   (1) 37° (2) 53° (3) 90° (4) 1°

3. The value of $\tan 72° \tan 18°$ is

   (1) 0 (2) 1 (3) 18° (4) 72°

4. The value of $\frac{2\tan 30°}{1 - \tan^2 30°}$ is equal to

   (1) $\cos 60°$ (2) $\sin 60°$ (3) $\tan 60°$ (4) $\sin 30°$

5. If $2\sin 2\theta = \sqrt{3}$, then the value of θ is

(1) 90° (2) 30° (3) 45° (4) 60°

6. The value of $3\sin 70^\circ \sec 20^\circ + 2\sin 49^\circ \sec 51^\circ$ is

(1) 2 (2) 3 (3) 5 (4) 6

7. The value of $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$ is

(1) 2 (2) 1 (3) 0 (4) $\frac{1}{2}$

8. The value of $\text{cosec}(70^\circ+\theta) - \sec(20^\circ-\theta) + \tan(65^\circ+\theta) - \cot(25^\circ-\theta)$ is

(1) 0 (2) 1 (3) 2 (4) 3

9. The value of $\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots \tan 89^\circ$ is

(1) 0 (2) 1 (3) 2 (4) $\frac{\sqrt{3}}{2}$

10. Given that $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then the value of $\alpha+\beta$ is

(1) 0° (2) 90° (3) 30° (4) 60°

Points to Remember

- Trigonometric ratios are

$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$	$\text{cosec}\,\theta = \frac{\text{hypotenuse}}{\text{opposite side}}$
$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$	$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent side}}$
$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$	$\cot\theta = \frac{\text{adjacent side}}{\text{opposite side}}$

- Reciprocal trigonometric ratios

$\sin\theta = \frac{1}{\text{cosec}\,\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\tan\theta = \frac{1}{\cot\theta}$

$\text{cosec}\,\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

- Complementary angles

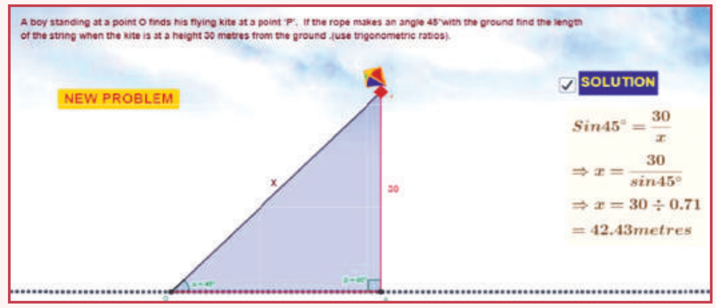
$\sin\theta = \cos(90^\circ-\theta)$ $\text{cosec}\,\theta = \sec(90^\circ-\theta)$

$\cos\theta = \sin(90^\circ-\theta)$ $\sec\theta = \text{cosec}(90^\circ-\theta)$

$\tan\theta = \cot(90^\circ-\theta)$ $\cot\theta = \tan(90^\circ-\theta)$

ICT Corner

Expected Result is shown in this picture



Step – 1

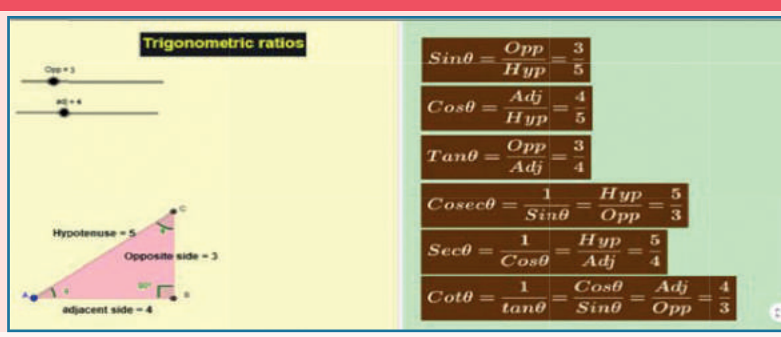
Open the Browser by typing the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Trigonometry" will open. There are three worksheets under the title Trigonometric ratios and Complementary angles and kite problem.

Step - 2

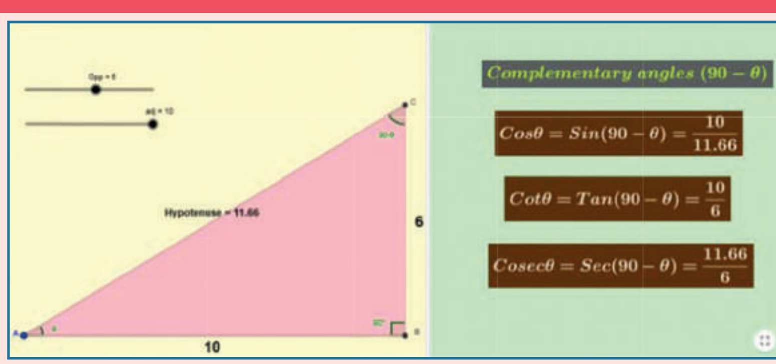
Move the sliders of the respective values to change the points and ratio. Work out the solution and check.

For the kite problem click on "NEW PROBLEM" to change the question and work it out. Click the check box for solution to check your answer.

Step 1



Step 2



Scan the QR Code.

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NATURAL SINES

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Difference				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
0	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	3	6	9	12	15
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	3	6	9	12	15
2	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506	3	6	9	12	15
3	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680	3	6	9	12	15
4	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854	3	6	9	12	15
5	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028	3	6	9	12	14
6	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201	3	6	9	12	14
7	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374	3	6	9	12	14
8	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547	3	6	9	12	14
9	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719	3	6	9	12	14
10	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891	3	6	9	12	14
11	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062	3	6	9	11	14
12	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2233	3	6	9	11	14
13	0.2250	0.2267	0.2284	0.2300	0.2317	0.2334	0.2351	0.2368	0.2385	0.2402	3	6	8	11	14
14	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571	3	6	8	11	14
15	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740	3	6	8	11	14
16	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907	3	6	8	11	14
17	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074	3	6	8	11	14
18	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239	3	6	8	11	14
19	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404	3	5	8	11	14
20	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567	3	5	8	11	14
21	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730	3	5	8	11	14
22	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891	3	5	8	11	14
23	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051	3	5	8	11	14
24	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210	3	5	8	11	13
25	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368	3	5	8	11	13
26	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524	3	5	8	10	13
27	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679	3	5	8	10	13
28	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833	3	5	8	10	13
29	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985	3	5	8	10	13
30	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135	3	5	8	10	13
31	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284	2	5	7	10	12
32	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432	2	5	7	10	12
33	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577	2	5	7	10	12
34	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721	2	5	7	10	12
35	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864	2	5	7	10	12
36	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004	2	5	7	9	12
37	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143	2	5	7	9	12
38	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280	2	5	7	9	11
39	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414	2	4	7	9	11
40	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547	2	4	7	9	11
41	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678	2	4	7	9	11
42	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807	2	4	6	9	11
43	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934	2	4	6	8	11
44	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059	2	4	6	8	10

NATURAL SINES

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Difference				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
45	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181	2	4	6	8	10
46	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302	2	4	6	8	10
47	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420	2	4	6	8	10
48	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536	2	4	6	8	10
49	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649	2	4	6	8	9
50	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760	2	4	6	7	9
51	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869	2	4	5	7	9
52	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976	2	4	5	7	9
53	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080	2	3	5	7	9
54	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181	2	3	5	7	8
55	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281	2	3	5	7	8
56	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377	2	3	5	6	8
57	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471	2	3	5	6	8
58	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563	2	3	5	6	8
59	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652	1	3	4	6	7
60	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738	1	3	4	6	7
61	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821	1	3	4	6	7
62	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902	1	3	4	5	7
63	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980	1	3	4	5	6
64	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056	1	3	4	5	6
65	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128	1	2	4	5	6
66	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198	1	2	3	5	6
67	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265	1	2	3	4	6
68	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330	1	2	3	4	5
69	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391	1	2	3	4	5
70	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449	1	2	3	4	5
71	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505	1	2	3	4	5
72	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558	1	2	3	3	4
73	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608	1	2	2	3	4
74	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655	1	2	2	3	4
75	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699	1	1	2	3	4
76	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740	1	1	2	3	3
77	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778	1	1	2	3	3
78	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813	1	1	2	2	3
79	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845	1	1	2	2	3
80	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874	0	1	1	2	2
81	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900	0	1	1	2	2
82	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923	0	1	1	2	2
83	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943	0	1	1	1	2
84	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960	0	1	1	1	2
85	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974	0	0	1	1	1
86	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985	0	0	1	1	1
87	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993	0	0	0	1	1
88	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998	0	0	0	0	0
89	0.9998	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0

NATURAL COSINES

(Numbers in mean difference columns to be subtracted, not added)

Degree	0´	6´	12´	18´	24´	30´	36´	42´	48´	54´	Mean Difference				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0	0	0	0	0
1	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995	0	0	0	0	0
2	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987	0	0	0	1	1
3	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977	0	0	1	1	1
4	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963	0	0	1	1	1
5	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0.9951	0.9949	0.9947	0	1	1	1	2
6	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928	0	1	1	1	2
7	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905	0	1	1	2	2
8	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9882	0.9880	0	1	1	2	2
9	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851	0	1	1	2	2
10	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820	1	1	2	2	3
11	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785	1	1	2	2	3
12	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748	1	1	2	3	3
13	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707	1	1	2	3	3
14	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664	1	1	2	3	4
15	0.9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617	1	2	2	3	4
16	0.9613	0.9608	0.9603	0.9598	0.9593	0.9588	0.9583	0.9578	0.9573	0.9568	1	2	2	3	4
17	0.9563	0.9558	0.9553	0.9548	0.9542	0.9537	0.9532	0.9527	0.9521	0.9516	1	2	3	3	4
18	0.9511	0.9505	0.9500	0.9494	0.9489	0.9483	0.9478	0.9472	0.9466	0.9461	1	2	3	4	5
19	0.9455	0.9449	0.9444	0.9438	0.9432	0.9426	0.9421	0.9415	0.9409	0.9403	1	2	3	4	5
20	0.9397	0.9391	0.9385	0.9379	0.9373	0.9367	0.9361	0.9354	0.9348	0.9342	1	2	3	4	5
21	0.9336	0.9330	0.9323	0.9317	0.9311	0.9304	0.9298	0.9291	0.9285	0.9278	1	2	3	4	5
22	0.9272	0.9265	0.9259	0.9252	0.9245	0.9239	0.9232	0.9225	0.9219	0.9212	1	2	3	4	6
23	0.9205	0.9198	0.9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0.9143	1	2	3	5	6
24	0.9135	0.9128	0.9121	0.9114	0.9107	0.9100	0.9092	0.9085	0.9078	0.9070	1	2	4	5	6
25	0.9063	0.9056	0.9048	0.9041	0.9033	0.9026	0.9018	0.9011	0.9003	0.8996	1	3	4	5	6
26	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.8934	0.8926	0.8918	1	3	4	5	6
27	0.8910	0.8902	0.8894	0.8886	0.8878	0.8870	0.8862	0.8854	0.8846	0.8838	1	3	4	5	7
28	0.8829	0.8821	0.8813	0.8805	0.8796	0.8788	0.8780	0.8771	0.8763	0.8755	1	3	4	6	7
29	0.8746	0.8738	0.8729	0.8721	0.8712	0.8704	0.8695	0.8686	0.8678	0.8669	1	3	4	6	7
30	0.8660	0.8652	0.8643	0.8634	0.8625	0.8616	0.8607	0.8599	0.8590	0.8581	1	3	4	6	7
31	0.8572	0.8563	0.8554	0.8545	0.8536	0.8526	0.8517	0.8508	0.8499	0.8490	2	3	5	6	8
32	0.8480	0.8471	0.8462	0.8453	0.8443	0.8434	0.8425	0.8415	0.8406	0.8396	2	3	5	6	8
33	0.8387	0.8377	0.8368	0.8358	0.8348	0.8339	0.8329	0.8320	0.8310	0.8300	2	3	5	6	8
34	0.8290	0.8281	0.8271	0.8261	0.8251	0.8241	0.8231	0.8221	0.8211	0.8202	2	3	5	7	8
35	0.8192	0.8181	0.8171	0.8161	0.8151	0.8141	0.8131	0.8121	0.8111	0.8100	2	3	5	7	8
36	0.8090	0.8080	0.8070	0.8059	0.8049	0.8039	0.8028	0.8018	0.8007	0.7997	2	3	5	7	9
37	0.7986	0.7976	0.7965	0.7955	0.7944	0.7934	0.7923	0.7912	0.7902	0.7891	2	4	5	7	9
38	0.7880	0.7869	0.7859	0.7848	0.7837	0.7826	0.7815	0.7804	0.7793	0.7782	2	4	5	7	9
39	0.7771	0.7760	0.7749	0.7738	0.7727	0.7716	0.7705	0.7694	0.7683	0.7672	2	4	6	7	9
40	0.7660	0.7649	0.7638	0.7627	0.7615	0.7604	0.7593	0.7581	0.7570	0.7559	2	4	6	8	9
41	0.7547	0.7536	0.7524	0.7513	0.7501	0.7490	0.7478	0.7466	0.7455	0.7443	2	4	6	8	10
42	0.7431	0.7420	0.7408	0.7396	0.7385	0.7373	0.7361	0.7349	0.7337	0.7325	2	4	6	8	10
43	0.7314	0.7302	0.7290	0.7278	0.7266	0.7254	0.7242	0.7230	0.7218	0.7206	2	4	6	8	10
44	0.7193	0.7181	0.7169	0.7157	0.7145	0.7133	0.7120	0.7108	0.7096	0.7083	2	4	6	8	10

NATURAL COSINES

(Numbers in mean difference columns to be subtracted, not added)

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Difference				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
45	0.7071	0.7059	0.7046	0.7034	0.7022	0.7009	0.6997	0.6984	0.6972	0.6959	2	4	6	8	10
46	0.6947	0.6934	0.6921	0.6909	0.6896	0.6884	0.6871	0.6858	0.6845	0.6833	2	4	6	8	11
47	0.6820	0.6807	0.6794	0.6782	0.6769	0.6756	0.6743	0.6730	0.6717	0.6704	2	4	6	9	11
48	0.6691	0.6678	0.6665	0.6652	0.6639	0.6626	0.6613	0.6600	0.6587	0.6574	2	4	7	9	11
49	0.6561	0.6547	0.6534	0.6521	0.6508	0.6494	0.6481	0.6468	0.6455	0.6441	2	4	7	9	11
50	0.6428	0.6414	0.6401	0.6388	0.6374	0.6361	0.6347	0.6334	0.6320	0.6307	2	4	7	9	11
51	0.6293	0.6280	0.6266	0.6252	0.6239	0.6225	0.6211	0.6198	0.6184	0.6170	2	5	7	9	11
52	0.6157	0.6143	0.6129	0.6115	0.6101	0.6088	0.6074	0.6060	0.6046	0.6032	2	5	7	9	12
53	0.6018	0.6004	0.5990	0.5976	0.5962	0.5948	0.5934	0.5920	0.5906	0.5892	2	5	7	9	12
54	0.5878	0.5864	0.5850	0.5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750	2	5	7	9	12
55	0.5736	0.5721	0.5707	0.5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606	2	5	7	10	12
56	0.5592	0.5577	0.5563	0.5548	0.5534	0.5519	0.5505	0.5490	0.5476	0.5461	2	5	7	10	12
57	0.5446	0.5432	0.5417	0.5402	0.5388	0.5373	0.5358	0.5344	0.5329	0.5314	2	5	7	10	12
58	0.5299	0.5284	0.5270	0.5255	0.5240	0.5225	0.5210	0.5195	0.5180	0.5165	2	5	7	10	12
59	0.5150	0.5135	0.5120	0.5105	0.5090	0.5075	0.5060	0.5045	0.5030	0.5015	3	5	8	10	13
60	0.5000	0.4985	0.4970	0.4955	0.4939	0.4924	0.4909	0.4894	0.4879	0.4863	3	5	8	10	13
61	0.4848	0.4833	0.4818	0.4802	0.4787	0.4772	0.4756	0.4741	0.4726	0.4710	3	5	8	10	13
62	0.4695	0.4679	0.4664	0.4648	0.4633	0.4617	0.4602	0.4586	0.4571	0.4555	3	5	8	10	13
63	0.4540	0.4524	0.4509	0.4493	0.4478	0.4462	0.4446	0.4431	0.4415	0.4399	3	5	8	10	13
64	0.4384	0.4368	0.4352	0.4337	0.4321	0.4305	0.4289	0.4274	0.4258	0.4242	3	5	8	11	13
65	0.4226	0.4210	0.4195	0.4179	0.4163	0.4147	0.4131	0.4115	0.4099	0.4083	3	5	8	11	13
66	0.4067	0.4051	0.4035	0.4019	0.4003	0.3987	0.3971	0.3955	0.3939	0.3923	3	5	8	11	14
67	0.3907	0.3891	0.3875	0.3859	0.3843	0.3827	0.3811	0.3795	0.3778	0.3762	3	5	8	11	14
68	0.3746	0.3730	0.3714	0.3697	0.3681	0.3665	0.3649	0.3633	0.3616	0.3600	3	5	8	11	14
69	0.3584	0.3567	0.3551	0.3535	0.3518	0.3502	0.3486	0.3469	0.3453	0.3437	3	5	8	11	14
70	0.3420	0.3404	0.3387	0.3371	0.3355	0.3338	0.3322	0.3305	0.3289	0.3272	3	5	8	11	14
71	0.3256	0.3239	0.3223	0.3206	0.3190	0.3173	0.3156	0.3140	0.3123	0.3107	3	6	8	11	14
72	0.3090	0.3074	0.3057	0.3040	0.3024	0.3007	0.2990	0.2974	0.2957	0.2940	3	6	8	11	14
73	0.2924	0.2907	0.2890	0.2874	0.2857	0.2840	0.2823	0.2807	0.2790	0.2773	3	6	8	11	14
74	0.2756	0.2740	0.2723	0.2706	0.2689	0.2672	0.2656	0.2639	0.2622	0.2605	3	6	8	11	14
75	0.2588	0.2571	0.2554	0.2538	0.2521	0.2504	0.2487	0.2470	0.2453	0.2436	3	6	8	11	14
76	0.2419	0.2402	0.2385	0.2368	0.2351	0.2334	0.2317	0.2300	0.2284	0.2267	3	6	8	11	14
77	0.2250	0.2233	0.2215	0.2198	0.2181	0.2164	0.2147	0.2130	0.2113	0.2096	3	6	9	11	14
78	0.2079	0.2062	0.2045	0.2028	0.2011	0.1994	0.1977	0.1959	0.1942	0.1925	3	6	9	11	14
79	0.1908	0.1891	0.1874	0.1857	0.1840	0.1822	0.1805	0.1788	0.1771	0.1754	3	6	9	11	14
80	0.1736	0.1719	0.1702	0.1685	0.1668	0.1650	0.1633	0.1616	0.1599	0.1582	3	6	9	12	14
81	0.1564	0.1547	0.1530	0.1513	0.1495	0.1478	0.1461	0.1444	0.1426	0.1409	3	6	9	12	14
82	0.1392	0.1374	0.1357	0.1340	0.1323	0.1305	0.1288	0.1271	0.1253	0.1236	3	6	9	12	14
83	0.1219	0.1201	0.1184	0.1167	0.1149	0.1132	0.1115	0.1097	0.1080	0.1063	3	6	9	12	14
84	0.1045	0.1028	0.1011	0.0993	0.0976	0.0958	0.0941	0.0924	0.0906	0.0889	3	6	9	12	14
85	0.0872	0.0854	0.0837	0.0819	0.0802	0.0785	0.0767	0.0750	0.0732	0.0715	3	6	9	12	15
86	0.0698	0.0680	0.0663	0.0645	0.0628	0.0610	0.0593	0.0576	0.0558	0.0541	3	6	9	12	15
87	0.0523	0.0506	0.0488	0.0471	0.0454	0.0436	0.0419	0.0401	0.0384	0.0366	3	6	9	12	15
88	0.0349	0.0332	0.0314	0.0297	0.0279	0.0262	0.0244	0.0227	0.0209	0.0192	3	6	9	12	15
89	0.0175	0.0157	0.0140	0.0122	0.0105	0.0087	0.0070	0.0052	0.0035	0.0017	3	6	9	12	15

NATURAL TANGENTS

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Difference				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
0	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	3	6	9	12	15
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	3	6	9	12	15
2	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507	3	6	9	12	15
3	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682	3	6	9	12	15
4	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857	3	6	9	12	15
5	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033	3	6	9	12	15
6	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210	3	6	9	12	15
7	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388	3	6	9	12	15
8	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566	3	6	9	12	15
9	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745	3	6	9	12	15
10	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926	3	6	9	12	15
11	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107	3	6	9	12	15
12	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290	3	6	9	12	15
13	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475	3	6	9	12	15
14	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661	3	6	9	12	16
15	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849	3	6	9	13	16
16	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038	3	6	9	13	16
17	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230	3	6	10	13	16
18	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424	3	6	10	13	16
19	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620	3	7	10	13	16
20	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819	3	7	10	13	17
21	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020	3	7	10	13	17
22	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224	3	7	10	14	17
23	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431	3	7	10	14	17
24	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642	4	7	11	14	18
25	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856	4	7	11	14	18
26	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073	4	7	11	15	18
27	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295	4	7	11	15	18
28	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520	4	8	11	15	19
29	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750	4	8	12	15	19
30	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985	4	8	12	16	20
31	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224	4	8	12	16	20
32	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469	4	8	12	16	20
33	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720	4	8	13	17	21
34	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976	4	9	13	17	21
35	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239	4	9	13	18	22
36	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508	5	9	14	18	23
37	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785	5	9	14	18	23
38	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069	5	9	14	19	24
39	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361	5	10	15	20	24
40	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662	5	10	15	20	25
41	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972	5	10	16	21	26
42	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293	5	11	16	21	27
43	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623	6	11	17	22	28
44	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965	6	11	17	23	29

NATURAL TANGENTS

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Difference				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
45	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319	6	12	18	24	30
46	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686	6	12	18	25	31
47	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067	6	13	19	25	32
48	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463	7	13	20	27	33
49	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875	7	14	21	28	34
50	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305	7	14	22	29	36
51	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753	8	15	23	30	38
52	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222	8	16	24	31	39
53	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713	8	16	25	33	41
54	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229	9	17	26	34	43
55	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770	9	18	27	36	45
56	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340	10	19	29	38	48
57	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941	10	20	30	40	50
58	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577	11	21	32	43	53
59	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251	11	23	34	45	56
60	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966	12	24	36	48	60
61	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728	13	26	38	51	64
62	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542	14	27	41	55	68
63	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413	15	29	44	58	73
64	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348	16	31	47	63	78
65	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355	17	34	51	68	85
66	2.2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445	18	37	55	73	92
67	2.3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627	20	40	60	79	99
68	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916	22	43	65	87	108
69	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326	24	47	71	95	119
70	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878	26	52	78	104	131
71	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595	29	58	87	116	145
72	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	3.2506	32	64	96	129	161
73	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646	36	72	108	144	180
74	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062	41	81	122	163	204
75	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812	46	93	139	186	232
76	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972	53	107	160	213	267
77	4.3315	4.3662	4.4015	4.4373	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646					
78	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970					
79	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140					
80	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432					
81	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264					
82	7.1154	7.2066	7.3002	7.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285					
83	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572					
84	9.5144	9.6768	9.8448	10.0187	10.1988	10.3854	10.5789	10.7797	10.9882	11.2048					
85	11.4301	11.6645	11.9087	12.1632	12.4288	12.7062	12.9962	13.2996	13.6174	13.9507					
86	14.3007	14.6685	15.0557	15.4638	15.8945	16.3499	16.8319	17.3432	17.8863	18.4645					
87	19.0811	19.7403	20.4465	21.2049	22.0217	22.9038	23.8593	24.8978	26.0307	27.2715					
88	28.6363	30.1446	31.8205	33.6935	35.8006	38.1885	40.9174	44.0661	47.7395	52.0807					
89	57.2900	63.6567	71.6151	81.8470	95.4895	114.5887	143.2371	190.9842	286.4777	572.9572					