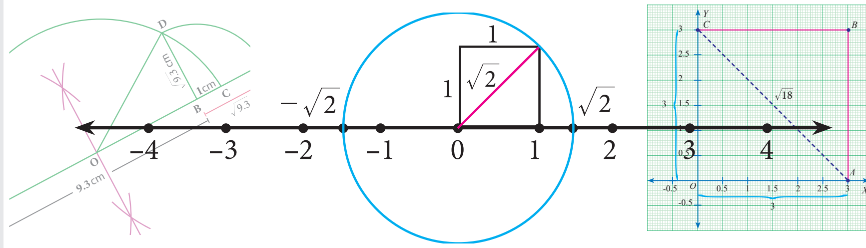


2 REAL NUMBERS

"When I consider what people generally want in calculating, I found that it always is a number"

- Al-Khwarizmi

Al-Khwarizmi
(A D (CE) 780 - 850)

Al-Khwarizmi, a Persian scholar, is credited with identification of surds as something noticeable in mathematics. He referred to the irrational numbers as 'inaudible', which was later translated to the Latin word 'surdus' (meaning 'deaf' or 'mute'). In mathematics, a surd came to mean a root (radical) that cannot be expressed (spoken) as a rational number.

E4JDHU

Learning Outcomes

- To know that there exists infinitely many rational numbers between two given rational numbers.
- To represent rational and irrational numbers on number line and express them in decimal form.
- To visualize the real numbers on the number line.
- To identify surds.
- To carry out basic operations of addition, subtraction, multiplication and division using surds.
- To rationalise denominators of surds.
- To understand the scientific notation.

2.1 Introduction

Numbers, numbers, everywhere!

- Do you have a phone at home? How many digits does its dial have?
- What is the Pin code of your locality? How is it useful?
- When you park a vehicle, do you get a 'token'? What is its purpose?
- Have you handled 24 'carat' gold? How do you decide its purity?
- How high is the 'power' of your spectacles?

- How much water does the overhead tank in your house can hold?
- Does your friend have fever? What is his body temperature?

You have learnt about many types of numbers so far. Now is the time to extend the ideas further.

2.2 Rational Numbers

When you want to count the number of books in your cupboard, you start with 1, 2, 3, … and so on. These counting numbers 1, 2, 3, … , are called Natural numbers. You know to show these numbers on a line (see Fig. 2.1).

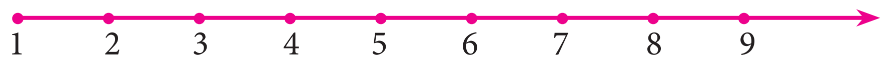


Fig. 2.1

We use $\mathbb{N}$ to denote the set of all **Natural numbers**.

$$\mathbb{N} = \{ 1, 2, 3, \ldots \}$$

If the cupboard is empty (since no books are there). To denote such a situation we use the symbol 0. Including zero as a digit you can now consider the numbers 0, 1, 2, 3, … and call them **Whole numbers**. With this additional entity, the number line will look as shown below

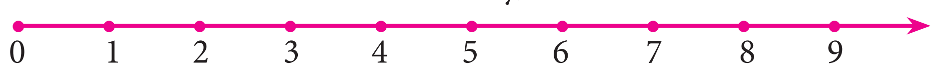


Fig. 2.2

We use $\mathbb{W}$ to denote the set of all Whole numbers.

$$\mathbb{W} = \{ 0, 1, 2, 3, \ldots \}$$

Certain conventions lead to more varieties of numbers. Let us agree that certain conventions may be thought of as **"positive"** denoted by a '+' sign. A thing that is 'up' or 'forward' or 'more' or 'increasing' is positive; and anything that is 'down' or 'backward' or 'less' or 'decreasing' is **"negative"** denoted by a '–' sign.

You can treat natural numbers as positive numbers and rename them as **positive integers**; thereby you have enabled the entry of negative integers –1, –2, –3, … .

Note that –2 is "more negative" than –1. Therefore, among –1 and –2, you find that –2 is smaller and –1 is bigger. Are –2 and –1 smaller or greater than –3? Think about it.

The number line at this stage may be given as follows:

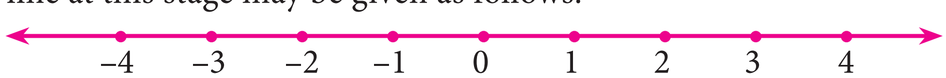


Fig. 2.3

We use $\mathbb{Z}$ to denote the set of all **Integers**.

$$\mathbb{Z} = \{ \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \}.$$

When you look at the figures (Fig. 2.2 and 2.3) above, you are sure to get amused by the gap between any pair of consecutive integers. Could there be some numbers in between?

You have come across fractions already. How will you mark the point that shows $\frac{1}{2}$ on $\mathbb{Z}$? It is just midway between 0 and 1. In the same way, you can plot $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, 2\frac{3}{4}$ ... etc. These are all fractions of the form $\frac{a}{b}$ where a and b are integers with one restriction that $b \neq 0$. (Why?) If a fraction is in decimal form, even then the setting is same.

Because of the connection between fractions and ratios of lengths, we name them as **Rational numbers**. Here is a rough picture of the situation:

Fig. 2.4

> A **rational number** is a fraction indicating the quotient of two integers, excluding division by zero.

Since a fraction can have many equivalent fractions , there are many possible names for the same rational number. Thus $\frac{1}{3}, \frac{2}{6}, \frac{8}{24}$ all these denote the same rational number.

2.2.1 Denseness Property of Rational Numbers

Consider a, b where $a > b$ and their AM(Arithmetic Mean) given by $\frac{a+b}{2}$. Is this AM a rational number? Let us see.

If $a = \frac{p}{q}$ (p, q integers and $q \neq 0$); $b = \frac{r}{s}$ (r, s integers and $s \neq 0$), then

$$\frac{a+b}{2} = \frac{\frac{p}{q} + \frac{r}{s}}{2} = \frac{ps + qr}{2qs} \text{ which is a rational number.}$$

We have to show that this rational number lies between a and b.

$$a - \left(\frac{a+b}{2}\right) = \frac{2a - a - b}{2} = \frac{a-b}{2} \text{ which is} > 0 \text{ since } a > b.$$

$$\text{Therefore, } a > \left(\frac{a+b}{2}\right) \qquad \ldots (1)$$

$$\left(\frac{a+b}{2}\right) - b = \frac{a + b - 2b}{2} = \frac{a-b}{2} \text{ which is} > 0 \text{ since } a > b.$$

$$\text{Therefore, } \left(\frac{a+b}{2}\right) > b \qquad \ldots (2)$$

From (1) and (2) we see that $a > \left(\frac{a+b}{2}\right) > b$, which can be visualized as follows:

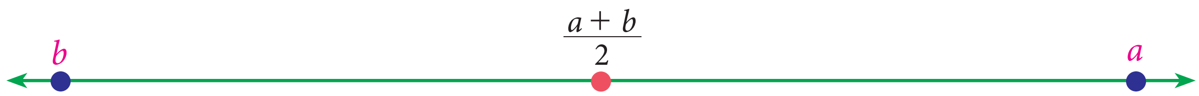


Fig. 2.5

Thus, for any two rational numbers, their average/mid point is rational. Proceeding similarly, we can generate infinitely many rational numbers.

Example 2.1 Find any two rational numbers between $\frac{1}{2}$ and $\frac{2}{3}$.

Solution 1

A rational number between $\frac{1}{2}$ and $\frac{2}{3} = \frac{1}{2}\left(\frac{1}{2}+\frac{2}{3}\right) = \frac{1}{2}\left(\frac{3+4}{6}\right) = \frac{1}{2}\left(\frac{7}{6}\right) = \frac{7}{12}$

A rational number between $\frac{1}{2}$ and $\frac{7}{12} = \frac{1}{2}\left(\frac{1}{2}+\frac{7}{12}\right) = \frac{1}{2}\left(\frac{6+7}{12}\right) = \frac{1}{2}\left(\frac{13}{12}\right) = \frac{13}{24}$

Hence two rational numbers between $\frac{1}{2}$ and $\frac{2}{3}$ are $\frac{7}{12}$ and $\frac{13}{24}$ (of course, there are many more!)

There is an interesting result that could help you to write instantly rational numbers between any two given rational numbers.

Result

If $\frac{p}{q}$ and $\frac{r}{s}$ are any two rational numbers such that $\frac{p}{q} < \frac{r}{s}$, then $\frac{p+r}{q+s}$ is a rational number, such that $\frac{p}{q} < \frac{p+r}{q+s} < \frac{r}{s}$.

Let us take the same example: Find any two rational numbers between $\frac{1}{2}$ and $\frac{2}{3}$

Solution 2

$\frac{1}{2} < \frac{2}{3}$ gives $\frac{1}{2} < \frac{1+2}{2+3} < \frac{2}{3}$ *or* $\frac{1}{2} < \frac{3}{5} < \frac{2}{3}$ gives $\frac{1}{2} < \frac{1+3}{2+5} < \frac{3}{5} < \frac{3+2}{5+3} < \frac{2}{3}$ *or* $\frac{1}{2} < \frac{4}{7} < \frac{3}{5} < \frac{5}{8} < \frac{2}{3}$

Solution 3

Any more new methods to solve? Yes, if decimals are your favourites, then the above example can be given an alternate solution as follows:

$$\frac{1}{2} = 0.5 \text{ and } \frac{2}{3} = 0.66...$$

Hence rational numbers between $\frac{1}{2}$ and $\frac{2}{3}$ can be listed as 0.51, 0.57,0.58,…

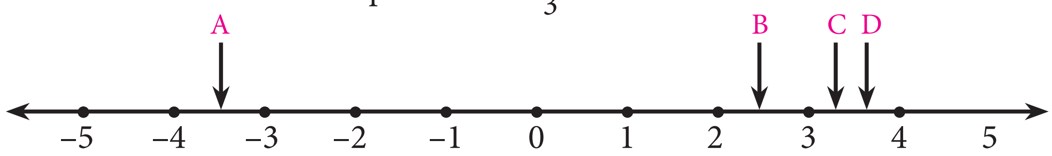
Solution 4

There is one more way to solve some problems. For example, to find four rational numbers between $\frac{4}{9}$ and $\frac{3}{5}$, note that the LCM of 9 and 5 is 45; so we can write $\frac{4}{9} = \frac{20}{45}$ and $\frac{3}{5} = \frac{27}{45}$.

Therefore, four rational numbers between $\frac{4}{9}$ and $\frac{3}{5}$ are $\frac{21}{45}, \frac{22}{45}, \frac{23}{45}, \frac{24}{45}, ...$

Exercise 2.1

1. Which arrow best shows the position of $\frac{11}{3}$ on the number line?



2. Find any three rational numbers between $\frac{-7}{11}$ and $\frac{2}{11}$.

3. Find any five rational numbers between (i) $\frac{1}{4}$ and $\frac{1}{5}$ (ii) 0.1 and 0.11 (iii) −1 and −2

2.3 Irrational Numbers

You know that each rational number is assigned to a point on the number line and learnt about the denseness property of the rational numbers. Does it mean that the line is entirely filled with the rational numbers and there are no more numbers on the number line? Let us explore.

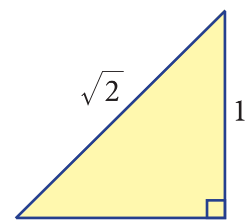


Fig.2.6

Consider an isosceles right-angled triangle whose base and height are each 1 unit long. Using Pythagoras theorem, the hypotenuse can be seen having a length $\sqrt{1^2+1^2} = \sqrt{2}$ (see Fig. 2.6). Greeks found that this $\sqrt{2}$ is neither a whole number nor an ordinary fraction. The belief of relationship between points on the number line and all numbers was shattered! $\sqrt{2}$ was called an irrational number.

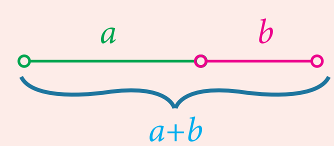
An irrational number is a number that cannot be expressed as an ordinary ratio of two integers.

Examples

1. Apart from $\sqrt{2}$, one can produce a number of examples for such irrational numbers. Here are a few: $\sqrt{5}, \sqrt{7}, 2\sqrt{3}, \ldots$
2. π, the ratio of the circumference of a circle to the diameter of that same circle, is another example for an irrational number.
3. e, also known as Euler's number, is another common irrational number.

GOLDEN RATIO

The Golden Ratio has been heralded as the most beautiful ratio in art and architecture.



$a+b : a = a : b$

Take a line segment and divide it into two smaller segments such that the ratio of the whole line segment ($a+b$) to segment a is the same as the ratio of segment a to the segment b.

This gives the proportion $\frac{a+b}{a} = \frac{a}{b}$

Notice that '**a**' is the geometric mean of $a+b$ and b.

4. The golden ratio, also known as golden mean, or golden section, is a number often stumbled upon when taking the ratios of distances in simple geometric figures such as the pentagon, the pentagram, decagon and dodecahedron, etc., it is an irrational number.

2.3.1 Irrational Numbers on the Number Line

Where are the points on the number line that correspond to the irrational numbers? As an example, let us locate $\sqrt{2}$ on the number line. This is easy.

Remember that $\sqrt{2}$ is the length of the diagonal of the square whose side is 1 unit (How?)Simply construct a square and transfer the length of one of its diagonals to our number line. (see Fig.2.7).

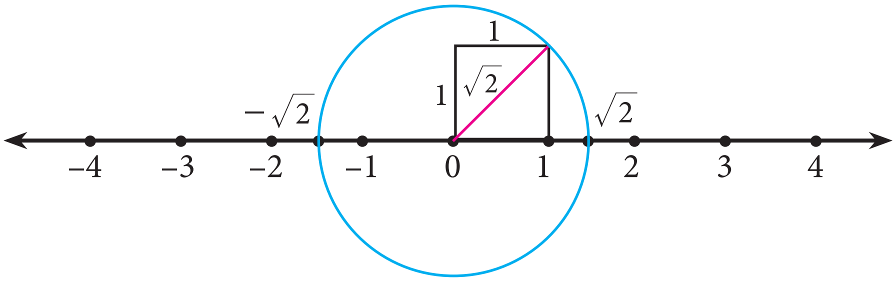


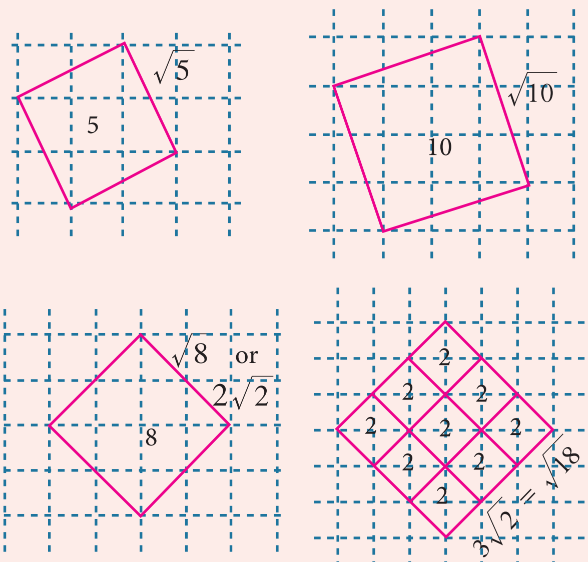
Fig.2.7

We draw a circle with centre at 0 on the number line,with a radius equal to that of diagonal of the square. This circle cuts the number line in two points, locating $\sqrt{2}$ on the right of 0 and $-\sqrt{2}$ on its left. (You wanted to locate $\sqrt{2}$; you have also got a bonus in $-\sqrt{2}$)

You started with Natural numbers and extended it to rational numbers and then irrational numbers. You may wonder if further extension on the number line waits for us. Fortunately it stops and you can learn about it in higher classes.

Squares on grid sheets can be used to produce irrational lengths.

Here are a few examples :



Representation of a Rational number as terminating and non terminating decimal helps us to understand irrational numbers. Let us see the decimal expansion of rational numbers.

2.3.2 Decimal Representation of a Rational Number

If you have a rational number written as a fraction, you get the decimal representation by long division. Study the following examples where the remainder is always zero.

Consider the examples,

$$\begin{array}{r|l} & 0.875 \\ \hline 8 & 7.000 \\ & 64 \\ \hline & \ \ 60 \\ & \ \ 56 \\ \hline & \ \ \ \ 40 \\ & \ \ \ \ 40 \\ \hline & \ \ \ \ \ \ 0 \end{array} \qquad \begin{array}{r|l} & 0.84 \\ \hline 25 & 21.00 \\ & 200 \\ \hline & \ \ 100 \\ & \ \ 100 \\ \hline & \ \ \ \ 0 \end{array} \qquad \begin{array}{r|l} & 2.71875 \\ \hline 32 & 87.00000 \\ & 64 \\ \hline & 230 \\ & 224 \\ \hline & \ \ 60 \\ & \ \ 32 \\ \hline & \ \ 280 \\ & \ \ 256 \\ \hline & \ \ \ \ 240 \\ & \ \ \ \ 224 \\ \hline & \ \ \ \ \ \ 160 \\ & \ \ \ \ \ \ 160 \\ \hline & \ \ \ \ \ \ \ \ 0 \end{array}$$

$\frac{7}{8} = 0.875$ $\qquad$ $\frac{21}{25} = 0.84$ $\qquad$ $\frac{-87}{32} = -2.71875$

Note

These show that the process could lead to a decimal with finite number of decimal places. They are called terminating decimals.

Can the decimal representation of a rational number lead to forms of decimals that do not terminate? The following examples (with non-zero remainder) throw some light on this point.

Example 2.2 Represent the following as decimal form (i) $\frac{-4}{11}$ (ii) $\frac{11}{75}$

Solution

$$\begin{array}{r|l} & 0.3636\ldots \\ \hline 11 & 4.0000 \\ & 33 \\ \hline & \ \ 70 \\ & \ \ 66 \\ \hline & \ \ \ \ 40 \\ & \ \ \ \ 33 \\ \hline & \ \ \ \ \ \ 70 \\ & \ \ \ \ \ \ 66 \\ \hline & \ \ \ \ \ \ \ \ 4 \\ & \ \ \ \ \ \ \ \ \vdots \end{array} \qquad \begin{array}{r|l} & 0.1466\ldots \\ \hline 75 & 11.0000 \\ & \ \ 7\,5 \\ \hline & \ \ \ \ 350 \\ & \ \ \ \ 300 \\ \hline & \ \ \ \ \ \ 500 \\ & \ \ \ \ \ \ 450 \\ \hline & \ \ \ \ \ \ 500 \\ & \ \ \ \ \ \ 450 \\ \hline & \ \ \ \ \ \ \ \ 50 \\ & \ \ \ \ \ \ \ \ \vdots \end{array}$$

Thus we see that, $\frac{-4}{11} = -0.\overline{36}$ $\frac{11}{75} = 0.1\overline{46}$

The reciprocals of Natural Numbers are Rational numbers. It is interesting to note their decimal forms. See the first ten.

S.No.	Reciprocal	Decimal Representation
1	$\frac{1}{1} = 1.0$	Terminating
2	$\frac{1}{2} = 0.5$	Terminating
3	$\frac{1}{3} = 0.\overline{3}$	Non-terminating Recurring
4	$\frac{1}{4} = 0.25$	Terminating
5	$\frac{1}{5} = 0.2$	Terminating
6	$\frac{1}{6} = 0.1\overline{6}$	Non-terminating Recurring

A rational number can be expressed by

(i) **either** a terminating

(ii) **or** a non-terminating and recurring (repeating) decimal expansion.

The converse of this statement is also true.

That is, if the decimal expansion of a number is terminating or non-terminating and recurring, then the number is a rational number.

7	$\frac{1}{7} = 0.\overline{142857}$	Non-terminating Recurring
8	$\frac{1}{8} = 0.125$	Terminating
9	$\frac{1}{9} = 0.\overline{1}$	Non-terminating Recurring
10	$\frac{1}{10} = 0.1$	Terminating

Note

In this case the decimal expansion does not terminate!

The remainders repeat again and again! We get non-terminating but recurring block of digits.

2.3.3 Period of Decimal

In the decimal expansion of the rational numbers, the number of repeating decimals is called the length of the period of decimals.

For example,

(i) $\frac{25}{7} = 3.\overline{571428}$ has the length of the period of decimal = 6

(ii) $\frac{27}{110} = 0.2\overline{45}$ has the length of the period of decimal = 2

Example 2.3 Express the rational number $\frac{1}{27}$ in recurring decimal form by using the recurring decimal expansion of $\frac{1}{3}$. Hence write $\frac{59}{27}$ in recurring decimal form.

Solution

We know that $\frac{1}{3} = 0.\overline{3}$

Therefore, $\frac{1}{27} = \frac{1}{9}\times\frac{1}{3} = \frac{1}{9}\times 0.333... = 0.037037... = 0.\overline{037}$

Also, $\frac{59}{27} = 2\frac{5}{27} = 2 + \frac{5}{27}$

$= 2 + \left(5\times\frac{1}{27}\right)$

$= 2 + (5\times 0.\overline{037}) = 2 + (5\times 0.037037037...) = 2 + 0.185185... = 2.185185... = 2.\overline{185}$

2.3.4 Conversion of Terminating Decimals into Rational Numbers

Let us now try to convert a terminating decimal, say 2.945 as rational number in the fraction form.

2.945 $= 2 + 0.945$

$$
\begin{aligned}
2.945 &= 2+\frac{9}{10}+\frac{4}{100}+\frac{5}{1000}\\
&= 2+\frac{900}{1000}+\frac{40}{1000}+\frac{5}{1000} \text{ (making denominators common)}\\
&= 2+\frac{945}{1000}\\
&= \frac{2945}{1000} \text{ or } \frac{589}{200} \text{ which is required}
\end{aligned}
$$

(In the above, is it possible to write directly $2.945 = \frac{2945}{1000}$?)

Example 2.4 Convert the following decimal numbers in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$: (i) 0.35 (ii) 2.176 (iii) – 0.0028

Solution

(i) $0.35 = \frac{35}{100} = \frac{7}{20}$

(ii) $2.176 = \frac{2176}{1000} = \frac{272}{125}$

(iii) $-0.0028 = \frac{-28}{10000} = \frac{-7}{2500}$

2.3.5 Conversion of Non-terminating and recurring decimals into Rational Numbers

It was very easy to handle a terminating decimal. When we come across a decimal such as 2.4, we get rid of the decimal point, by just divide it by 10.

Thus $2.4 = \frac{24}{10}$, which is simplified as $\frac{12}{5}$. But, when we have a decimal such as $2.\overline{4}$, the problem is that we have infinite number of 4s and hence will need infinite number of 0s in the denominator. For example,

$$
\begin{aligned}
2.4 &= 2+\frac{4}{10}\\
2.44 &= 2+\frac{4}{10}+\frac{4}{100}\\
2.444 &= 2+\frac{4}{10}+\frac{4}{100}+\frac{4}{1000}
\end{aligned}
$$

How tough it is to have infinite 4's and work with them. We need to get rid of the infinite sequence in some way. The good thing about the infinite sequence is that even if we pull away one , two or more 4 out of it, the sequence still remains infinite.

Let $x = 2.\overline{4}$ ...(1)

Then, $10x = 24.\overline{4}$ ...(2) [When you multiply by 10, the decimal moves one place to the right but you still have infinite 4s left over).

Subtract the first equation from the second to get,

$9x = 24.\overline{4} - 2.\overline{4} = 22$ (Infinite 4s subtract out the infinite 4s and the left out is $24 - 2 = 22$)

$$x = \frac{22}{9}, \text{ the required value.}$$

We use the same exact logic to convert any number with a non terminating repeating part into a fraction.

Example 2.5 Convert the following decimal numbers in the form of $\frac{p}{q}$ $(p, q \in Z \text{ and } q \neq 0)$.

(i) $0.\overline{3}$ (ii) $2.\overline{124}$ (iii) $0.4\overline{5}$ (iv) $0.5\overline{68}$

Solution

(i) Let $x = 0.\overline{3} = 0.3333\ldots$ (1)

(Here period of decimal is 1, multiply equation (1) by 10)

$10\,x = 3.3333\ldots$ (2)

(2) – (1): $9\,x = 3$ or $x = \frac{1}{3}$

(ii) Let $x = 2.\overline{124} = 2.124124124\ldots$ (1)

(Here period of decimal is 3, multiply equation (1) by 1000.)

$1000\,x = 2124.124124124\ldots$ (2)

(2)–(1): $999\,x = 2122$ $x = \frac{2122}{999}$

(iii) Let $x = 0.4\overline{5} = 0.45555\ldots$ (1)

(Here the repeating decimal digit is 5, which is the second digit after the decimal point, multiply equation (1) by 10)

$10\,x = 4.5555\ldots$ (2)

(Now period of decimal is 1, multiply equation (2) by 10)

$100\,x = 45.5555\ldots$ (3)

(3) – (2): $90\,x = 41$ or $x = \frac{41}{90}$

(iv) Let $x = 0.5\overline{68} = 0.5686868\ldots$ (1)

(Here the repeating decimal digit is 68, which is the second digit after the decimal point, so multiply equation (1) by 10)

$10\,x = 5.686868\ldots$ (2)

(Now period of decimal is 2, multiply equation (2) by 100)

$1000\,x = 568.686868\ldots$ (3)

(3) – (2): $990\,x = 563$ or $x = \frac{563}{990}$.

Note

To determine whether the decimal form of a rational number will terminate or non - terminate, we can make use of the following rule

If a rational number $\frac{p}{q}$, $q \neq 0$ can be expressed in the form $\frac{p}{2^m \times 5^n}$, where $p \in \mathbb{Z}$ and $m, n \in \mathbb{W}$, then rational number will have a terminating decimal expansion. Otherwise, the rational number will have a non- terminating and recurring decimal expansion

Example 2.6 Without actual division, classify the decimal expansion of the following numbers as terminating or non – terminating and recurring.

(i) $\frac{13}{64}$ (ii) $\frac{-71}{125}$ (iii) $\frac{43}{375}$ (iv) $\frac{31}{400}$

Solution

(a) $\frac{13}{64} = \frac{13}{2^6}$, So $\frac{13}{64}$ has a terminating decimal expansion.

(b) $\frac{-71}{125} = \frac{-71}{5^3}$, So $\frac{-71}{125}$ has a terminating decimal expansion.

(c) $\frac{43}{375} = \frac{43}{3^1 \times 5^3}$, So $\frac{43}{375}$ has a non – terminating recurring decimal expansion.

(d) $\frac{31}{400} = \frac{31}{2^4 \times 5^2}$, So $\frac{31}{400}$ has a terminating decimal expansion.

Example 2.7 Verify that $1 = 0.\overline{9}$

Solution

Let $x = 0.\overline{9} = 0.99999\ldots$ (1)

(Multiply equation (1) by 10)

$10\,x = 9.99999\ldots$ (2)

Subtract (1) from (2)

$9x = 9$ or $x = 1$

Thus, $0.\overline{9} = 1$

$1 = 0.9999\ldots$

$7 = 6.9999\ldots$

$3.7 = 3.6999\ldots$

The pattern suggests that any terminating decimal can be represented as a non-terminating and recurring decimal expansion with an endless block of 9's.

Exercise 2.2

1. Express the following rational numbers into decimal and state the kind of decimal expansion

(i) $\frac{2}{7}$ (ii) $-5\frac{3}{11}$ (iii) $\frac{22}{3}$ (iv) $\frac{327}{200}$

2. Express $\frac{1}{13}$ in decimal form. Find the length of the period of decimals.

3. Express the rational number $\frac{1}{33}$ in recurring decimal form by using the recurring decimal expansion of $\frac{1}{11}$. Hence write $\frac{71}{33}$ in recurring decimal form.

4. Express the following decimal expression into rational numbers.

   (i) $0.\overline{24}$ (ii) $2.\overline{327}$ (iii) -5.132

   (iv) $3.1\overline{7}$ (v) $17.2\overline{15}$ (vi) $-21.213\overline{7}$

5. Without actual division, find which of the following rational numbers have terminating decimal expansion.

   (i) $\frac{7}{128}$ (ii) $\frac{21}{15}$ (iii) $4\frac{9}{35}$ (iv) $\frac{219}{2200}$

2.3.6 Decimal Representation to Identify Irrational Numbers

It can be shown that irrational numbers, when expressed as decimal numbers, do not terminate, nor do they repeat. For example, the decimal representation of the number π starts with 3.14159265358979..., but no finite number of digits can represent π exactly, nor does it repeat.

Consider the following decimal expansions:

(i) 0.101100111000 1111… (ii) 3.012012120121212…

(iii) 12.230223300222333000… (in) $\sqrt{2} = 1.4142135624\ldots$

Are the above numbers terminating (or) recurring and non- terminating? No… They are neither terminating, nor non–terminating and recurring. Hence they are not rational numbers. They cannot be written in the form of $\frac{p}{q}$,where p, q ,$\in \mathbb{Z}$ and $q \neq 0$. They are irrational numbers.

A number having non- terminating and non- recurring decimal expansion is an irrational number.

Example 2.8 Find the decimal expansion of $\sqrt{3}$.

Solution

	1.7320508…
1	3.00,00,00,00,00,…
	1
27	200
	189
343	1100
	1029
3462	7100
	6924
346405	1760000
	1732025
34641008	279750000
	277128064
	2621936

We often write $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\pi = 3.14$ etc. These are only approximate values and not exact values. In the case of the irrational number π, we take frequently $\frac{22}{7}$ (which gives the value $3.\overline{142857}$) to be its correct value but in reality these are only approximations. This is because, the decimal expansion of an irrational number is non-terminating and non-recurring. None of them gives an exact value!

Thus, by division method, $\sqrt{3} = 1.7320508\ldots$

It is found that the square root of every positive non perfect square number is an irrational number. $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots$ are all irrational numbers.

Example 2.9 Classify the numbers as rational or irrational:

(i) $\sqrt{10}$ (ii) $\sqrt{49}$ (iii) 0.025 (iv) $0.7\overline{6}$ (v) 2.505500555... (vi) $\frac{\sqrt{2}}{2}$

Solution

(i) $\sqrt{10}$ is an irrational number (since 10 is not a perfect square number).

(ii) $\sqrt{49} = 7 = \frac{7}{1}$, a rational number(since 49 is a perfect square number).

(iii) 0.025 is a rational number (since it is a terminating decimal).

(iv) $0.7\overline{6} = 0.7666\ldots$ is a rational number (since it is a non – terminating and recurring decimal expansion).

(v) 2.505500555…. is an irrational number (since it is a non – terminating and non–recurring decimal).

(vi) $\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{1}{\sqrt{2}}$ is an irrational number (since 2 is not a perfect square number).

Note

The above example(vi) it is not to be misunderstood as $\frac{p}{q}$ form, because both p and q must be integers and not an irrational number.

Example 2.10 Find any 3 irrational numbers between 0.12 and 0.13 .

E4T9YH

Solution

Three irrational numbers between 0.12 and 0.13 are 0.12010010001…, 0.12040040004…, 0.12070070007…

Note

We state (without proof) an important result worth remembering.

If 'a' is a rational number and $\sqrt{b}$ is an irrational number then each one of the following is an irrational number:

(i) $a+\sqrt{b}$; (ii) $a-\sqrt{b}$; (iii) $a\sqrt{b}$; (iv) $\frac{a}{\sqrt{b}}$; (v) $\frac{\sqrt{b}}{a}$.

For example, when you consider the rational number 4 and the irrational number $\sqrt{5}$, then $4+\sqrt{5}$, $4-\sqrt{5}$, $4\sqrt{5}$, $\frac{4}{\sqrt{5}}$, $\frac{\sqrt{5}}{4}$ ... are all irrational numbers.

Example 2.11 Give any two rational numbers lying between 0.5151151115…. and 0.5353353335…

Solution Two rational numbers between the given two irrational numbers are 0.5152 and 0.5352

Example 2.12 Find whether x and y are rational or irrational in the following.

(i) $a = 2+\sqrt{3}$, $b = 2-\sqrt{3}$; $x = a+b$, $y = a-b$

(ii) $a = \sqrt{2}+7$, $b = \sqrt{2}-7$; $x = a+b$, $y = a-b$

(iii) $a = \sqrt{75}$, $b = \sqrt{3}$; $x = ab$, $y = \frac{a}{b}$

(iv) $a = \sqrt{18}$, $b = \sqrt{3}$; $x = ab$, $y = \frac{a}{b}$

> **Note** From these examples, it is clear that the sum , difference, product, quotient of any two irrational numbers could be rational or irrational.

Solution

(i) Given that $a = 2+\sqrt{3}, b = 2-\sqrt{3}$

$x = a + b = (2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$, a rational number.

$y = a - b = (2 + \sqrt{3}) - (2 - \sqrt{3}) = 2\sqrt{3}$, an irrational number.

(ii) Given that $a = \sqrt{2}+7$, $b = \sqrt{2}-7$

$x = a + b = (\sqrt{2}+7) + (\sqrt{2}-7) = 2\sqrt{2}$, an irrational number.

$y = a - b = (\sqrt{2}+7)-(\sqrt{2}-7) = 14$, a rational number.

(iii) Given that $a = \sqrt{75}$, $b = \sqrt{3}$

$x = ab = \sqrt{75}\times\sqrt{3} = \sqrt{75\times 3} = \sqrt{5\times5\times3\times3} = 5\times3 = 15$, a rational number.

$y = \frac{a}{b} = \frac{\sqrt{75}}{\sqrt{3}} = \sqrt{\frac{75}{3}} = \sqrt{25} = 5$, rational number.

(iv) Given that $a = \sqrt{18}$, $b = \sqrt{3}$

$x = ab = \sqrt{18}\times\sqrt{3} = \sqrt{18\times3} = \sqrt{6\times3\times3} = 3\sqrt{6}$, an irrational number.

$y = \frac{a}{b} = \frac{\sqrt{18}}{\sqrt{3}} = \sqrt{\frac{18}{3}} = \sqrt{6}$, an irrational number.

Example 2.13 Represent $\sqrt{9.3}$ on a number line.

Solution

- Draw a line and mark a point A on it.
- Mark a point B such that $AB = 9.3$ cm.
- Mark a point C on this line such that $BC = 1$ cm.
- Find the midpoint of AC by drawing perpendicular bisector of AC and let it be O
- With O as center and $OC = OA$ as radius, draw a semicircle.

- Draw a line BD, which is perpendicular to AB at B.
- Now $BD = \sqrt{9.3}$, which can be marked in the number line as the value of $BE = BD = \sqrt{9.3}$

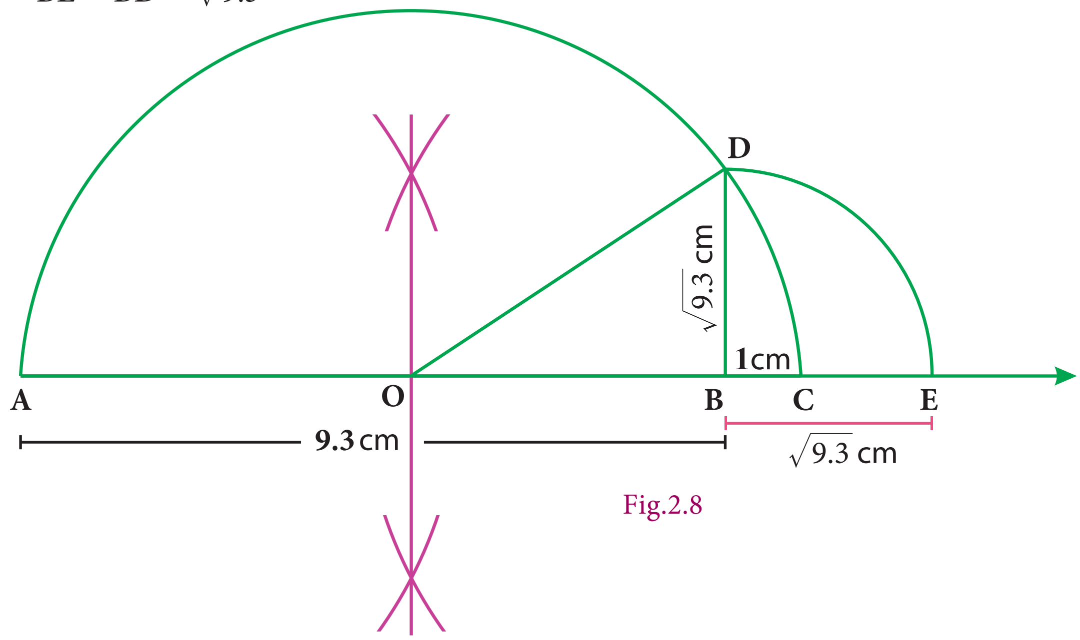


Fig.2.8

Exercise 2.3

1. Represent the following irrational numbers on the number line.

   (i) $\sqrt{3}$ (ii) $\sqrt{4.7}$ (iii) $\sqrt{6.5}$

2. Find any two irrational numbers between

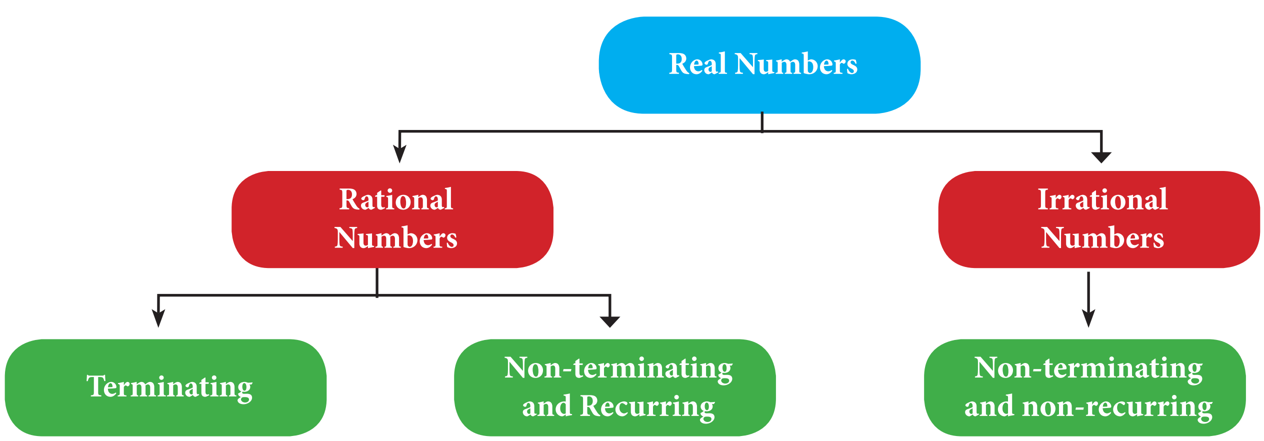
   (i) 0.3010011000111…. and 0.3020020002…. (ii) $\frac{6}{7}$ and $\frac{12}{13}$ (iii) $\sqrt{2}$ and $\sqrt{3}$

3. Find any two rational numbers between 2.2360679….. and 2.236505500….

2.4 Real Numbers

The real numbers consist of all the rational numbers and all the irrational numbers.

Real numbers can be thought of as points on an infinitely long number line called the real line, where the points corresponding to integers are equally spaced.



Any real number can be determined by a possibly infinite decimal representation, (as we have already seen decimal representation of the rational numbers and the irrational numbers).

2.4.1 The Real Number Line

Visualisation through Successive Magnification.

We can visualise the representation of numbers on the number line, as if we glimpse through a magnifying glass.

Example 2.14 Represent 4.863 on the number line.

Solution

4.863 lies between 4 and 5(see Fig. 2.9)

(i) Divide the distance between 4 and 5 into 10 equal intervals.

(ii) Mark the point 4.8 which is second from the left of 5 and eighth from the right of 4

(iii) 4.86 lies between 4.8 and 4.9. Divide the distance into 10 equal intervals.

(iv) Mark the point 4.86 which is fourth from the left of 4.9 and sixth from the right of 4.8

(v) 4.863 lies between 4.86 and 4.87. Divide the distance into 10 equal intervals.

(vi) Mark point 4.863 which is seventh from the left of 4.87 and third from the right of 4.86.

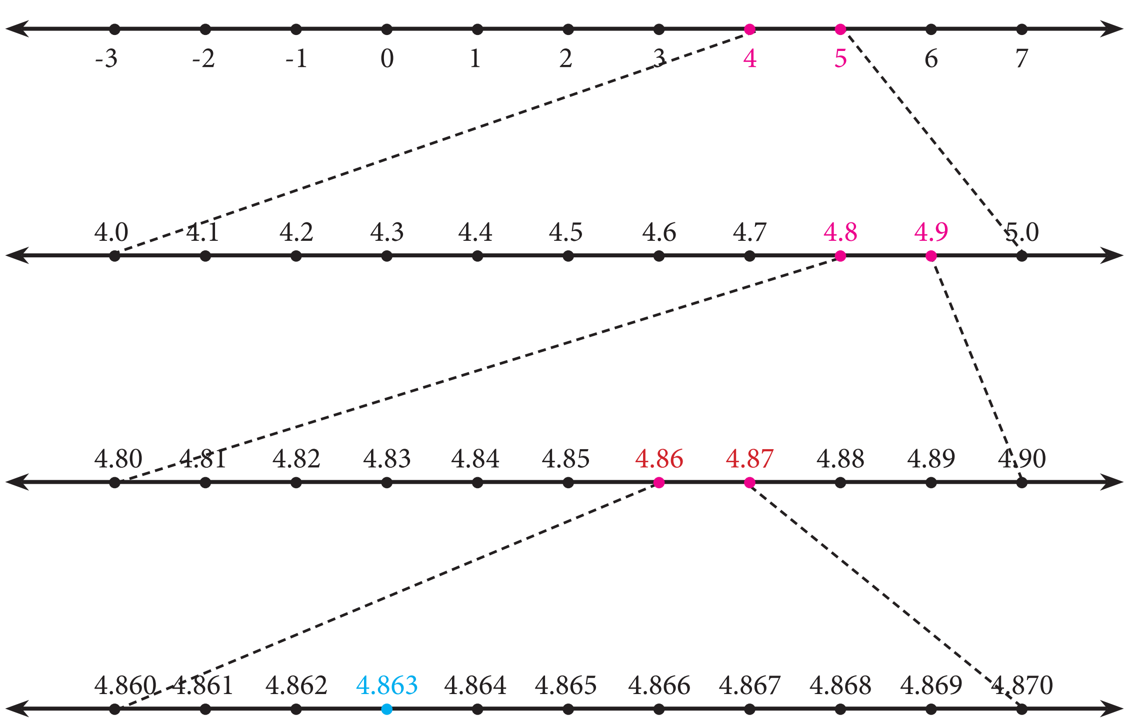


Fig. 2.9

Example 2.15 Represent $3.\overline{45}$ on the number line upto 4 decimal places.

Solution

$3.\overline{45} = 3.45454545\ldots..$

= 3.4545 (correct to 4 decimal places).

The number lies between 3 and 4

E53615

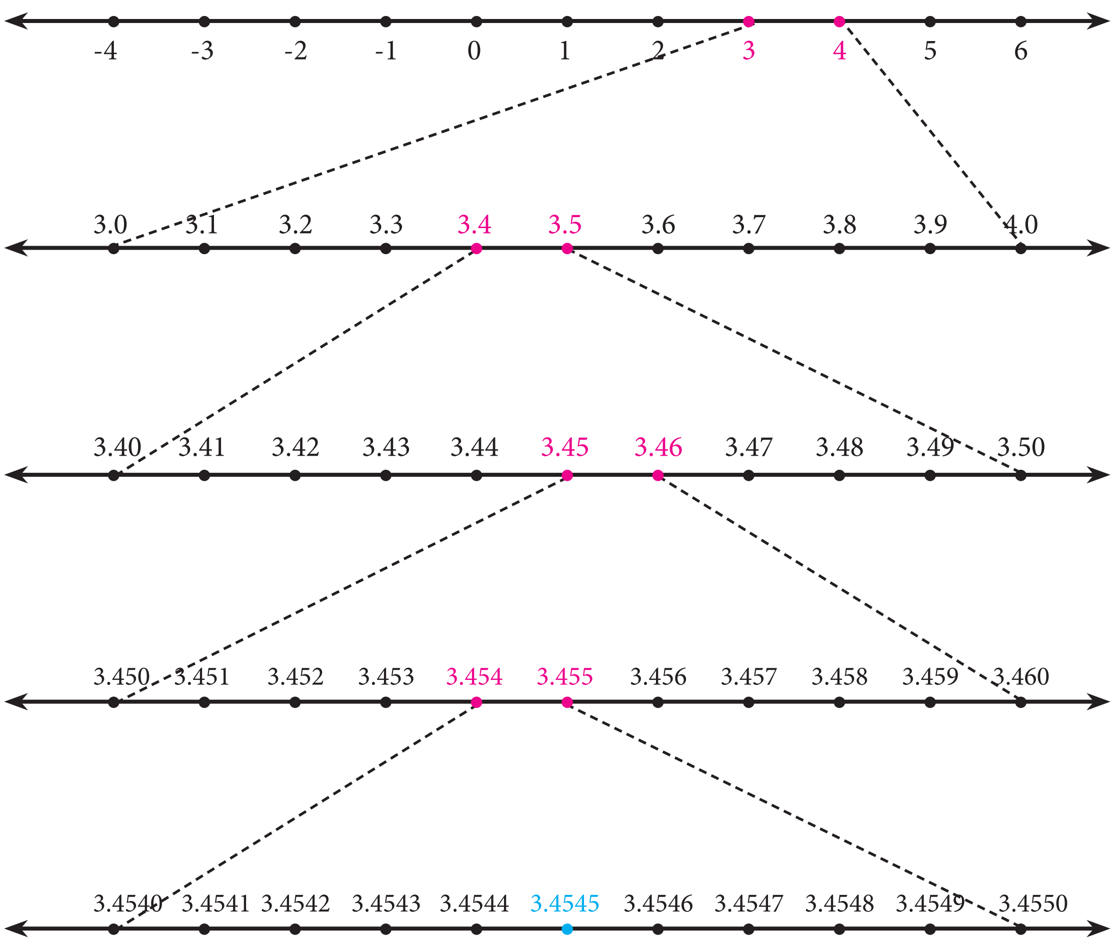


Fig. 2.10

Exercise 2.4

1. Represent the following numbers on the number line.

   (i) 5.348 (ii) $6.\overline{4}$ upto 3 decimal places. (iii) $4.\overline{73}$ upto 4 decimal places.

2.5 Radical Notation

Let n be a positive integer and r be a real number. If $r^n = x$, then r is called the n^{th} root of x and we write

$$\sqrt[n]{x} = r$$

The symbol $\sqrt[n]{\ }$ (read as n^{th} root) is called a radical; n is the index of the radical (hitherto we named it as exponent); and x is called the radicand.

Note

It is worth spending some time on the concepts of the 'square root' and the 'cube root', for better understanding of surds.

What happens when $n = 2$? Then we get $r^2 = x$, so that r is $\sqrt[2]{x}$, our good old friend, the square root of x. Thus $\sqrt[2]{16}$ is written as $\sqrt{16}$, and when $n = 3$, we get the cube root of x, namely $\sqrt[3]{x}$. For example, $\sqrt[3]{8}$ is cube root of 8, giving 2. (Is not $8 = 2^3$?)

How many square roots are there for 4? Since $(+2)\times(+2) = 4$ and also $(-2)\times(-2) = 4$, we can say that both +2 and −2 are square roots of 4. But it is incorrect to write that $\sqrt{4} = \pm 2$. This is because, when n is even, it is an accepted convention to reserve the symbol $\sqrt[n]{x}$ for the positive n^{th} root and to denote the negative n^{th} root by $-\sqrt[n]{x}$. Therefore we need to write $\sqrt{4} = 2$ and $-\sqrt{4} = -2$.

When n is odd, for any value of x, there is exactly one real n^{th} root. For example, $\sqrt[3]{8} = 2$ and $\sqrt[5]{-32} = -2$.

Thinking Corner

Which one of the following is false?

(1) The square root of 9 is 3 or −3.

(2) $\sqrt{9} = 3$

(3) $-\sqrt{9} = -3$

(4) $\sqrt{9} = \pm 3$

2.5.1 Fractional Index

Consider again results of the form $r = \sqrt[n]{x}$.

Root Index Radical Symbol

$\sqrt[3]{64} = 4$

Radicand Number

In the adjacent notation, the index of the radical (namely n which is 3 here) tells you how many times the answer (that is 4) must be multiplied with itself to yield the radicand.

To express the powers and roots, there is one more way of representation. It involves the use of fractional indices.

$$\text{We write } \sqrt[n]{x} \text{ as } x^{\frac{1}{n}}.$$

With this notation, for example

$$\sqrt[3]{64} \text{ is } 64^{\frac{1}{3}} \text{ and } \sqrt{25} \text{ is } 25^{\frac{1}{2}}.$$

Observe in the following table just some representative patterns arising out of this new acquaintance:

Power	Radical Notation	Index Notation	Read as
$2^6 = 64$	$2 = \sqrt[6]{64}$	$2 = 64^{\frac{1}{6}}$	2 is the 6^{th} root of 64
$2^5 = 32$	$2 = \sqrt[5]{32}$	$2 = 32^{\frac{1}{5}}$	2 is the 5^{th} root of 32
$2^4 = 16$	$2 = \sqrt[4]{16}$	$2 = 16^{\frac{1}{4}}$	2 is the 4^{th} root of 16
$2^3 = 8$	$2 = \sqrt[3]{8}$	$2 = 8^{\frac{1}{3}}$	2 is the cube root of 8 meaning 2 is the 3^{rd} root of 8
$2^2 = 4$	$2 = \sqrt[2]{4}$ or simply $2 = \sqrt{4}$	$2 = 4^{\frac{1}{2}}$	2 is the square root of 4 meaning 2 is the 2^{nd} root of 4

Example 2.16 Express the following in the form 2^n :

(i) 8 (ii) 32 (iii) $\frac{1}{4}$ (iv) $\sqrt{2}$ (v) $\sqrt{8}$.

Solution

(i) $8 = 2\times2\times2$; therefore $8 = 2^3$

(ii) $32 = 2\times2\times2\times2\times2 = 2^5$

(iii) $\frac{1}{4} = \frac{1}{2\times2} = \frac{1}{2^2} = 2^{-2}$

(iv) $\sqrt{2} = 2^{1/2}$

(v) $\sqrt{8} = \sqrt{2}\times\sqrt{2}\times\sqrt{2} = \left(2^{\frac{1}{2}}\right)^3$ which may be written as $2^{\frac{3}{2}}$

Meaning of $x^{\frac{m}{n}}$, (where m and n are Positive Integers)

We interpret $x^{\frac{m}{n}}$ either as the n^{th} root of the m^{th} power of x or as the m^{th} power of the n^{th} root of x.

In symbols, $x^{\frac{m}{n}} = (x^m)^{\frac{1}{n}}$ or $(x^{\frac{1}{n}})^m = \sqrt[n]{x^m}$ or $(\sqrt[n]{x})^m$

Example 2.17 Find the value of (i) $81^{\frac{5}{4}}$ (ii) $64^{\frac{-2}{3}}$

Solution

(i) $81^{\frac{5}{4}} = \left(\sqrt[4]{81}\right)^5 = \left(\sqrt[4]{3^4}\right)^5 = 3^5 = 3\times3\times3\times3\times3 = 243$

(ii) $64^{\frac{-2}{3}} = \frac{1}{64^{\frac{2}{3}}} = \frac{1}{\left(\sqrt[3]{64}\right)^2} = \frac{1}{4^2}$ (How?) $= \frac{1}{16}$

Exercise 2.5

1. Write the following in the form of 5^n:

   (i) 625 (ii) $\frac{1}{5}$ (iii) $\sqrt{5}$ (iv) $\sqrt{125}$

2. Write the following in the form of 4^n:

   (i) 16 (ii) 8 (iii) 32

3. Find the value of

   (i) $(49)^{\frac{1}{2}}$ (ii) $(243)^{\frac{2}{5}}$ (iii) $9^{\frac{-3}{2}}$ (vi) $\left(\frac{64}{125}\right)^{\frac{-2}{3}}$

4. Use a fractional index to write:

   (i) $\sqrt{5}$ (ii) $\sqrt[2]{7}$ (iii) $\left(\sqrt[3]{49}\right)^5$ (iv) $\left(\frac{1}{\sqrt[3]{100}}\right)^7$

5. Find the 5^{th} root of

(i) 32 (ii) 243 (iii) 100000 (iv) $\frac{1024}{3125}$

2.6 Surds

Having familiarized with the concept of Real numbers, representing them on the number line and manipulating them, we now learn about surds, a distinctive way of representing certain approximate values.

Can you simplify $\sqrt{4}$ and remove the $\sqrt{\ }$ symbol? Yes; one can replace $\sqrt{4}$ by the number 2. How about $\sqrt{\frac{1}{9}}$? It is easy; without $\sqrt{\ }$ symbol, the answer is $\frac{1}{3}$. What about $\sqrt{0.01}$?. This is also easy and the solution is 0.1

In the cases of $\sqrt{4}$, $\sqrt{\frac{1}{9}}$ and $\sqrt{0.01}$, you can resolve to get a solution and make sure that the symbol $\sqrt{\ }$ is not seen in your solution. Is this possible at all times?

Consider $\sqrt{18}$. Can you evaluate it and also remove the radical symbol? Surds are unresolved radicals, such as square root of 2, cube root of 5, etc.

They are irrational roots of equations with rational coefficients.

A surd is an irrational root of a rational number. $\sqrt[n]{a}$ is a surd, provided $n \in \mathbb{N},\ n > 1$, 'a' is rational.

Examples : $\sqrt{2}$ is a surd. It is an irrational root of the equation $x^2 = 2$. (Note that $x^2 - 2 = 0$ is an equation with rational coefficients. $\sqrt{2}$ is irrational and may be shown as 1.4142135… a non-recurring, non-terminating decimal).

$\sqrt[3]{3}$ (which is same as $3^{\frac{1}{3}}$) is a surd since it is an irrational root of the equation $x^3 - 3 = 0$. ($\sqrt{3}$ is irrational and may be shown as 1.7320508… a non-recurring, non-terminating decimal).

You will learn solving (quadratic) equations like $x^2 - 6x + 7 = 0$ in your next class. This is an equation with rational coefficients and one of its roots is $3+\sqrt{2}$, which is a surd.

Is $\sqrt{\frac{1}{25}}$ a surd? No; it can be simplified and written as rational number $\frac{1}{5}$. How about $\sqrt[4]{\frac{16}{81}}$? It is not a surd because it can be simplified as $\frac{2}{3}$.

The famous irrational number π is not a surd! Though it is irrational, it cannot be expressed as a rational number under the $\sqrt{\ }$ symbol. (In other words, it is not a root of any equation with rational co-efficients).

Why surds are important? For calculation purposes we assume approximate value as $\sqrt{2}=1.414, \sqrt{3}=1.732$ and so on.

$$\left(\sqrt{2}\right)^2=\left(1.414\right)^2=1.99936\neq 2\ ; \quad \left(\sqrt{3}\right)^2=\left(1.732\right)^2=3.999824\neq 3$$

Hence, we observe that $\sqrt{2}$ and $\sqrt{3}$ represent the more accurate and precise values than their assumed values. Engineers and scientists need more accurate values while constructing the bridges and for architectural works. Thus it becomes essential to learn surds.

Progress Check

1. Which is the odd one out? Justify your answer.

   (i) $\sqrt{36}$, $\sqrt{\dfrac{50}{98}}$, $\sqrt{1}$, $\sqrt{1.44}$, $\sqrt[5]{32}$, $\sqrt{120}$

   (ii) $\sqrt{7}$, $\sqrt{48}$, $\sqrt[3]{36}$, $\sqrt{5}+\sqrt{3}$, $\sqrt{1.21}$, $\sqrt{\dfrac{1}{10}}$

2. Are all surds irrational numbers? - Discuss with your answer.
3. Are all irrational numbers surds? Verify with some examples.

2.6.1 Order of a Surd

The order of a surd is the index of the root to be extracted. The order of the surd $\sqrt[n]{a}$ is n. What is the order of $\sqrt[5]{99}$? It is 5.

Surds can be classified in different ways:

(i) Surds of same order : Surds of same order are surds for which the index of the root to be extracted is same. (They are also called equiradical surds).

For example, $\sqrt{x},\ a^{\frac{3}{2}},\ \sqrt[2]{m}$ are all 2nd order (called quadratic) surds .

$\sqrt[3]{5},\ \sqrt[3]{(x-2)},\ (ab)^{\frac{1}{3}}$ are all 3rd order (called cubic) surds.

$\sqrt{3},\ \sqrt[3]{10},\ \sqrt[4]{6}$ and $8^{\frac{2}{5}}$ are surds of different order.

(ii) Simplest form of a surd : A surd is said to be in simplest form, when it is expressed as the product of a rational factor and an irrational factor. In this form the surd has

(a) the smallest possible index of the radical sign.

(b) no fraction under the radical sign.

(c) no factor is of the form a^n, where a is a positive integer under index n.

Example 2.18 Can you reduce the following numbers to surds of same order :

(i) $\sqrt{3}$ (ii) $\sqrt[4]{3}$ (iii) $\sqrt[3]{3}$

Solution

(i) $\sqrt{3} = 3^{\frac{1}{2}} = 3^{\frac{6}{12}} = \sqrt[12]{3^6} = \sqrt[12]{729}$

(ii) $\sqrt[4]{3} = 3^{\frac{1}{4}} = 3^{\frac{3}{12}} = \sqrt[12]{3^3} = \sqrt[12]{27}$

(iii) $\sqrt[3]{3} = 3^{\frac{1}{3}} = 3^{\frac{4}{12}} = \sqrt[12]{3^4} = \sqrt[12]{81}$

The last row has surds of same order.

Example 2.19

1. Express the surds in the simplest form: i) $\sqrt{8}$ ii) $\sqrt[3]{192}$
2. Show that $\sqrt[3]{7} > \sqrt[4]{5}$.

Solution

1. (i) $\sqrt{8} = \sqrt{4\times 2} = 2\sqrt{2}$

   (ii) $\sqrt[3]{192} = \sqrt[3]{4\times 4\times 4\times 3} = 4\sqrt[3]{3}$

2. $\sqrt[3]{7} = \sqrt[12]{7^4} = \sqrt[12]{2401}$

   $\sqrt[4]{5} = 5^{\frac{1}{4}} = 5^{\frac{3}{12}} = \sqrt[12]{5^3} = \sqrt[12]{125}$

   $\sqrt[12]{2401} > \sqrt[12]{125}$

   Therefore, $\sqrt[3]{7} > \sqrt[4]{5}$.

(iii) Pure and Mixed Surds : A surd is called a pure surd if its coefficient in its simplest form is 1. For example, $\sqrt{3}$, $\sqrt[3]{6}$, $\sqrt[4]{7}$, $\sqrt[5]{49}$ are pure surds. A surd is called a mixed surd if its co-efficient in its simplest form is other than 1. For example, $5\sqrt{3}$, $2\sqrt[4]{5}$, $3\sqrt[4]{6}$ are mixed surds.

(iv) Simple and Compound Surds : A surd with a single term is said to be a simple surd. For example, $\sqrt{3}$, $2\sqrt{5}$ are simple surds. The algebraic sum of two (or more) surds is called a compound surd. For example, $\sqrt{5}+3\sqrt{2}, \sqrt{3}-2\sqrt{7}, \sqrt{5}-7\sqrt{2}+6\sqrt{3}$ are compound surds.

(v) **Binomial Surd :** A binomial surd is an algebraic sum (or difference) of 2 terms both of which could be surds or one could be a rational number and another a surd. For example, $\frac{1}{2}-\sqrt{19}, 5+3\sqrt{2}, \sqrt{3}-2\sqrt{7}$ are binomial surds.

Example 2.20 Arrange in ascending order: $\sqrt[3]{2},\ \sqrt[2]{4},\ \sqrt[4]{3}$

Solution

The order of the surds $\sqrt[3]{2},\ \sqrt[2]{4}$ and $\sqrt[4]{3}$ are 3, 2, 4.

L.C.M. of 3, 2, 4 = 12.

$$\sqrt[3]{2}=\left(2^{\frac{1}{3}}\right)=\left(2^{\frac{4}{12}}\right)=\sqrt[12]{2^4}=\sqrt[12]{16}\ ; \qquad \sqrt[2]{4}=\left(4^{\frac{1}{2}}\right)=\left(4^{\frac{6}{12}}\right)=\sqrt[12]{4^6}=\sqrt[12]{4096}$$

$$\sqrt[4]{3}=\left(3^{\frac{1}{4}}\right)=\left(3^{\frac{3}{12}}\right)=\sqrt[12]{3^3}=\sqrt[12]{27}$$

The ascending order of the surds $\sqrt[3]{2},\ \sqrt[4]{3},\ \sqrt[2]{4}$ is $\sqrt[12]{16}<\sqrt[12]{27}<\sqrt[12]{4096}$

that is, $\sqrt[3]{2}$, $\sqrt[4]{3}$, $\sqrt[2]{4}$.

2.6.2 Laws of Radicals

For positive integers m, n and positive rational numbers a and b, it is worth remembering the following properties of radicals:

S.No.	Radical Notation	Index Notation
1.	$\left(\sqrt[n]{a}\right)^n=a=\sqrt[n]{a^n}$	$\left(a^{\frac{1}{n}}\right)^n=a=\left(a^n\right)^{\frac{1}{n}}$
2.	$\sqrt[n]{a}\times\sqrt[n]{b}=\sqrt[n]{ab}$	$a^{\frac{1}{n}}\times b^{\frac{1}{n}}=\left(ab\right)^{\frac{1}{n}}$
3.	$\sqrt[m]{\sqrt[n]{a}}=\sqrt[mn]{a}=\sqrt[n]{\sqrt[m]{a}}$	$\left(a^{\frac{1}{n}}\right)^{\frac{1}{m}}=a^{\frac{1}{mn}}=\left(a^{\frac{1}{m}}\right)^{\frac{1}{n}}$
4.	$\frac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\frac{a}{b}}$	$\frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}}=\left(\frac{a}{b}\right)^{\frac{1}{n}}$

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We shall now discuss certain problems which require the laws of radicals for simplifying.

Example 2.21 Express each of the following surds in its simplest form (i) $\sqrt[3]{108}$ (ii) $\sqrt[3]{\left(1024\right)^{-2}}$ and find its order, radicand and coefficient.

Solution

(i) $\sqrt[3]{108} = \sqrt[3]{27\times 4}$

$= \sqrt[3]{3^3\times 4}$

$= \sqrt[3]{3^3}\times\sqrt[3]{4}$ (Laws of radicals - ii)

$= 3\times\sqrt[3]{4}$ (Laws of radicals- i)

$$\begin{array}{r|r} 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

order = 3; radicand = 4; coefficient = 3

(ii) $\sqrt[3]{(1024)^{-2}} = \left[\sqrt[3]{\left(2^3\times 2^3\times 2^3\times 2\right)^{-2}}\right]$

$= \left[\sqrt[3]{\left(2^3\times 2^3\times 2^3\times 2\right)}\right]^{-2}$ [Laws of radicals - (i)]

$= \left[\sqrt[3]{2^3}\times\sqrt[3]{2^3}\times\sqrt[3]{2^3}\times\sqrt[3]{2}\right]^{-2}$ [Laws of radicals – (ii)]

$= \left[2\times 2\times 2\times\sqrt[3]{2}\right]^{-2}$ [Laws of radicals – (i)]

$= \left[8\times\sqrt[3]{2}\right]^{-2} = \left[\frac{1}{8}\right]^2\times\left(\frac{1}{\sqrt[3]{2}}\right)^2$

$= \frac{1}{64}\sqrt[3]{\frac{1}{4}}$

$$\begin{array}{r|r} 2 & 1024 \\ \hline 2 & 512 \\ \hline 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

order $= 3$; radicand $= \frac{1}{4}$; coefficient $= \frac{1}{64}$

(These results can also be obtained using index notation).

Note

Consider the numbers 5 and 6. As $5=\sqrt{25}$ and $6=\sqrt{36}$

Therefore, $\sqrt{26}$, $\sqrt{27}$, $\sqrt{28}$, $\sqrt{29}$, $\sqrt{30}$, $\sqrt{31}$, $\sqrt{32}$, $\sqrt{33}$, $\sqrt{34}$, and $\sqrt{35}$ are surds between 5 and 6.

Consider $3\sqrt{2}=\sqrt{3^2\times 2}=\sqrt{18}$, $2\sqrt{3}=\sqrt{2^2\times 3}=\sqrt{12}$

Therefore, $\sqrt{17}$, $\sqrt{15}$, $\sqrt{14}$, $\sqrt{13}$ are surds between $2\sqrt{3}$ and $3\sqrt{2}$.

2.6.3 Four Basic Operations on Surds

(i) **Addition and subtraction of surds :** Like surds can be added and subtracted using the following rules:

$$a\sqrt[n]{b}\pm c\sqrt[n]{b}=(a\pm c)\sqrt[n]{b},\ \text{where } b>0.$$

Example 2.22 (i) Add $3\sqrt{7}$ and $5\sqrt{7}$. Check whether the sum is rational or irrational. (ii) Subtract $4\sqrt{5}$ from $7\sqrt{5}$. Is the answer rational or irrational?

Solution

(i) $3\sqrt{7}+5\sqrt{7} = (3+5)\sqrt{7} = 8\sqrt{7}$. The answer is irrational.

(ii) $7\sqrt{5}-4\sqrt{5} = (7-4)\sqrt{5} = 3\sqrt{5}$. The answer is irrational.

Example 2.23 Simplify the following:

(i) $\sqrt{63}-\sqrt{175}+\sqrt{28}$ (ii) $2\sqrt[3]{40}+3\sqrt[3]{625}-4\sqrt[3]{320}$

Solution

$$\begin{aligned}
\text{(i) } \sqrt{63}-\sqrt{175}+\sqrt{28} &= \sqrt{9\times 7}-\sqrt{25\times 7}+\sqrt{4\times 7}\\
&= 3\sqrt{7}-5\sqrt{7}+2\sqrt{7}\\
&= \left(3\sqrt{7}+2\sqrt{7}\right)-5\sqrt{7}\\
&= 5\sqrt{7}-5\sqrt{7}\\
&= 0
\end{aligned}$$

$$\begin{aligned}
\text{(ii) } & 2\sqrt[3]{40}+3\sqrt[3]{625}-4\sqrt[3]{320}\\
&= 2\sqrt[3]{8\times 5}+3\sqrt[3]{125\times 5}-4\sqrt[3]{64\times 5}\\
&= 2\sqrt[3]{2^3\times 5}+3\sqrt[3]{5^3\times 5}-4\sqrt[3]{4^3\times 5}\\
&= 2\times 2\sqrt[3]{5}+3\times 5\sqrt[3]{5}-4\times 4\sqrt[3]{5}\\
&= 4\sqrt[3]{5}+15\sqrt[3]{5}-16\sqrt[3]{5}\\
&= (4+15-16)\sqrt[3]{5} = 3\sqrt[3]{5}
\end{aligned}$$

(ii) Multiplication and division of surds

Like surds can be multiplied or divided by using the following rules:

Multiplication property of surds	Division property of surds
(i) $\sqrt[n]{a}\times\sqrt[n]{b}=\sqrt[n]{ab}$	(iii) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\dfrac{a}{b}}$
(ii) $a\sqrt[n]{b}\times c\sqrt[n]{d}=ac\sqrt[n]{bd}$ where $b,d>0$	(iv) $\dfrac{a\sqrt[n]{b}}{c\sqrt[n]{d}}=\dfrac{a}{c}\sqrt[n]{\dfrac{b}{d}}$ where $b,d>0$

Example 2.24

Multiply $\sqrt[3]{40}$ and $\sqrt[3]{16}$.

Solution

$$\sqrt[3]{40} \times \sqrt[3]{16} = \left(\sqrt[3]{2\times2\times2\times5}\right) \times \left(\sqrt[3]{2\times2\times2\times2}\right)$$

$$= \left(2\times\sqrt[3]{5}\right)\times\left(2\times\sqrt[3]{2}\right) = 4\times\left(\sqrt[3]{2}\times\sqrt[3]{5}\right) = 4\times\sqrt[3]{2\times5}$$

$$= 4\sqrt[3]{10}$$

Example 2.25

Compute and give the answer in the simplest form: $2\sqrt{72}\times5\sqrt{32}\times3\sqrt{50}$

Solution

$$2\sqrt{72}\times5\sqrt{32}\times3\sqrt{50} = \left(2\times6\sqrt{2}\right)\times\left(5\times4\sqrt{2}\right)\times\left(3\times5\sqrt{2}\right)$$

$$= 2\times5\times3\times6\times4\times5\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}$$

$$= 3600\times2\sqrt{2}$$

$$= 7200\sqrt{2}$$

Let us simplify:

$\sqrt{72} = \sqrt{36\times2} = 6\sqrt{2}$

$\sqrt{32} = \sqrt{16\times2} = 4\sqrt{2}$

$\sqrt{50} = \sqrt{25\times2} = 5\sqrt{2}$

Example 2.26

Divide $\sqrt[9]{8}$ by $\sqrt[6]{6}$.

Solution

$$\frac{\sqrt[9]{8}}{\sqrt[6]{6}} = \frac{8^{\frac{1}{9}}}{6^{\frac{1}{6}}} \text{ (Note that 18 is the LCM of 6 and 9)}$$

$$= \frac{8^{\frac{2}{18}}}{6^{\frac{3}{18}}} \text{ (How?)}$$

$$= \left(\frac{8^2}{6^3}\right)^{\frac{1}{18}} \text{ (How ?)} = \left(\frac{8\times8}{6\times6\times6}\right)^{\frac{1}{18}}$$

$$= \left(\frac{8}{27}\right)^{\frac{1}{18}} = \left[\left(\frac{2}{3}\right)^3\right]^{\frac{1}{18}} = \left(\frac{2}{3}\right)^{\frac{1}{6}} = \sqrt[6]{\frac{2}{3}}$$

Activity - 1

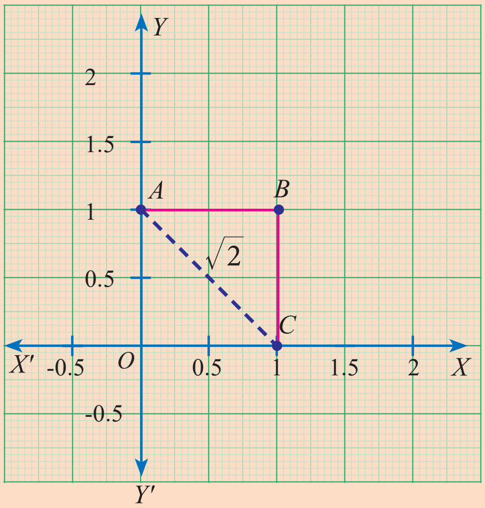
Is it interesting to see this pattern ?

$$\sqrt{4\frac{4}{15}} = 4\sqrt{\frac{4}{15}} \text{ and } \sqrt{5\frac{5}{24}} = 5\sqrt{\frac{5}{24}}$$

Verify it. Can you frame 4 such new surds?

Activity - 2

Take a graph sheet and mark O, A, B, C as follows.



In the square $OABC$,

$$OA = AB = BC = OC = 1\text{unit}$$

Consider right angled ΔOAC

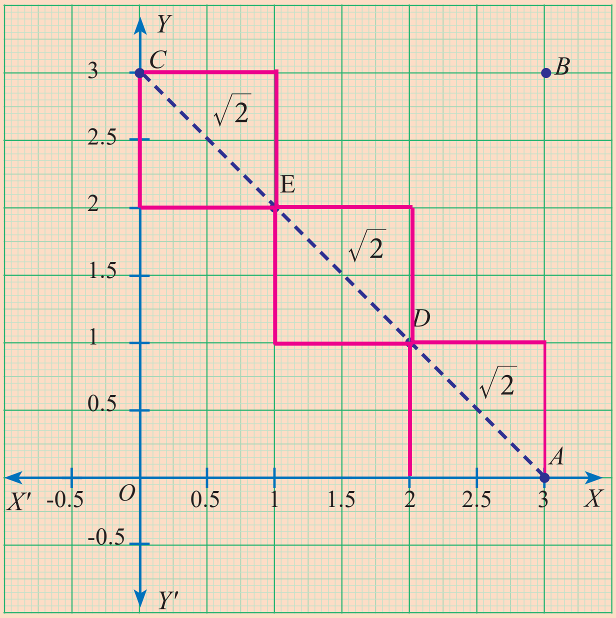
$$AC = \sqrt{1^2 + 1^2}$$

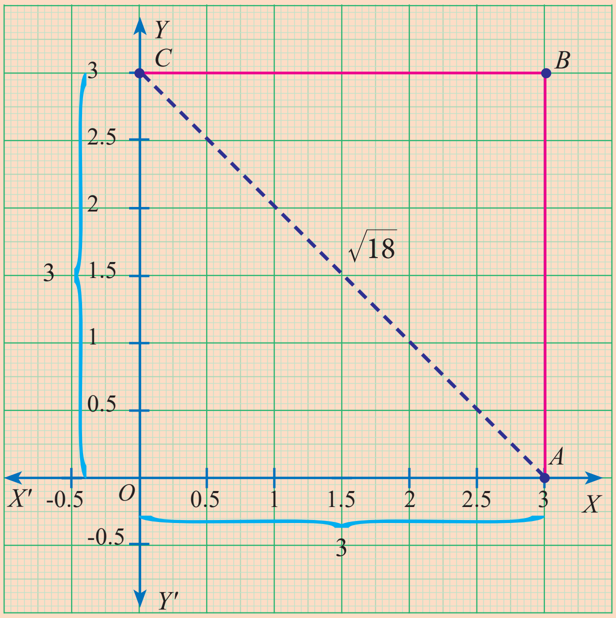
$$= \sqrt{2} \text{ unit} \quad \text{[By Pythagoras theorem]}$$

The length of the diagonal (hypotenuse)

$AC = \sqrt{2}$, which is a surd.

Consider the following graphs:





Let us try to find the length of AC in two different ways :

$AC = AD + DE + EC$

(diagonals of units squares)

$$= \sqrt{2} + \sqrt{2} + \sqrt{2}$$

$$AC = 3\sqrt{2} \text{ units}$$

$$AC = \sqrt{OA^2 + OC^2} = \sqrt{3^2 + 3^2}$$

$$= \sqrt{9+9}$$

$$AC = \sqrt{18} \text{ units}$$

Are they equal? Discuss. Can you verify the same by taking different squares of different lengths?

Exercise 2.6

1. Simplify the following using addition and subtraction properties of surds:

   (i) $5\sqrt{3}+18\sqrt{3}-2\sqrt{3}$ (ii) $4\sqrt[3]{5}+2\sqrt[3]{5}-3\sqrt[3]{5}$

   (iii) $3\sqrt{75}+5\sqrt{48}-\sqrt{243}$ (iv) $5\sqrt[3]{40}+2\sqrt[3]{625}-3\sqrt[3]{320}$

2. Simplify the following using multiplication and division properties of surds:

   (i) $\sqrt{3}\times\sqrt{5}\times\sqrt{2}$ (ii) $\sqrt{35}\div\sqrt{7}$ (iii) $\sqrt[3]{27}\times\sqrt[3]{8}\times\sqrt[3]{125}$

   (iv) $\left(7\sqrt{a}-5\sqrt{b}\right)\left(7\sqrt{a}+5\sqrt{b}\right)$ (v) $\left[\sqrt{\frac{225}{729}}-\sqrt{\frac{25}{144}}\right]\div\sqrt{\frac{16}{81}}$

3. If $\sqrt{2}=1.414$, $\sqrt{3}=1.732$, $\sqrt{5}=2.236$, $\sqrt{10}=3.162$, then find the values of the following correct to 3 places of decimals.

   (i) $\sqrt{40}-\sqrt{20}$ (ii) $\sqrt{300}+\sqrt{90}-\sqrt{8}$

4. Arrange surds in descending order : (i) $\sqrt[3]{5}, \sqrt[9]{4}, \sqrt[6]{3}$ (ii) $\sqrt[2]{\sqrt[3]{5}}, \sqrt[3]{\sqrt[4]{7}}, \sqrt{\sqrt{3}}$

5. Can you get a pure surd when you find

   (i) the sum of two surds (ii) the difference of two surds

   (iii) the product of two surds (iv) the quotient of two surds

   Justify each answer with an example.

6. Can you get a rational number when you compute

   (i) the sum of two surds (ii) the difference of two surds

   (iii) the product of two surds (iv) the quotient of two surds

   Justify each answer with an example.

2.7 Rationalisation of Surds

Rationalising factor is a term with which a term is multiplied or divided to make the whole term rational.

Examples:

(i) $\sqrt{3}$ is a rationalising factor of $\sqrt{3}$ (since $\sqrt{3}\times\sqrt{3}$ = the rational number 3)

(ii) $\sqrt[7]{5^4}$ is a rationalising factor of $\sqrt[7]{5^3}$ (since their product $=\sqrt[7]{5^7}=5$, a rational)

Thinking Corner

1. In the example (i) above, can $\sqrt{12}$ also be a rationalising factor? Can you think of any other number as a rationalising factor for $\sqrt{3}$?
2. Can you think of any other number as a rationalising factor for $\sqrt[7]{5^3}$ in example (ii) ?
3. If there can be many rationalising factors for an expression containing a surd, is there any advantage in choosing the smallest among them for manipulation?

Progress Check

Identify a rationalising factor for each one of the following surds and verify the same in each case:

(i) $\sqrt{18}$ (ii) $5\sqrt{12}$ (iii) $\sqrt[3]{49}$ (iv) $\dfrac{1}{\sqrt{8}}$

2.7.1 Conjugate Surds

Can you guess a rationalising factor for $3+\sqrt{2}$? This surd has one rational part and one radical part. In such cases, the rationalising factor has an interesting form.

A rationalising factor for $3+\sqrt{2}$ is $3-\sqrt{2}$. You can very easily check this.

$$\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right) = 3^2 - \left(\sqrt{2}\right)^2$$
$$= 9-2$$
$$= 7, \text{ a rational.}$$

What is the rationalising factor for $a+\sqrt{b}$ where a and b are rational numbers? Is it $a-\sqrt{b}$? Check it. What could be the rationalising factor for $\sqrt{a}+\sqrt{b}$ where a and b are rational numbers? Is it $\sqrt{a}-\sqrt{b}$? Or, is it $-\sqrt{a}+\sqrt{b}$? Investigate.

Surds like $a+\sqrt{b}$ and $a-\sqrt{b}$ are called conjugate surds. What is the conjugate of $\sqrt{b}+a$? It is $-\sqrt{b}+a$. You would have perhaps noted by now that a conjugate is usually obtained by changing the sign in front of the surd!

Example 2.27 Rationalise the denominator of (i) $\dfrac{7}{\sqrt{14}}$ (ii) $\dfrac{5+\sqrt{3}}{5-\sqrt{3}}$

Solution

(i) Multiply both numerator and denominator by the rationalising factor $\sqrt{14}$.

$$\frac{7}{\sqrt{14}} = \frac{7}{\sqrt{14}} \times \frac{\sqrt{14}}{\sqrt{14}} = \frac{7\sqrt{14}}{14} = \frac{\sqrt{14}}{2}$$

(ii)
$$\frac{5+\sqrt{3}}{5-\sqrt{3}} = \frac{\left(5+\sqrt{3}\right)}{\left(5-\sqrt{3}\right)} \times \frac{\left(5+\sqrt{3}\right)}{\left(5+\sqrt{3}\right)} = \frac{\left(5+\sqrt{3}\right)^2}{5^2-\left(\sqrt{3}\right)^2}$$
$$= \frac{5^2+\left(\sqrt{3}\right)^2+2\times 5\times\sqrt{3}}{25-3}$$
$$= \frac{25+3+10\sqrt{3}}{22} = \frac{28+10\sqrt{3}}{22} = \frac{2\times[14+5\sqrt{3}]}{22}$$
$$= \frac{14+5\sqrt{3}}{11}$$

Exercise 2.7

1. Rationalise the denominator

(i) $\dfrac{1}{\sqrt{50}}$ (ii) $\dfrac{5}{3\sqrt{5}}$ (iii) $\dfrac{\sqrt{75}}{\sqrt{18}}$ (iv) $\dfrac{3\sqrt{5}}{\sqrt{6}}$

2. Rationalise the denominator and simplify

(i) $\dfrac{\sqrt{48}+\sqrt{32}}{\sqrt{27}-\sqrt{18}}$ (ii) $\dfrac{5\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

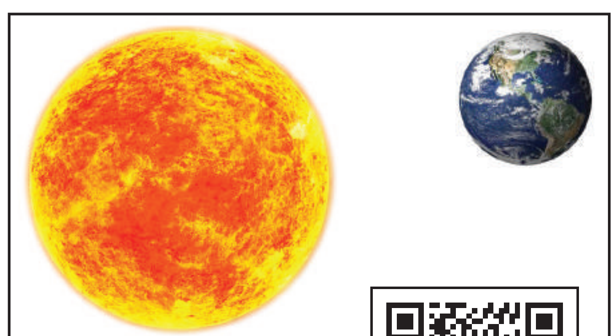
(iii) $\dfrac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$ (iv) $\dfrac{\sqrt{5}}{\sqrt{6}+2}-\dfrac{\sqrt{5}}{\sqrt{6}-2}$

3. Find the value of a and b if $\dfrac{\sqrt{7}-2}{\sqrt{7}+2}=a\sqrt{7}+b$

4. If $x=\sqrt{5}+2$, then find the value of $x^2+\dfrac{1}{x^2}$

5. Given $\sqrt{2}=1.414$, find the value of $\dfrac{8-5\sqrt{2}}{3-2\sqrt{2}}$ (to 3 places of decimals).

2.8 Scientific Notation



BBZAX5

Suppose you are told that the diameter of the Sun is 13,92,000 km and that of the Earth is 12,740 km, it would seem to be a daunting task to compare them. In contrast, if 13,92,000 is written as 1.392×10^6 and 12,740 as 1.274×10^4, one will feel comfortable. This sort of representation is known as scientific notation.

$$\text{Since } \frac{1.392\times10^6}{1.274\times10^4}\approx\frac{14}{13}\times10^2\approx108 .$$

You can imagine 108 Earths could line up across the face of the sun.

Scientific notation is a way of representing numbers that are too large or too small, to be conveniently written in decimal form. It allows the numbers to be easily recorded and handled.

2.8.1 Writing a Decimal Number in Scientific Notation

Here are steps to help you to represent a number in scientific form:

(i) Move the decimal point so that there is only one non-zero digit to its left.

(ii) Count the number of digits between the old and new decimal point. This gives 'n', the power of 10.

(iii) If the decimal is shifted to the left, the exponent n is positive. If the decimal is shifted to the right, the exponent n is negative.

Expressing a number N in the form of $N = a \times 10^n$ where, $1 \leq a < 10$ and 'n' is an integer is called as Scientific Notation.

The following table of base 10 examples may make things clearer:

Decimal notation	Scientific notation	Decimal notation	Scientific notation
100	1×10^2	0.01	1×10^{-2}
1,000	1×10^3	0.001	1×10^{-3}
10,000	1×10^4	0.0001	1×10^{-4}
1,00,000	1×10^5	0.00001	1×10^{-5}
10,00,000	1×10^6	0.000001	1×10^{-6}
1,00,00,000	1×10^7	0.0000001	1×10^{-7}

Let us look into few more examples.

Example 2.28 Express in scientific notation (i) 9768854 (ii) 0.04567891 (iii) 72006865.48

Solution

(i) 9 7 6 8 8 5 4 . 0 $= 9.768854 \times 10^6$

6 5 4 3 2 1

The decimal point is to be moved six places to the left. Therefore $n = 6$.

(ii) 0 . 0 4 5 6 7 8 9 1 $= 4.567891 \times 10^{-2}$

1 2

The decimal point is to be moved two places to the right. Therefore $n = -2$.

(iii) 7 2 0 0 6 8 6 5 . 4 8 $= 7.200686548 \times 10^7$

7 6 5 4 3 2 1

The decimal point is to be moved seven places to the left. Therefore $n = 7$.

2.8.2 Converting Scientific Notation to Decimal Form

The reverse process of converting a number in scientific notation to the decimal form is easily done when the following steps are followed:

(i) Write the decimal number.

(ii) Move the decimal point by the number of places specified by the power of 10, to the right if positive, or to the left if negative. Add zeros if necessary.

(iii) Rewrite the number in decimal form.

Example 2.29 Write the following numbers in decimal form:

(i) 6.34×10^{4} (ii) 2.00367×10^{-5}

Solution

(i)V 6.34×10^{4}

$$\Rightarrow \quad 6\,.\quad 3 \quad 4 \quad 0 \quad 0 \quad = 63400$$

1 2 3 4

(ii) 2.00367×10^{-5}

$$0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2 \quad .00367$$

5 4 3 2 1

$= 0.0000200367$

2.8.3 Arithmetic of Numbers in Scientific Notation

(i) If the indices in the scientific notation of two numbers are the same, addition (or subtraction) is easily performed.

Example 2.30 The mass of the Earth is 5.97×10^{24} kg and that of the Moon is 0.073×10^{24} kg. What is their total mass?

Solution

$$\begin{aligned} \text{Total mass} &= 5.97 \times 10^{24}\text{ kg} + 0.073 \times 10^{24}\text{ kg} \\ &= (5.97 + 0.073) \times 10^{24}\text{ kg} \\ &= 6.043 \times 10^{24}\text{ kg} \end{aligned}$$

(ii) The product or quotient of numbers in scientific notation can be easily done if we make use of the laws of radicals appropriately.

Example 2.31 Write the following in scientific notation :

(i) $(50000000)^{4}$ (ii) $(0.00000005)^{3}$

(iii) $(300000)^{3} \times (2000)^{4}$ (iv) $(4000000)^{3} \div (0.00002)^{4}$

Solution

(i) $\left(50000000\right)^4 = \left(5.0\times10^7\right)^4$

$= \left(5.0\right)^4\times\left(10^7\right)^4$

$= 625.0\times10^{28}$

$= 6.25\times10^2\times10^{28}$

$= 6.25\times10^{30}$

(ii) $\left(0.00000005\right)^3 = \left(5.0\times10^{-8}\right)^3$

$= \left(5.0\right)^3\times\left(10^{-8}\right)^3$

$= \left(125.0\right)\times\left(10\right)^{-24}$

$= 1.25\times10^2\times10^{-24}$

$= 1.25\times10^{-22}$

(iii) $\left(300000\right)^3\times\left(2000\right)^4$

$= \left(3.0\times10^5\right)^3\times\left(2.0\times10^3\right)^4$

$= \left(3.0\right)^3\times\left(10^5\right)^3\times\left(2.0\right)^4\times\left(10^3\right)^4$

$= \left(27.0\right)\times\left(10^{15}\right)\times\left(16.0\right)\times\left(10^{12}\right)$

$= \left(2.7\times10^1\right)\times\left(10^{15}\right)\times\left(1.6\times10^1\right)\times\left(10^{12}\right)$

$= 2.7\times1.6\times10^1\times10^{15}\times10^1\times10^{12}$

$= 4.32\times10^{1+15+1+12} = 4.32\times10^{29}$

(iv) $\left(4000000\right)^3\div\left(0.00002\right)^4$

$= \left(4.0\times10^6\right)^3\div\left(2.0\times10^{-5}\right)^4$

$= \left(4.0\right)^3\times\left(10^6\right)^3\div\left(2.0\right)^4\times\left(10^{-5}\right)^4$

$= \dfrac{64.0\times10^{18}}{16.0\times10^{-20}}$

$= 4\times10^{18}\times10^{+20}$

$= 4.0\times10^{38}$

Thinking Corner

1. Write two numbers in scientific notation whose product is 2. 83104.
2. Write two numbers in scientific notation whose quotient is 2. 83104.

Exercise 2.8

1. Represent the following numbers in the scientific notation:
   (i) 569430000000 (ii) 2000.57
   (iii) 0.0000006000 (iv) 0.0009000002

2. Write the following numbers in decimal form:
   (i) 3.459×10^6 (ii) 5.678×10^4
   (iii) 1.00005×10^{-5} (iv) 2.530009×10^{-7}

3. Represent the following numbers in scientific notation:
   (i) $\left(300000\right)^2\times\left(20000\right)^4$ (ii) $\left(0.000001\right)^{11}\div\left(0.005\right)^3$
   (iii) $\left\{\left(0.00003\right)^6\times\left(0.00005\right)^4\right\}\div\left\{\left(0.009\right)^3\times\left(0.05\right)^2\right\}$

4. Represent the following information in scientific notation:

(i) The world population is nearly 7000,000,000.

(ii) One light year means the distance 9460528400000000 km.

(iii) Mass of an electron is 0.000 000 000 000 000 000 000 000 000 00091093822 kg.

5. Simplify:

(i) $(2.75 \times 10^{7}) + (1.23 \times 10^{8})$ (ii) $(1.598\times10^{17}) - (4.58 \times10^{15})$

(iii) $(1.02 \times 10^{10}) \times (1.20 \times 10^{-3})$ (iv) $(8.41 \times 10^{4}) \div (4.3 \times 10^{5})$

Activity - 3

The following list shows the mean distance of the planets of the solar system from the Sun. Complete the following table. Then arrange in order of magnitude starting with the distance of the planet closest to the Sun.

Planet	Decimal form (in Km)	Scientific Notation (in Km)
Jupiter		7.78×10^{8}
Mercury	58000000	
Mars		2.28×10^{8}
Uranus	2870000000	
Venus	108000000	
Neptune	4500000000	
Earth		1.5×10^{8}
Saturn		1.43×10^{8}

Exercise 2.9

BC3XL2

Multiple Choice Questions

1. If n is a natural number then $\sqrt{n}$ is

(1) always a natural number. (2) always an irrational number.

(3) always a rational number (4) may be rational or irrational

2. Which of the following is not true?.

(1) Every rational number is a real number. (2) Every integer is a rational number.

(3) Every real number is an irrational number. (4) Every natural number is a whole number.

3. Which one of the following, regarding sum of two irrational numbers, is true?

(1) always an irrational number. (2) may be a rational or irrational number.

(3) always a rational number. (4) always an integer.

4. Which one of the following has a terminating decimal expansion?.

(1) $\frac{5}{64}$ (2) $\frac{8}{9}$ (3) $\frac{14}{15}$ (4) $\frac{1}{12}$

5. Which one of the following is an irrational number

(1) $\sqrt{25}$ (2) $\sqrt{\frac{9}{4}}$ (3) $\frac{7}{11}$ (4) π

6. An irrational number between 2 and 2.5 is

(1) $\sqrt{11}$ (2) $\sqrt{5}$ (3) $\sqrt{2.5}$ (4) $\sqrt{8}$

7. The smallest rational number by which $\frac{1}{3}$ should be multiplied so that its decimal expansion terminates with one place of decimal is

(1) $\frac{1}{10}$ (2) $\frac{3}{10}$ (3) 3 (4) 30

8. If $\frac{1}{7} = 0.\overline{142857}$ then the value of $\frac{5}{7}$ is

(1) $0.\overline{142857}$ (2) $0.\overline{714285}$ (3) $0.\overline{571428}$ (4) 0.714285

9. Find the odd one out of the following.

(1) $\sqrt{32} \times \sqrt{2}$ (2) $\frac{\sqrt{27}}{\sqrt{3}}$ (3) $\sqrt{72} \times \sqrt{8}$ (4) $\frac{\sqrt{54}}{\sqrt{18}}$

10. $0.\overline{34} + 0.3\overline{4} =$

(1) $0.6\overline{87}$ (2) $0.\overline{68}$ (3) $0.6\overline{8}$ (4) $0.68\overline{7}$

11. Which of the following statement is false?

(1) The square root of 25 is 5 or -5

(2) $-\sqrt{25} = -5$

(3) $\sqrt{25} = 5$

(4) $\sqrt{25} = \pm 5$

12. Which one of the following is not a rational number?

(1) $\sqrt{\frac{8}{18}}$ (2) $\frac{7}{3}$ (3) $\sqrt{0.01}$ (4) $\sqrt{13}$

13. $\sqrt{27} + \sqrt{12} =$

(1) $\sqrt{39}$ (2) $5\sqrt{6}$ (3) $5\sqrt{3}$ (4) $3\sqrt{5}$

14. If $\sqrt{80} = k\sqrt{5}$, then $k =$

(1) 2 (2) 4 (3) 8 (4) 16

15. $4\sqrt{7} \times 2\sqrt{3} =$

(1) $6\sqrt{10}$ (2) $8\sqrt{21}$ (3) $8\sqrt{10}$ (4) $6\sqrt{21}$

16. When written with a rational denominator, the expression $\frac{2\sqrt{3}}{3\sqrt{2}}$ can be simplified as

(1) $\frac{\sqrt{2}}{3}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{\sqrt{6}}{3}$ (4) $\frac{2}{3}$

17. When $\left(2\sqrt{5}-\sqrt{2}\right)^2$ is simplified, we get

(1) $4\sqrt{5}+2\sqrt{2}$ (2) $22-4\sqrt{10}$ (3) $8-4\sqrt{10}$ (4) $2\sqrt{10}-2$

18. $\left(0.000729\right)^{\frac{-3}{4}} \times \left(0.09\right)^{\frac{-3}{4}} =$ ______

(1) $\frac{10^3}{3^3}$ (2) $\frac{10^5}{3^5}$ (3) $\frac{10^2}{3^2}$ (4) $\frac{10^6}{3^6}$

19. If $\sqrt{9^x} = \sqrt[3]{9^2}$, then $x =$ ______

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{1}{3}$ (4) $\frac{5}{3}$

20. The length and breadth of a rectangular plot are 5×10^5 and 4×10^4 metres respectively. Its area is ______.

(1) $9\times10^1\ m^2$ (2) $9\times10^9\ m^2$ (3) $2\times10^{10}\ m^2$ (4) $20\times10^{20}\ m^2$

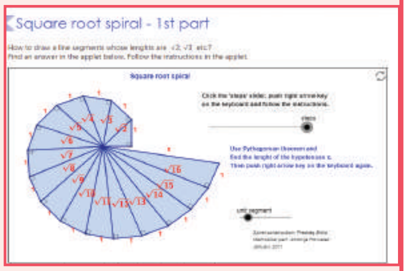
Points to Remember

- When the decimal expansion of $\frac{p}{q}$, q≠0 terminates that is, comes to an end, the decimal is called a terminating decimal.
- In the decimal expansion of $\frac{p}{q}$, q≠0 when the remainder is not zero, we have a repeating (recurring) block of digits in the quotient. In this case, the decimal expansion is called non-terminating and recurring.
- If a rational number $\frac{p}{q}$, q≠ 0 can be expressed in the form $\frac{p}{2^m \times 5^n}$, where $p \in \mathbb{Z}$ and $m, n \in \mathbb{W}$, then the rational number will have a terminating decimals. Otherwise, the rational number will have a non-terminating repeating (recurring) decimal.
- A rational number can be expressed either a terminating or a non- terminating recurring decimal.
- An irrational number is a non-terminating and non-recuring decimal, i.e. it cannot be written in form $\frac{p}{q}$, where p and q are both integers and q≠ 0.
- The union of all rational numbers and all irrational numbers is called the set of real numbers.
- Every real number is either a rational number or an irratonal number.
- If a real number is not rational number, then it must be an irrational number.
- If 'a' is a positive rational number, 'n' is a positive integer and if $\sqrt[n]{a}$ is an irrational number, then $\sqrt[n]{a}$ is called as a surd.
- If 'm', 'n' are positive integers and a, b are positive rational numbers, then

(i) $\left(\sqrt[n]{a}\right)^n = a = \sqrt[n]{a^n}$ (ii) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$ (iii) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$ (iv) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

- The process of multiplying a surd by another surd to get a rational number is called Rationalisation.
- Expressing a number N in the form of $N = a \times 10^n$ where, $1 \le a < 10$ and 'n' is an integer is called as Scientific Notation.

ICT Corner-1



Expected Result is shown in this picture

Step – 1

Open the Browser and copy and paste the Link given below (or) by typing the URL given (or) Scan the QR Code.

Step - 2

GeoGebra workbook named "Real Numbers" will open. There are several worksheets in the workbook. Open the worksheet named "Square root spiral – 1st part"

Step-3

Drag the slider named "Steps ". The construction of Square root of numbers 2,3,4,5,…. will appear step by step.

Step-4

By dragging the slider named "Unit segment" you can enlarge the diagram for more clarity. Now you can draw the same in a paper and measure the values obtained

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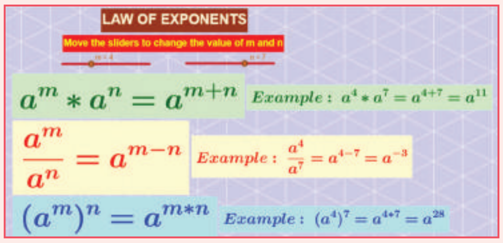
ICT Corner-2

Expected Result is shown in this picture



Step – 1

Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Real Numbers" will open. In the work sheet there are two activities. 1. Rationalising the denominator for surds and 2. Law of exponents.

In the first activity procedure for rationalising the denominator is given. Also, example is given under. To change the values of a and b enter the value in the input box given.

Step - 2

In the second activity law of exponents is given. Also, example is given on right side. To change the value of m and n move the sliders and check the answers.

Scan the QR Code.

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