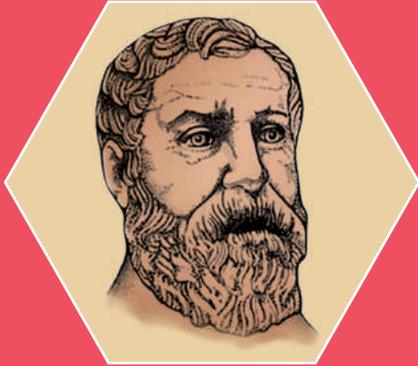


7



MENSURATION

The most beautiful plane figure is the circle and the most beautiful solid figure is the sphere. - Pythagoras.



Heron
A.D (C.E) 10-75

Heron of Alexandria was a Greek mathematician. He wrote books on mathematics, mechanics and physics. His famous book 'Metrica' consists of three volumes. This book shows the way to calculate area and volume of plane and solid figures. Heron has derived the formula for the area of triangle when three sides are given.

Learning Outcomes



- To use Heron's formula for calculating area of triangles and quadrilaterals.
- To find Total Surface Area (TSA), Lateral Surface Area (LSA) and Volume of cuboids and cubes.



7.1 Introduction

Mensuration is the branch of mathematics which deals with the study of areas and volumes of different kinds of geometrical shapes. In the broadest sense, it is all about the process of measurement.

Mensuration is used in the field of architecture, medicine, construction, etc. It is necessary for everyone to learn formulae used to find the perimeter and area of two dimensional figures as well as the surface area and volume of three dimensional solids in day to day life. In this chapter we deal with finding the area of triangles (using Heron's formula), surface area and volume of cuboids and cubes.

For a closed plane figure (a quadrilateral or a triangle), what do we call the distance around its boundary? What is the measure of the region covered inside the boundary?

In general, the area of a triangle is calculated by the formula

$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height} \text{ sq. units}$$

$$\text{That is, } A = \frac{1}{2} \times b \times h \text{ sq. units}$$

where, b is base and h is height of the triangle.

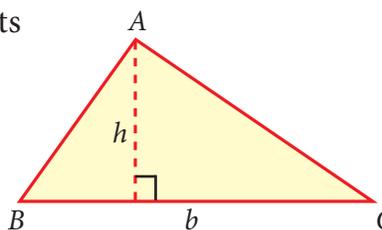


Fig. 7.1

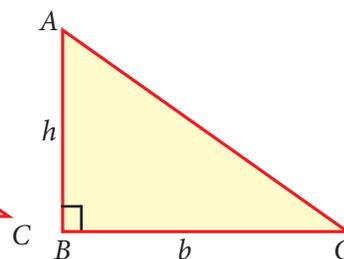


Fig. 7.2

From the above, we know how to find the area of a triangle when its 'base' and 'height' (that is altitude) are given.

7.2 Heron's Formula

How will you find the area of a triangle, if the height is not known but the lengths of the three sides are known?

For this, Heron has given a formula to find the area of a triangle.



If a , b and c are the sides of a triangle, then

the area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ sq.units.

where $s = \frac{a+b+c}{2}$, ' s ' is the semi-perimeter (that is half of the perimeter) of the triangle.

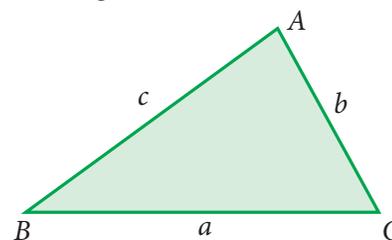


Fig. 7.3

Note



If we assume that the sides are of equal length that is $a = b = c$, then Heron's formula will be $\frac{\sqrt{3}}{4} a^2$ sq.units, which is the area of an equilateral triangle.

Example 7.1

The lengths of sides of a triangular field are 28 m, 15 m and 41 m. Calculate the area of the field. Find the cost of levelling the field at the rate of ₹ 20 per m^2 .

Solution

Let $a = 28$ m, $b = 15$ m and $c = 41$ m

$$\text{Then, } s = \frac{a+b+c}{2} = \frac{28+15+41}{2} = \frac{84}{2} = 42 \text{ m}$$

$$\begin{aligned} \text{Area of triangular field} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{42(42-28)(42-15)(42-41)} \\ &= \sqrt{42 \times 14 \times 27 \times 1} \end{aligned}$$

$$\begin{aligned}
 &= \sqrt{2 \times 3 \times 7 \times 7 \times 2 \times 3 \times 3 \times 3 \times 1} \\
 &= 2 \times 3 \times 7 \times 3 \\
 &= 126 \text{ m}^2
 \end{aligned}$$

Given the cost of levelling is ₹ 20 per m^2 .

The total cost of levelling the field = $20 \times 126 = ₹ 2520$.

Example 7.2

Three different triangular plots are available for sale in a locality. Each plot has a perimeter of 120 m. The side lengths are also given:

| Shape of plot | Perimeter | Length of sides |
|-----------------------|-----------|------------------|
| Right angled triangle | 120 m | 30 m, 40 m, 50 m |
| Acute angled triangle | 120 m | 35 m, 40 m, 45 m |
| Equilateral triangle | 120 m | 40 m, 40 m, 40 m |

Help the buyer to decide which among these will be more spacious.

Solution

For clarity, let us draw a rough figure indicating the measurements:

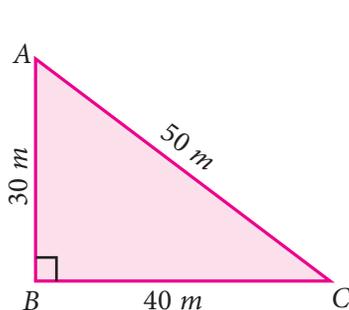


Fig. 7.4

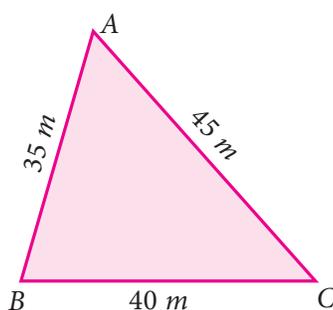


Fig. 7.5

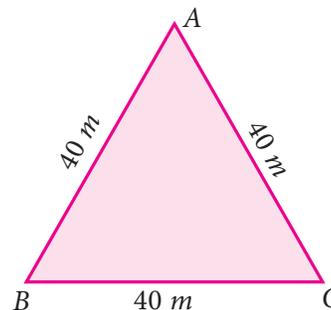


Fig. 7.6

(i) The semi-perimeter of Fig.7.4, $s = \frac{30 + 40 + 50}{2} = 60 \text{ m}$

Fig.7.5, $s = \frac{35 + 40 + 45}{2} = 60 \text{ m}$

Fig.7.6, $s = \frac{40 + 40 + 40}{2} = 60 \text{ m}$

Note that all the semi-perimeters are equal.

(ii) Area of triangle using Heron's formula:

In Fig.7.4, Area of triangle = $\sqrt{60(60 - 30)(60 - 40)(60 - 50)}$
 $= \sqrt{60 \times 30 \times 20 \times 10}$



$$= \sqrt{30 \times 2 \times 30 \times 2 \times 10 \times 10}$$

$$= 600 \text{ m}^2$$

In Fig.7.5, Area of triangle

$$= \sqrt{60(60 - 35)(60 - 40)(60 - 45)}$$

$$= \sqrt{60 \times 25 \times 20 \times 15}$$

$$= \sqrt{20 \times 3 \times 5 \times 5 \times 20 \times 3 \times 5}$$

$$= 300\sqrt{5} \quad (\text{Since } \sqrt{5} = 2.236)$$

$$= 670.8 \text{ m}^2$$

In Fig.7.6, Area of triangle

$$= \sqrt{60(60 - 40)(60 - 40)(60 - 40)}$$

$$= \sqrt{60 \times 20 \times 20 \times 20}$$

$$= \sqrt{3 \times 20 \times 20 \times 20 \times 20}$$

$$= 400\sqrt{3} \quad (\text{Since } \sqrt{3} = 1.732)$$

$$= 692.8 \text{ m}^2$$

We find that though the perimeters are same, the areas of the three triangular plots are different. The area of the triangle in Fig 7.6 is the greatest among these; the buyer can be suggested to choose this since it is more spacious.

Note



If the perimeter of different types of triangles have the same value, among all the types of triangles, the equilateral triangle possess the greatest area. We will learn more about maximum areas in higher classes.

7.3 Application of Heron's Formula in Finding Areas of Quadrilaterals

A plane figure bounded by four line segments is called a **quadrilateral**.

Let $ABCD$ be a quadrilateral. To find the area of a quadrilateral, we divide the quadrilateral into two triangular parts and use Heron's formula to calculate the area of the triangular parts.

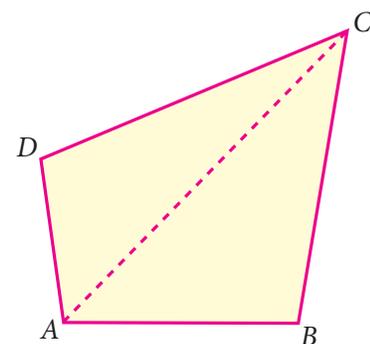


Fig. 7.7

In Fig 7.7,

$$\text{Area of quadrilateral } ABCD = \text{Area of triangle } ABC + \text{Area of triangle } ACD$$



Example 7.3

A farmer has a field in the shape of a rhombus. The perimeter of the field is $400m$ and one of its diagonal is $120m$. He wants to divide the field into two equal parts to grow two different types of vegetables. Find the area of the field.

Solution

Let $ABCD$ be the rhombus.

Its perimeter = $4 \times \text{side} = 400 m$

Therefore, each side of the rhombus = $100 m$

Given the length of the diagonal $AC = 120 m$

In $\triangle ABC$, let $a = 100 m$, $b = 100 m$, $c = 120 m$

$$s = \frac{a+b+c}{2} = \frac{100+100+120}{2} = 160 m$$

$$\begin{aligned} \text{Area of } \triangle ABC &= \sqrt{160(160-100)(160-100)(160-120)} \\ &= \sqrt{160 \times 60 \times 60 \times 40} \\ &= \sqrt{40 \times 2 \times 2 \times 60 \times 60 \times 40} \\ &= 40 \times 2 \times 60 = 4800 m^2 \end{aligned}$$

Therefore, Area of the field $ABCD = 2 \times \text{Area of } \triangle ABC = 2 \times 4800 = 9600 m^2$

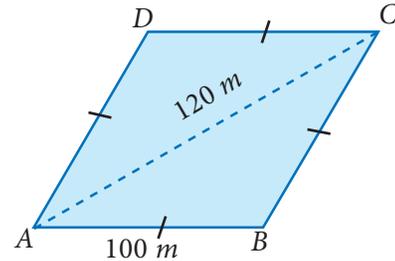
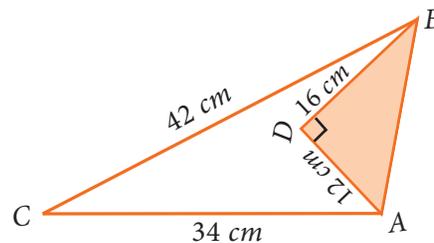


Fig. 7.8



Exercise 7.1

- Using Heron's formula, find the area of a triangle whose sides are
(i) $10 cm$, $24 cm$, $26 cm$ (ii) $1.8m$, $8 m$, $8.2 m$
- The sides of the triangular ground are $22 m$, $120 m$ and $122 m$. Find the area and cost of levelling the ground at the rate of ₹ 20 per m^2 .
- The perimeter of a triangular plot is $600 m$. If the sides are in the ratio $5:12:13$, then find the area of the plot.
- Find the area of an equilateral triangle whose perimeter is $180 cm$.
- An advertisement board is in the form of an isosceles triangle with perimeter $36m$ and each of the equal sides are $13 m$. Find the cost of painting it at ₹ 17.50 per square metre.
- Find the area of the unshaded region.



7. Find the area of a quadrilateral $ABCD$ whose sides are $AB = 13\text{ cm}$, $BC = 12\text{ cm}$, $CD = 9\text{ cm}$, $AD = 14\text{ cm}$ and diagonal $BD = 15\text{ cm}$.
8. A park is in the shape of a quadrilateral. The sides of the park are 15 m , 20 m , 26 m and 17 m and the angle between the first two sides is a right angle. Find the area of the park.
9. A land is in the shape of rhombus. The perimeter of the land is 160 m and one of the diagonal is 48 m . Find the area of the land.
10. The adjacent sides of a parallelogram measures 34 m , 20 m and the measure of one of the diagonal is 42 m . Find the area of parallelogram.

7.4 Surface Area of Cuboid and Cube

We have learnt in the earlier classes about 3-Dimension structures. The 3D shapes are those which do not lie completely in a plane. Any 3D shape has dimensions namely length, breadth and height.



7.4.1 Cuboid and its Surface Area

Cuboid: A cuboid is a closed solid figure bounded by six rectangular plane regions. For example, match box, Brick, Book.

A cuboid has 6 faces, 12 edges and 8 vertices. Ultimately, a cuboid has the shape of a rectangular box.

Total Surface Area (TSA) of a cuboid is the sum of the areas of all the faces that enclose the cuboid. If we leave out the areas of the top and bottom of the cuboid we get what is known as its **Lateral Surface Area (LSA)**.

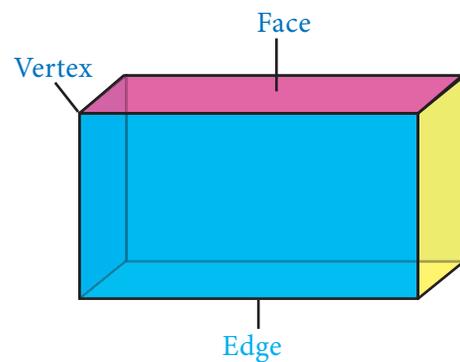


Fig. 7.9

In the Fig 7.10, l , b and h represents length, breadth and height respectively.

(i) Total Surface Area (TSA) of a cuboid

| | |
|----------------------|---------------|
| Top and bottom | $2 \times lb$ |
| Front and back | $2 \times bh$ |
| Left and Right sides | $2 \times lh$ |

$$= 2 (lb + bh + lh) \text{ sq. units.}$$

(ii) Lateral Surface Area (LSA) of a cuboid

| | |
|----------------------|---------------|
| Front and back | $2 \times bh$ |
| Left and Right sides | $2 \times lh$ |

$$= 2 (l+b)h \text{ sq. units.}$$

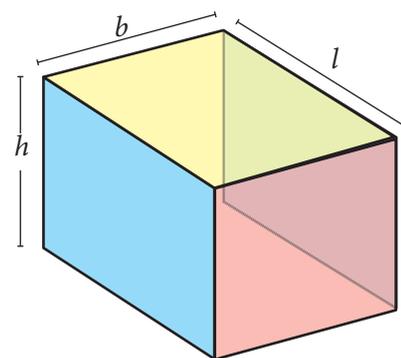


Fig. 7.10

We are using the concept of Lateral Surface Area (LSA) and Total Surface Area (TSA) in real life situations. For instance a room can be cuboidal in shape that has different length, breadth and height. If we require to find areas of only the walls of a room, avoiding floor and ceiling then we can use LSA. However if we want to find the surface area of the whole room then we have to calculate the TSA.

If the length, breadth and height of a cuboid are l , b and h respectively. Then

(i) Total Surface Area = $2(lb + bh + lh)$ sq.units.

(ii) Lateral Surface Area = $2(l+b)h$ sq.units.

Note



- The top and bottom area in a cuboid is independent of height. The total area of top and bottom is $2lb$. Hence LSA is obtained by removing $2lb$ from $2(lb+bh+lh)$.
- The units of length, breadth and height should be same while calculating surface area of the cuboid.

Example 7.4

Find the TSA and LSA of a cuboid whose length, breadth and height are 7.5 m , 3 m and 5 m respectively.

Solution

Given the dimensions of the cuboid;

that is length (l) = 7.5 m , breadth (b) = 3 m and height (h) = 5 m .

$$\begin{aligned} \text{TSA} &= 2(lb + bh + lh) \\ &= 2[(7.5 \times 3) + (3 \times 5) + (7.5 \times 5)] \\ &= 2(22.5 + 15 + 37.5) \\ &= 2 \times 75 \\ &= 150\text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{LSA} &= 2(l + b) \times h \\ &= 2(7.5 + 3) \times 5 \\ &= 2 \times 10.5 \times 5 \\ &= 105\text{ m}^2 \end{aligned}$$

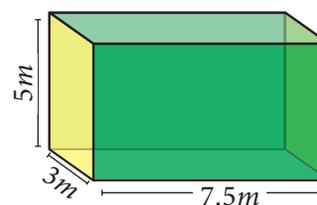


Fig. 7.11

Example 7.5

The length, breadth and height of a hall are 25 m, 15 m and 5 m respectively. Find the cost of renovating its floor and four walls at the rate of ₹80 per m^2 .

Solution

Here, length (l) = 25 m, breadth (b) = 15 m, height (h) = 5 m.

Area of four walls = LSA of cuboid

$$\begin{aligned} &= 2(l + b) \times h \\ &= 2(25 + 15) \times 5 \\ &= 80 \times 5 = 400 \text{ m}^2 \end{aligned}$$

Area of the floor = $l \times b$

$$\begin{aligned} &= 25 \times 15 \\ &= 375 \text{ m}^2 \end{aligned}$$



Fig. 7.12

Total renovating area of the hall

$$\begin{aligned} &= (\text{Area of four walls} + \text{Area of the floor}) \\ &= (400 + 375) \text{ m}^2 = 775 \text{ m}^2 \end{aligned}$$

Therefore, cost of renovating at the rate of ₹80 per m^2 = 80×775

$$= ₹ 62,000$$

7.4.2 Cube and its Surface Area

Cube: A cuboid whose length, breadth and height are all equal is called as a **cube**.

That is a cube is a solid having six square faces. Here are some real-life examples.



Dice



Ice cubes



Sugar cubes

Fig. 7.13

A cube being a cuboid has 6 faces, 12 edges and 8 vertices.

Consider a cube whose sides are 'a' units as shown in the Fig 7.14. Now,

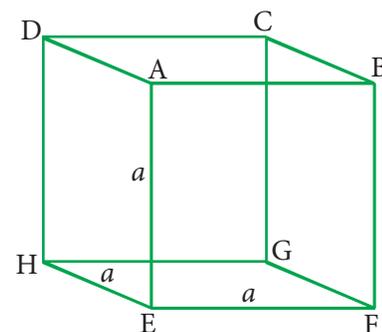


Fig. 7.14

(i) Total Surface Area of the cube

= sum of area of the faces ($ABCD+EFGH+AEHD+BFGC+ABFE+CDHG$)

$$=(a^2 + a^2 + a^2 + a^2 + a^2 + a^2)$$

$$=6a^2 \text{ sq. units}$$

(ii) Lateral Surface Area of the cube

= sum of area of the faces ($AEHD+BFGC+ABFE+CDHG$)

$$=(a^2 + a^2 + a^2 + a^2)$$

$$=4a^2 \text{ sq. units}$$

If the side of a cube is a units, then,

(i) The Total Surface Area = $6a^2$ sq.units

(ii) The Lateral Surface Area = $4a^2$ sq.units

Thinking Corner



Can you get these formulae from the corresponding formula of Cuboid?

Example 7.6

Find the Total Surface Area and Lateral Surface Area of the cube, whose side is 5 cm .

Solution

The side of the cube (a) = 5 cm

Total Surface Area = $6a^2 = 6(5^2) = 150 \text{ sq. cm}$

Lateral Surface Area = $4a^2 = 4(5^2) = 100 \text{ sq. cm}$

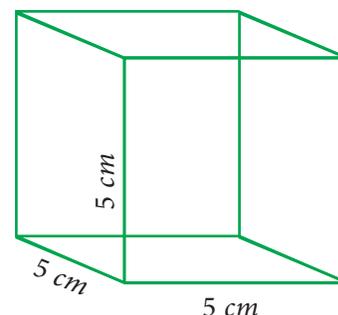


Fig. 7.15

Example 7.7

A cube has the Total Surface Area of 486 cm^2 . Find its lateral surface area.

Solution

Here, Total Surface Area of the cube = 486 cm^2

$$6a^2 = 486 \Rightarrow a^2 = \frac{486}{6} \text{ and so, } a^2 = 81. \text{ This gives } a = 9.$$

The side of the cube = 9 cm

$$\text{Lateral Surface Area} = 4a^2 = 4 \times 9^2 = 4 \times 81 = 324 \text{ cm}^2$$

Example 7.8

Two identical cubes of side 7 cm are joined end to end. Find the Total and Lateral surface area of the new resulting cuboid.

Solution

Side of a cube = 7 cm

Now length of the resulting cuboid (l) = $7+7 = 14\text{ cm}$

Breadth (b) = 7 cm , Height (h) = 7 cm

So, Total Surface Area = $2(lb + bh + lh)$

$$= 2[(14 \times 7) + (7 \times 7) + (14 \times 7)]$$

$$= 2(98 + 49 + 98)$$

$$= 2 \times 245$$

$$= 490\text{ cm}^2$$

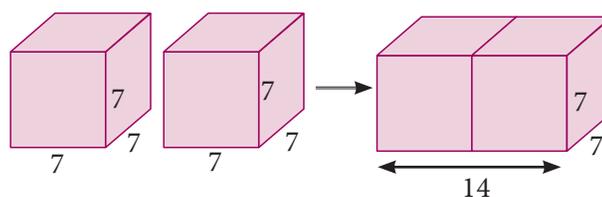


Fig. 7.16

Lateral Surface Area = $2(l + b) \times h$

$$= 2(14 + 7) \times 7 = 2 \times 21 \times 7$$

$$= 294\text{ cm}^2$$



Exercise 7.2

1. Find the Total Surface Area and the Lateral Surface Area of a cuboid whose dimensions are: length = 20 cm , breadth = 15 cm and height = 8 cm
2. The dimensions of a cuboidal box are $6\text{ m} \times 400\text{ cm} \times 1.5\text{ m}$. Find the cost of painting its entire outer surface at the rate of ₹22 per m^2 .
3. The dimensions of a hall is $10\text{ m} \times 9\text{ m} \times 8\text{ m}$. Find the cost of white washing the walls and ceiling at the rate of ₹8.50 per m^2 .
4. Find the TSA and LSA of the cube whose side is (i) 8 m (ii) 21 cm (iii) 7.5 cm
5. If the total surface area of a cube is 2400 cm^2 then, find its lateral surface area.
6. A cubical container of side 6.5 m is to be painted on the entire outer surface. Find the area to be painted and the total cost of painting it at the rate of ₹24 per m^2 .
7. Three identical cubes of side 4 cm are joined end to end. Find the total surface area and lateral surface area of the new resulting cuboid.

7.5 Volume of Cuboid and Cube

All of us have tasted 50 ml and 100 ml of ice cream. Take one such 100 ml ice cream cup. This cup can contain 100 ml of water, which means that the capacity or volume of that cup is 100 ml. Take a 100 ml cup and find out how many such cups of water can fill a jug. If 10 such 100 ml cups can fill a jug then the capacity or volume of the jug is 1 litre ($10 \times 100 \text{ ml} = 1000 \text{ ml} = 1 \text{ l}$). Further check how many such jug of water can fill a bucket. That is the capacity or volume of the bucket. Likewise we can calculate the volume or capacity of any such things.



Volume is the measure of the amount of space occupied by a three dimensional solid. Cubic centimetres (cm^3), cubic metres (m^3) are some cubic units to measure volume.

Note

Unit Cube :

A cube with side 1 unit.

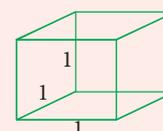


Fig. 7.17

Volume of the solid is the product of 'base area' and 'height'. This can easily be understood from a practical situation. You might have seen the bundles of A4 size paper. Each paper is rectangular in shape and has an area ($=lb$). When you pile them up, it becomes a bundle in the form of a cuboid; h times lb make the cuboid.

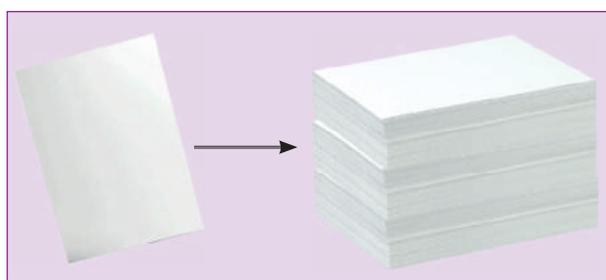


Fig. 7.18

7.5.1 Volume of a Cuboid

Let the length, breadth and height of a cuboid be l , b and h respectively.

Then, volume of the cuboid

$$\begin{aligned} V &= (\text{cuboid's base area}) \times \text{height} \\ &= (l \times b) \times h = lbh \text{ cubic units} \end{aligned}$$

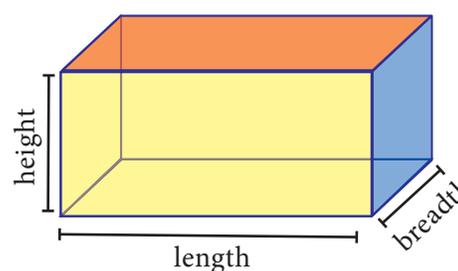


Fig. 7.19

Note

The units of length, breadth and height should be same while calculating the volume of a cuboid.

Example 7.9

The length, breadth and height of a cuboid is 120 mm , 10 cm and 8 cm respectively. Find the volume of 10 such cuboids.

Solution

Since both breadth and height are given in cm , it is necessary to convert the length also in cm .

So we get, $l = 120\text{ mm} = \frac{120}{10} = 12\text{ cm}$ and take $b = 10\text{ cm}$, $h = 8\text{ cm}$ as such.

$$\begin{aligned}\text{Volume of a cuboid} &= l \times b \times h \\ &= 12 \times 10 \times 8\end{aligned}$$

$$= 960\text{ cm}^3$$

$$\begin{aligned}\text{Volume of 10 such cuboids} &= 10 \times 960 \\ &= 9600\text{ cm}^3\end{aligned}$$

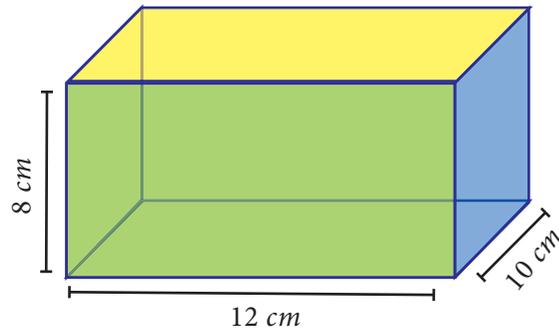


Fig. 7.20

Example 7.10

The length, breadth and height of a cuboid are in the ratio $7:5:2$. Its volume is 35840 cm^3 . Find its dimensions.

Solution

Let the dimensions of the cuboid be

$$l = 7x, b = 5x \text{ and } h = 2x.$$

Given that volume of cuboid = 35840 cm^3

$$l \times b \times h = 35840$$

$$(7x)(5x)(2x) = 35840$$

$$70x^3 = 35840$$

$$x^3 = \frac{35840}{70}$$

$$x^3 = 512$$

$$x = \sqrt[3]{8 \times 8 \times 8}$$

$$x = 8\text{ cm}$$

Length of cuboid = $7x = 7 \times 8 = 56\text{ cm}$

Breadth of cuboid = $5x = 5 \times 8 = 40\text{ cm}$

Height of cuboid = $2x = 2 \times 8 = 16\text{ cm}$

THINKING CORNER



Each cuboid given below has the same volume 120 cm^3 . Can you find the missing dimensions?

(i)

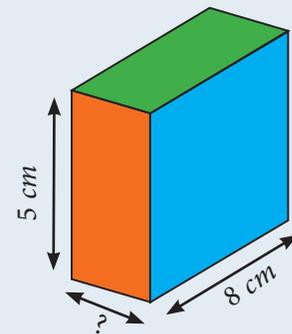


Fig. 7.21

(ii)

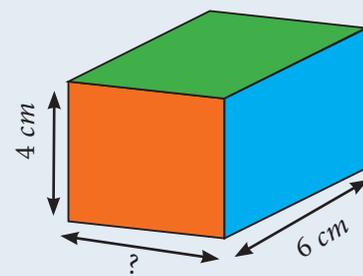


Fig. 7.22

Example 7.11

The dimensions of a fish tank are $3.8\text{ m} \times 2.5\text{ m} \times 1.6\text{ m}$. How many litres of water it can hold?

Solution

Length of the fish tank $l = 3.8\text{ m}$

Breadth of the fish tank $b = 2.5\text{ m}$, Height of the fish tank $h = 1.6\text{ m}$

$$\begin{aligned}\text{Volume of the fish tank} &= l \times b \times h \\ &= 3.8 \times 2.5 \times 1.6 \\ &= 15.2\text{ m}^3 \\ &= 15.2 \times 1000\text{ litres} \\ &= 15200\text{ litres}\end{aligned}$$



Fig. 7.23

Note

A few important conversions
 $1\text{ cm}^3 = 1\text{ ml}$, $1000\text{ cm}^3 = 1\text{ litre}$,
 $1\text{ m}^3 = 1000\text{ litres}$

Example 7.12

The dimensions of a sweet box are $22\text{ cm} \times 18\text{ cm} \times 10\text{ cm}$. How many such boxes can be packed in a carton of dimensions $1\text{ m} \times 88\text{ cm} \times 63\text{ cm}$?

Solution

Here, the dimensions of a sweet box are Length (l) = 22cm, breadth (b) = 18cm, height (h) = 10 cm.

$$\begin{aligned}\text{Volume of a sweet box} &= l \times b \times h \\ &= 22 \times 18 \times 10\text{ cm}^3\end{aligned}$$

The dimensions of a carton are

Length (l) = $1\text{ m} = 100\text{ cm}$, breadth (b) = 88 cm
height (h) = 63 cm.

$$\begin{aligned}\text{Volume of the carton} &= l \times b \times h \\ &= 100 \times 88 \times 63\text{ cm}^3\end{aligned}$$

$$\begin{aligned}\text{The number of sweet boxes packed} &= \frac{\text{volume of the carton}}{\text{volume of a sweet box}} \\ &= \frac{100 \times 88 \times 63}{22 \times 18 \times 10} \\ &= 140\text{ boxes}\end{aligned}$$



Fig. 7.24

7.5.2 Volume of a Cube

It is easy to get the volume of a cube whose side is a units. Simply put $l = b = h = a$ in the formula for the volume of a cuboid. We get volume of cube to be a^3 cubic units.

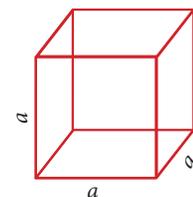


Fig. 7.25

If the side of a cube is 'a' units then the Volume of the cube (V) = a^3 cubic units.

Note



For any two cubes, the following results are true.

- Ratio of surface areas = (Ratio of sides)²
- Ratio of volumes = (Ratio of sides)³
- (Ratio of surface areas)³ = (Ratio of volumes)²

Example 7.13

Find the volume of cube whose side is 10 cm.

Solution

Given that side (a) = 10 cm

$$\begin{aligned}\text{volume of the cube} &= a^3 \\ &= 10 \times 10 \times 10 \\ &= 1000 \text{ cm}^3\end{aligned}$$

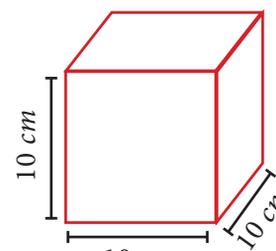


Fig. 7.26

Example 7.14

A cubical tank can hold 64,000 litres of water. Find the length of its side in metres.

Solution

Let 'a' be the side of cubical tank.

Here, volume of the tank = 64,000 litres

$$\text{i.e., } a^3 = 64,000 = \frac{64000}{1000} \text{ [since, } 1000 \text{ litres} = 1 \text{ m}^3 \text{]}$$

$$a^3 = 64 \text{ m}^3$$

$$a = \sqrt[3]{64} \quad a = 4 \text{ m}$$

Therefore, length of the side of the tank is 4 metres.

Example 7.15

The side of a metallic cube is 12 cm. It is melted and formed into a cuboid whose length and breadth are 18 cm and 16 cm respectively. Find the height of the cuboid.

Solution

Cube

Side (a) = 12 cm

Cuboid

length (l) = 18 cm

breadth (b) = 16 cm

height (h) = ?

Here, Volume of the Cuboid = Volume of the Cube

$$l \times b \times h = a^3$$

$$18 \times 16 \times h = 12 \times 12 \times 12$$

$$h = \frac{12 \times 12 \times 12}{18 \times 16}$$

$$h = 6\text{ cm}$$

Therefore, the height of the cuboid is 6 cm .

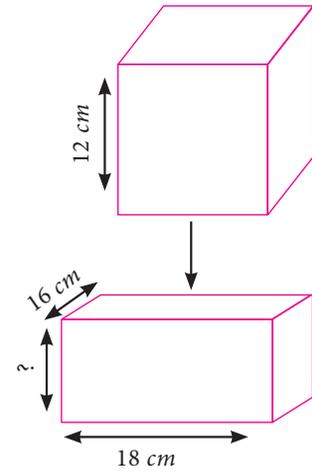


Fig. 7.27



Activity

Take some square sheets of paper / chart paper of given dimension $18\text{ cm} \times 18\text{ cm}$. Remove the squares of same sizes from each corner of the given square paper and fold up the flaps to make an open cuboidal box. Then tabulate the dimensions of each of the cuboidal boxes made. Also find the volume each time and complete the table. The side measures of corner squares that are to be removed is given in the table below.

| Side of the corner square | Dimensions of boxes | | | Volume V |
|---------------------------|---------------------|-----|-----|------------|
| | l | b | h | |
| 2 cm | | | | |
| 3 cm | | | | |
| 4 cm | | | | |
| 5 cm | | | | |

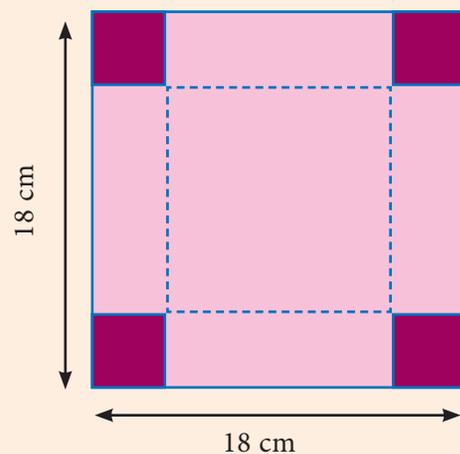


Fig. 7.28

Observe the above table and answer the following:

- (i) What is the greatest possible volume?
- (ii) What is the side of the square that when removed produces the greatest volume?



Exercise 7.3

- Find the volume of a cuboid whose dimensions are
 - length = 12 cm, breadth = 8 cm, height = 6 cm
 - length = 60 m, breadth = 25 m, height = 1.5 m
- The dimensions of a match box are 6 cm × 3.5 cm × 2.5 cm. Find the volume of a packet containing 12 such match boxes.
- The length, breadth and height of a chocolate box are in the ratio 5:4:3. If its volume is 7500 cm³, then find its dimensions.
- The length, breadth and depth of a pond are 20.5 m, 16 m and 8 m respectively. Find the capacity of the pond in litres.
- The dimensions of a brick are 24 cm × 12 cm × 8 cm. How many such bricks will be required to build a wall of 20 m length, 48 cm breadth and 6 m height?
- The volume of a container is 1440 m³. The length and breadth of the container are 15 m and 8 m respectively. Find its height.
- Find the volume of a cube each of whose side is (i) 5 cm (ii) 3.5 m (iii) 21 cm
- A cubical milk tank can hold 125000 litres of milk. Find the length of its side in metres.
- A metallic cube with side 15 cm is melted and formed into a cuboid. If the length and height of the cuboid is 25 cm and 9 cm respectively then find the breadth of the cuboid.



Exercise 7.4



Multiple choice questions

- The semi-perimeter of a triangle having sides 15 cm, 20 cm and 25 cm is
 - 60 cm
 - 45 cm
 - 30 cm
 - 15 cm
- If the sides of a triangle are 3 cm, 4 cm and 5 cm, then the area is
 - 3 cm²
 - 6 cm²
 - 9 cm²
 - 12 cm²
- The perimeter of an equilateral triangle is 30 cm. The area is
 - 10√3 cm²
 - 12√3 cm²
 - 15√3 cm²
 - 25√3 cm²
- The lateral surface area of a cube of side 12 cm is
 - 144 cm²
 - 196 cm²
 - 576 cm²
 - 664 cm²





5. If the lateral surface area of a cube is 600 cm^2 , then the total surface area is
(1) 150 cm^2 (2) 400 cm^2 (3) 900 cm^2 (4) 1350 cm^2
6. The total surface area of a cuboid with dimension $10 \text{ cm} \times 6 \text{ cm} \times 5 \text{ cm}$ is
(1) 280 cm^2 (2) 300 cm^2 (3) 360 cm^2 (4) 600 cm^2
7. If the ratio of the sides of two cubes are 2:3, then ratio of their surface areas will be
(1) 4:6 (2) 4:9 (3) 6:9 (4) 16:36
8. The volume of a cuboid is 660 cm^3 and the area of the base is 33 cm^2 . Its height is
(1) 10 cm (2) 12 cm (3) 20 cm (4) 22 cm
9. The capacity of a water tank of dimensions $10 \text{ m} \times 5 \text{ m} \times 1.5 \text{ m}$ is
(1) 75 litres (2) 750 litres (3) 7500 litres (4) 75000 litres
10. The number of bricks each measuring $50 \text{ cm} \times 30 \text{ cm} \times 20 \text{ cm}$ that will be required to build a wall whose dimensions are $5 \text{ m} \times 3 \text{ m} \times 2 \text{ m}$ is
(1) 1000 (2) 2000 (3) 3000 (4) 5000

Points to Remember

- If a , b and c are the sides of a triangle, then the area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ sq.units, where $s = \frac{a+b+c}{2}$.
- If the length, breadth and height of the cuboid are l , b and h respectively, then
 - (i) Total Surface Area(TSA) $= 2(lb + bh + lh)$ sq.units
 - (ii) Lateral Surface Area(LSA) $= 2(l+b)h$ sq.units
- If the side of a cube is ' a ' units, then
 - (i) Total Surface Area(TSA) $= 6a^2$ sq.units
 - (ii) Lateral Surface Area(LSA) $= 4a^2$ sq.units
- If the length, breadth and height of the cuboid are l , b and h respectively, then the Volume of the cuboid $(V) = lbh$ cu.units
- If the side of a cube is ' a ' units then, the Volume of the cube $(V) = a^3$ cu.units.





ICT Corner

Expected Result is shown
in this picture

New Problem [Click here for New Problem](#)

Find the Volume and Surface Area of a Cuboid with Length 6 units, Breadth 4 Units, and Height 5 Units.

Solution: Length = $l = 6$ units
Breadth = $b = 4$ units
Height = $h = 5$ units

Volume = $l \times b \times h$ Cubic Units
 Next = $6 \times 4 \times 5 = 120$ cubic units

Lateral Surface Area = $2(lb + bh)$ Square units
 Next = $2(6 \times 4 + 4 \times 5)$ Sq. Units = 100 Sq. Units

Total Surface Area = $2(lb + bh + lh)$ Square units
 Next = $2(6 \times 4 + 4 \times 5 + 6 \times 5)$ Square units = 148 Square Units

Volume and Surface Area of a Cuboid

Step - 1

Open the Browser by typing the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named “Mensuration” will open. There are two worksheets under the title CUBE and CUBOID.

Step - 2

Click on “New Problem”. Volume, Lateral surface and Total surface area are asked. Work out the solution, and click on the respective check box and check the answer.

Step 1

New Problem [Click here for New Problem](#)

Find the Volume and Surface Area of a Cube with side 4 units.

Solution: Side = $S = 4$ units

Volume = $S^3 = S \times S \times S$ Cubic Units
 Next

Lateral Surface Area = $4S^2$ Square units
 Next

Total Surface Area = $6S^2$ Square units
 Next

Volume and Surface Area of a Cube

Step 2

New Problem [Click here for New Problem](#)

Find the Volume and Surface Area of a Cuboid with Length 6 units, Breadth 6 Units, and Height 2 Units.

Solution: Length = $l = 6$ units
Breadth = $b = 6$ units
Height = $h = 2$ units

Volume = $l \times b \times h$ Cubic Units
 Next

Lateral Surface Area = $2(lb + bh)$ Square units
 Next

Total Surface Area = $2(lb + bh + lh)$ Square units
 Next

Volume and Surface Area of a Cuboid

Scan the QR Code.



B566_9_MAT_EM_T3