GOVERNMENT OF TAMIL NADU

HIGHER SECONDARY SECOND YEAR

PHYSICS

VOLUME - I

A publication under Free Textbook Programme of Government of Tamil Nadu

Department of School Education

Untouchability is Inhuman and a Crime

Government of Tamil Nadu

First Edition - 2019

(Published Under New Syllabus)

Content Creation

State Council of Educational Research and Training

Printing & Publishing

Tamil NaduTextbook and Educational Services Corporation

www.textbooksonline.tn.nic.in

CONTENTS

PHYSICS

GPTWUZ

E-book

4T8364

Assessment

B263_12_PHYSICS_EM

DIGI links

QR GUIDE

Let's use the QR code in the text books ! How ?
- Download the QR code scanner from the Google PlayStore/ Apple App Store into your smartphone
- Open the QR code scanner application
- Once the scanner button in the application is clicked, camera opens and then bring it closer to the QR code in the text book.
- Once the camera detects the QR code, a url appears in the screen.Click the url and goto the content page.

HOW TO USE THE BOOK

Scope of Physics	• Awareness on higher learning - courses, institutions and required competitive exams • Financial assistance possible to help students to climb academic ladder
Learning Objectives:	• Overview of the unit • Gives clarity on the goals and objective of the topics
DO YOU KNOW?	• Additional facts related to the topics covered to facilitate curiosity driven learning
Example problems	• To ensure understanding, problems/illustrations are given at every stage before advancing to next level
QR GUIDE	• Visual representation of concepts with illustrations • Videos, animations, and tutorials
ICT	• To harness the digital skills to class room learning and experimenting
Summary	• Recap of salient points of the lesson
Concept Map	• Schematic outline of salient learning of the unit
Evaluation	• Evaluate students' understanding and get them acquainted with the application of physical concepts to numerical and conceptual questions
Books for Reference	• List of relevant books for further reading
Solved examples	• Solutions to exercise problems are accessible here. In addition, a few solved examples are given to facilitate students to apply the concepts learnt.
Competitive Exam corner	• Model Questions - To motivate students aspiring to take up competitive examinations such as NEET, JEE, Physics Olympiad, JIPMER etc
Practical	• List of practical and the description of each is appended for easy access.
Glossary	• Scientific terms frequently used with their Tamil equivalents
Content Focus	• Sequential understanding of the stationary charges, moving charges, electric current, magnetism and the interlink between electric and magnetic phenomena.

Back Wrapper: NIKOLA TESLA a Serbian-American Engineer (10 July 1856 – 7 January 1943)

Nikola Tesla made breakthroughs in the production, transmission and application of electric power. He invented the first alternating current (AC) motor and developed AC generation and transmission technology. In 1884, he was hired by Edison (discoverer of DC dynamos) later on Tesla became his competitor in this field.

Tesla conducted a range of experiments with mechanical oscillators/generators, electrical discharge tubes, and early X – ray imaging. Tesla in 1890 itself conducted research for wireless lighting and worldwide wireless electric power distribution in his high – voltage, high-frequency power experiments in New York. Unfortunately Tesla could not put his ideas in practical use due to lack of funds.

Scope of Physics - Higher Education

Entrance Examinations After +2

- Physics Olympiad Exam
- NEET-National Eligibility cum Entrance Test
- IIT JEE-Joint Entrance Examination(Mains & Advanced)
- NEST- National Entrance Screening Test
- KVPY-Kishore Vaigyanik Protsahan Yojana
- JEE Mains Paper II for B.Arch
- AIIMS - All Indian Institute of Medical Science's Examination
- Chennai Mathematical Institute Entrance Examination
- BITSAT- Birla Institute of Science And Technology Admission Test
- AIEEE – All India Engineering Entrance Exam
- CUCET – Central Universities Common Entrance Test
- JIPMER - Jawaharlal Institute of Postgraduate Medical Education & Research
- CLAT – Common Law Admission Test
- HSEE- Humanities and Social Sciences Entrance Examination
- AIPVT -All India Pre-Veterinary Test
- NDA – National Defence Academy Examination

After Graduation

- JAM- Joint Admission Test
- JEST – Joint Entrance Screening Test
- GATE- Graduate Aptitude Test in Engineering
- CAT – Common Admission Test(for MBA)
- Exams conducted by Respective Universities

After Post Graduation

- CSIR - National Eligibility Test for JRF and Lectureship

Education Career

After Completing +2

- Integrated Msc. Physics
- Central Uiversities through CUCET
- Central Research Institutes like IISER using KVPY, JEE Advanced , IISER Aptitude Test
- Top 1% students in State board are eligible for IISER Aptitude Test
- Admission in NISER through NEST
- B.Sc Photonics
- B.Sc Hons in Mathematics and Physics in CMI
- B.Sc Hons in Mathematics and Computer Science in CMI
- Five-Year Dual degree In IIST (B.Tech + Master of Science)
- Master of Science (Astronomy and Astrophysics, Solid State Physics)

Note

- Students admitted to IISc, IIT's, NIT's IISER's, IIST, will get a Scholorship equivalent to INSPIRE
- Assured placement in ISRO and other divisions for the students of IIST
- Institutes and their ranking can be found in www.nirfindia.org

After B.Sc Physics

- M.Sc Physics in IIT's and NIT's through JAM
- Integrated Phd in IISER's and IISc through JAM and JEST
- M.Sc Physics in Central Universities throught CUCET
- M.Sc in Energy Physics, Applied Physics in IIT's through JAM
- Integrated Phd in IMSc, TIFR, JNCASR through JEST score
- Integrated Phd in TIFR through JEST and TIFR exam
- M.Sc Photonics, Reactor physics, Nuclear Engineering ,
- M.Sc Medical Physics
- M.Sc Biophyiscs
- Research Institutes in abroad like CERN, NASA, LIGO offer Summer internship programmes for motivated Indian students pursuing Undergraduate course in physics
- Indian Academy of Science & various other research institutes offer paid Summer Internship for science students to get an hands on experience in research.

Opportunities after B.Sc. Physics

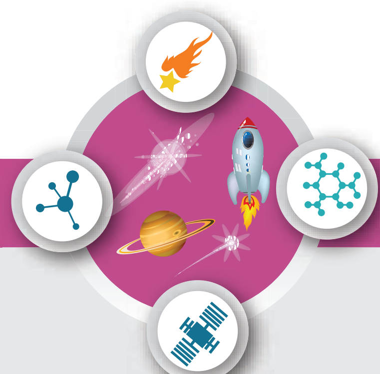
Jobs in Government Sector

- Scientific Officer and Scientific Assistant Jobs
- CSIR Labs
- DRDO – Defence Research and Development Organisation
- DAE -Department of Atomic Energy
- DoS - Department of Science
- IMD- Indian Meteorological Department
- ONGC -Oil and Natural Gas Corporation
- ATC – Air Traffic Controller
- Teaching faculty in schools and colleges through SET, NET,TET
- Scientist post in various research institutes in India

Scholarships

- INSPIRE Scholarship - Scholarship for Higher Education(SHE) - 80000 per annum, for B.Sc/B.S/Int M.Sc/Int M.S
  Eligibility Criteria:- Top 1% students in their plus 2 board exam
  Top 10000 rank holders in JEE or NEET
- Students studying at NIISER, IISER, Department of Atomic Energy Centre for Basic Science
  NTSE, KVPY, JBNSTS Scholars
- International Olympiad Medalists
- Indira Gandhi Scholarship for single girl child for full time regular Master's Degree
- Post Graduate Merit Scholarship for University rank holders in UG level
- Women Scientist Scheme (WOS -A)
  Eligibility Criteria:- Women who are pursuing M.Sc or Ph.D
- Mathematics Training and Talent Search(MTTS) Programme
  Eligibility Criteria:- Students who studied Maths at UG or PG level
- Dr. K S Krishnan Research Associateship (KSKRA)
  Eligibility Criteria:- Students who posses Master's Degree or Ph.D in science or engineering
- IGCAR JRF
  Eligibility Criteria:- Passing JEST, GATE, NET Exams
- Promotion of Science Education (POSE) Scholarship Scheme
- Dhirubhai Ambani Scholarship Programme
- Foundation for Academic Excellence and Access Scholarship(FAEA)
- Central Sector Scheme of National Fellowship and Scholarship for Higher Education of ST students
- Pre - Matric and Post - Matric Scholarship for students belonging to minority communities to pursue their School and Collegiate education by the Ministry of Minority affairs, Government of India.
- Pre Matric and Post Matric Scholarship for students with Disabilities to pursue their School and Collegiate Education by the Department of Empowerment of Persons with Disabilities, Government of India.

Institutes in india to pursue research in physics



Research Areas

General Relativity and Cosmology
Astronomy and Astrophysics
Quantum Optics and Information theory
Plasma physics
Meteorology and Atmospheric science
String Theory, Quantum Gravity
Optics and Photonics
Condensed Matter Theory, Material Science and Spintronics
Cryptography
Mathematical Physics,Statistical Physics
Crystal Growth and Crystallography
Atomic and Molecular Physics
Biophysics, Medical Physics
Nuclear and High energy Particle Physics
Energy and Environmental Studies
Geophysics
Quantum Biology and Quantum Thermodynamics and Cymatics

Famous Research Institutes for Physics in India

Name of the Institution	Website
Institute of Mathematical Sciences,Chennai(IMSc)	www.imsc.res.in
Saha Institute of Nuclear Physics, Kolkata	www.saha.ac.in
International Centre for Theoretical Sciences, Bangalore	www.icts.res.in
Harish chandra Research Institute,Allahabad	www.hri.res.in
Aryabhatta Research Institute of Observational Sciences, Nainital	www.aries.res.in
Jawaharlal Nehru Centre for Advanced Scientific Research (JNCASR)	www.jncasr.ac.in
Institute of Physics (IOP), Bhubaneshwar	www.iopb.res.in
Indian Association for the Cultivation of Sciences (IACS), Kolkata	www.iacs.res.in
Vikram Sarabhai Space Centre (VSSC), Thiruvananthapuram	www.vssc.gov.in
National Physical Laboratory (NPL),Delhi	www.nplindia.in
National Institute of Science Education and Research (NISER), Bhubaneshwar	www.niser.ac.in
Indian Institute of Science(IISc),Bangalore	www.iisc.ac.in
Raman Research Institute(RRI),Bangalore	www.rri.res.in
Tata Institute of Fundamental Research (TIFR)	www.tifr.res.in
Bhaba Atomic Research Centre(BARC)	www.barc.gov.in
Indira Gandhi Centre for Atomic Research(IGCAR)	www.igcar.gov.in
Inter University Centre for Astronomy and Astrophysics(IUCAA),Pune	www.iucaa.in
Indian Institute of Space Science and Technology(IIST), Trivandrum	www.iist.ac.in
Institute of Plasma Research(IPR),Gujarat	www.ipr.res.in
Physical Research Laboratory(PRL),Ahmedabad	www.prl.res.in
Inter-University Accelerator Center (IUAC)	www.iuac.res.in
Indian Institute of Astrophysics(IIA),Bangalore	www.iiap.res.in
Chennai Mathematical Institute(CMI),Chennai	www.cmi.ac.in
Liquid Propulsion Systems Centre	www.lpsc.gov.in
S.N.Bose Centre for Basic Sciences	www.bose.res.in
CSIR National laboratories	
Indian Institute of Technology(IIT) in various places	
IISER's in various places	
National Institute of Technology(NIT) in various places	
Indian Institute of Information Technology (IIITs) at various places	
Central and State Universities	

UNIT 1

ELECTROSTATICS

Electricity is really just organized lightning

– George Carlin

LEARNING OBJECTIVES

In this unit, student is exposed to

- Historical background of electricity and magnetism
- The role of electrostatic force in day – to-day life
- Coulomb's law and superposition principle
- The concept of electric field
- Calculation of electric field for various charge configurations
- Electrostatic potential and electrostatic potential energy
- Electric dipole and dipole moment
- Electric field and electrostatic potential for a dipole
- Electric flux
- Gauss law and its various applications
- Electrostatic properties of conductors and dielectrics
- Polarisation
- Capacitors in series and parallel combinations
- Effect of a dielectric in a capacitor
- Distribution of charges in conductors, corona discharge
- Working of a Van de Graaff generator

TLFRJV

1.1 INTRODUCTION

Electromagnetism is one of the most important branches of physics. The technological developments of the modern 21st century are primarily due to our understanding of electromagnetism. The forces we experience in everyday life are electromagnetic in nature except gravity.

In standard XI, we studied about the gravitational force, tension, friction, normal force etc. Newton treated them to be independent of each other with each force being a separate natural force. But what is the origin of all these forces? It is now understood that except gravity, all forces which we experience in every day life (tension in the string, normal force from the surface, friction etc.) arise from electromagnetic forces within the atoms. Some examples are

(i) When an object is pushed, the atoms in our hand interact with the atoms in the object and this interaction is basically electromagnetic in nature.

(ii) When we stand on Earth's surface, the gravitational force on us acts downwards and the normal force acts upward to counter balance the gravitational force. What is the origin of this normal force?

It arises due to the electromagnetic interaction of atoms on the surface of the Earth with the atoms present in the feet of the person. Though, we are attracted by the gravitational force of the Earth, we stand on Earth only because of electromagnetic force of atoms.

(iii) When an object is moved on a surface, static friction resists the motion of the object. This static friction arises due to electromagnetic interaction between the atoms present in the object and atoms on the surface. Kinetic friction also has similar origin.

From these examples, it is clear that understanding electromagnetism is very essential to understand the universe in a holistic manner. The basic principles of electromagnetism are dealt in XII physics volume 1. This unit deals with the behaviour and other related phenomena of charges at rest. This **branch of electricity which deals with stationary charges is called Electrostatics.**

1.1.1 Historical background of electric charges

Two millenniums ago, Greeks noticed that amber (a solid, translucent material formed from the resin of a fossilized tree) after rubbing with animal fur attracted small pieces of leaves and dust. The amber possessing this property is said to be 'charged'. It was initially thought that amber has this special property. Later people found that not only amber but even a glass rod rubbed with silk cloth, attracts pieces of papers. So glass rod also becomes 'charged' when rubbed with a suitable material.

Consider a charged rubber rod hanging from a thread as shown in Figure 1.1. Suppose another charged rubber rod is brought near the first rubber rod; the rods repel each other. Now if we bring a charged glass rod close to the charged rubber rod, they attract each other. At the same time, if a charged glass rod is brought near another charged glass rod, both the rods repel each other.

From these observations, the following inferences are made

(i) The charging of rubber rod and that of glass rod are different from one another.

(ii) The charged rubber rod repels another charged rubber rod, which implies that 'like charges repel each other'. We can also arrive at the same inference by observing that a charged glass rod repels another charged glass rod.

(iii) The charged amber rod attracts the charged glass rod, implying that the charge in the glass rod is not the same kind of charge present in the rubber. Thus unlike charges attract each other.

Therefore, two kinds of charges exist in the universe. In the 18th century, Benjamin Franklin called one type of charge as positive (+) and another type of charge as negative (-). Based on Franklin's convention, rubber and amber rods are negatively charged while the glass rod is positively charged. **If the net charge is zero in the object, it is said to be electrically neutral.**

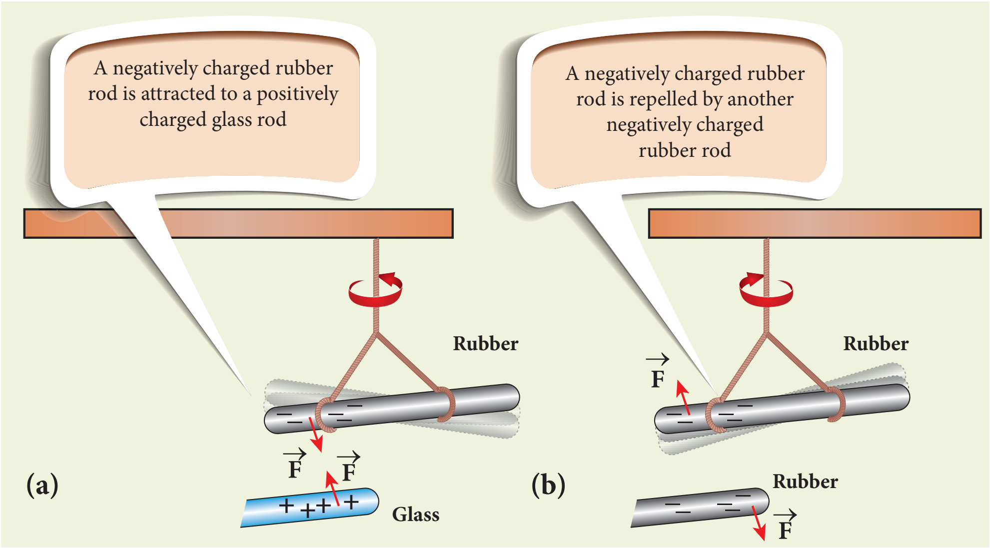


Figure 1.1 (a) Unlike charges attract each other (b) Like charges repel each other

Following the pioneering work of J. J. Thomson and E. Rutherford, in the late 19th century and in the beginning of 20th century, we now understand that the atom is electrically neutral and is made up of the negatively charged electrons, positively charged protons, and neutrons which have zero charge. The material objects made up of atoms are neutral in general. When an object is rubbed with another object (for example rubber with silk cloth), some amount of charge is transferred from one object to another due to the friction between them and the object is then said to be electrically charged. **Charging the objects through rubbing is called triboelectric charging.**

1.1.2 Basic properties of charges

(i) Electric charge

Most objects in the universe are made up of atoms, which in turn are made up of protons, neutrons and electrons. These particles have mass, an inherent property of particles. Similarly, the electric charge is another intrinsic and fundamental property of particles. The nature of charges is understood through various experiments performed in the 19th and 20th century. The SI unit of charge is coulomb.

(ii) Conservation of charges

Benjamin Franklin argued that when one object is rubbed with another object, charges get transferred from one to the other. Before rubbing, both objects are electrically neutral and rubbing simply transfers the charges from one object to the other. (For example, when a glass rod is rubbed against silk cloth, some negative charge are transferred from glass to silk. As a result, the glass rod is positively charged and silk cloth becomes negatively charged). From these observations, he concluded that charges are neither created or nor destroyed but can only be transferred from one object to other. This is called conservation of total charges and is one of the fundamental conservation laws in physics. It is stated more generally in the following way.

'The total electric charge in the universe is constant and charge can neither be created nor be destroyed. In any physical

process, the net change in charge will always be zero.

(iii) Quantisation of charges

What is the smallest amount of charge that can be found in nature? Experiments show that the charge on an electron is $-e$ and the charge on the proton is $+e$. Here, e denotes the fundamental unit of charge. The charge q on any object is equal to an integral multiple of this fundamental unit of charge e.

$$q = ne \qquad (1.1)$$

Here n is any integer (0, ±1, ±2, ±3, ±4..........). This is called quantisation of electric charge.

Robert Millikan in his famous experiment found that the value of $e = 1.6 \times 10^{-19}$C. The charge of an electron is -1.6×10^{-19} C and the charge of the proton is $+1.6 \times 10^{-19}$C.

When a glass rod is rubbed with silk cloth, the number of charges transferred is usually very large, typically of the order of 10^{10}. So the charge quantisation is not appreciable at the macroscopic level. Hence the charges are treated to be continuous (not discrete). But at the microscopic level, quantisation of charge plays a vital role.

EXAMPLE 1.1

Calculate the number of electrons in one coulomb of negative charge.

Solution

According to the quantisation of charge

$$q = ne$$

Here q = 1C. So the number of electrons in 1 coulomb of charge is

$$n = \frac{q}{e} = \frac{1C}{1.6\times10^{-19}} = 6.25\times10^{18}\text{electrons}$$

1.2 COULOMB'S LAW

In the year 1786, Coulomb deduced the expression for the force between two stationary point charges in vacuum or free space. Consider two point charges q_1 and q_2 at rest in vacuum, and separated by a distance of r, as shown in Figure 1.2. According to Coulomb, the force on the point charge q_2 exerted by another point charge q_1 is

$$\vec{F}_{21} = k\frac{q_1 q_2}{r^2}\hat{r}_{12} \qquad (1.2)$$

where $\hat{r}_{12}$ is the unit vector directed from charge q_1 to charge q_2 and k is the proportionality constant.

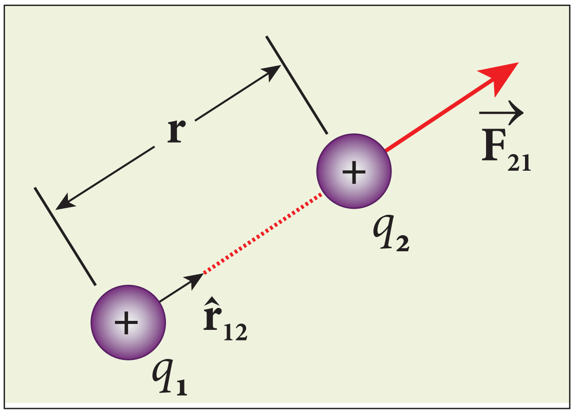


Figure 1.2 Coulomb force between two point charges

Important aspects of Coulomb's law

(i) Coulomb's law states that the electrostatic force is directly proportional to the product of the magnitude of the two point charges and is inversely proportional to the square of the distance between the two point charges.

(ii) The force on the charge q_2 exerted by the charge q_1 always lies along the line joining the two charges. $\hat{r}_{12}$ is the unit vector pointing from charge q_1 to q_2 .It is shown in the Figure 1.2. Likewise, the force on the charge q_1 exerted by q_2 is along $-\hat{r}_{12}$ (i.e., in the direction opposite to $\hat{r}_{12}$).

(iii) In SI units, $k = \frac{1}{4\pi\varepsilon_o}$ and its value is 9×10^9 N m^2 C^{-2}. Here ε_o is the permittivity of free space or vacuum and the value of $\varepsilon_o = \frac{1}{4\pi k} = 8.85\times10^{-12} C^2N^{-1}m^{-2}$.

(iv) The magnitude of the electrostatic force between two charges each of one coulomb and separated by a distance of 1 m is calculated as follows:

$|F| = \frac{9\times10^9\times1\times1}{1^2} = 9\times10^9 N$. This is a huge quantity, almost equivalent to the weight of one million ton. We never come across 1 coulomb of charge in practice. Most of the electrical phenomena in day-to-day life involve electrical charges of the order of μC (micro coulomb) or nC (nano coulomb).

(v) In SI units, Coulomb's law in vacuum takes the form $\vec{F}_{21} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}\hat{r}_{12}$. In a medium of permittivity ε, the force between two point charges is given by $\vec{F}_{21} = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}\hat{r}_{12}$. Since $\varepsilon>\varepsilon_o$, the force between two point charges in a medium other than vacuum is always less than that in vacuum. We define the relative permittivity for a given medium as $\varepsilon_r = \frac{\varepsilon}{\varepsilon_o}$. For vacuum or air, ε_r = 1 and for all other media ε_r> 1.

(vi) Coulomb's law has same structure as Newton's law of gravitation. Both are inversely proportional to the square of the distance between the particles. The electrostatic force is directly proportional to the product of the magnitude of two point charges and gravitational force is directly proportional to the product of two masses. But there are some important differences between these two laws.

- The gravitational force between two masses is always attractive but Coulomb force between two charges can be attractive or repulsive, depending on the nature of charges.
- The value of the gravitational constant G = 6.626 × 10^{-11} N m^2 kg^{-2}. The value of the constant k in Coulomb law is k = 9 × 10^9 N m^2 C^{-2}. Since k is much more greater than G, the electrostatic force is always greater in magnitude than gravitational force for smaller size objects.
- The gravitational force between two masses is independent of the medium. For example, if 1 kg of two masses are kept in air or inside water, the gravitational force between two masses remains the same. But the electrostatic force between the two charges depends on nature of the medium in which the two charges are kept at rest.
- The gravitational force between two point masses is the same whether two masses are at rest or in motion. If the charges are in motion, yet another force (Lorentz force) comes into play in addition to coulomb force.

(vii) The force on a charge q_1 exerted by a point charge q_2 is given by

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}\hat{r}_{21}$$

Here $\hat{r}_{21}$ is the unit vector from charge q_2 to q_1.

But $\hat{r}_{21} = -\hat{r}_{12}$,

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_\circ}\frac{q_1q_2}{r^2}(-\hat{r}_{12}) = -\frac{1}{4\pi\varepsilon_\circ}\frac{q_1q_2}{r^2}(\hat{r}_{12})$$

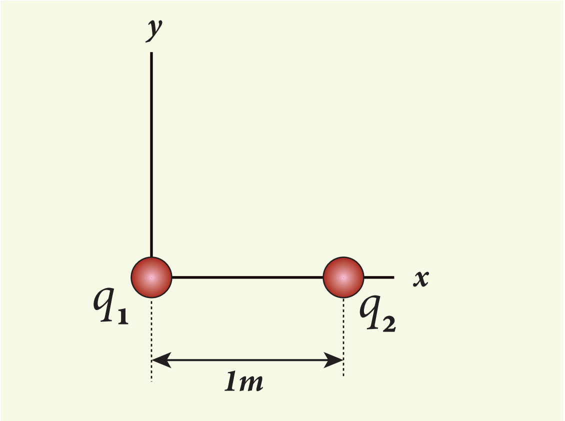
(or) $\vec{F}_{12} = -\vec{F}_{21}$

Therefore, the electrostatic force obeys Newton's third law.

(viii) The expression for Coulomb force is true only for point charges. But the point charge is an ideal concept. However we can apply Coulomb's law for two charged objects whose sizes are very much smaller than the distance between them. In fact, Coulomb discovered his law by considering the charged spheres in the torsion balance as point charges. The distance between the two charged spheres is much greater than the radii of the spheres.

EXAMPLE 1.2

Consider two point charges q_1 and q_2 at rest as shown in the figure.



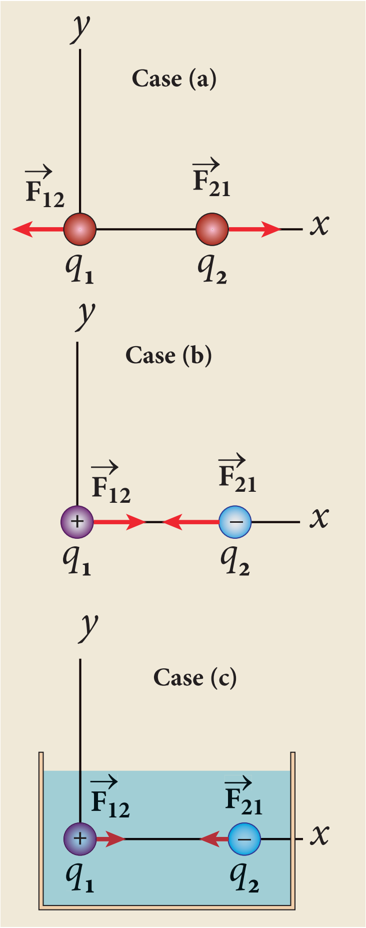
They are separated by a distance of 1m. Calculate the force experienced by the two charges for the following cases:

(a) $q_1 = +2\mu C$ and $q_2 = +3\mu C$

(b) $q_1 = +2\mu C$ and $q_2 = -3\mu C$

(c) $q_1 = +2\mu C$ and $q_2 = -3\mu C$ kept in water ($\varepsilon_r = 80$)

Solution



(a) $q_1 = +2\ \mu C$, $q_2 = +3\ \mu C$, and r = 1m. Both are positive charges. so the force will be repulsive

Force experienced by the charge q_2 due to q_1 is given by

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_\circ}\frac{q_1q_2}{r^2}\hat{r}_{12}$$

Here $\hat{r}_{12}$ is the unit vector from q_1 to q_2. Since q_2 is located on the right of q_1, we have

$\hat{r}_{12} = \hat{i}$, so that

$$\vec{F}_{21} = \frac{9\times10^9 \times 2\times10^{-6}\times3\times10^{-6}}{1^2}\hat{i} \quad \left[\frac{1}{4\pi\varepsilon_\circ} = 9\times10^9\right]$$

$$= 54\times10^{-3}\,N\hat{i}$$

According to Newton's third law, the force experienced by the charge q_1 due to q_2 is $\vec{F}_{12} = -\vec{F}_{21}$

So that $\vec{F}_{12} = -54 \times 10^{-3} N\hat{i}$.

The directions of $\vec{F}_{21}$ and $\vec{F}_{12}$ are shown in the above figure in case (a)

(b) $q_1 = +2\ \mu C$, $q_2 = -3\ \mu C$, and $r = 1m$. They are unlike charges. So the force will be attractive.

Force experienced by the charge q_2 due to q_1 is given by

$$\vec{F}_{21} = \frac{9\times10^{9}\times\left(2\times10^{-6}\right)\times\left(-3\times10^{-6}\right)}{1^2}\hat{r}_{12}$$
$$= -54\times10^{-3} N\hat{i} \quad (\text{Using } \hat{r}_{12} = \hat{i})$$

The charge q_2 will experience an attractive force towards q_1 which is in the negative x direction.

According to Newton's third law, the force experienced by the charge q_1 due to q_2 is

$$\vec{F}_{12} = -\vec{F}_{21}$$

so that $\vec{F}_{12} = 54\times10^{-3} N\hat{i}$

The directions of $\vec{F}_{21}$ and $\vec{F}_{12}$ are shown in the figure (case (b)).

(c) If these two charges are kept inside the water, then the force experienced by q_2 due to q_1

$$\vec{F}_{21}^{W} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

since $\varepsilon = \varepsilon_r \varepsilon_o$,

we have $$\vec{F}_{21}^{W} = \frac{1}{4\pi\varepsilon_r\varepsilon_\circ}\frac{q_1 q_2}{r^2}\hat{r}_{12} = \frac{\vec{F}_{21}}{\varepsilon_r}$$

Therefore,

$$\vec{F}_{21}^{W} = -\frac{54\times10^{-3} N}{80}\hat{i} = -0.675\times10^{-3} N\hat{i}$$

Note

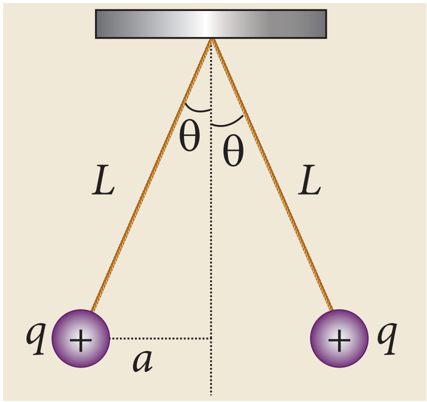
Note that the strength of the force between the two charges in water is reduced by 80 times compared to the force between the same two charges in vacuum.

When common salt (NaCl) is taken in water, the electrostatic force between Na and Cl ions is reduced due to the high relative permittivity of water ($\varepsilon_r = 80$). This is the reason water acts as a good solvent.

EXAMPLE 1.3

Two small-sized identical equally charged spheres, each having mass 1 mg are hanging in equilibrium as shown in the figure. The length of each string is 10 cm and the angle θ is 7° with the vertical. Calculate the magnitude of the charge in each sphere.

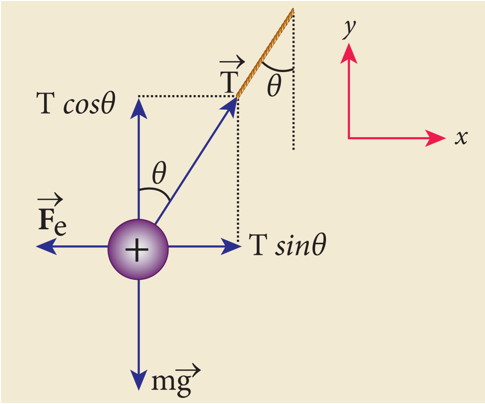
(Take $g = 10\ ms^{-2}$)



Solution

If the two spheres are neutral, the angle between them will be 0° when hanged vertically. Since they are positively charged spheres, there will be a repulsive force between them and they will be at equilibrium with each other at an angle of 7° with the vertical. At equilibrium, each charge experiences zero net force in each direction. We can draw a free body diagram for one of the charged spheres and apply Newton's second law for both vertical and horizontal directions.

The free body diagram is shown below.



In the x-direction, the acceleration of the charged sphere is zero.

Using Newton's second law $\left(\vec{F}_{tot} = m\vec{a}\right)$, we have

$$T\sin\theta\hat{i} - F_e\hat{i} = 0$$

$$T\sin\theta = F_e \qquad (1)$$

Here T is the tension acting on the charge due to the string and F_e is the electrostatic force between the two charges.

In the y-direction also, the net acceleration experienced by the charge is zero.

$$T\cos\theta\hat{j} - mg\hat{j} = 0$$

Therefore, $T\cos\theta = mg$. (2)

By dividing equation (1) by equation (2),

$$\tan\theta = \frac{F_e}{mg} \qquad (3)$$

Since they are equally charged, the magnitude of the electrostatic force is

$$F_e = k\frac{q^2}{r^2} \text{ where } k = \frac{1}{4\pi\varepsilon_0}$$

Here r = 2a = 2Lsinθ. By substituting these values in equation (3),

$$\tan\theta = k\frac{q^2}{mg\left(2L\sin\theta\right)^2} \qquad (4)$$

Rearranging the equation (4) to get q

$$q = 2L\sin\theta\sqrt{\frac{mg\tan\theta}{k}}$$

$$= 2\times0.1\times\sin7^\circ\times\sqrt{\frac{10^{-3}\times10\times\tan7^\circ}{9\times10^9}}$$

$q = 8.9 \times 10^{-9}$ C = 8.9 nC

EXAMPLE 1.4

Calculate the electrostatic force and gravitational force between the proton and the electron in a hydrogen atom. They are separated by a distance of 5.3×10^{-11} m. The magnitude of charges on the electron and proton are 1.6×10^{-19} C. Mass of the electron is $m_e = 9.1 \times 10^{-31}$ kg and mass of proton is $m_p = 1.6 \times 10^{-27}$ kg.

Solution

The proton and the electron attract each other. The magnitude of the electrostatic force between these two particles is given by

$$F_e = \frac{ke^2}{r^2} = \frac{9\times10^9\times\left(1.6\times10^{-19}\right)^2}{\left(5.3\times10^{-11}\right)^2}$$

$$= \frac{9\times2.56}{28.09}\times10^{-7} = 8.2 \times 10^{-8}\text{ N}$$

The gravitational force between the proton and the electron is attractive. The magnitude of the gravitational force between these particles is

$$F_G = \frac{Gm_e m_p}{r^2}$$

$$= \frac{6.67\times10^{-11}\times9.1\times10^{-31}\times1.6\times10^{-27}}{\left(5.3\times10^{-11}\right)^2}$$

$$= \frac{97.11}{28.09}\times10^{-47} = 3.4 \text{ x } 10^{-47}\text{ N}$$

The ratio of the two forces $\frac{F_e}{F_G} = \frac{8.2\times10^{-8}}{3.4\times10^{-47}}$

$= 2.41 \times 10^{39}$

Note that $F_e \approx 10^{39} F_G$

The electrostatic force between a proton and an electron is enormously greater than the gravitational force between them. Thus the gravitational force is negligible when compared with the electrostatic force in many situations such as for small size objects and in the atomic domain. This is the reason why a charged comb attracts an uncharged piece of paper with greater force even though the piece of paper is attracted downward by the Earth. This is shown in Figure 1.3

6I2L9R

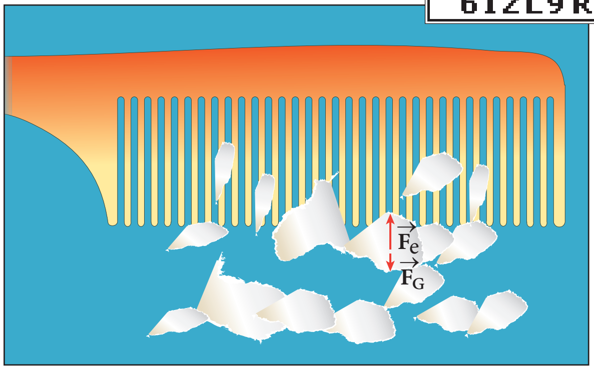


Figure 1.3 Electrostatic attraction between a comb and pieces of papers

1.2.1 Superposition principle

Coulomb's law explains the interaction between two point charges. If there are more than two charges, the force on one charge due to all the other charges needs to be calculated. Coulomb's law alone does not give the answer. The superposition principle explains the interaction between multiple charges.

According to this superposition principle, **the total force acting on a given charge is equal to the vector sum of forces exerted on it by all the other charges.**

Consider a system of n charges, namely q_1, q_2, q_3 ….q_n. The force on q_1 exerted by the charge q_2

$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21}$$

Here $\hat{r}_{21}$ is the unit vector from q_2 to q_1 along the line joining the two charges and r_{21} is the distance between the charges q_1 and q_2. The electrostatic force between two charges is not affected by the presence of other charges in the neighbourhood.

The force on q_1 exerted by the charge q_3 is

$$\vec{F}_{13} = k\frac{q_1 q_3}{r_{31}^2}\hat{r}_{31}$$

By continuing this, the total force acting on the charge q_1 due to all other charges is given by

$$\vec{F}_1^{tot} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + ...\vec{F}_{1n}$$

$$\vec{F}_1^{tot} = k\left\{\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} + \frac{q_1 q_3}{r_{31}^2}\hat{r}_{31} + \frac{q_1 q_4}{r_{41}^2}\hat{r}_{41} + ... \right.$$

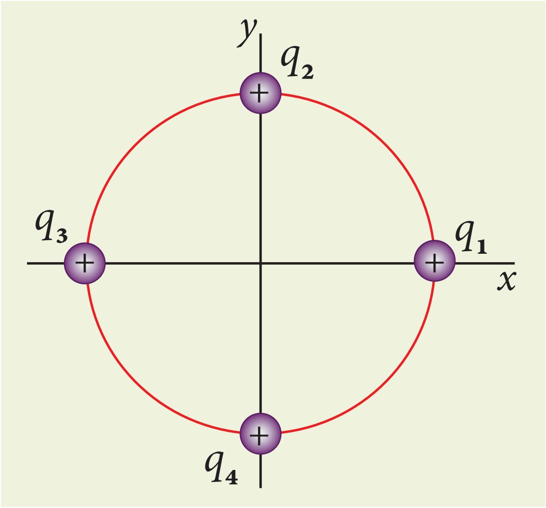
$$\left. .... + \frac{q_1 q_2}{r_{n1}^2}\hat{r}_{n1}\right\} \quad (1.3)$$

Note

Without the superposition principle, Coulomb's law will be incomplete when applied to more than two charges. Both the superposition principle and Coulomb's law form fundamental principles of electrostatics and explain all the phenomena in electrostatics. But they are not derivable from each other.

EXAMPLE 1.5

Consider four equal charges q_1,q_2, q_3 and q_4 = q = +1μC located at four different points on a circle of radius 1m, as shown in the figure. Calculate the total force acting on the charge q_1 due to all the other charges.

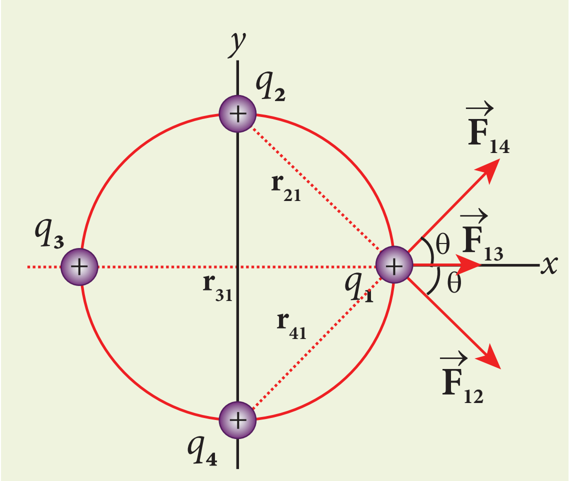


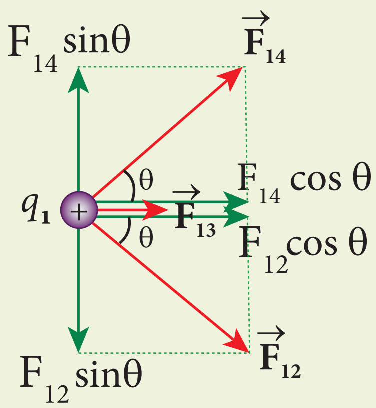
Solution

According to the superposition principle, the total electrostatic force on charge q_1 is the vector sum of the forces due to the other charges,

$$\vec{F}_1^{tot} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

The following diagram shows the direction of each force on the charge q_1.





The charges q_2 and q_4 are equi-distant from q_1. As a result the strengths (magnitude) of the forces $\vec{F}_{12}$ and $\vec{F}_{14}$ are the same even though their directions are different. Therefore the vectors representing these two forces are drawn with equal lengths. But the charge q_3 is located farther compared to q_2 and q_4. Since the strength of the electrostatic force decreases as distance increases, the strength of the force $\vec{F}_{13}$ is lesser than that of forces $\vec{F}_{12}$ and $\vec{F}_{14}$. Hence the vector representing the force $\vec{F}_{13}$ is drawn with smaller length compared to that for forces $\vec{F}_{12}$ and $\vec{F}_{14}$.

From the figure, $r_{21} = \sqrt{2}\,m = r_{41}$ and $r_{31} = 2m$

The magnitudes of the forces are given by

$$F_{13} = \frac{kq^2}{r_{31}^2} = \frac{9\times10^9\times10^{-12}}{4}$$

$F_{13} = 2.25 \times 10^{-3}$ N

$$F_{12} = \frac{kq^2}{r_{21}^2} = F_{14} = \frac{9\times10^9\times10^{-12}}{2}$$

$= 4.5 \times 10^{-3}$ N

From the figure, the angle $\theta = 45^o$. In terms of the components, we have

$$\vec{F}_{12} = F_{12}\cos\theta\hat{i} - F_{12}\sin\theta\hat{j}$$

$$= 4.5\times10^{-3}\times\frac{1}{\sqrt{2}}\hat{i} - 4.5\times10^{-3}\times\frac{1}{\sqrt{2}}\hat{j}$$

$$\vec{F}_{13} = F_{13}\hat{i} = 2.25\times10^{-3}N\hat{i}$$

$$\vec{F}_{14} = F_{14}\cos\theta\hat{i} + F_{14}\sin\theta\hat{j}$$

$$= 4.5\times10^{-3}\times\frac{1}{\sqrt{2}}\hat{i} + 4.5\times10^{-3}\times\frac{1}{\sqrt{2}}\hat{j}$$

Then the total force on q_1 is,

$$\vec{F}_1^{tot} = \left(F_{12}\cos\theta\hat{i} - F_{12}\sin\theta\hat{j}\right) + F_{13}\hat{i} + \left(F_{14}\cos\theta\hat{i} + F_{14}\sin\theta\hat{j}\right)$$

$$\vec{F}_1^{tot} = \left(F_{12}\cos\theta + F_{13} + F_{14}\cos\theta\right)\hat{i} + \left(-F_{12}\sin\theta + F_{14}\sin\theta\right)\hat{j}$$

Since $F_{12} = F_{14}$, the j^{th} component is zero. Hence we have

$$\vec{F}_1^{tot} = \left(F_{12}\cos\theta + F_{13} + F_{14}\cos\theta\right)\hat{i}$$

substituting the values in the above equation,

$$= \left(\frac{4.5}{\sqrt{2}} + 2.25 + \frac{4.5}{\sqrt{2}}\right)\hat{i} = \left(4.5\sqrt{2} + 2.25\right)\hat{i}$$

$$\vec{F}_1^{tot} = 8.61\times10^{-3}N\hat{i}$$

The resultant force is along the positive x axis.

1.3 ELECTRIC FIELD AND ELECTRIC FIELD LINES

1.3.1 Electric Field

The interaction between two charges is determined by Coulomb's law. How does the interaction itself occur? Consider a point charge kept at a point in space. If another point charge is placed at some distance from the first point charge, it experiences either an attractive force or repulsive force. This is called 'action at a distance'. But how does the second charge know about existence of the first charge which is located at some distance away from it? To answer this question, Michael Faraday introduced the concept of **field**.

According to Faraday, every charge in the universe creates an electric field in the surrounding space, and if another charge is brought into its field, it will interact with the electric field at that point and will experience a force. It may be recalled that the interaction of two masses is similarly explained using the concept of gravitational field (Refer unit 6, volume 2, XI physics). Both the electric and gravitational forces are non-contact forces, hence the field concept is required to explain action at a distance.

Consider a source point charge q located at a point in space. Another point charge q_o (test charge) is placed at some point P which is at a distance r from the charge q. The electrostatic force experienced by the charge q_o due to q is given by Coulomb's law.

$$\vec{F} = \frac{kqq_0}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{r} \quad \text{where} \quad k = \frac{1}{4\pi\varepsilon_0}$$

The charge q creates an electric field in the surrounding space. The electric field at the point P at a distance r from the point charge q is the force experienced by a unit charge and is given by

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{kq}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \qquad (1.4)$$

Here $\hat{r}$ is the unit vector pointing from q to the point of interest P. The electric field is a vector quantity and its SI unit is Newton per Coulomb (NC^{-1}).

Important aspects of Electric field

(i) If the charge q is positive then the electric field points away from the source charge and if q is negative, the electric field points towards the source charge q. This is shown in the Figure 1.4.

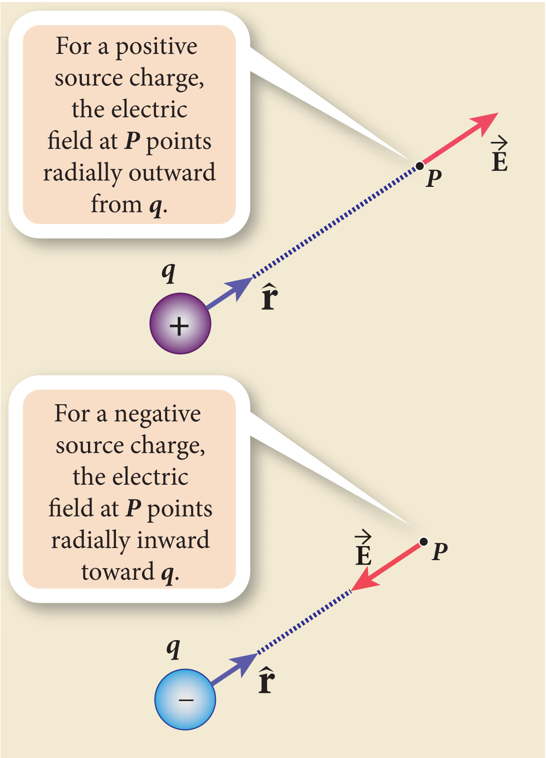


Figure 1.4 Electric field of positive and negative charges

(ii) If the electric field at a point P is $\vec{E}$,then the force experienced by the test charge q_o placed at the point P is

$$\vec{F} = q_0\vec{E} \qquad (1.5)$$

This is Coulomb's law in terms of electric field. This is shown in Figure 1.5

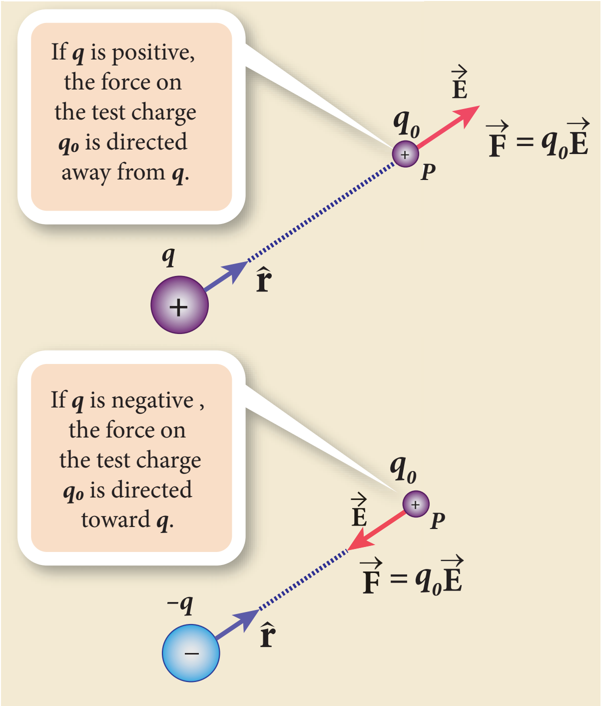


Figure 1.5 Coulomb's law in terms of electric field

(iii) The equation (1.4) implies that the electric field is independent of the test charge q_o and it depends only on the source charge q.

(iv) Since the electric field is a vector quantity, at every point in space, this field has unique direction and magnitude as shown in Figures 1.6(a) and (b). From equation (1.4), we can infer that as distance increases, the electric field decreases in magnitude.

Note that in Figures 1.6 (a) and (b) the length of the electric field vector is shown for three different points. The strength or magnitude of the electric field at point P is stronger than at the points Q and R because the point P is closer to the source charge.

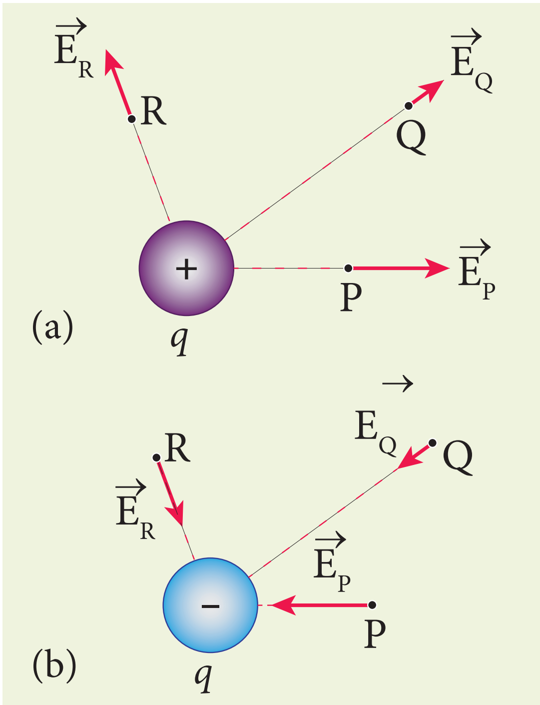


Figure 1.6 (a) Electric field due to positive charge (b) Electric field due to negative charge

(v) In the definition of electric field, it is assumed that the test charge q_0 is taken sufficiently small, so that bringing this test charge will not move the source charge. In other words, the test charge is made sufficiently small such that it will not modify the electric field of the source charge.

(vi) The expression (1.4) is valid only for point charges. For continuous and finite size charge distributions, integration techniques must be used These will be explained later in the same section. However, this expression can be used as an approximation for a finite-sized charge if the test point is very far away from the finite sized source charge. Note that we similarly treat the Earth as a point mass when we calculate the gravitational field of the Sun on the Earth (refer unit 6, volume 2, XI physics).

(vii) There are two kinds of the electric field: uniform (constant) electric field and non-uniform electric field. Uniform electric field will have the same direction and constant magnitude at all points in space. Non-uniform electric field will have different directions or different magnitudes or both at different points in space. The electric field created by a point charge is basically a non uniform electric field. This non-uniformity arises, both in direction and magnitude, with the direction being radially outward (or inward) and the magnitude changes as distance increases. These are shown in Figure 1.7.

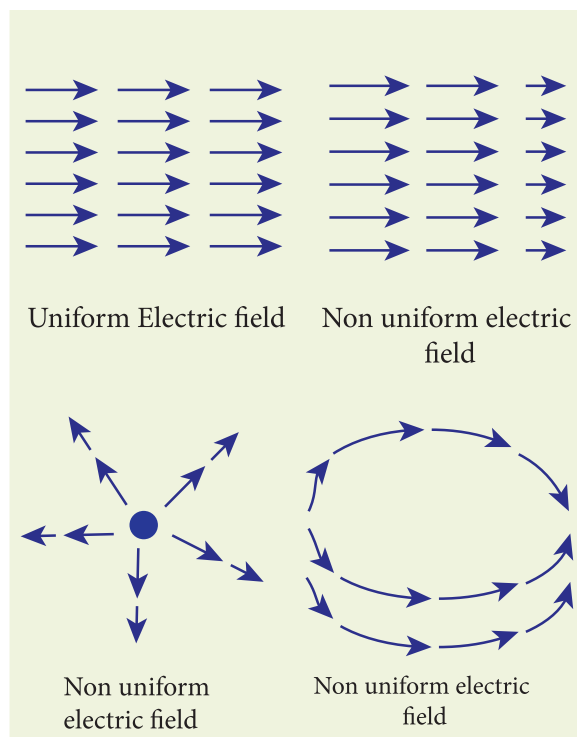


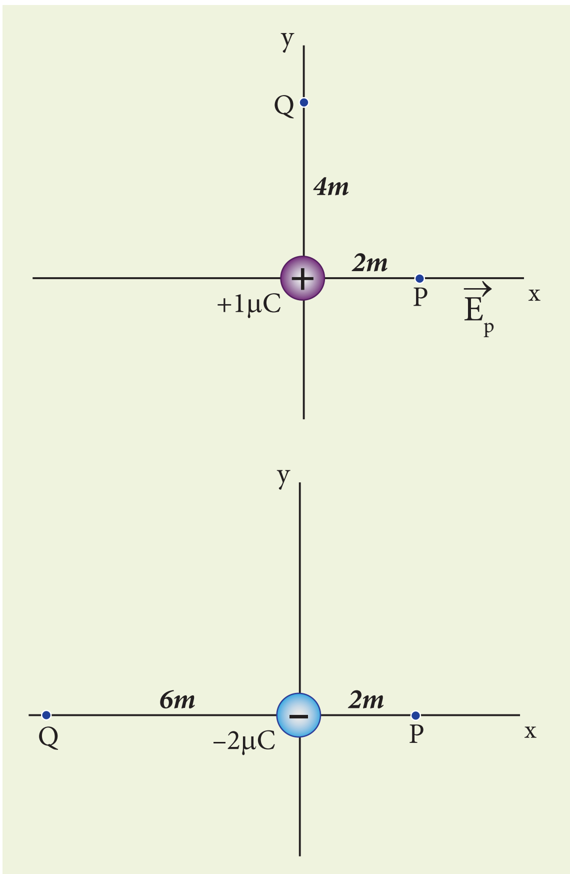
Figure 1.7 Uniform and non-uniform electric field

EXAMPLE 1.6

Calculate the electric field at points P, Q for the following two cases, as shown in the figure.

(a) A positive point charge +1 μC is placed at the origin

(b) A negative point charge -2 μC is placed at the origin



Solution

Case (a)

The magnitude of the electric field at point P is

$$E_P = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{9\times10^9\times1\times10^{-6}}{4}$$

$$= 2.25\times10^3\,NC^{-1}$$

Since the source charge is positive, the electric field points away from the charge. So the electric field at the point P is given by

$$\vec{E}_P = 2.25\times10^3 NC^{-1}\,\hat{i}$$

For the point Q

$$\left|\vec{E}_Q\right| = \frac{9\times10^9\times1\times10^{-6}}{16} = 0.56\times10^3\,NC^{-1}$$

Hence $\quad \vec{E}_Q = 0.56\times10^3\,\hat{j}$

Case (b)

The magnitude of the electric field at point P

$$\left|\vec{E}_P\right| = \frac{kq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{9\times10^9\times2\times10^{-6}}{4}$$

$= 4.5 \times 10^3$ N C^{-1}

Since the source charge is negative, the electric field points towards the charge. So the electric field at the point P is given by

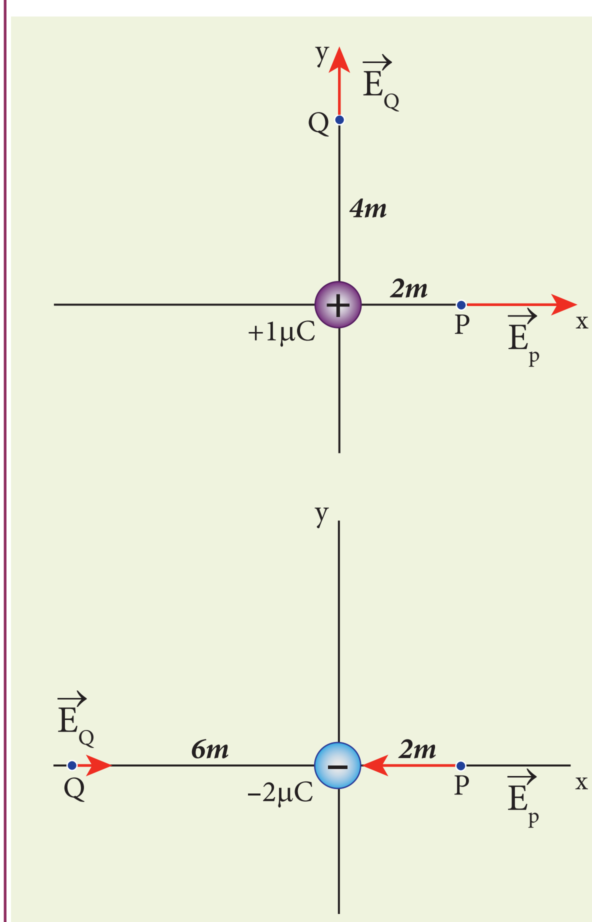
$$\vec{E}_P = -4.5\times10^3\,\hat{i}\,NC^{-1}$$

For the point Q, $\quad \left|\vec{E}_Q\right| = \dfrac{9\times10^9\times2\times10^{-6}}{36}$

$= 0.5 \times 10^3$ N C^{-1}

$$\vec{E}_R = 0.56\times10^3\,\hat{i}\,NC^{-1}$$

At the point Q the electric field is directed along the positive x-axis.



1.3.2 Electric field due to the system of point charges

Suppose a number of point charges are distributed in space. To find the electric field at some point P due to this collection of point charges, superposition principle is used. The electric field at an arbitrary point due to a collection of point charges is simply equal to the vector sum of the electric fields created by the individual point charges. This is called superposition of electric fields.

Consider a collection of point charges $q_1, q_2, q_3 ....q_n$ located at various points in space. The total electric field at some point P due to all these n charges is given by

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ... + \vec{E}_n \quad (1.6)$$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\left\{\frac{q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{q_2}{r_{2P}^2}\hat{r}_{2P} + \frac{q_3}{r_{3P}^2}\hat{r}_{3P} + ... \right.$$
$$\left. .... + \frac{q_n}{r_{nP}^2}\hat{r}_{nP}\right\} \quad (1.7)$$

Here $r_{1P}, r_{2P}, r_{3P} ....r_{nP}$ are the distance of the the charges $q_1, q_2, q_3 ....q_n$ from the point P respectively. Also $\hat{r}_{1P}, \hat{r}_{2P}, \hat{r}_{3P} ....\hat{r}_{nP}$ are the corresponding unit vectors directed from $q_1, q_2, q_3 ....q_n$ to P.
Equation (1.7) can be re-written as,

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_\circ}\sum_{i=1}^{n}\left(\frac{q_i}{r_{iP}^2}\hat{r}_{iP}\right) \quad (1.8)$$

For example in Figure 1.8, the resultant electric field due to three point charges q_1, q_2, q_3 at point P is shown.
Note that the relative lengths of the electric field vectors for the charges depend on relative distances of the charges to the point P.

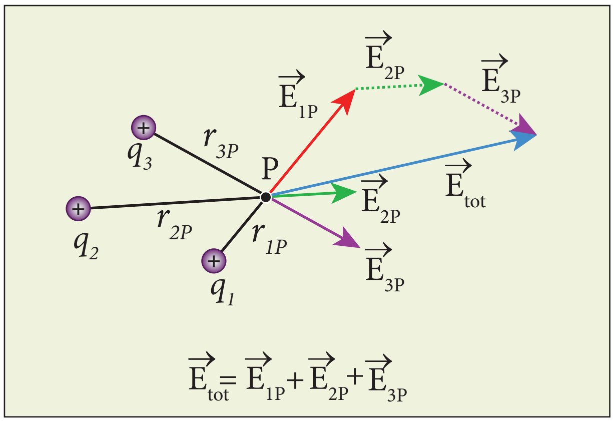
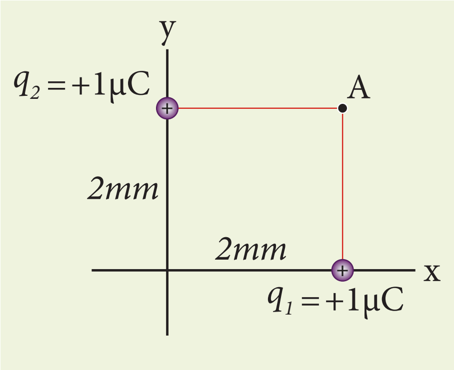


Figure 1.8 Superposition of Electric field

EXAMPLE 1.7

Consider the charge configuration as shown in the figure. Calculate the electric field at point A. If an electron is placed at points A, what is the acceleration experienced by this electron? (mass of the electron = 9.1 × 10^{-31} kg and charge of electron = −1.6 × 10^{-19} C)



Solution

By using superposition principle, the net electric field at point A is

$$\vec{E}_A = \frac{1}{4\pi\varepsilon_\circ}\frac{q_1}{r_{1A}^2}\hat{r}_{1A} + \frac{1}{4\pi\varepsilon_\circ}\frac{q_2}{r_{2A}^2}\hat{r}_{2A},$$

where r_{1A} and r_{2A} are the distances of point A from the two charges respectively.

$$\vec{E}_A = \frac{9\times10^9\times1\times10^{-6}}{\left(2\times10^{-3}\right)^2}(\hat{j}) + \frac{9\times10^9\times1\times10^{-6}}{\left(2\times10^{-3}\right)^2}(\hat{i})$$

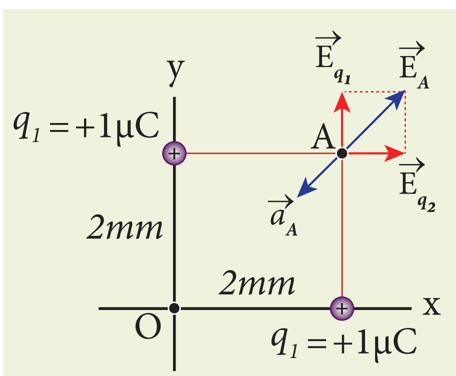
$$= 2.25\times10^9\ \hat{j} + 2.25\times10^9\hat{i} = 2.25\times10^9\left(\hat{i}+\hat{j}\right)$$

The magnitude of electric field

$$\left|\vec{E}_A\right| = \sqrt{\left(2.25\times10^9\right)^2 + \left(2.25\times10^9\right)^2}$$

$$= 2.25\times\sqrt{2}\times10^9\ NC^{-1}$$

The direction of $\vec{E}_A$ is given by $\frac{\vec{E}_A}{\left|\vec{E}_A\right|} = \frac{2.25\times10^9\left(\hat{i}+\hat{j}\right)}{2.25\times\sqrt{2}\times10^9} = \frac{\left(\hat{i}+\hat{j}\right)}{\sqrt{2}}$, which is the unit vector along OA as shown in the figure.



The acceleration experienced by an electron placed at point A is

$$\vec{a}_A = \frac{\vec{F}}{m} = \frac{q\vec{E}_A}{m}$$

$$= \frac{\left(-1.6\times10^{-19}\right)\times\left(2.25\times10^9\right)\left(\hat{i}+\hat{j}\right)}{9.1\times10^{-31}}$$

$$= -3.95\times10^{20}\left(\hat{i}+\hat{j}\right)N$$

The electron is accelerated in a direction exactly opposite to $\vec{E}_A$.

1.3.3 Electric field due to continuous charge distribution

The electric charge is quantized microscopically. The expressions (1.2), (1.3), (1.4) are applicable to only point charges. While dealing with the electric field due to a charged sphere or a charged wire etc., it is very difficult to look at individual charges in these charged bodies. Therefore, it is assumed that charge is distributed continuously on the charged bodies and the discrete nature of charges is not considered here. The electric field due to such continuous charge distributions is found by invoking the method of calculus.

Consider the following charged object of irregular shape as shown in Figure 1.9. The entire charged object is divided into a large number of charge elements $\Delta q_1, \Delta q_2, \Delta q_3 ......\Delta q_n$ and each charge element Δq is taken as a point charge.

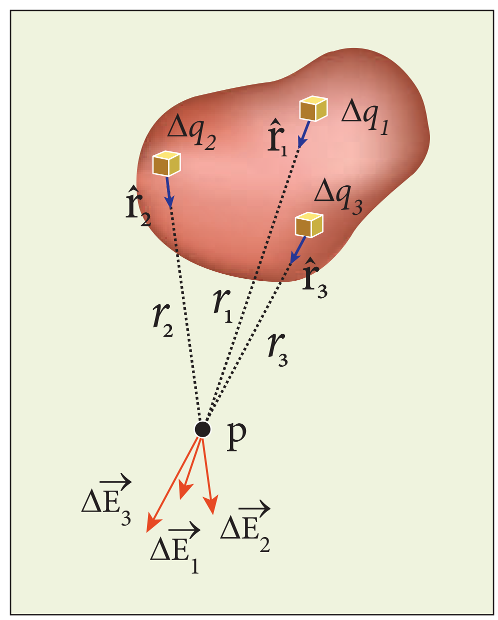


Figure 1.9 Continuous charge distributions

The electric field at a point P due to a charged object is approximately given by the sum of the fields at P due to all such charge elements.

$$\vec{E} \approx \frac{1}{4\pi\varepsilon_0}\left(\frac{\Delta q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{\Delta q_2}{r_{2P}^2}\hat{r}_{2P} + ...+ \frac{\Delta q_n}{r_{nP}^2}\hat{r}_{nP}\right)$$

$$\approx \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{\Delta q_i}{r_{iP}^2}\hat{r}_{iP} \quad (1.9)$$

Here Δq_i is the i^{th} charge element, r_{iP} is the distance of the point P from the i^{th} charge element and $\hat{r}_{iP}$ is the unit vector from i^{th} charge element to the point P.

However the equation (1.9) is only an approximation. To incorporate the continuous distribution of charge, we take the limit $\Delta q \to 0 (= dq)$. In this limit, the summation in the equation (1.9) becomes an integration and takes the following form

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\hat{r} \qquad (1.10)$$

Here r is the distance of the point P from the infinitesimal charge dq and $\hat{r}$ is the unit vector from dq to point P. Even though the electric field for a continuous charge distribution is difficult to evaluate, the force experienced by some test charge q in this electric field is still given by $\vec{F} = q\vec{E}$.

(a) If the charge Q is uniformly distributed along the wire of length L, then linear charge density (charge per unit length) is $\lambda = \frac{Q}{L}$. Its unit is coulomb per meter (Cm^{-1}).

The charge present in the infinitesimal length dl is $dq = \lambda dl$. This is shown in Figure 1.10 (a).

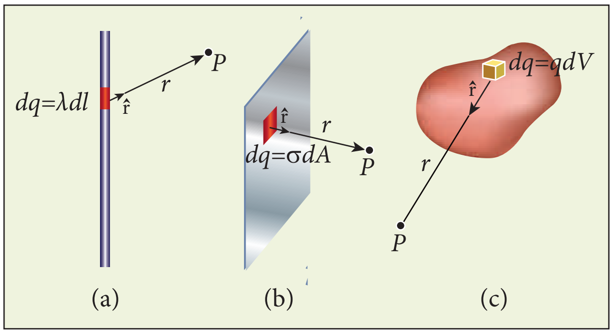


Figure 1.10 Line, surface and volume charge distribution

The electric field due to the line of total charge Q is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda dl}{r^2}\hat{r} = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{dl}{r^2}\hat{r}$$

(b) If the charge Q is uniformly distributed on a surface of area A, then surface charge density (charge per unit area) is $\sigma = \frac{Q}{A}$. Its unit is coulomb per square meter ($C\,m^{-2}$).

The charge present in the infinitesimal area dA is $dq = \sigma dA$. This is shown in the figure 1.10 (b).

The electric field due to a of total charge Q is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_\circ}\int \frac{\sigma da}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_\circ}\sigma\int \frac{da}{r^2}\hat{r}$$

This is shown in Figure 1.10(b).

(c) If the charge Q is uniformly distributed in a volume V, then volume charge density (charge per unit volume) is given by $\rho = \frac{Q}{V}$. Its unit is coulomb per cubic meter ($C\,m^{-3}$).

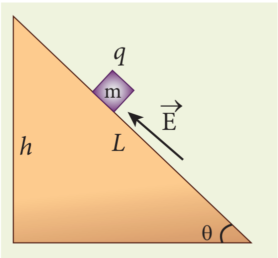
The charge present in the infinitesimal volume element dV is $dq = \rho dV$. This is shown in Figure 1.10(c).

The electric field due to a volume of total charge Q is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_\circ}\int \frac{\rho dV}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_\circ}\rho\int \frac{dV}{r^2}\hat{r}.$$

EXAMPLE 1.8

A block of mass m and positive charge q is placed on an insulated frictionless inclined plane as shown in the figure. A uniform electric field E is applied parallel to the inclined surface such that the block is at rest. Calculate the magnitude of the electric field E.



Solution

Note: A similar problem is solved in XI[th] Physics volume I, unit 3 section 3.3.2.

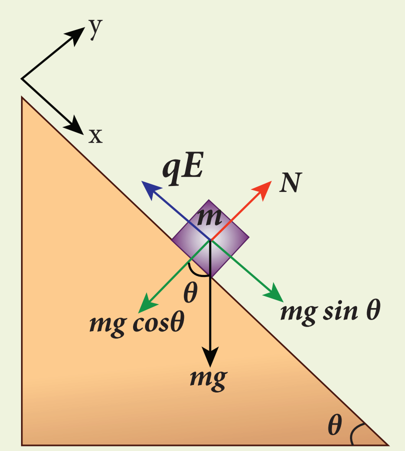
There are three forces that acts on the mass m:

(i) The downward gravitational force exerted by the Earth (mg)

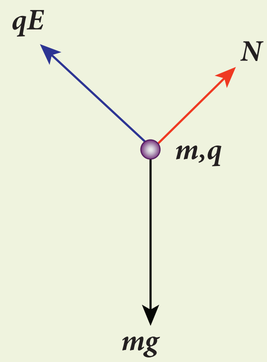
(ii) The normal force exerted by the inclined surface (N)

(iii) The Coulomb force given by uniform electric field (qE)

The free body diagram for the mass m is drawn below.



Forces acting on the mass *m*



Free body diagram

A convenient inertial coordinate system is located in the inclined surface as shown in the figure. The mass m has zero net acceleration both in x and y-direction.

Along x-direction, applying Newton's second law, we have

$$mg\sin\theta\hat{i} - qE\hat{i} = 0$$

$$mg\sin\theta - qE = 0$$

$$\text{or,} \qquad E = \frac{mg\sin\theta}{q}$$

Note that the magnitude of the electric field is directly proportional to the mass m and inversely proportional to the charge q. It implies that, if the mass is increased by keeping the charge constant, then a strong electric field is required to stop the object from sliding. If the charge is increased by keeping the mass constant, then a weak electric field is sufficient to stop the mass from sliding down the plane.

The electric field also can be expressed in terms of height and the length of the inclined surface of the plane.

$$E = \frac{mg\,h}{qL}$$

1.3.4 Electric field lines

Electric field vectors are visualized by the concept of electric field lines. They form a set of continuous lines which are the visual representation of the electric field in some region of space. The following rules are followed while drawing electric field lines for charges.

- The electric field lines start from a positive charge and end at negative charges or at infinity. For a positive point charge the electric field lines point radially outward and for a negative point charge, the electric field lines point radially inward. These are shown in Figure 1.11 (a) and (b).

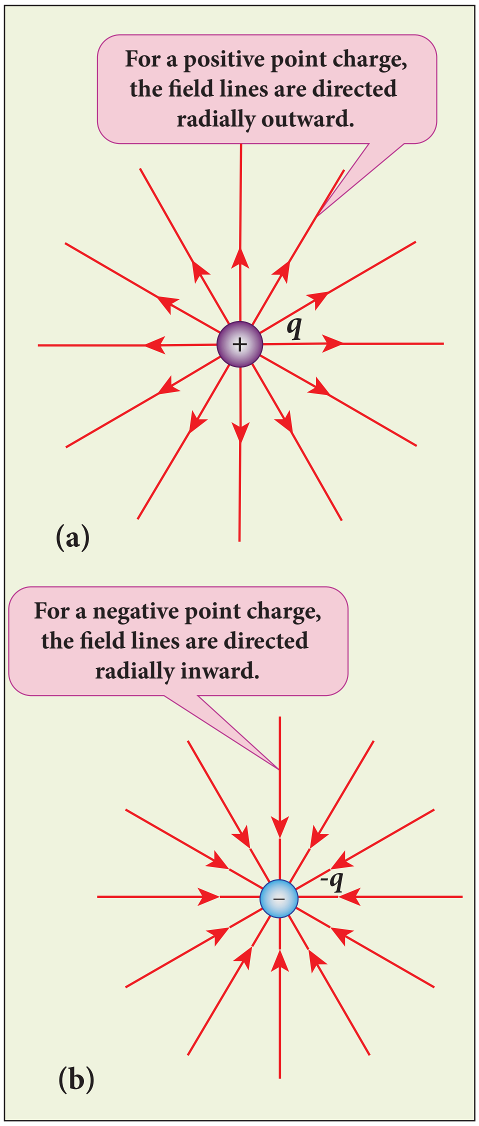


Figure 1.11 Electric field lines for isolated positive and negative charges

Note that for an isolated positive point charge the electric field line starts from the charge and ends only at infinity. For an isolated negative point charge the electric field lines start at infinity and end at the negative charge.

- The electric field vector at a point in space is tangential to the electric field line at that point. This is shown in Figure 1.12

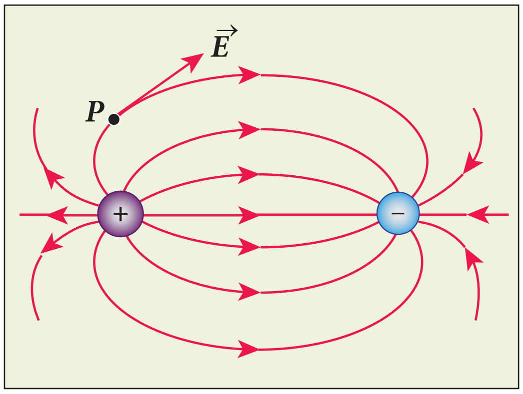


Figure 1.12 Electric field at a point P

- The electric field lines are denser (more closer) in a region where the electric field has larger magnitude and less dense in a region where the electric field is of smaller magnitude. In other words, the number of lines passing through a given surface area perpendicular to the lines is proportional to the magnitude of the electric field in that region. This is shown in Figure1.13

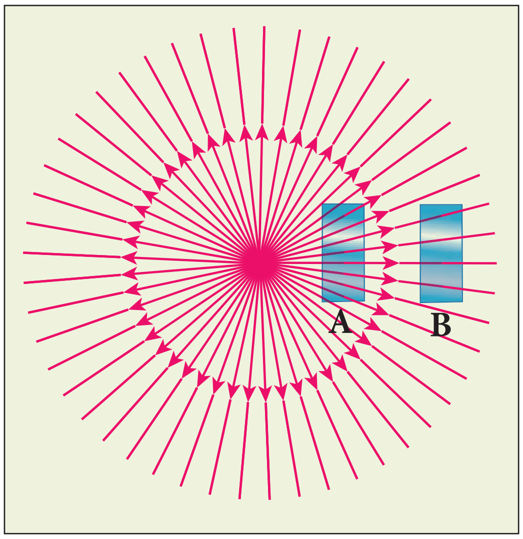


Figure 1.13 Electric field has larger magnitude at surface A than B

Figure 1.13 shows electric field lines from a positive point charge. The magnitude of the electric field for a point charge decreases as the distance increases $\left(\left|\vec{E}\right| \propto \frac{1}{r^2}\right)$. So the electric field has greater magnitude at the surface A than at B. Therefore, the number of lines crossing the surface A is greater than the number of lines crossing the surface B. Note that at surface B the electric field lines are farther apart compared to the electric field lines at the surface A.

- No two electric field lines intersect each other. If two lines cross at a point, then there will be two different electric field vectors at the same point, as shown in Figure 1.14.

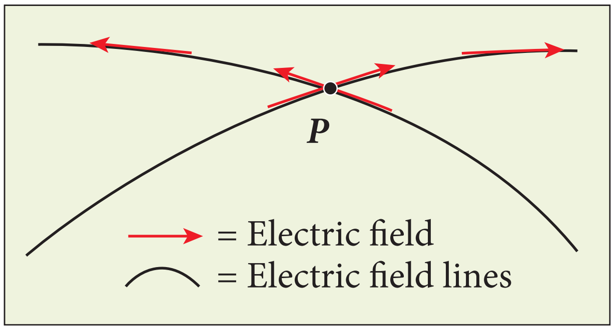


Figure 1.14 Two electric field lines never intersect each other

As a consequence, if some charge is placed in the intersection point, then it has to move in two different directions at the same time, which is physically impossible. Hence, electric field lines do not intersect.

- The number of electric field lines that emanate from the positive charge or end at a negative charge is directly proportional to the magnitude of the charges.

For example in the Figure 1.15, the electric field lines are drawn for charges +q and -2q. Note that the number of field lines emanating from +q is 8 and the number of field lines ending at -2q is 16. Since the magnitude of the second charge is twice that of the first charge, the number of field lines drawn for -2q is twice in number than that for charge +q.

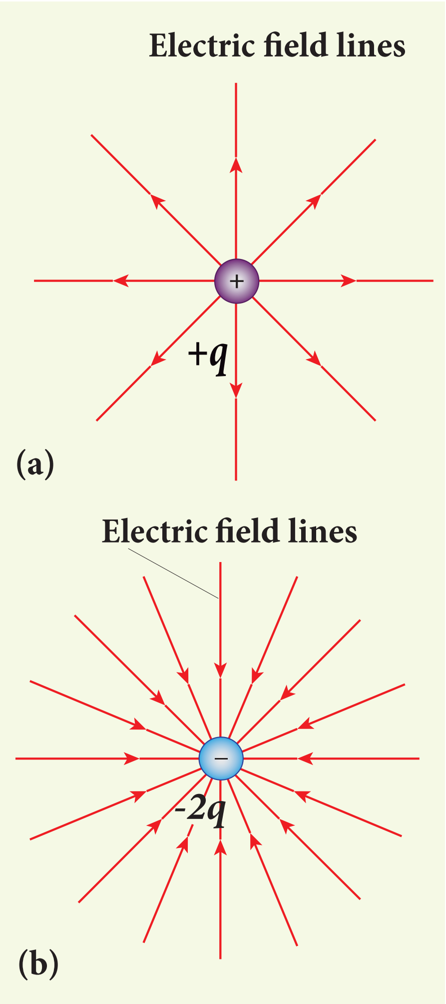
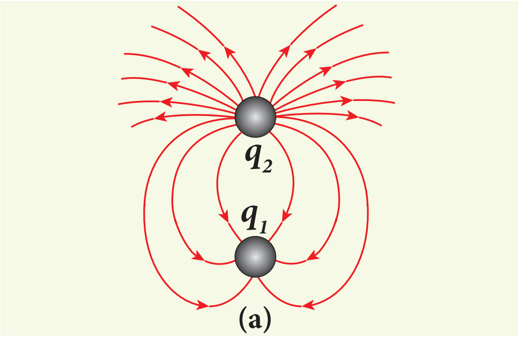
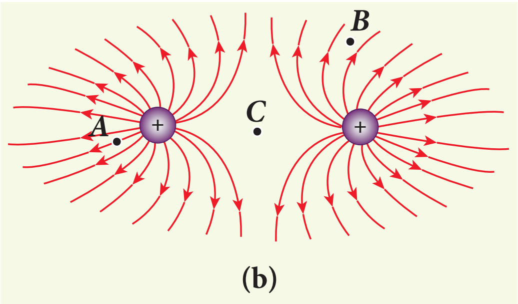


Figure 1.15 Electric field lines and magnitude of the charge

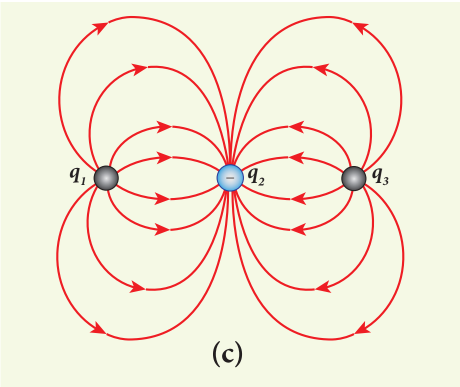
EXAMPLE 1.9

The following pictures depict electric field lines for various charge configurations.





(b)



(c)

(i) In figure (a) identify the signs of two charges and find the ratio $\left|\frac{q_1}{q_2}\right|$

(ii) In figure (b), calculate the ratio of two positive charges and identify the strength of the electric field at three points A, B, and C

(iii) Figure (c) represents the electric field lines for three charges. If q_2 = -20 nC, then calculate the values of q_1 and q_3

Solution

(i) The electric field lines start at q_2 and end at q_1. In figure (a), q_2 is positive and q_1 is negative. The number of lines starting from q_2 is 18 and number of the lines ending at q_1 is 6. So q_2 has greater magnitude. The ratio of $\left|\frac{q_1}{q_2}\right| = \frac{N_1}{N_2} = \frac{6}{18} = \frac{1}{3}$. It implies that $|q_2| = 3|q_1|$

(ii) In figure (b), the number of field lines emanating from both positive charges are equal (N=18). So the charges are equal. At point A, the electric field lines are denser compared to the lines at point B. So the electric field at point A is greater in magnitude compared to the field at point B. Further, no electric field line passes through C, which implies that the resultant electric field at C due to these two charges is zero.

(iii) In the figure (c), the electric field lines start at q_1 and q_3 and end at q_2. This implies that q_1 and q_3 are positive charges. The ratio of the number of field lines is $\left|\frac{q_1}{q_2}\right| = \frac{8}{16} = \left|\frac{q_3}{q_2}\right| = \frac{1}{2}$, implying that q_1 and q_3 are half of the magnitude of q_2. So $q_1 = q_3 = +10$ nC.

1.4 ELECTRIC DIPOLE AND ITS PROPERTIES

1.4.1 Electric dipole

Two equal and opposite charges separated by a small distance constitute an electric dipole. In many molecules, the centers of positive and negative charge do not coincide. Such molecules behave as permanent dipoles. Examples: CO, water, ammonia, HCl etc.

Consider two equal and opposite point charges (+q, -q) that are separated by a distance 2a as shown in Figure 1.16(a).
The electric dipole moment is defined as $\vec{p} = q\vec{r}_+ - q\vec{r}_-$.
Here $\vec{r}_+$ is the position vector of +q from the origin and $\vec{r}_-$ is the position vector of -q from the origin. Then, from Figure 1.16 (a),

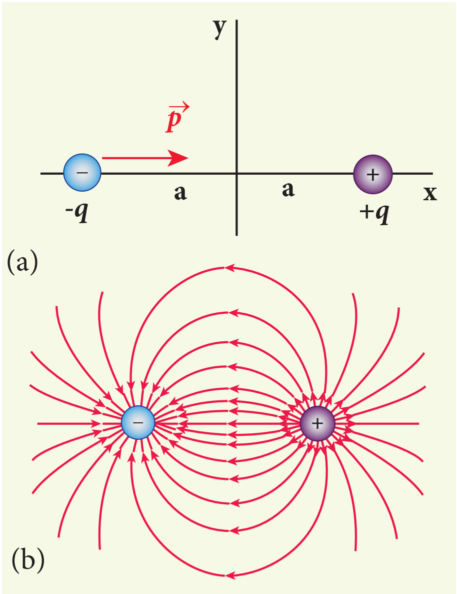


Figure 1.16 (a) Electric dipole (b) Electric field lines for the electric dipole

$$\vec{p} = qa\hat{i} - qa(-\hat{i}) = 2qa\hat{i} \qquad (1.11)$$

The electric dipole moment vector lies along the line joining two charges and is directed from –q to +q. The SI unit of dipole moment is coulomb meter (Cm). The electric field lines for an electric dipole are shown in Figure 1.16 (b).

- For simplicity, the two charges are placed on the x-axis. Even if the two charges are placed on y or z-axies, dipole moment will point from –q to +q. The magnitude of the electric dipole moment is equal to the product of the magnitude of one of the charges and the distance between them, $|\vec{p}| = 2qa$
- Though the electric dipole moment for two equal and opposite charges is defined, it is very general. It is possible to define and calculate the electric dipole moment for a single charge, two positive charges, two negative charges and also for more than two charges.

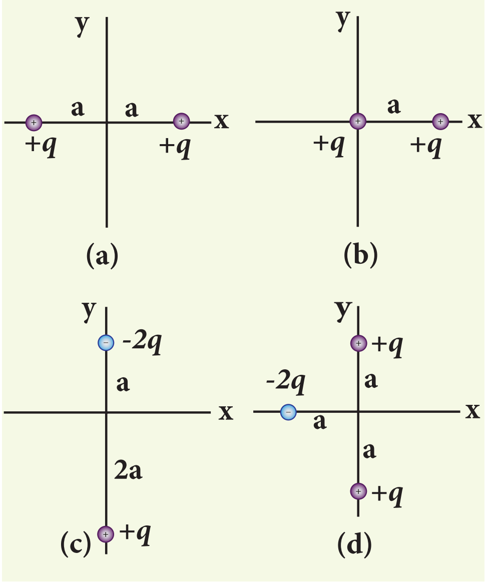
For a collection of n point charges, the electric dipole moment is defined as follows:

$$\vec{p} = \sum_{i=1}^{n} q_i \vec{r}_i \qquad (1.12)$$

where $\vec{r}_i$ is the position vector of charge q_i from the origin.

EXAMPLE 1.10

Calculate the electric dipole moment for the following charge configurations.



Solution

Case (a) The position vector for the +q on the positive x-axis is $a\hat{i}$ and position vector for the +q charge the negative x axis is $-a\hat{i}$. So the dipole moment is,

$$\vec{p} = (+q)(a\hat{i}) + (+q)(-a\hat{i}) = 0$$

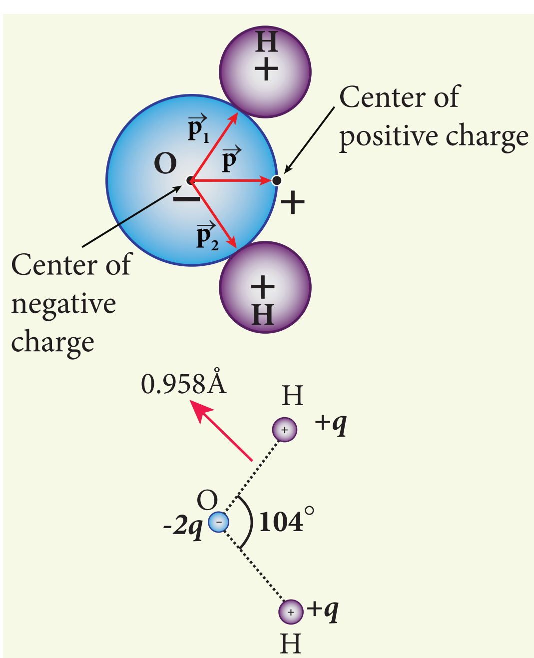
Case (b) In this case one charge is placed at the origin, so its position vector is zero. Hence only the second charge +q with position vector $a\hat{i}$ contributes to the dipole moment, which is $\vec{p} = qa\hat{i}$.

From both cases (a) and (b), we can infer that in general the electric dipole moment depends on the choice of the origin and charge configuration. But for one special case, the electric dipole moment is independent of the origin. If the total charge is zero, then the electric dipole moment will be the same irrespective of the choice of the origin. It is because of this reason that the electric dipole moment of an electric dipole (total charge is zero) is always directed from –q to +q, independent of the choice of the origin.

Case (c) $\vec{p} = (-2q)a\hat{j} + q(2a)(-\hat{j}) = -4qa\hat{j}$.
Note that in this case $\vec{p}$ is directed from -2q to +q.

Case (d) $\vec{p} = -2qa(-\hat{i}) + qa\hat{j} + qa(-\hat{j})$
$= 2qa\,\hat{i}$

The water molecule (H_2O) has this charge configuration. The water molecule has three atoms (two H atom and one O atom). The centers of positive (H) and negative (O) charges of a water molecule lie at different points, hence it possess permanent dipole moment. The O-H bond length is 0.958×10^{-10} m due to which the electric dipole moment of water molecule has the magnitude $p = 6.1 \times 10^{-30}$ Cm. The electric dipole moment $\vec{p}$ is directed from center of negative charge to the center of positive charge, as shown in the figure.



1.4.2 Electric field due to a dipole

Case (i) Electric field due to an electric dipole at points on the axial line

Consider an electric dipole placed on the x-axis as shown in Figure 1.17. A point C is located at a distance of *r* from the midpoint O of the dipole along the axial line.

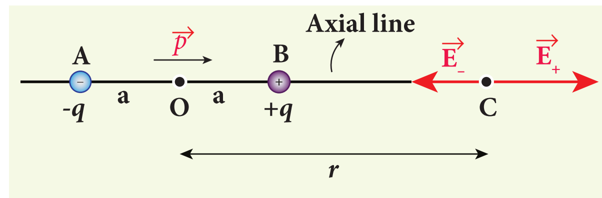


Figure 1.17 Electric field of the dipole along the axial line

The electric field at a point C due to +q is

$$\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2} \text{ along BC}$$

Since the electric dipole moment vector $\vec{p}$ is from –q to +q and is directed along BC, the above equation is rewritten as

$$\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p} \qquad (1.13)$$

where $\hat{p}$ is the electric dipole moment unit vector from –q to +q.

The electric field at a point C due to –q is

$$\vec{E}_- = -\frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}\hat{p} \qquad (1.14)$$

Since +q is located closer to the point C than –q, $\vec{E}_+$ is stronger than $\vec{E}_-$. Therefore, the length of the $\vec{E}_+$ vector is drawn larger than that of $\vec{E}_-$ vector.

The total electric field at point C is calculated using the superposition principle of the electric field.

$$\vec{E}_{tot} = \vec{E}_+ + \vec{E}_-$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}\hat{p}$$

$$\vec{E}_{tot} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right)\hat{p} \quad (1.15)$$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0} q\left(\frac{4ra}{(r^2-a^2)^2}\right)\hat{p} \quad (1.16)$$

Note that the total electric field is along $\vec{E}_+$, since $+q$ is closer to C than −q.

The direction of $\vec{E}_{tot}$ is shown in Figure 1.18.

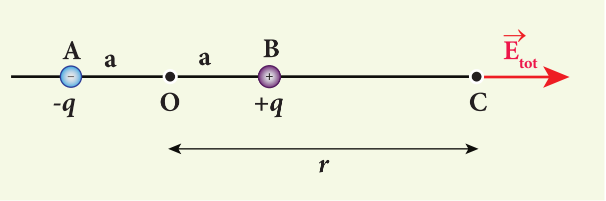


Figure 1.18 Total electric field of the dipole on the axial line

If the point C is very far away from the dipole then (r >> a). Under this limit the term $(r^2 - a^2)^2 \approx r^4$. Substituting this into equation (1.16), we get

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\left(\frac{4aq}{r^3}\right)\hat{p} \quad (r >> a)$$

since $2aq\,\hat{p} = \vec{p}$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3} \quad (r >> a) \quad (1.17)$$

If the point C is chosen on the left side of the dipole, the total electric field is still in the direction of $\vec{p}$. We infer this result by examining the electric field lines of the dipole shown in Figure 1.16(b).

Case (ii) Electric field due to an electric dipole at a point on the equatorial plane

Consider a point C at a distance r from the midpoint O of the dipole on the equatorial plane as shown in Figure 1.19.

Since the point C is equi-distant from +q and −q, the magnitude of the electric fields of +q and −q are the same. The direction of $\vec{E}_+$ is along BC and the direction of $\vec{E}_-$ is along CA. $\vec{E}_+$ and $\vec{E}_-$ are resolved into two components; one component parallel to the dipole axis and the other perpendicular to it. The perpendicular components $|\vec{E}_+|\sin\theta$ and $|\vec{E}_-|\sin\theta$ are oppositely directed and cancel each other. The magnitude of the total electric field at point C is the sum of the parallel components of $\vec{E}_+$ and $\vec{E}_-$ and its direction is along $-\hat{p}$ as shown in the Figure 1.19.

$$\vec{E}_{tot} = -|\vec{E}_+|\cos\theta\,\hat{p} - |\vec{E}_-|\cos\theta\,\hat{p} \quad (1.18)$$

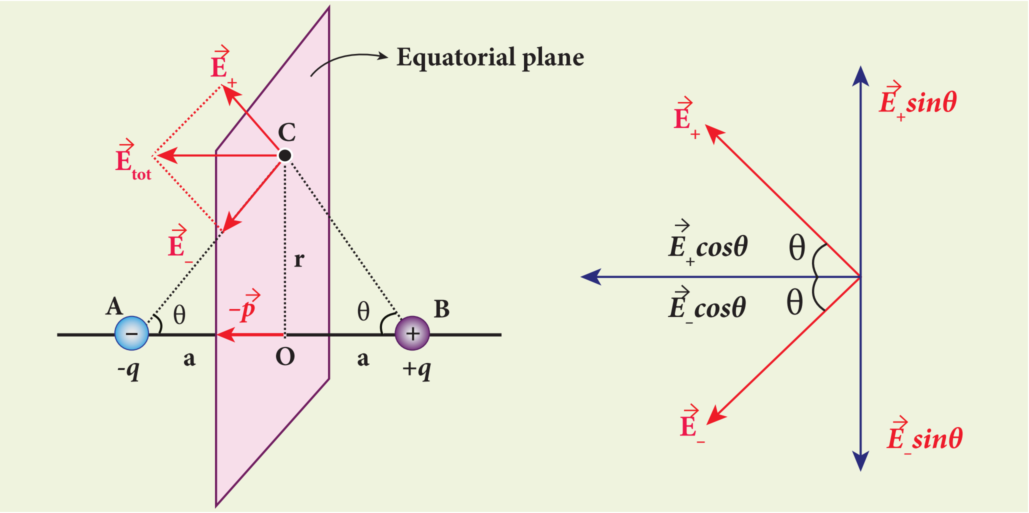


Figure 1.19 Electric field due to a dipole at a point on the equatorial plane

The magnitudes $\vec{E}_+$ and $\vec{E}_-$ are the same and are given by

$$\left|\vec{E}_+\right| = \left|\vec{E}_-\right| = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r^2+a^2\right)} \qquad (1.19)$$

By substituting equation (1.19) into equation (1.18), we get

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_0}\frac{2q\cos\theta}{\left(r^2+a^2\right)}\hat{p}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{2qa}{\left(r^2+a^2\right)^{\frac{3}{2}}}\hat{p}$$

$$\text{since } \cos\theta = \frac{a}{\sqrt{r^2+a^2}}$$

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{\left(r^2+a^2\right)^{\frac{3}{2}}}$$

$$\text{since } \vec{p} = 2qa\hat{p} \qquad (1.20)$$

At very large distances (r>>a), the equation (1.20) becomes

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_o}\frac{\vec{p}}{r^3} \qquad (r >> a) \qquad (1.21)$$

Important inferences

(i) From equations (1.17) and (1.21), it is inferred that for very large distances, the magnitude of the electric field at points on the dipole axis is twice the magnitude of the electric field at points on the equatorial plane. The direction of the electric field at points on the dipole axis is directed along the direction of dipole moment vector $\vec{p}$ but at points on the equatorial plane it is directed opposite to the dipole moment vector, that is along $-\vec{p}$.

(ii) At very large distances, the electric field due to a dipole varies as $\frac{1}{r^3}$. Note that for a point charge, the electric field varies as $\frac{1}{r^2}$. This implies that the electric field due to a dipole at very large distances goes to zero faster than the electric field due to a point charge. The reason for this behavior is that at very large distance, the two charges appear to be close to each other and neutralize each other.

(iii) The equations (1.17) and (1.21) are valid only at very large distances (r>>a). Suppose the distance 2a approaches zero and q approaches infinity such that the product of $2aq = p$ is finite, then the dipole is called a point dipole. For such point dipoles, equations (1.17) and (1.21) are exact and hold true for any r.

1.4.3 Torque experienced by an electric dipole in the uniform electric field

Consider an electric dipole of dipole moment $\vec{p}$ placed in a uniform electric field $\vec{E}$ whose field lines are equally spaced and point in the same direction. The charge +q will experience a force q$\vec{E}$ in the direction of the field and charge –q will experience a force -q$\vec{E}$ in a direction opposite to the field. Since the external field $\vec{E}$ is uniform, the total force acting on the dipole is zero. These two forces acting at different points will constitute a couple and the dipole experience a torque as shown in Figure 1.20. This torque tends to rotate the dipole. (Note that electric field lines of a uniform field are equally spaced and point in the same direction).

The total torque on the dipole about the point O

$$\vec{\tau} = \overrightarrow{OA}\times\left(-q\vec{E}\right)+\overrightarrow{OB}\times q\vec{E} \qquad (1.22)$$

Using right-hand corkscrew rule (Refer XI, volume 1, unit 2), it is found that total

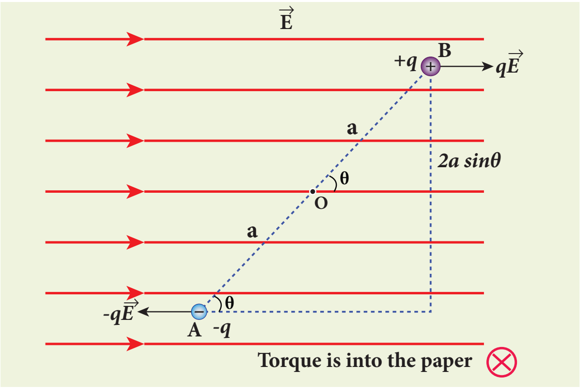


Figure 1.20 Torque on dipole

torque is perpendicular to the plane of the paper and is directed into it.

The magnitude of the total torque

$$\vec{\tau} = \left|\overline{OA}\right|\left|\left(-q\vec{E}\right)\right|\sin\theta + \left|\overline{OB}\right|\left|q\vec{E}\right|\sin\theta$$

$$\tau = qE \cdot 2a\sin\theta \qquad (1.23)$$

where θ is the angle made by $\vec{p}$ with $\vec{E}$. Since p = 2aq, the torque is written in terms of the vector product as

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad (1.24)$$

The magnitude of this torque is $\tau = pE\sin\theta$ and is maximum when $\theta = 90^\circ$.

This torque tends to rotate the dipole and align it with the electric field $\vec{E}$. Once $\vec{p}$ is aligned with $\vec{E}$, the total torque on the dipole becomes zero.

If the electric field is not uniform, then the force experienced by +q is different from that experienced by –q. In addition to the torque, there will be net force acting on the dipole. This is shown in Figure 1.21.

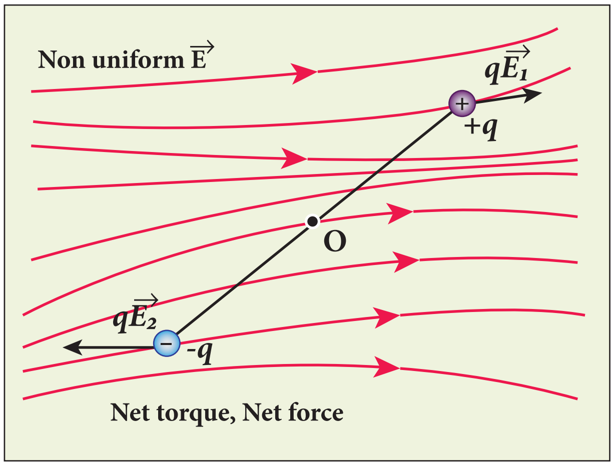


Figure 1.21 The dipole in a non-uniform electric field

EXAMPLE 1.11

A sample of HCl gas is placed in a uniform electric field of magnitude 3×10^4 N C^{-1}. The dipole moment of each HCl molecule is 3.4×10^{-30} Cm. Calculate the maximum torque experienced by each HCl molecule.

Solution

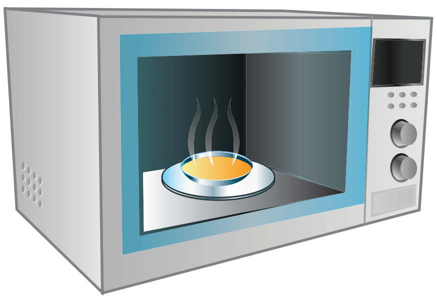
The maximum torque experienced by the dipole is when it is aligned perpendicular to the applied field.

$$\tau_{max} = pE\sin 90^\circ = 3.4\times10^{-30}\times3\times10^{4}\,Nm$$

$$\tau_{max} = 10.2\times10^{-26}\,Nm$$

DO YOU KNOW?

Microwave oven works on the principle of torque acting on an electric dipole. The food we consume has water molecules which are permanent electric dipoles. Oven produces microwaves that are oscillating electromagnetic fields and produce torque on the water molecules. Due to this torque on each water molecule, the molecules rotate very fast and produce thermal energy. Thus, heat generated is used to heat the food.



UNIT-1(XII-Physics_Vol-1).indd 26
04-03-2019 10:25:27

1.5 ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

Introduction

In mechanics, potential energy is defined for conservative forces. Since gravitational force is a conservative force, its gravitational potential energy is defined in XI standard physics (Unit 6). Since Coulomb force is an inverse-square-law force, its also a conservative force like gravitational force. Therefore, we can define potential energy for charge configurations.

1.5.1 Electrostatic Potential energy and Electrostatic potential

Consider a positive charge q kept fixed at the origin which produces an electric field $\vec{E}$ around it. A positive test charge q′ is brought from point R to point P against the repulsive force between q and q′ as shown in Figure 1.22. Work must be done to overcome this repulsion. This work done is stored as potential energy.

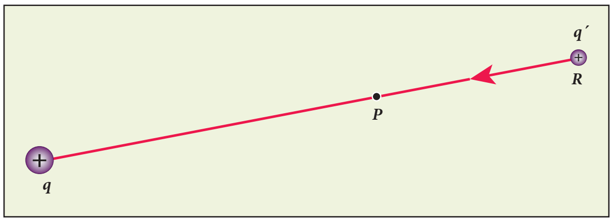


Figure 1.22 Work done is equal to potential energy

The test charge q′ is brought from R to P with constant velocity which means that external force used to bring the test charge q′ from R to P must be equal and opposite to the coulomb force $\left(\vec{F}_{ext} = -\vec{F}_{coloumb}\right)$. The work done is

$$W = \int_R^P \vec{F}_{ext} \cdot d\vec{r} \quad (1.25)$$

Since coulomb force is conservative, work done is independent of the path and it depends only on the initial and final positions of the test charge. If potential energy associated with q′ at P is U_P and that at R is U_R, then difference in potential energy is defined as the work done to bring a test charge q′ from point R to P and is given as $U_P - U_R = W = \Delta U$

$$\Delta U = \int_R^P \vec{F}_{ext} \cdot d\vec{r} \quad (1.26)$$

$$\text{Since } \vec{F}_{ext} = -\vec{F}_{coloumb} = -q'\vec{E} \quad (1.27)$$

$$\Delta U = \int_R^P -\left(q'\vec{E}\right) \cdot d\vec{r} = q'\int_R^P \left(-\vec{E}\right) \cdot d\vec{r} \quad (1.28)$$

The potential energy difference per unit charge is given by

$$\frac{\Delta U}{q'} = \frac{q'\int_R^P \left(-\vec{E}\right) \cdot d\vec{r}}{q'} = -\int_R^P \vec{E} \cdot d\vec{r} \quad (1.29)$$

The above equation (1.29) is independent of q′. The quantity $\frac{\Delta U}{q'} = -\int_R^P \vec{E} \cdot d\vec{r}$ is called electric potential difference between P and R and is denoted as $V_P - V_R = \Delta V$.

In otherwords, the electric potential difference is defined as the work done by an external force to bring unit positive charge from point R to point P.

$$V_P - V_R = \Delta V = \int_R^P -\vec{E} \cdot d\vec{r} \quad (1.30)$$

The electric potential energy difference can be written as ΔU = q′ ΔV. Physically potential difference between two points is a meaningful quantity. The value of the potential itself at one point is not meaningful. Therefore the point R is taken at infinity and its potential is considered as zero $\left(V_\infty = 0\right)$.

Then **the electric potential at a point P is equal to the work done by an external force to bring a unit positive charge with constant velocity from infinity to the point P in the region of the external electric field** $\vec{E}$. Mathematically this is written as

$$V_P = -\int_\infty^P \vec{E}\cdot d\vec{r} \qquad (1.31)$$

Important points

1. Electric potential at point P depends only on the electric field which is due to the source charge q and not on the test charge q′. Unit positive charge is brought from infinity to the point P with constant velocity because external agency should not impart any kinetic energy to the test charge.
2. From equation (1.29), the unit of electric potential is Joule per coulomb. The practical unit is volt (V) named after Alessandro Volta (1745-1827) who invented the electrical battery. The potential difference between two points is expressed in terms of voltage.

1.5.2 Electric potential due to a point charge

Consider a positive charge q kept fixed at the origin. Let P be a point at distance r from the charge q. This is shown in Figure 1.23.

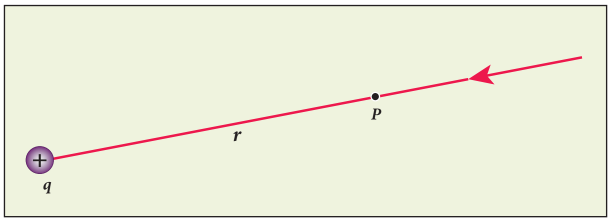


Figure 1.23 Electrostatic potential at a point P

The electric potential at the point P is

$$V = \int_\infty^r \left(-\vec{E}\right)\cdot d\vec{r} = -\int_\infty^r \vec{E}\cdot d\vec{r} \qquad (1.32)$$

Electric field due to positive point charge q is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$V = \frac{-1}{4\pi\varepsilon_0}\int_\infty^r \frac{q}{r^2}\hat{r}.d\vec{r}$$

The infinitesimal displacement vector, $d\vec{r} = dr\hat{r}$ and using $\hat{r}.\hat{r} = 1$, we have

$$V = -\frac{1}{4\pi\varepsilon_\circ}\int_\infty^r \frac{q}{r^2}\hat{r}\cdot dr\hat{r} = -\frac{1}{4\pi\varepsilon_\circ}\int_\infty^r \frac{q}{r^2}dr$$

After the integration,

$$V = -\frac{1}{4\pi\varepsilon_\circ}q\left\{-\frac{1}{r}\right\}_\infty^r = \frac{1}{4\pi\varepsilon_\circ}\frac{q}{r}$$

Hence the electric potential due to a point charge q at a distance r is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad (1.33)$$

Important points

(i) If the source charge q is positive, V > 0. If q is negative, then V is negative and equal to $V = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

(ii) The description of motion of objects using the concept of potential or potential energy is simpler than that using the concept of field.

(iii) From expression (1.33), it is clear that the potential due to positive charge decreases as the distance increases, but for a negative charge the potential increases as the distance is increased. At infinity $(r = \infty)$ electrostatic potential is zero (V = 0).

In the case of gravitational force, mass moves from a point of higher gravitational potential to a point of lower gravitational potential. Similarly a positive charge moves from a point of higher electrostatic potential to lower electrostatic potential. However a negative charge moves from lower electrostatic potential to higher electrostatic potential. This comparison is shown in Figure 1.24.

(iv) The electric potential at a point P due to a collection of charges $q_1, q_2, q_3 \ldots . q_n$ is equal to sum of the electric potentials due to individual charges.

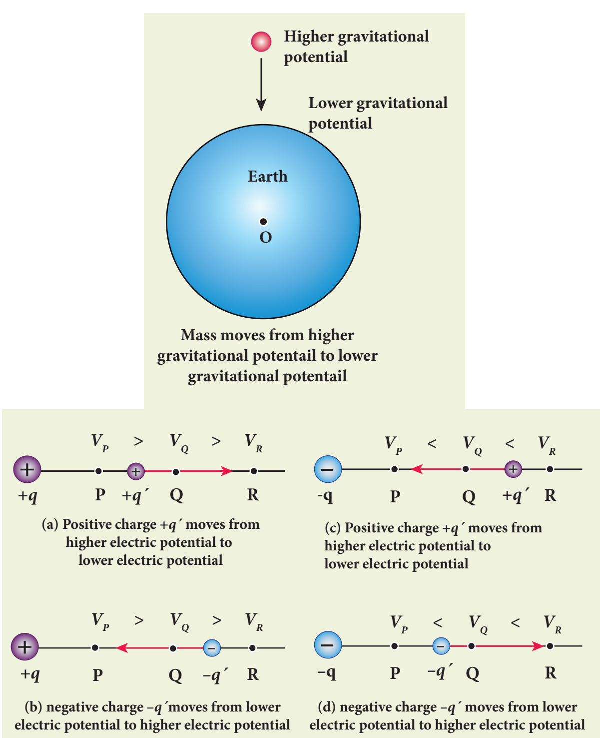


Figure 1.24 Motion of charges in terms of electric potential

$$V_{tot} = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3} + \ldots$$
$$\ldots . + \frac{kq_n}{r_n} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i} \qquad (1.34)$$

where $r_1, r_2, r_3 \ldots \ldots r_n$ are the distances of q_1, q_2, q_3 ….. q_n respectively from P (Figure 1.25).

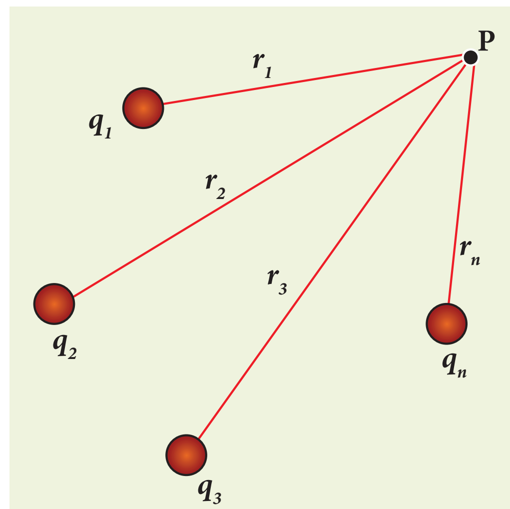
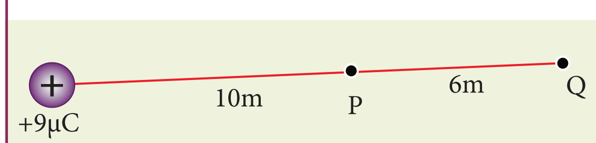


Figure 1.25 Electrostatic potential due to collection of charges

EXAMPLE 1.12

(a) Calculate the electric potential at points P and Q as shown in the figure below.

(b) Suppose the charge +9μC is replaced by -9μC find the electrostatic potentials at points P and Q



(c) Calculate the work done to bring a test charge +2μC from infinity to the point P. Assume the charge +9μC is held fixed at origin and +2μC is brought from infinity to P.

Solution

(a) Electric potential at point P is given by

$$V_P = \frac{1}{4\pi\varepsilon_\circ}\frac{q}{r_P} = \frac{9\times10^9\times9\times10^{-6}}{10} = 8.1\times10^3\,V$$

Electric potential at point Q is given by

$$V_Q = \frac{1}{4\pi\varepsilon_\circ}\frac{q}{r_Q} = \frac{9\times10^9\times9\times10^{-6}}{16} = 5.06\times10^3\,V$$

Note that the electric potential at point Q is less than the electric potential at point P. If we put a positive charge at P, it moves from P to Q. However if we place a negative charge at P it will move towards the charge +9μC.

The potential difference between the points P and Q is given by

$$\Delta V = V_P - V_Q = +3.04\times10^3\,V$$

(b) Suppose we replace the charge +9 μC by -9 μC, then the corresponding potentials at the points P and Q are,

$$V_P = -8.1\times10^3\,V,\ V_Q = -5.06\times10^3\,V$$

Note that in this case electric potential at the point Q is higher than at point P.

The potential difference or voltage between the points P and Q is given by

$$\Delta V = V_P - V_Q = -3.04\times10^3\,V$$

(c) The electric potential V at a point P due to some charge is defined as the work done by an external force to bring a unit positive charge from infinity to P. So to bring the q amount of charge from infinity to the point P, work done is given as follows.

$$W = qV$$

$$W_Q = 2\times10^{-6}\times5.06\times10^3\,J = 10.12\times10^{-3}\,J.$$

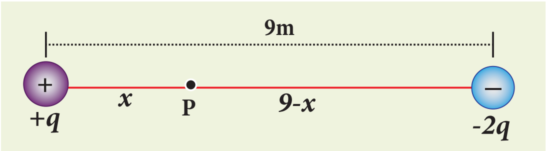
EXAMPLE 1.13

Consider a point charge +q placed at the origin and another point charge -2q placed at a distance of 9 m from the charge +q. Determine the point between the two charges at which electric potential is zero.

Solution

According to the superposition principle, the total electric potential at a point is equal to the sum of the potentials due to each charge at that point.

Consider the point at which the total potential zero is located at a distance x from the charge +q as shown in the figure.



The total electric potential at P is zero.

$$V_{tot} = \frac{1}{4\pi\varepsilon_\circ}\left(\frac{q}{x} - \frac{2q}{(9-x)}\right) = 0$$

which gives $\frac{q}{x} = \frac{2q}{(9-x)}$

or $\frac{1}{x} = \frac{2}{(9-x)}$

Hence, $x = 3$ m

1.5.3 Electrostatic potential at a point due to an electric dipole

Consider two equal and opposite charges separated by a small distance 2a as shown in Figure 1.26. The point P is located at a distance r from the midpoint of the dipole. Let θ be the angle between the line OP and dipole axis AB.

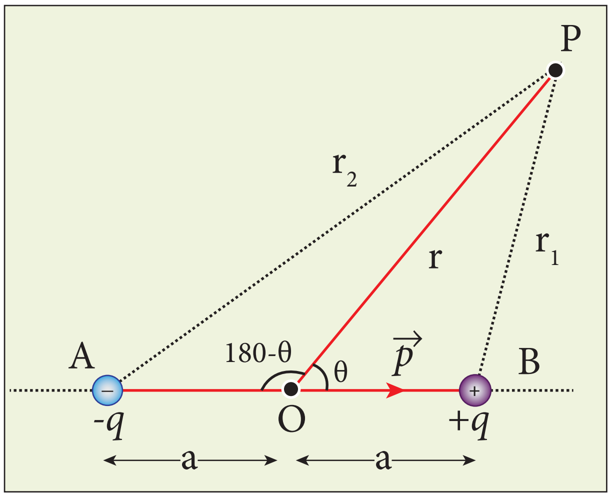


Figure 1.26 Potential due to electric dipole

Let r_1 be the distance of point P from +q and r_2 be the distance of point P from −q.

Potential at P due to charge +q $= \frac{1}{4\pi\varepsilon_\circ}\frac{q}{r_1}$

Potential at P due to charge −q $= -\frac{1}{4\pi\varepsilon_\circ}\frac{q}{r_2}$

Total potential at the point P,

$$V = \frac{1}{4\pi\varepsilon_\circ}q\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad (1.35)$$

Suppose if the point P is far away from the dipole, such that r>>a, then equation (1.35) can be expressed in terms of r.
By the cosine law for triangle BOP,

$$r_1^2 = r^2 + a^2 - 2ra\cos\theta$$

$$r_1^2 = r^2\left(1 + \frac{a^2}{r^2} - \frac{2a}{r}\cos\theta\right)$$

Since the point P is very far from dipole, then r>>a. As a result the term $\frac{a^2}{r^2}$ is very small and can be neglected. Therefore

$$r_1^2 = r^2\left(1 - 2a\frac{\cos\theta}{r}\right)$$

$$\text{(or)} \quad r_1 = r\left(1 - \frac{2a}{r}\cos\theta\right)^{\frac{1}{2}}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta\right)^{-\frac{1}{2}}$$

Since $\frac{a}{r} << 1$, we can use binomial theorem and retain the terms up to first order

$$\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{a}{r}\cos\theta\right) \quad (1.36)$$

Similarly applying the cosine law for triangle AOP,

$$r_2^2 = r^2 + a^2 - 2ra\cos(180 - \theta)$$

since $\cos(180-\theta) = -\cos\theta$ we get

$$r_2^2 = r^2 + a^2 + 2ra\cos\theta$$

Neglecting the term $\frac{a^2}{r^2}$ (because r>>a)

$$r_2^2 = r^2\left(1 + \frac{2a\cos\theta}{r}\right)$$

$$r_2 = r\left(1 + \frac{2a\cos\theta}{r}\right)^{\frac{1}{2}}$$

Using Binomial theorem, we get

$$\frac{1}{r_2} = \frac{1}{r}\left(1 - a\frac{\cos\theta}{r}\right) \quad (1.37)$$

Substituting equation (1.37) and (1.36) in equation (1.35),

$$V = \frac{1}{4\pi\varepsilon_\circ}q\left(\frac{1}{r}\left(1 + a\frac{\cos\theta}{r}\right) - \frac{1}{r}\left(1 - a\frac{\cos\theta}{r}\right)\right)$$

$$V = \frac{q}{4\pi\varepsilon_\circ}\left(\frac{1}{r}\left(1 + a\frac{\cos\theta}{r} - 1 + a\frac{\cos\theta}{r}\right)\right)$$

$$V = \frac{1}{4\pi\varepsilon_\circ}\frac{2aq}{r^2}\cos\theta$$

But the electric dipole moment $p = 2qa$ and we get,

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{p\cos\theta}{r^2}\right)$$

Now we can write p cosθ = $\vec{p}\cdot\hat{r}$, where $\hat{r}$ is the unit vector from the point O to point P. Hence the electric potential at a point P due to an electric dipole is given by

$$V = \frac{1}{4\pi\varepsilon_\circ}\frac{\vec{p}\cdot\hat{r}}{r^2} \qquad (r>>a) \quad (1.38)$$

Equation (1.38) is valid for distances very large compared to the size of the dipole. But for a point dipole, the equation (1.38) is valid for any distance.

Special cases

Case (i) If the point P lies on the axial line of the dipole on the side of +q, then θ = 0. Then the electric potential becomes

$$V = \frac{1}{4\pi\varepsilon_\circ}\frac{p}{r^2} \qquad (1.39)$$

Case (ii) If the point P lies on the axial line of the dipole on the side of –q, then θ = 180°, then

$$V = -\frac{1}{4\pi\varepsilon_\circ}\frac{p}{r^2} \qquad (1.40)$$

Case (iii) If the point P lies on the equatorial line of the dipole, then θ = 90°. Hence

$$V = 0 \qquad (1.41)$$

Important points

(i) The potential due to an electric dipole falls as $\frac{1}{r^2}$ and the potential due to a single point charge falls as $\frac{1}{r}$. Thus the potential due to the dipole falls faster than that due to a monopole (point charge). As the distance increases from electric dipole, the effects of positive and negative charges nullify each other.

(ii) The potential due to a point charge is spherically symmetric since it depends only on the distance r. But the potential due to a dipole is not spherically symmetric because the potential depends on the angle between $\vec{p}$ and position vector $\vec{r}$ of the point.
However the dipole potential is axially symmetric. If the position vector $\vec{r}$ is rotated about $\vec{p}$ by keeping θ fixed, then all points on the cone at the same distance r will have the same potential as shown in Figure 1.27. In this figure, all the points located on the blue curve will have the same potential.

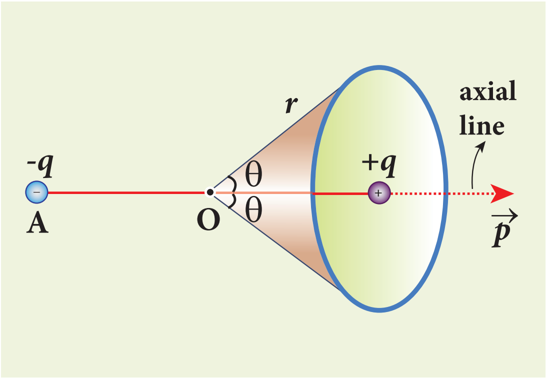


Figure 1.27 Dipole potential is axially symmetric

1.5.4 Equi-potential Surface

Consider a point charge q located at some point in space and an imaginary sphere of radius r is chosen by keeping the charge q at its center (Figure 1.28(a)). The electric potential at all points on the surface of the given sphere is the same. Such a surface is called an equipotential surface.

An equipotential surface is a surface on which all the points are at the same potential.

For a point charge the equipotential surfaces are concentric spherical surfaces as shown in Figure 1.28(b). Each spherical surface is an equipotential surface but the value of the

UNIT-1(XII-Physics_Vol-1).indd 32 04-03-2019 10:25:46

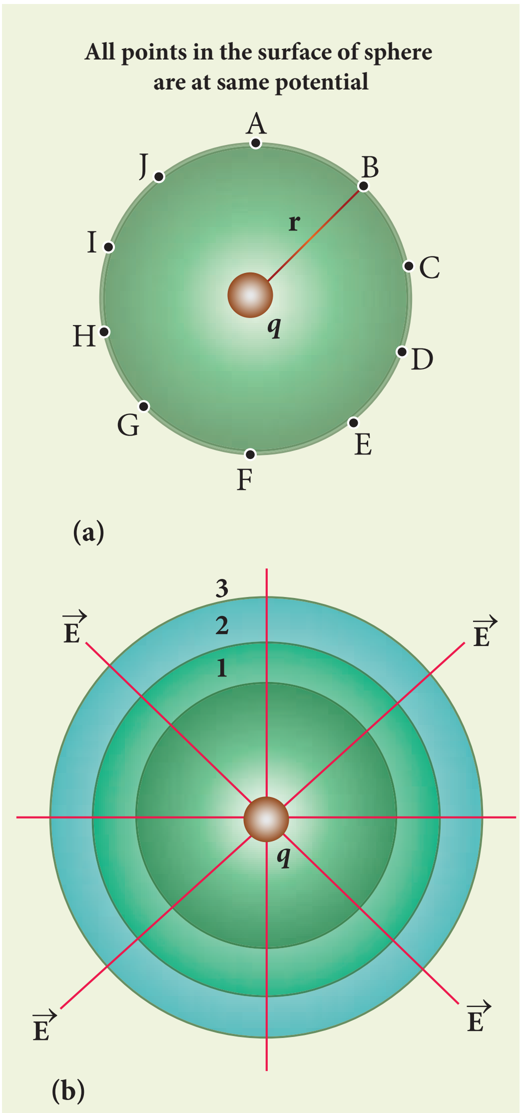


Figure 1.28 Equipotential surface of point Charge

potential is different for different spherical surfaces.

For a uniform electric field, the equipotential surfaces form a set of planes normal to the electric field $\vec{E}$. This is shown in the Figure 1.29.

Properties of equipotential surfaces

(i) The work done to move a charge q between any two points A and B, $W = q\,(V_B - V_A)$. If the points A and B lie on the same equipotential surface, work done is zero because $V_A = V_B$.

(ii) The electric field is normal to an equipotential surface. If it is not normal, then there is a component of the field parallel to the surface. Then work must be done to move a charge between two points on the same surface. This is a contradiction. Therefore the electric field must always be normal to equipotential surface.

1.5.5 Relation between electric field and potential

Consider a positive charge q kept fixed at the origin. To move a unit positive charge by a small distance dx in the electric field E,

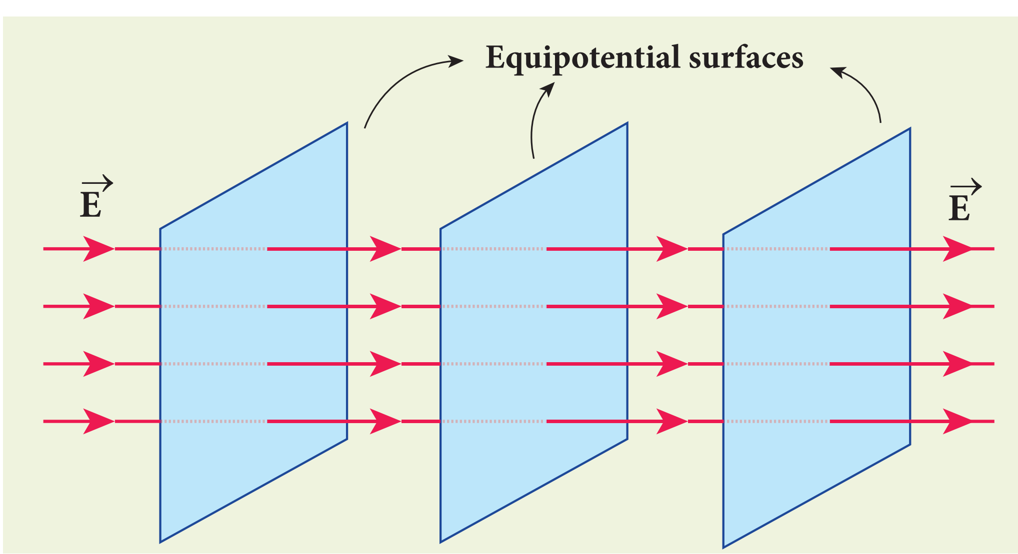


Figure 1.29 Equipotential surface for uniform electric field

the work done is given by $dW = -E\,dx$. The minus sign implies that work is done against the electric field. This work done is equal to electric potential difference. Therefore,

$$dW = dV.$$

$$\text{(or)} \quad dV = -E\,dx \qquad (1.42)$$

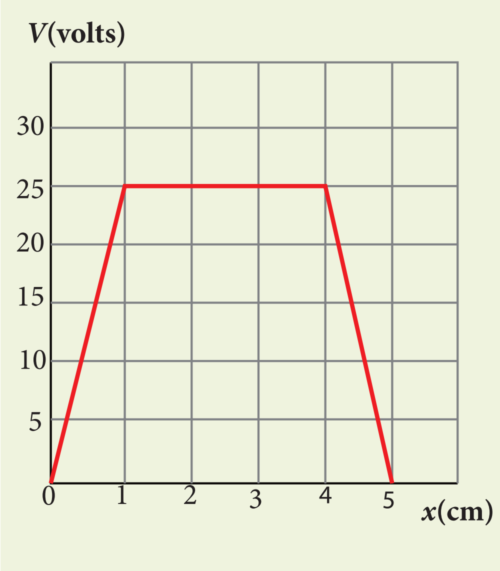
$$\text{Hence } E = -\frac{dV}{dx} \qquad (1.43)$$

The electric field is the negative gradient of the electric potential. In general,

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial k}\hat{k}\right) \qquad (1.44)$$

EXAMPLE 1.14

The following figure represents the electric potential as a function of x – coordinate. Plot the corresponding electric field as a function of x.



Solution

In the given problem, since the potential depends only on x, we can use $\vec{E} = -\frac{dV}{dx}\hat{i}$

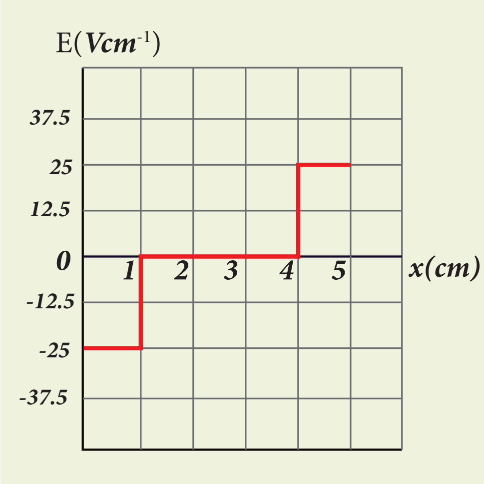
(the other two terms $\frac{\partial V}{\partial y}$ and $\frac{\partial V}{\partial z}$ are zero)

From 0 to 1 cm, the slope is constant and so $\frac{dV}{dx} = 25\,V\,cm^{-1}$. So $\vec{E} = -25\,V\,cm^{-1}\hat{i}$

From 1 to 4 cm, the potential is constant, V = 25 V. It implies that $\frac{dV}{dx} = 0$. So $\vec{E} = 0$

From 4 to 5 cm, the slope $\frac{dV}{dx} = -25\,V\,cm^{-1}$. So $\vec{E} = +25\,V\,cm^{-1}\hat{i}$.

The plot of electric field for the various points along the *x* axis is given below.



1.5.6 Electrostatic potential energy for collection of point charges

The electric potential at a point at a distance *r* from point charge q_1 is given by

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r}$$

This potential *V* is the work done to bring a unit positive charge from infinity to the point. Now if the charge q_2 is brought from infinity to that point at a distance *r* from q_1, the work done is the product of q_2 and the electric potential at that point. Thus we have

$$W = q_2 V$$

This work done is stored as the electrostatic potential energy U of a system of charges q_1 and q_2 separated by a distance r. Thus we have

$$U = q_2 V = \frac{1}{4\pi\varepsilon_\circ}\frac{q_1 q_2}{r} \quad (1.45)$$

The electrostatic potential energy depends only on the distance between the two point charges. In fact, the expression (1.45) is derived by assuming that q_1 is fixed and q_2 is brought from infinity. The equation (1.45) holds true when q_2 is fixed and q_1 is brought from infinity or both q_1 and q_2 are simultaneously brought from infinity to a distance *r* between them.

Three charges are arranged in the following configuration as shown in Figure 1.30.

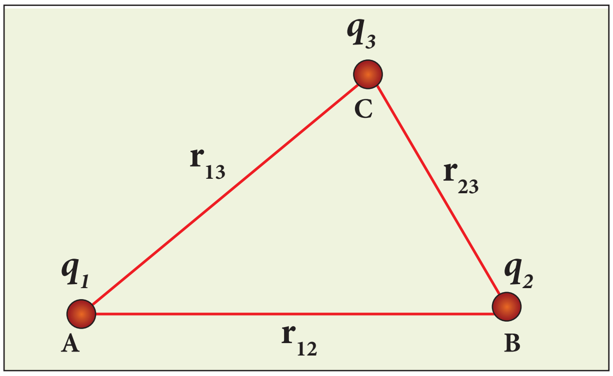


Figure 1.30 Electrostatic potential energy for Collection of point charges

To calculate the total electrostatic potential energy, we use the following procedure. We bring all the charges one by one and arrange them according to the configuration as shown in Figure 1.30.

(i) Bringing a charge q_1 from infinity to the point A requires no work, because there are no other charges already present in the vicinity of charge q_1.

(ii) To bring the second charge q_2 to the point B, work must be done against the electric field created by the charge q_1. So the work done on the charge q_2 is $W = q_2\ V_{1B.}$ Here V_{1B} is the electrostatic potential due to the charge q_1 at point *B*.

$$U = \frac{1}{4\pi\varepsilon_\circ}\frac{q_1 q_2}{r_{12}} \quad (1.46)$$

Note that the expression is same when q_2 is brought first and then q_1 later.

(iii) Similarly to bring the charge q_3 to the point C, work has to be done against the total electric field due to both charges q_1 and q_2. So the work done to bring the charge q_3 is $= q_3\ (V_{1C} + V_{2C})$. Here V_{1C} is the electrostatic potential due to charge q_1 at point C and V_{2C} is the electrostatic potential due to charge q_2 at point C.

The electrostatic potential is

$$U = \frac{1}{4\pi\varepsilon_\circ}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) \quad (1.47)$$

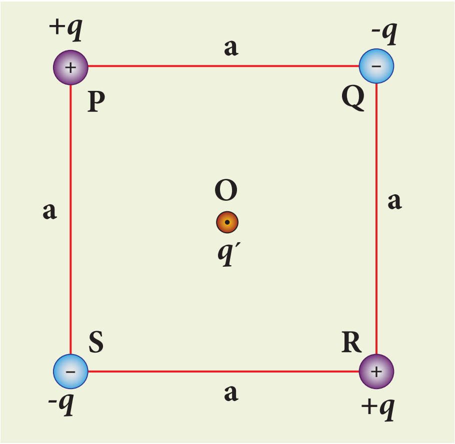
(iv) Adding equations (1.46) and (1.47), the total electrostatic potential energy for the system of three charges q_1, q_2 and q_3 is

$$U = \frac{1}{4\pi\varepsilon_\circ}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) \quad (1.48)$$

Note that this stored potential energy U is equal to the total external work done to assemble the three charges at the given locations. The expression (1.48) is same if the charges are brought to their positions in any other order. Since the Coulomb force is a conservative force, the electrostatic potential energy is independent of the manner in which the configuration of charges is arrived at.

EXAMPLE 1.15

Four charges are arranged at the corners of the square *PQRS* of side *a* as shown in the figure.(a) Find the work required to assemble these charges in the given configuration. (b) Suppose a charge q′ is brought to the center of the square, by keeping the four charges fixed at the corners, how much extra work is required for this?



Solution

(a) The work done to arrange the charges in the corners of the square is independent of the way they are arranged. We can follow any order.

(i) First, the charge +q is brought to the corner P. This requires no work since no charge is already present, $W_P = 0$

(ii) Work required to bring the charge –q to the corner Q = (-q) × potential at a point Q due to +q located at a point P.

$$W_Q = -q \times \frac{1}{4\pi\varepsilon_\circ}\frac{q}{a} = -\frac{1}{4\pi\varepsilon_\circ}\frac{q^2}{a}$$

(iii) Work required to bring the charge +q to the corner R= q × potential at the point R due to charges at the point P and Q.

$$W_R = q \times \frac{1}{4\pi\varepsilon_\circ}\left(-\frac{q}{a} + \frac{q}{\sqrt{2}a}\right)$$

$$= \frac{1}{4\pi\varepsilon_\circ}\frac{q^2}{a}\left(-1 + \frac{1}{\sqrt{2}}\right)$$

(iv) Work required to bring the fourth charge –q at the position S = q × potential at the point S due the all the three charges at the point P, Q and R

$$W_S = -q \times \frac{1}{4\pi\varepsilon_\circ}\left(\frac{q}{a} + \frac{q}{a} - \frac{q}{\sqrt{2}a}\right)$$

$$W_S = -\frac{1}{4\pi\varepsilon_\circ}\frac{q}{a}\left(2 - \frac{1}{\sqrt{2}}\right)$$

(b) Work required to bring the charge q′ to the center of the square = q′ × potential at the center point O due to all the four charges in the four corners

The potential created by the two +*q* charges are canceled by the potential created by the -*q* charges which are located in the opposite corners. Therefore the net electric potential at the center O due to all the charges in the corners is zero.

Hence no work is required to bring any charge to the point O. Physically this implies that if any charge *q*′ when brought close to O, then it moves to the point O without any external force.

1.5.7 Electrostatic potential energy of a dipole in a uniform electric field

Consider a dipole placed in the uniform electric field $\vec{E}$ as shown in the Figure 1.31. A dipole experiences a torque when kept in an uniform electric field $\vec{E}$. This torque rotates the dipole to align it with the

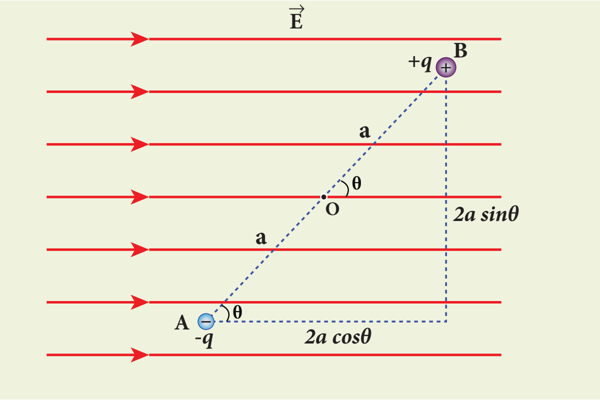


Figure 1.31 The dipole in a uniform electric field

direction of the electric field. To rotate the dipole (at constant angular velocity) from its initial angle θ' to another angle θ against the torque exerted by the electric field, an equal and opposite external torque must be applied on the dipole.

The work done by the external torque to rotate the dipole from angle θ' to θ at constant angular velocity is

$$W = \int_{\theta'}^{\theta} \tau_{ext} d\theta \qquad (1.49)$$

Since $\vec{\tau}_{ext}$ is equal and opposite to $\vec{\tau}_E = \vec{p} \times \vec{E}$, we have

$$|\vec{\tau}_{ext}| = |\vec{\tau}_E| = |\vec{p} \times \vec{E}| \qquad (1.50)$$

Substituting equation (1.50) in equation (1.49), we get

$$W = \int_{\theta'}^{\theta} pE \sin\theta \, d\theta$$

$$W = pE(\cos\theta' - \cos\theta)$$

This work done is equal to the potential energy difference between the angular positions θ and θ'.

$$U(\theta) - U(\theta') = \Delta U = -pE\cos\theta + pE\cos\theta'$$

If the initial angle is $\theta' = 90^\circ$ and is taken as reference point, then $U(\theta') = pE\cos 90^\circ = 0$.

The potential energy stored in the system of dipole kept in the uniform electric field is given by

$$U = -pE\cos\theta = -\vec{p} \cdot \vec{E} \qquad (1.51)$$

In addition to p and E, the potential energy also depends on the orientation θ of the electric dipole with respect to the external electric field.

The potential energy is maximum when the dipole is aligned anti-parallel ($\theta = \pi$) to the external electric field and minimum when the dipole is aligned parallel ($\theta = 0$) to the external electric field.

EXAMPLE 1.16

A water molecule has an electric dipole moment of 6.3×10^{-30} Cm. A sample contains 10^{22} water molecules, with all the dipole moments aligned parallel to the external electric field of magnitude 3×10^5 N C^{-1}. How much work is required to rotate all the water molecules from $\theta = 0^\circ$ to 90°?

Solution

When the water molecules are aligned in the direction of the electric field, it has minimum potential energy. The work done to rotate the dipole from $\theta = 0^\circ$ to 90° is equal to the potential energy difference between these two configurations.

$$W = \Delta U = U(90^\circ) - U(0^\circ)$$

From the equation (1.51), we write $U = -pE\cos\theta$, Next we calculate the work done to rotate one water molecule from $\theta = 0^\circ$ to 90°.

For one water molecule

$$W = -pE\cos 90^\circ + pE\cos 0^\circ = pE$$

$$W = 6.3\times10^{-30}\times3\times10^{5} = 18.9\times10^{-25}\,J$$

For 10^{22} water molecules, the total work done is

$$W_{tot} = 18.9\times10^{-25}\times10^{22} = 18.9\times10^{-3}\,J$$

1.6 GAUSS LAW AND ITS APPLICATIONS

1.6.1 Electric Flux

The number of electric field lines crossing a given area kept normal to the electric field lines is called electric flux. It is usually denoted by the Greek letter Φ_E and its unit is N m^2 C^{-1}. Electric flux is a scalar quantity and it can be positive or negative. For a simpler understanding of electric flux, the following Figure 1.32 is useful.

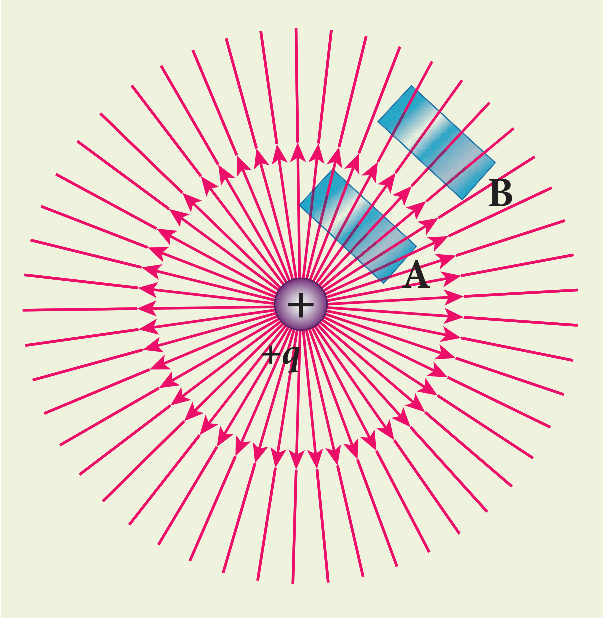


Figure 1.32 Electric flux

The electric field of a point charge is drawn in this figure. Consider two small rectangular area elements placed normal to the field at regions A and B. Even though these elements have the same area, the number of electric field lines crossing the element in region A is more than that crossing the element in region B. Therfore the electric flux in region A is more than that in region B. The electric field strength for a point charge decreases as the distance increases, then for a point charge electric flux also decreases as the distance increases. The above discussion gives a qualitative idea of electric flux. However a precise definition of electric flux is needed.

Electric flux for uniform Electric field

Consider a uniform electric field in a region of space. Let us choose an area A normal to the electric field lines as shown in Figure 1.33 (a). The electric flux for this case is

$$\Phi_E = EA \qquad (1.52)$$

Suppose the same area A is kept parallel to the uniform electric field, then no electric field lines pierce through the area A , as shown in Figure 1.33(b). The electric flux for this case is zero.

$$\Phi_E = 0 \qquad (1.53)$$

If the area is inclined at an angle θ with the field, then the component of the electric field perpendicular to the area alone contributes to the electric flux. The electric field component parallel to the surface area will not contribute to the electric flux. This is shown in Figure 1.33 (c). For this case, the electric flux

$$\Phi_E = (E\cos\theta)\,A \qquad (1.54)$$

Further, θ is also the angle between the electric field and the direction normal to the area. Hence in general, for uniform electric field, the electric flux is defined as

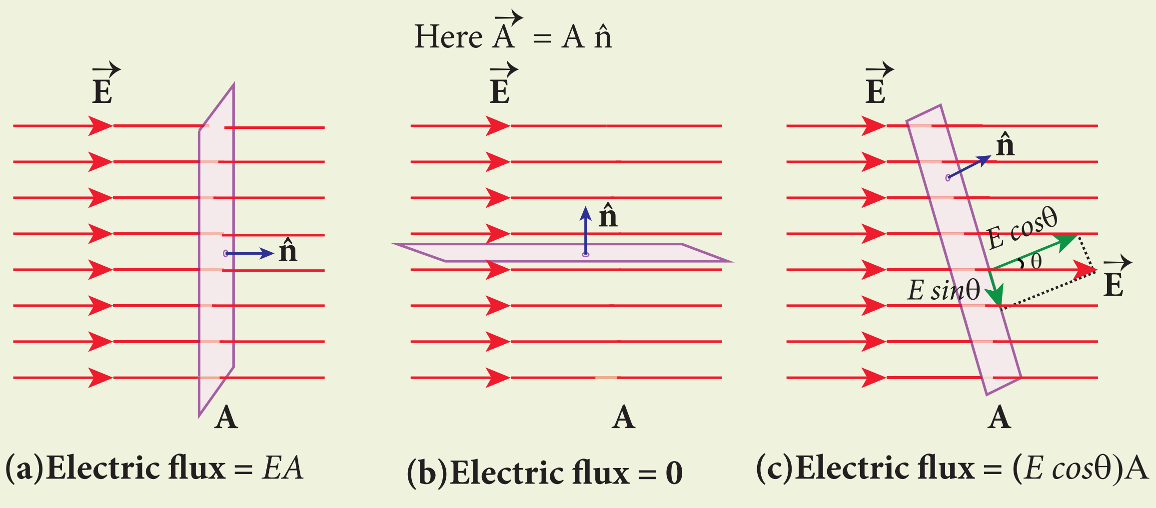


Figure 1.33 The electric flux for Uniform electric field

$$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta \qquad (1.55)$$

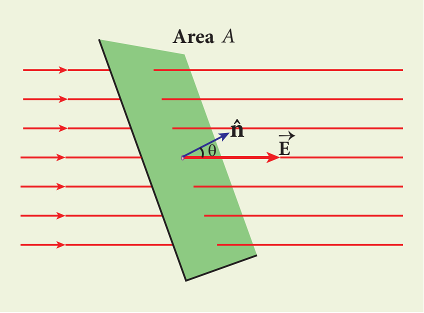
Here, note that $\vec{A}$ is the area vector $\vec{A} = A\hat{n}$. Its magnitude is simply the area A and the direction is along the unit vector $\hat{n}$ perpendicular to the area as shown in Figure 1.33. Using this definition for flux, $\Phi_E = \vec{E}\cdot\vec{A}$, equations (1.53) and (1.54) can be obtained as special cases.

In Figure 1.33 (a), θ = 0° so $\Phi_E = \vec{E}\cdot\vec{A} = EA$

In Figure 1.33 (b), θ = 90° so $\Phi_E = \vec{E}\cdot\vec{A} = 0$

EXAMPLE 1.17

Calculate the electric flux through the rectangle of sides 5 cm and 10 cm kept in the region of a uniform electric field 100 NC^{-1}. The angle θ is 60°. Suppose θ becomes zero, what is the electric flux?



Solution

The electric flux

$$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta = 100\times5\times10\times10^{-4}\times\cos60^\circ$$

$$\Rightarrow \qquad \Phi_E = 0.25\,N.m^2C^{-1}$$

For θ = 0°,

$$\Phi_E = \vec{E}\cdot\vec{A} = EA = 100\times5\times10\times10^{-4} = 0.5\,N.m^2C^{-1}$$

Electric flux in a non uniform electric field and an arbitrarily shaped area

Suppose the electric field is not uniform and the area A is not flat (Figure 1.34), then the entire area is divided into n small area segments $\Delta\vec{A}_1, \Delta\vec{A}_2, \Delta\vec{A}_3 .... \Delta\vec{A}_n$ such that each area element is almost flat and the electric field over each area element is considered to be uniform.

The electric flux for the entire area A is approximately written as

$$\Phi_E = \vec{E}_1\cdot\Delta\vec{A}_1 + \vec{E}_2\cdot\Delta\vec{A}_2 + \vec{E}_3\cdot\Delta\vec{A}_3 ....\vec{E}_n\cdot\Delta\vec{A}_n = \sum_{i=1}^{n}\vec{E}_i\cdot\Delta\vec{A}_i \qquad (1.56)$$

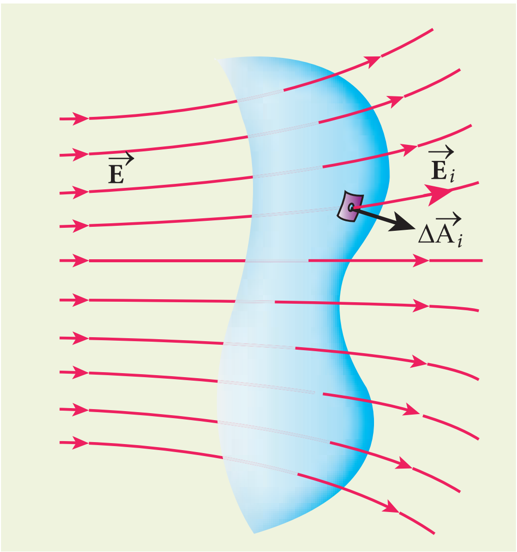


Figure 1.34 Electric flux for non-uniform electric Field

By taking the limit $\Delta\vec{A}_i \to 0$ (for all i) the summation in equation (1.56) becomes integration. The total electric flux for the entire area is given by

$$\Phi_E = \int \vec{E}\cdot d\vec{A} \qquad (1.57)$$

From Equation (1.57), it is clear that the electric flux for a given surface depends on both the electric field pattern on the surface area and orientation of the surface with respect to the electric field.

1.6.2 Electric flux for closed surfaces

In the previous section, the electric flux for any arbitrary curved surface is discussed. Suppose a closed surface is present in the region of the non-uniform electric field as shown in Figure 1.35 (a).

The total electric flux over this closed surface is written as

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} \qquad (1.58)$$

Note the difference between equations (1.57) and (1.58). The integration in equation

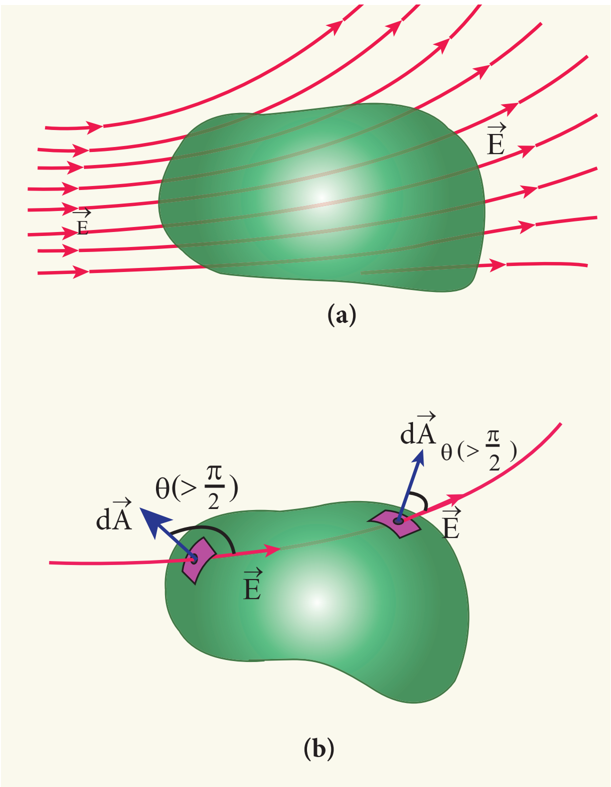


Figure 1.35 Electric flux over a closed surface

(1.58) is a closed surface integration and for each areal element, the outward normal is the direction of $d\vec{A}$ as shown in the Figure 1.35(b).

The total electric flux over a closed surface can be negative, positive or zero. In the Figure 1.35(b), it is shown that in one area element, the angle between $d\vec{A}$ and $\vec{E}$ is less than 90°, then the electric flux is positive and in another areal element, the angle between $d\vec{A}$ and $\vec{E}$ is greater than 90°, then the electric flux is negative.

In general, the electric flux is negative if the electric field lines enter the closed surface and positive if the electric field lines leave the closed surface.

1.6.3 Gauss law

A positive point charge Q is surrounded by an imaginary sphere of radius r as shown in Figure 1.36. We can calculate the total electric flux through the closed surface of the sphere using the equation (1.58).

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E\, dA \cos\theta$$

The electric field of the point charge is directed radially outward at all points on the surface of the sphere. Therefore, the direction of the area element $d\vec{A}$ is along the electric field $\vec{E}$ and $\theta = 0°$.

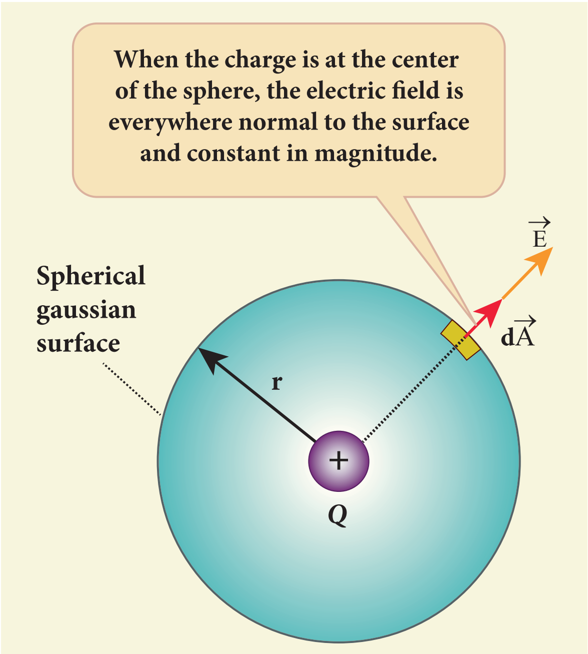


Figure 1.36 Total electric flux of point charge

$$\Phi_E = \oint E\, dA \quad \text{since } \cos 0^0 = 1 \qquad (1.59)$$

E is uniform on the surface of the sphere,

$$\Phi_E = E \oint dA \qquad (1.60)$$

Substituting for $\oint dA = 4\pi r^2$ and $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ in equation 1.60, we get

$$\Phi_E = \frac{1}{4\pi\varepsilon_\circ}\frac{Q}{r^2} \times 4\pi r^2 = 4\pi \frac{1}{4\pi\varepsilon_\circ} Q$$

$$\Phi_E = \frac{Q}{\varepsilon_\circ} \qquad (1.61)$$

The equation (1.61) is called as Gauss's law. The remarkable point about this result is that the equation (1.61) is equally true for any arbitrary shaped surface which encloses the charge Q and as shown in the Figure 1.37. It is seen that the total electric flux is the same for closed surfaces A_1, A_2 and A_3 as shown in the Figure 1.37.

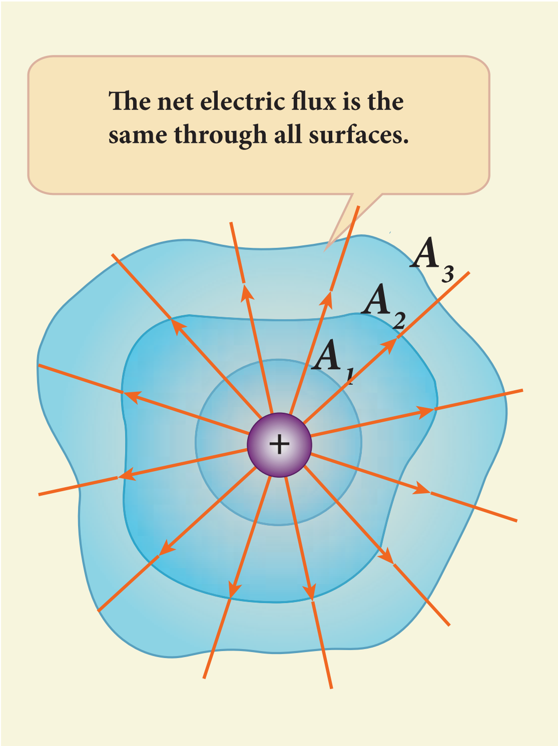


Figure 1.37 Gauss law for arbitrarily shaped surface

Gauss's law states that **if a charge Q is enclosed by an arbitrary closed surface, then the total electric flux Φ_E through the closed surface is**

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_\circ} \qquad (1.62)$$

Here Q_{encl} denotes the charges inside the closed surface.

Discussion of Gauss law

(i) The total electric flux through the closed surface depends only on the charges enclosed by the surface and the charges present outside the surface will not contribute to the flux and the shape of the closed surface which can be chosen arbitrarily.

(ii) The total electric flux is independent of the location of the charges inside the closed surface.

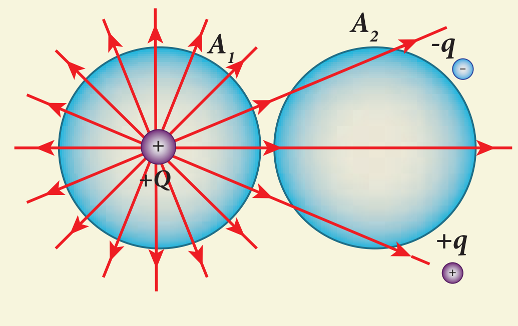
(iii) To arrive at equation (1.62), we have chosen a spherical surface. This imaginary surface is called a Gaussian surface. The shape of the Gaussian surface to be chosen depends on the type of charge configuration and the kind of symmetry existing in that charge configuration. The electric field is spherically symmetric for a point charge, therefore spherical Gaussian surface is chosen. Cylindrical and planar Gaussian surfaces can be chosen for other kinds of charge configurations.

(iv) In the LHS of equation (1.62), the electric field $\vec{E}$ is due to charges present inside and outside the Gaussian surface but the charge Q_{encl} denotes the charges which lie only inside the Gaussian surface.

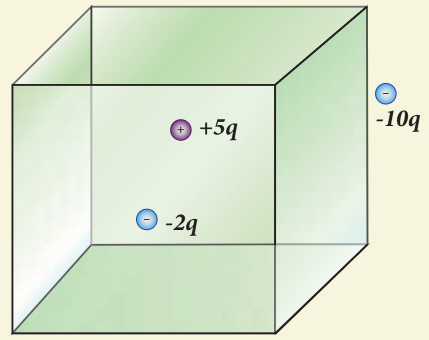
(v) The Gaussian surface cannot pass through any discrete charge but it can pass through continuous charge distributions. It is because, very close to the discrete charges, the electric field is not well defined.

(vi) Gauss law is another form of Coulomb's law and it is also applicable to the charges in motion. Because of this reason, Gauss law is treated as much more general law than Coulomb's law.

EXAMPLE 1.18



(a)



(b)

(i) In figure (a), calculate the electric flux through the closed areas A_1 and A_2.

(ii) In figure (b), calculate the electric flux through the cube

Solution

(i) In figure (a), the area A_1 encloses the charge Q. So electric flux through this closed surface A_1 is $\frac{Q}{\varepsilon_\circ}$. But the closed surface A_2 contains no charges inside, so electric flux through A_2 is zero.

(ii) In figure (b), the net charge inside the cube is 3q and the total electric flux in the cube is therefore $\Phi_E = \frac{3q}{\varepsilon_\circ}$.

Note that the charge -10 q lies outside the cube and it will not contribute the total flux through the surface of the cube.

1.6.4 Applications of Gauss law

Electric field due to any arbitrary charge configuration can be calculated using Coulomb's law or Gauss law. If the charge configuration possesses some kind of symmetry, then Gauss law is a very efficient way to calculate the electric field. It is illustrated in the following cases.

(i) Electric field due to an infinitely long charged wire

Consider an infinitely long straight wire having uniform linear charge density λ. Let P be a point located at a perpendicular distance r from the wire (Figure 1.38(a)).

The electric field at the point P can be found using Gauss law. We choose two small charge elements A_1 and A_2 on the wire which are at equal distances from the point P. The resultant electric field due to these two charge elements points radially away from the charged wire and the magnitude of electric field is same at all points on the circle of radius r. This is shown in the Figure 1.38(b). From this property, we can infer that the charged wire possesses a cylindrical symmetry.

Let us choose a cylindrical Gaussian surface of radius r and length L as shown in the Figure 1.39.

The total electric flux in this closed surface is calculated as follows.

$$\Phi_E = \oint \vec{E}\cdot d\vec{A}$$
$$= \int_{\substack{\text{Curved}\\\text{surface}}} \vec{E}\cdot d\vec{A} + \int_{\substack{\text{top}\\\text{surface}}} \vec{E}\cdot d\vec{A} + \int_{\substack{\text{bottom}\\\text{surface}}} \vec{E}\cdot d\vec{A} \quad (1.63)$$

It is seen from Figure (1.39) that for the curved surface, $\vec{E}$ is parallel to $\vec{A}$ and $\vec{E}\cdot d\vec{A} = E\,dA$. For the top and bottom surfaces, $\vec{E}$ is perpendicular to $\vec{A}$ and $\vec{E}\cdot d\vec{A} = 0$

Substituting these values in the equation (1.63) and applying Gauss law to the cylindrical surface, we have

$$\Phi_E = \int_{\substack{\text{Curved}\\\text{surface}}} E\,dA = \frac{Q_{encl}}{\varepsilon_\circ} \quad (1.64)$$

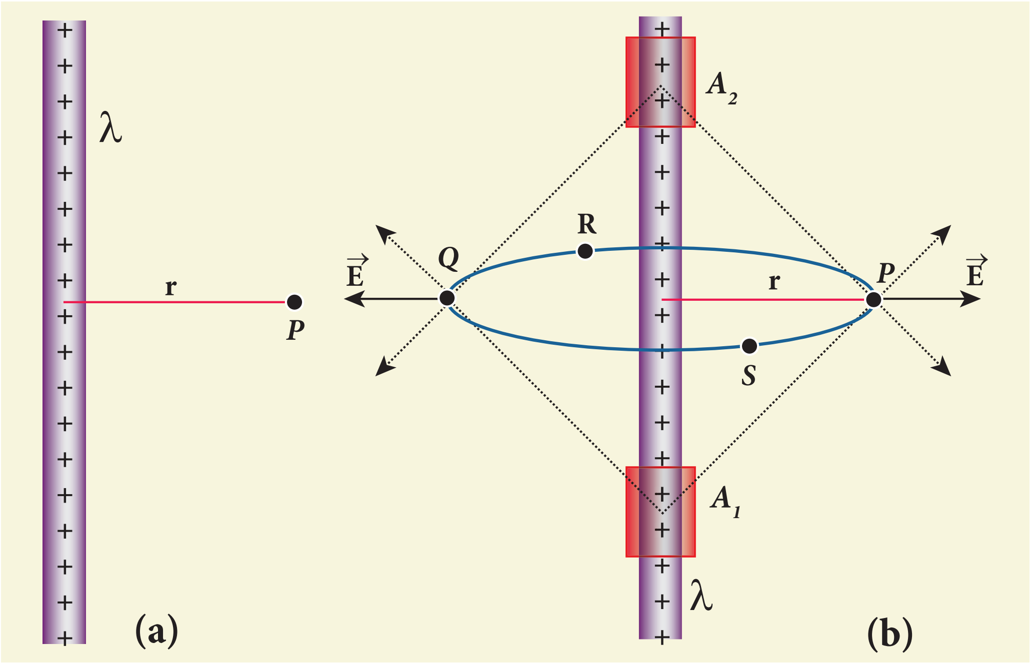


Figure 1.38 Electric field due to infinite long charged wire

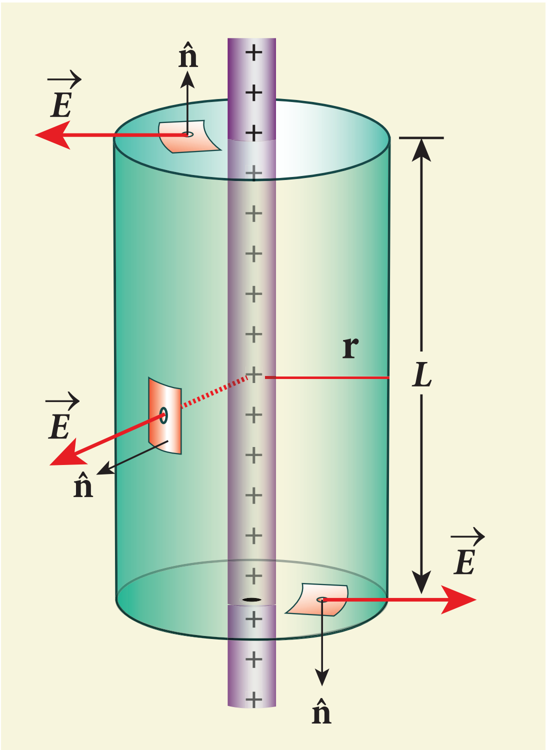


Figure 1.39 Cylindrical Gaussian surface

Since the magnitude of the electric field for the entire curved surface is constant, E is taken out of the integration and Q_{encl} is given by $Q_{encl} = \lambda L$.

$$E \int_{\substack{Curved \\ surface}} dA = \frac{\lambda L}{\varepsilon_\circ} \qquad (1.65)$$

Here $\Phi_E = \int_{\substack{Curved \\ surface}} dA =$ total area of the curved surface = 2πrL. Substituting this in equation (1.65), we get

$$E \cdot 2\pi r L = \frac{\lambda L}{\varepsilon_\circ}$$

$$E = \frac{1}{2\pi\varepsilon_\circ} \frac{\lambda}{r} \qquad (1.66)$$

In vector form $\vec{E} = \frac{1}{2\pi\varepsilon_\circ} \frac{\lambda}{r} \hat{r}$ (1.67)

The electric field due to the infinite charged wire depends on $\frac{1}{r}$ rather than $\frac{1}{r^2}$ for a point charge.

Equation (1.67) indicates that the electric field is always along the perpendicular direction ($\hat{r}$) to wire. In fact, if $\lambda > 0$ then $\vec{E}$ points perpendicular outward ($\hat{r}$) from the wire and if $\lambda < 0$, then $\vec{E}$ points perpendicular inward $(-\hat{r})$.

The equation (1.67) is true only for an infinitely long charged wire. For a charged wire of finite length, the electric field need not be radial at all points. However, equation (1.67) for such a wire is taken approximately true around the mid-point of the wire and far away from the both ends of the wire

(ii) Electric field due to charged infinite plane sheet

Consider an infinite plane sheet of charges with uniform surface charge density σ. Let P be a point at a distance of r from the sheet as shown in the Figure 1.40.

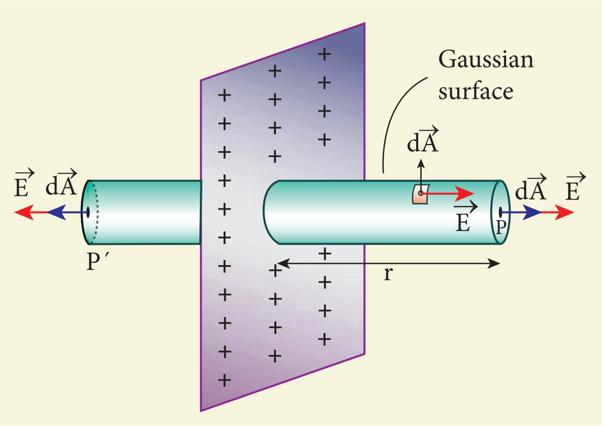


Figure 1.40 Electric field due to charged infinite planar sheet

Since the plane is infinitely large, the electric field should be same at all points equidistant from the plane and radially directed at all points. A cylindrical shaped Gaussian surface of length 2r and area A of the flat surfaces is chosen such that the infinite plane sheet passes perpendicularly through the middle part of the Gaussian surface.

Applying Gauss law for this cylindrical surface,

$$\Phi_E = \oint \vec{E}\cdot d\vec{A}$$

$$= \int_{\substack{Curved\\surface}} \vec{E}\cdot d\vec{A} + \int_{P} \vec{E}\cdot d\vec{A} + \int_{P'} \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \quad (1.68)$$

The electric field is perpendicular to the area element at all points on the curved surface and is parallel to the surface areas at P and P' (Figure 1.40). Then,

$$\Phi_E = \int_P E\,dA + \int_{P'} E\,dA = \frac{Q_{encl}}{\varepsilon_0} \quad (1.69)$$

Since the magnitude of the electric field at these two equal surfaces is uniform, E is taken out of the integration and Q_{encl} is given by $Q_{encl} = \sigma A$, we get

$$2E\int_P dA = \frac{\sigma A}{\varepsilon_0}$$

The total area of surface either at P or P′

$$\int_P dA = A$$

$$\text{Hence } 2EA = \frac{\sigma A}{\varepsilon_0} \quad \text{or} \quad E = \frac{\sigma}{2\varepsilon_0} \quad (1.70)$$

$$\text{In vector form, } \vec{E} = \frac{\sigma}{2\varepsilon_\circ}\hat{n} \quad (1.71)$$

Here $\hat{n}$ is the outward unit vector normal to the plane. Note that the electric field due to an infinite plane sheet of charge depends on the surface charge density and is independent of the distance r.

The electric field will be the same at any point farther away from the charged plane. Equation (1.71) implies that if $\sigma > 0$ the electric field at any point P is outward perpendicular $\hat{n}$ to the plane and if $\sigma < 0$ the electric field points inward perpendicularly $(-\hat{n})$ to the plane.

For a finite charged plane sheet, equation (1.71) is approximately true only in the middle region of the plane and at points far away from both ends.

(iii) Electric field due to two parallel charged infinite sheets

Consider two infinitely large charged plane sheets with equal and opposite charge densities +σ and -σ which are placed parallel to each other as shown in the Figure 1.41.

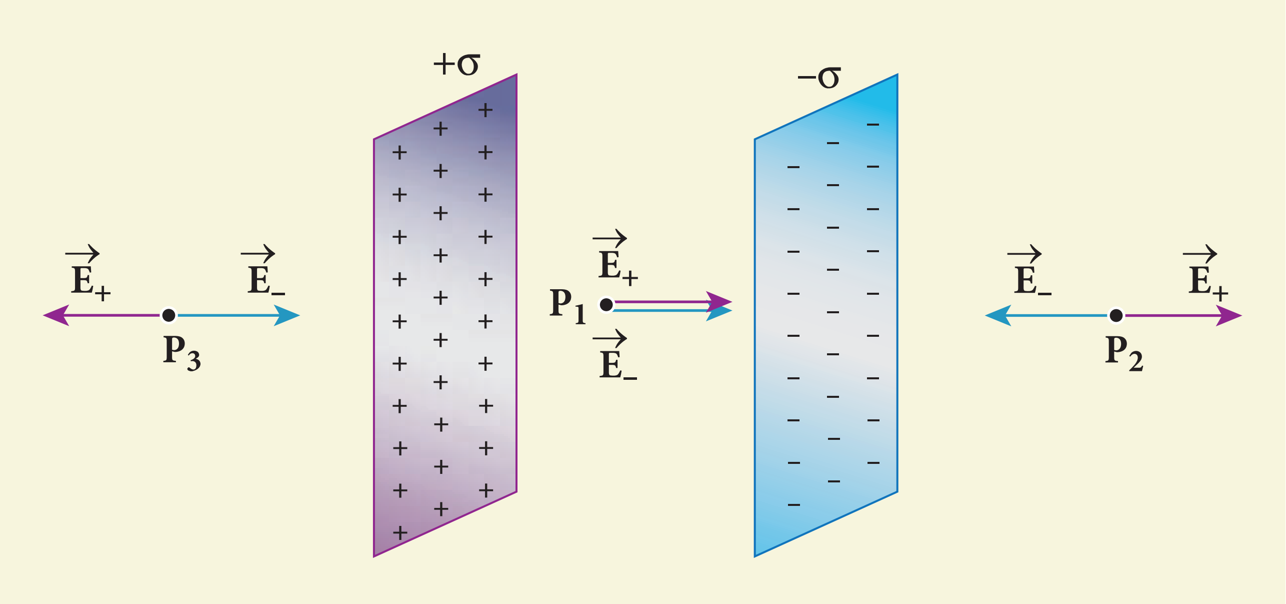


Figure 1.41 Electric field due to two parallel charged sheets

The electric field between the plates and outside the plates is found using Gauss law. The magnitude of the electric field due to an infinite charged plane sheet is $\frac{\sigma}{2\varepsilon_\circ}$ and it points perpendicularly outward if $\sigma > 0$ and points inward if $\sigma < 0$.

At the points P_2 and P_3, the electric field due to both plates are equal in magnitude and opposite in direction (Figure 1.41). As a result, electric field at a point outside the plates is zero. But inside the plate, electric fields are in same direction i.e., towards the right, the total electric field at a point P_1

$$E_{inside} = \frac{\sigma}{2\varepsilon_\circ} + \frac{\sigma}{2\varepsilon_\circ} = \frac{\sigma}{\varepsilon_\circ} \qquad (1.72)$$

The direction of the electric field inside the plates is directed from positively charged plate to negatively charged plate and is uniform everywhere inside the plate.

(iv) Electric field due to a uniformly charged spherical shell

Consider a uniformly charged spherical shell of radius R and total charge Q as shown in Figure 1.42. The electric field at points outside and inside the sphere is found using Gauss law.

Case (a) At a point outside the shell (r > R)

Let us choose a point P outside the shell at a distance r from the center as shown in Figure 1.42 (a). The charge is uniformly distributed on the surface of the sphere (spherical symmetry). Hence the electric field must point radially outward if Q > 0 and point radially inward if Q < 0. So we choose a spherical Gaussian surface of radius r is chosen and the total charge enclosed by this Gaussian surface is Q. Applying Gauss law

$$\oint_{\substack{Gaussian\\surface}} \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_\circ} \qquad (1.73)$$

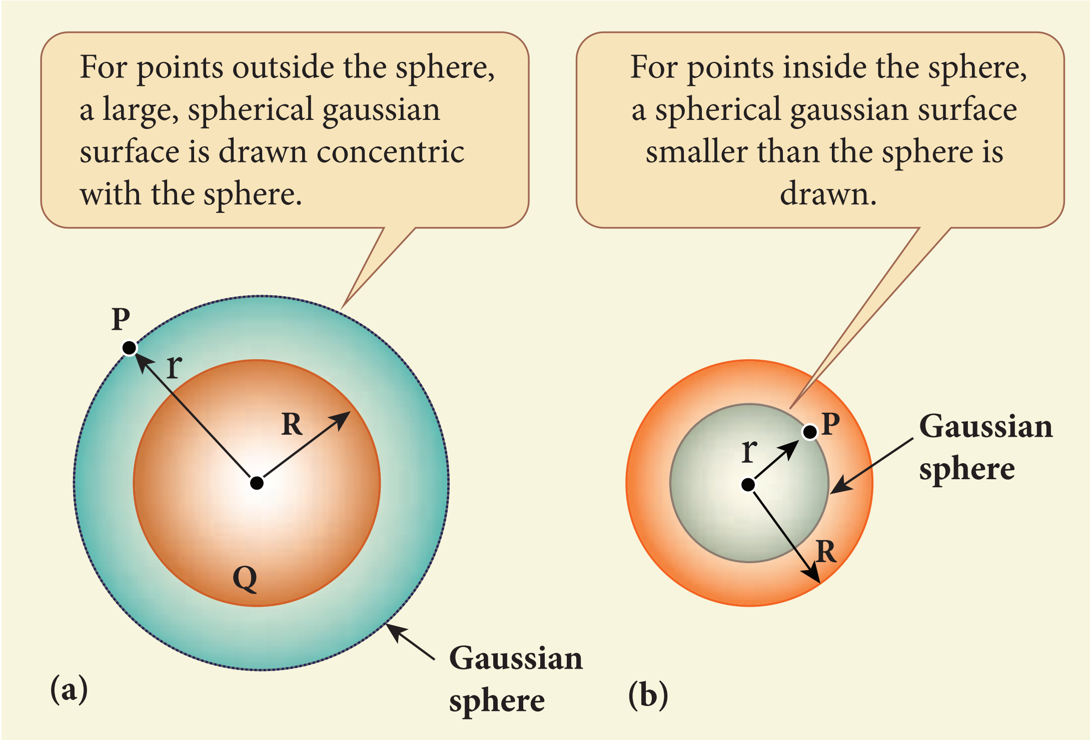


Figure 1.42 The electric field due to a charged spherical shell

The electric field $\vec{E}$ and $d\vec{A}$ point in the same direction (outward normal) at all the points on the Gaussian surface. The magnitude of $\vec{E}$ is also the same at all points due to the spherical symmetry of the charge distribution.

$$\text{Hence } E \oint_{\substack{\text{Gaussian}\\ \text{surface}}} dA = \frac{Q}{\varepsilon_\circ} \quad (1.74)$$

But $\oint_{\substack{\text{Gaussian}\\ \text{surface}}} dA =$ total area of Gaussian surface $= 4\pi r^2$. Substituting this value in equation (1.74)

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_\circ}$$

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_\circ} \quad \text{(or)} \quad E = \frac{1}{4\pi\varepsilon_\circ}\frac{Q}{r^2}$$

In vector form $$\vec{E} = \frac{1}{4\pi\varepsilon_\circ}\frac{Q}{r^2}\hat{r} \quad (1.75)$$

The electric field is radially outward if Q > 0 and radially inward if Q < 0. From equation (1.75), we infer that the electric field at a point outside the shell will be same as if the entire charge Q is concentrated at the center of the spherical shell. (A similar result is observed in gravitation, for gravitational force due to a spherical shell with mass M)

Case (b): At a point on the surface of the spherical shell (r = R)

The electrical field at points on the spherical shell (r = R) is given by

$$\vec{E} = \frac{Q}{4\pi\varepsilon_\circ R^2}\hat{r} \quad (1.76)$$

Case (c) At a point inside the spherical shell (r < R)

Consider a point P inside the shell at a distance r from the center. A Gaussian sphere of radius r is constructed as shown in the Figure 1.42 (b). Applying Gauss law

$$\oint_{\substack{\text{Gaussian}\\ \text{surface}}} \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_\circ}$$

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_\circ} \quad (1.77)$$

Since Gaussian surface encloses no charge, So Q = 0. The equation (1.77) becomes

$$E = 0 \qquad (r < R) \quad (1.78)$$

The electric field due to the uniformly charged spherical shell is zero at all points inside the shell.

A graph is plotted between the electric field and radial distance. This is shown in Figure 1.43.

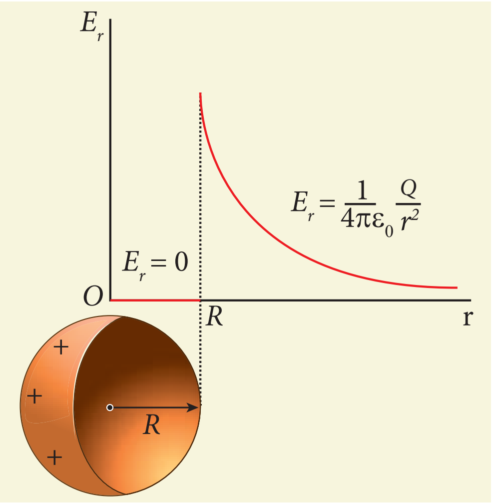


Figure 1.43 Electric field versus distance for a spherical shell of radius R

Note Gauss law is a powerful technique whenever a given charge configuration possesses spherical, cylindrical or planer symmetry, then the electric field due to such a charge configuration can be easily found. If there is no such symmetry, the direct method (Coulomb's law and calculus) can be used. For example, it is difficult to use Gauss law to find the electric field for a dipole since it has no spherical, cylindrical or planar symmetry.

1.7 ELECTROSTATICS OF CONDUCTORS AND DIELECTRICS

1.7.1 Conductors at electrostatic equilibrium

An electrical conductor has a large number of mobile charges which are free to move in the material. In a metallic conductor, these mobile charges are free electrons which are not bound to any atom and therefore are free to move on the surface of the conductor. When there is no external electric field, the free electrons are in continuous random motion in all directions. As a result, there is no net motion of electrons along any particular direction which implies that the conductor is in electrostatic equilibrium. Thus at electrostatic equilibrium, there is no net current in the conductor. A conductor at electrostatic equilibrium has the following properties.

(i) The electric field is zero everywhere inside the conductor. This is true regardless of whether the conductor is solid or hollow.

This is an experimental fact. Suppose the electric field is not zero inside the metal, then there will be a force on the mobile charge carriers due to this electric field. As a result, there will be a net motion of the mobile charges, which contradicts the conductors being in electrostatic equilibrium. Thus the electric field is zero everywhere inside the conductor. We can also understand this fact by applying an external uniform electric field on the conductor. This is shown in Figure 1.44.

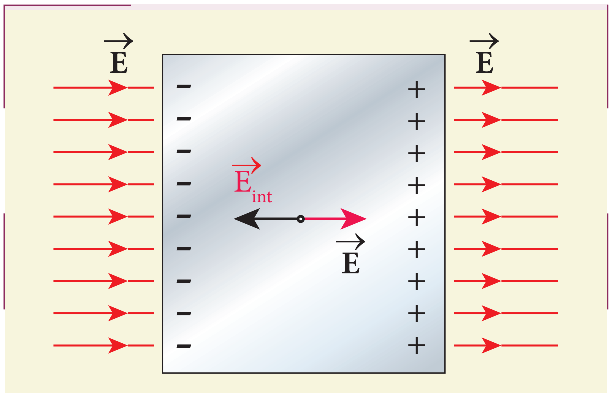


Figure 1.44 Electric field of conductors

Before applying the external electric field, the free electrons in the conductor are uniformly distributed in the conductor. When an electric field is applied, the free electrons accelerate to the left causing the left plate to be negatively charged and the right plate to be positively charged as shown in Figure 1.44.

Due to this realignment of free electrons, there will be an internal electric field created inside the conductor which increases until it nullifies the external electric field. Once the external electric field is nullified the conductor is said to be in electrostatic equilibrium. The time taken by a conductor to reach electrostatic equilibrium is in the order of 10^{-16}s, which can be taken as almost instantaneous.

(ii) There is no net charge inside the conductors. The charges must reside only on the surface of the conductors.

We can prove this property using Gauss law. Consider an arbitrarily shaped conductor as shown in Figure 1.45.

A Gaussian surface is drawn inside the conductor such that it is very close to the surface of the conductor. Since the electric field is zero everywhere inside

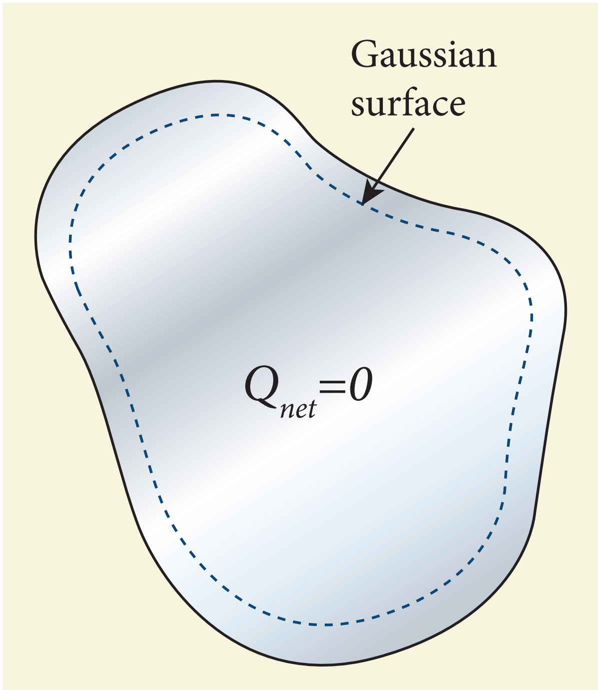


Figure 1.45 No net charge inside the conductor

the conductor, the net electric flux is also zero over this Gaussian surface. From Gauss's law, this implies that there is no net charge inside the conductor. Even if some charge is introduced inside the conductor, it immediately reaches the surface of the conductor.

(iii) The electric field outside the conductor is perpendicular to the surface of the conductor and has a magnitude of $\frac{\sigma}{\varepsilon_\circ}$ where σ is the surface charge density at that point.

If the electric field has components parallel to the surface of the conductor, then free electrons on the surface of the conductor would experience acceleration (Figure 1.46(a)). This means that the conductor is not in equilibrium. Therefore at electrostatic equilibrium, the electric field must be perpendicular to the surface of the conductor. This is shown in Figure 1.46 (b).

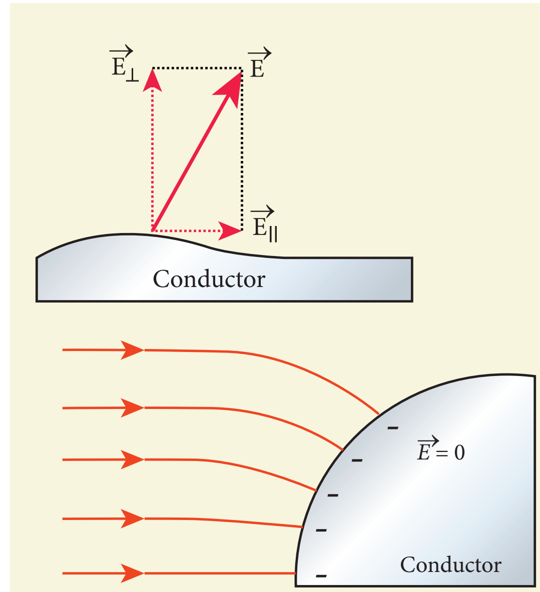


Figure 1.46 (a) Electric field is along the surface (b)Electric field is perpendicular to the surface of the conductor

We now prove that the electric field has magnitude $\frac{\sigma}{\varepsilon_\circ}$ just outside the conductor's surface. Consider a small cylindrical Gaussian surface, as shown in the Figure 1.47. One half of this cylinder is embedded inside the conductor.

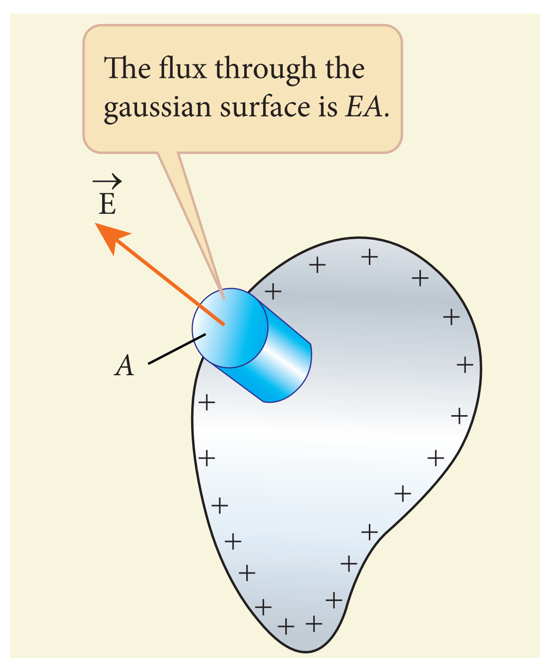


Figure 1.47 The electric field on the surface of the conductor

Since electric field is normal to the surface of the conductor, the curved part of the cylinder has zero electric flux. Also inside the conductor, the electric field is zero. Hence the bottom flat part of the Gaussian surface has no electric flux.

Therefore the top flat surface alone contributes to the electric flux. The electric field is parallel to the area vector and the total charge inside the surface is σA. By applying Gaus's law,

$$EA = \frac{\sigma A}{\varepsilon_\circ}$$

In vector form, $$\vec{E} = \frac{\sigma}{\varepsilon_\circ}\hat{n} \qquad (1.79)$$

Here $\hat{n}$ represents the unit vector outward normal to the surface of the conductor. Suppose $\sigma < 0$, then electric field points inward perpendicular to the surface.

(iv) The electrostatic potential has the same value on the surface and inside of the conductor.

We know that the conductor has no parallel electric component on the surface which means that charges can be moved on the surface without doing any work. This is possible only if the electrostatic potential is constant at all points on the surface and there is no potential difference between any two points on the surface.

Since the electric field is zero inside the conductor, the potential is the same as the surface of the conductor. Thus at electrostatic equilibrium, the conductor is always at equipotential.

1.7.2 Electrostatic shielding

Using Gauss law, we proved that the electric field inside the charged spherical shell is zero, Further, we showed that the electric field inside both hollow and solid conductors is zero. It is a very interesting property which has an important consequence.

Consider a cavity inside the conductor as shown in Figure 1.48 (a). Whatever the charges at the surfaces and whatever the electrical disturbances outside, the electric field inside the cavity is zero. A sensitive electrical instrument which is to be protected from external electrical disturbance is kept inside this cavity. This is called electrostatic shielding.

Faraday cage is an instrument used to demonstrate this effect. It is made up of metal bars configured as shown in Figure1.48 (b). If an artificial lightning jolt is created outside, the person inside is not affected.

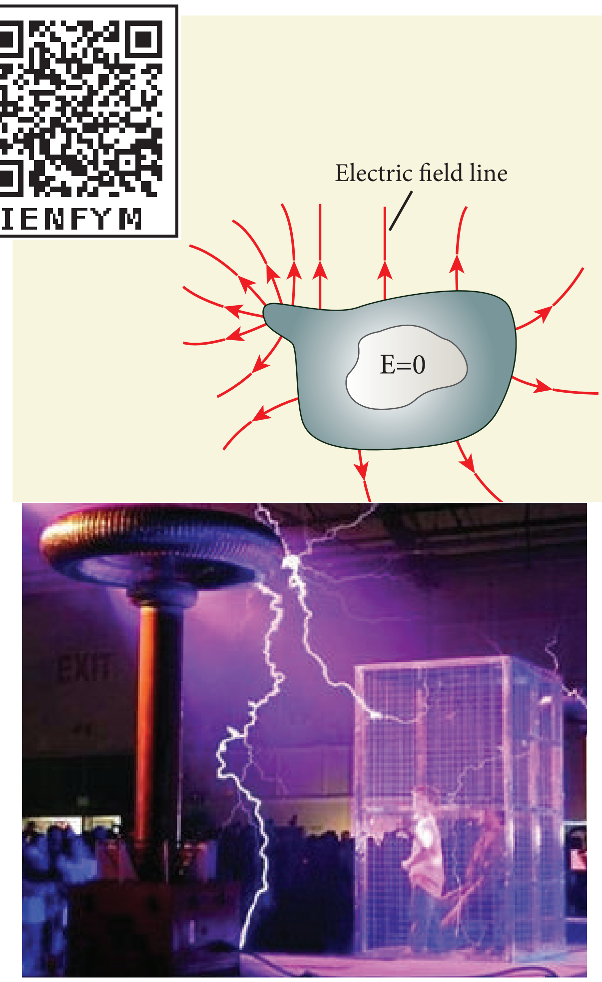


Figure 1.48 (a) Electric field inside the cavity (b) Faraday cage

During lightning accompanied by a thunderstorm, it is always safer to sit inside a bus than in open ground or under a tree. The metal body of the bus provides electrostatic shielding, since the electric field inside is zero. During lightning, the charges flow through the body of the conductor to the ground with no effect on the person inside that bus.

1.7.3 Electrostatic induction

In section 1.1, we have learnt that an object can be charged by rubbing using an appropriate material. Whenever a charged rod is touched by another conductor, charges start to flow from charged rod to the conductor. Is it possible to charge a conductor without any contact? The answer is yes. This type of **charging without actual contact is called electrostatic induction.**

(i) Consider an uncharged (neutral) conducting sphere at rest on an insulating stand. Suppose a negatively charged rod is brought near the conductor without touching it, as shown in Figure 1.49(a).

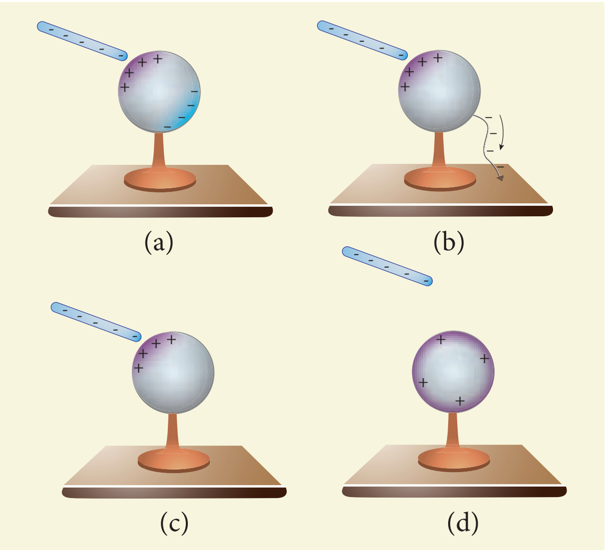


Figure 1.49 Various steps in electrostatic induction

The negative charge of the rod repels the electrons in the conductor to the opposite side. As a result, positive charges are induced near the region of the charged rod while negative charges on the farther side.

Before introducing the charged rod, the free electrons were distributed uniformly on the surface of the conductor and the net charge is zero. Once the charged rod is brought near the conductor, the distribution is no longer uniform with more electrons located on the farther side of the rod and positive charges are located closer to the rod. But the total charge is zero.

(ii) Now the conducting sphere is connected to the ground through a conducting wire. This is called grounding. Since the ground can always receive any amount of electrons, grounding removes the electron from the conducting sphere. Note that positive charges will not flow to the ground because they are attracted by the negative charges of the rod (Figure 1.49(b)).

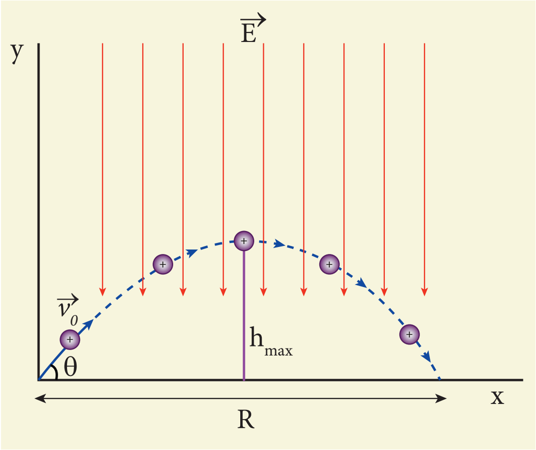
(iii) When the grounding wire is removed from the conductor, the positive charges remain near the charged rod (Figure 1.49(c))

(iv) Now the charged rod is taken away from the conductor. As soon as the charged rod is removed, the positive charge gets distributed uniformly on the surface of the conductor (Figure 1.49 (d)). By this process, the neutral conducting sphere becomes positively charged.

For an arbitrary shaped conductor, the intermediate steps and conclusion are the same except the final step. The distribution of positive charges is not uniform for arbitrarily-shaped conductors. Why is it not uniform? The reason for it is discussed in the section 1.9

EXAMPLE 1.19

A small ball of conducting material having a charge +q and mass m is thrown upward at an angle θ to horizontal surface with an initial speed v_o as shown in the figure. There exists an uniform electric field E downward along with the gravitational field g. Calculate the range, maximum height and time of flight in the motion of this charged ball. Neglect the effect of air and treat the ball as a point mass.



Solution

If the conductor has no net charge, then its motion is the same as usual projectile motion of a mass m which we studied in Kinematics (unit 2, vol-1 XI physics). Here, in this problem, in addition to downward gravitational force, the charge also will experience a downward uniform electrostatic force.

The acceleration of the charged ball due to gravity = $-g\hat{j}$

The acceleration of the charged ball due to uniform electric field = $-\frac{qE}{m}\hat{j}$

The total acceleration of charged ball in downward direction $\vec{a} = -\left(g + \frac{qE}{m}\right)\hat{j}$

It is important here to note that the acceleration depends on the mass of the object. Galileo's conclusion that all objects fall at the same rate towards the Earth is true only in a uniform gravitational field. When a uniform electric field is included, the acceleration of a charged object depends on both mass and charge.

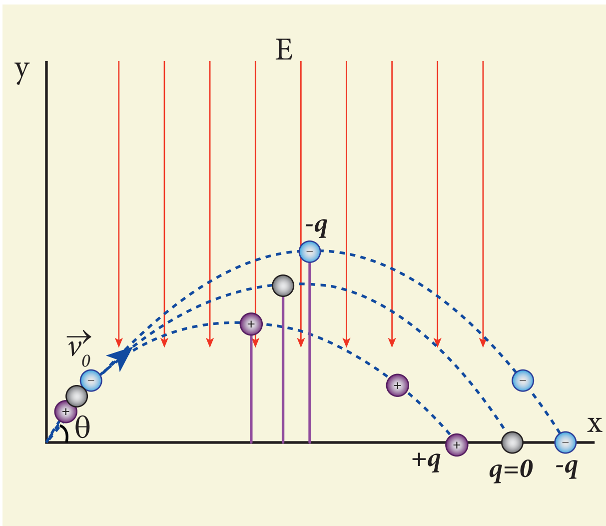
But still the acceleration $a = \left(g + \frac{qE}{m}\right)$ is constant throughout the motion. Hence we use kinematic equations to calculate the range, maximum height and time of flight.

In fact we can simply replace g by $g + \frac{qE}{m}$ in the usual expressions of range, maximum height and time of flight of a projectile.

	Without charge	With the charge +q
Time of flight T	$\frac{2v_o \sin\theta}{g}$	$\frac{2v_o \sin\theta}{\left(g + \frac{qE}{m}\right)}$
Maximum height h_{max}	$\frac{v_o^2 \sin^2\theta}{2g}$	$\frac{v_o^2 \sin^2\theta}{2\left(g + \frac{qE}{m}\right)}$
Range R	$\frac{v_o^2 \sin 2\theta}{g}$	$\frac{v_o^2 \sin 2\theta}{\left(g + \frac{qE}{m}\right)}$

Note that the time of flight, maximum height, range are all inversely proportional to the acceleration of the object. Since $\left(g + \frac{qE}{m}\right) > g$ for charge +q, the quantities T, h_{max}, and R will decrease when compared to the motion of an object of mass m and zero net charge. Suppose the charge is −q, then $\left(g - \frac{qE}{m}\right) < g$, and the quantities T, h_{max} and

R will increase. Interestingly the trajectory is still parabolic as shown in the figure.



1.7.4 Dielectrics or insulators

A dielectric is a non-conducting material and has no free electrons. The electrons in a dielectric are bound within the atoms. Ebonite, glass and mica are some examples of dielectrics. When an external electric field is applied, the electrons are not free to move anywhere but they are realigned in a specific way. A dielectric is made up of either polar molecules or non-polar molecules.

Non-polar molecules

A non-polar molecule is one in which centers of positive and negative charges coincide. As a result, it has no permanent dipole moment. Examples of non-polar molecules are hydrogen (H_2), oxygen (O_2), and carbon dioxide (CO_2) etc.

When an external electric field is applied, the centers of positive and negative charges are separated by a small distance which induces dipole moment in the direction of the external electric field. Then the dielectric is said to be polarized by an external electric field. This is shown in Figure 1.50.

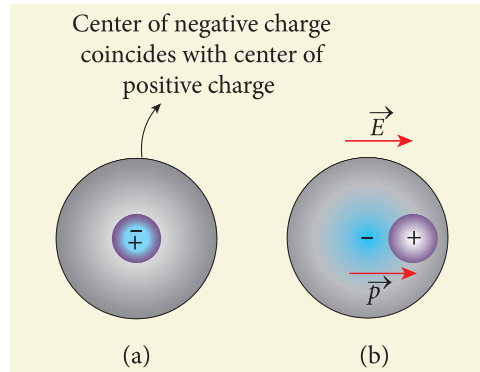


Figure 1.50 (a) Non polar molecules without external field (b) With the external field

Polar molecules

In polar molecules, the centers of the positive and negative charges are separated even in the absence of an external electric field. They have a permanent dipole moment. Due to thermal motion, the direction of each dipole moment is oriented randomly (Figure 1.51(a)). Hence the net dipole moment is zero in the absence of an external electric field. Examples of polar molecules are H_2O, N_2O, HCl, NH_3.

When an external electric field is applied, the dipoles inside the polar molecule tend to align in the direction of the electric field. Hence a net dipole moment is induced in it. Then the dielectric is said to be polarized by an external electric field (Figure 1.51(b)).

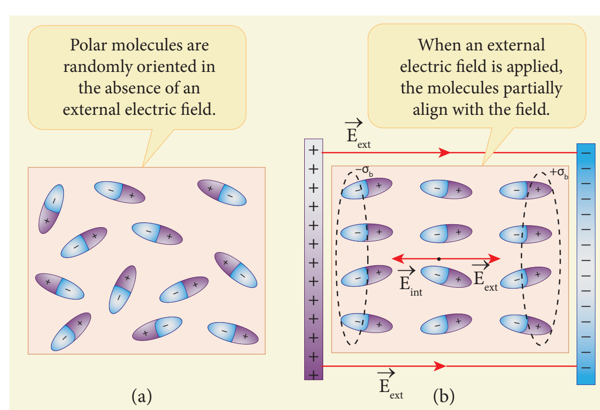


Figure 1.51 (a) Randomly oriented polar molecules (b) Align with the external electric field

Polarisation

In the presence of an external electric field, the dipole moment is induced in the dielectric material. **Polarisation $\vec{P}$ is defined as the total dipole moment per unit volume of the dielectric.** For most dielectrics (linear isotropic), the Polarisation is directly proportional to the strength of the external electric field. This is written as

$$\vec{P} = \chi_e \vec{E}_{ext} \qquad (1.80)$$

where χ_e is a constant called the electric susceptibility which is a characteristic of each dielectric.

1.7.5 Induced Electric field inside the dielectric

When an external electric field is applied on a conductor, the charges are aligned in such a way that an internal electric field is created which cancels the external electric field. But in the case of a dielectric, which has no free electrons, the external electric field only realigns the charges so that an internal electric field is produced. The magnitude of the internal electric field is smaller than that of external electric field. Therefore the net electric field inside the dielectric is not zero but is parallel to an external electric field with magnitude less than that of the external electric field. For example, let us consider a rectangular dielectric slab placed between two oppositely charged plates (capacitor) as shown in the Figure 1.52(b).

The uniform electric field between the plates acts as an external electric field $\vec{E}_{ext}$ which polarizes the dielectric placed between plates. The positive charges are induced on one side surface and negative charges are induced on the other side of surface.
But inside the dielectric, the net charge is zero even in a small volume. So the dielectric in the external field is equivalent to two oppositely charged sheets with the surface charge densities $+\sigma_b$ and $-\sigma_b$. These charges are called bound charges. They are not free to move like free electrons in conductors. This is shown in the Figure 1.52(b).

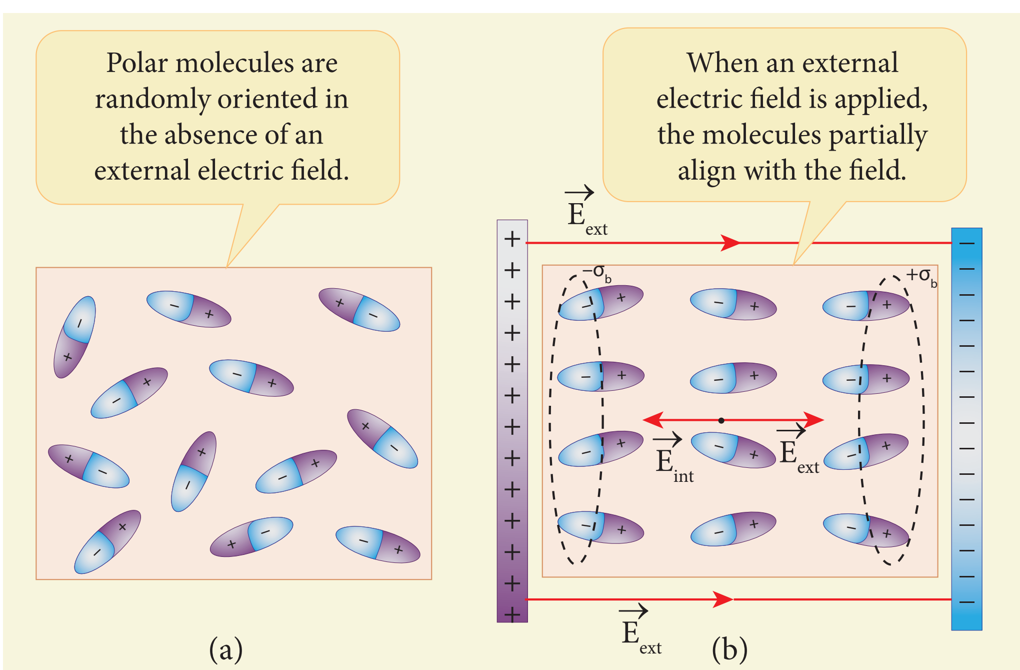


Figure 1.52 Induced electric field lines inside the dielectric

For example, the charged balloon after rubbing sticks onto a wall. The reason is that the negatively charged balloon is brought near the wall, it polarizes opposite charges on the surface of the wall, which attracts the balloon. This is shown in Figure 1.53.

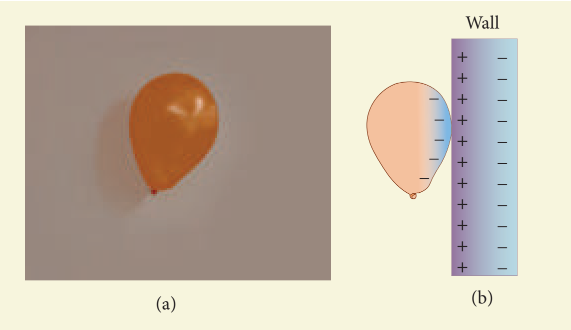


Figure 1.53 (a) Balloon sticks to the wall (b) Polarisation of wall due to the electric field created by the balloon

1.7.6 Dielectric strength

When the external electric field applied to a dielectric is very large, it tears the atoms apart so that the bound charges become free charges. Then the dielectric starts to conduct electricity. This is called dielectric breakdown. The maximum electric field the dielectric can withstand before it breakdowns is called dielectric strength. For example, the dielectric strength of air is 3×10^6 V m^{-1}. If the applied electric field increases beyond this, a spark is produced in the air. The dielectric strengths of some dielectrics are given in the Table 1.1.

Table 1.1 Dielectric strength

Substance	Dielectric strength (Vm^{-1})
Mica	100×10^6
Teflon	60×10^6
Paper	16×10^6
Air	3×10^6
Pyrex glass	14×10^6

1.8 CAPACITORS AND CAPACITANCE

1.8.1 Capacitors

Capacitor is a device used to store electric charge and electrical energy. It consists of two conducting objects (usually plates or sheets) separated by some distance. Capacitors are widely used in many electronic circuits and have applications in many areas of science and technology.

A simple capacitor consists of two parallel metal plates separated by a small distance as shown in Figure 1.54 (a).

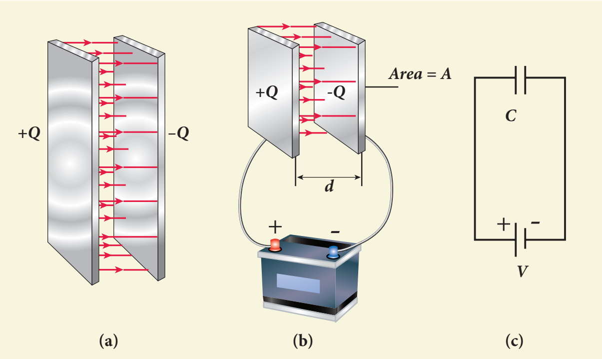


Figure 1.54 (a) Parallel plate capacitor (b) Capacitor connected with a battery (c) Symbolic representation of capacitor.

When a capacitor is connected to a battery of potential difference V, the electrons are transferred from one plate to the other plate by battery so that one plate becomes negatively charged with a charge of –Q and the other plate positively charged with +Q. The potential difference between the plates is equivalent to the battery's terminal voltage. This is shown in Figure 1.54(b). If the battery voltage is increased, the amount of charges stored in the plates also increase. In general, the charge stored in the capacitor is proportional to the potential difference between the plates.

$$Q \propto V$$

so that $Q = CV$

where the C is the proportionality constant called capacitance. **The capacitance C of a capacitor is defined as the ratio of the magnitude of charge on either of the conductor plates to the potential difference existing between the conductors.**

$$C = \frac{Q}{V} \qquad (1.81)$$

The SI unit of capacitance is *coulomb per volt* or *farad (F)* in honor of Michael Faraday. Farad is a very large unit of capacitance. In practice, capacitors are available in the range of microfarad ($1\mu F = 10^{-6}$ F) to picofarad ($1pf = 10^{-12}$ F). A capacitor is represented by the symbol or . Note that the total charge stored in the capacitor is zero (Q - Q = 0). When we say the capacitor stores charges, it means the amount of charge that can be stored in any one of the plates.

Nowadays there are capacitors available in various shapes (cylindrical, disk) and types (tantalum, ceramic and electrolytic), as shown in Figure 1.55. These capacitors are extensively used in various kinds of electronic circuits.

Figure 1.55 Various types of capacitors

Capacitance of a parallel plate capacitor

Consider a capacitor with two parallel plates each of cross-sectional area A and separated by a distance d as shown in Figure 1.56.

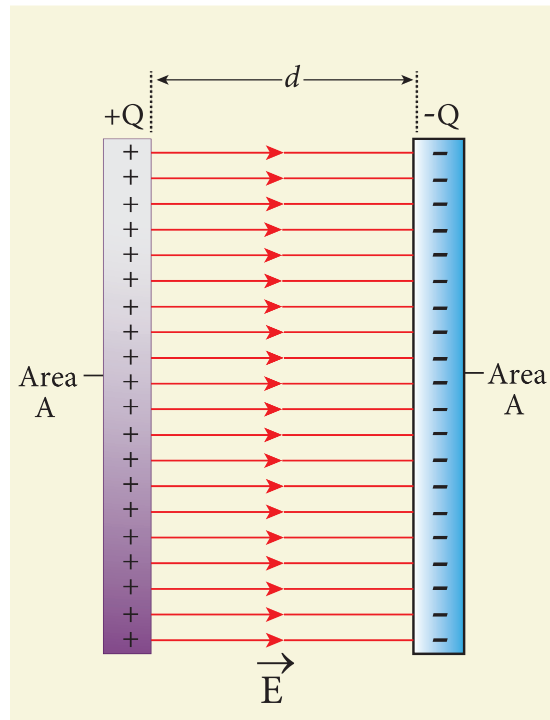


Figure 1.56 Capacitance of a parallel plate capacitor

The electric field between two infinite parallel plates is uniform and is given by $E = \frac{\sigma}{\varepsilon_\circ}$ where σ is the surface charge density on the plates $\left(\sigma = \frac{Q}{A}\right)$. If the separation distance d is very much smaller than the size of the plate ($d^2 << A$), then the above result is used even for finite–sized parallel plate capacitor.

The electric field between the plates is

$$E = \frac{Q}{A\varepsilon_\circ} \qquad (1.82)$$

Since the electric field is uniform, the electric potential between the plates having separation d is given by

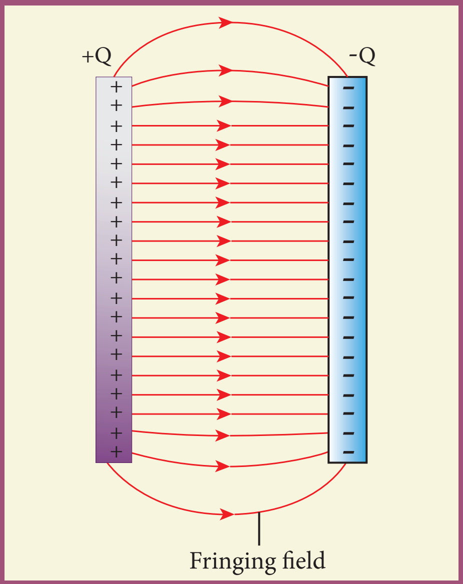
$$V = Ed = \frac{Qd}{A\varepsilon_0} \quad (1.83)$$

Therefore the capacitance of the capacitor is given by

$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{Qd}{A\varepsilon_0}\right)} = \frac{\varepsilon_0 A}{d} \quad (1.84)$$

Note

While deriving an expression for capacitance of the parallel plate capacitor, the expression of the electric field for infinite plates is used. But for finite sized plates, the electric field is not strictly uniform between the plates. At both edges, the electric field is bent outwards as shown in the Figure.



This is called "fringing field". However under the condition ($d^2 << A$), this effect can be ignored.

From equation (1.84), it is evident that capacitance is directly proportional to the area of cross section and is inversely proportional to the distance between the plates. This can be understood from the following.

(i) If the area of cross-section of the capacitor plates is increased, more charges can be distributed for the same potential difference. As a result, the capacitance is increased.

(ii) If the distance d between the two plates is reduced, the potential difference between the plates ($V = Ed$) decreases with E constant. As a result, voltage difference between the terminals of the battery increases which in turn leads to an additional flow of charge to the plates from the battery, till the voltage on the capacitor equals to the battery's terminal voltage. Suppose the distance is increased, the capacitor voltage increases and becomes greater than the battery voltage. Then, the charges flow from capacitor plates to battery till both voltages becomes equal.

EXAMPLE 1.20

A parallel plate capacitor has square plates of side 5 cm and separated by a distance of 1 mm. (a) Calculate the capacitance of this capacitor. (b) If a 10 V battery is connected to the capacitor, what is the charge stored in any one of the plates? (The value of $\varepsilon_o = 8.85 \times 10^{-12}$ Nm^2 C^{-2})

Solution

(a) The capacitance of the capacitor is

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\times25\times10^{-4}}{1\times10^{-3}}$$
$$= 221.2\times10^{-13}F$$
$$C = 22.12\times10^{-12}F = 22.12\,pF$$

(b) The charge stored in any one of the plates is Q = CV, Then

$$Q = 22.12\times10^{-12}\times10 = 221.2\times10^{-12}C = 221.2\,pC$$

Sometimes we notice that the ceiling fan does not start rotating as soon as it is switched on. But when we rotate the blades, it starts to rotate as usual. Why it is so? We know that to rotate any object, there must be a torque applied on the object. For the ceiling fan, the initial torque is given by the capacitor widely known as a condenser. If the condenser is faulty, it will not give sufficient initial torque to rotate the blades when the fan is switched on.

1.8.2 Energy stored in the capacitor

Capacitor not only stores the charge but also it stores energy. When a battery is connected to the capacitor, electrons of total charge -Q are transferred from one plate to the other plate. To transfer the charge, work is done by the battery. This work done is stored as electrostatic potential energy in the capacitor.

To transfer an infinitesimal charge dQ for a potential difference V, the work done is given by

$$dW = V\,dQ \qquad (1.85)$$

where $V = \frac{Q}{C}$

The total work done to charge a capacitor is

$$W = \int_0^Q \frac{Q}{C}\,dQ = \frac{Q^2}{2C} \qquad (1.86)$$

This work done is stored as electrostatic potential energy (U_E) in the capacitor.

$$U_E = \frac{Q^2}{2C} = \frac{1}{2}CV^2 \quad (\because Q = CV) \qquad (1.87)$$

where Q = CV is used. This stored energy is thus directly proportional to the capacitance of the capacitor and the square of the voltage between the plates of the capacitor. But where is this energy stored in the capacitor? To understand this question, the equation (1.87) is rewritten as follows using the results $C = \frac{\varepsilon_0 A}{d}$ and V = Ed

$$U_E = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\varepsilon_0 (Ad)E^2 \qquad (1.88)$$

where Ad = volume of the space between the capacitor plates. **The energy stored per unit volume of space is defined as energy density** $u_E = \frac{U}{Volume}$ From equation (1.88), we get

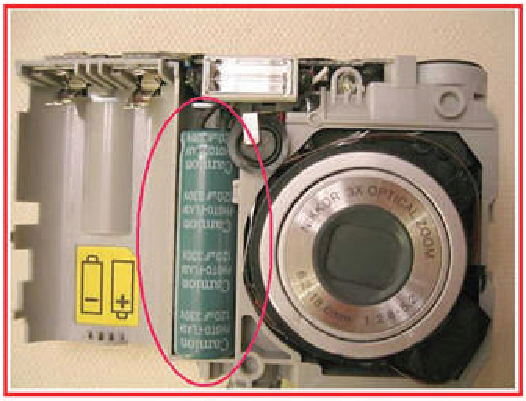
$$u_E = \frac{1}{2}\varepsilon_0 E^2 \qquad (1.89)$$

From equation (1.89), we infer that the energy is stored in the electric field existing between the plates of the capacitor. Once the capacitor is allowed to discharge, the energy is retrieved.

It is important to note that the energy density depends only on the electric field and not on the size of the plates of the capacitor. In fact, expression (1.89) is true for the electric field due to any type of charge configuration.

1.8.3 Applications of capacitors

Capacitors are used in various electronics circuits. A few of the applications.



(a)

(b)

Figure 1.57 (a) Flash capacitor in camera (b) Heart defibrillator

(a) Most people are now familiar with the digital camera. The flash which comes from the camera when we take photographs is due to the energy released from the capacitor, called a flash capacitor (Figure 1.57 (a))

(b) During cardiac arrest, a device called heart defibrillator is used to give a sudden surge of a large amount of electrical energy to the patient's chest to retrieve the normal heart function. This defibrillator uses a capacitor of 175 μF charged to a high voltage of around 2000 V. This is shown in Figure 1.57(b).

(c) Capacitors are used in the ignition system of automobile engines to eliminate sparking

(d) Capacitors are used to reduce power fluctuations in power supplies and to increase the efficiency of power transmission.

However, capacitors have disadvantage as well. Even after the battery or power supply is removed, the capacitor stores charges and energy for some time. For example if the TV is switched off, it is always advisable to not touch the back side of the TV panel.

1.8.4 Effect of dielectrics in capacitors

In earlier discussions, we assumed that the space between the parallel plates of a capacitor is either empty or filled with air. Suppose dielectrics like mica, glass or paper are introduced between the plates, then the capacitance of the capacitor is altered. The dielectric can be inserted into the plates in two different ways. (i) when the capacitor is disconnected from the battery. (ii) when the capacitor is connected to the battery.

(i) when the capacitor is disconnected from the battery

Consider a capacitor with two parallel plates each of cross-sectional area A and are separated by a distance d. The capacitor is charged by a battery of voltage V_0 and the charge stored is Q_0. The capacitance of the capacitor without the dielectric is

$$C_0 = \frac{Q_0}{V_0} \tag{1.90}$$

The battery is then disconnected from the capacitor and the dielectric is inserted between the plates. This is shown in Figure 1.58.

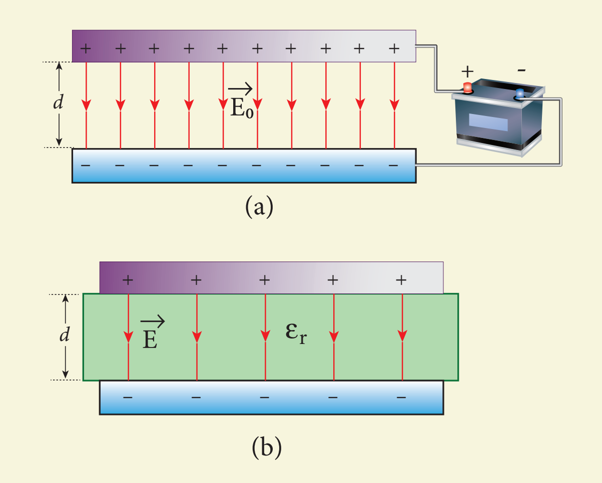


Figure 1.58 (a) Capacitor is charged with a battery (b) Dielectric is inserted after the battery is disconnected

The introduction of dielectric between the plates will decrease the electric field. Experimentally it is found that the modified electric field is given by

$$E = \frac{E_0}{\varepsilon_r} \qquad (1.91)$$

Here E_o is the electric field inside the capacitors when there is no dielectric and ε_r is the relative permeability of the dielectric or simply known as the dielectric constant. Since $\varepsilon_r > 1$, the electric field $E < E_o$.

As a result, the electrostatic potential difference between the plates (V = Ed) is also reduced. But at the same time, the charge Q_o will remain constant once the battery is disconnected.

Hence the new potential difference is

$$V = Ed = \frac{E_0}{\varepsilon_r} d = \frac{V_0}{\varepsilon_r} \qquad (1.92)$$

We know that capacitance is inversely proportional to the potential difference. Therefore as V decreases, C increases.

Thus new capacitance in the presence of a dielectric is

$$C = \frac{Q_0}{V} = \varepsilon_r \frac{Q_0}{V_0} = \varepsilon_r C_0 \qquad (1.93)$$

Since $\varepsilon_r > 1$, we have $C > C_o$. Thus insertion of the dielectric constant ε_r increases the capacitance.

Using equation (1.84),

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{\varepsilon A}{d} \qquad (1.94)$$

where $\varepsilon = \varepsilon_r \varepsilon_o$ is the permittivity of the dielectric medium.

The energy stored in the capacitor before the insertion of a dielectric is given by

$$U_0 = \frac{1}{2}\frac{Q_0^2}{C_0} \qquad (1.95)$$

After the dielectric is inserted, the charge Q_0 remains constant but the capacitance is increased. As a result, the stored energy is decreased.

$$U = \frac{1}{2}\frac{Q_0^2}{C} = \frac{1}{2}\frac{Q_0^2}{\varepsilon_r C_0} = \frac{U_0}{\varepsilon_r} \qquad (1.96)$$

Since $\varepsilon_r > 1$ we get $U < U_o$. There is a decrease in energy because,when the dielectric is inserted, the capacitor spends some energy in pulling the dielectric inside.

(ii) When the battery remains connected to the capacitor

Let us now consider what happens when the battery of voltage V_0 remains connected to the capacitor when the dielectric is inserted into the capacitor. This is shown in Figure1.59.

The potential difference V_0 across the plates remains constant. But it is found experimentally (first shown by Faraday) that when dielectric is inserted, the charge stored in the capacitor is increased by a factor ε_r.

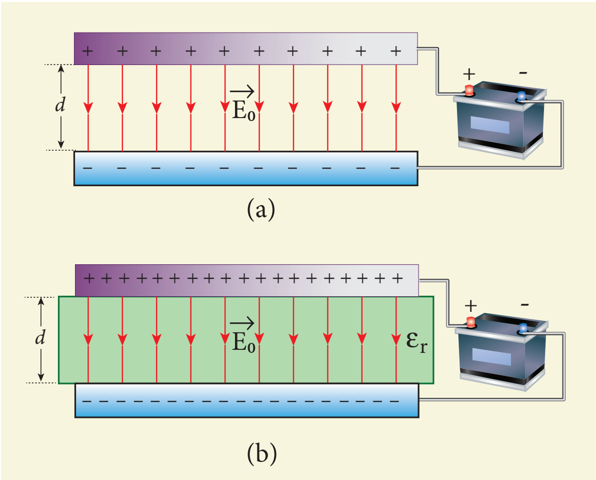


Figure 1.59 (a) Capacitor is charged through a battery (b) Dielectric is inserted when the battery is connected.

$$Q = \varepsilon_r Q_o \qquad (1.97)$$

Due to this increased charge, the capacitance is also increased. The new capacitance is

$$C = \frac{Q}{V_0} = \varepsilon_r \frac{Q_0}{V_0} = \varepsilon_r C_0 \qquad (1.98)$$

However the reason for the increase in capacitance in this case when the battery remains connected is different from the case when the battery is disconnected before introducing the dielectric.

$$\text{Now,} \quad C_0 = \frac{\varepsilon_0 A}{d}$$

$$\text{and} \quad C = \frac{\varepsilon A}{d} \qquad (1.99)$$

The energy stored in the capacitor before the insertion of a dielectric is given by

$$U_0 = \frac{1}{2} C_0 V_0^2 \qquad (1.100)$$

Note that here we have not used the expression $U_0 = \frac{1}{2}\frac{Q_0^2}{C_0}$ because here, both charge and capacitance are changed, whereas in equation (1.100), V_o remains constant.

After the dielectric is inserted, the capacitance is increased; hence the stored energy is also increased.

$$U = \frac{1}{2} C V_0^2 = \frac{1}{2} \varepsilon_r C_0 V_0^2 = \varepsilon_r U_0 \qquad (1.101)$$

Since $\varepsilon_r > 1$ we have $U > U_o$.

It may be noted here that since voltage between the capacitor V_0 is constant, the electric field between the plates also remains constant.

The energy density is given by

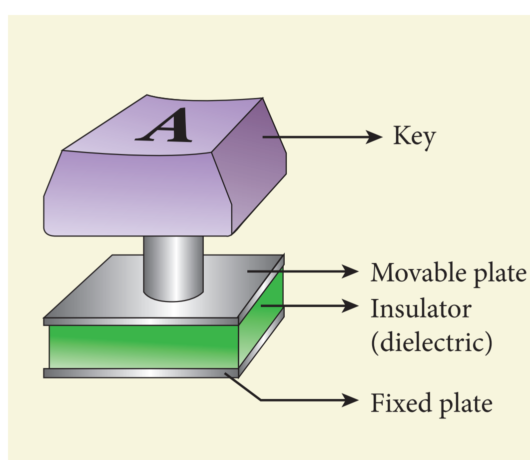
$$u = \frac{1}{2} \varepsilon E_0^2 \qquad (1.102)$$

where ε is the permittivity of the given dielectric material.

The results of the above discussions are summarised in the following Table 1.2

DO YOU KNOW?

Computer keyboard keys are constructed using capacitors with a dielectric as shown in the figure.



When the key is pressed, the separation between the plates decreases leading to an increase in the capacitance. This in turn triggers the electronic circuits in the computer to identify which key is pressed.

Table 1.2

S. No	Dielectric is inserted	Charge Q	Voltage V	Electric field E	Capacitance C	Energy U
1	When the battery is disconnected	Constant	decreases	Decreases	Increases	Decreases
2	When the battery is connected	Increases	Constant	Constant	Increases	Increases

EXAMPLE 1.21

A parallel plate capacitor filled with mica having $\varepsilon_r = 5$ is connected to a 10 V battery. The area of the parallel plate is 6 m^2 and separation distance is 6 mm. (a) Find the capacitance and stored charge.

(b) After the capacitor is fully charged, the battery is disconnected and the dielectric is removed carefully.

Calculate the new values of capacitance, stored energy and charge.

Solution

(a) The capacitance of the capacitor in the presence of dielectric is

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{5 \times 8.85 \times 10^{-12} \times 6}{6 \times 10^{-3}}$$
$$= 44.25 \times 10^{-9} F = 44.25\text{nF}$$

The stored charge is

$$Q = CV = 44.25 \times 10^{-9} \times 10$$
$$= 442.5 \times 10^{-9} C = 442.5nC$$

The stored energy is

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times 44.25C \times 10^{-9} \times 100$$
$$= 2.21 \times 10^{-6} J = 2.21\ \mu J$$

(b) After the removal of the dielectric, since the battery is already disconnected the total charge will not change. But the potential difference between the plates increases. As a result, the capacitance is decreased.

New capacitance is

$$C_0 = \frac{C}{\varepsilon_r} = \frac{44.25 \times 10^{-9}}{5}$$
$$= 8.85 \times 10^{-9} F = 8.85nF$$

The stored charge remains same and 442.5 nC. Hence newly stored energy is

$$U_0 = \frac{Q^2}{2C_0} = \frac{Q^2 \varepsilon_r}{2C} = \varepsilon_r U$$
$$= 5 \times 2.21\ \mu J = 11.05\ \mu J$$

The increased energy is

$$\Delta U = 11.05 \mu J - 2.21 \mu J = 8.84 \mu J$$

When the dielectric is removed, it experiences an inward pulling force due to the plates. To remove the dielectric, an external agency has to do work on the dielectric which is stored as additional energy. This is the source for the extra energy 8.84 μJ.

1.8.5 Capacitor in series and parallel

(i) Capacitor in series

Consider three capacitors of capacitance C_1, C_2 and C_3 connected in series with a battery of voltage V as shown in the Figure 1.60 (a).

As soon as the battery is connected to the capacitors in series, the electrons of charge

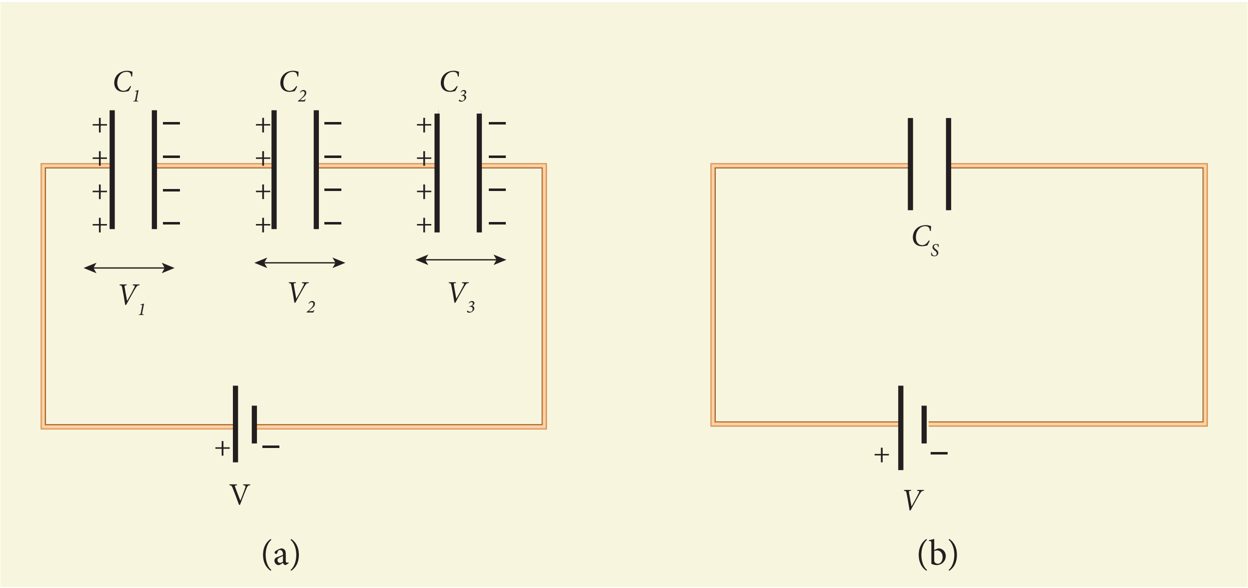


Figure 1.60 (a) Capacitors connected in series (b) Equivalence capacitors C_S

–Q are transferred from negative terminal to the right plate of C_3 which pushes the electrons of same amount -Q from left plate of C_3 to the right plate of C_2 due to electrostatic induction. Similarly, the left plate of C_2 pushes the charges of –Q to the right plate of C_1 which induces the positive charge +Q on the left plate of C_1. At the same time, electrons of charge –Q are transferred from left plate of C_1 to positive terminal of the battery.

By these processes, each capacitor stores the same amount of charge Q. The capacitances of the capacitors are in general different, so that the voltage across each capacitor is also different and are denoted as V_1, V_2 and V_3 respectively.

The total voltage across each capacitor must be equal to the voltage of the battery.

$$V = V_1 + V_2 + V_3 \quad (1.103)$$

Since, $Q = CV$, we have $V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

$$= Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \quad (1.104)$$

If three capacitors in series are considered to form an equivalent single capacitor C_s shown in Figure 1.60(b), then we have $V = \frac{Q}{C_S}$. Substituting this expression into equation (1.104), we get

$$\frac{Q}{C_S} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad (1.105)$$

Thus, the inverse of the equivalent capacitance C_S of three capacitors connected in series is equal to the sum of the inverses of each capacitance. This equivalent capacitance C_S is always less than the smallest individual capacitance in the series.

(ii) Capacitance in parallel

Consider three capacitors of capacitance C_1, C_2 and C_3 connected in parallel with a battery of voltage V as shown in Figure 1.61 (a).

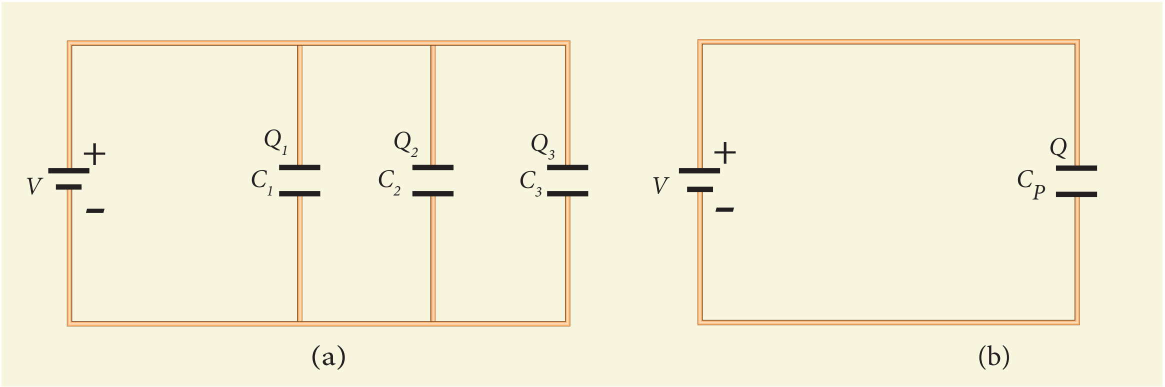


Figure 1.61 (a) capacitors in parallel (b) equivalent capacitance with the same total charge

Since corresponding sides of the capacitors are connected to the same positive and negative terminals of the battery, the voltage across each capacitor is equal to the battery's voltage. Since capacitance of the capacitors is different, the charge stored in each capacitor is not the same. Let the charge stored in the three capacitors be Q_1, Q_2 and Q_3 respectively. According to the law of conservation of total charge, the sum of these three charges is equal to the charge Q transferred by the battery,

$$Q = Q_1 + Q_2 + Q_3 \qquad (1.106)$$

Now, since Q=CV, we have

$$Q = C_1V + C_2V + C_3V \qquad (1.107)$$

If these three capacitors are considered to form a single capacitance C_P which stores the total charge Q as shown in the Figure 1.61(b), then we can write $Q = C_PV$. Substituting this in equation (1.107), we get

$$C_PV = C_1V + C_2V + C_3V$$

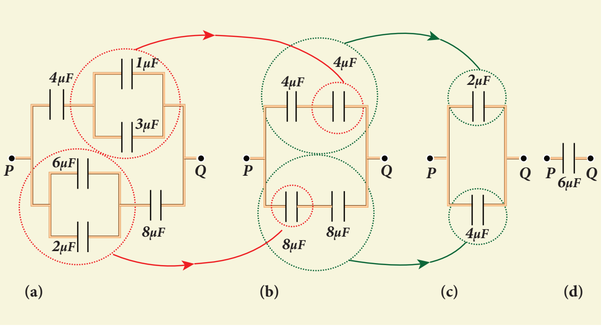
$$C_P = C_1 + C_2 + C_3 \qquad (1.108)$$

Thus, the equivalent capacitance of capacitors connected in parallel is equal to the sum of the individual capacitances.

The equivalent capacitance C_P in a parallel connection is always greater than the largest individual capacitance. In a parallel connection, it is equivalent as area of each capacitance adds to give more effective area such that total capacitance increases.

EXAMPLE 1.22

Find the equivalent capacitance between P and Q for the configuration shown below in the figure (a).



Solution

The capacitors 1 μF and 3μF are connected in parallel and 6μF and 2 μF are also separately connected in parallel. So these parallel combinations reduced to equivalent single capacitances in their respective positions, as shown in the figure (b).

$C_{eq} = 1\mu F + 3\mu F = 4\mu F$

$C_{eq} = 6\mu F + 2\mu F = 8\mu F$

From the figure (b), we infer that the two 4 μF capacitors are connected in series and the two 8 μF capacitors are connected in series. By using formula for the series, we can reduce to their equivalent capacitances as shown in figure (c).

$$\frac{1}{C_{eq}} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \qquad \Rightarrow C_{eq} = 2\ \mu F$$

and

$$\frac{1}{C_{eq}} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4} \qquad \Rightarrow C_{eq} = 4\ \mu F$$

From the figure (c), we infer that 2μF and 4μF are connected in parallel. So the equivalent capacitance is given in the figure (d).

$C_{eq} = 2\mu F + 4\mu F = 6\mu F$

Thus the combination of capacitances in figure (a) can be replaced by a single capacitance 6 μF.

1.9 DISTRIBUTION OF CHARGES IN A CONDUCTOR AND ACTION AT POINTS

1.9.1 Distribution of charges in a conductor

Consider two conducting spheres A and B of radii r_1 and r_2 respectively connected to each other by a thin conducting wire as shown in the Figure 1.62. The distance between the spheres is much greater than the radii of either spheres.

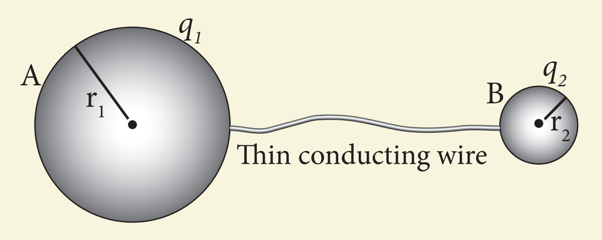


Figure 1.62 Two conductors are connected through conducting wire

If a charge Q is introduced into any one of the spheres, this charge Q is redistributed into both the spheres such that the electrostatic potential is same in both the spheres. They are now uniformly charged and attain electrostatic equilibrium. Let q_1 be the charge residing on the surface of sphere A and q_2 is the charge residing on the surface of sphere B such that $Q = q_1 + q_2$. The charges are distributed only on the surface and there is no net charge inside the conductor.

The electrostatic potential at the surface of the sphere A is given by

$$V_A = \frac{1}{4\pi\varepsilon_\circ}\frac{q_1}{r_1} \qquad (1.110)$$

The electrostatic potential at the surface of the sphere B is given by

$$V_B = \frac{1}{4\pi\varepsilon_\circ}\frac{q_2}{r_2} \qquad (1.111)$$

The surface of the conductor is an equipotential. Since the spheres are connected by the conducting wire, the surfaces of both the spheres together form an equipotential surface. This implies that

$$V_A = V_B$$

$$\text{or} \quad \frac{q_1}{r_1} = \frac{q_2}{r_2} \qquad (1.112)$$

Let us take the charge density on the surface of sphere A is σ_1 and charge density on the surface of sphere B is σ_2. This implies that $q_1 = 4\pi r_1^2\sigma_1$ and

$q_2 = 4\pi r_2^2\sigma_2$. Substituting these values into equation (1.112), we get

$$\sigma_1 r_1 = \sigma_2 r_2 \qquad (1.113)$$

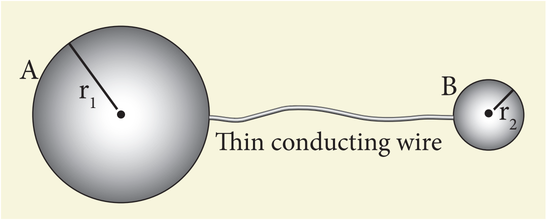
from which we conclude that

$$\sigma r = \text{constant} \qquad (1.114)$$

Thus the surface charge density σ is inversely proportional to the radius of the sphere. For a smaller radius, the charge density will be larger and vice versa.

EXAMPLE 1.23

Two conducting spheres of radius r_1 = 8 cm and r_2 = 2 cm are separated by a distance much larger than 8 cm and are connected by a thin conducting wire as shown in the figure. A total charge of Q = +100 nC is placed on one of the spheres. After a fraction of a second, the charge Q is redistributed and both the spheres attain electrostatic equilibrium.



(a) Calculate the charge and surface charge density on each sphere.

(b) Calculate the potential at the surface of each sphere.

Solution

(a) The electrostatic potential on the surface of the sphere A is $V_A = \frac{1}{4\pi\varepsilon_\circ}\frac{q_1}{r_1}$

The electrostatic potential on the surface of the sphere A is $V_B = \frac{1}{4\pi\varepsilon_\circ}\frac{q_2}{r_2}$

Since $V_A = V_B$. We have

$$\frac{q_1}{r_1} = \frac{q_2}{r_2} \Rightarrow q_1 = \left(\frac{r_1}{r_2}\right)q_2$$

But from the conservation of total charge, Q = $q_1 + q_2$, we get $q_1 = Q - q_2$. By substituting this in the above equation,

$$Q - q_2 = \left(\frac{r_1}{r_2}\right)q_2$$

so that $$q_2 = Q\left(\frac{r_2}{r_1 + r_2}\right)$$

Therefore,

$$q_2 = 100\times10^{-9}\times\left(\frac{2}{10}\right) = 20nC$$

and $q_1 = Q - q_2 = 80nC$

The electric charge density for sphere A is

$$\sigma_1 = \frac{q_1}{4\pi r_1^2}$$

The electric charge density for sphere B is

$$\sigma_2 = \frac{q_2}{4\pi r_2^2}$$

Therefore,

$$\sigma_1 = \frac{80\times10^{-9}}{4\times64\times10^{-4}} = 0.99\times10^{-6}Cm^{-2}$$

and

$$\sigma_2 = \frac{20\times10^{-9}}{4\pi\times4\times10^{-4}} = 3.9\times10^{-6}Cm^{-2}$$

Note that the surface charge density is greater on the smaller sphere compared to the larger sphere ($\sigma_2 \approx 4\sigma_1$) which confirms the result $\frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}$.

The potential on both spheres is the same. So we can calculate the potential on any one of the spheres.

$$V_A = \frac{1}{4\pi\varepsilon_\circ}\frac{q_1}{r_1} = \frac{9\times10^9\times80\times10^{-9}}{8\times10^{-2}} = 9kV$$

1.9.2 Action at points or Corona discharge

Consider a charged conductor of irregular shape as shown in Figure 1.63 (a).

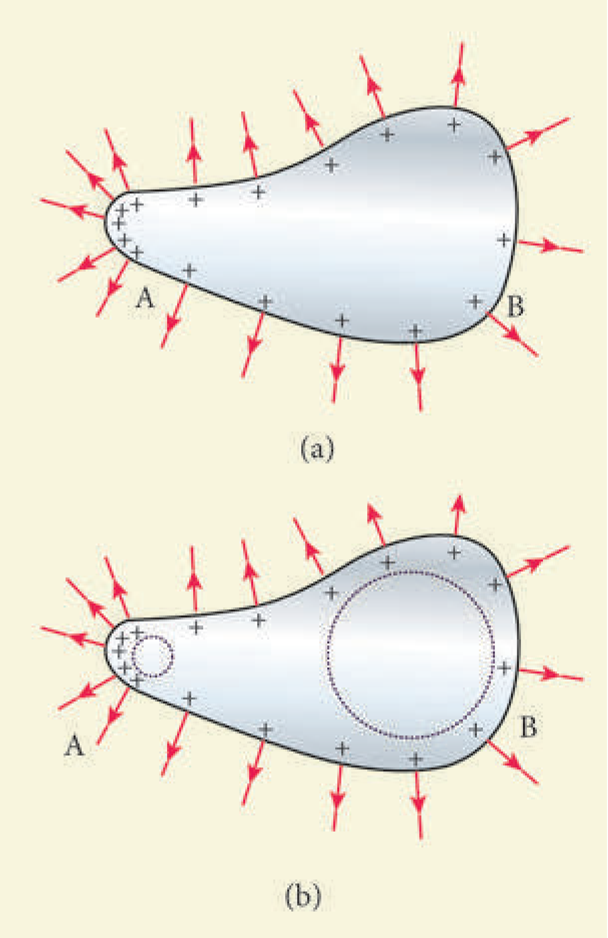


Figure 1.63 Action at a points or corona discharge

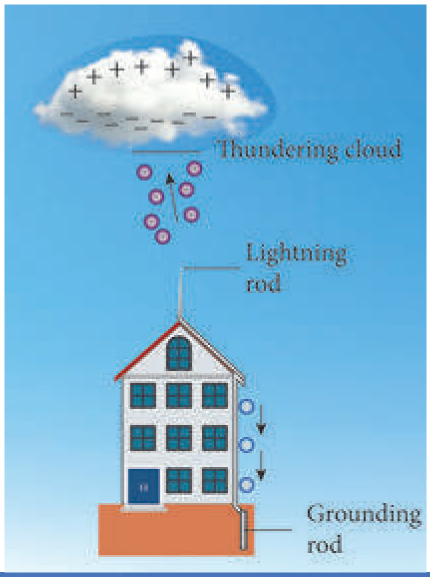
We know that smaller the radius of curvature, the larger is the charge density. The end of the conductor which has larger curvature (smaller radius) has a large charge accumulation as shown in Figure 1.63 (b).

As a result, the electric field near this edge is very high and it ionizes the surrounding air. The positive ions are repelled at the sharp edge and negative ions are attracted towards the sharper edge. This reduces the total charge of the conductor near the sharp edge. This is called action at points or corona discharge.

1.9.3 Lightning arrester or lightning conductor

This is a device used to protect tall buildings from lightning strikes. It works on the principle of action at points or corona discharge.

This device consists of a long thick copper rod passing from top of the building to the ground. The upper end of the rod has a sharp spike or a sharp needle as shown in Figure 1.64 (a) and (b).



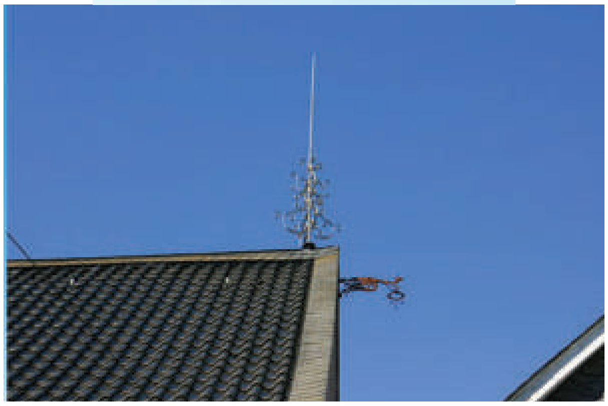


Figure 1.64 (a) Schematic diagram of a lightning arrestor. (b) A house with a lightning arrestor

The lower end of the rod is connected to the copper plate which is buried deep into the ground. When a negatively charged cloud is passing above the building, it induces a positive charge on the spike. Since the induced charge density on thin sharp spike is large, it results in a corona discharge. This positive charge ionizes the surrounding air which in turn neutralizes the negative charge in the cloud. The negative charge pushed to the spikes passes through the copper rod and is safely diverted to the Earth. The lightning arrester does not stop the lightning; rather it diverts the lightning to the ground safely.

1.9.4 Van de Graaff Generator

In the year 1929, Robert Van de Graaff designed a machine which produces a large amount of electrostatic potential difference, up to several million volts (10^7 V). This Van de Graff generator works on the principle of electrostatic induction and action at points.

A large hollow spherical conductor is fixed on the insulating stand as shown in Figure 1.65. A pulley B is mounted at the center of the hollow sphere and another pulley C is fixed at the bottom. A belt made up of insulating materials like silk or rubber runs over both pulleys. The pulley C is driven continuously by the electric motor. Two comb shaped metallic conductors E and D are fixed near the pulleys.

The comb D is maintained at a positive potential of 10^4 V by a power supply. The upper comb E is connected to the inner side of the hollow metal sphere.

Due to the high electric field near comb D, air between the belt and comb D gets ionized. The positive charges are pushed towards the belt and negative charges are attracted towards the comb D. The positive charges stick to the belt and move up. When the positive charges reach the comb E, a large amount of negative and positive charges are induced on either side of comb E due to electrostatic induction. As a result, the positive charges are pushed away from the comb E and they reach the outer surface of the sphere. Since the sphere is a conductor, the positive charges are distributed uniformly on the outer surface of the hollow sphere. At the same time, the negative charges nullify the positive charges in the belt due to corona discharge before it passes over the pulley.

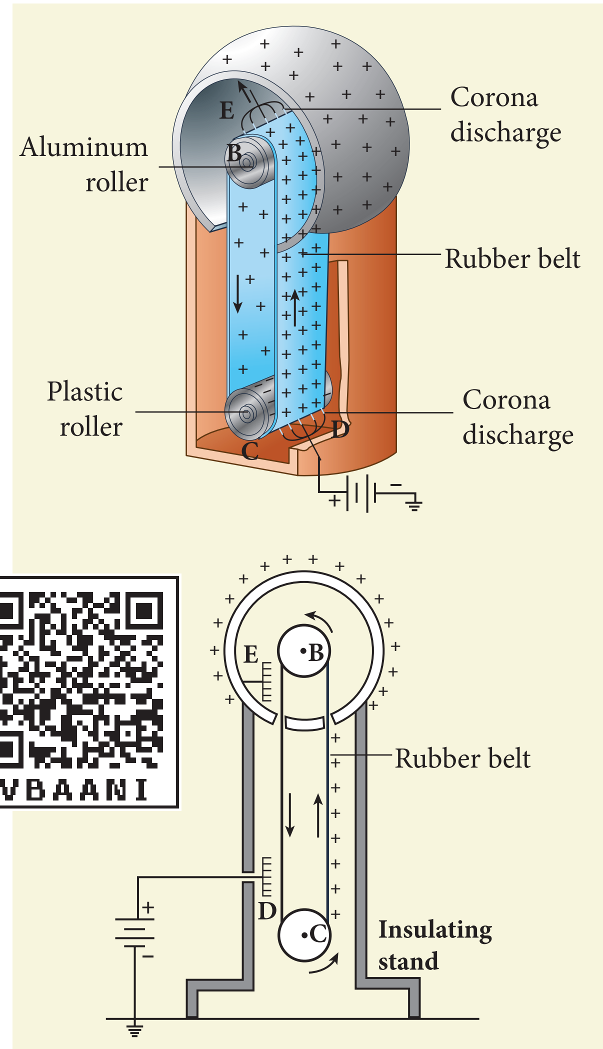


Figure 1.65 Van de Graaff generator

When the belt descends, it has almost no net charge. At the bottom, it again gains a large positive charge. The belt goes up and delivers the positive charges to the outer surface of the sphere. This process continues until the outer surface produces the potential difference of the order of 10^7 which is the limiting value. We cannot store charges beyond this limit since the extra charge starts leaking to the surroundings due to ionization of air. The leakage of charges can be reduced by enclosing the machine in a gas filled steel chamber at very high pressure.

The high voltage produced in this Van de Graaff generator is used to accelerate positive ions (protons and deuterons) for nuclear disintegrations and other applications.

EXAMPLE 1.24

Dielectric strength of air is 3×10^6 V m^{-1}. Suppose the radius of a hollow sphere in the Van de Graff generator is R = 0.5 m, calculate the maximum potential difference created by this Van de Graaff generator.

The electric field on the surface of the sphere(by Gauss law) is given by

$$E = \frac{1}{4\pi\varepsilon_\circ}\frac{Q}{R^2}$$

The potential on the surface of the hollow metallic sphere is given by

$$V = \frac{1}{4\pi\varepsilon_\circ}\frac{Q}{R} = ER$$

with $V_{max} = E_{max}R$

Here $E_{max} = 3\times10^6 \frac{V}{m}$. So the maximum potential difference created is given by

$V_{max} = 3 \times 10^6 \times 0.5$

$= 1.5 \times 10^6$ V (or) 1.5 million volt

SUMMARY

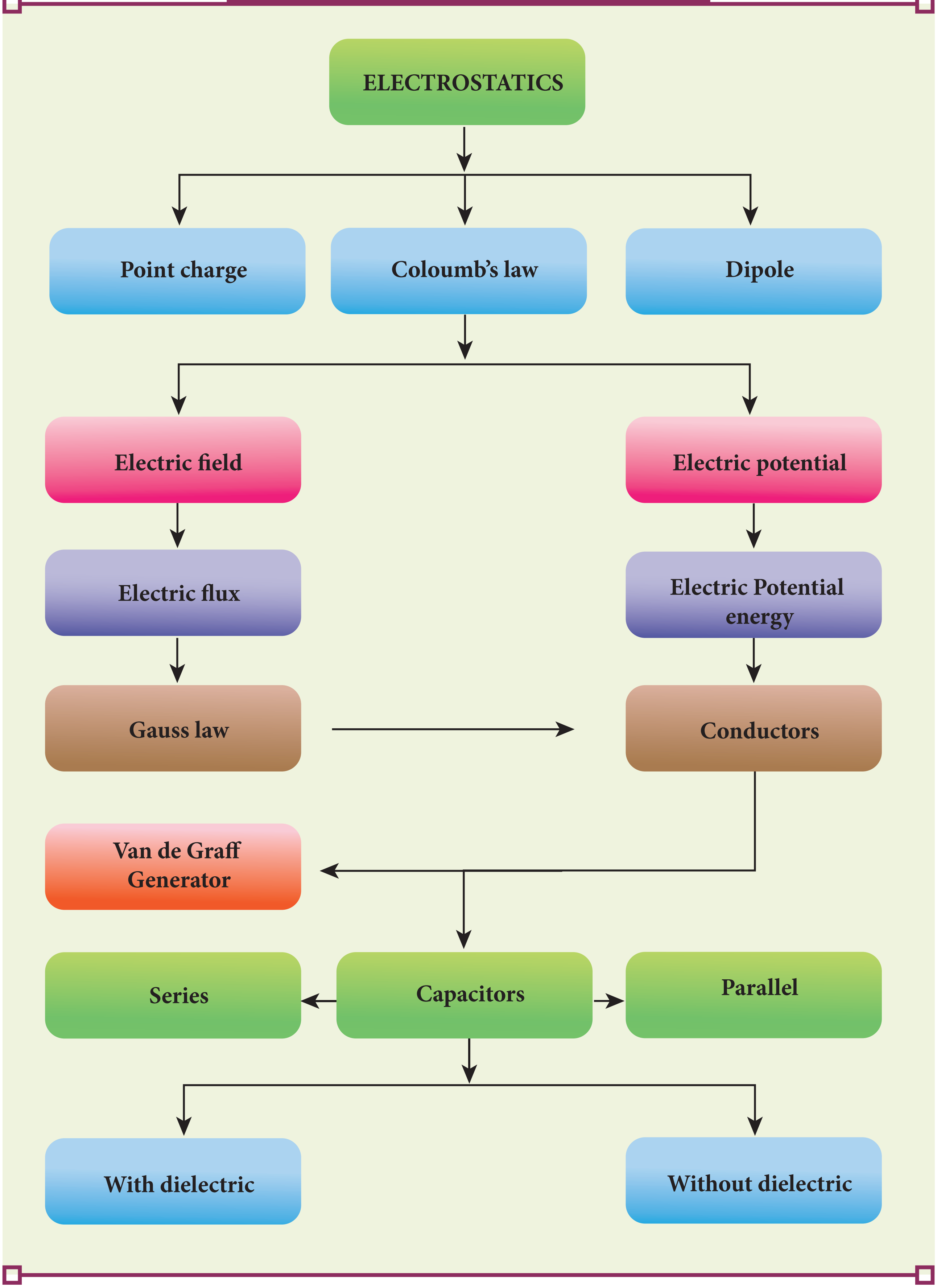
- Like charges repel and unlike charges attract
- The total charge in the universe is conserved
- Charge is quantized. Total charge in an object q = ne where n = 0,1,2,3… and e is electron charge.
- Coulomb's law in vector form: $\vec{F} = \frac{1}{4\pi\varepsilon_\circ}\frac{q_1 q_2}{r^2}\hat{r}$ ($\hat{r}$ is unit vector along joining q_1, q_2)
- For continuous charge distributions, integration methods can be used.
- Electrostatic force obeys the superposition principle.
- Electric field at a distance r from a point charge: $\vec{E} = \frac{1}{4\pi\varepsilon_\circ}\frac{q}{r^2}\hat{r}$
- Electric field lines starts at a positive charge and end at a negative charge or at infinity
- Electric field due to electric dipole at points on the axial line : $\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_\circ}\left(\frac{2\vec{p}}{r^3}\right)$
- Electric field due to electric dipole at points on the equatorial line: $\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_\circ}\left(\frac{\vec{p}}{r^3}\right)$
- Torque experienced by a dipole in a uniform electric field: $\vec{\tau} = \vec{p}\times\vec{E}$
- Electrostatic potential at a distance r from the point charge: $V = \frac{1}{4\pi\varepsilon_\circ}\frac{q}{r}$
- Electrostatic potential due to an electric dipole: $V = \frac{1}{4\pi\varepsilon_\circ}\frac{\vec{p}\cdot\hat{r}}{r^2}$
- The electrostatic potential is the same at all points on an equipotential surface.
- The relation between electric field and electrostatic potential:

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

- Electrostatic potential energy for system of charges is equal to the work done to arrange the charges in the given configuration.
- Electrostatic potential energy stored in a dipole system in a uniform electric field: $U = -\vec{p}\cdot\vec{E}$
- The total electric flux through a closed surface : $\Phi_E = \frac{Q}{\varepsilon_\circ}$ where Q is the net charge enclosed by the surface
- Electric field due to a charged infinite wire : $\vec{E} = \frac{1}{2\pi\varepsilon_\circ}\frac{\lambda}{r}\hat{r}$
- Electric field due to a charged infinite plane : $\vec{E} = \frac{\sigma}{\varepsilon_\circ}\hat{n}$ ($\hat{n}$ is normal to the plane)
- Electric field inside a charged spherical shell is zero. For points outside: $\vec{E} = \frac{Q}{4\pi\varepsilon_\circ r^2}\hat{r}$

- Electric field inside a conductor is zero. The electric field at the surface of the conductor is normal to the surface and has magnitude $E = \frac{\sigma}{\varepsilon_\circ}$.
- The surface of the conductor has the same potential, at all points on the surface.
- Conductor can be charged using the process of induction.
- A dielectric or insulator has no free electrons. When an electric field is applied, the dielectric is polarised.
- Capacitance is given by $C = \frac{Q}{V}$.
- Capacitance of a parallel plate capacitor: $C = \frac{\varepsilon_\circ A}{d}$
- Electrostatic energy stored in a capacitor: $U = \frac{1}{2}CV^2$
- The equivalent capacitance for parallel combination is equal to the sum of individual capacitance of capacitors.
- For a series combination: The inverse of equivalent capacitance is equal to sum of inverse of individual capacitances of capacitors.
- The distribution of charges in the conductors depends on the shape of conductor. For sharper edge, the surface charge density is greater. This principle is used in the lightning arrestor
- To create a large potential difference, a Van de Graaff generator is used.

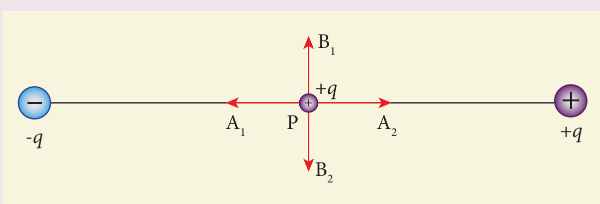
CONCEPT MAP


ELECTROSTATICS
Point charge
Coloumb's law
Dipole
Electric field
Electric potential
Electric flux
Electric Potential energy
Gauss law
Conductors
Van de Graff Generator
Series
Capacitors
Parallel
With dielectric
Without dielectric

EVALUATION

I Multiple choice questions

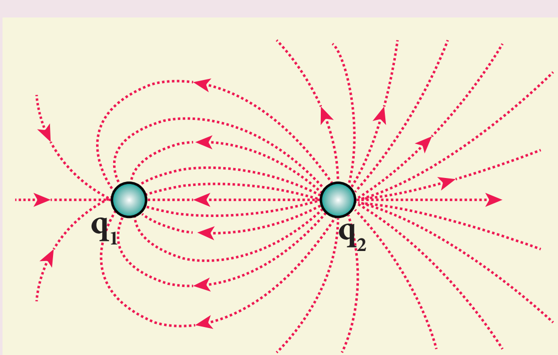
1. Two identical point charges of magnitude –q are fixed as shown in the figure below. A third charge +q is placed midway between the two charges at the point P. Suppose this charge +q is displaced a small distance from the point P in the directions indicated by the arrows, in which direction(s) will +q be stable with respect to the displacement?



(a) A_1 and A_2 (b) B_1 and B_2
(c) both directions (d) No stable

2. Which charge configuration produces a uniform electric field?
(a) point Charge
(b) infinite uniform line charge
(c) uniformly charged infinite plane
(d) uniformly charged spherical shell

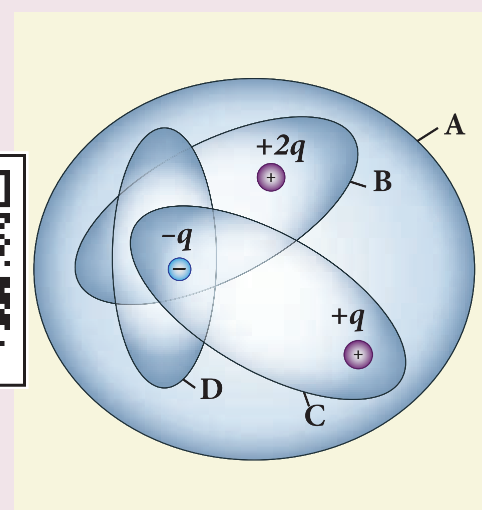
3. What is the ratio of the charges $\left|\frac{q_1}{q_2}\right|$ for the following electric field line pattern?



(a) $\frac{1}{5}$ (b) $\frac{25}{11}$
(c) 5 (d) $\frac{11}{25}$

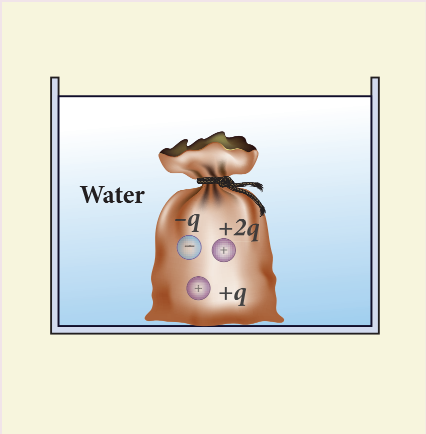
4. An electric dipole is placed at an alignment angle of 30° with an electric field of 2×10^5 N C^{-1}. It experiences a torque equal to 8 N m. The charge on the dipole if the dipole length is 1 cm is
(a) 4 mC (b) 8 mC
(c) 5 mC (d) 7 mC

5. Four Gaussian surfaces are given below with charges inside each Gaussian surface. Rank the electric flux through each Gaussian surface in increasing order.



(a) D < C < B < A
(b) A < B = C < D
(c) C < A = B < D
(d) D > C > B > A

6. The total electric flux for the following closed surface which is kept inside water



(a) $\frac{80q}{\varepsilon_o}$ (b) $\frac{q}{40\varepsilon_o}$

(c) $\frac{q}{80\varepsilon_o}$ (d) $\frac{q}{160\varepsilon_o}$

7. Two identical conducting balls having positive charges q_1 and q_2 are separated by a center to center distance r. If they are made to touch each other and then separated to the same distance, the force between them will be
(NSEP 04-05)

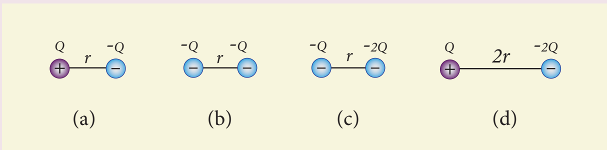
(a) less than before

(b) same as before

(c) more than before

(d) zero

8. Rank the electrostatic potential energies for the given system of charges in increasing order.



(a) 1 = 4 < 2 < 3 (b) 2 = 4 < 3 < 1

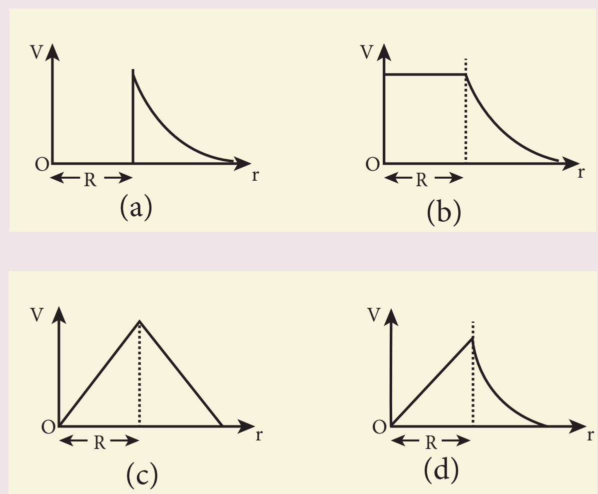
(c) 2 = 3 < 1 < 4 (d) 3 < 1 < 2 < 4

9. An electric field $\vec{E} = 10x\hat{i}$ exists in a certain region of space. Then the potential difference $V = V_o - V_A$, where V_o is the potential at the origin and V_A is the potential at x = 2 m is:

(a) 10 J (b) – 20 J

(c) +20 J (d) -10J

10. A thin conducting spherical shell of radius R has a charge Q which is uniformly distributed on its surface. The correct plot for electrostatic potential due to this spherical shell is



11. Two points A and B are maintained at a potential of 7 V and -4 V respectively. The work done in moving 50 electrons from A to B is

(a) 8.80×10^{-17} J

(b) -8.80×10^{-17} J

(c) 4.40×10^{-17} J

(d) 5.80×10^{-17} J

12. If voltage applied on a capacitor is increased from V to 2V, choose the correct conclusion.

(a) Q remains the same, C is doubled

(b) Q is doubled, C doubled

(c) C remains same, Q doubled

(d) Both Q and C remain same

13. A parallel plate capacitor stores a charge Q at a voltage V. Suppose the area of the parallel plate capacitor and the distance between the plates are each doubled then which is the quantity that will change?

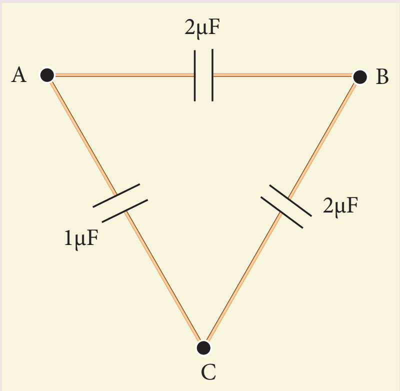
(a) Capacitance

(b) Charge

(c) Voltage

(d) Energy density

14. Three capacitors are connected in triangle as shown in the figure. The equivalent capacitance between the points A and C is



(a) 1μF (b) 2 μF

(c) 3 μF (d) $\frac{1}{4}\mu F$

15. Two metallic spheres of radii 1 cm and 3 cm are given charges of -1 x 10^{-2} C and 5 x 10^{-2} C respectively. If these are connected by a conducting wire, the final charge on the bigger sphere is (AIIPMT -2012)

(a) 3×10^{-2} C

(b) 4×10^{-2} C

(c) 1×10^{-2} C

(d) 2×10^{-2} C

Answers

1) b	**2)** c	**3)** d	**4)** b	**5)** a
6) b	**7)** c	**8)** a	**9)** b	**10)** b
11) a	**12)** c	**13)** d	**14)** b	**15)** a

II Short Answer Questions

1. What is meant by quantisation of charges?
2. Write down Coulomb's law in vector form and mention what each term represents.
3. What are the differences between Coulomb force and gravitational force?
4. Write a short note on superposition principle.
5. Define 'Electric field'.
6. What is mean by 'Electric field lines'?
7. The electric field lines never intersect. Justify.
8. Define 'Electric dipole'
9. What is the general definition of electric dipole moment?
10. Define 'electrostatic potential".
11. What is an equipotential surface?
12. What are the properties of an equipotential surface?
13. Give the relation between electric field and electric potential.
14. Define 'electrostatic potential energy'.
15. Define 'electric flux'
16. What is meant by electrostatic energy density?
17. Write a short note on 'electrostatic shielding'.
18. What is Polarisation?

19. What is dielectric strength?

20. Define 'capacitance'. Give its unit.

21. What is corona discharge?

III Long Answer questions

1. Discuss the basic properties of electric charges.
2. Explain in detail Coulomb's law and its various aspects.
3. Define 'Electric field' and discuss its various aspects.
4. How do we determine the electric field due to a continuous charge distribution? Explain.
5. Calculate the electric field due to a dipole on its axial line and equatorial plane.
6. Derive an expression for the torque experienced by a dipole due to a uniform electric field.
7. Derive an expression for electrostatic potential due to a point charge.
8. Derive an expression for electrostatic potential due to an electric dipole.
9. Obtain an expression for potential energy due to a collection of three point charges which are separated by finite distances.
10. Derive an expression for electrostatic potential energy of the dipole in a uniform electric field.
11. Obtain Gauss law from Coulomb's law.
12. Obtain the expression for electric field due to an infinitely long charged wire.
13. Obtain the expression for electric field due to an charged infinite plane sheet.
14. Obtain the expression for electric field due to an uniformly charged spherical shell.
15. Discuss the various properties of conductors in electrostatic equilibrium.
16. Explain the process of electrostatic induction.
17. Explain dielectrics in detail and how an electric field is induced inside a dielectric.
18. Obtain the expression for capacitance for a parallel plate capacitor.
19. Obtain the expression for energy stored in the parallel plate capacitor.
20. Explain in detail the effect of a dielectric placed in a parallel plate capacitor.
21. Derive the expression for resultant capacitance, when capacitors are connected in series and in parallel.
22. Explain in detail how charges are distributed in a conductor, and the principle behind the lightning conductor.
23. Explain in detail the construction and working of a Van de Graaff generator.

Exercises

1. When two objects are rubbed with each other, approximately a charge of 50 nC can be produced in each object. Calculate the number of electrons that must be transferred to produce this charge.

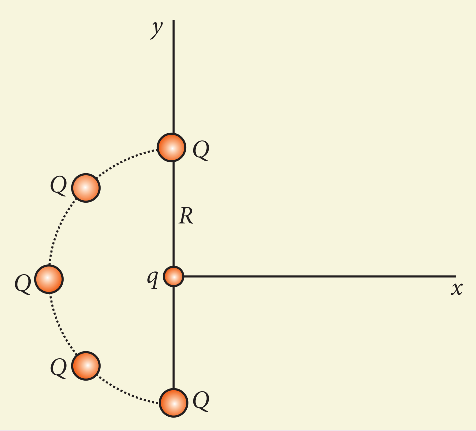
   Ans: 31.25×10^{10} electrons

2. The total number of electrons in the human body is typically in the order of 10^{28}. Suppose, due to some reason, you and your friend lost 1% of this number

of electrons. Calculate the electrostatic force between you and your friend separated at a distance of 1m. Compare this with your weight. Assume mass of each person is 60 kg and use point charge approximation.

Ans: $F_e = 9 \times 10^{61}$ N, W = 588 N

3. Five identical charges Q are placed equidistant on a semicircle as shown in the figure. Another point charge q is kept at the center of the circle of radius R. Calculate the electrostatic force experienced by the charge q.



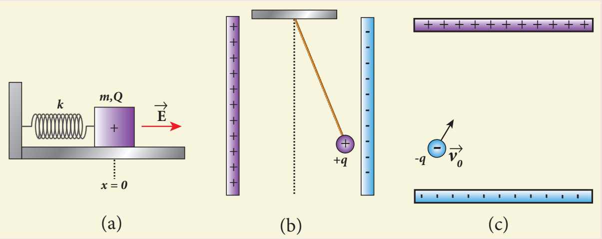
Ans: $\vec{F} = \frac{1}{4\pi\varepsilon_\circ} \frac{qQ}{R^2}\left(1+\sqrt{2}\right) N\,\hat{i}$

4. Suppose a charge +q on Earth's surface and another +q charge is placed on the surface of the Moon. (a) Calculate the value of q required to balance the gravitational attraction between Earth and Moon (b) Suppose the distance between the Moon and Earth is halved, would the charge q change?

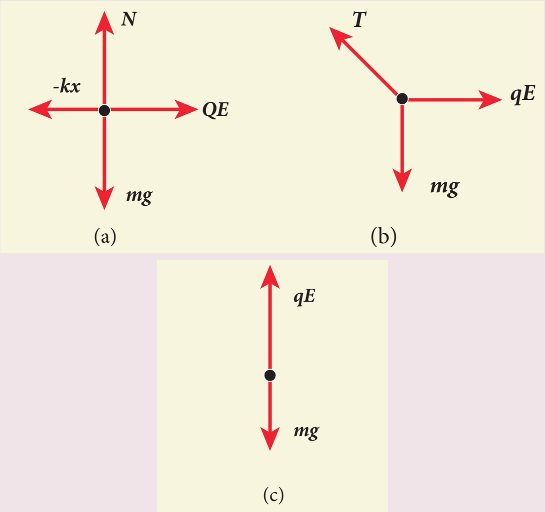
(Take $m_E = 5.9 \times 10^{24}$ kg, $m_M = 7.9 \times 10^{22}$ kg)

Ans: (a) q ≈ $+5.64 \times 10^{13}$ C, (b) no change

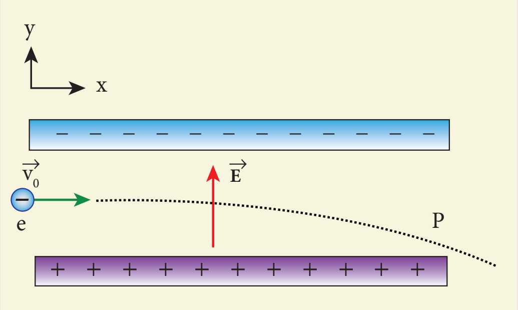
5. Draw the free body diagram for the following charges as shown in the figure (a), (b) and (c).



Ans:



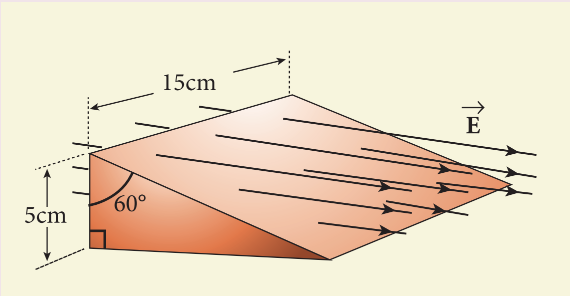
6. Consider an electron travelling with a speed v_o and entering into a uniform electric field $\vec{E}$ which is perpendicular to $\vec{v}_\circ$ as shown in the Figure. Ignoring gravity, obtain the electron's acceleration, velocity and position as functions of time.



Ans :

$\vec{a} = -\frac{eE}{m}\hat{j}, \vec{v} = v_\circ\hat{i} - \frac{eE}{m}t\hat{j}, \vec{r} = v_\circ t\,\hat{i} - \frac{1}{2}\frac{eE}{m}t^2\hat{j}$

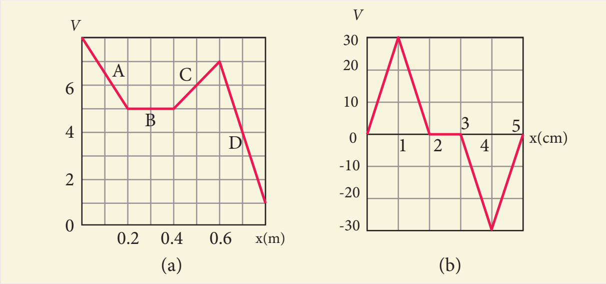
7. A closed triangular box is kept in an electric field of magnitude $E = 2 \times 10^3$ N C^{-1} as shown in the figure.



Calculate the electric flux through the (a) vertical rectangular surface (b) slanted surface and (c) entire surface.

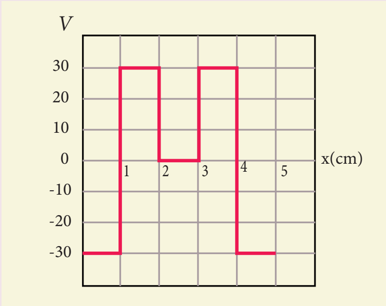
Ans: (a) 15 $Nm^2 C^{-1}$ (b) 15 $Nm^2 C^{-1}$ (c) zero

8. The electrostatic potential is given as a function of x in figure (a) and (b). Calculate the corresponding electric fields in regions A, B, C and D. Plot the electric field as a function of x for the figure (b).



Ans: (a) E_X = 15 Vm^{-1} (region A), E_X = -10 Vm^{-1} (region C)

E_X = 0 (region B), E_X = 30 Vm^{-1} (region D)



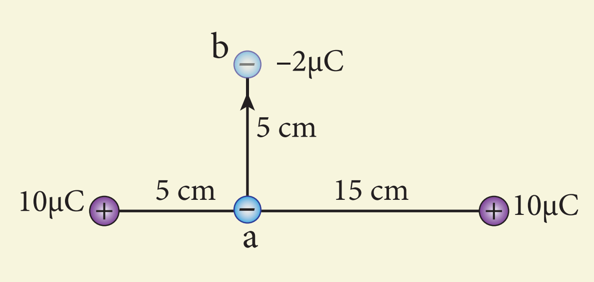
(b)

9. A spark plug in a bike or a car is used to ignite the air-fuel mixture in the engine. It consists of two electrodes separated by a gap of around 0.6 mm gap as shown in the figure.

To create the spark, an electric field of magnitude 3×10^6 Vm^{-1} is required. (a) What potential difference must be applied to produce the spark? (b) If the gap is increased, does the potential difference increase, decrease or remains the same? (c) find the potential difference if the gap is 1 mm.

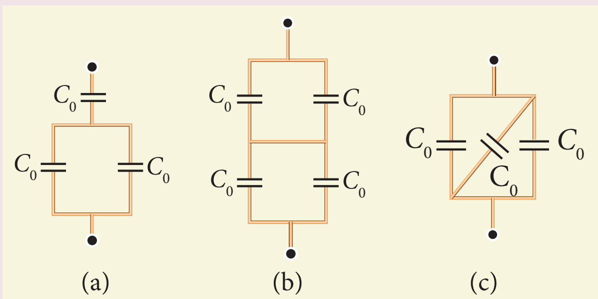
Ans: (a) 1800 V, (b) increases (c) 3000 V

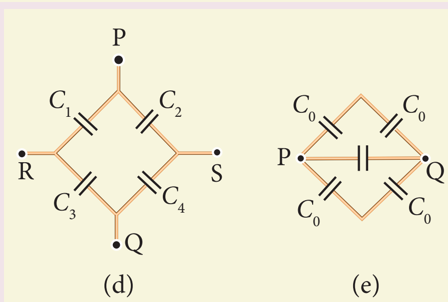
10. A point charge of +10 μC is placed at a distance of 20 cm from another identical point charge of +10 μC. A point charge of -2 μC is moved from point a to b as shown in the figure. Calculate the change in potential energy of the system? Interpret your result.



Ans: ΔU = -3.246 J, negative sign implies that to move the charge -2μC no external work is required. System spends its stored energy to move the charge from point a to point b.

11. Calculate the resultant capacitances for each of the following combinations of capacitors.





Ans: (a) $\frac{2}{3}C_{\circ}$ (b) $C_{\circ}$ (c) $3C_{\circ}$

(d) across PQ:

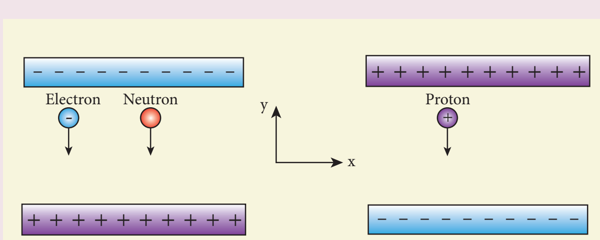
$$\frac{C_1C_2C_3 + C_2C_3C_4 + C_1C_2C_4 + C_1C_3C_4}{(C_1+C_2)(C_3+C_4)}$$

across RS:

$$\frac{C_1C_2C_3 + C_2C_3C_4 + C_1C_2C_4 + C_1C_3C_4}{(C_1+C_2)(C_3+C_4)}$$

(e) across PQ: 2 C_o

12. An electron and a proton are allowed to fall through the separation between the plates of a parallel plate capacitor of voltage 5 V and separation distance h = 1 mm as shown in the figure.



(a) Calculate the time of flight for both electron and proton (b) Suppose if a neutron is allowed to fall, what is the time of flight? (c) Among the three, which one will reach the bottom first? (Take $m_p = 1.6 \times 10^{-27}$ kg, $m_e = 9.1 \times 10^{-31}$ kg and $g = 10$ m s^{-2})

Ans:

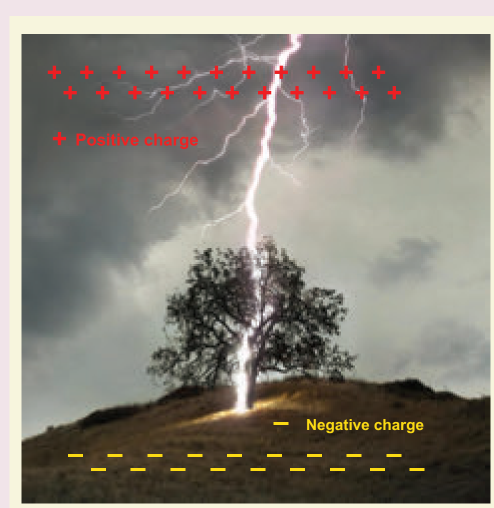
(a) $t_e = \sqrt{\frac{2hm_e}{eE}} \approx 1.5ns$ (ignoring the gravity),

$t_p = \sqrt{\frac{2hm_e}{eE}} \approx 63ns$ (ignoring the gravity)

(b) $t_n = \sqrt{\frac{2h}{g}} \approx 14.1ms$

(c) electron will reach first

13. During a thunder storm, the movement of water molecules within the clouds creates friction, partially causing the bottom part of the clouds to become negatively charged. This implies that the bottom of the cloud and the ground act as a parallel plate capacitor. If the electric field between the cloud and ground exceeds the dielectric breakdown of the air (3×10^6 Vm^{-1}), lightning will occur.

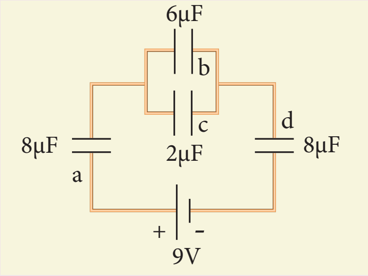


(a) If the bottom part of the cloud is 1000 m above the ground, determine the electric potential difference that exists between the cloud and ground.

(b) In a typical lightning phenomenon, around 25C of electrons are transferred from cloud to ground. How much electrostatic potential energy is transferred to the ground?

Ans: (a) $V = 3 \times 10^9$ V, (b) $U = 75 \times 10^9$ J

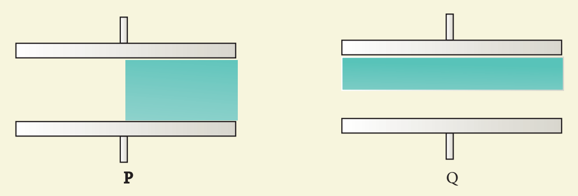
14. For the given capacitor configuration
(a) Find the charges on each capacitor
(b) potential difference across them
(c) energy stored in each capacitor



Ans:

$Q_a = 24\ \mu C$, $Q_b = 18\ \mu C$,
$Q_c = 6\ \mu C$, $Q_d = 24\ \mu C$
$V_a = 3V$, $V_b = 3V$,
$V_c = 3V$, $V_d = 3V$,
$U_a = 36\ \mu J$, $U_b = 27\ \mu J$,
$U_c = 9\ \mu J$, $U_d = 36\ \mu J$

15. Capacitors P and Q have identical cross sectional areas A and separation d. The space between the capacitors is filled with a dielectric of dielectric constant ε_r as shown in the figure. Calculate the capacitance of capacitors P and Q.



$$\text{Ans}: C_P = \frac{\varepsilon_\circ A}{2d}(1+\varepsilon_r),\ C_Q = \frac{2\varepsilon_\circ A}{d}\left(\frac{\varepsilon_r}{1+\varepsilon_r}\right)$$

BOOKS FOR REFERENCE

1. Douglas C.Giancoli, , "*Physics for Scientist & Engineers with Modern Physics*", Pearson Prentice Hall, Fourth edition
2. James Walker, "*Physics*", Pearson- Addison Wesley Publishers, Fourth Edition
3. Purcell, Morin, "*Electricity and Magnetism*", "Cambridge University Press, Third Edition.
4. Serway and Jewett, "*Physics for Scientist and Engineers with Modern Physics*", Brook/Coole Publishers, Eighth Edition
5. Tipler, Mosca, "*Physics for scientist and Engineers with Modern Physics*", Freeman and Company, Sixth Edition
6. Tarasov and Tarasova, "*Questions and problems in School Physics*", Mir Publishers
7. H.C.Verma, "*Concepts of Physics: Vol 2*, Bharthi Bhawan Publishers
8. Eric Roger, *Physics for the Inquiring Mind*, Princeton University Press

ICT CORNER

Electrostatics

In this activity you will be able to learn about capacitor and the factors affecting capacitance.

Topic: Capacitor lab

STEPS:

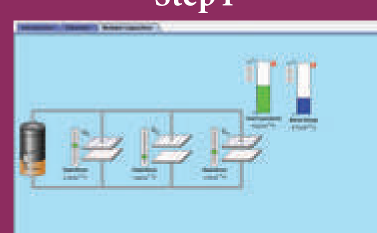
- Open the browser and type “phet.colorado.edu/en/simulation/legacy/capacitor-lab” in the address bar. Go to the tab ‘Dielectric’.
- Change the plate area, distance between the plate and dielectric. Identify what you would maximize or minimize to make a capacitor with the greatest capacitance.
- Explore the relationships between charge, voltage, and stored energy for a capacitor. Design a capacitor system to store the greatest energy.
- Charge the capacitor with 1.0 v using the battery. Disconnect the battery. Now insert a dielectric between the plates. Discus how electric field changes in between the plates when dielectric is introduced.
- What is the effect of introducing a dielectric between plates? (Change dielectric materials)

Step1

Step2

Step3

Step4



Connect capacitors parallel and series combination and find the effective capacitance.

B263_12_PHYSICS_EM

UNIT
2

CURRENT ELECTRICITY

We will make electricity so cheap that only the rich will burn candles
– Thomas A. Edison

Learning Objectives

In this unit, the student is exposed to

- Flow of charges in a metallic conductor
- Ohm's law, electrical resistance, V-I characteristics
- Carbon resistors and combination of resistors
- Kirchhoff's laws - Wheatstone's bridge and its applications
- Electric power and Electric energy
- Heating effect - Joule's law – Experimental verification and applications
- Thermoelectric effects – Seebeck effect – Peltier effect – Thomson effect

K4HZ3A

INTRODUCTION

In unit 1, we studied the properties of charges when it is at rest. In reality, the charges are always moving within the materials. For example, the electrons in a copper wire are never at rest and are continuously in random motion. Therefore it is important to analyse the behaviour of charges when it is at motion. The motion of charges is called 'electric current'. Current electricity is the study of flow of electric charges. It owes its origin to Alessandro Volta (1745-1827), who invented the electric battery which produced the first steady flow of electric current. Modern world depends heavily on the use of electricity. It is used to operate machines, communication systems, electronic devices, home appliances etc., In this unit, we will study about the electric current, resistance and related phenomenon in materials.

2.1 ELECTRIC CURRENT

Matter is made up of atoms. Each atom consists of a positively charged nucleus with negatively charged electrons moving around the nucleus. Atoms in metals have one or more electrons which are loosely bound to the nucleus. These electrons are called free electrons and can be easily detached from the atoms. The substances which have an abundance of these free electrons are called conductors. These free electrons move at random throughout the conductor at a given temperature. In general due to this random motion, there is no net transfer of charges from one end of the conductor to other end and hence no current. When a potential difference is applied by the battery across the ends of the conductor, the free electrons drift towards the positive terminal of the battery, producing a net electric current. This is easily understandable from the analogy given in the Figure 2.1.

In the XI Volume 2, unit 6, we studied, that the mass move from higher gravitational potential to lower gravitational potential. Likewise, positive charge flows from higher electric potential to lower electric potential and negative charge flows from lower electric potential to higher electric potential. So battery or electric cell simply creates potential difference across the conductor.

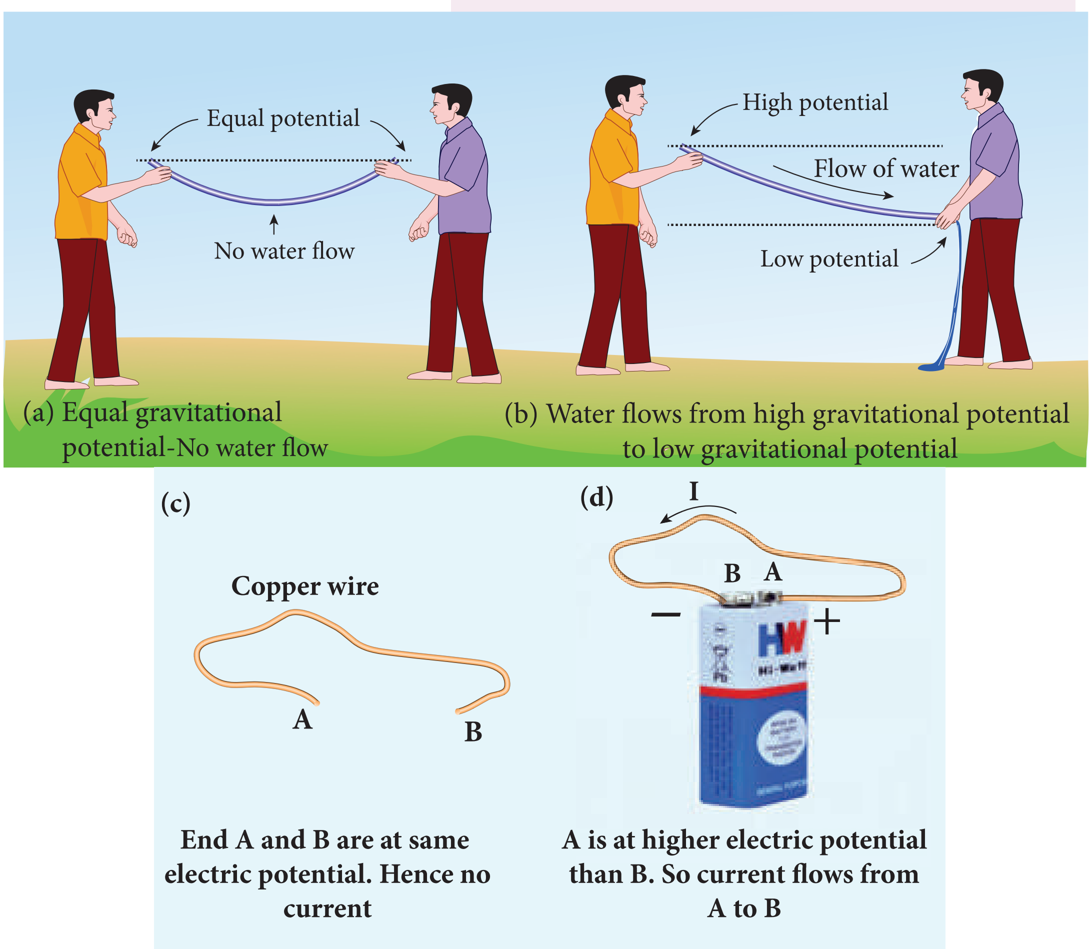


Figure 2.1 Water current and Electric current

The electric current in a conductor is defined as the rate of flow of charges through a given cross-sectional area A. It is shown in the Figure 2.2.

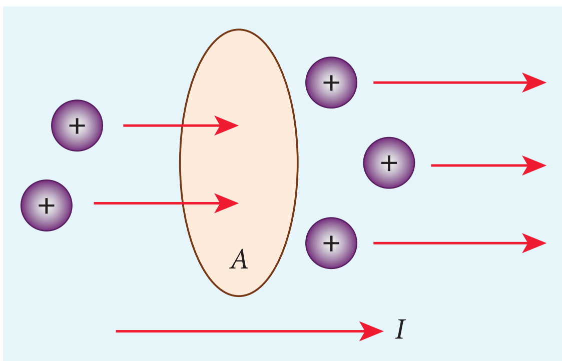


Figure 2.2 Charges flow across the area A

If a net charge Q passes through any cross section of a conductor in time t, then the current is defined as $I = \frac{Q}{t}$**.** But charge flow is not always constant. Hence current can more generally be defined as

$$I_{avg} = \frac{\Delta Q}{\Delta t} \quad (2.1)$$

Where ΔQ is the amount of charge that passes through the conductor at any cross section during the time interval Δt. If the rate at which charge flows changes in time, the current also changes. The instantaneous current I is defined as the limit of the average current, as $\Delta t \to 0$

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \quad (2.2)$$

The SI unit of current is the **ampere** (A)

$$1A = \frac{1C}{1s}$$

That is, 1A of current is equivalent to 1 Coulomb of charge passing through a perpendicular cross section in 1second. The electric current is a scalar quantity.

EXAMPLE 2.1

Compute the current in the wire if a charge of 120 C is flowing through a copper wire in 1 minute.

Solution

The current (rate of flow of charge) in the wire is

$$I = \frac{Q}{t} = \frac{120}{60} = 2A$$

2.1.1 Conventional Current

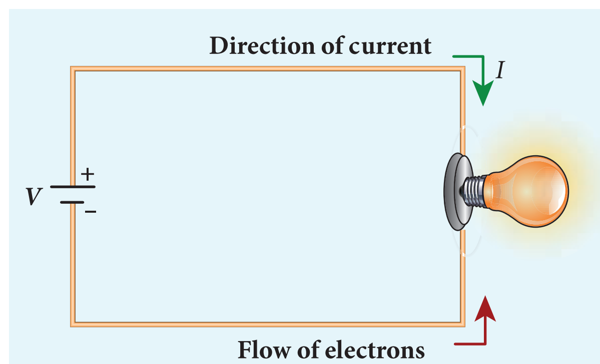


Figure 2.3 Direction of conventional current and electron flow

In an electric circuit, arrow heads are used to indicate the direction of flow of current. By convention, this flow in the circuit should be from the positive terminal of the battery to the negative terminal. This current is called the conventional current or simply current and is in the direction in which a positive test charge would move. In typical circuits the charges that flow are actually electrons, from the negative terminal of the battery to the positive. As a result, the flow of electrons and the direction of conventional current points in opposite direction as shown in Figure 2.3. Mathematically, a transfer of positive charge

X14TS6

is the same as a transfer of negative charge in the opposite direction.

Electric current is not only produced by batteries. In nature, lightning bolt produces enormous electric current in a short time. During lightning, very high potential difference is created between the clouds and ground so charges flow between the clouds and ground.

2.1.2 Drift velocity

In a conductor the charge carriers are free electrons. These electrons move freely through the conductor and collide repeatedly with the positive ions. If there is no electric field, the electrons move in random directions, so the directions of their velocities are also completely random direction. On an average, the number of electrons travelling in any direction will be equal to the number of electrons travelling in the opposite direction. As a result, there is no net flow of electrons in any direction and hence there will not be any current.

Suppose a potential difference is set across the conductor by connecting a battery, an electric field $\vec{E}$ is created in the conductor. This electric field exerts a force on the electrons, producing a current. The electric field accelerates the electrons, while ions scatter the electrons and change the direction of motion. Thus, we have zigzag paths of electrons. In addition to the zigzag motion due to the collisions, the electrons move slowly along the conductor in a direction opposite to that of $\vec{E}$ as shown in the Figure 2.4.

Ions

Any material is made up of neutral atoms with equal number of electrons and protons. If the outermost electrons leave the atoms, they become free electrons and are responsible for electric current. The atoms after losing their outer most electrons will have more positive charges and hence are called positive ions. These ions will not move freely within the material like the free electrons. Hence the positive ions will not give rise to current.

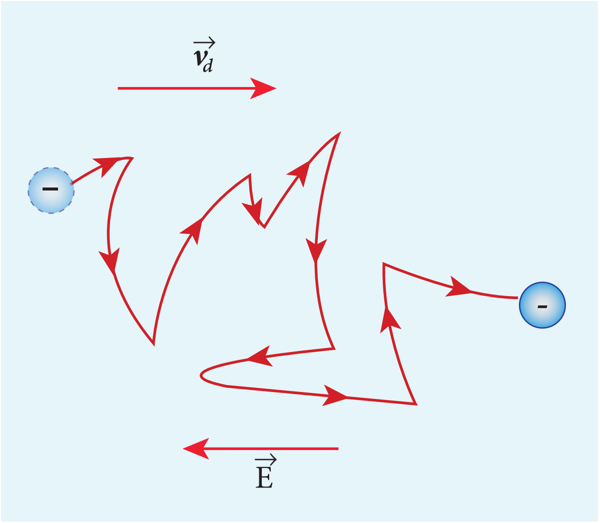


Figure 2.4 Electric current

This velocity is called drift velocity $\vec{v}_d$. The drift velocity is the average velocity acquired by the electrons inside the conductor when it is subjected to an electric field. The average time between successive collisions is called the mean free time denoted by τ. The acceleration $\vec{a}$ experienced by the electron in an electric field $\vec{E}$ is given by

$$\vec{a} = \frac{-e\vec{E}}{m} \qquad \left(\text{since } \vec{F} = -e\vec{E}\right) \qquad (2.3)$$

The drift velocity $\vec{v}_d$ is given by

$$\vec{v}_d = \vec{a}\,\tau$$

$$\vec{v}_d = -\frac{e\tau}{m}\vec{E} \qquad (2.4)$$

$$\vec{v}_d = -\mu\vec{E} \qquad (2.5)$$

Here $\mu = \frac{e\tau}{m}$ is the mobility of the electron and it is defined as the magnitude of the drift velocity per unit electric field.

$$\mu = \frac{|\vec{v}_d|}{|\vec{E}|} \qquad (2.6)$$

The SI unit of mobility is $\frac{m^2}{V\,s}$.

Note The typical drift velocity of electrons in the wire is 10^{-4} m s^{-1}. If an electron drifts with this speed, then the electrons leaving the battery will take hours to reach the light bulb. Then how electric bulbs glow as soon as we switch on the battery? When battery is switched on, the electrons begin to move away from the negative terminal of the battery and this electron exerts force on the nearby electrons. This process creates a propagating influence (electric field) that travels through the wire at the speed of light. In other words, the energy is transported from the battery to light bulb at the speed of light through propagating influence (electric field). Due to this reason, the light bulb glows as soon as the battery is switched on.

EXAMPLE 2.2

If an electric field of magnitude 570 N C^{-1}, is applied in the copper wire, find the acceleration experienced by the electron.

Solution:

E = 570 N C^{-1}, e = 1.6 × 10^{-19} C,
m = 9.11 × 10^{-31} kg and a = ?

F = ma = eE

$$a = \frac{eE}{m} = \frac{570 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}}$$

$$= \frac{912 \times 10^{-19} \times 10^{31}}{9.11}$$

= 1.001 × 10^{14} m s^{-2}

Misconception

(i) There is a common misconception that the battery is the source of electrons. It is not true. When a battery is connected across the given wire, the electrons in the closed circuit resulting the current. Battery sets the potential difference (electrical energy) due to which these electrons in the conducting wire flow in a particular direction. The resulting electrical energy is used by electric bulb, electric fan etc. Similarly the electricity board is supplying the electrical energy to our home.

(ii) We often use the phrases like 'charging the battery in my mobile' and 'my mobile phone battery has no charge' etc. These sentences are not correct.

When we say 'battery has no charge', it means, that the battery has lost ability to provide energy or provide potential difference to the electrons in the circuit. When we say 'mobile is charging', it implies that the battery is receiving energy from AC power supply and not electrons.

2.1.3 Microscopic model of current

Consider a conductor with area of cross section A and an electric field $\vec{E}$ applied from right to left. Suppose there are n electrons per unit volume in the conductor and assume that all the electrons move with the same drift velocity $\vec{v}_d$ as shown in Figure 2.5.

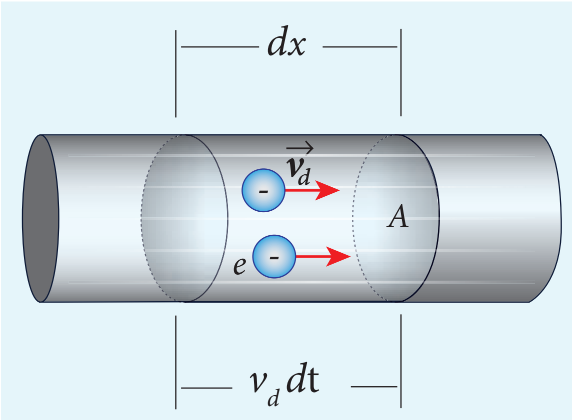


Figure 2.5 Microscopic model of current

The drift velocity of the electrons = v_d

The electrons move through a distance dx within a small interval of dt

$$v_d = \frac{dx}{dt}\ ; \quad dx = v_d\,dt \qquad (2.7)$$

Since A is the area of cross section of the conductor, the electrons available in the volume of length dx is

= volume × number per unit volume

$$= A\,dx \times n \qquad (2.8)$$

Substituting for dx from equation (2.7) in (2.8)

$$= (A\,v_d\,dt)\,n$$

Total charge in volume element dQ = (charge) × (number of electrons in the volume element)

$$dQ = (e)(Av_d dt)n$$

Hence the current $I = \frac{dQ}{dt} = \frac{neAv_d dt}{dt}$

$$I = ne\,Av_d. \qquad (2.9)$$

Current density (J)

The current density (J) is defined as the current per unit area of cross section of the conductor.

$$J = \frac{I}{A}$$

The S.I unit of current density is $\frac{A}{m^2}$ (or) A m^{-2}

$$J = \frac{neAv_d}{A} \text{ (from equation 2.9)}$$

$$J = nev_d \qquad (2.10)$$

The above expression is valid only when the direction of the current is perpendicular to the area A. In general, the current density is a vector quantity and it is given by

$$\vec{J} = ne\vec{v}_d$$

Substituting $\vec{v}_d$ from equation (2.4)

$$\vec{J} = -\frac{n \cdot e^2 \tau}{m}\vec{E} \qquad (2.11)$$

$$\vec{J} = -\sigma\vec{E}$$

But conventionally, we take the direction of (conventional) current density as the direction of electric field. So the above equation becomes

$$\vec{J} = \sigma\vec{E} \qquad (2.12)$$

where $\sigma = \frac{ne^2\tau}{m}$ is called conductivity. The equation 2.12 is called microscopic form of ohm's law.

The inverse of conductivity is called resistivity (ρ) [Refer section 2.2.1].

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau} \quad (2.13)$$

EXAMPLE 2.3

A copper wire of cross-sectional area 0.5 mm^2 carries a current of 0.2 A. If the free electron density of copper is 8.4 × 10^{28} m^{-3} then compute the drift velocity of free electrons.

Solution

The relation between drift velocity of electrons and current in a wire of cross-sectional area A is

$$v_d = \frac{I}{neA} = \frac{0.2}{8.4\times10^{28}\times1.6\times10^{-19}\times0.5\times10^{-6}}$$

v_d = 0.03 x 10^{-3} m s^{-1}

Note

Why current density is a vector but current is a scalar?

In general, the current I is defined as the scalar product of the current density and area vector in which the charges cross.

$$I = \vec{J}\cdot\vec{A}$$

The current I can be positive or negative depending on the choice of the unit vector normal to the surface area A.

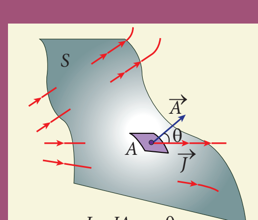


Figure 2.6 Current is a scalar

EXAMPLE 2.4

Determine the number of electrons flowing per second through a conductor, when a current of 32 A flows through it.

Solution

I = 32 A , t = 1 s

Charge of an electron, e = 1.6 × 10^{-19} C

The number of electrons flowing per second, n =?

$$I = \frac{q}{t} = \frac{ne}{t}$$

$$n = \frac{It}{e}$$

$$n = \frac{32\times1}{1.6\times10^{-19}C}$$

n = 20 × 10^{19} = 2 × 10^{20} electrons

2.2 OHM'S LAW

The ohm's law can be derived from the equation J = σE. Consider a segment of wire of length l and cross sectional area A as shown in Figure 2.7.

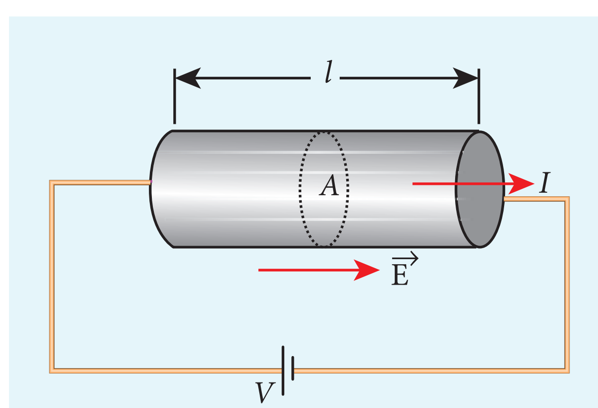


Figure 2.7 Current through the conductor

When a potential difference V is applied across the wire, a net electric field is created in the wire which constitutes the current. For simplicity, we assume that the electric field is uniform in the entire length of the wire, the potential difference (voltage V) can be written as

$$V = El$$

As we know, the magnitude of current density

$$J = \sigma E = \sigma \frac{V}{l} \qquad (2.14)$$

But $J = \frac{I}{A}$, so we write the equation (2.14) as

$$\frac{I}{A} = \sigma \frac{V}{l}.$$

By rearranging the above equation, we get

$$V = I\left(\frac{l}{\sigma A}\right) \qquad (2.15)$$

The quantity $\frac{l}{\sigma A}$ is called resistance of the conductor and it is denoted as R. Note that the resistance is directly proportional to the length of the conductor and inversely proportional to area of cross section.

Therefore, the macroscopic form of ohm's law can be stated as

$$V = IR \qquad (2.16)$$

From the above equation, **the resistance is the ratio of potential difference across the given conductor to the current passing through the conductor.**

$$R = \frac{V}{I} \qquad (2.17)$$

The SI unit of resistance is ohm (Ω). From the equation (2.16), we infer that the graph between current versus voltage is straight line with a slope equal to the inverse of resistance R of the conductor. It is shown in the Figure 2.8 (a).

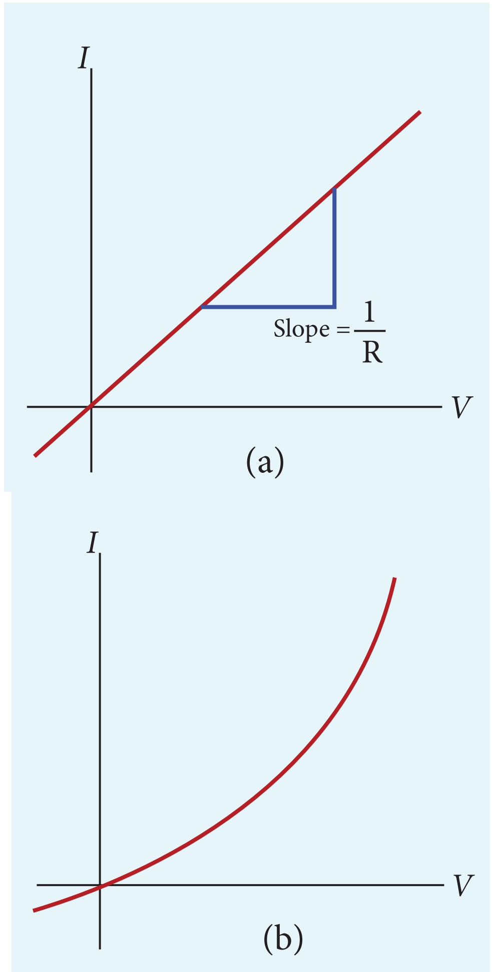


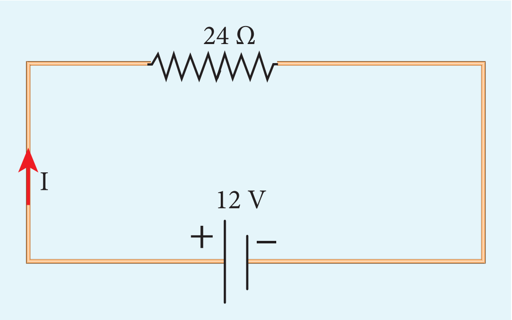
Figure 2.8 Current against voltage for (a) a conductor which obey Ohm's law and (b) for a non-ohmic device (Diode given in XII physics, unit 9 is an example of a non-ohmic device)

Materials for which the current against voltage graph is a straight line through the origin, are said to obey Ohm's law and their behaviour is said to be ohmic as shown in Figure 2.8(a). Materials or devices that do not follow Ohm's law are said to be non-ohmic. These materials have more complex relationships between voltage and current. A plot of I against V for a non-ohmic material is non-linear and they do not have a constant resistance (Figure 2.8(b)).

EXAMPLE 2.5

A potential difference across 24 Ω resistor is 12 V. What is the current through the resistor?

Solution



V = 12 V and R = 24 Ω

Current, I = ?

From Ohm's law, $I = \frac{V}{R} = \frac{12}{24} = 0.5\,A$

2.2.1 Resistivity

In the previous section, we have seen that the resistance R of any conductor is given by

$$R = \frac{l}{\sigma A} \qquad (2.18)$$

where σ is called the conductivity of the material and it depends only on the type of the material used and not on its dimension.

The resistivity of a material is equal to the reciprocal of its conductivity.

$$\rho = \frac{1}{\sigma} \qquad (2.19)$$

Now we can rewrite equation (2.18) using equation (2.19)

$$R = \rho \frac{l}{A} \qquad (2.20)$$

The resistance of a material is directly proportional to the length of the conductor and inversely proportional to the area of cross section of the conductor. The proportionality constant ρ is called the resistivity of the material.

If l = 1 m and A = 1 m^2, then the resistance R = ρ. In other words, the **electrical resistivity of a material is defined as the resistance offered to current flow by a conductor of unit length having unit area of cross section.** The SI unit of ρ is ohm-metre (Ω m). Based on the resistivity, materials are classified as conductors, insulators and semi-conductors. The conductors have lowest resistivity, insulators have highest resistivity and semiconductors have resistivity greater than conductors but less than insulators. The typical resistivity values of some conductors, insulators and semiconductors are given in the Table 2.1

Table 2.1 Resistivity for various materials

Material	Resistivity, ρ (Ω m) at 20^{0}C
Insulators	
Pure Water	2.5×10^5
Glass	10^{10} - 10^{14}
Hard Rubber	10^{13} - 10^{16}
NaCl	- 10^{14}
Fused Quartz	- 10^{16}
Semiconductors	
Germanium	0.46
Silicon	640
Conductors	
Silver	1.6×10^{-8}
Copper	17×10^{-8}
Aluminium	2.7×10^{-8}
Tungsten	5.6×10^{-8}
Iron	10×10^{-8}

EXAMPLE 2.6

The resistance of a wire is 20 Ω. What will be new resistance, if it is stretched uniformly 8 times its original length?

Solution

$R_1 = 20\ \Omega$, $R_2 = ?$

Let the original length (l_1) be l.

The new length, $l_2 = 8l_1$ (i.,e) $l_2 = 8l$

The original resistance, $R_1 = \rho \frac{l_1}{A_1}$

The new resistance $R_2 = \rho \frac{l_2}{A_2} = \frac{\rho(8l)}{A_2}$

Though the wire is stretched, its volume is unchanged.

Initial volume = Final volume

$A_1 l_1 = A_2 l_2$, $\quad A_1 l = A_2 8l$

$$\frac{A_1}{A_2} = \frac{8l}{l} = 8$$

By dividing equation R_2 by equation R_1, we get

$$\frac{R_2}{R_1} = \frac{\rho(8l)}{A_2} \times \frac{A_1}{\rho l}$$

$$\frac{R_2}{R_1} = \frac{A_1}{A_2} \times 8$$

Substituting the value of $\frac{A_1}{A_2}$, we get

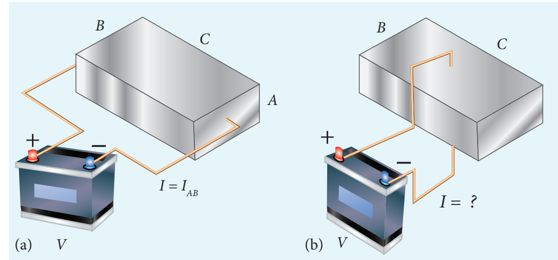
$$\frac{R_2}{R_1} = 8 \times 8 = 64$$

$R_2 = 64 \times 20 = 1280\ \Omega$

Hence, stretching the length of the wire has increased its resistance.

EXAMPLE 2.7

Consider a rectangular block of metal of height A, width B and length C as shown in the figure.



If a potential difference of V is applied between the two faces A and B of the block (figure (a)), the current I_{AB} is observed. Find the current that flows if the same potential difference V is applied between the two faces B and C of the block (figure (b)). Give your answers in terms of I_{AB}.

Solution

In the first case, the resistance of the block

$$R_{AB} = \rho \frac{\text{length}}{\text{Area}} = \rho \frac{C}{AB}$$

The current $I_{AB} = \frac{V}{R_{AB}} = \frac{V}{\rho} \cdot \frac{AB}{C}$ $\quad$ (1)

In the second case, the resistance of the block $R_{BC} = \rho \frac{A}{BC}$

The current $I_{BC} = \frac{V}{R_{BC}} = \frac{V}{\rho} \cdot \frac{BC}{A}$ $\quad$ (2)

To express I_{BC} interms of I_{AB}, we multiply and divide equation (2) by AC, we get

$$I_{BC} = \frac{V}{\rho} \cdot \frac{BC}{A} \frac{AC}{AC} = \left(\frac{V}{\rho} \cdot \frac{AB}{C}\right) \frac{C^2}{A^2} = \frac{C^2}{A^2} \cdot I_{AB}$$

Since C > A, the current $I_{BC} > I_{AB}$

DO YOU KNOW?

The human body contains a large amount of water which has low resistance of around 200 Ω and the dry skin has high resistance of around 500 k Ω. But when the skin is wet, the resistance is reduced to around1000 Ω. This is the reason, repairing the electrical connection with the wet skin is always dangerous.

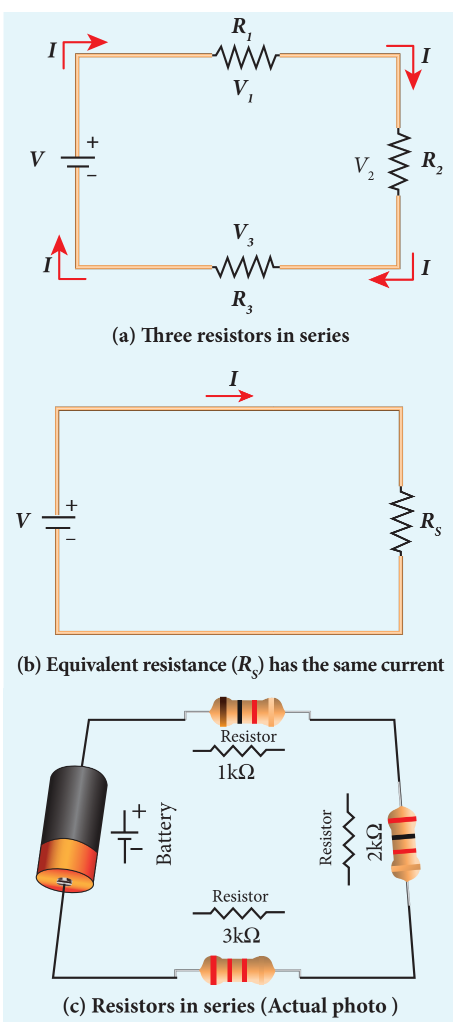
2.2.2 Resistors in series and parallel

An electric circuit may contain a number of resistors which can be connected in different ways. For each type of circuit, we can calculate the equivalent resistance produced by a group of individual resistors.

Resistors in series

When two or more resistors are connected end to end, they are said to be in series. The resistors could be simple resistors or bulbs or heating elements or other devices. Figure 2.9 (a) shows three resistors R_1, R_2 and R_3 connected in series.

The amount of charge passing through resistor R_1 must also pass through resistors R_2 and R_3 since the charges cannot accumulate anywhere in the circuit. Due to this reason, the current I passing through all the three resistors is the same. According to Ohm's law, if same current pass through different resistors of different values, then the potential difference across each resistor must be different. Let V_1, V_2 and V_3 be the potential difference (voltage) across each of the resistors R_1, R_2 and R_3 respectively, then we can write $V_1 = IR_1$, $V_2 = IR_2$ and $V_3 = IR_3$. But the total voltage V is equal to the sum of voltages across each resistor.



(a) Three resistors in series

(b) Equivalent resistance (R_S) has the same current

(c) Resistors in series (Actual photo)

Figure 2.9 Resistors in series

$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 \quad (2.21)$$

$$V = I\,(R_1 + R_2 + R_3)$$

$$V = I.R_S \quad (2.22)$$

where R_S is the equivalent resistance,

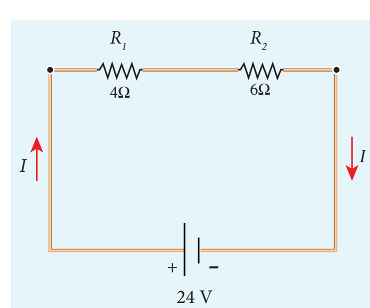
$$R_S = R_1 + R_2 + R_3 \quad (2.23)$$

When several resistances are connected in series, the total or equivalent resistance is the sum of the individual resistances as shown in the Figure 2.9 (b).

Note: The value of equivalent resistance in series connection will be greater than each individual resistance.

EXAMPLE 2.8

Calculate the equivalent resistance for the circuit which is connected to 24 V battery and also find the potential difference across 4 Ω and 6 Ω resistors in the circuit.



UNIT-2(XII-Physics_Vol-1).indd 92 04-03-2019 11:01:18

Solution

Since the resistors are connected in series, the effective resistance in the circuit

$= 4\ \Omega + 6\ \Omega = 10\ \Omega$

The Current I in the circuit $= \frac{V}{R_{eq}} = \frac{24}{10} = 2.4\,A$

Voltage across 4Ω resistor

$V_1 = IR_1 = 2.4\,A \times 4\Omega = 9.6\,V$

Voltage across $6\ \Omega$ resistor

$V_2 = IR_1 = 2.4\,A \times 6\Omega = 14.4\,V$

Resistors in parallel

Resistors are in parallel when they are connected across the same potential difference as shown in Figure 2.10 (a).

In this case, the total current I that leaves the battery is split into three separate paths. Let I_1, I_2 and I_3 be the current through the resistors R_1, R_2 and R_3 respectively. Due to the conservation of charge, total current in the circuit I is equal to sum of the currents through each of the three resistors.

$$I = I_1 + I_2 + I_3 \qquad (2.24)$$

Since the voltage across each resistor is the same, applying Ohm's law to each resistor, we have

$$I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3} \qquad (2.25)$$

Substituting these values in equation (2.24),we get

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$I = \frac{V}{R_P}$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad (2.26)$$

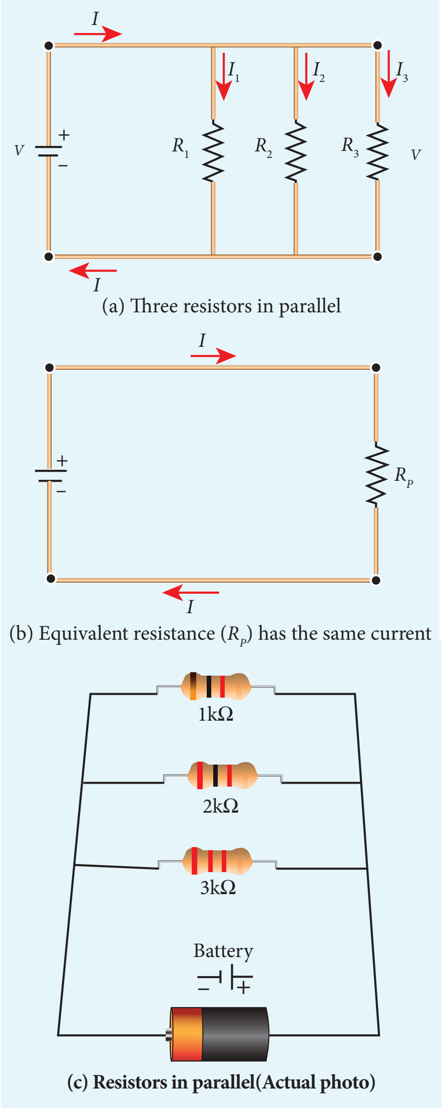


Figure 2.10 Resistors in parallel

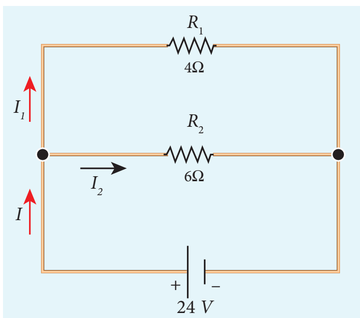
Here R_P is the equivalent resistance of the parallel combination of the resistors. Thus, when a number of resistors are connected in parallel, the sum of the reciprocal of the values of resistance of the individual resistor is equal to the reciprocal of the effective resistance of the combination as shown in the Figure 2.10 (b)

Note: The value of equivalent resistance in parallel connection will be lesser than each individual resistance.

House hold appliances are always connected in parallel so that even if one is switched off, the other devices could function properly.

EXAMPLE 2.9

Calculate the equivalent resistance in the following circuit and also find the current I, I_1 and I_2 in the given circuit.



Solution

Since the resistances are connected in parallel, therefore, the equivalent resistance in the circuit is

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4} + \frac{1}{6}$$

$$\frac{1}{R_P} = \frac{5}{12}\Omega \quad \text{or } R_P = \frac{12}{5}\Omega$$

The resistors are connected in parallel, the potential (voltage) across each resistor is the same.

$$I_1 = \frac{V}{R_1} = \frac{24V}{6\Omega} = 6\,A$$

$$I_2 = \frac{V}{R_2} = \frac{24}{6} = 4\,A$$

The current I is the total of the currents in the two branches. Then,

$$I = I_1 + I_2 = 6\text{ A} + 4\text{ A} = 10\text{ A}$$

EXAMPLE 2.10

When two resistances connected in series and parallel their equivalent resistances are 15 Ω and $\frac{56}{15}\Omega$ respectively. Find the individual resistances.

Solution

$R_s = R_1 + R_2 = 15\ \Omega$ (1)

$R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{56}{15}\Omega$ (2)

From equation (1) substituting for $R_1 + R_2$ in equation (2)

$$\frac{R_1 R_2}{15} = \frac{56}{15}\Omega$$

$\therefore\ R_1 R_2 = 56$

$R_2 = \frac{56}{15}\Omega$ (3)

Substituting for R_2 in equation (1) from equation (3)

$$R_1 + \frac{56}{R_1} = 15$$

Then, $\frac{R_1^2 + 56}{R_1} = 15$

$R_1^2 + 56 = 15\,R_1$

$R_1^2 - 15\,R_1 + 56 = 0$

The above equation can be solved using factorisation.

$R_1^2 - 8\,R_1 - 7\,R_1 + 56 = 0$

$R_1\,(R_1 - 8) - 7\,(R_1 - 8) = 0$

$(R_1 - 8)\,(R_1 - 7) = 0$

If $(R_1 = 8\ \Omega)$

using in equation (1)

$8 + R_2 = 15$

$R_2 = 15 - 8 = 7\ \Omega$,

$R_2 = 7\ \Omega$ i.e , (when $R_1 = 8\ \Omega$; $R_2 = 7\ \Omega$)

If $(R_1 = 7\ \Omega)$

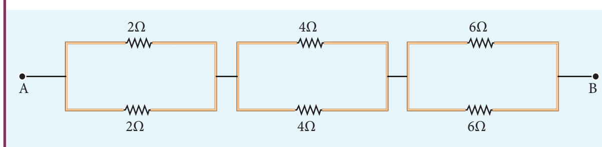
Substituting in equation (1)

$7 + R_2 = 15$

$R_2 = 8\ \Omega$, i.e , (when $R_1 = 8\ \Omega$; $R_2 = 7\ \Omega$)

EXAMPLE 2.11

Calculate the equivalent resistance between A and B in the given circuit.



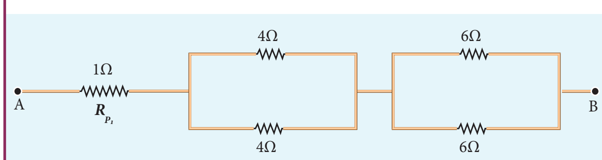
Solution

Parallel connection

Part 1

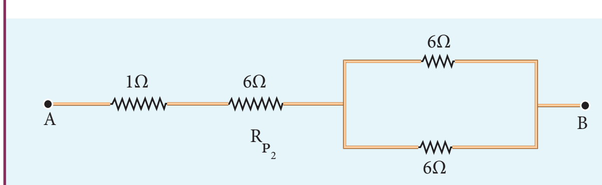
$$\frac{1}{R_{p_1}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{p_1}} = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} \qquad R_{p_1} = 1\Omega$$



Part II

$$\frac{1}{R_{p_2}} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4}, \quad \frac{1}{R_{p_2}} = \frac{1}{2}, \quad R_{p_2} = 2\Omega$$



Part III

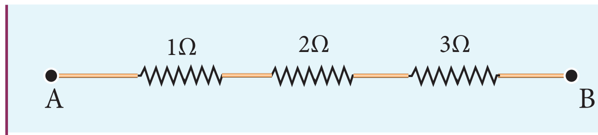
$$\frac{1}{R_{p_3}} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

$$\frac{1}{R_{p_3}} = \frac{1}{3}, \qquad R_{p_3} = 3\Omega$$

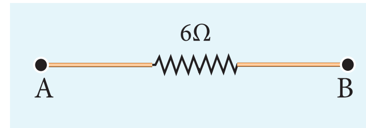
$$R = R_{p_1} + R_{p_2} + R_{p_3}$$

$$R = 1 + 2 + 3 \quad R = 6\ \Omega$$

The circuit became:

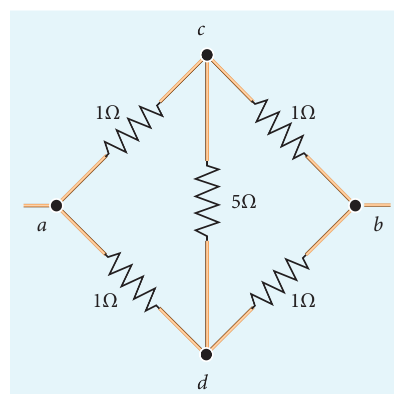


Equivalent resistance between A and B is



EXAMPLE 2.12

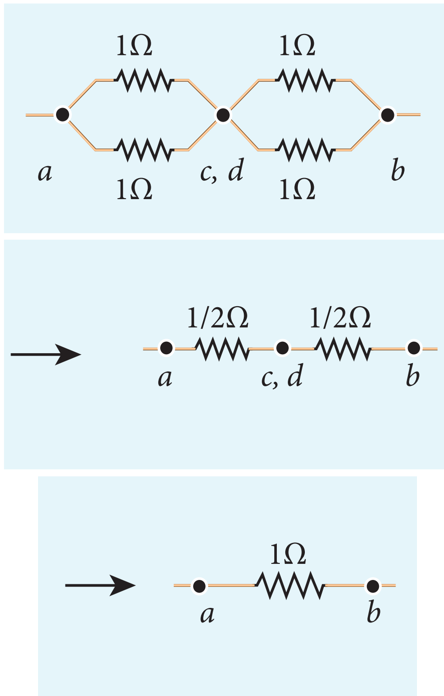
Five resistors are connected in the configuration as shown in the figure. Calculate the equivalent resistance between the points a and b.



Solution

Case (a)

To find the equivalent resistance between the points a and b, we assume that current is entering the junction a. Since all the resistances in the outside loop are the same (1Ω), the current in the branches ac and ad must be equal. So the electric potential at the point c and d is the same hence no current flows into 5 Ω resistance. It implies that the 5 Ω has no role in determining the equivalent resistance and it can be removed. So the circuit is simplified as shown in the figure.



The equivalent resistance of the circuit between a and b is $R_{eq} = 1\Omega$

2.2.3 Color code for Carbon resistors

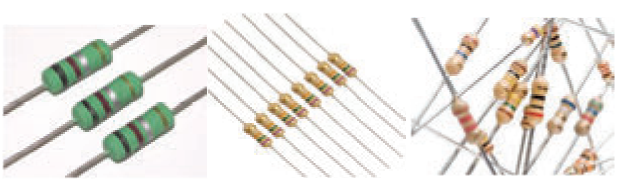


Figure 2.11 Resistance used in our laboratory

Carbon resistors consists of a ceramic core, on which a thin layer of crystalline carbon is deposited as shown in Figure 2.11. These resistors are inexpensive, stable and compact in size. Color rings are used to indicate the value of the resistance according to the rules given in the Table 2.2.

Three coloured rings are used to indicate the values of a resistor: the first two rings are significant figures of resistances, the third ring indicates the decimal multiplier after them. The fourth color, silver or gold,

Table 2.2 Color Coding for Resistors

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Sliver		10^{-2}	10%
Colorless			20%

shows the tolerance of the resistor at 10% or 5% as shown in the Figure 2.12 .If there is no fourth ring, the tolerance is 20%.

For the resistor shown in Figure 2.12, the first digit = 5 (green), the second digit = 6 (blue), decimal multiplier = 10^3 (orange) and tolerance = 5% (gold). The value of resistance = $56 \times 10^3\ \Omega$ or 56 kΩ with the tolerance value 5%.

Note While reading the colour code, hold the resistor with colour bands to your left. Resistors never start with a metallic band on the left.

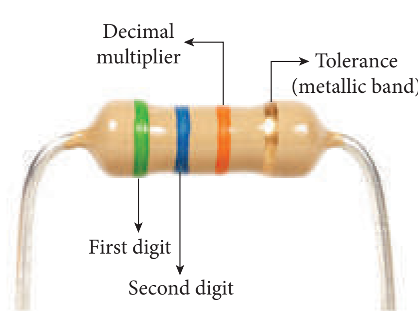


Figure 2.12 Resistor color coding

UNIT-2(XII-Physics_Vol-1).indd 96 04-03-2019 11:01:27

DO YOU KNOW?

Measuring current

Measuring resistance

Measuring voltage

A multimeter is a very useful electronic instrument used to measure voltage, current, resistance and capacitance. In fact, it can also measure AC voltage and AC current. The circular slider has to be kept in appropriate position to measure each electrical quantity.

2.2.4 Temperature dependence of resistivity

The resistivity of a material is dependent on temperature. It is experimentally found that for a wide range of temperatures, the resistivity of a conductor increases with increase in temperature according to the expression,

$$\rho_T = \rho_\circ\left[1+\alpha\left(T-T_\circ\right)\right] \qquad (2.27)$$

where ρ_T is the resistivity of a conductor at T °C, ρ_o is the resistivity of the conductor at some reference temperature T_o (usually at 20°C) and α is the temperature coefficient of resistivity. **It is defined as the ratio of increase in resistivity per degree rise in temperature to its resistivity at T_o.**

From the equation (2.27), we can write

$$\rho_T - \rho_\circ = \alpha\rho_\circ\left(T-T_\circ\right)$$

$$\therefore \alpha = \frac{\rho_T-\rho_\circ}{\rho_\circ\left(T-T_\circ\right)} = \frac{\Delta\rho}{\rho_\circ \Delta T}$$

where $\Delta\rho = \rho_T - \rho_o$ is change in resistivity for a change in temperature $\Delta T = T - T_o$. Its unit is per °C.

α of conductors

For conductors α *is positive.* If the temperature of a conductor increases, the average kinetic energy of electrons in the conductor increases. This results in more frequent collisions and hence the resistivity increases. The graph of the equation (2.27) is shown in Figure 2.13

Even though, the resistivity of conductors like metals varies linearly for wide range of temperatures, there also exists a non-linear region at very low temperatures. The resistivity approaches some finite value as the temperature approaches absolute zero as shown in Figure 2.13(b).

As the resistance is directly proportional to resistivity of the material, we can also write the resistance of a conductor at temperature T °C as

$$R_T = R_\circ\left[1+\alpha\left(T-T_\circ\right)\right] \qquad (2.28)$$

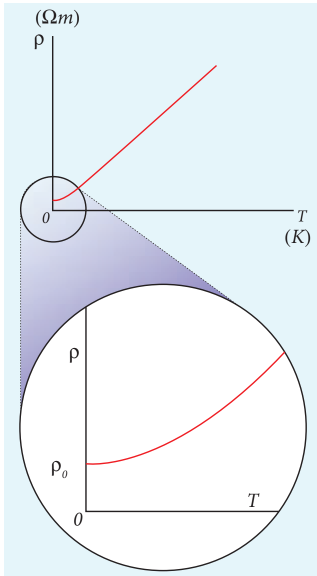


Figure 2.13 (a) Temperature dependence of resistivity for a conductor (b) Non linear region at low temperature

The temperature coefficient can be also be obtained from the equation (2.28),

$$R_T - R_\circ = \alpha R_\circ (T - T_\circ)$$

$$\therefore \alpha = \frac{R_T - R_\circ}{R_\circ (T - T_\circ)} = \frac{1}{R_\circ} \frac{\Delta R}{\Delta T}$$

$$\alpha = \frac{1}{R_\circ} \frac{\Delta R}{\Delta T} \qquad (2.29)$$

where $\Delta R = R_T - R_\circ$ is change in resistance during the change in temperature $\Delta T = T - T_\circ$

α of semiconductors

For semiconductors, the resistivity decreases with increase in temperature. As the temperature increases, more electrons will be liberated from their atoms (Refer unit 9 for conduction in semi conductors). Hence the current increases and therefore the resistivity decreases as shown in Figure 2.14. A semiconductor with a negative temperature coefficient of resistance is called a thermistor.

The typical values of temperature coefficients of various materials are given in table 2.3.

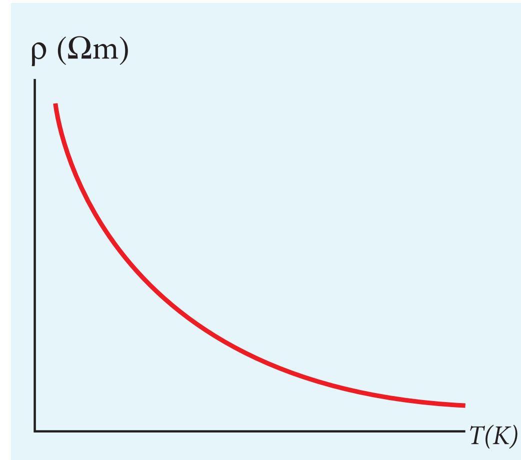


Figure 2.14 Temperature dependence of resistivity for a semiconductor

Table 2.3

Color	Temperature Coefficient α [(°C)$^{-1}$]
Silver	3.8×10^{-3}
Copper	3.9×10^{-3}
Gold	3.4×10^{-3}
Aluminum	3.9×10^{-3}
Tungsten	4.5×10^{-3}
Iron	5.0×10^{-3}
Platinum	3.92×10^{-3}
Lead	3.9×10^{-3}
Nichrome	0.4×10^{-3}
Carbon	-0.5×10^{-3}
Germanium	-48×10^{-3}
Silicon	-75×10^{-3}

We can understand the temperature dependence of resistivity in the following way. In section 2.1.3, we have shown that the electrical conductivity, $\sigma = \frac{ne^2\tau}{m}$. As the resistivity is inverse of σ, it can be written as,

$$\rho = \frac{m}{ne^2\tau} \qquad (2.30)$$

The resistivity of materials is

i) inversely proportional to the number density (n) of the electrons

ii) inversely proportional to the average time between the collisions (τ).

In metals, if the temperature increases, the average time between the collision (τ) decreases and n is independent of temperature. In semiconductors when temperature increases, n increases and τ decreases, but increase in n is dominant than decreasing τ, so that overall resistivity decreases.

DO YOU KNOW? The resistance of certain materials become zero below certain temperature T_c. This temperature is known as critical temperature or transition temperature. The materials which exhibit this property are known as superconductors. This phenomenon was first observed by Kammerlingh Onnes in 1911. He found that mercury exhibits superconductor behaviour at 4.2 K. Since R = 0, current once induced in a superconductor persists without any potential difference.

EXAMPLE 2.13

If the resistance of coil is 3 Ω at 20 ^{0}C and α = 0.004/^{0}C then determine its resistance at 100 ^{0}C.

Solution

$R_0 = 3\ \Omega$, $T = 100^0C$, $T_0 = 20^0C$

$\alpha = 0.004/^0C$, $R_T = ?$

$R_T = R_0(1 + \alpha(T - T_0))$

$R_{100} = 3(1 + 0.004 \times 80)$

$R_{100} = 3(1 + 0.32)$

$R_{100} = 3(1.32)$

$R_{100} = 3.96\ \Omega$

EXAMPLE 2.14

Resistance of a material at 10^{0}C and 40^{0}C are 45 Ω and 85 Ω respectively. Find its temperature co-efficient of resistance.

Solution

$T_0 = 10\ ^0C$, $T = 40\ ^0C$, $R_0 = 45\ \Omega$, $R = 85\ \Omega$

$$\alpha = \frac{1}{R_o}\frac{\Delta R}{\Delta T}$$

$$\alpha = \frac{1}{45}\left(\frac{85-45}{40-10}\right) = \frac{1}{45}\left(\frac{40}{30}\right)$$

$$\alpha = 0.0296\, per^\circ C$$

2.3 ENERGY AND POWER IN ELECTRICAL CIRCUITS

When a battery is connected between the ends of a conductor, a current is established. The battery is transporting energy to the device which is connected in the circuit. Consider a circuit in which a battery of voltage V is connected to the resistor as shown in Figure 2.15.

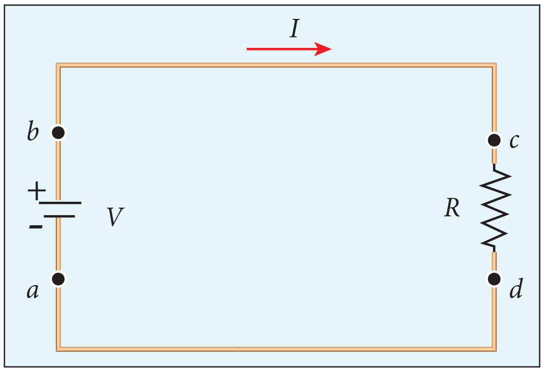


Figure 2.15 Energy given by the battery

Assume that a positive charge of dQ moves from point a to b through the battery and moves from point c to d through the resistor and back to point a. When the charge moves from point a to b, it gains potential energy $dU = V.dQ$ and the chemical potential energy of the battery decreases by the same amount. When this charge dQ passes through resistor it loses the potential energy $dU = V.dQ$ due to collision with atoms in the resistor and again reaches the point a. This process occurs continuously till the battery is connected in the circuit. The rate at which the charge loses its electrical potential energy in the resistor can be calculated.

The electrical power P is the rate at which the electrical potential energy is delivered,

$$P = \frac{dV}{dt} = \frac{d}{dt}\left(V.dQ\right) = V\frac{dQ}{dt} \qquad (2.31)$$

Since the electric current $I = \frac{dQ}{dt}$.

So the equation (2.31) can be rewritten as

$$P = VI \qquad (2.32)$$

This expression gives the power delivered by the battery to any electrical system, where I is the current passing through it and V is the potential difference across it. The SI unit of electrical power is watt ($1W = 1\ J s^{-1}$). Commercially, the electrical bulbs used in houses come with the power and voltage rating of 5W-220V, 30W-220V, 60W-220V etc. (Figure 2.16).

Figure 2.16 Electrical bulbs with power rating

Usually these voltage rating refers AC RMS voltages. For a given bulb, if the voltage drop across the bulb is greater than voltage rating, the bulb will fuse.

Using Ohm's law, power delivered to the resistance R is expressed in other forms

$$P = IV = I\,(IR) = I^2R \qquad (2.33)$$

$$P = IV = \frac{V}{R}V = \frac{V^2}{R} \qquad (2.34)$$

DO YOU KNOW?

The electrical power produced (dissipated) by a resistor is I^2R. It depends on the square of the current. Hence, if current is doubled, the power will increase by four times. Similar explanation holds true for voltage also.

The total energy used by any device is obtained by multiplying the power and duration of the time when it is ON. If the power is in watts and the time is in seconds, the energy will be in joules. In practice, electrical energy is measured in kilowatt hour (kWh). 1 kWh is known as 1 unit of electrical energy.

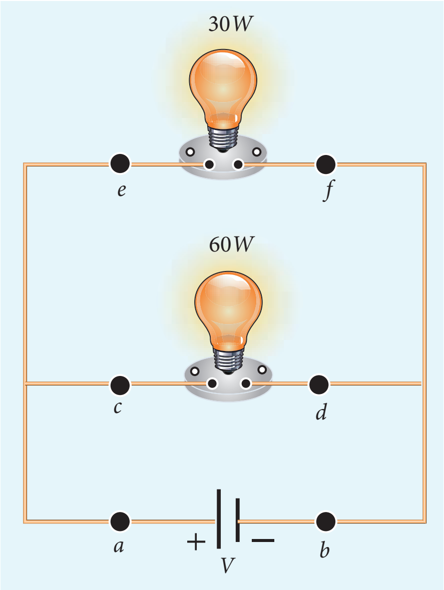
($1\ kWh = 1000\ Wh = (1000\ W)\ (3600\ s) = 3.6 \times 10^6\ J$)

DO YOU KNOW?

The Tamilnadu Electricity Board is charging for the amount of energy you use and not for the power. A current of 1A flowing through a potential difference of 1V produces a power of 1W.

EXAMPLE 2.15

A battery of voltage V is connected to 30 W bulb and 60 W bulb as shown in the figure. (a) Identify brightest bulb (b) which bulb has greater resistance? (c) Suppose the two bulbs are connected in series, which bulb will glow brighter?



Solution

(a) The power delivered by the battery P = VI. Since the bulbs are connected in parallel, the voltage drop across each bulb is the same. If the voltage is kept fixed, then the power is directly proportional to current (P ∝ I). So 60 W bulb draws twice as much as current as 30 W and it will glow brighter then others.

(b) To calculate the resistance of the bulbs, we use the relation $P=\frac{V^2}{R}$. In both the bulbs, the voltage drop is the same, so the power is inversely proportional to the resistance or resistance is inversely proportional to the power $\left(R \propto \frac{1}{P}\right)$. It implies that, the 30W has twice as much as resistance as 60 W bulb.

(c) When these two bulbs are connected in series, the current passing through each bulb is the same. It is equivalent to two resistors connected in series. The bulb which has higher resistance has higher voltage drop. So 30W bulb will glow brighter than 60W bulb. So the higher power rating does not always imply more brightness and it depends whether bulbs are connected in series or parallel.

EXAMPLE 2.16

Two electric bulbs marked 20 W – 220 V and 100 W – 220 V are connected in series to 440 V supply. Which bulb will be fused?

Solution

To check which bulb will be fused, the voltage drop across each bulb has to be calculated.

The resistance of a bulb,

$$R=\frac{V^2}{P}=\frac{(\text{Rated voltage})^2}{\text{Rated power}}$$

For 20W-220V bulb,

$$R_1=\frac{(220)^2}{20}\Omega=2420\Omega$$

DQNPP1

For 100W-220V bulb,

$$R_2=\frac{(220)^2}{100}\Omega=484\Omega$$

Both the bulbs are connected in series. So the current which passes through both the bulbs are same. The current that passes through the circuit, $I=\frac{V}{R_{tot}}$.

$$R_{tot} = (R_1 + R_2)$$
$$R_{tot} = (484 + 2420)\Omega = 2904\Omega$$
$$I = \frac{440V}{2904\Omega} \approx 0.151A$$
The voltage drop across the 20W bulb is
$$V_1 = IR_1 = \frac{440}{2904} \times 2420 \approx 366.6\ V$$
The voltage drop across the 100W bulb is
$$V_2 = IR_2 = \frac{440}{2904} \times 484 \approx 73.3\ V$$
The 20 W bulb will be fused because its voltage rating is only 220 V and 366.6 V is dropped across it.

2.4 ELECTRIC CELLS AND BATTERIES

An electric cell converts chemical energy into electrical energy to produce electricity. It contains two electrodes immersed in an electrolyte as shown in Figure 2.17.

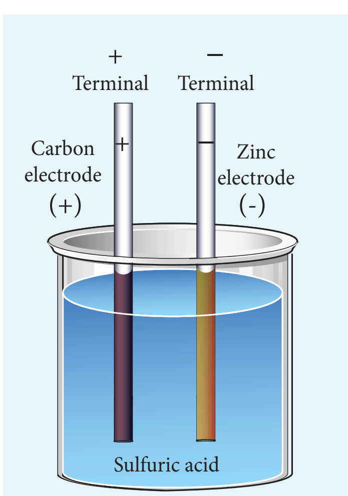
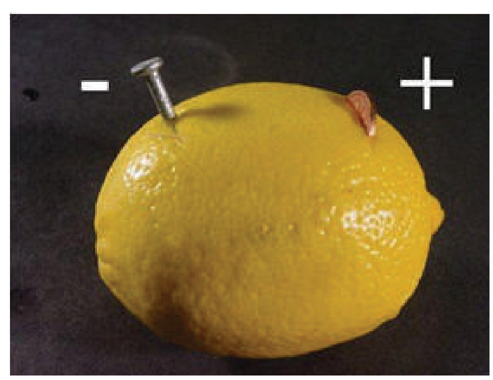


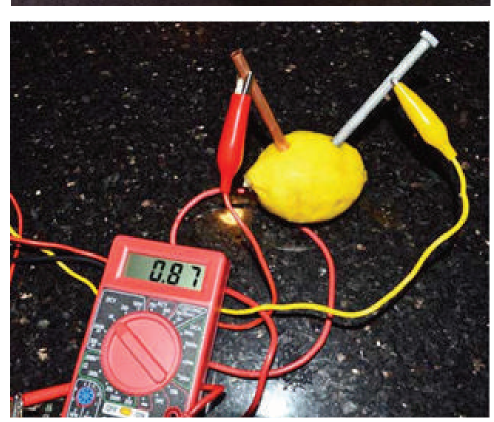
Figure 2.17 Simple electric cell

Several electric cells connected together form a battery. When a cell or battery is connected to a circuit, electrons flow from the negative terminal to the positive terminal through the circuit. By using chemical reactions, a battery produces potential difference across its terminals. This potential difference provides the energy to move the electrons through the circuit. Commercially available electric cells and batteries are shown in Figure 2.18

Figure 2.18 Electric cells and Batteries

DO YOU KNOW? If we connect copper and zinc rod in a lemon, it acts as an electric cell. The citric acid in the lemon acts as an electrolyte. The potential can be measured using a multimeter.





2.4.1 Electromotive force and internal resistance

A battery or cell is called a source of electromotive force (emf). The term 'electromotive force' is a misnomer since it does not really refer to a force but describes a potential difference in volts. The emf of a battery or cell is the voltage provided by the battery when no current flows in the external circuit. It is shown in Figure 2.19.

Figure 2.19 Measuring the emf of a cell

Electromotive force determines the amount of work a battery or cell does to move a certain amount of charge around the circuit. It is denoted by the symbol ξ and to be pronounced as 'xi'. An ideal battery has zero internal resistance and the potential difference (terminal voltage) across the battery equals to its emf. But a real battery is made of electrodes and electrolyte, there is resistance to the flow of charges within the battery. This resistance is called internal resistance *r*. For a real battery, the terminal voltage is not equal to the emf of the battery. A freshly prepared cell has low internal resistance and it increases with ageing.

2.4.2 Determination of internal resistance

The circuit connections are made as shown in Figure 2.20.

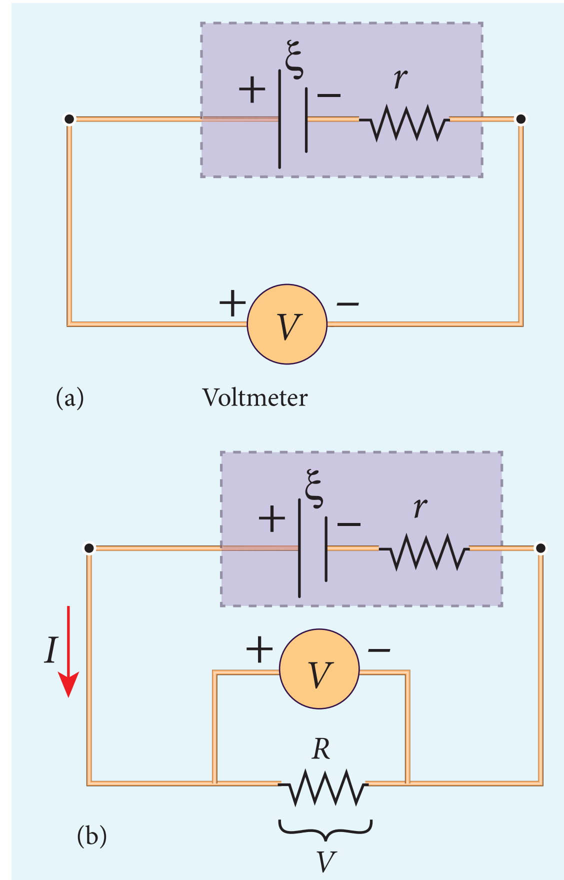


Figure 2.20 Internal resistance of the cell

The emf of cell ξ is measured by connecting a high resistance voltmeter across it without connecting the external resistance R as shown in Figure 2.20(a). Since the voltmeter draws very little current for deflection, the circuit may be considered as open. Hence the voltmeter reading gives the emf of the cell. Then, external resistance R is included in the circuit and current I is established in the circuit. The potential difference across R is equal to the potential difference across the cell (V) as shown in Figure 2.20(b).

The potential drop across the resistor R is

$$V = IR \qquad (2.35)$$

Due to internal resistance r of the cell, the voltmeter reads a value V, which is less than the emf of cell ξ. It is because, certain

UNIT-2(XII-Physics_Vol-1).indd 103 04-03-2019 11:01:37

amount of voltage (Ir) has dropped across the internal resistance r.

Then $V = \xi - Ir$

$$Ir = \xi - V \quad (2.36)$$

Dividing equation (2.36) by equation (2.35), we get

$$\frac{Ir}{IR} = \frac{\xi - V}{V}$$

$$r = \left[\frac{\xi - V}{V}\right]R \quad (2.37)$$

Since ξ, V and R are known, internal resistance r can be determined. We can also find the total current that flows in the circuit.

Due to this internal resistance, the power delivered to the circuit is not equal to power rating mentioned in the battery. For a battery of emf ξ, with an internal resistance r, the power delivered to the circuit of resistance R is given by

$$P = I\xi = I(V + Ir) \quad \text{(from equation 2.36)}$$

Here V is the voltage drop across the resistance R and it is equal to IR.

Therefore, $P = I(IR + Ir)$

$$P = I^2 R + I^2 r \quad (2.38)$$

Here $I^2 r$ is the power delivered to the internal resistance and $I^2 R$ is the power delivered to the electrical device (here it is the resistance R). For a good battery, the internal resistance r is very small, then $I^2 r << I^2 R$ and almost entire power is delivered to the resistance.

EXAMPLE 2.17

A battery has an emf of 12 V and connected to a resistor of 3 Ω. The current in the circuit is 3.93 A. Calculate (a) terminal voltage and the internal resistance of the battery (b) power delivered by the battery and power delivered to the resistor

Solution

The given values $I = 3.93$ A, $\xi = 12$ V, $R = 3\,\Omega$

(a) The terminal voltage of the battery is equal to voltage drop across the resistor

$V = IR = 3.93 \times 3 = 11.79$ V

The internal resistance of the battery,

$$r = \left[\frac{\xi - V}{V}\right]R = \left[\frac{12 - 11.79}{11.79}\right] \times 3 = 0.05\,\Omega$$

(b) The power delivered by the battery $P = I\xi = 3.93 \times 12 = 47.1$ W

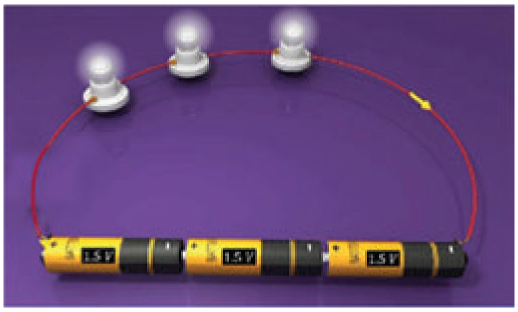
The power delivered to the resistor $= I^2 R = 46.3$ W

The remaining power $= (47.1 - 46.3)$ $P = 0.772$ W is delivered to the internal resistance and cannot be used to do useful work. (it is equal to $I^2 r$).

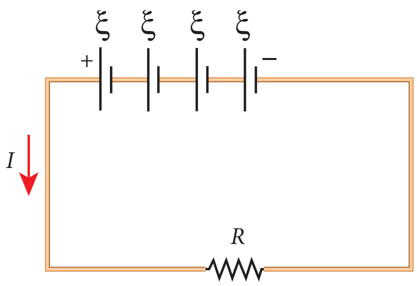
2.4.3 Cells in series

Several cells can be connected to form a battery. In series connection, the negative terminal of one cell is connected to the positive terminal of the second cell, the negative terminal of second cell is connected to the positive terminal of the third cell and so on. The free positive terminal of the first cell and the free negative terminal of the last cell become the terminals of the battery.

Suppose n cells, each of emf ξ volts and internal resistance r ohms are connected in series with an external resistance R as shown in Figure 2.21



Cella in series (Schematic diagram)



Cells in series (circuit diagram)

Figure 2.21 cells in series

The total emf of the battery = $n\xi$

The total resistance in the circuit = $nr + R$

By Ohm's law, the current in the circuit is

$$I = \frac{\text{total emf}}{\text{total resistance}} = \frac{n\xi}{nr + R} \qquad (2.39)$$

Case (a) If r << R, then,

$$I = \frac{n\xi}{R} \approx nI_1 \qquad (2.40)$$

where, I_1 is the current due to a single cell

$$\left(I_1 = \frac{\xi}{R}\right)$$

Thus, if r is negligible when compared to R the current supplied by the battery is n times that supplied by a single cell.

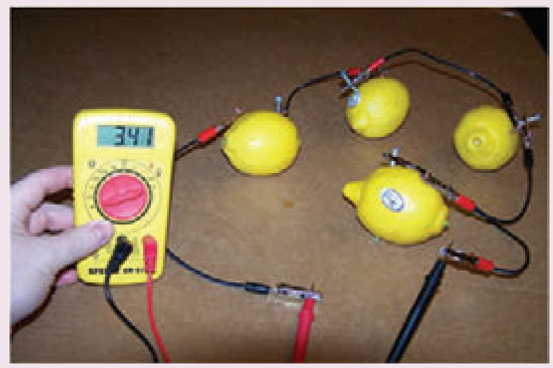
$$\text{Case (b) If r>>R, } I = \frac{n\xi}{nr} \approx \frac{\xi}{r} \qquad (2.41)$$

It is the current due to a single cell. That is, current due to the whole battery is the same as that due to a single cell and hence there is no advantage in connecting several cells.

Thus series connection of cells is advantageous only when the effective internal resistance of the cells is negligibly small compared with R.

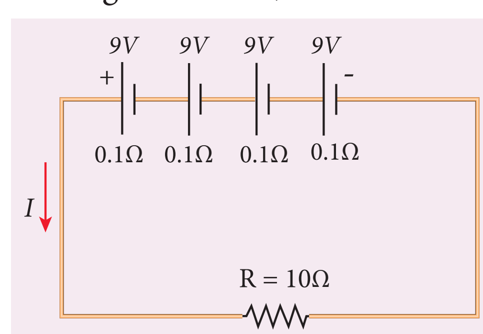
Activity

Construct lemon cells in series and observe the potential of this combination



EXAMPLE 2.18

From the given circuit,



Find

i) Equivalent emf of the combination

ii) Equivalent internal resistance

iii) Total current

iv) Potential difference across external resistance

v) Potential difference across each cell

Solution

i) Equivalent emf of the combination

$\xi_{eq} = n\xi = 4\ 9 = 36$ V

ii) Equivalent internal resistance r_{eq} = nr = 4 × 0.1 = 0.4 Ω

iii) Total current $I = \frac{n\xi}{R + nr}$

$= \frac{4 \times 9}{10 + (4 \times 0.1)}$

$= \frac{4 \times 9}{10 + 0.4} = \frac{36}{10.4}$

I = 3.46 A

iv) Potential difference across external resistance V = IR = 3.46 × 10 = 34.6 V. The remaining 1.4 V is dropped across the internal resistance of cells.

v) Potential difference across each cell $\frac{V}{n} = \frac{34.6}{4} = 8.65V$

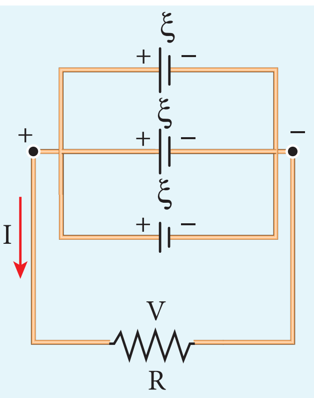
2.4.4 Cells in parallel

In parallel connection all the positive terminals of the cells are connected to one point and all the negative terminals to a second point. These two points form the positive and negative terminals of the battery.

Let n cells be connected in parallel between the points A and B and a resistance R is connected between the points A and B as shown in Figure 2.22. Let ξ be the emf and r the internal resistance of each cell.

The equivalent internal resistance of the battery is $\frac{1}{r_{eq}} = \frac{1}{r} + \frac{1}{r} + \ldots \frac{1}{r}(n terms) = \frac{n}{r}$. So $r_{eq} = \frac{r}{n}$ and the total resistance in the circuit $= R + \frac{r}{n}$. The total emf is the potential difference between the points A and B, which is equal to ξ. The current in the circuit is given by

$$I = \frac{\xi}{\frac{r}{n} + R}$$



cells in parallel (Circuit diagram)


Cells in parallel (Schematic diagram)

Figure 2.22 Cells in parallel

$$I = \frac{n\xi}{r + nR} \quad (2.42)$$

Case (a) If r >> R, $I = \frac{n\xi}{r} = nI_1$ (2.43)

where I_1 is the current due to a single cell and is equal to $\frac{\xi}{r}$ when R is negligible. Thus, the current through the external resistance due to the whole battery is n times the current due to a single cell.

Case (b) If r<<R, I = $\frac{\xi}{R}$ (2.44)

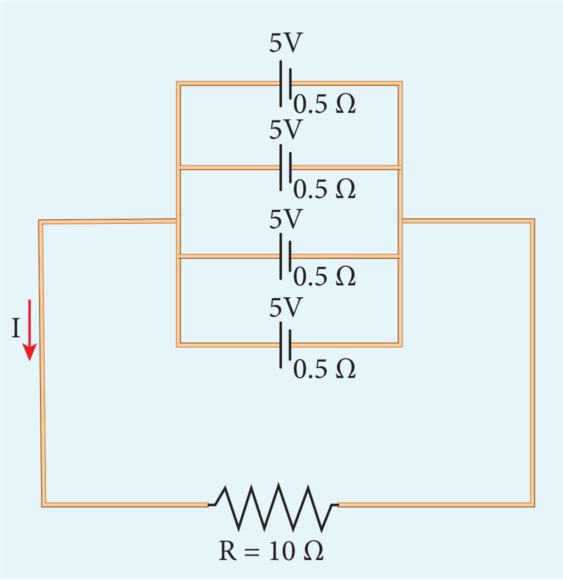
DO YOU KNOW?

When the car engine is started with headlights turned on, they sometimes become dim. This is due to the internal resistance of the car battery.

The above equation implies that current due to the whole battery is the same as that due to a single cell. Hence it is advantageous to connect cells in parallel when the external resistance is very small compared to the internal resistance of the cells.

EXAMPLE 2.19

From the given circuit



Find

i) Equivalent emf

ii) Equivalent internal resistance

iii) Total current (I)

iv) Potential difference across each cell

v) Current from each cell

Solution

i) Equivalent emf $\xi_{eq} = 5$ V

ii) Equivalent internal resistance,

$$R_{eq} = \frac{r}{n} = \frac{0.5}{4} = 0.125\Omega$$

iii) total current, $I = \dfrac{\xi}{R + {}^{r}/_{n}}$

$$I = \frac{5}{10 + 0.125} = \frac{5}{10.125}$$

$I \approx 0.5$ A

iv) Potential difference across each cell $V = IR = 0.5 \times 10 = 5$ V

v) Current from each cell, $I' = \dfrac{I}{n}$

$$I' = \frac{0.5}{4} = 0.125\,A$$

2.5 KIRCHHOFF'S RULES

Ohm's law is useful only for simple circuits. For more complex circuits, Kirchhoff's rules can be used to find current and voltage. There are two generalized rules: i) Kirchhoff's current rule ii) Kirchhoff's voltage rule.

2.5.1 Kirchhoff's first rule (Current rule or Junction rule)

It states that the algebraic sum of the currents at any junction of a circuit is zero. It is a statement of conservation of electric charge. All charges that enter a given junction in a circuit must leave that junction since charge cannot build up or disappear at a junction. Current entering the junction is taken as positive and current leaving the junction is taken as negative.

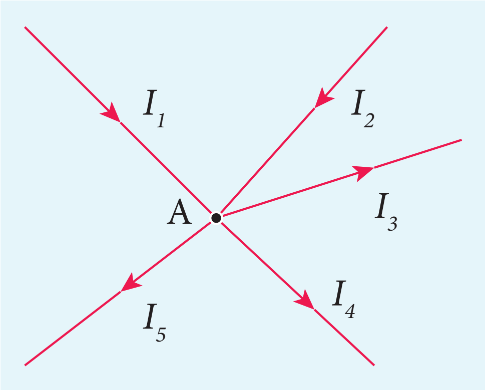


Figure 2.23 Kirchhoff's current rule

Applying this law to the junction A in Figure 2.23

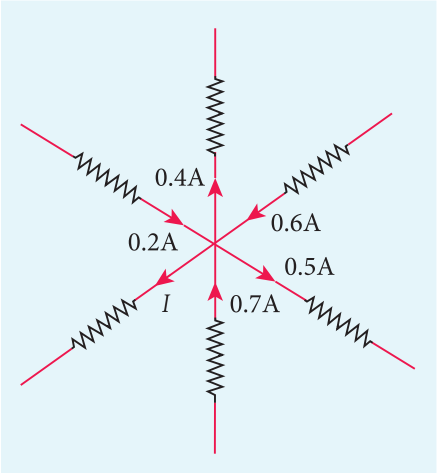
$I_1 + I_2 - I_3 - I_4 - I_5 = 0$

(or)

$I_1 + I_2 = I_3 + I_4 + I_5$

EXAMPLE 2.20

From the given circuit find the value of I.



Solution

Applying Kirchoff's rule to the point P in the circuit,

The arrows pointing towards P are positive and away from P are negative.

Therefore, 0.2A – 0.4A + 0.6A – 0.5A + 0.7A – I = 0

1.5A – 0.9A – I = 0

0.6A – I = 0

I = 0.6 A

2.5.2 Kirchhoff's Second rule (Voltage rule or Loop rule)

It states that in a closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf included in the circuit. This rule follows from the law of conservation of energy for an isolated system (The energy supplied by the emf sources is equal to the sum of the energy delivered to all resistors).

The product of current and resistance is taken as positive when the direction of the current is followed. Suppose if the direction of current is opposite to the direction of the loop, then product of current and voltage across the resistor is negative. It is shown in Figure 2.24 (a) and (b). The emf is considered positive when proceeding from the negative to the positive terminal of the cell. It is shown in Figure 2.24 (c) and (d).

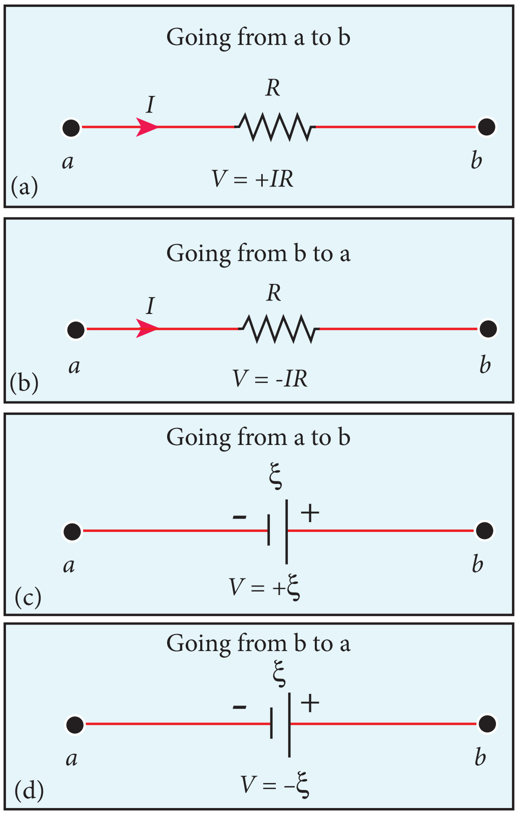
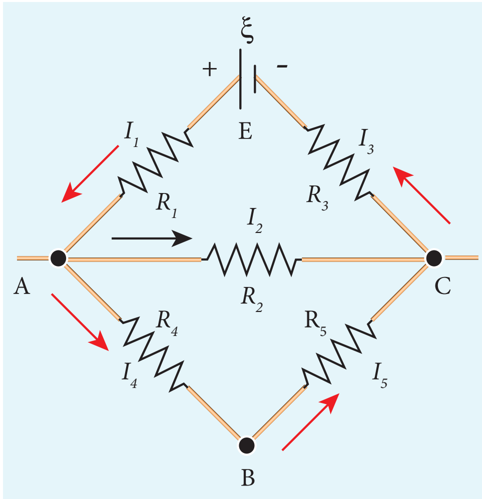


Figure 2.24 Kirchhoff voltage rule

Kirchhoff voltage rule has to be applied only when all currents in the circuit reach a steady state condition (the current in various branches are constant).

EXAMPLE 2.21

The following figure shows a complex network of conductors which can be divided into two closed loops like ACE and ABC. Apply Kirchoff's voltage rule.



Solution

Thus applying Kirchoff's second law to the closed loop EACE

$$I_1R_1 + I_2R_2 + I_3R_3 = \xi$$

and for the closed loop ABCA

$$I_4R_4 + I_5R_5 - I_2R_2 = 0$$

We can denote the current that flows from 9V battery as I_1 and it splits into I_2 and $I_1 - I_2$ in the junction according Kirchoff's current rule (KCR). It is shown below.

Now consider the loop EFCBE and apply KVR, we get

$$1I_2 + 3I_1 + 2I_1 = 9$$

$$5I_1 + I_2 = 9 \qquad (1)$$

Applying KVR to the loop EADFE, we get

$$3\,(I_1 - I_2) - 1I_2 = 6$$

$$3I_1 - 4I_2 = 6 \qquad (2)$$

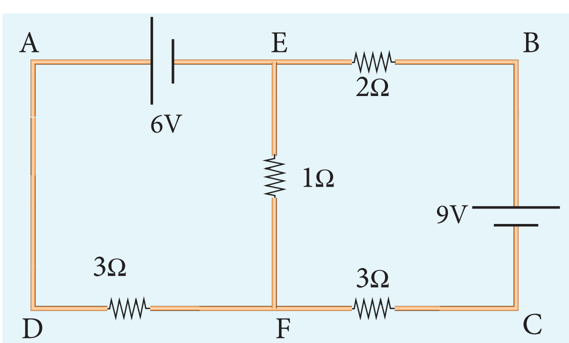
Solving equation (1) and (2), we get

$$I_1 = 1.83 \text{ A and } I_2 = -0.13 \text{ A}$$

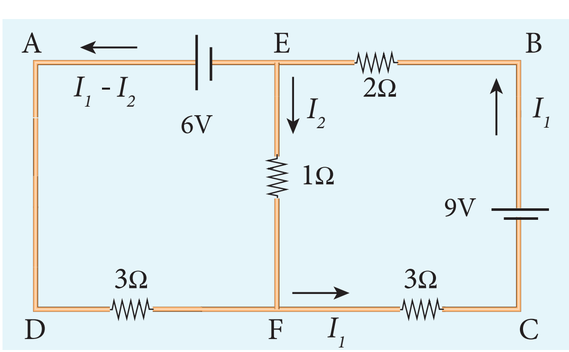
It implies that the current in the 1 ohm resistor flows from F to E.

EXAMPLE 2.22

Calculate the current that flows in the 1 Ω resistor in the following circuit.



Solution



2.5.3 Wheatstone's bridge

An important application of Kirchhoff's rules is the Wheatstone's bridge. It is used to compare resistances and also helps in determining the unknown resistance in electrical network. The bridge consists of four resistances P, Q, R and S connected as shown in Figure 2.25. A galvanometer G is connected between the points B and D. The battery is connected between the points A and C. The current through the galvanometer is I_G and its resistance is G.

Applying Kirchhoff's current rule to junction B

$$I_1 - I_G - I_3 = 0 \qquad (2.45)$$

Applying Kirchhoff's current rule to junction D,

$$I_2 + I_G - I_4 = 0 \qquad (2.46)$$

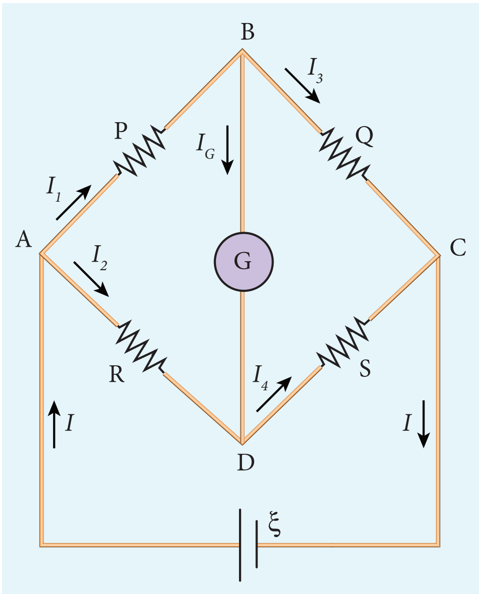


Figure 2.25 Wheatstone's bridge

Applying Kirchhoff's voltage rule to loop ABDA,

$$I_1P + I_GG - I_2R = 0 \qquad (2.47)$$

Applying Kirchhoff's voltage rule to loop ABCDA,

$$I_1P + I_3Q - I_4S - I_2R = 0 \qquad (2.48)$$

When the points B and D are at the same potential, the bridge is said to be balanced. As there is no potential difference between B and D, no current flows through galvanometer ($I_G = 0$). Substituting $I_G = 0$ in equation (2.45), (2.46) and (2.47), we get

$$I_1 = I_3 \qquad (2.49)$$

$$I_2 = I_4 \qquad (2.50)$$

$$I_1P = I_2R \qquad (2.51)$$

Substituting the equation (2.49) and (2.50) in equation (2.48)

$$I_1P + I_1Q - I_2S - I_2R = 0$$

$$I_1 (P + Q) = I_2 (R + S) \qquad (2.52)$$

Dividing equation (2.52) by equation (2.51), we get

$$\frac{P+Q}{P} = \frac{R+S}{R}$$

$$1+\frac{Q}{P} = 1+\frac{S}{R}$$

$$\frac{Q}{P} = \frac{S}{R}$$

$$\frac{P}{Q} = \frac{R}{S} \qquad (2.53)$$

This is the bridge balance condition. Only under this condition, galvanometer shows null deflection. Suppose we know the values of two adjacent resistances, the other two resistances can be compared. If three of the resistances are known, the value of unknown resistance (fourth one) can be determined.

DO YOU KNOW?

A galvanometer is an instrument used for detecting and measuring even very small electric currents. It is extensively useful to compare the potential difference between various parts of the circuit.

EXAMPLE 2.23

In a Wheatstone's bridge P = 100 Ω, Q = 1000 Ω and R = 40 Ω. If the galvanometer shows zero deflection, determine the value of S.

Solution

$$\frac{P}{Q}=\frac{R}{S}$$

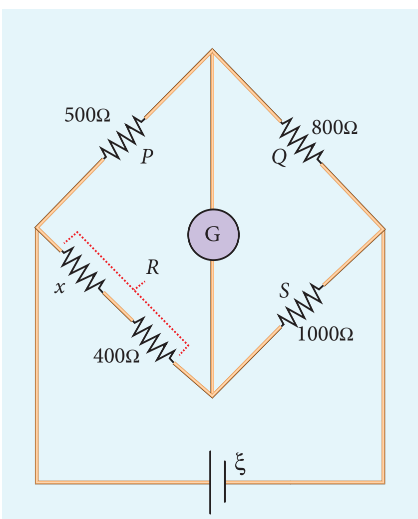
$$S=\frac{Q}{P}\times R$$

$$S=\frac{1000}{100}\times 40 \quad S = 400\ \Omega$$

EXAMPLE 2.24

What is the value of x when the Wheatstone's network is balanced?

P = 500 Ω, Q = 800 Ω, R = x + 400,
S = 1000 Ω



Solution

$$\frac{P}{Q}=\frac{R}{S}$$

$$\frac{500}{800}=\frac{x+400}{1000}$$

$$\frac{x+400}{1000}=\frac{500}{800}$$

$$x+400=\frac{500}{800}\times 1000$$

$$x+400=\frac{5}{8}\times 1000$$

x + 400 = 0.625 × 1000

x + 400 = 625

x = 625 – 400

x = 225 Ω

2.5.4 Meter bridge

The meter bridge is another form of Wheatstone's bridge. It consists of a uniform manganin wire AB of one meter length. This wire is stretched along a meter scale on a wooden board between two copper strips C and D. Between these two copper strips another copper strip E is mounted to enclose two gaps G_1 and G_2 as shown in Figure 2.26. An unknown resistance P is connected in G_1 and a standard resistance Q is connected in G_2. A jockey (conducting wire) is connected to the terminal E on the central copper strip through a galvanometer (G) and a high resistance (HR). The exact position of jockey on the wire can be read on the scale. A Lechlanche cell and a key (K) are connected across the ends of the bridge wire.

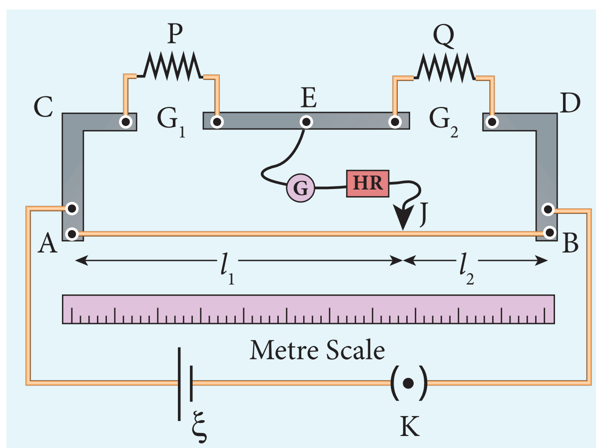


Figure 2.26 Meter bridge

The position of the jockey on the wire is adjusted so that the galvanometer shows zero deflection. Let the point be J. The lengths AJ and JB of the bridge wire now replace the resistance R and S of the Wheatstone's bridge. Then

$$\frac{P}{Q}=\frac{R}{S}=\frac{R'.AJ}{R'.JB} \qquad (2.54)$$

where R′ is the resistance per unit length of wire

$$\frac{P}{Q} = \frac{AJ}{JB} = \frac{l_1}{l_2} \quad (2.55)$$

$$P = Q\frac{l_1}{l_2} \quad (2.56)$$

The bridge wire is soldered at the ends of the copper strips. Due to imperfect contact, some resistance might be introduced at the contact. These are called end resistances. This error can be eliminated, if another set of readings are taken with P and Q interchanged and the average value of P is found.

To find the specific resistance of the material of the wire in the coil P, the radius r and length l of the wire is measured. The specific resistance or resistivity ρ can be calculated using the relation

$$\text{Resistance} = \rho\frac{l}{A}$$

By rearranging the above equation, we get

$$\rho = \text{Resistance} \times \frac{A}{l} \quad (2.57)$$

If P is the unknown resistance equation (2.57) becomes,

$$\rho = P\frac{\pi r^2}{l}$$

EXAMPLE 2.25

In a meter bridge with a standard resistance of 15 Ω in the right gap, the ratio of balancing length is 3:2. Find the value of the other resistance.

Solution

Q = 15 Ω, $l_1 : l_2 = 3:2$

$$\frac{l_1}{l_2} = \frac{3}{2}$$

$$\frac{P}{Q} = \frac{l_1}{l_2}$$

$$P = Q\frac{l_1}{l_2}$$

$$P = 15\frac{3}{2} = 22.5\ \Omega$$

EXAMPLE 2.26

In a meter bridge, the value of resistance in the resistance box is 10 Ω. The balancing length is l_1 = 55 cm. Find the value of unknown resistance.

Solution

Q = 10 Ω

$$\frac{P}{Q} = \frac{l_1}{100 - l_1} = \frac{l_1}{l_2}$$

$$P = Q \times \frac{l_1}{100 - l_1}$$

$$P = \frac{10 \times 55}{100 - 55}$$

$$P = \frac{550}{45} = 12.2\ \Omega$$

2.5.5 Potentiometer

Potentiometer is used for the accurate measurement of potential differences, current and resistances. It consists of ten meter long uniform wire of manganin or constantan stretched in parallel rows each of 1 meter length, on a wooden board. The two free ends A and B are brought to the same side and fixed to copper strips with binding screws. A meter scale is fixed parallel to the wire. A jockey is provided for making contact.

The principle of the potentiometer is illustrated in Figure 2.27. A steady current is maintained across the wire CD by a battery

Bt. The battery, key and the potentiometer wire are connected in series forms the primary circuit. The positive terminal of a primary cell of emf ξ is connected to the point C and negative terminal is connected to the jockey through a galvanometer G and a high resistance HR. This forms the secondary circuit.

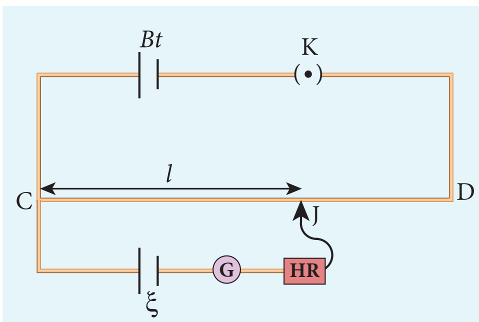


Figure 2.27 Potentiometer

Let contact be made at any point J on the wire by jockey. If the potential difference across CJ is equal to the emf of the cell ξ then no current will flow through the galvanometer and it will show zero deflection. CJ is the balancing length *l*. The potential difference across CJ is equal to *Irl* where *I* is the current flowing through the wire and *r* is the resistance per unit length of the wire.

Hence $\xi = Irl$ (2.58)

Since *I* and *r* are constants, $\xi \propto l$. The emf of the cell is directly proportional to the balancing length.

2.5.6 Comparison of emf of two cells with a potentiometer

To compare the emf of two cells, the circuit connections are made as shown in Figure 2.28. Potentiometer wire CD is connected to a battery *Bt* and a key K in series. This is the primary circuit. The end C of the wire is connected to the terminal M of a DPDT (Double Pole Double Throw) switch and the other terminal N is connected to a jockey through a galvanometer G and a high resistance HR. The cells whose emf ξ_1 and ξ_2 to be compared are connected to the terminals M_1,N_1 and M_2,N_2 of the DPDT switch. The positive terminals of *Bt*, ξ_1 and ξ_2 should be connected to the same end C.

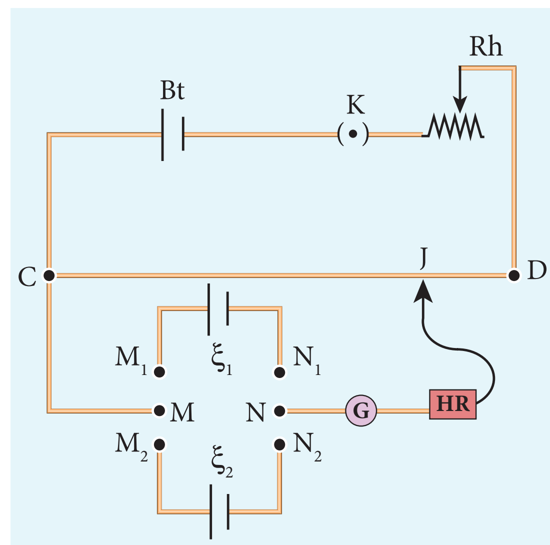


Figure 2.28 Comparison of emf of two cells

The DPDT switch is pressed towards M_1, N_1 so that cell ξ_1 is included in the secondary circuit and the balancing length l_1 is found by adjusting the jockey for zero deflection. Then the second cell ξ_2 is included in the circuit and the balancing length l_2 is determined. Let *r* be the resistance per unit length of the potentiometer wire and *I* be the current flowing through the wire.

we have $\xi_1 = Irl_1$ (2.59)

$\xi_2 = Irl_2$ (2.60)

By dividing equation (2.59) by (2.60)

$$\frac{\xi_1}{\xi_2} = \frac{l_1}{l_2} \quad (2.61)$$

By including a rheostat (Rh) in the primary circuit, the experiment can be repeated several times by changing the current flowing through it.

2.5.7 Measurement of internal resistance of a cell by potentiometer

To measure the internal resistance of a cell, the circuit connections are made as shown in Figure 2.29. The end C of the potentiometer wire is connected to the positive terminal of the battery Bt and the negative terminal of the battery is connected to the end D through a key K_1. This forms the primary circuit.

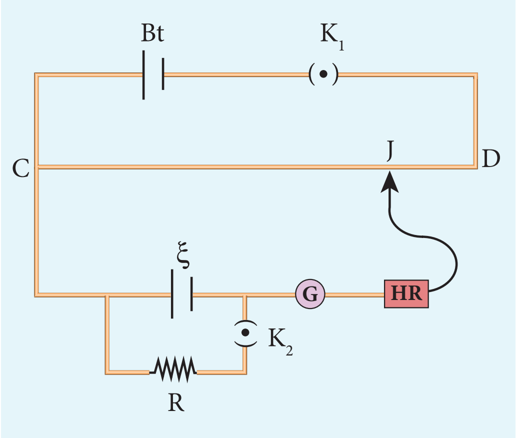


Figure 2.29 measurement of internal resistance

The positive terminal of the cell ξ whose internal resistance is to be determined is also connected to the end C of the wire. The negative terminal of the cell ξ is connected to a jockey through a galvanometer and a high resistance. A resistance box R and key K_2 are connected across the cell ξ. With K_2 open, the balancing point J is obtained and the balancing length CJ = l_1 is measured. Since the cell is in open circuit, its emf is

$$\xi \propto l_1 \qquad (2.62)$$

A suitable resistance (say, 10 Ω) is included in the resistance box and key K_2 is closed. Let r be the internal resistance of the cell. The current passing through the cell and the resistance R is given by

$$I = \frac{\xi}{R+r}$$

The potential difference across R is

$$V = \frac{\xi R}{R+r}$$

When this potential difference is balanced on the potentiometer wire, let l_2 be the balancing length.

$$\text{Then } \frac{\xi R}{R+r} \propto l_2 \qquad (2.63)$$

From equations (2.62) and (2.63)

$$\frac{R+r}{R} = \frac{l_1}{l_2} \qquad (2.64)$$

$$1 + \frac{r}{R} = \frac{l_1}{l_2};$$

$$r = R\left[\frac{l_1}{l_2} - 1\right]$$

$$\therefore\ r = R\left(\frac{l_1 - l_2}{l_2}\right) \qquad (2.65)$$

Substituting the values of the R, l_1 and l_2, the internal resistance of the cell is determined. The experiment can be repeated for different values of *R*. It is found that the internal resistance of the cell is not constant but increases with increase of external resistance connected across its terminals.

2.6 HEATING EFFECT OF ELECTRIC CURRENT

When current flows through a resistor, some of the electrical energy delivered to the resistor is converted into heat energy and it is dissipated. This heating effect of

current is known as Joule's heating effect. Just as current produces thermal energy, thermal energy may also be suitably used to produce an electromotive force. This is known as thermoelectric effect.

2.6.1 Joule's law

If a current I flows through a conductor kept across a potential difference V for a time t, the work done or the electric potential energy spent is

$$W = VIt \qquad (2.66)$$

In the absence of any other external effect, this energy is spent in heating the conductor. The amount of heat(H) produced is

$$H = VIt \qquad (2.67)$$

For a resistance R,

$$H = I^2Rt \qquad (2.68)$$

This relation was experimentally verified by Joule and is known as Joule's law of heating. It states that **the heat developed in an electrical circuit due to the flow of current varies directly as**

(i) the square of the current

(ii) the resistance of the circuit and

(iii) the time of flow.

EXAMPLE 2.27

Find the heat energy produced in a resistance of 10 Ω when 5 A current flows through it for 5 minutes.

Solution

R = 10 Ω, I = 5 A, t = 5 minutes = 5 × 60 s

$H = I^2 R t$

$= 5^2 \times 10 \times 5 \times 60$

$= 25 \times 10 \times 300$

$= 25 \times 3000$

= 75000 J (or) 75 kJ

2.6.2 Application of Joule's heating effect

1. Electric heaters

Electric iron, electric heater, electric toaster shown in Figure 2.30 are some of the home appliances that utilize the heating effect of current. In these appliances, the heating elements are made of nichrome, an alloy of nickel and chromium. Nichrome has a high specific resistance and can be heated to very high temperatures without oxidation.

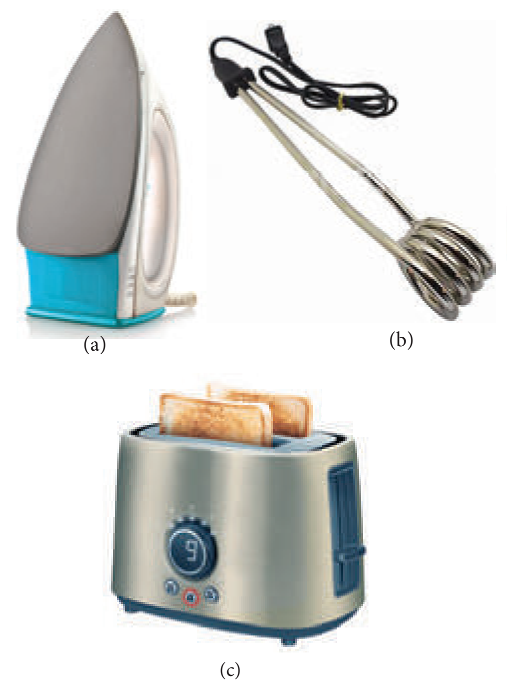


Figure 2.30 (a) Electric Iron box, (b) electric heater (c) electric Toaster

EXAMPLE 2.28

An electric heater of resistance 10 Ω connected to 220 V power supply is immersed in the water of 1 kg. How long the electrical heater has to be switched on to increase its temperature from 30°C to 60°C. (The specific heat of water is s = 4200 J kg^{-1})

Solution

According to Joule's heating law $H = I^2 Rt$

The current passed through the electrical heater $= \frac{220V}{10\Omega} = 22\,A$

The heat produced in one second by the electrical heater $H = I^2 R$

The heat produced in one second $H = (22)^2 \times 10 = 4840$ J = 4.84 k J. In fact the power rating of this electrical heater is 4.84 k W.

The amount of energy to increase the temperature of 1kg water from 30°C to 60°C is

$Q = ms\,\Delta T$ (Refer XI physics vol 2, unit 8)

Here m = 1 kg,

s = 4200 J kg^{-1},

$\Delta T = 30$,

so Q = 1 × 4200 x 30 = 126 kJ

The time required to produce this heat energy $t = \frac{Q}{I^2 R} = \frac{126\times 10^3}{4840} \approx 26.03\,s$

2. Electric fuses

Fuses as shown in Figure 2.31, are connected in series in a circuit to protect the electric devices from the heat developed by the passage of excessive current. It is a short length of a wire made of a low melting point material. It melts and breaks the circuit if current exceeds a certain value. Lead and copper wire melts and burns out when the current increases above 5 A and 35 A respectively.

The only disadvantage with the above fuses is that once fuse wire is burnt due to excessive current, they need to be replaced. Nowadays in houses, circuit breakers (trippers) are also used instead of fuses.

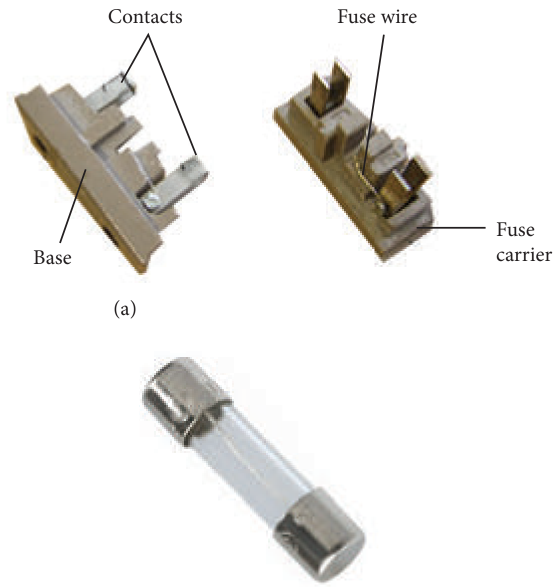


Figure 2.31 Electric Fuse

Whenever there is an excessive current produced due to faulty wire connection, the circuit breaker switch opens. After repairing the faulty connection, we can close the circuit breaker switch. It is shown in the Figure 2.32.

Figure 2.32 circuit breakers

3. Electric furnace

Furnaces as shown in Figure 2.33 are used to manufacture a large number of technologically important materials such as steel, silicon carbide, quartz, gallium arsenide, etc). To produce temperatures up to 1500°C, molybdenum-nichrome wire wound on a silica tube is used. Carbon arc furnaces produce temperatures up to 3000 °C.

Figure 2.33 Electric furnace

4. Electrical lamp

It consists of a tungsten filament (melting point 3380 ^{0}C) kept inside a glass bulb and heated to incandescence by current. In incandescent electric lamps only about 5% of electrical energy is converted into light and the rest is wasted as heat. Electric discharge lamps, electric welding and electric arc also utilize the heating effect of current as shown in Figure 2.34.

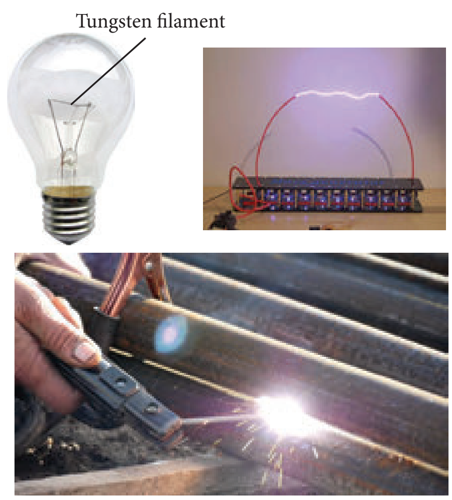


Figure 2.34 Electric bulb, electric arc and electric welding

2.7 THERMOELECTRIC EFFECT

Conversion of temperature differences into electrical voltage and vice versa is known as thermoelectric effect. A thermoelectric device generates voltage when there is a temperature difference on each side. If a voltage is applied, it generates a temperature difference.

2.7.1 Seebeck effect

Seebeck discovered that in a closed circuit consisting of two dissimilar metals, when the junctions are maintained at different temperatures an emf (potential difference) is developed. The current that flows due to the emf developed is called thermoelectric current. The two dissimilar metals connected to form two junctions is known as thermocouple (Figure 2.35).

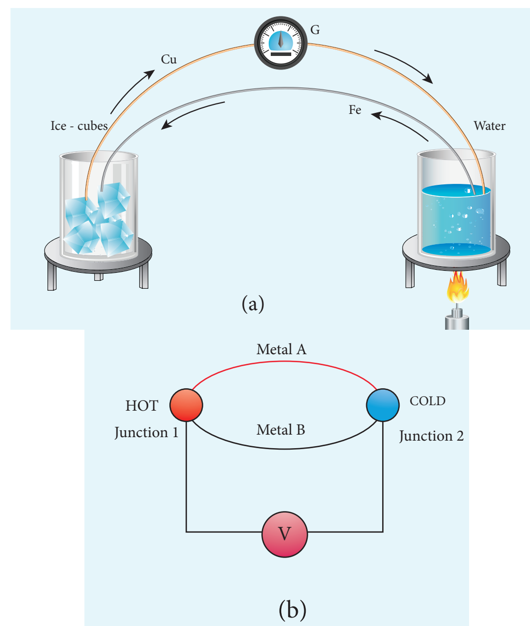


Figure 2.35 Seebeck effect (Thermocouple)

If the hot and cold junctions are interchanged, the direction of current also reverses. Hence the effect is reversible.

The magnitude of the emf developed in a thermocouple depends on (i) the nature of the metals forming the couple and (ii) the temperature difference between the junctions.

Applications of Seebeck effect

1. Seebeck effect is used in thermoelectric generators (Seebeck generators).These thermoelectric generators are used in power plants to convert waste heat into electricity.

2. This effect is utilized in automobiles as automotive thermoelectric generators for increasing fuel efficiency.
3. Seebeck effect is used in thermocouples and thermopiles to measure the temperature difference between the two objects.

2.7.2 Peltier effect

In 1834, Peltier discovered that **when an electric current is passed through a circuit of a thermocouple, heat is evolved at one junction and absorbed at the other junction. This is known as Peltier effect.**

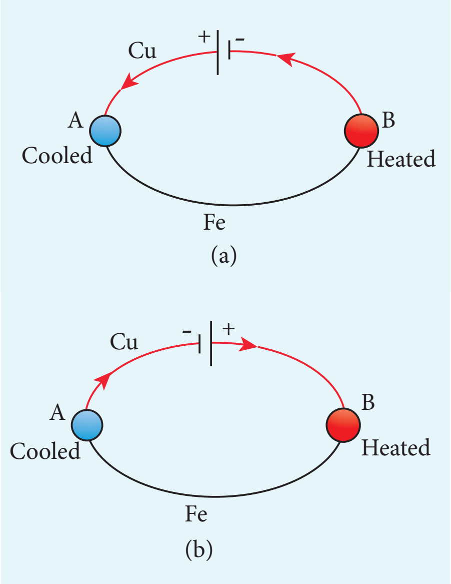


Figure 2.36 Peltier effect: Cu – Fe thermocouple

In the Cu-Fe thermocouple the junctions A and B are maintained at the same temperature. Let a current from a battery flow through the thermocouple (Figure 2.36 (a)). At the junction A, where the current flows from Cu to Fe, heat is absorbed and the junction A becomes cold. At the junction B, where the current flows from Fe to Cu heat is liberated and it becomes hot. When the direction of current is reversed, junction A gets heated and junction B gets cooled as shown in the Figure 2.36(b). Hence Peltier effect is reversible.

2.7.3 Thomson effect

Thomson showed that **if two points in a conductor are at different temperatures, the density of electrons at these points will differ and as a result the potential difference is created between these points. Thomson effect is also reversible.**

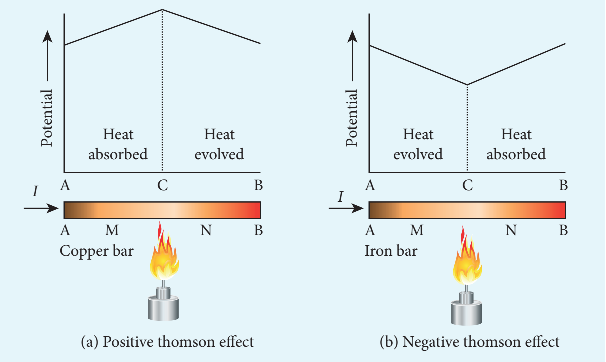


Figure 2.37 (a) Positive Thomson effect (b) Negative Thomson effect

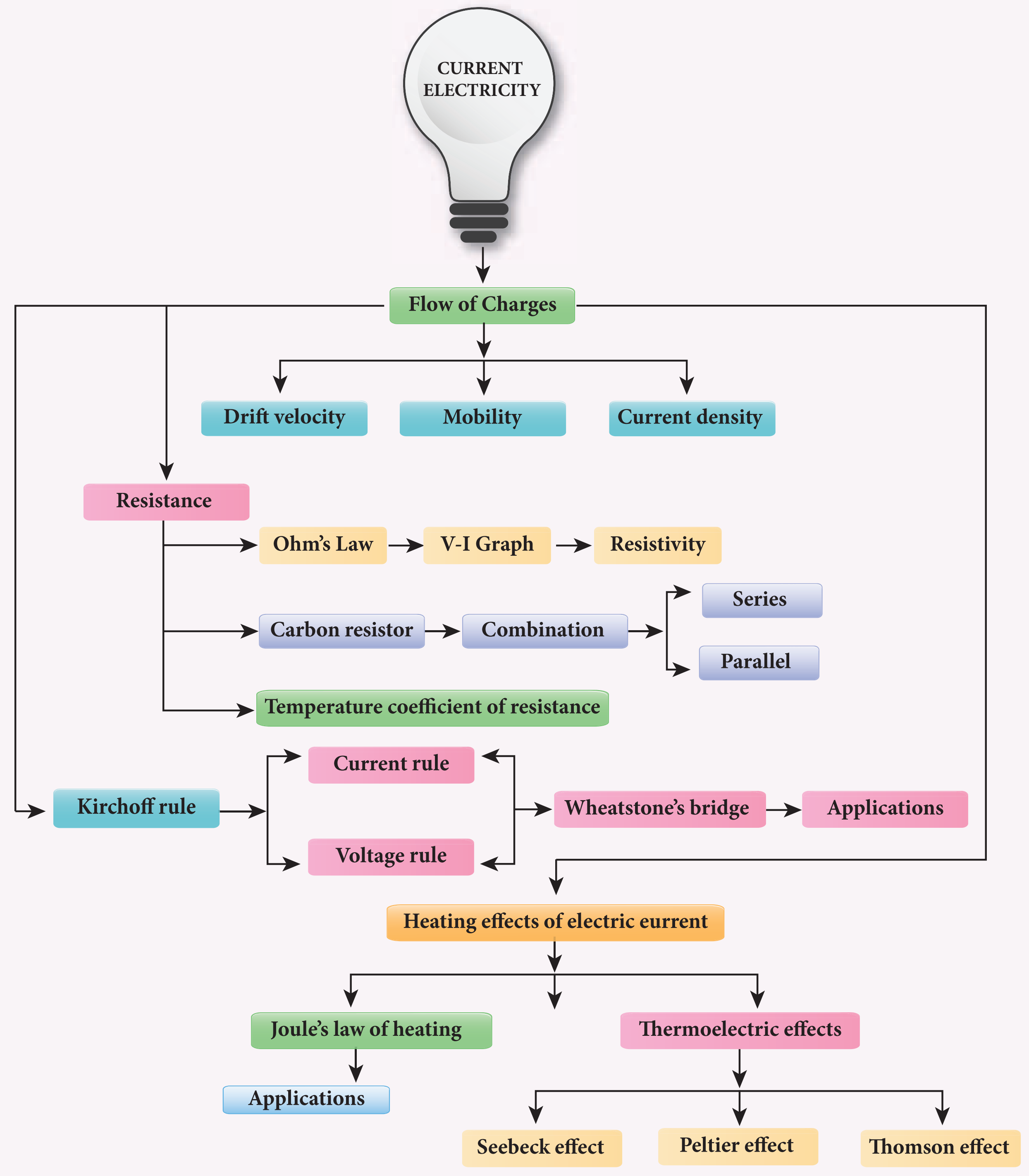
If current is passed through a copper bar AB which is heated at the middle point C, the point C will be at higher potential. This indicates that the heat is absorbed along AC and evolved along CB of the conductor as shown in Figure 2.37(a). Thus heat is transferred due to the current flow in the direction of the current. It is called positive Thomson effect. Similar effect is observed in metals like silver, zinc, and cadmium.

When the copper bar is replaced by an iron bar, heat is evolved along CA and absorbed along BC. Thus heat is transferred due to the current flow in the direction opposite to the direction of current. It is called negative Thomson effect as shown in the Figure 2.37(b). Similar effect is observed in metals like platinum, nickel, cobalt, and mercury.

SUMMARY

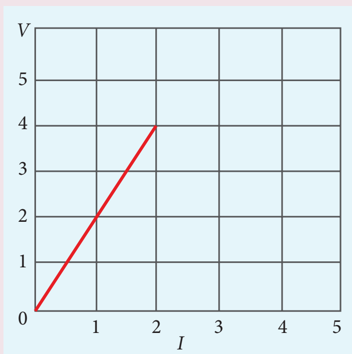
- The current, I flowing in a conductor $I = \frac{dQ}{dt}$, where dQ is the charge that flows through a cross-section in a time interval dt. SI unit of current is ampere (A). $1A = 1\ C\ s^{-1}$.
- The current density J in a conductor is the current flowing per unit area. $\left(J = \frac{I}{A}\right)$
- Current is a scalar but current density is a vector.
- The general form of Ohm's law $\vec{J} = \sigma \vec{E}$
- Practical form of Ohm's law states that $V \propto I$, or $V = IR$ where I is the current.
- The resistance R of a conductor is $R = \frac{V}{I}$. SI unit of resistance is ohm (Ω) and $1\ \Omega = \frac{1V}{1A}$
- The resistance of a material $R = \rho \frac{l}{A}$ where l is length of the material and A is the area of cross section.
- The resistivity of a material determines how much resistance it offers to the flow of current.
- The equivalent resistance (R_S) of several resistances (R_1, R_2, R_3........) connected in series combination is $R_s = (R_1 + R_2 + R_3 ....)$
- The equivalent resistance (R_P) of several resistances (R_1, R_2, R_3........) connected in parallel combination is $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$
- Kirchoff's first rule (Current rule or junction rule): The algebraic sum of the currents at any junction is zero.
- Kirchoff's second rule (Voltage rule or loop rule): In a closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf included in the circuit.
- Electric power is the rate at which energy is transformed.
- If a current I flows across a potential difference V, the power delivered to the circuit is P = IV.
- In a resistor R, the electrical power converted to heat is $P = I^2R = \frac{V^2}{R}$
- The energy equivalent of one kilowatt-hour (kWh) is $1kWh = 3.6 \times 10^6$ J.
- Metre bridge is one form of Wheatstone's bridge.
- Potentiometer is used to compare potential differences.
- Joule's law of heating is H = VIt (or) $H = I^2Rt$.
- Thermoelectric effect: Conversion of temperature differences into electrical voltage and vice versa.

CONCEPT MAP


CURRENT ELECTRICITY
Flow of Charges
Drift velocity
Mobility
Current density
Resistance
Ohm's Law
V-I Graph
Resistivity
Carbon resistor
Combination
Series
Parallel
Temperature coefficient of resistance
Kirchoff rule
Current rule
Voltage rule
Wheatstone's bridge
Applications
Heating effects of electric current
Joule's law of heating
Applications
Thermoelectric effects
Seebeck effect
Peltier effect
Thomson effect

I Multiple Choice Questions

1. The following graph shows current versus voltage values of some unknown conductor. What is the resistance of this conductor?

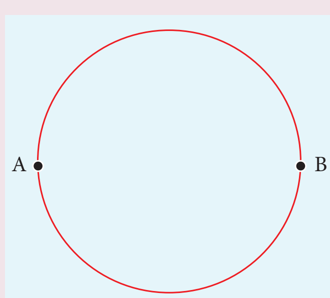


QMAJDW

(a) 2 ohm (b) 4 ohm

(c) 8 ohm (d)1 ohm

2. A wire of resistance 2 ohms per meter is bent to form a circle of radius 1m. The equivalent resistance between its two diametrically opposite points, A and B as shown in the figure is



(a) $\pi\ \Omega$ (b) $\frac{\pi}{2}\Omega$

(c) $2\pi\Omega$ (d) $\frac{\pi}{4}\Omega$

3. A toaster operating at 240 V has a resistance of 120 Ω. The power is

a) 400 W b) 2 W

c) 480 W d) 240 W

4. A carbon resistor of (47 ± 4.7) k Ω to be marked with rings of different colours for its identification. The colour code sequence will be

a) Yellow – Green – Violet – Gold

b) Yellow – Violet – Orange – Silver

c) Violet – Yellow – Orange – Silver

d) Green – Orange – Violet - Gold

5. What is the value of resistance of the following resistor?

(a)100 k Ω (b)10 k Ω

(c) 1k Ω (d)1000 k Ω

6. Two wires of A and B with circular cross section made up of the same material with equal lengths. Suppose $R_A = 3\ R_B$, then what is the ratio of radius of wire A to that of B?

(a) 3 (b) $\sqrt{3}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{3}$

7. A wire connected to a power supply of 230 V has power dissipation P_1. Suppose the wire is cut into two equal pieces and connected parallel to the same power supply. In this case power dissipation is P_2. The ratio $\frac{P_2}{P_1}$ is

(a)1 (b) 2

(c) 3 (d) 4

8. In India electricity is supplied for domestic use at 220 V. It is supplied at 110 V in USA. If the resistance of a 60W bulb for use in India is R, the resistance of a 60W bulb for use in USA will be

(a) R (b) 2R

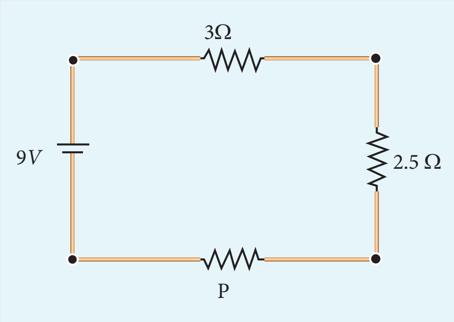
(c) $\frac{R}{4}$ (d) $\frac{R}{2}$

9. In a large building, there are 15 bulbs of 40W, 5 bulbs of 100W, 5 fans of 80W and 1 heater of 1kW are connected. The voltage of electric mains is 220V. The minimum capacity of the main fuse of the building will be (IIT-JEE 2014)

(a) 14 A (b) 8 A

(c) 10 A (d) 12 A

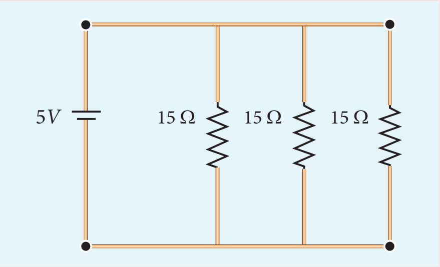
10. There is a current of 1.0 A in the circuit shown below. What is the resistance of P ?



a) 1.5 Ω b) 2.5 Ω

c) 3.5 Ω d) 4.5 Ω

11. What is the current out of the battery?



a) 1A b) 2A

c) 3A d) 4A

12. The temperature coefficient of resistance of a wire is 0.00125 per °C. At 300 K, its resistance is 1 Ω. The resistance of the wire will be 2 Ω at

a) 1154 K b) 1100 K

c) 1400 K d) 1127 K

13. The internal resistance of a 2.1 V cell which gives a current of 0.2 A through a resistance of 10 Ω is

a) 0.2 Ω b) 0.5 Ω

c) 0.8 Ω d) 1.0 Ω

14. A piece of copper and another of germanium are cooled from room temperature to 80 K. The resistance of

a) each of them increases

b) each of them decreases

c) copper increases and germanium decreases

d) copper decreases and germanium increases

15. In Joule's heating law, when I and t are constant, if the H is taken along the y axis and I^2 along the x axis, the graph is

a) straight line b) parabola

c) circle d) ellipse

Answers

1) a	**2)** b	**3)** c	**4)** b	**5)** a
6) c	**7)** d	**8)** c	**9)** d	**10)** c
11) a	**12)** d	**13)** b	**14)** d	**15)** a

II Short Answer Questions

1. Why current is a scalar?
2. Distinguish between drift velocity and mobility.
3. State microscopic form of Ohm's law.

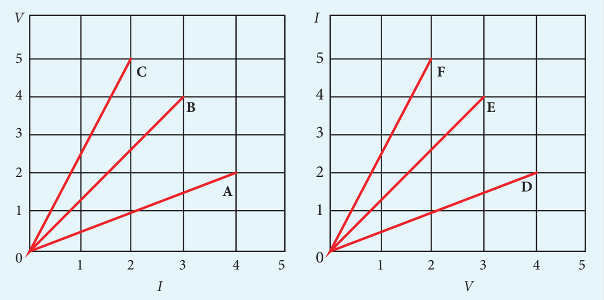
4. State macroscopic form of Ohm's law.
5. What are ohmic and non ohmic devices?
6. Define electrical resistivity.
7. Define temperature coefficient of resistance.
8. What is superconductivity?
9. What is electric power and electric energy?
10. Define current density.
11. Derive the expression for power P=VI in electrical circuit.
12. Write down the various forms of expression for power in electrical circuit.
13. State Kirchhoff's current rule.
14. State Kirchhoff's voltage rule.
15. State the principle of potentiometer.
16. What do you mean by internal resistance of a cell?
17. State Joule's law of heating.
18. What is Seebeck effect?
19. What is Thomson effect?
20. What is Peltier effect?
21. State the applications of Seebeck effect.

III Long Answer Questions

1. Describe the microscopic model of current and obtain general form of Ohm's law
2. Obtain the macroscopic form of Ohm's law from its microscopic form and discuss its limitation.
3. Explain the equivalent resistance of a series and parallel resistor network
4. Explain the determination of the internal resistance of a cell using voltmeter.
5. State and explain Kirchhoff's rules.
6. Obtain the condition for bridge balance in Wheatstone's bridge.
7. Explain the determination of unknown resistance using meter bridge.
8. How the emf of two cells are compared using potentiometer?

IV Numerical problems

1. The following graphs represent the current versus voltage and voltage versus current for the six conductors A,B,C,D,E and F. Which conductor has least resistance and which has maximum resistance?



Ans: Least: $R_F = 0.4\ \Omega$, maximum $R_C = 2.5\ \Omega$

2. Lightning is very good example of natural current. In typical lightning, there is 10^9 J energy transfer across the potential difference of 5×10^7 V during a time interval of 0.2 s.

Using this information, estimate the following quantities (a) total amount of charge transferred between cloud and ground (b) the current in the lightning bolt (c) the power delivered in 0.2 s.

Ans: charge = 20 C, I = 100 A, P = 5 GW

3. A copper wire of 10^{-6} m^2 area of cross section, carries a current of 2 A. If the number of electrons per cubic meter is 8×10^{28}, calculate the current density and average drift velocity.

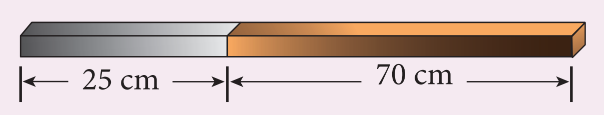
Ans: $J = 2 \times 10^6$ Am^{-2} ; $v_d = 15.6 \times 10^{-5}$ ms^{-1}

4. The resistance of a nichrome wire at 0 ^{0}C is 10 Ω. If its temperature coefficient of resistance is 0.004/^{0}C, find its resistance at boiling point of water. Comment on the result.

Ans: $R_T = 14$ Ω.

As the temperature increases the resistance of the wire also increases.

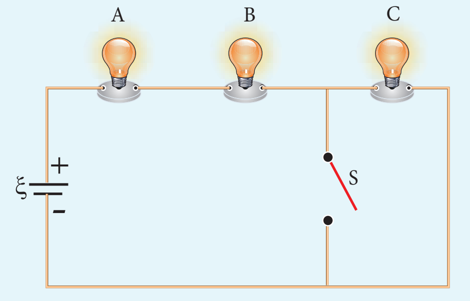
5. The rod given in the figure is made up of two different materials.



Both have square cross sections of 3 mm side. The resistivity of the first material is 4 x 10^{-3} Ω.m and it is 25 cm long while second material has resistivity of 5 x 10^{-3} Ω.m and is of 70 cm long. What is the resistivity of rod between its ends?

Ans: 500 Ω

6. Three identical lamps each having a resistance R are connected to the battery of emf as shown in the figure.

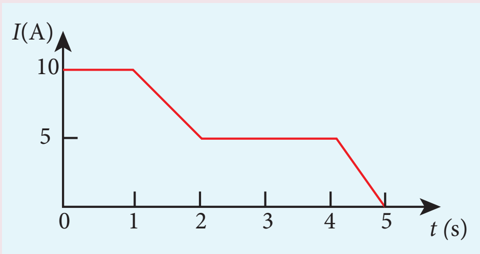


Suddenly the switch S is closed. (a) Calculate the current in the circuit when S is open and closed (b) What happens to the intensities of the bulbs A,B and C. (c) Calculate the voltage across the three bulbs when S is open and closed (d) Calculate the power delivered to the circuit when S is opened and closed (e) Does the power delivered to the circuit decreases, increases or remain same?

Ans:

Electrical quantities	Switch S is open	Switch S is closed
Current	$\frac{\xi}{3R}$	$\frac{\xi}{2R}$
Voltage	$V_A = \frac{\xi}{3R}$, $V_B = \frac{\xi}{3R}$, $V_C = \frac{\xi}{3R}$	$V_A = \frac{\xi}{2R}$, $V_B = \frac{\xi}{2R}$, $V_C = 0$
Power	$P_A = \frac{\xi^2}{9R}$, $P_B = \frac{\xi^2}{9R}$, $P_C = \frac{\xi^2}{9R}$	$P_A = \frac{\xi^2}{4R}$, $P_B = \frac{\xi^2}{4R}$, $P_C = 0$ Total power increases
Intensity	All the bulbs glow with equal intensity	The intensities of the bulbs A and B equally increase. Bulb C will not glow since no current pass through it.

7. The current through an element is shown in the figure. Determine the total charge that pass through the element at a) t = 0 s, b) t = 2 s, c) t = 5s



Ans: At t= 0s,dq = 0 C, At t=2 s,
dq = 10 C; At t=5 s, dq = 0 C

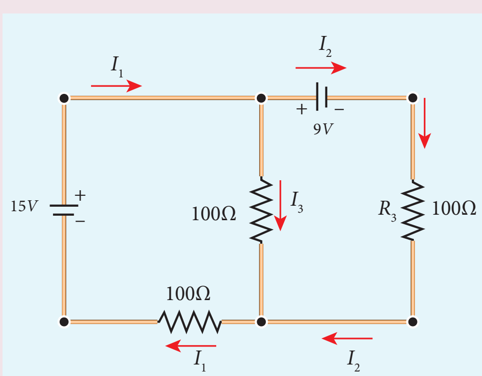
8. An electronics hobbyist is building a radio which requires 150 Ω in her circuit, but she has only 220 Ω, 79 Ω and 92 Ω resistors available. How can she connect the available resistors to get desired value of resistance?

Ans: Parallel combination of 220 Ω and 79 Ω in series with 92 Ω

9. A cell supplies a current of 0.9 A through a 2 Ω resistor and a current of 0.3 A through a 7 Ω resistor. Calculate the internal resistance of the cell.

Ans: 0.5 Ω

10. Calculate the currents in the following circuit.

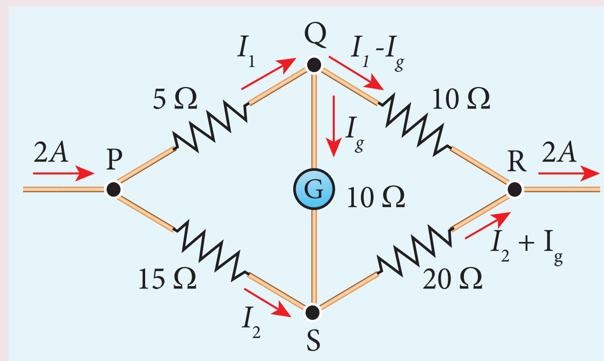


Ans : I_1 = 0.070 A, I_2 = -0.010 A and I_3 = 0.080 A

11. A potentiometer wire has a length of 4 m and resistance of 20 Ω. It is connected in series with resistance of 2980 Ω and a cell of emf 4 V. Calculate the potential along the wire.

Ans: Potential = 0.65×10^{-2} V m^{-1}.

12. Determine the current flowing through the galvanometer (G) as shown in the figure.



Ans: $I_g = \frac{1}{11} A$

13. Two cells each of 5V are connected in series across a 8 Ω resistor and three parallel resistors of 4 Ω, 6 Ω and 12 Ω. Draw a circuit diagram for the above arrangement. Calculate i) the current drawn from the cell (ii) current through each resistor

Ans: The current at 4 Ω ,$I = \frac{2}{4} = 0.5A$,

the current at 6 Ω, $I = \frac{2}{6} = 0.33\,A$,

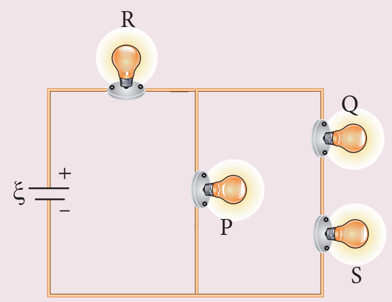
the current at 12 Ω, $I = \frac{2}{12} = 0.17\,A$

14. Four light bulbs P, Q, R, S are connected in a circuit of unknown arrangement. When each bulb is removed one at a time and replaced, the following behavior is observed.

	P	Q	R	S
P removed	*	on	on	on
Q removed	on	*	on	off
R removed	off	off	*	off
S removed	on	off	on	*

Draw the circuit diagram for these bulbs.

Ans:



15. In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63 cm, what is the emf of the second cell?

Ans: emf of the second cell is 2.25 V

BOOKS FOR REFERENCE:

1. Douglas C.Giancoli, , "*Physics for Scientist &Engineers with Modern Physics*", Pearson Prentice Hall, Fourth edition
2. James Walker, *Physics*, Pearson- Addison Wesley publishers, Fourth edition
3. Tipler, Mosca, "*Physics for scientist and Engineers with Modern Physics*", Freeman and Company, sixth edition
4. Purcell, Morin, *Electricity and magnetism*, Cambridge university press, third edition
5. Serway and Jewett, "*Physics for Scientist and Engineers with Modern Physics*", Brook/Coole publishers, eighth edition
6. Tarasov and Tarasova, "*Questions and problems in School Physics*", Mir Publishers
7. H.C.Verma, "*Concepts of Physics Vol 2*, Bharthi Bhawan publishers
8. Eric Roger, *Physics for the Inquiring Mind*, Princeton University press

ICT CORNER

Electric current

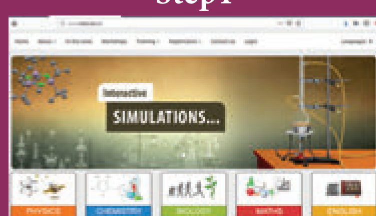
In this activity you will be able to
(a) measure the potential difference of cells
(b) measure the internal resistance of a given primary cell

Topic: Potentiometer

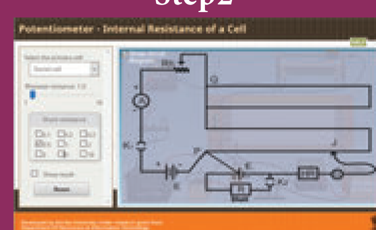
STEPS:

- Open the browser and type "olabs.edu.in" in the address bar. Click physics tab and then click "Potentiometer-Internal Resistance of a Cell" in class 12 section. Go to "simulator" tab to do the experiment.
- Construct the electric circuit as per the connection diagram by clicking "show circuit diagram" tab. You can connect wires between electric component by dragging the mouse between the component.
- To check whether the connections are correct or not, drag the jockey and place it at the two end points of the wire. If the galvanometer shows opposite deflections, the connections are correct. (keep both keys on)

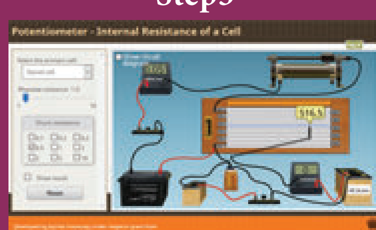
Step1



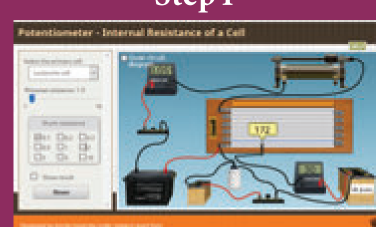
Step2



Step3



Step4



Find the balancing length. Calculate the internal resistance for the observed balancing lengths. Repeat the experiment for five times and take the average.

Note:

1. One time sign up is needed to do simulation. Then login using that username and password.
2. Read theory, procedure and animation to get the theory by clicking the corresponding tab.

URL:

http://amrita.olabs.edu.in/?sub=1&brch=6&sim=147&cnt=4

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

B263_12_PHYSICS_EM

UNIT 3

MAGNETISM AND MAGNETIC EFFECTS OF ELECTRIC CURRENT

"The magnetic force is animate, or imitates a soul; in many respects it surpasses the human soul while it is united to an organic body" – William Gilbert

Learning Objectives

In this unit, the student is exposed to

- Earth's magnetic field and magnetic elements
- Basic property of magnets
- Statement of Coulomb inverse square law of magnetism
- Magnetic dipole
- Magnetic induction at a point due to axial line and equatorial line
- Torque acting on a bar magnet in a uniform magnetic field
- Potential energy of a bar magnet placed in a uniform magnetic field
- Tangent law and tangent Galvanometer
- Magnetic properties – permeability, susceptibility etc
- Classification of magnetic materials – dia, para and ferro magnetic materials
- Concept of Hysteresis
- Magnetic effects of electric current – long straight conductor and circular coil
- Right hand thumb rule and Maxwell's right hand cork screw rule
- Biot-Savart's law – applications
- Current loop as a magnetic dipole
- Magnetic dipole moment of revolving electron
- Ampère's circuital law – applications
- Solenoid and toroid
- Lorentz force – charged particle moving in an electromagnetic field
- Cyclotron
- Force on a current carrying conductor in a magnetic field
- Force between two long parallel current carrying conductor
- Torque on a current loop in a magnetic field
- Moving coil Galvanometer

3IWE3S

UNIT-3(XII-Physics_Vol-1).indd 128 04-03-2019 10:33:14

3.1 INTRODUCTION TO MAGNETISM

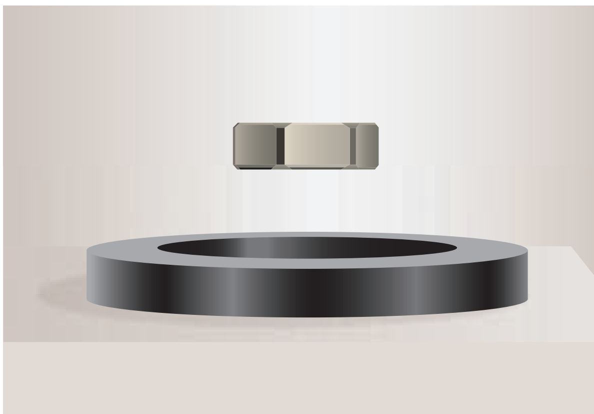


Figure 3.1: Magnetic levitation

Magnets! no doubt, its behaviour will attract everyone (see Figure 3.1). The world enjoys its benefits, to lead a modern luxurious life. The study of magnets fascinated scientists around our globe for many centuries and even now, door for research on magnets is still open.

Many birds and animals have magnetic sense in their eyes using Earth's magnetic field for navigation.

Magnetic sensing in eyes - for Zebrafinches bird, due to protein cryptochromes Cry4 present in retina, it uses Earth magnetic field for navigation

Magnetism is everywhere from tiny particles like electrons to the entire universe. Historically the word 'magnetism' was derived from iron ore magnetite ($Fe_3 O_4$). In olden days, magnets were used as magnetic compass for navigation, magnetic therapy for treatment and also used in magic shows.

In modern days, most of the things we use in our daily life contain magnets (Figure 3.2). Motors, cycle dynamo, loudspeakers, magnetic tapes used in audio and video recording, mobile phones, head phones, CD, pen-drive, hard disc of laptop, refrigerator door, generator are a few examples.

Earlier, both electricity and magnetism were thought to be two independent branches in physics. In 1820, H.C. Oersted observed the deflection of magnetic compass needle kept near a current carrying wire. This unified the two different branches, electricity and magnetism as a single subject 'electromagnetism' in physics.

In this unit, basics of magnets and their properties are given. Later, how a current carrying conductor (here only steady current, not time-varying current is considered) behaves like a magnet is presented.

(a) (b) (c) (d)

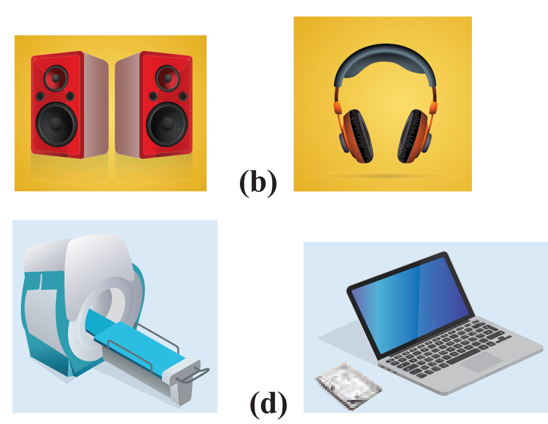


Figure 3.2 Uses of magnets in modern world – (a) speakers (b) head phones (c) MRI scan (d) Hard disc of laptop

3.1.1 Earth's magnetic field and magnetic elements

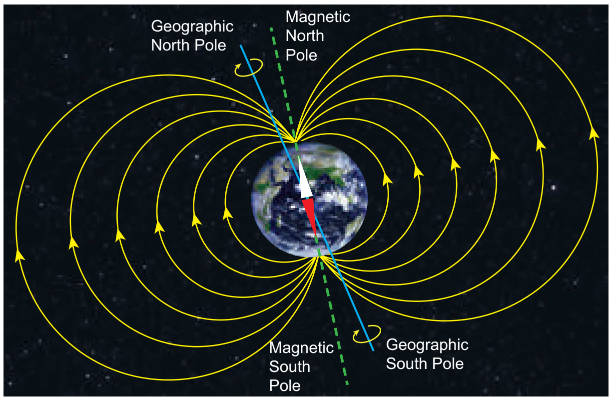


Figure 3.3 Earth's magnetic field

From the activities performed in lower classes, we have noticed that the needle in a magnetic compass or freely suspended magnet comes to rest in a position which is approximately along the geographical north-south direction of the Earth. William Gilbert in 1600 proposed that Earth itself behaves like a gigantic powerful bar magnet. But this theory is not successful because the temperature inside the Earth is very high and so it will not be possible for a magnet to retain its magnetism.

Gover suggested that the Earth's magnetic field is due to hot rays coming out from the Sun. These rays will heat up the air near equatorial region. Once air becomes hotter, it rises above and will move towards northern and southern hemispheres and get electrified. This may be responsible to magnetize the ferromagnetic materials near the Earth's surface. Till date, so many theories have been proposed. But none of the theory completely explains the cause for the Earth's magnetism.

The north pole of magnetic compass needle is attracted towards the magnetic south pole of the Earth which is near the geographic north pole (Figure 3.3). Similarly, the south pole of magnetic compass needle is attracted towards the geographic north pole of the Earth which is near magnetic north-pole. **The branch of physics which deals with the Earth's magnetic field is called Geomagnetism or Terrestrial magnetism.**

There are three quantities required to specify the magnetic field of the Earth on its surface, which are often called as the elements of the Earth's magnetic field. They are

(a) magnetic declination (D)
(b) magnetic dip or inclination (I)
(c) the horizontal component of the Earth's magnetic field (B_H)

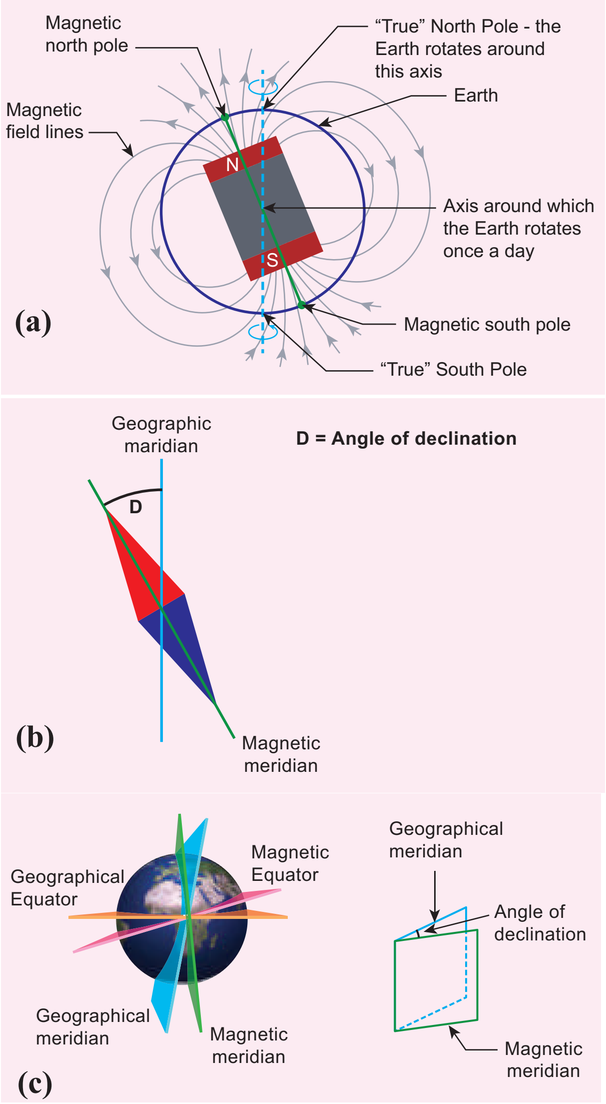


Figure 3.4 Declination angle

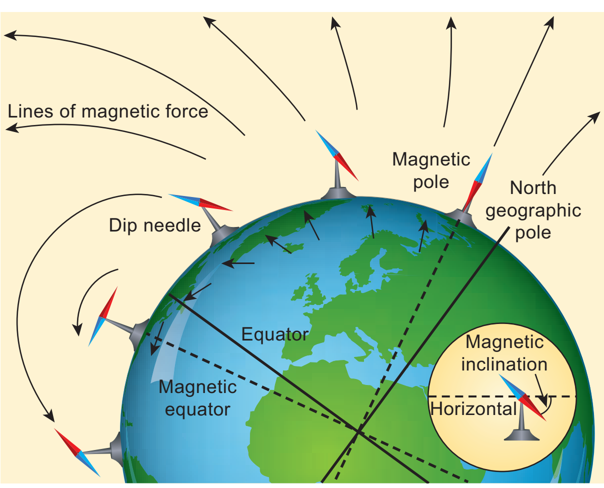
UNIT-3(XII-Physics_Vol-1).indd 130 04-03-2019 10:33:15

Day and night occur because Earth spins about an axis called geographic axis. A vertical plane passing through the geographic axis is called geographic meridian and a great circle perpendicular to Earth's geographic axis is called geographic equator.

The straight line which connects magnetic poles of Earth is known as magnetic axis. A vertical plane passing through magnetic axis is called magnetic meridian and a great circle perpendicular to Earth's magnetic axis is called magnetic equator.

When a magnetic needle is freely suspended, the alignment of the magnet does not exactly lie along the geographic meridian as shown in Figure 3.4. **The angle between magnetic meridian at a point and geographical meridian is called the *declination or magnetic declination (D)*.** At higher latitudes, the declination is greater whereas near the equator, the declination is smaller. In India, declination angle is very small and for Chennai, magnetic declination angle is -1° 8' (which is negative (west)).

The angle subtended by the Earth's total magnetic field $\vec{B}$ with the horizontal direction in the magnetic meridian is called dip or magnetic inclination (I) at that point (Figure 3.5). For Chennai, inclination angle is 14° 16'. **The component of Earth's magnetic field along the horizontal direction in the magnetic meridian is called horizontal component of Earth's magnetic field, denoted by B_H.**



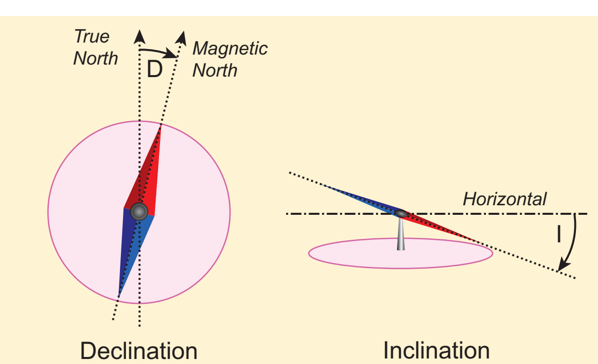


Figure 3.5 Inclination angle

Let B_E be the net Earth's magnetic field at a point P on the surface of the Earth. B_E can be resolved into two perpendicular components.

Horizontal component $B_H = B_E \cos I$ (3.1)

Vertical component $B_V = B_E \sin I$ (3.2)

Dividing equation (3.2) and (3.1), we get

$$\tan I = \frac{B_V}{B_H} \qquad (3.3)$$

(i) At magnetic equator

The Earth's magnetic field is parallel to the surface of the Earth (i.e., horizontal) which implies that the needle of magnetic compass rests horizontally at an angle of dip, I = 0° as shown in figure 3.6.

$B_H = B_E$

$B_V = 0$

This implies that the horizontal component is maximum at equator and vertical component is zero at equator.

(ii) At magnetic poles

The Earth's magnetic field is perpendicular to the surface of the Earth (i.e., vertical) which implies that the needle

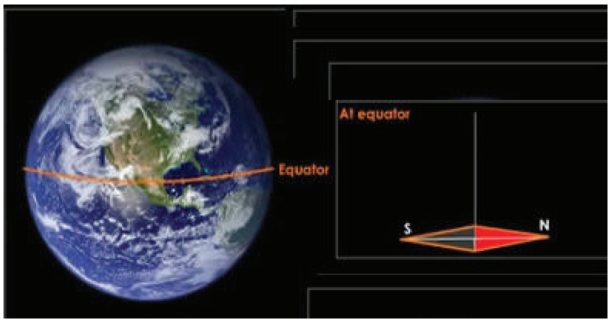


Figure 3.6: Needle of magnetic compass rests horizontally at an angle of dip – at magnetic equator

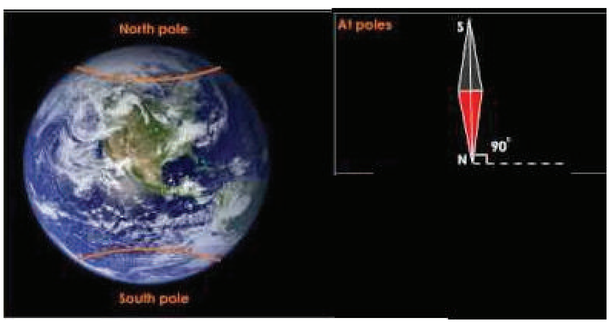


Figure 3.7: Needle of magnetic compass rests vertically at an angle of dip – at magnetic poles

of magnetic compass rests vertically at an angle of dip, I = 90° as shown in Figure 3.7. Hence,

$B_H = 0$

$B_V = B_E$

This implies that the vertical component is maximum at poles and horizontal component is zero at poles.

EXAMPLE 3.1

The horizontal component and vertical components of Earth's magnetic field at a place are 0.15 *G* and 0.26 *G* respectively. Calculate the angle of dip and resultant magnetic field.

Solution:

$B_H = 0.15$ G and $B_V = 0.26$ G

$$\tan I = \frac{0.26}{0.15} \Rightarrow I = \tan^{-1}(1.732) = 60^\circ$$

The resultant magnetic field of the Earth is

$$B = \sqrt{B_H^2 + B_V^2} = 0.3G$$

3.1.2 Basic properties of magnets

Some basic terminologies and properties used in describing bar magnet.

(a) Magnetic dipole moment

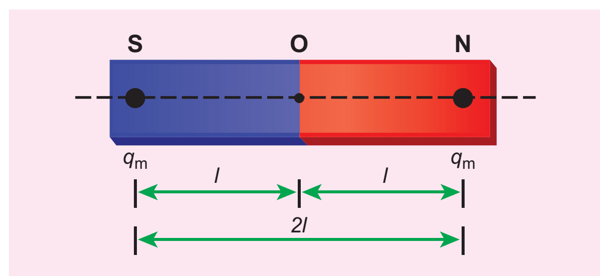


Figure 3.8 **A bar magnet**

Consider a bar magnet as shown in Figure 3.8. Let q_m be the pole strength (it is also called as magnetic charge) of the magnetic pole and let l be the distance between the geometrical center of bar magnet O and one end of the pole. **The magnetic dipole moment is defined as the product of its pole strength and magnetic length**. It is a vector quantity, denoted by $\vec{p}_m$.

$$\vec{p}_m = q_m \vec{d} \qquad (3.4)$$

where $\vec{d}$ is the vector drawn from south pole to north pole and its magnitude $|\vec{d}| = 2l$.

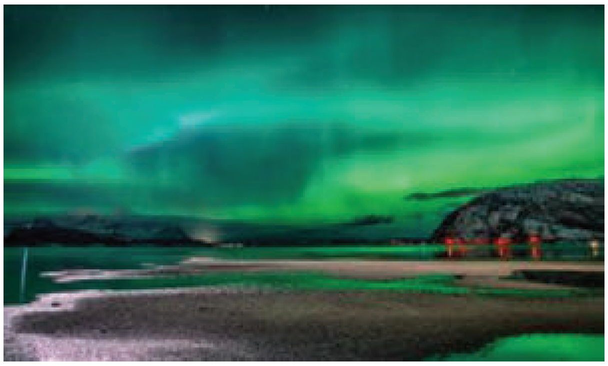
The magnitude of magnetic dipole moment is $p_m = 2q_m l$

DO YOU KNOW?

Aurora Borealis and Aurora Australis

People living at high latitude regions (near Arctic or Antarctic) might experience dazzling coloured natural lights across the night sky. This ethereal display on the sky is known as aurora borealis (northern lights) or



aurora australis (southern lights). These lights are often called as polar lights. The lights are seen above the magnetic poles of the northern and southern hemispheres. They are called as "Aurora borealis" in the north and "Aurora australis" in the south. This occurs as a result of interaction between the gaseous particles in the Earth's atmosphere with highly charged particles released from the Sun's atmosphere through solar wind. These particles emit light due to collision and variations in colour are due to the type of the gas particles that take part in the collisions. A pale yellowish – green colour is produced when the ionized oxygen takes part in the collision and a blue or purplish – red aurora is produced due to ionized nitrogen molecules.

The SI unit of magnetic moment is A m^2. Note that the direction of magnetic moment is from South pole to North pole.

(b) Magnetic field

Magnetic field is the region or space around every magnet within which its influence can be felt by keeping another magnet in that region. The magnetic field $\vec{B}$ at a point is defined as a force experienced by the bar magnet of unit pole strength.

$$\vec{B} = \frac{1}{q_m}\vec{F} \qquad (3.5)$$

Its unit is N A^{-1} m^{-1}.

(c) Types of magnets

Magnets are classified into natural magnets and artificial magnets. For example, iron, cobalt, nickel, etc. are natural magnets. Strengths of natural magnets are very weak and the shapes of the magnet are irregular. Artificial magnets are made by us in order to have desired shape and strength. If the magnet is in the form of rectangular shape or cylindrical shape, then it is known as bar magnet.

Properties of magnet

The following are the properties of bar magnet (Figure 3.9)

1. A freely suspended bar magnet will always point along the north-south direction.
2. A magnet attracts another magnet or magnetic substances towards itself. The attractive force is maximum near the end of the bar magnet. When a bar magnet is dipped into iron filling, they cling to the ends of the magnet.

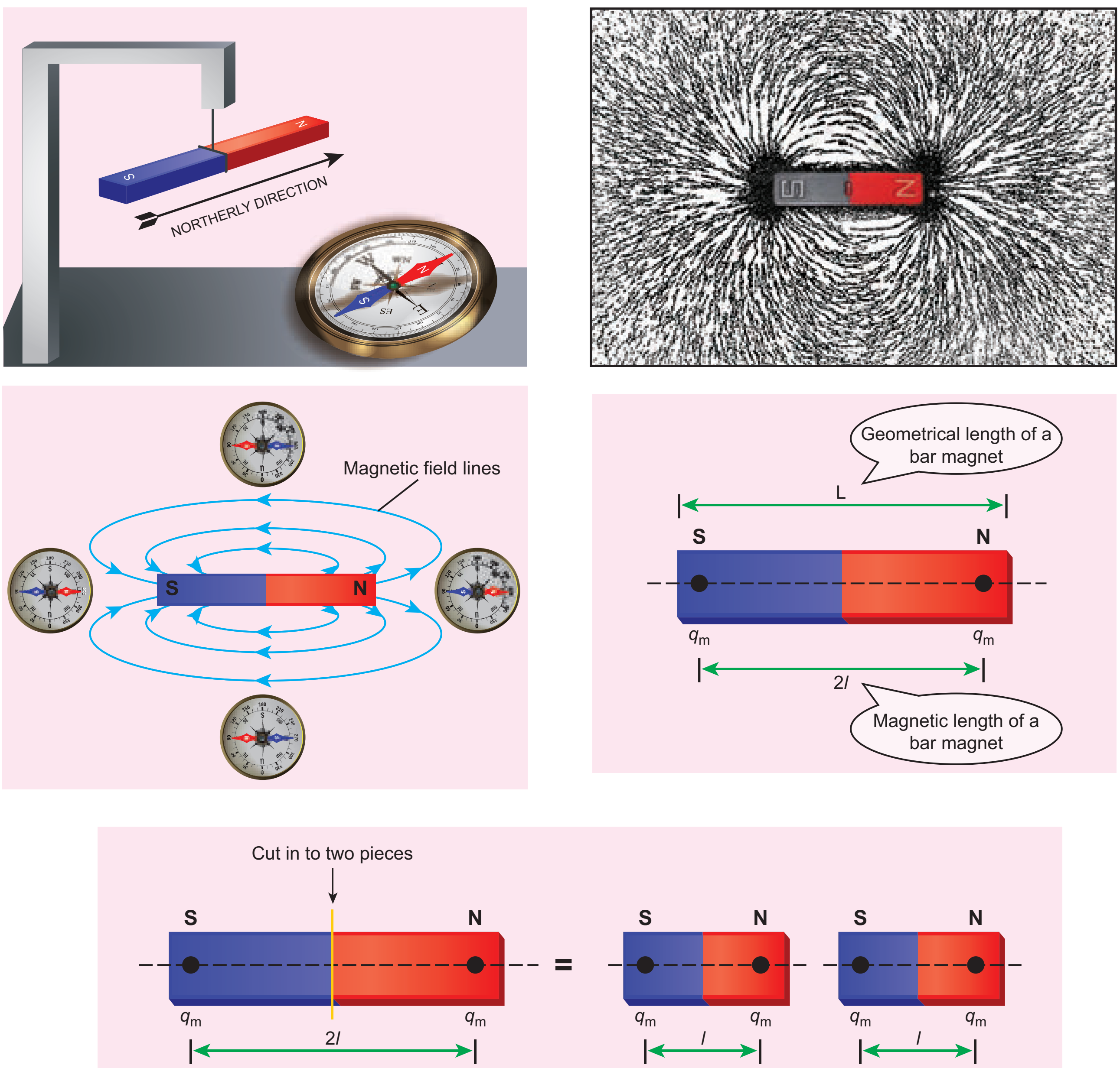


Figure 3.9 Properties of bar magnet

3. When a magnet is broken into pieces, each piece behaves like a magnet with poles at its ends.
4. Two poles of a magnet have pole strength equal to one another.
5. The length of the bar magnet is called geometrical length and the length between two magnetic poles in a bar magnet is called magnetic length. Magnetic length is always slightly smaller than geometrical length. The ratio of magnetic length and geometrical length is $\frac{5}{6}$.

$$\frac{\textit{Magnetic length}}{\textit{Geometrical length}} = \frac{5}{6} = 0.833$$

EXAMPLE 3.2

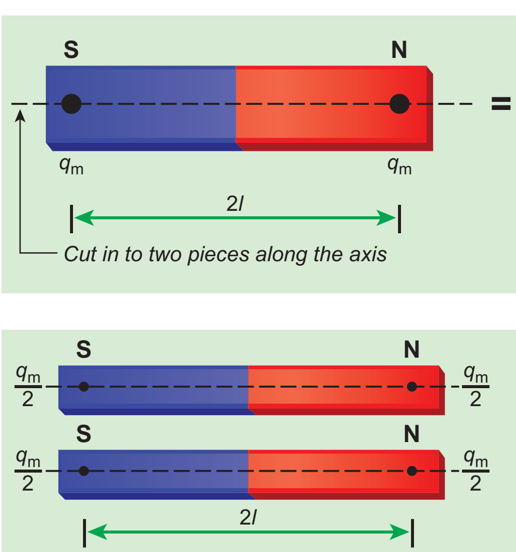
Let the magnetic moment of a bar magnet be $\vec{p}_m$ whose magnetic length is d = 2l and pole strength is q_m. Compute the magnetic moment of the bar magnet when it is cut into two pieces

(a) along its length

(b) perpendicular to its length.

Solution

(a) a bar magnet cut into two pieces along its length:



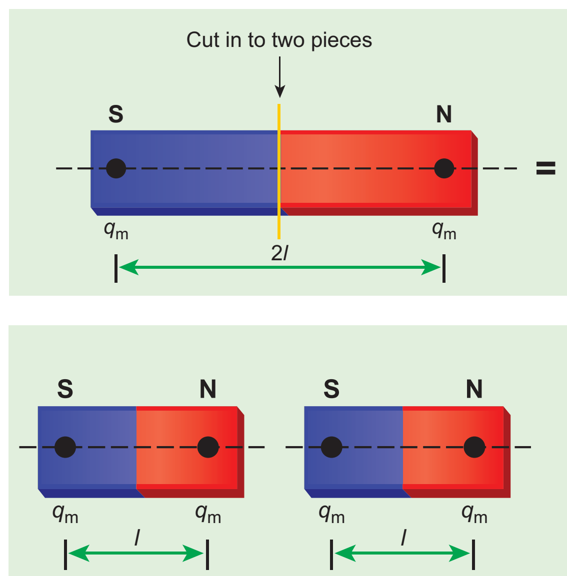
When the bar magnet is cut along the axis into two pieces, new magnetic pole strength is $q'_m = \frac{q_m}{2}$ but magnetic length does not change. So, the magnetic moment is

$$p'_m = q'_m 2l$$

$$p'_m = \frac{q_m}{2} 2l = \frac{1}{2}(q_m 2l) = \frac{1}{2} p_m$$

In vector notation, $\vec{p}'_m = \frac{1}{2}\vec{p}_m$

(b) a bar magnet cut into two pieces perpendicular to the axis:



When the bar magnet is cut perpendicular to the axis into two pieces, magnetic pole strength will not change but magnetic length will be halved. So the magnetic moment is

$$p'_m = q_m \times \frac{1}{2}(2l) = \frac{1}{2}(q_m \cdot 2l) = \frac{1}{2} p_m$$

In vector notation $\vec{p}'_m = \frac{1}{2}\vec{p}_m$

Note

(i) Pole strength is a scalar quantity with dimension $[M^0LT^0A]$. Its SI unit is $N\,T^{-1}$ (newton per tesla) or A m (ampere-metre).

(ii) Like positive and negative charges in electrostatics, north pole of a magnet experiences a force in the direction of magnetic field while south pole of a magnet experiences force opposite to the magnetic field.

(iii) Pole strength depends on the nature of materials of the magnet, area of cross-section and the state of magnetization.

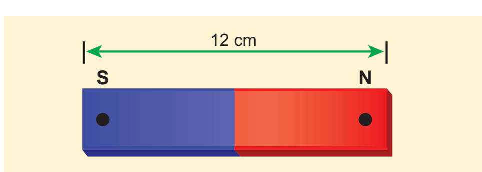
(iv) If a magnet is cut into two equal halves along the length then pole strength is reduced to half.

(v) If a magnet is cut into two equal halves perpendicular to the length, then pole strength remains same.

(vi) If a magnet is cut into two pieces, we will not get separate north and south poles. Instead, we get two magnets. In other words, isolated monopole does not exist in nature.

EXAMPLE 3.3

Compute the magnetic length of a uniform bar magnet if the geometrical length of the magnet is 12 cm. Mark the positions of magnetic pole points.



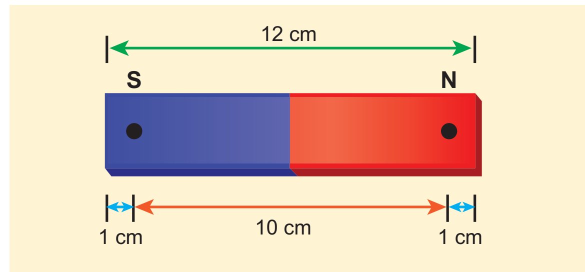
Solution

Geometrical length of the bar magnet is 12 cm

$$Magnetic\ length = \frac{5}{6}\times\left(geometrical\,length\right)$$

$$= \frac{5}{6}\times 12 = 10\,cm$$

In this figure, the dot implies the pole points.



Magnetic field lines

1. Magnetic field lines are continuous closed curves. The direction of magnetic field lines is from North pole to South pole outside the magnet (Figure 3.10) and South pole to North pole inside the magnet.

2. The direction of magnetic field at any point on the curve is known by drawing tangent to the magnetic line of force at that point. In the Figure No. 3.10 (b), the tangent drawn at points P, Q and R gives the direction of magnetic field $\vec{B}$ at that point.

3. Magnetic field lines never intersect each other. Otherwise, the magnetic compass needle would point towards two directions, which is not possible.

4. The degree of closeness of the field lines determines the relative strength of the magnetic field. The magnetic field is strong where magnetic field lines crowd and weak where magnetic field lines thin out.

(d) Magnetic flux

The number of magnetic field lines crossing per unit area is called magnetic flux Φ_B. Mathematically, the magnetic flux through a surface of area A in a uniform magnetic field is defined as

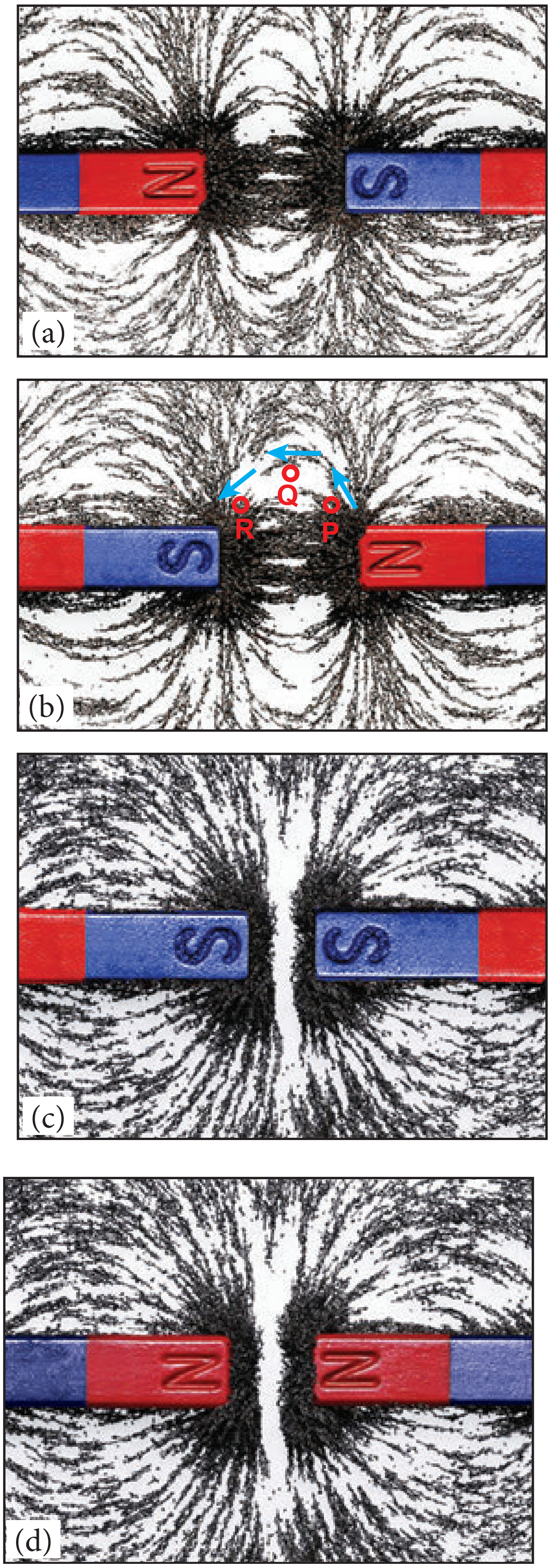


Figure 3.10 Properties of magnetic field lines– unlike poles attracts each other shown in picture (a) and (b) like poles repel each other-shown in picture (c) and (d)

$$\Phi_B = \vec{B}.\vec{A} = BA\cos\theta = B_\perp A \qquad (3.6)$$

where θ is the angle between $\vec{B}$ and $\vec{A}$ as shown in Figure 3.11.

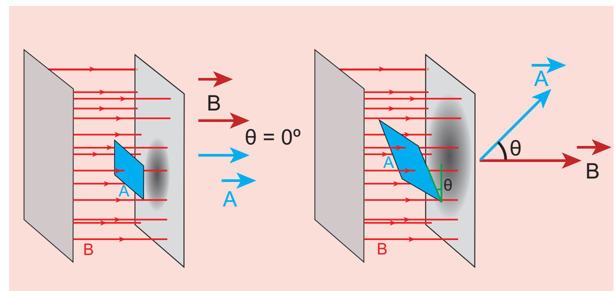


Figure 3.11 Magnetic flux

Special cases

(a) When $\vec{B}$ is normal to the surface i.e., $\theta = 0^o$, the magnetic flux is Φ_B = BA (maximum).

(b) When $\vec{B}$ is parallel to the surface i.e., $\theta = 90^o$, the magnetic flux is $\Phi_B = 0$.

Suppose the magnetic field is not uniform over the surface, the equation (3.6) can be written as

$$\Phi_B = \int \vec{B}.d\vec{A}$$

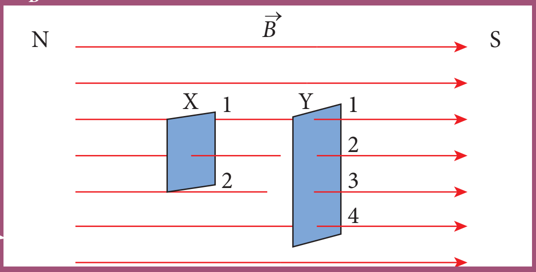
Magnetic flux is a scalar quantity. The SI unit for magnetic flux is weber, which is denoted by symbol Wb. Dimensional formula for magnetic flux is $\left[ML^2T^{-2}A^{-1}\right]$. The CGS unit of magnetic flux is Maxwell.

1 weber = 10^8 maxwell

The *magnetic flux density* can also be defined as the number of magnetic field lines crossing unit area kept normal to the direction of line of force. Its unit is Wb m^{-2} or tesla.

Note Here the integral is taken over area.

Let X and Y be two planar strips whose orientation is such that the direction of area vector of planar strips is parallel to the direction of the magnetic field $\vec{B}$ as shown in figure. The number of magnetic field lines passing through area of the strip X is two. Therefore, the flux passing through area X is $\Phi_B = 2$ Wb. Similarly, the number of magnetic field lines passing through area of strip Y is $\Phi_B = 4$ Wb.



(e) Uniform magnetic field and Non-uniform magnetic field

Uniform magnetic field

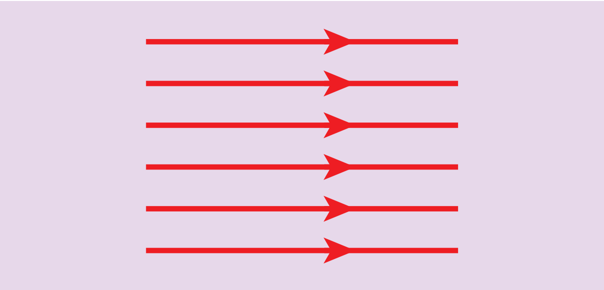


Figure 3.12 Uniform magnetic field

Magnetic field is said to be uniform if it has same magnitude and direction at all the points in a given region. Example, locally Earth's magnetic field is uniform.

The magnetic field of Earth has same value over the entire area of your school!

Non-uniform magnetic field

Magnetic field is said to be non-uniform if the magnitude or direction or both varies at all its points. Example: magnetic field of a bar magnet

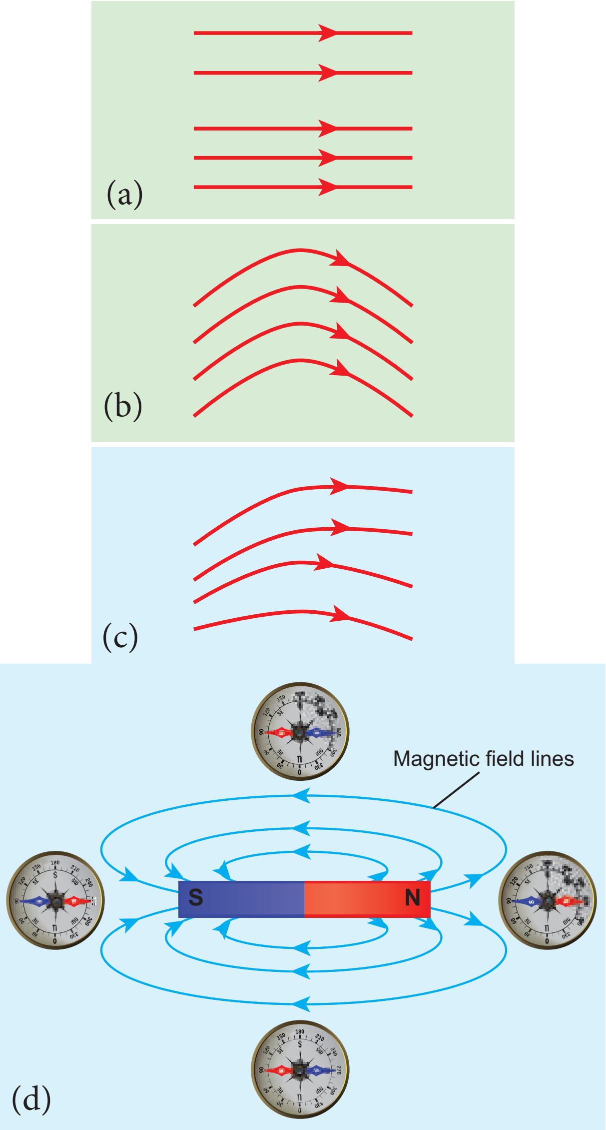
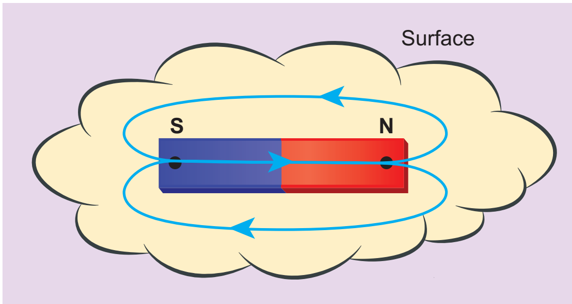


Figure 3.13 Non-uniform magnetic field – (a) direction is constant (b) direction is not a constant (c) both magnitude and direction are not constant (d) magnetic field of a bar magnet

EXAMPLE 3.4

Calculate the magnetic flux coming out from the surface containing magnetic dipole (say, a bar magnet) as shown in figure.



Solution

Magnetic dipole is kept, the total flux emanating from the closed surface S is zero. So,

$$\Phi_B = \oint \vec{B}.d\vec{A} = 0$$

Here the integral is taken over closed surface. Since no isolated magnetic pole (called magnetic monopole) exists, this integral is always zero,

$$\oint \vec{B}.d\vec{A} = 0$$

This is similar to Gauss's law in electrostatics. (Refer unit 1)

3.2 COULOMB'S INVERSE SQUARE LAW OF MAGNETISM

Consider two bar magnets A and B as shown in Figure 3.14.

When the north pole of magnet A and the north pole of magnet B or the south pole of magnet A and the south pole of magnet B are brought closer, they repel each other. On the other hand, when the north pole of magnet A and the south pole of magnet B or the south pole of magnet A and the north pole of magnet B are brought closer, their poles attract each other. This looks similar to Coulomb's law for static charges studied

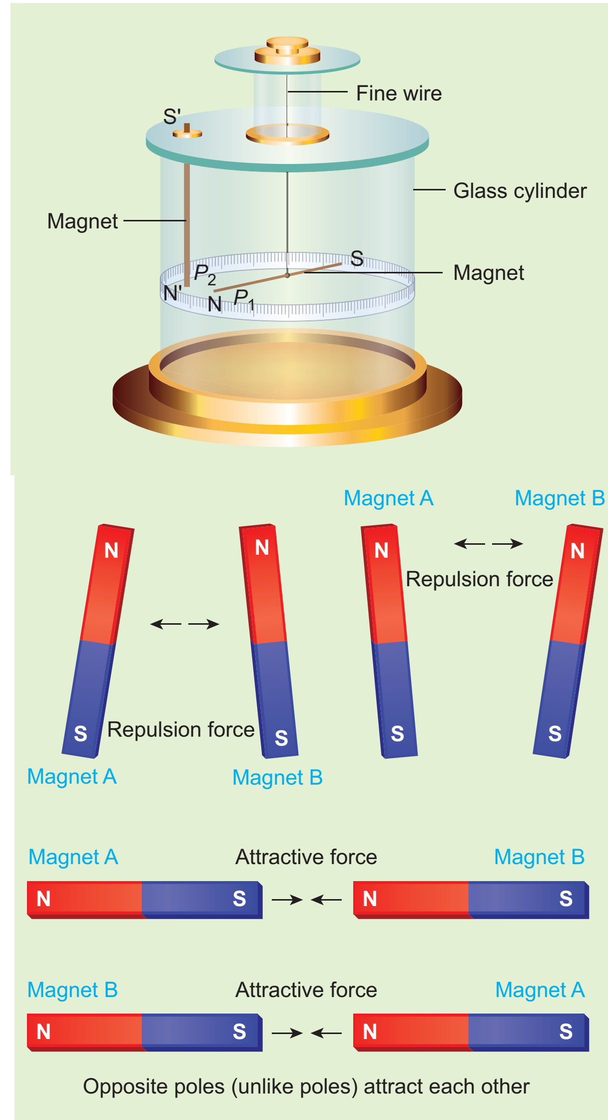


Figure 3.14: Magnetic poles behave like electric charges – like poles repel and unlike poles attract

in Unit I (opposite charges attract and like charges repel each other). So analogous to Coulomb's law in electrostatics, (Refer unit 1) we can state Coulomb's law for magnetism (Figure 3.15) as follows:

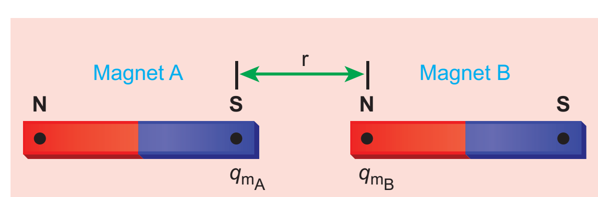


Figure 3.15 Coulomb's law – force between two magnetic pole strength

The force of attraction or repulsion between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them.

Mathematically, we can write

$$\vec{F} \propto \frac{q_{m_A} q_{m_B}}{r^2} \hat{r}$$

where m_A and m_B are pole strengths of two poles and *r* is the distance between two magnetic poles.

$$\vec{F} = k \frac{q_{m_A} q_{m_B}}{r^2} \hat{r} \quad (3.7)$$

In magnitude, $$\vec{F} = k \frac{q_{m_A} q_{m_B}}{r^2} \quad (3.8)$$

where k is a proportionality constant whose value depends on the surrounding medium. In S.I. unit, the value of k for free space is $k = \frac{\mu_o}{4\pi} \approx 10^{-7} H m^{-1}$, where μ_o is the absolute permeability of free space (air or vacuum).

EXAMPLE 3.5

The repulsive force between two magnetic poles in air is 9×10^{-3} N. If the two poles are equal in strength and are separated by a distance of 10 cm, calculate the pole strength of each pole.

***Solution*:**

The force between two poles are given by

$$\vec{F} = k \frac{q_{m_A} q_{m_B}}{r^2} \hat{r}$$

The magnitude of the force is

$$F = k \frac{q_{m_A} q_{m_B}}{r^2}$$

Given : $F = 9 \times 10^{-3}$N, r = 10 cm = 10×10^{-2} m

Therefore,

$$9 \times 10^{-3} = 10^{-7} \times \frac{q_m^2}{(10 \times 10^{-2})^2} \Rightarrow q_m = 30\, N T^{-1}$$

3.2.1 Magnetic field at a point along the axial line of the magnetic dipole (bar magnet)

Consider a bar magnet NS as shown in Figure 3.16. Let N be the North Pole and S be the south pole of the bar magnet, each of pole strength q_m and separated by a distance of 2*l*. The magnetic field at a point C (lies along the axis of the magnet) at a distance from the geometrical center O of the bar magnet can be computed by keeping unit north pole (q_{mc} = 1 A m) at C. The force experienced by the unit north pole at C due to pole strength can be computed using Coulomb's law of magnetism as follows:

The force of repulsion between north pole of the bar magnet and unit north pole at point C (in free space) is

$$\vec{F}_N = \frac{\mu_\circ}{4\pi} \frac{q_m}{(r-l)^2} \hat{i} \quad (3.9)$$

where r – *l* is the distance between north pole of the bar magnet and unit north pole at C.

The force of attraction between South Pole of the bar magnet and unit North Pole at point C (in free space) is

$$\vec{F}_S = -\frac{\mu_\circ}{4\pi} \frac{q_m}{(r+l)^2} \hat{i} \quad (3.10)$$

where r + *l* is the distance between south pole of the bar magnet and unit north pole at C.

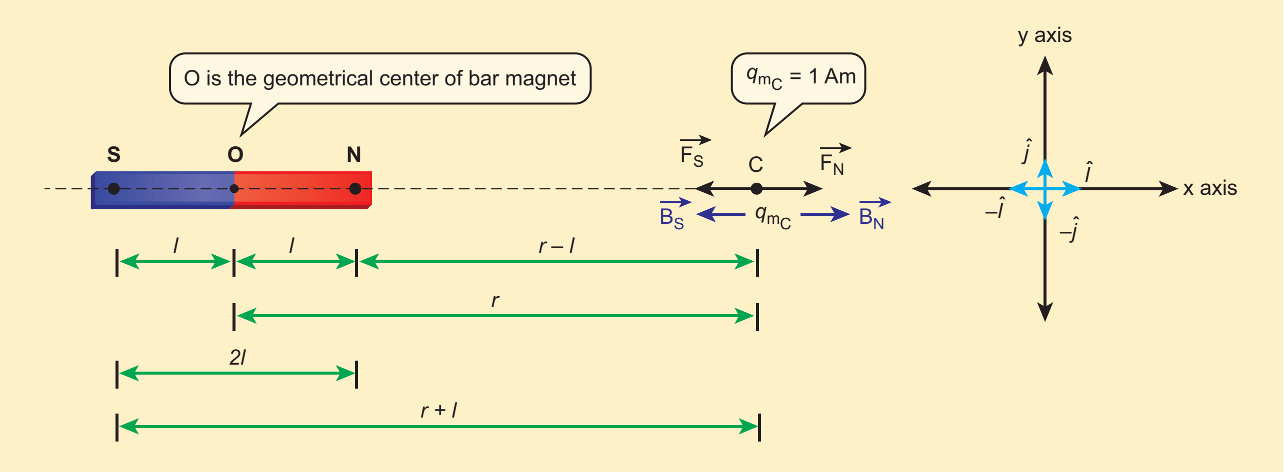


Figure 3.16 Magnetic field at a point along the axial line due to magnetic dipole

From equation (3.9) and (3.10), the net force at point C is $\vec{F} = \vec{F}_N + \vec{F}_S$. From definition, this net force is the magnetic field due to magnetic dipole at a point C $(\vec{F} = \vec{B})$

$$\vec{B} = \frac{\mu_\circ}{4\pi}\frac{q_m}{(r-l)^2}\hat{i} + \left(-\frac{\mu_\circ}{4\pi}\frac{q_m}{(r+l)^2}\hat{i}\right)$$

$$\vec{B} = \frac{\mu_\circ q_m}{4\pi}\left(\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right)\hat{i}$$

$$\vec{B} = \frac{\mu_\circ 2r}{4\pi}\left(\frac{q_m \cdot (2l)}{(r^2-l^2)^2}\right)\hat{i} \quad (3.11)$$

Since, magnitude of magnetic dipole moment is $|\vec{p}_m| = p_m = q_m \cdot 2l$ the magnetic field at a point C equation (3.11) can be written as

$$\vec{B}_{axial} = \frac{\mu_\circ}{4\pi}\left(\frac{2rp_m}{(r^2-l^2)^2}\right)\hat{i} \quad (3.12)$$

If the distance between two poles in a bar magnet are small (looks like short magnet) compared to the distance between geometrical centre O of bar magnet and the location of point C i.e., r >>l then,

$$(r^2-l^2)^2 \approx r^4 \quad (3.13)$$

Therefore, using equation (3.13) in equation (3.12), we get

$$\vec{B}_{axial} = \frac{\mu_\circ}{4\pi}\left(\frac{2p_m}{r^3}\right)\hat{i} = \frac{\mu_\circ}{4\pi}\frac{2}{r^3}\vec{p}_m \quad (3.14)$$

where $\vec{p}_m = p_m\hat{i}$.

3.2.2. Magnetic field at a point along the equatorial line due to a magnetic dipole (bar magnet)

Consider a bar magnet NS as shown in Figure 3.17. Let N be the north pole and S be the south pole of the bar magnet, each with pole strength q_m and separated by a distance of $2l$. The magnetic field at a point C (lies along the equatorial line) at a distance r from the geometrical center O of the bar magnet can be computed by keeping unit north pole (q_{mC} = 1 A m) at C. The force experienced by the unit north pole at C due to pole strength N-S can be computed using Coulomb's law of magnetism as follows:

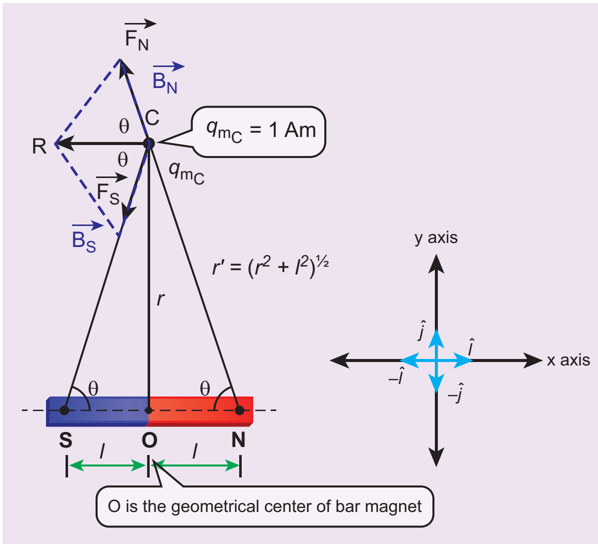


Figure 3.17 Magnetic field at a point along the equatorial line due to a magnetic dipole

The force of repulsion between North Pole of the bar magnet and unit north pole at point C (in free space) is

$$\vec{F}_N = -F_N \cos\theta \hat{i} + F_N \sin\theta \hat{j} \qquad (3.15)$$

where $F_N = \frac{\mu_\circ}{4\pi} \frac{q_m}{r'^2}$

The force of attraction (in free space) between south pole of the bar magnet and unit north pole at point C is (Figure 3.18) is

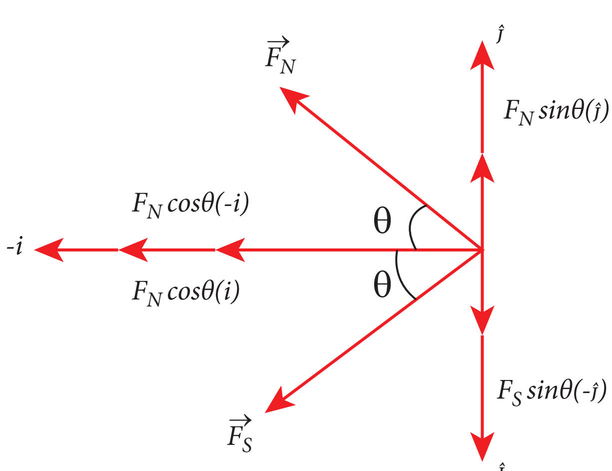


Figure 3.18 Components of force

$$\vec{F}_S = -F_S \cos\theta \hat{i} - F_S \sin\theta \hat{j} \qquad (3.16)$$

where, $F_S = \frac{\mu_\circ}{4\pi} \frac{q_m}{r'^2}$

From equation (3.15) and equation (3.16), the net force at point C is $\vec{F} = \vec{F}_N + \vec{F}_S$. This net force is equal to the magnetic field at the point C.

$$\vec{B} = -(F_N + F_S)\cos\theta \, \hat{i}$$

Since, $F_N = F_S$

$$\vec{B} = -\frac{2\mu_\circ}{4\pi} \frac{q_m}{r'^2} \cos\theta \, \hat{i} = -\frac{2\mu_\circ}{4\pi} \frac{q_m}{(r^2 + l^2)} \cos\theta \, \hat{i} \qquad (3.17)$$

In a right angle triangle NOC as shown in the Figure 3.17

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{l}{r'} = \frac{l}{(r^2 + l^2)^{\frac{1}{2}}} \qquad (3.18)$$

Substituting equation (3.18) in equation (3.17) we get

$$\vec{B} = -\frac{\mu_\circ}{4\pi} \frac{q_m \times (2l)}{(r^2 + l^2)^{\frac{3}{2}}} \hat{i} \qquad (3.19)$$

Since, magnitude of magnetic dipole moment is $|\vec{p}_m| = p_m = q_m \cdot 2l$ and substituting in equation (3.19), the magnetic field at a point C is

$$\vec{B}_{equatorial} = -\frac{\mu_\circ}{4\pi} \frac{p_m}{(r^2 + l^2)^{\frac{3}{2}}} \hat{i} \qquad (3.20)$$

If the distance between two poles in a bar magnet are small (looks like short magnet) when compared to the distance between geometrical center O of bar magnet and the location of point C i.e., r >>*l*, then,

$$(r^2 + l^2)^{\frac{3}{2}} \approx r^3 \quad (3.21)$$

Therefore, using equation (3.21) in equation (3.20), we get

$$\vec{B}_{equatorial} = -\frac{\mu_\circ}{4\pi}\frac{p_m}{r^3}\hat{i}$$

Since $p_m\hat{i} = \vec{p}_m$, In general, the magnetic field at equatorial point is given by

$$\vec{B}_{equatorial} = -\frac{\mu_\circ}{4\pi}\frac{\vec{p}_m}{r^3} \quad (3.22)$$

Note that magnitude of B_{axial} is twice that of magnitude of $B_{equatorial}$ and the direction of B_{axial} and $B_{equatorial}$ are opposite.

EXAMPLE 3.6

A short bar magnet has a magnetic moment of 0.5 J T^{-1}. Calculate magnitude and direction of the magnetic field produced by the bar magnet which is kept at a distance of 0.1 m from the center of the bar magnet along (a) axial line of the bar magnet and (b) normal bisector of the bar magnet.

Solution

Given magnetic moment 0.5 J T^{-1} and distance r = 0.1 m

(a) When the point lies on the axial line of the bar magnet, the magnetic field for short magnet is given by

$$\vec{B}_{axial} = \frac{\mu_\circ}{4\pi}\left(\frac{2p_m}{r^3}\right)\hat{i}$$

$$\vec{B}_{axial} = 10^{-7} \times \left(\frac{2\times 0.5}{(0.1)^3}\right)\hat{i} = 1\times 10^{-4}\,T\,\hat{i}$$

Hence, the magnitude of the magnetic field along axial is B_{axial} = 1 x 10^{-4} T and direction is towards South to North.

(b) When the point lies on the normal bisector (equatorial) line of the bar magnet, the magnetic field for short magnet is given by

$$\vec{B}_{equatorial} = -\frac{\mu_\circ}{4\pi}\frac{p_m}{r^3}\hat{i}$$

$$\vec{B}_{equatorial} = -10^{-7}\left(\frac{0.5}{(0.1)^3}\right)\hat{i} = -0.5\times 10^{-4}\,T\,\hat{i}$$

Hence, the magnitude of the magnetic field along axial is $B_{equatorial}$ = 0.5 x 10^{-4} T and direction is towards North to South.

Note that magnitude of B_{axial} is twice that of magnitude of $B_{equatorial}$ and the direction of B_{axial} and $B_{equatorial}$ are opposite.

3.3 TORQUE ACTING ON A BAR MAGNET IN UNIFORM MAGNETIC FIELD

Consider a magnet of length $2l$ of pole strength q_m kept in a uniform magnetic field $\vec{B}$ as shown in Figure 3.19. Each pole experiences a force of magnitude $q_m B$ but acts in opposite direction. Therefore, the net force exerted on the magnet is zero, so that there is no translatory motion. These two forces constitute a couple (about midpoint of bar magnet) which will rotate and try to align in the direction of the magnetic field $\vec{B}$.

The force experienced by north pole,

$$\vec{F}_N = q_m\vec{B} \quad (3.23)$$

The force experienced by south pole,

$$\vec{F}_S = -q_m\vec{B} \quad (3.24)$$

Adding equations (3.23) and (3.24), we get the net force acting on the dipole as

$$\vec{F} = \vec{F}_N + \vec{F}_S = \vec{0}$$

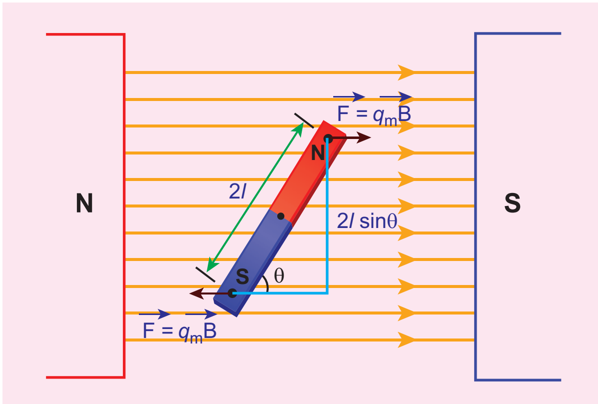


Figure 3.19 Magnetic dipole kept in a uniform magnetic field

This implies, that the net force acting on the dipole is zero, but forms a couple which tends to rotate the bar magnet clockwise (here) in order to align it along $\vec{B}$.

The moment of force or torque experienced by north and south pole about point O is

$$\vec{\tau} = \overrightarrow{ON} \times \vec{F}_N + \overrightarrow{OS} \times \vec{F}_S$$

$$\vec{\tau} = \overrightarrow{ON} \times q_m\vec{B} + \overrightarrow{OS} \times \left(-q_m\vec{B}\right)$$

By using right hand cork screw rule, we conclude that the total torque is pointing into the paper. Since the magnitudes $\left|\overrightarrow{ON}\right| = \left|\overrightarrow{OS}\right| = l \ and \ \left|q_m\vec{B}\right| = \left|-q_m\vec{B}\right|$, the magnitude of total torque about point O

$$\tau = l \times q_m B\sin\theta + l \times q_m B\sin\theta$$

$$\tau = 2l \times q_m B\sin\theta$$

$$\tau = p_m B\sin\theta \qquad (\therefore q_m \times 2l = p_m)$$

In vector notation, $\vec{\tau} = \vec{p}_m \times \vec{B}$ (3.25)

EXAMPLE 3.7

Show the time period of oscillation when a bar magnet is kept in a uniform magnetic field is $T = 2\pi\sqrt{\frac{1}{p_m B}}$ in second, where I represents moment of inertia of the bar magnet, p_m is the magnetic moment and is the magnetic field.

Solution

The magnitude of deflecting torque (the torque which makes the object rotate) acting on the bar magnet which will tend to align the bar magnet parallel to the direction of the uniform magnetic field $\vec{B}$ is

$$|\vec{\tau}| = p_m B\sin\theta$$

The magnitude of restoring torque acting on the bar magnet can be written as

$$|\vec{\tau}| = I\frac{d^2\theta}{dt^2}$$

Under equilibrium conditions, both magnitude of deflecting torque and restoring torque will be equal but act in the opposite directions, which means

$$I\frac{d^2\theta}{dt^2} = -p_m B\sin\theta$$

DO YOU KNOW?

(a) Why a freely suspended bar magnet in your laboratory experiences only torque (rotational motion) but not any translatory motion even though Earth has non-uniform magnetic field?

It is because Earth's magnetic field is locally(physics laboratory)uniform.

(b) Suppose we keep a freely suspended bar magnet in a non-uniform magnetic field. What will happen?

It will undergo translatory motion (net force) and rotational motion (torque).

The negative sign implies that both are in opposite directions. The above equation can be written as

$$\frac{d^2\theta}{dt^2} = -\frac{p_m B}{I}\sin\theta$$

This is non-linear second order homogeneous differential equation. In order to make it linear, we use small angle approximation as we did in XI volume II (Unit 10 – oscillations, Refer section 10.4.4) i.e., $\sin\theta \approx \theta$, we get

$$\frac{d^2\theta}{dt^2} = -\frac{p_m B}{I}\theta$$

This linear second order homogeneous differential equation is a Simple Harmonic differential equation. Therefore,

> Comparing with Simple Harmonic Motion (SHM) differential equation
>
> $$\frac{d^2x}{dt^2} = -\omega^2 x$$
>
> where ω is the angular frequency of the oscillation.

$$\omega^2 = \frac{p_m B}{I} \Rightarrow \omega = \sqrt{\frac{p_m B}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{p_m B}}$$

$$T = 2\pi\sqrt{\frac{I}{p_m B_H}} \text{ in second}$$

where, B_H is the horizontal component of Earth's magnetic field.

3.3.1. Potential energy of a bar magnet in a uniform magnetic field

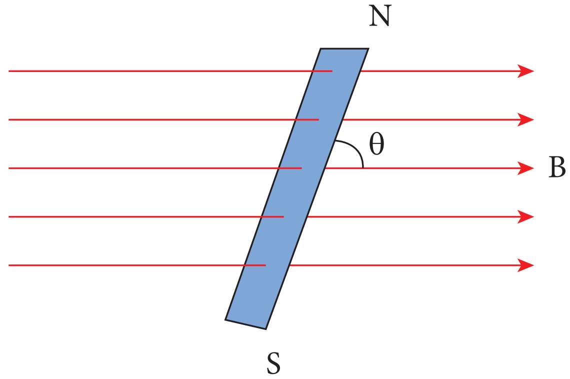


Figure 3.20: A bar magnet (magnetic dipole) in a uniform magnetic field

When a bar magnet (magnetic dipole) of dipole moment $\vec{p}_m$ is held at an angle θ with the direction of a uniform magnetic field $\vec{B}$, as shown in Figure 3.20 the magnitude of the torque acting on the dipole is

$$|\vec{\tau}_B| = |\vec{p}_m||\vec{B}|\sin\theta$$

If the dipole is rotated through a very small angular displacement dθ against the torque τ_B at constant angular velocity, then the work done by external torque $(\vec{\tau}_{ext})$ for this small angular displacement is given by

$$dW = |\vec{\tau}_{ext}|\,d\theta$$

The bar magnet has to be moved at constant angular velocity, which implies that $|\vec{\tau}_B| = |\vec{\tau}_{ext}|$

$$dW = p_m B\sin\theta\, d\theta$$

Total work done in rotating the dipole from θ′ to θ is

$$W = \int_{\theta'}^{\theta} \tau d\theta = \int_{\theta'}^{\theta} p_m B\sin\theta\, d\theta = p_m B\left[-\cos\theta\, d\theta\right]_{\theta'}^{\theta}$$

$$W = -p_m B(\cos\theta - \cos\theta')$$

This work done is stored as potential energy in bar magnet at an angle θ when it is rotated from θ′ to θ and it can be written as

$$U = -p_m B(\cos\theta - \cos\theta') \quad (3.26)$$

In fact, the equation (3.26) gives the difference in potential energy between the angular positions θ′ and θ. We can choose the reference point θ′ = 90°, so that second term in the equation becomes zero and the equation (3.26) can be written as

$$U = -p_m B\,(\cos\theta) \quad (3.27)$$

The potential energy stored in a bar magnet in a uniform magnetic field is given by

$$U = -\vec{p}_m \cdot \vec{B} \quad (3.28)$$

Case 1

(i) If θ = 0°, then

$$U = -p_m B\,(\cos 0^\circ) = -p_m\,\mathrm{B}$$

(ii) If θ = 180°, then

$$U = -p_m B\,(\cos 180^\circ) = p_m\,\mathrm{B}$$

We can infer from the above two results, the potential energy of the bar magnet is minimum when it is aligned along the external magnetic field and maximum when the bar magnet is aligned anti-parallel to external magnetic field.

EXAMPLE 3.8

Consider a magnetic dipole which on switching ON external magnetic field orient only in two possible ways i.e., one along the direction of the magnetic field (parallel to the field) and another anti-parallel to magnetic field. Compute the energy for the possible orientation. Sketch the graph.

Solution

Let $\vec{p}_m$ be the dipole and before switching ON the external magnetic field, there is no orientation. Therefore, the energy U = 0.

As soon as external magnetic field is switched ON, the magnetic dipole orient parallel (θ = 0°) to the magnetic field with energy,

$$U_{parallel} = U_{minimum} = -p_m B\cos 0$$
$$U_{parallel} = -p_m B$$

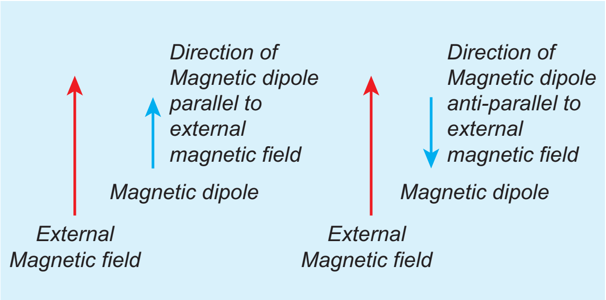
since cos 0° = 1

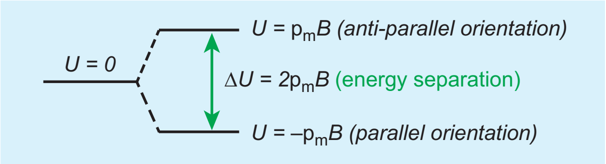
Otherwise, the magnetic dipole orients anti-parallel (θ = 180°) to the magnetic field with energy,

$$U_{anti-parallel} = U_{maximum} = -p_m B\cos 180$$

$$\Rightarrow U_{anti-parallel} = p_m B$$

since cos 180° = -1





3.3.2 Tangent law and Tangent Galvanometer

Tangent Galvanometer (Figure 3.21) is a device used to measure very small currents. It is a moving magnet type galvanometer. Its working is based on tangent law.

Figure 3.21 Tangent Galvanometer

Tangent law

When a magnetic needle or magnet is freely suspended in two mutually perpendicular uniform magnetic fields, it will come to rest in the direction of the resultant of the two fields.

Let B be the magnetic field produced by passing current through the coil of the tangent Galvanometer and B_H be the horizontal component of earth's magnetic field. Under the action of two magnetic fields, the needle comes to rest making angle θ with B_H, such that

$$B = B_H \tan\theta \qquad (3.29)$$

Construction

Tangent Galvanometer (TG) consists of copper coil wounded on a non-magnetic circular frame. The frame is made up of brass or wood which is mounted vertically on a horizontal base table (turn table) with three levelling screws as shown in Figure 3.22. The TG is provided with two or more coils of different number of turns. Most of the equipment we use in laboratory consists of 2 turns, 5 turns and 50 turns which are of different thickness and are used for measuring currents of different strengths.

At the center of turn table, a small upright projection is seen on which compass box (also known as magnetometre box) is placed. Compass box consists of a small magnetic needle which is pivoted at the center, such that arrangement shows the center of both magnetic needle and circular coil exactly coincide. A thin aluminium pointer is attached to the magnetic needle normally and moves over circular scale. The circular scale is divided into four quadrants and graduated in degrees which are used to measure the deflection of aluminium pointer on a circular degree scale. In order to avoid parallax error in measurement, a mirror is placed below the aluminium pointer.

Figure 3.22 Tangent Galvanometer and its parts

Precautions

1. All the nearby magnets and magnetic materials are kept away from the instrument.
2. Using spirit level, the levelling screws at the base are adjustedso that the small

magnetic needle is exactly horizontal and also coil (mounted on the frame) is exactly vertical.

3. The plane of the coil is kept parallel to the small magnetic needle by rotating the coil about its vertical axis. So, the coil remains in magnetic meridian.

Figure 3.23 Compass box

4. The compass box (as shown in Figure 3.23) is rotated such that the pointer reads $0^\circ - 0^\circ$

Theory

The circuit connection for Tangent Galvanometer (TG) experiment is shown in Figure 3.24. When no current is passed through the coil, the small magnetic needle lies along horizontal component of Earth's magnetic field. When the circuit is switched ON, the electric current will pass through the circular coil and produce magnetic field. The magnetic field produced due to the circulatory electric current is discussed **(in section 3.8.3).** Now there are two fields which are acting mutually perpendicular to each other. They are:

(1) the magnetic field (B) due to the electric current in the coil acting normal to the plane of the coil.

(2) the horizontal component of Earth's magnetic field (B_H)

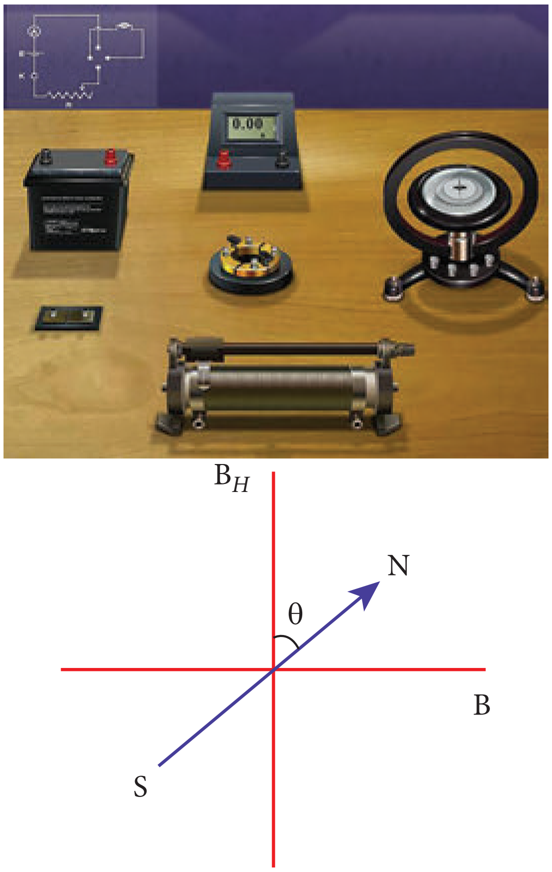


Figure 3.24 (a) circuit connection (b) resultant position of pivoted needle

Because of these crossed fields, the pivoted magnetic needle deflects through an angle θ. From tangent law (equation 3.29),

$$B = B_H \tan\theta$$

When an electric current is passed through a circular coil of radius R having N turns, the magnitude of magnetic field at the center is

$$B = \mu_\circ \frac{NI}{2R} \qquad (3.30)$$

From equation (3.29) and equation (3.30), we get

$$\mu_\circ \frac{NI}{2R} = B_H \tan\theta$$

The horizontal component of Earth's magnetic field can be determined as

$$B_H = \mu_\circ \frac{NI}{2R}\frac{1}{\tan\theta} \text{ in tesla} \qquad (3.31)$$

Note

1. The current in circuit can be calculated from I = K tan θ, where K is called reduction factor of tangent Galvanometer, where

$$K = \frac{2RB_H}{\mu_\circ N}$$

2. Sensitivity measures the change in the deflection produced by a unit current, mathematically

$$\frac{d\theta}{dI} = \frac{1}{K\left(1+\frac{I^2}{K^2}\right)}$$

3. The tangent Galvanometer is most sensitive at a deflection of 45°. Generally the deflection is taken between 30° and 60°.

EXAMPLE 3.9

A coil of a tangent galvanometer of diametre 0.24 m has 100 turns. If the horizontal component of Earth's magnetic field is 25×10^{-6} T then, calculate the current which gives a deflection of 60°.

Solution

The diameter of the coil is 0.24 m. Therefore, radius of the coil is 0.12 m.

Number of turns is 100 turns.

Earth's magnetic field is 25 x 10^{-6} T

Deflection is

$$\theta = 60^\circ \Rightarrow \tan 60^\circ = \sqrt{3} = 1.732$$

$$I = \frac{2RB_H}{\mu_\circ N}\tan\theta$$

$$= \frac{2\times 0.12\times 25\times 10^{-6}}{4\times 10^{-7}\times 3.14\times 100}\times 1.732 = 0.82\times 10^{-1} A.$$

$$I = 0.082\ A$$

3.4 MAGNETIC PROPERTIES

All the materials we use are not magnetic materials. Further, all the magnetic materials will not behave identically. So, in order to differentiate one magnetic material from another, we need to know some basic parameters. They are:

(a) Magnetising field

The magnetic field which is used to magnetize a sample or specimen is called the magnetising field. Magnetising field is a vector quantity and it denoted by $\vec{H}$ and its unit is A m^{-1}.

(b) Magnetic permeability

The magnetic permeability can be defined as **the measure of ability of the material to allow the passage of magnetic field lines through it or measure of the capacity of the substance to take magnetisation or the degree of penetration of magnetic field through the substance.**

In free space, the permeability (or absolute permeability) is denoted by μ_0 and for any medium it is denoted by μ. **The relative permeability μ_r is defined as the ratio between absolute permeability of the medium to the permeability of free space.**

$$\mu_r = \frac{\mu}{\mu_\circ} \qquad (3.32)$$

Relative permeability is a dimensionless number and has no units. For free space (air or vacuum), the relative permeability is unity i.e., μ_r = 1. In isotropic medium, μ is a scalar but for non-isotropic medium, μ is a tensor.

UNIT-3(XII-Physics_Vol-1).indd 149 04-03-2019 10:33:51

DO YOU KNOW?

Physical quantity	Component	Direction
Scalar	1 Component	No direction (no unit vector)
Vector	Each Component	1 direction (one unit vector)
Tensor	Each Component	More than one direction (more than one unit vectors)

Physical quantity	Component	Rank
Scalar	1 Component with zero direction	Zero
Vector	Each Component has one direction	One
Tensor of rank two	Each Component associated with two directions	Two
Tensor of rank three	Each Component associated with three directions	Three
Tensor of rank n	Each Component associated with n directions	n

(c) Intensity of magnetisation

Any bulk material (any object of finite size) contains a large number of atoms. Each atom consists of electrons which undergo orbital motion. Due to orbital motion, electron has magnetic moment which is a vector quantity. In general, these magnetic moments orient randomly, therefore, the net magnetic moment is zero per unit volume of the material.

When such a material is kept in an external magnetic field, atomic dipoles are created and hence, it will try to align partially or fully along the direction of external field. The net magnetic moment per unit volume of the material is known **as intensity of magnetisation or magnetisation vector or magnetisation.** It is a vector quantity. Mathematically,

$$\vec{M} = \frac{magnetic\,moment}{volume} = \frac{1}{V}\vec{p}_m \qquad (3.33)$$

The SI unit of intensity of magnetisation is ampere metre^{-1}. For a bar magnet of pole strength q_m, length $2l$ and area of cross-section A, the magnetic moment of the bar magnet is $\vec{p}_m = q_m \vec{2l}$ and volume of the bar magnet is $V = A\left|\vec{2l}\right| = 2l\,A$. The intensity of magnetisation for a bar magnet is

$$\vec{M} = \frac{magnetic\,moment}{volume} = \frac{q_m \vec{2l}}{2l\,A} \qquad (3.34)$$

In magnitude, equation (3.34) is

$$\left|\vec{M}\right| = M = \frac{q_m \times 2l}{2l \times A} \Rightarrow M = \frac{q_m}{A}$$

This means, **for a bar magnet the intensity of magnetisation can be defined as the pole strength per unit area (face area).**

(d) Magnetic induction or total magnetic field

When a substance like soft iron bar is placed in an uniform magnetising field $\vec{H}$, it becomes a magnet, which means that the substance gets magnetised. The magnetic induction (**total magnetic field**) inside the specimen $\vec{B}$ is equal to the sum of the magnetic field $\vec{B}_\circ$ produced in vacuum due to the magnetising field and the magnetic field $\vec{B}_m$ due to the induced magnetisation of the substance.

$$\vec{B} = \vec{B}_o + \vec{B}_m = \mu_\circ \vec{H} + \mu_\circ \vec{I}$$

$$\Rightarrow \vec{B} = \vec{B}_o + \vec{B}_m = \mu_\circ(\vec{H} + \vec{I}) \qquad (3.35)$$

(e) Magnetic susceptibility

When a substance is kept in a magnetising field $\vec{H}$, magnetic susceptibility gives information about how a material respond to the external (applied) magnetic field. In other words, the magnetic susceptibility measures, how easily and how strongly a material can be magnetised. It is defined as the ratio of the intensity of magnetisation $(\vec{M})$ induced in the material due to the magnetising field $(\vec{H})$

$$\chi_m = \frac{|\vec{M}|}{|\vec{H}|} \qquad (3.36)$$

It is a dimensionless quantity. For an isotropic medium, susceptibility is a scalar but for non-isotropic medium, susceptibility is a tensor. Magnetic susceptibility for some of the isotropic substances is given in Table 3.1.

Table 3.1 Magnetic susceptibility for various materials

Material	Magnetic susceptibility (χ_m)
Aluminium	2.3×10^{-5}
Copper	-0.98×10^{-5}
Diamond	-2.2×10^{-5}
Gold	-3.6×10^{-5}
Mercury	-3.2×10^{-5}
Silver	-2.6×10^{-5}
Titanium	7.06×10^{-5}
Tungsten	6.8×10^{-5}
Carbon dioxide (1 atm)	-2.3×10^{-9}
Oxygen (1 atm)	2090×10^{-9}

EXAMPLE 3.10

Compute the intensity of magnetisation of the bar magnet whose mass, magnetic moment and density are 200 g, 2 A m^2 and 8 g cm^{-3}, respectively.

Solution

Density of the magnet is

$$Density = \frac{Mass}{Volume} \Rightarrow Volume = \frac{Mass}{Density}$$

$$Volume = \frac{200\times10^{-3}\,kg}{\left(8\times10^{-3}\,kg\right)\times10^{6}\,m^{-3}} = 25\times10^{-6}\,m^3$$

Magnitude of magnetic moment $p_m = 2\,A\,m^2$

Intensity of magnetization,

$$I = \frac{Magnetic\,moment}{Volume} = \frac{2}{25\times10^{-6}}$$

$$M = 0.8\times10^5\,A\,m^{-1}$$

EXAMPLE 3.11

Using the relation $\vec{B} = \mu_\circ(\vec{H}+\vec{M})$, show that $\chi_m = \mu_r - 1$.

Solution

$\vec{B} = \mu_\circ(\vec{H}+\vec{M})$,

But from equation (3.36), in vector form,

$\vec{M} = \chi_m \vec{H}$

Hence, $\vec{B} = \mu_\circ(\chi_m + 1)\vec{H} \Rightarrow \vec{B} = \mu\vec{H}$

where, $\mu = \mu_\circ(\chi_m+1) \Rightarrow \chi_m + 1 = \frac{\mu}{\mu_\circ} = \mu_r$

$\Rightarrow \chi_m = \mu_r - 1$

EXAMPLE 3.12

Two materials X and Y are magnetised, whose intensity of magnetisation are 500 A m^{-1} and 2000 A m^{-1}, respectively. If the magnetising field is 1000 A m^{-1}, then which one among these materials can be easily magnetized?.

Solution

The susceptibility of material X is

$$\chi_{m,X} = \frac{|\vec{M}|}{|\vec{H}|} = \frac{500}{1000} = 0.5$$

The susceptibility of material Y is

$$\chi_{m,Y} = \frac{|\vec{M}|}{|\vec{H}|} = \frac{2000}{1000} = 2$$

Since, susceptibility of material Y is greater than that of material X, material Y can be easily magnetized than X.

3.5 CLASSIFICATION OF MAGNETIC MATERIALS

The magnetic materials are generally classified into three types based on the behaviour of materials in a magnetising field. They are diamagnetic, paramagnetic and ferromagnetic materials which are dealt with in this section.

(a) Diamagnetic materials

The orbital motion of electrons around the nucleus produces a magnetic field perpendicular to the plane of the orbit. Thus each electron orbit has finite orbital magnetic dipole moment. Since the orbital planes are oriented in random manner, the vector sum of magnetic moments is zero and there is no resultant magnetic moment for each atom.

In the presence of an external magnetic field, some electrons are speeded up and some are slowed down. The electrons whose moments were anti-parallel are speeded up according to Lenz's law and this produces an induced magnetic moment in a direction opposite to the field. The induced moment disappears as soon as the external field is removed.

When placed in a non-uniform magnetic field, the interaction between induced magnetic moment and the external field creates a force which tends to move the material from stronger part to weaker part of the external field. It means that diamagnetic material is repelled by the field.

This action is called diamagnetic action and such materials are known as diamagnetic materials. Examples: Bismuth, Copper and Water etc.

The properties of diamagnetic materials are

i) Magnetic susceptibility is negative.
ii) Relative permeability is slightly less than unity.
iii) The magnetic field lines are repelled or expelled by diamagnetic materials when placed in a magnetic field.
iv) Susceptibility is nearly temperature independent.

Note Superconductors are perfect diamagnetic materials. The expulsion of magnetic flux from a superconductor during its transition to the superconducting state is known as Meissner effect. (see figure 3.25)

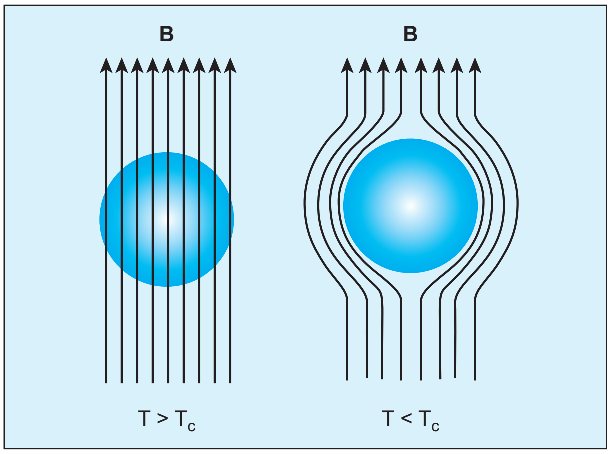


Figure 3.25 Meissner effect – superconductors behaves like perfect diamagnetic materials below transition temperature T_C.

DO YOU KNOW?

Magnetic levitated train

Magnetic levitated train is also called as Maglev train. This train floats above few centimetre from the guideway because of electromagnet used. Maglev train does not need wheels and also achieve greater speed. The basic mechanism of working of Maglev train involves two sets of magnets. One set is used to repel which makes train to float above the track and another set is used to move the floating train ahead at very great speed. These trains are quieter, smoother and environmental friendly compared conventional trains and have potential for moving with much higher speeds with technology in future.

(b) Paramagnetic materials

In some magnetic materials, each atom or molecule has net magnetic dipole moment which is the vector sum of orbital and spin magnetic moments of electrons. Due to the random orientation of these magnetic moments, the net magnetic moment of the materials is zero.

In the presence of an external magnetic field, the torque acting on the atomic dipoles will align them in the field direction. As a result, there is net magnetic dipole moment induced in the direction of the applied field. The induced dipole moment is present as long as the external field exists.

When placed in a non-uniform magnetic field, the paramagnetic materials will have a tendency to move from weaker to stronger part of the field. Materials which exhibit weak magnetism in the direction of the applied field are known as paramagnetic materials. Examples: Aluminium, Platinum and chromium etc.

The properties of paramagnetic materials are:

i) Magnetic susceptibility is positive and small.
ii) Relative permeability is greater than unity.
iii) The magnetic field lines are attracted into the paramagnetic materials when placed in a magnetic field.
iv) Susceptibility is inversely proportional to temperature.

Curie's law

When temperature is increased, thermal vibration will upset the alignment of magnetic dipole moments. Therefore, the magnetic susceptibility decreases with increase in temperature. In many cases, the susceptibility of the materials is

$$\chi_m \propto \frac{1}{T} \quad \text{or} \quad \chi_m = \frac{C}{T}$$

This relation is called Curie's law. Here C is called Curie constant and temperature T is in kelvin. The graph drawn between magnetic susceptibility and temperature is shown in Figure 3.26, which is a rectangular hyperbola.

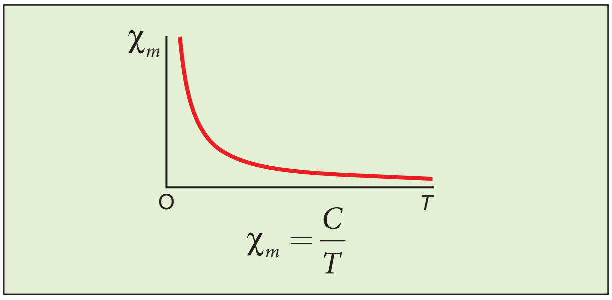


Figure 3.26 Curie's law – susceptibility vs temperature

(c) Ferromagnetic materials

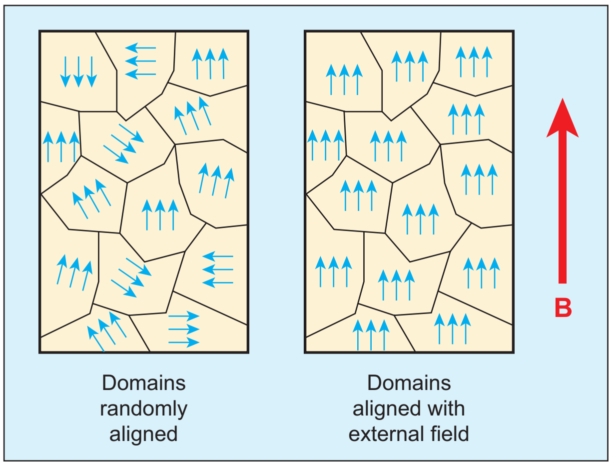


Figure 3.27 magnetic domains – ferromagnetic materials

An atom or a molecule in a ferromagnetic material possesses net magnetic dipole moment as in a paramagnetic material. A ferromagnetic material is made up of smaller regions, called ferromagnetic domain (Figure 3.27). Within each domain, the magnetic moments are spontaneously aligned in a direction. This alignment is caused by strong interaction arising from electron spin which depends on the inter-atomic distance. Each domain has net magnetisation in a direction. However the direction of magnetisation varies from domain to domain and thus net magnetisation of the specimen is zero.

In the presence of external magnetic field, two processes take place

(1) the domains having magnetic moments parallel to the field grow in size

(2) the other domains (not parallel to field) are rotated so that they are aligned with the field.

As a result of these mechanisms, there is a strong net magnetisation of the material in the direction of the applied field (Figure 3.28).

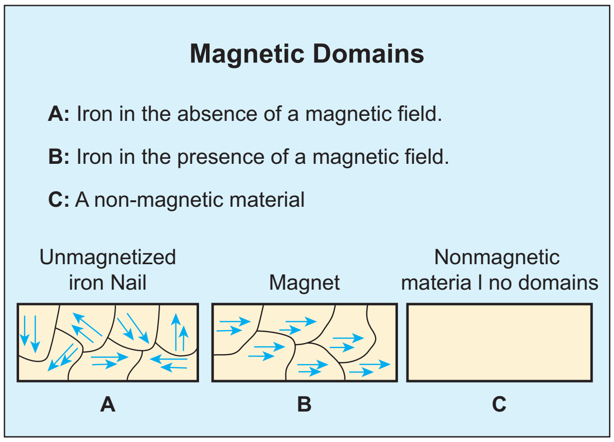


Figure 3.28 Processes of domain magnetization

When placed in a non-uniform magnetic field, the ferromagnetic materials will have a strong tendency to move from weaker to stronger part of the field. Materials which exhibit strong magnetism in the direction of applied field are called ferromagnetic materials. Examples: Iron, Nickel and Cobalt.

The properties of ferromagnetic materials are:

i) Magnetic susceptibility is positive and large.

ii) Relative permeability is large.

iii) The magnetic field lines are strongly attracted into the ferromagnetic materials when placed in a magnetic field.

iv) Susceptibility is inversely proportional to temperature.

Curie-Weiss law

As temperature increases, the ferromagnetism decreases due to the increased thermal agitation of the atomic dipoles. At a particular temperature, ferromagnetic material becomes paramagnetic. This temperature is known as Curie temperature T_C. The susceptibility of the material above the Curie temperature is given by

$$\chi_m = \frac{C}{T - T_C}$$

This relation is called Curie-Weiss law. The constant C is called Curie constant and temperature T is in kelvin. A plot of magnetic susceptibility with temperature is as shown in Figure 3.29.

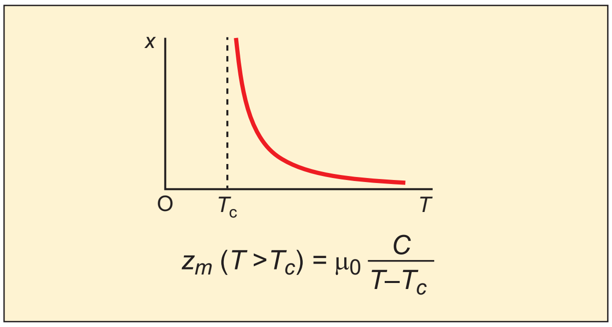
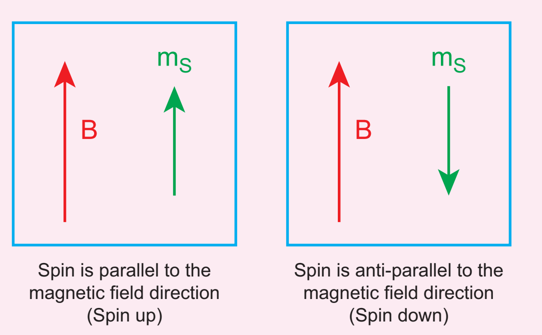


Figure 3.29 Curie-Weiss law – Susceptibility vs temperature

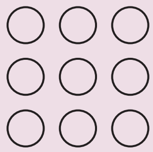
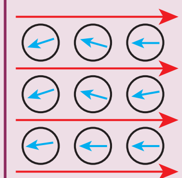
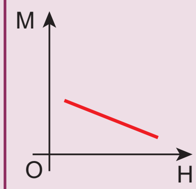
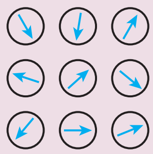
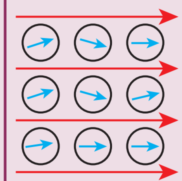
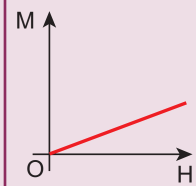
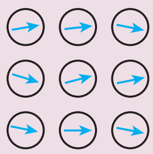
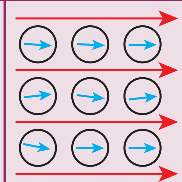
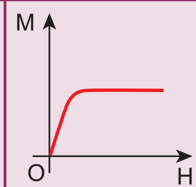
Spin

DO YOU KNOW?

Like mass and charge for particles, spin is also another important attribute for an elementary particle. Spin is a quantum mechanical phenomenon (this is discussed in Volume 2) which is responsible for magnetic properties of the material. Spin in quantum mechanics is entirely different from spin we encounter in classical mechanics. Spin in quantum mechanics does not mean rotation; it is intrinsic angular momentum which does not have classical analogue. For historical reason, the name spin is retained. Spin of a particle takes only positive values but the orientation of the spin vector takes plus or minus values in an external magnetic field. For an example, electron has spin $s = \frac{1}{2}$. In the presence of magnetic field, the spin will orient either parallel or anti-parallel to the direction of magnetic field.

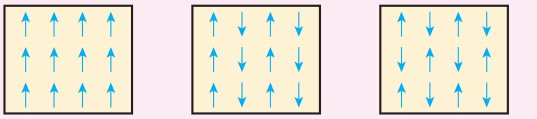


This implies that the magnetic spin m_s takes two values for an electron, such as $m_s = \frac{1}{2}$ (spin up) and $m_s = -\frac{1}{2}$ (spin down). Spin for proton and neutron is $s = \frac{1}{2}$. For a photon is spin s = 1.

Type of magnetism	Magnetising field is absent (H = 0)	Magnetising field is present ($H \neq 0$)	Magnetisation of the material	Susceptibility	Relative permeability
Diamagnetism	 (Zero magnetic moment)	 (Aligned opposite to the field)		Negative	Less than unity
Paramagnetism	 (Net magnetic moment but random alignment)	 (Aligned with the field)		Positive and small	Greater than unity
Ferromagnetism	 (Net magnetic moment in a domain but random alignment of domains)	 (Aligned with the field)		Positive and large	Very large

DO YOU KNOW?

Three simple types of ordering of atomic magnetic moments



Ferromagnetic (Adjacent magnetic moments are aligned) (a)

Antiferromagnetic (Adjacent magnetic moments are antiparallel and of equal magnitude) (b)

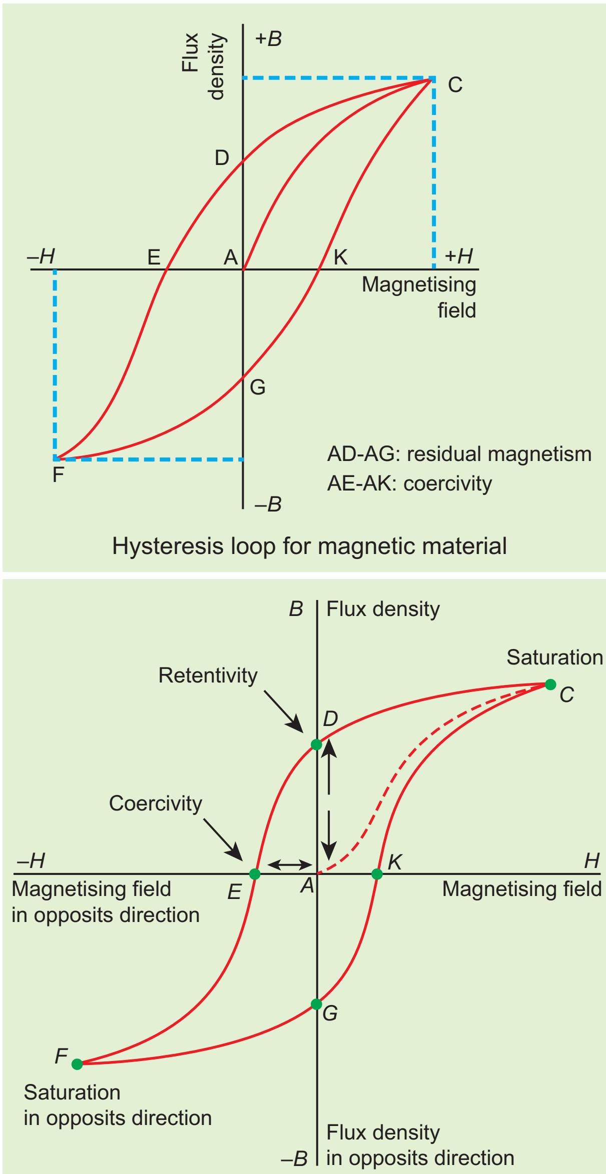
Ferrimagnetic (Adjacent magnetic moments are antiparallel and of unequal magnitude) (c)

3.6 HYSTERESIS

When a ferromagnetic material is kept in a magnetising field, the material gets magnetised by induction. An important characteristic of ferromagnetic material is that the variation of magnetic induction $\vec{B}$ with magnetising field $\vec{H}$ is not linear. It means that the ratio $\frac{B}{H} = \mu$ is not a constant. Let us study this behaviour in detail.

A ferromagnetic material (example, Iron) is magnetised slowly by a magnetising field $\vec{H}$. The magnetic induction $\vec{B}$ of the material increases from point A with the magnitude of the magnetising field and then attains a saturated level. This response of the material is depicted by the path AC as shown in Figure 3.30. Saturation magnetization is defined as the maximum point up to which the material can be magnetised by applying the magnetising field.

If the magnetising field is now reduced, the magnetic induction also decreases but does not retrace the original path CA. It takes different path CD. When the magnetising field is zero, the magnetic induction is not zero and it has positive value. This implies that some



Hysteresis loop for magnetic material

Figure 3.30 Hysteresis – plot for B vs H

magnetism is left in the specimen even when H = 0. The **residual magnetism AD present in the specimen is called remanence or retentivity. It is defined as the ability of the materials to retain the magnetism in them even magnetising field vanishes.**

In order to demagnetise the material, the magnetising field is gradually increased in the reverse direction. Now the magnetic induction decreases along DE and becomes zero at E. The magnetising field AE in the reverse direction is required to bring residual magnetism to zero. **The magnitude of the reverse magnetising field for which the residual magnetism of the material vanishes is called its coercivity.**

Further increase of $\vec{H}$ in the reverse direction, the magnetic induction increases along EF until it reaches saturation at F in the reverse direction. If magnetising field is decreased and then increased with direction reversed, the magnetic induction traces the path FGKC. This closed curve ACDEFGKC is called hysteresis loop and it represents a cycle of magnetisation.

In the entire cycle, the magnetic induction B lags behind the magnetising field H. This **phenomenon of lagging of magnetic induction behind the magnetising field is called hysteresis.** Hysteresis means 'lagging behind'.

Hysteresis loss

During the magnetisation of the specimen through a cycle, there is loss of energy in the form of heat. This loss is attributed to the rotation and orientation of molecular magnets in various directions. It is found that the energy lost (or dissipated) per unit volume of the material when it is carried through one cycle of magnetisation is equal to the area of the hysteresis loop. Thus, the loss of energy for a complete cycle is ΔE,

$$\Delta E = \oint \vec{H}.d\vec{B}$$

where $\vec{B}$ is in ampere – metre^2 and $\vec{H}$ is in ampere per meter. The loss in energy is measured in joules.

Hard and soft magnetic materials

Based on the shape and size of the hysteresis loop, ferromagnetic materials are classified as soft magnetic materials with smaller area and hard magnetic materials with larger area. The comparison of the hysteresis loops for two magnetic materials is shown in Figure 3.31. Properties of soft and hard magnetic materials are compared in Table 3.2.

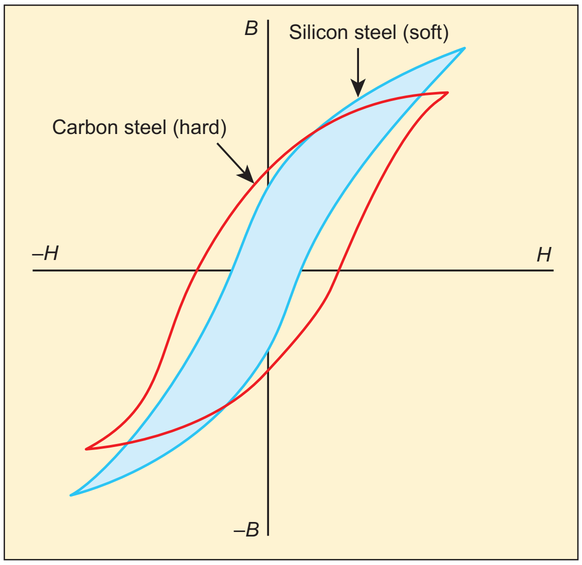


Figure 3.31 Comparison of two ferromagnetic materials – hysteresis loop

Applications of hysteresis loop

The significance of hysteresis loop is that it provides information such as retentivity, coercivity, permeability, susceptibility and energy loss during one cycle of magnetisation for each ferromagnetic material. Therefore, the study of hysteresis loop will help us in selecting proper and suitable material for a given purpose. Some examples:

i) Permanent magnets:

The materials with high retentivity, high coercivity and high permeability are suitable for making permanent magnets.
Examples: Steel and Alnico

ii) Electromagnets:

The materials with high initial permeability, low retentivity, low coercivity and thin hysteresis loop with smaller area are preferred to make electromagnets.

Table 3.2 Difference between soft and hard ferromagnetic materials

S.No.	Properties	Soft ferromagnetic materials	Hard ferromagnetic materials
1	When external field is removed	Magnetisation disappears	Magnetisation persists
2	Area of the loop	Small	Large
3	Retentivity	Low	High
4	Coercivity	Low	High
5	Susceptibility and magnetic permeability	High	Low
6	Hysteresis loss	Less	More
7	Uses	Solenoid core, transformer core and electromagnets	Permanent magnets
8	Examples	Soft iron, Mumetal, Stalloy etc.	Steel, Alnico, Lodestone etc.

Examples: Soft iron and Mumetal (Nickel Iron alloy).

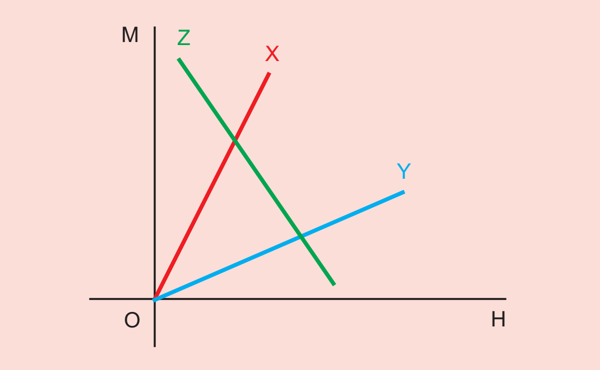
iii) Core of the transformer:

The materials with high initial permeability, large magnetic induction and thin hysteresis loop with smaller area are needed to design transformer cores.

Examples: Soft iron

EXAMPLE 3.13

The following figure shows the variation of intensity of magnetisation with the applied magnetic field intensity for three magnetic materials X, Y and Z. Identify the materials X,Y and Z.



Solution

The slope of M-H graph measures the magnetic susceptibility, which is

$$\chi_m = \frac{M}{H}$$

Material X: Slope is positive and larger value. So, it is a ferromagnetic material.

Material Y: Slope is positive and lesser value than X. So, it could be a paramagnetic material.

Material Z: Slope is negative and hence, it is a diamagnetic material.

3.3 MAGNETIC EFFECTS OF CURRENT

3.7.1 Oersted experiment

In 1820, Hans Christian Oersted while preparing for his lecture in physics noticed that electric current passing through a wire deflects the nearby magnetic compass. By proper investigation, he observed that the deflection of magnetic compass is due to the change in magnetic field produced around current carrying conductor (Figure 3.32). When the direction of current is reversed, the magnetic compass deflects in opposite direction. This lead to the development of the theory 'electromagnetism' which unifies the two branches in physics, namely electricity and magnetism.

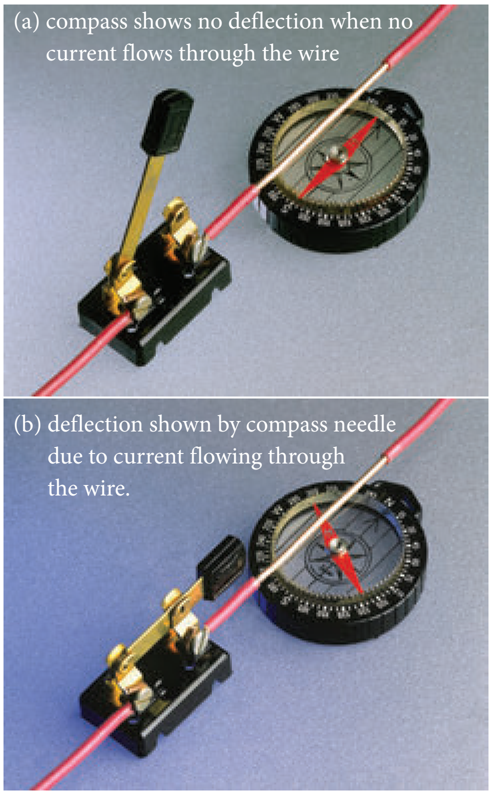
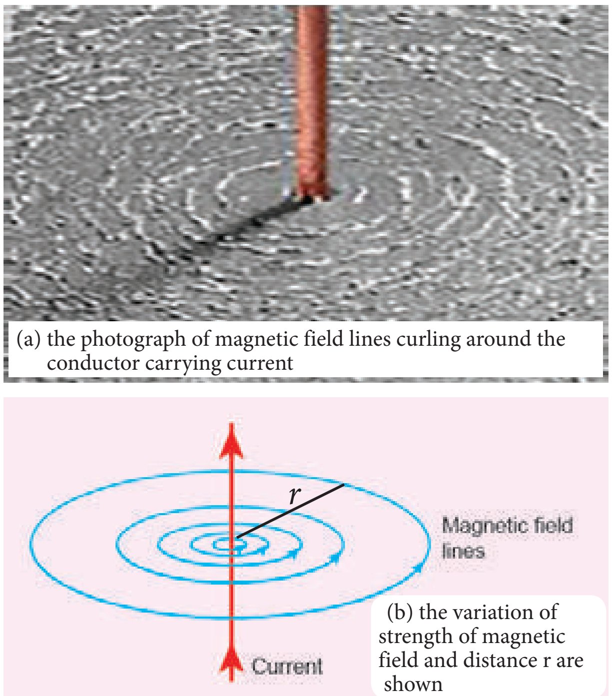


Figure 3.32 Oersted's experiment - current carrying wire and deflection of magnetic needle

3.7.2 Magnetic field around a straight current carrying conductor and circular loop

(a) Current carrying straight conductor:



(a) the photograph of magnetic field lines curling around the conductor carrying current

(b) the variation of strength of magnetic field and distance r are shown

Figure 3.33 Magnetic field lines around straight, long wire carrying current

Suppose we keep a magnetic compass near a current carrying straight conductor, then the needle of the magnetic compass experiences a torque and deflects to align in the direction of the magnetic field at that point. Tracing out the direction shown by magnetic compass, we can draw the magnetic field lines at a distance. For a straight current carrying conductor, the nature of magnetic field is like concentric circles having their center at the axis of the conductor as shown in Figure 3.33 (a).

The direction of circular magnetic field lines will be clockwise or anticlockwise depending on the direction of current in the conductor. If the strength (or magnitude) of the current is increased then the density of the magnetic field will also increase. The strength of the magnetic field (B) decreases as the distance (r) from the conductor increases are shown in Figure 3.33 (b).

(b) Circular coil carrying current

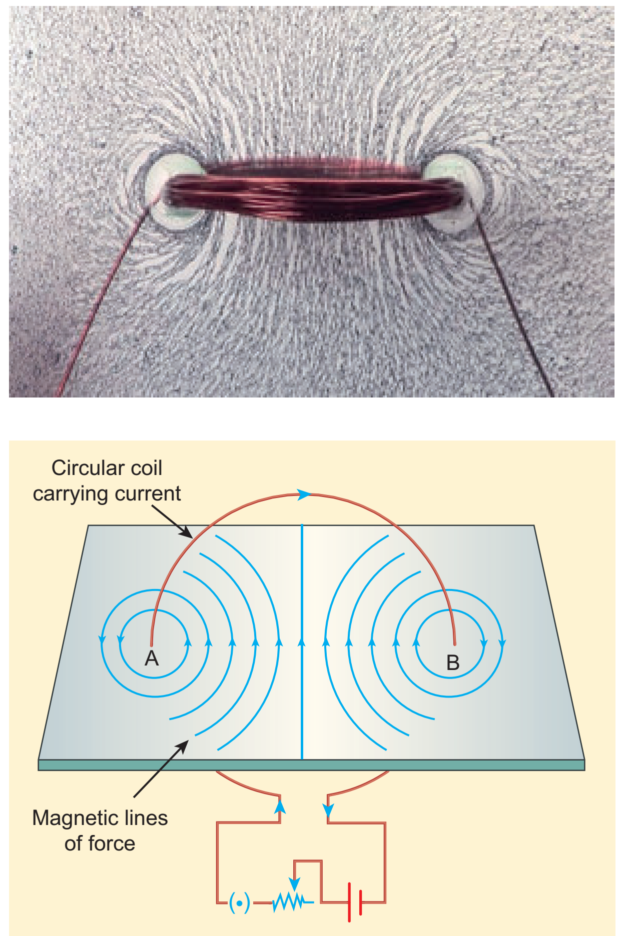


Figure 3.34 The magnetic field lines curling around the circular coil carrying current.

Suppose we keep a magnetic compass near a current carrying circular conductor, then the needle of the magnetic compass experiences a torque and deflects to align in the direction of the magnetic field at that point. We can notice that at the points A and B in the vicinity of the coil, the magnetic field lines are circular. The magnetic field lines are nearly parallel to each other near the center of the loop, indicating that the field present near the center of the coil is almost uniform (Figure 3.34).

The strength of the magnetic field is increased if either the current in the coil or the number of turns or both are increased. The polarity (north pole or south pole) depends on the direction of current in the loop.

3.7.3 Right hand thumb rule

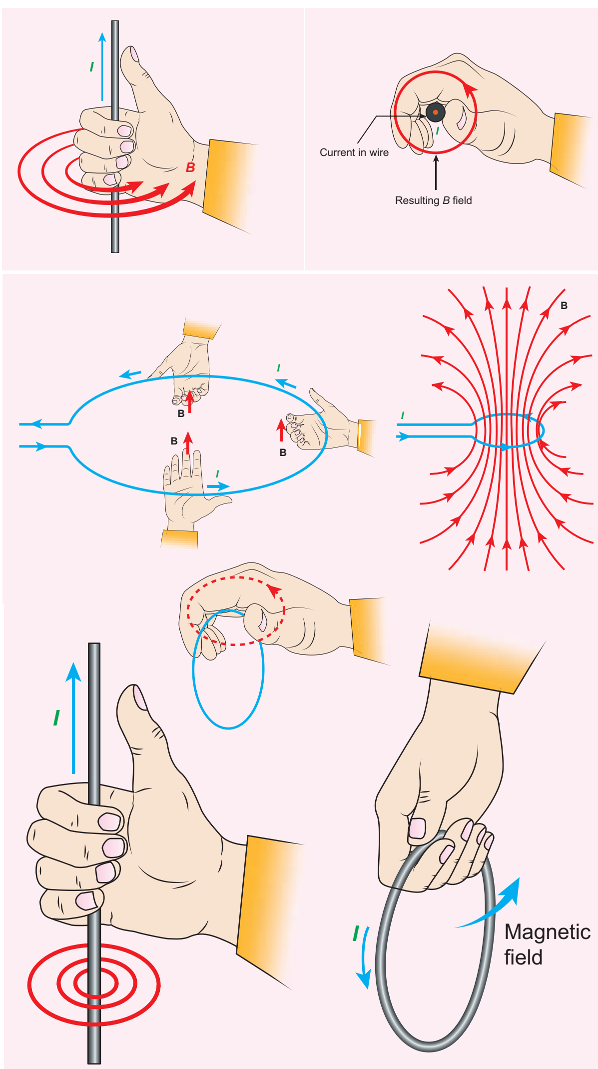


Figure 3.35 Right hand rule – straight conductor and circular loop

The right hand rule is a mnemonic to find the direction of magnetic field when the direction of current in a conductor is known.

If we hold the current carrying conductor in our right hand such that the thumb points in the direction of current flow, then the fingers encircling the wire points in the direction of the magnetic field lines produced.

The Figure 3.35 shows the right hand rule for current carrying straight conductor and circular coil.

Note *Mnemonic means that it is a special word or a collection of words used to help a person to remember something.*

3.7.4 Maxwell's right hand cork screw rule

This rule is used to determine the direction of the magnetic field. If we rotate a right-handed screw using a screw driver, then the direction of current is same as the direction in which screw advances and the direction of rotation of the screw gives the direction of the magnetic field. (Figure 3.36)

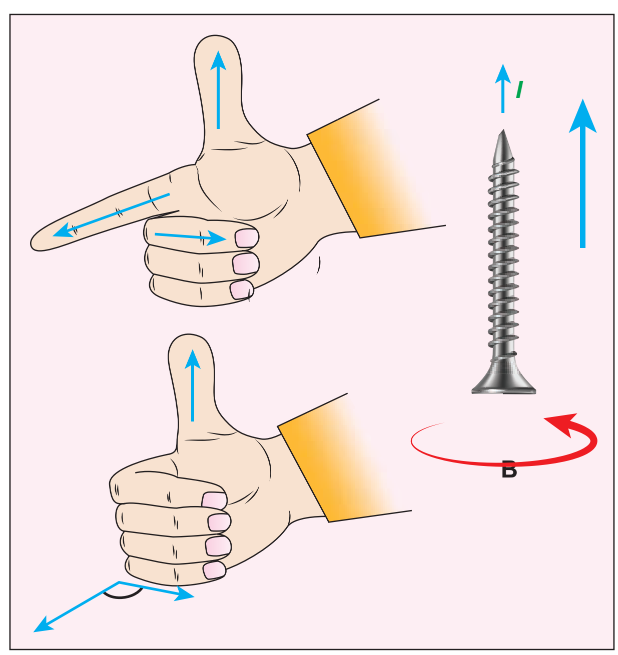
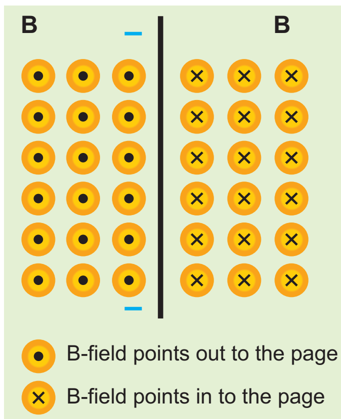


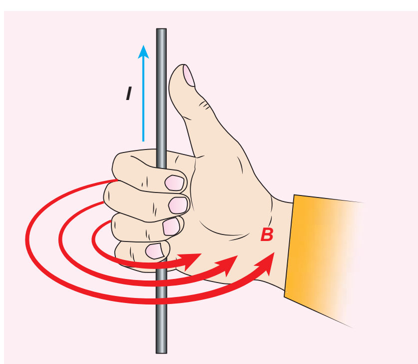
Figure 3.36 Maxwell's right hand cork screw rule

EXAMPLE 3.14

The magnetic field shown in the figure is due to the current carrying wire. In which direction does the current flow in the wire?.



Solution



Using right hand rule, current flows upwards.

3.8 BIOT - SAVART LAW

Soon after the Oersted's discovery, both Jean-Baptiste Biot and Felix Savart in 1819 did quantitative experiments on the force experienced by a magnet kept near current carrying wire and arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the magnetic field. This is true for any shape of the conductor.

3.8.1 Definition and explanation of Biot- Savart law

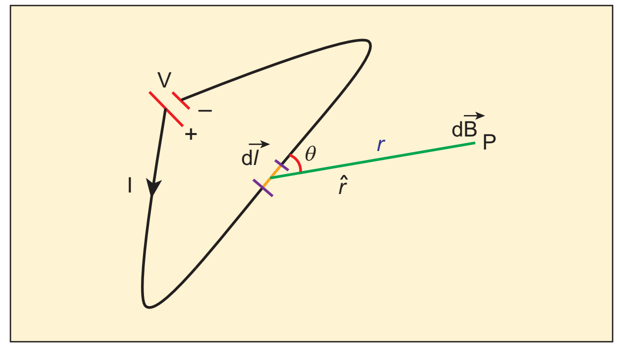


Figure 3.37 Magnetic field at a point P due to current carrying conductor

Biot and Savart experimentally observed that the magnitude of magnetic field $d\vec{B}$ at a point P (Figure 3.37) at a distance r from the small elemental length taken on a conductor carrying current varies

(i) directly as the strength of the current I

(ii) directly as the magnitude of the length element $d\vec{l}$

(iii) directly as the sine of the angle (say,θ) between $d\vec{l}$ and $\hat{r}$.

(iv) inversely as the square of the distance between the point P and length element $d\vec{l}$.

This is expressed as

$$dB \propto \frac{Idl}{r^2}\sin\theta$$

$$dB = k\frac{I\,dl}{r^2}\sin\theta$$

where $k = \frac{\mu_\circ}{4\pi}$ in SI units and k = 1 in CGS units. In vector notation,

$$d\vec{B} = \frac{\mu_\circ}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} \qquad (3.37)$$

Here vector $d\vec{B}$ is perpendicular to both I $d\vec{l}$ (pointing the direction of current flow) and the unit vector $\hat{r}$ directed from $d\vec{l}$ toward point P (Figure 3.38).

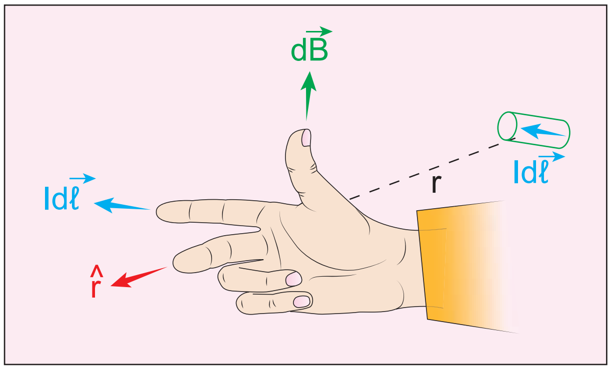


Figure 3.38 The direction of magnetic field using right hand rule

The equation (3.37) is used to compute the magnetic field only due to a small elemental length d*l* of the conductor. The net magnetic field at P due to the conductor is obtained from principle of superposition by considering the contribution from all current elements $I\,d\vec{l}$. Hence integrating equation (3.37), we get

$$\vec{B} = \int d\vec{B} = \frac{\mu_\circ I}{4\pi}\int \frac{d\vec{l}\times\hat{r}}{r^2} \qquad (3.38)$$

where the integral is taken over the entire current distribution.

Cases

1. If the point P lies on the conductor, then θ = 0°. Therefore, $d\vec{B}$ is zero.
2. If the point lies perpendicular to the conductor, then θ = 90°. Therefore, $d\vec{B}$ is maximum and is given by $d\vec{B} = \frac{I\,dl}{r^2}\hat{n}$

where $\hat{n}$ is the unit vector perpendicular to both $I\,d\vec{l}$ and $\hat{r}$

Note Electric current is not a vector quantity. It is a scalar quantity. But electric current in a conductor has direction of flow. Therefore, the electric current flowing in a small elemental conductor can be taken as vector quantity i.e. $I\,d\vec{l}$

Similarities between Coulomb's law and Biot-Savort's law

Electric and magnetic fields

- obey inverse square law, so they are long range fields.
- obey the principle of superposition and are linear with respect to source. In magnitude,

$$E \propto q$$

$$B \propto Idl$$

Difference between Coulomb's law and Biot-Savort's law

S. No.	Electric field	Magnetic field
1	Produced by a scalar source i.e., an electric charge q	Produced by a vector source i.e., current element $I\,d\vec{l}$
2	It is directed along the position vector joining the source and the point at which the field is calculated	It is directed perpendicular to the position vector $\vec{r}$ and the current element $I\,d\vec{l}$
3	Does not depend on angle	Depends on the angle between the position vector $\vec{r}$ and the current element $I\,d\vec{l}$

Note that the exponent of charge q (source) and exponent of electric field E is unity. Similarly, the exponent of current element Idl (source) and exponent of magnetic field B is unity. In other words, electric field $\vec{E}$ is proportional only to charge (source) and not on higher powers of charge $\left(q^2, q^3, etc\right)$. Similarly, magnetic field $\vec{B}$ is proportional to current element $Id\vec{l}$ (source) and not on square or cube or higher powers of current element. The cause and effect have linear relationship.

3.8.2 Magnetic field due to long straight conductor carrying current

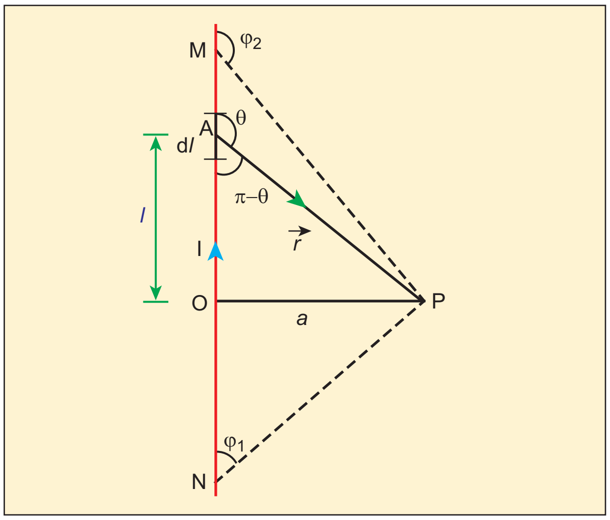


Figure 3.39 Magnetic field due to a long straight current carrying conductor

Consider a long straight wire NM with current I flowing from N to M as shown in Figure 3.39. Let P be the point at a distance a from point O. Consider an element of length dl of the wire at a distance l from point O and $\vec{r}$ be the vector joining the element dl with the point P. Let θ be the angle between $\vec{dl}$ and $\vec{r}$. Then, the magnetic field at P due to the element is

$$d\vec{B} = \frac{\mu_\circ I}{4\pi}\frac{d\vec{l}}{r^2}\sin\theta \begin{pmatrix} \textit{unit vector perpendicular} \\ \textit{to } d\vec{l} \textit{ and } \vec{r} \end{pmatrix}$$

The direction of the field is perpendicular to the plane of the paper and going into it. This can be determined by taking the cross product between two vectors $\vec{dl}$ and $\vec{r}$ (let it be $\hat{n}$). The net magnetic field can be determined by integrating equation (3.38) with proper limits.

From the Figure 3.39, in a right angle triangle PAO,

$$\tan(\pi-\theta) = \frac{a}{l}$$

$$l = -\frac{a}{\tan\theta} \quad \text{(since tan (π- θ) = - tan θ)}$$

$$l = -a\cot\theta \quad and \quad r = a\,cosec\,\theta$$

Differentiating,

$$dl = a\,cosec^2\theta\, d\theta$$

$$d\vec{B} = \frac{\mu_\circ I}{4\pi}\frac{\left(a\,cosec^2\theta\, d\theta\right)}{\left(a\,cosec\theta\right)^2}\sin\theta\, d\theta\,\hat{n}$$

$$d\vec{B} = \frac{\mu_\circ I}{4\pi}\frac{\left(a\,cosec^2\theta\, d\theta\right)}{a^2\,cosec^2\,\theta}\sin\theta\, d\theta\,\hat{n}$$

$$= \frac{\mu_\circ I}{4\pi a}\sin\theta\, d\theta\,\hat{n}$$

This is the magnetic field at a point P due to the current in small elemental length. Note that we have expressed the magnetic field OP in terms of angular coordinate i.e. θ. Therefore, the net magnetic field at the point P which can be obtained by integrating $d\vec{B}$ by varying the angle from $\theta = \varphi_1$ to $\theta = \varphi_2$ is

$$\vec{B} = \frac{\mu_\circ I}{4\pi a}\int_{\varphi_1}^{\varphi_2}\sin\theta\, d\theta\,\hat{n} = \frac{\mu_\circ I}{4\pi a}(\cos\varphi_1 - \cos\varphi_2)\,\hat{n}$$

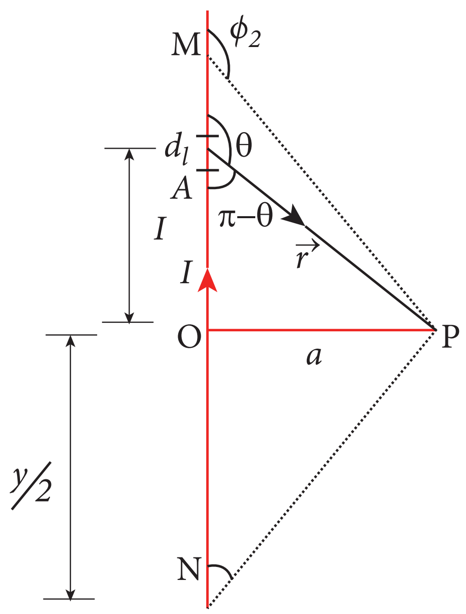
For a an infinitely long straight wire, $\varphi_1 = 0$ and $\varphi_2 = \pi$, the magnetic field is

$$\vec{B} = \frac{\mu_\circ I}{2\pi a}\hat{n} \qquad (3.39)$$

Note that here $\hat{n}$ represents the unit vector from the point O to P.

EXAMPLE 3.15

Calculate the magnetic field at a point P which is perpendicular bisector to current carrying straight wire as shown in figure.



Solution

Let the length MN = y and the point P is on its perpendicular bisector. Let O be the point on the conductor as shown in figure.

Therefore, $OM = ON = \frac{y}{2}$, then

$$\cos\varphi_1 = \frac{\frac{y}{2}}{\sqrt{\frac{y^2}{4} + a^2}} = \frac{\text{adjacent length}}{\text{hypotenuse length}}$$

$$= \frac{ON}{PN} = -\frac{\frac{y}{2}}{\sqrt{\frac{y^2}{4} + a^2}} = -\frac{y}{\sqrt{y^2 + 4a^2}}$$

$$\cos\varphi_2 = \frac{\text{adjacent length}}{\text{hypotenuse length}} = \frac{OM}{PM}$$

$$= -\frac{\frac{y}{2}}{\sqrt{\frac{y^2}{4} + a^2}} = -\frac{y}{\sqrt{y^2 + 4a^2}}$$

Hence,

$$\vec{B} = \frac{\mu_\circ I}{4\pi a}\frac{2y}{\sqrt{y^2 + 4a^2}}\hat{n}$$

For long straight wire, $y \to \infty$,

$$\vec{B} = \frac{\mu_\circ I}{2\pi a}\hat{n}$$

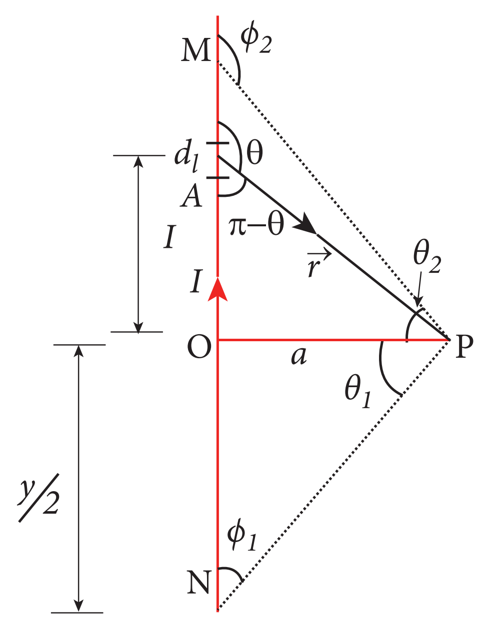
The result obtained is same as we obtained in equation (3.39).

EXAMPLE 3.16

Show that for a straight conductor, the magnetic field

$$\vec{B} = \frac{\mu_\circ I}{4\pi a}(\cos\varphi_1 - \cos\varphi_2)\hat{n}$$

$$= \frac{\mu_\circ I}{4\pi a}(\sin\theta_1 + \sin\theta_2)\hat{n}$$



Solution:

In a right angle triangle OPN, let the angle $\angle OPN = \theta_1$ which implies, $\varphi_1 = \frac{\pi}{2} - \theta_1$

and also in a right angle triangle OPM, $\angle OPM = \theta_2$ which implies, $\varphi_2 = \frac{\pi}{2} + \theta_2$

Hence,

$$\vec{B} = \frac{\mu_\circ I}{4\pi a}\left(\cos\left(\frac{\pi}{2} - \theta_1\right) - \cos\left(\frac{\pi}{2} + \theta_2\right)\right)\hat{n}$$

$$= \frac{\mu_\circ I}{4\pi a}(\sin\theta_1 + \sin\theta_2)\hat{n}$$

3.8.3 Magnetic field produced along the axis of the current carrying circular coil

Consider a current carrying circular loop of radius R and let I be the current flowing through the wire in the direction as shown in Figure 3.40. The magnetic field at a point P on the axis of the circular coil at a distance z from its center of the coil O. It is computed by taking two diametrically opposite line elements of the coil each of length $d\vec{l}$ at C and D. Let $\vec{r}$ be the vector joining the current element (I $d\vec{l}$) at C to the point P.

$$PC = PD = r = \sqrt{R^2 + Z^2} \text{ and}$$
$$\text{angle } \angle CPO = \angle DPO = \theta$$

According to Biot-Savart's law, the magnetic field at P due to the current element $I\,d\vec{l}$ is

$$d\vec{B} = \frac{\mu_\circ}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}$$

The magnitude of magnetic field due to current element $I\,d\vec{l}$ at C and D are equal because of equal distance from the coil. The magnetic field $d\vec{B}$ due to each current element $I\,d\vec{l}$ is resolved into two components; dB sin θ along y - direction and dB cos θ along z - direction. Horizontal components of each current element cancels out while the vertical components (dB cos θ $\hat{k}$) alone contribute to total magnetic field at the point P.

If we integrate $d\vec{l}$ around the loop, $d\vec{B}$ sweeps out a cone as shown in Figure 3.40, then the net magnetic field $\vec{B}$ at point P is

$$\vec{B} = \int d\vec{B} = \int dB\cos\theta\,\hat{k}$$

$$\vec{B} = \frac{\mu_\circ I}{4\pi}\int\frac{dl}{r^2}\cos\theta\,\hat{k}$$

But $\cos\theta = \frac{R}{(R^2 + Z^2)^{\frac{1}{2}}}$, using Pythagorous theorem $r^2 = R^2 + Z^2$ and integrating line element from 0 to 2πR, we get

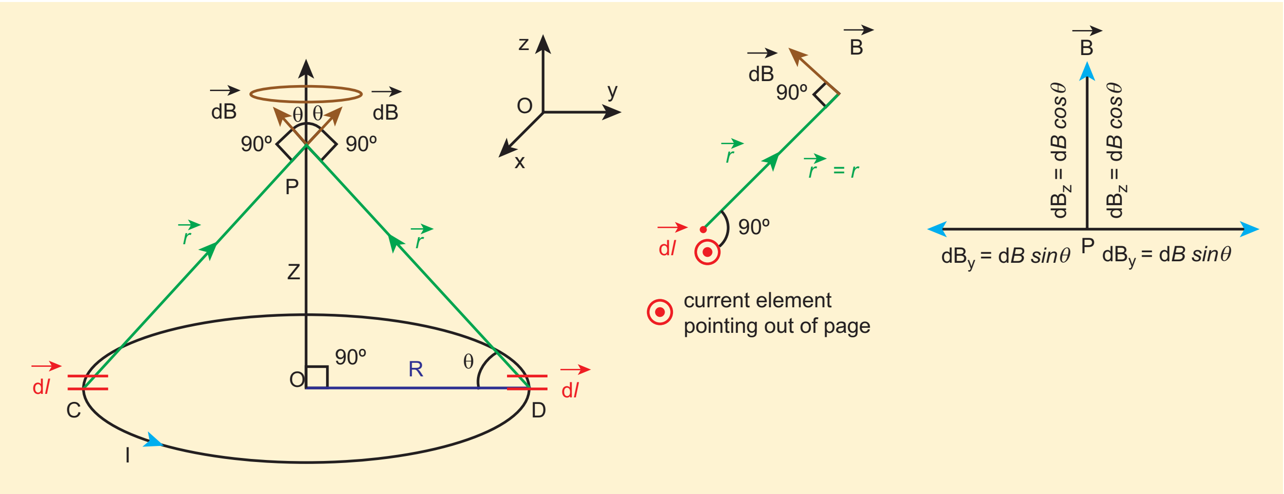


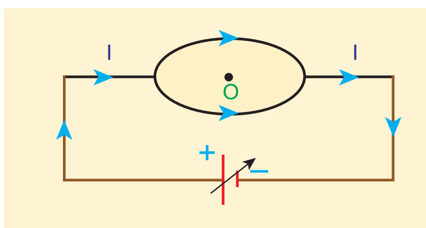
Figure 3.40 Current carrying circular loop using Biot-Savart's law

$$\vec{B} = \frac{\mu_\circ I}{2\pi}\frac{R^2}{\left(R^2+Z^2\right)^{\frac{3}{2}}}\hat{k} \qquad (3.40)$$

Note that the magnetic field $\vec{B}$ points along the direction from the point O to P. Suppose if the current flows in clockwise direction, then magnetic field points in the direction from the point P to O.

EXAMPLE 3.17

What is the magnetic field at the center of the loop shown in figure?



Solution

The magnetic field due to current in the upper hemisphere and lower hemisphere of the circular coil are equal in magnitude but opposite in direction. Hence, the net magnetic field at the center of the loop (at point O) is zero $\vec{B} = \vec{0}$.

3.8.4 Current loop as a magnetic dipole

The magnetic field from the centre of a circular loop of radius R along the axis is given by

$$\vec{B} = \frac{\mu_\circ I}{2}\frac{R^2}{\left(R^2+z^2\right)^{\frac{3}{2}}}\hat{k}$$

At larger distance Z >> R, therefore $R^2 + Z^2 \approx Z^2$, we have

$$\vec{B} = \frac{\mu_\circ I}{2}\frac{R^2}{Z^3}\hat{k} \qquad (3.41)$$

Let A be the area of the circular loop A = π R^2. So rewriting the equation (3.41) in terms of area of the loop, we have

$$\vec{B} = \frac{\mu_\circ I}{2\pi}\frac{A}{Z^3}\hat{k}$$

(multiply and divide by 2)

$$\vec{B} = \frac{\mu_\circ}{4\pi}\frac{2IA}{Z^3}\hat{k} \qquad (3.42)$$

Comparing equation (3.42) with equation (3.14) dimensionally, we get

$$p_m = I\,A$$

where p_m is called magnetic dipole moment. In vector notation,

$$\vec{p}_m = I\,\vec{A} \qquad (3.43)$$

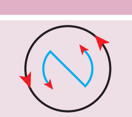
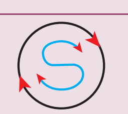
This implies that a current carrying circular loop behaves as a magnetic dipole of magnetic moment $\vec{p}_m$. So, **the magnetic dipole moment of any current loop is equal to the product of the current and area of the loop.**

Right hand thumb rule

In order to determine the direction of magnetic moment, we use right hand thumb rule (mnemonic) which states that

If we curl the fingers of right hand in the direction of current in the loop, then the stretched thumb gives the direction of the magnetic moment associated with the loop.

Table 3.3 End rule – polarity with direction of current in circular loop

Current in circular loop	Polarity	Picture
Anti-clockwise current	North Pole	 Anti-clockwise current Polarity: North Pole
Clockwise current	South Pole	 Clockwise current Polarity: South Pole

3.8.5 Magnetic dipole moment of revolving electron

Suppose an electron undergoes circular motion around the nucleus as shown in Figure 3.41. The circulating electron in a loop is like current in a circular loop (since flow of charge is current). The magnetic dipole moment due to current carrying circular loop is

$$\vec{\mu}_L = I\vec{A} \quad (3.44)$$

In magnitude,

$$\mu_L = IA$$

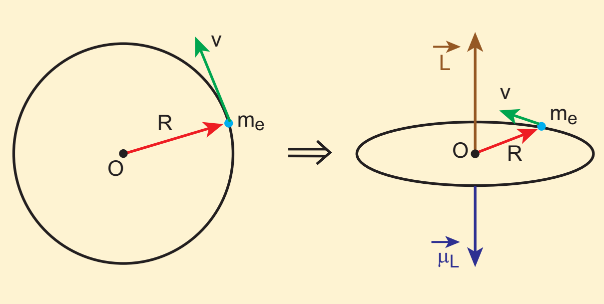


Figure 3.41 (a) Electron revolving in a circular orbit (b) Direction of magnetic dipole moment vector and orbital angular momentum vector are opposite

If T is the time period of an electron, the current due to circular motion of the electron is

$$I = \frac{-e}{T} \quad (3.45)$$

where $-e$ is the charge of an electron. If R is the radius of the circular orbit and v is the velocity of the electron in the circular orbit, then

$$T = \frac{2\pi R}{v} \quad (3.46)$$

Using equation (3.45) and equation (3.46) in equation (3.44), we get

$$\mu_L = -\frac{e}{\frac{2\pi R}{v}}\pi R^2 = -\frac{evR}{2} \quad (3.47)$$

where A = πR^2 is the area of the circular loop. By definition, angular momentum of the electron about O is

$$\vec{L} = \vec{R} \times \vec{p}$$

In magnitude,

$$L = Rp = mvR \quad (3.48)$$

Using equation (3.47) and equation (3.48), we get

$$\frac{\mu_L}{L} = -\frac{\frac{evR}{2}}{mvR} = -\frac{e}{2m} \Rightarrow \vec{\mu}_L = -\frac{e}{2m}\vec{L} \quad (3.49)$$

The negative sign indicates that the magnetic moment and angular momentum are in opposite direction.

In magnitude,

$$\frac{\mu_L}{L} = \frac{e}{2m} = \frac{1.60 \times 10^{-19}}{2 \times 9.11 \times 10^{-31}} = 0.0878 \times 10^{12}$$

$$\frac{\mu_L}{L} = 8.78 \times 10^{10}\, C\, kg^{-1} = \text{constant}$$

The ratio $\frac{\mu_L}{L}$ is a constant and also known as gyro-magnetic ratio $\left(\frac{e}{2m}\right)$. It must be noted that the gyro-magnetic ratio is a constant of proportionality which connects angular momentum of the electron and the magnetic moment of the electron.

According to Neil's Bohr quantization rule, the angular momentum of an electron moving in a stationary orbit is quantized, which means,

$$L = n\hbar = n\frac{h}{2\pi}$$

where, h is the Planck's constant (h = 6.63 x 10^{-34} J s) and number n takes natural numbers

(i.e., n = 1,2,3,....). Hence,

$$\mu_L = \frac{e}{2m}L = n\frac{eh}{4\pi m}Am^2$$

$$\mu_L = n\frac{(1.60\times10^{-19})h}{4\pi m}Am^2$$

$$= n\frac{(1.60\times10^{-19})(6.63\times10^{-34})}{4\times3.14\times(9.11\times10^{-31})}$$

$$\mu_L = n\times9.27\times10^{-24}Am^2$$

The minimum magnetic moment can be obtained by substituting n = 1,

$$\mu_L = 9.27\times10^{-24}Am^2 = 9.27\times10^{-24}JT^{-1}$$
$$= (\mu_L)_{min} = \mu_B$$

where, $\mu_B = \frac{eh}{4\pi m} = 9.27\times10^{-24}Am^2$ is called Bohr magneton. This is a convenient unit with which one can measure atomic magnetic moments.

Note: Bohr quantization rule will be discussed in unit 8 of second volume

3.9 AMPÈRE'S CIRCUITAL LAW

Ampère's circuital law is used to calculate magnetic field at a point whenever there is a symmetry in the problem. This is similar to Gauss's law in electrostatics. These are powerful methods whenever there is symmetry in the problem.

3.9.1 Definition and explanation of Ampère's circuital law

Ampère's law: The line integral of magnetic field over a closed loop is μ_0 times net current enclosed by the loop.

$$\oint_C \vec{B}.\vec{dl} = \mu_\circ I_{enclosed} \quad (3.50)$$

where $I_{enclosed}$ is the net current linked by the closed loop C. Note that the line integral does not depend on the shape of the path or the position of the conductor with the magnetic field.

Note

Line integral means integral over a line or curve, symbol used is $\int$.

Closed line integral means integral over a closed curve (or line), symbol is $\oint$ or $\oint_c$

3.9.2 Magnetic field due to the current carrying wire of infinite length using Ampère's law

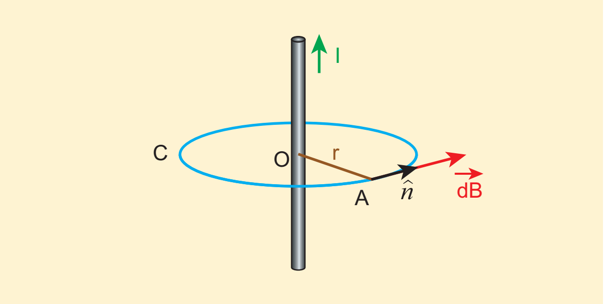


Figure 3.42 Ampèrian loop for current carrying straight wire

Consider a straight conductor of infinite length carrying current I and the direction of magnetic field lines is shown in Figure 3.42. Since the wire is geometrically cylindrical in shape and symmetrical about its axis, we construct an Ampèrian loop in the form of a circular shape at a distance *r* from the centre of the conductor as shown in Figure 3.42. From the Ampère's law, we get

$$\oint_C \vec{B}.d\vec{l} = \mu_\circ I$$

where $d\vec{l}$ is the line element along the amperian loop (tangent to the circular loop). Hence, the angle between magnetic field vector and line element is zero. Therefore,

$$\oint_C B\,dl = \mu_\circ I$$

where I is the current enclosed by the Ampèrian loop. Due to the symmetry, the magnitude of the magnetic field is uniform over the Ampèrian loop, we can take B out of the integration.

$$B\oint_C dl = \mu_\circ I$$

For a circular loop, the circumference is 2πr, which implies,

$$B\int_0^{2\pi r} dl = \mu_\circ I$$

$$\vec{B}\cdot 2\pi r = \mu_\circ I$$

$$B = \frac{\mu_\circ I}{2\pi r}$$

In vector form, the magnetic field is

$$\vec{B} = \frac{\mu_\circ I}{2\pi r}\hat{n}$$

where $\hat{n}$ is the unit vector along the tangent to the Ampèrian loop as shown in the Figure 3.42. This perfectly agrees with the result obtained from Biot-Savart's law as given in equation (3.39).

EXAMPLE 3.18

Compute the magnitude of the magnetic field of a long, straight wire carrying a current of 1 A at distance of 1m from it. Compare it with Earth's magnetic field.

Solution

Given that 1 = 1 A and radius r = 1 m

$$B_{straightwire} = \frac{\mu_\circ I}{2\pi r} = \frac{4\pi\times 10^{-7}\times 1}{2\pi \times 1} = 2\times 10^{-7}\,T$$

But the Earth's magnetic field is $B_{Earth} \sim 10^{-5}\,T$

So, $B_{straightwire}$ is one hundred times smaller than B_{Earth}.

Solenoid

A solenoid is a long coil of wire closely wound in the form of helix as shown in Figure 3.43. When electric current is passed through the solenoid, the magnetic field is produced. The magnetic field of the solenoid is due to the superposition of magnetic fields of each turn of the solenoid. The direction of magnetic field due to solenoid is given by right hand palm-rule (mnemonic).

Inside the solenoid, the magnetic field is nearly uniform and parallel to its axis whereas, outside the solenoid the field is negligibly small. Based on the direction of the current, one end of the solenoid behaves like North Pole and the other end behaves like South Pole.

The current carrying solenoid is held in right hand. If the fingers curl in the direction of current, then extended thumb gives the direction of magnetic field of current carrying solenoid. It is shown in Figure 3.44. Hence, the magnetic field of a

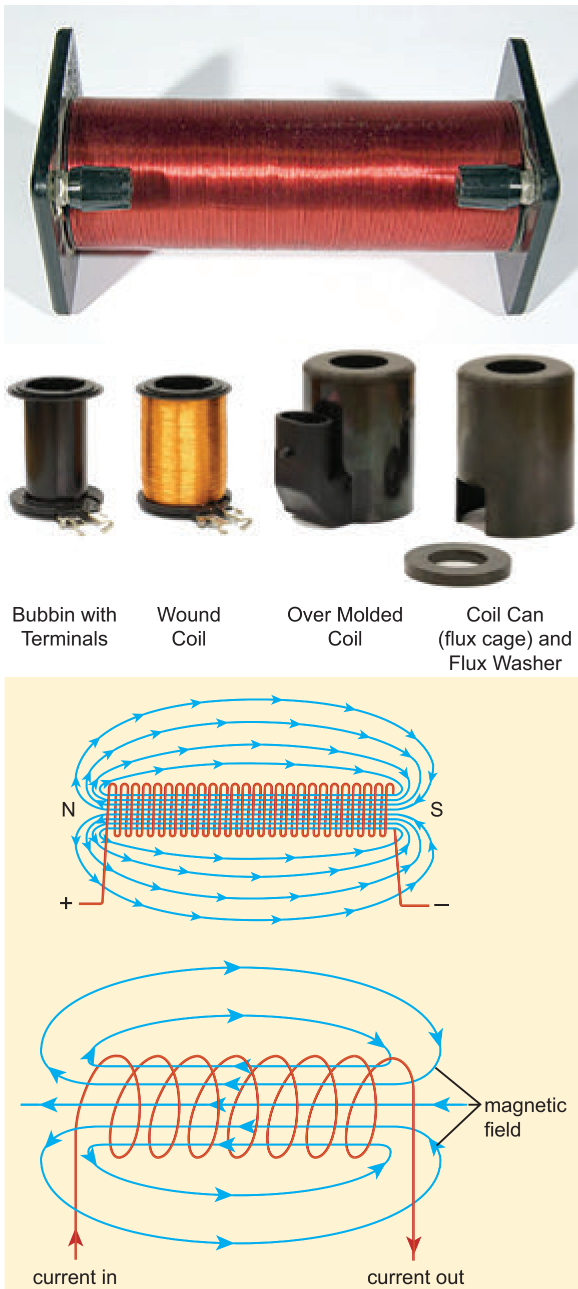


Figure 3.43 Solenoid

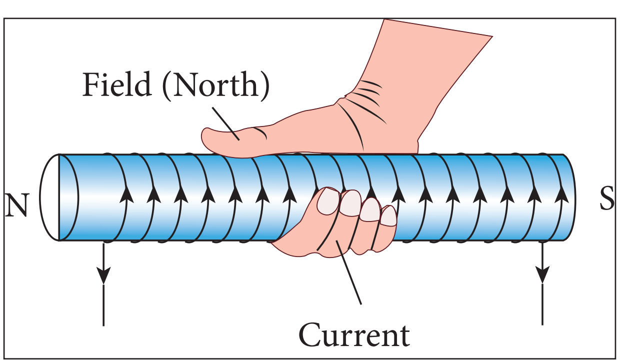


Figure 3.44 The direction of magnetic field of solenoid

solenoid looks like the magnetic field of a bar magnet.

The solenoid is assumed to be long which means that the length of the solenoid is large when compared to its diameter. The winding need not to be always circular, it can also be in other shapes. We consider here only circularly wound solenoid as shown in Figure 3.45.

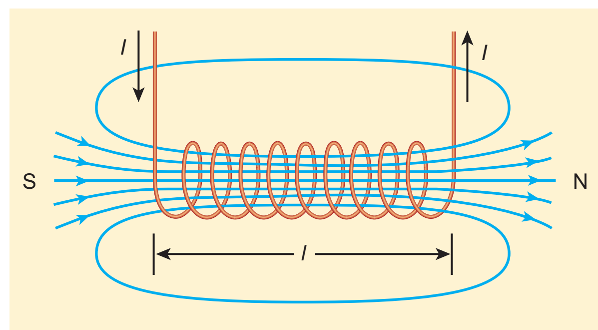


Figure 3.45 Solenoid as a bar magnet

3.9.3 Magnetic field due to a long current carrying solenoid

Consider a solenoid of length L having N turns. The diametre of the solenoid is assumed to be much smaller when compared to its length and the coil is wound very closely.

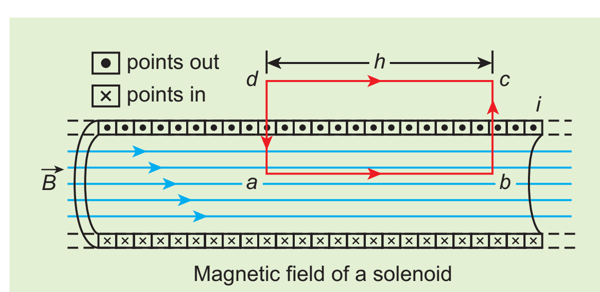


Figure 3.46 Amperian loop for solenoid

In order to calculate the magnetic field at any point inside the solenoid, we use Ampere's circuital law. Consider a rectangular loop abcd as shown in Figure 3.46. Then from Ampère's circuital law,

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_\circ I_{enclosed}$$

$$= \mu_\circ \times (total\, current\, enclosed\, by\, Amperian\, loop)$$

The left hand side of the equation is

$$\oint_C \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}.d\vec{l} + \int_b^c \vec{B}.d\vec{l} + \int_c^d \vec{B}.d\vec{l} + \int_d^a \vec{B}.d\vec{l}$$

Since the elemental lengths along bc and da are perpendicular to the magnetic field which is along the axis of the solenoid, the integrals

$$\int_b^c \vec{B}.d\vec{l} = \int_b^c |\vec{B}||d\vec{l}| \cos 90^\circ = 0$$

$$\int_d^a \vec{B}.d\vec{l} = 0$$

Since the magnetic field outside the solenoid is zero, the integral $\int_c^d \vec{B}.d\vec{l} = 0$

For the path along ab, the integral is

$$\int_a^b \vec{B}.d\vec{l} = B\int_a^b dl \cos 0^\circ = B\int_a^b dl$$

where the length of the loop ab as shown in the Figure 3.46 is h. But the choice of length of the loop ab is arbitrary. We can take very large loop such that it is equal to the length of the solenoid L. Therefore the integral is

$$\int_a^b \vec{B}.d\vec{l} = BL$$

Let N I be the current passing through the solenoid of N turns, then

$$\int_a^b \vec{B}.d\vec{l} = BL = \mu_\circ NI \Rightarrow B = \mu_\circ \frac{NI}{L}$$

The number of turns per unit length is given by $\frac{N}{L} = n$, Then

$$B = \mu_\circ \frac{nLI}{L} = \mu_\circ nI \qquad (3.51)$$

Since n is a constant for a given solenoid and μ_0 is also constant. For a fixed current I, the magnetic field inside the solenoid is also a constant.

Note

Solenoid can be used as electromagnets. It produces strong magnetic field that can be turned ON or OFF. This is not possible in case of permanent magnet. Further the strength of the magnetic field can be increased by keeping iron bar inside the solenoid. This is because the magnetic field of the solenoid magnetizes the iron bar and hence the net magnetic field is the sum of magnetic field of the solenoid and magnetic field of magnetised iron. Because of these properties, solenoids are useful in designing variety of electrical appliances.

EXAMPLE 3.19

Calculate the magnetic field inside a solenoid, when

(a) the length of the solenoid becomes twice and fixed number of turns

(b) both the length of the solenoid and number of turns are double

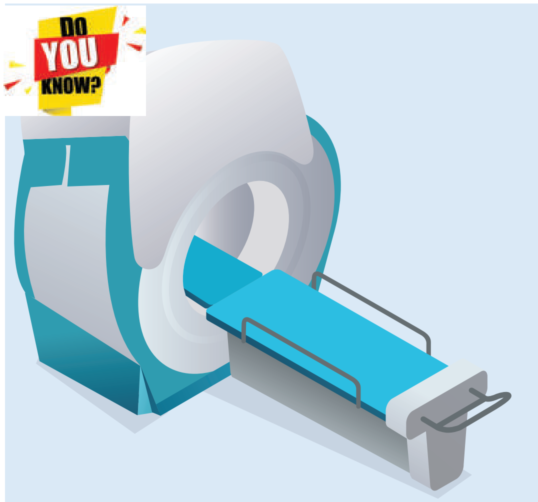
(c) the number of turns becomes twice for the fixed length of the solenoid

Compare the results.

Solution

The magnetic field of a solenoid (inside) is

$$B_{L,N} = \mu_\circ \frac{NI}{L}$$



MRI is Magnetic Resonance Imaging which helps the physicians to diagonise or monitor treatment for a variety of abnormal conditions happening within the head, chest, abdomen and pelvis. It is a non invasive medical test. The patient is placed in a circular opening (actually interior of a solenoid which is made up of superconducting wire) and large current is sent through the superconducting wire to produce a strong magnetic field. So, it uses more powerful magnet, radio frequency pulses and a computer to produce pictures of organs which helps the physicians to examine various parts of the body.

(a) length of the solenoid becomes twice and fixed number of turns

L→2L (length becomes twice)

N→N (number of turns are fixed)

The magnetic field is

$$B_{2L,N} = \mu_\circ \frac{NI}{2L} = \frac{1}{2} B_{L,N}$$

(b) both the length of the solenoid and number of turns are double

L→2L (length becomes twice)

N→2N (number of turns becomes twice)

The magnetic field is

$$B_{2L,2N} = \mu_\circ \frac{2NI}{2L} = B_{L,N}$$

(c) the number of turns becomes twice but for the fixed length of the solenoid

L→L (length is fixed)

N→2N (number of turns becomes twice)

The magnetic field is

$$B_{L,2N} = \mu_\circ \frac{2NI}{L} = 2B_{L,N}$$

From the above results,

$$B_{L,2N} > B_{2L,2N} > B_{2L,N}$$

Thus, strength of the magnetic field is increased when we pack more loops into the same length for a given current.

3.9.5 Toroid

A solenoid is bent in such a way its ends are joined together to form a closed ring shape, is called a toroid which is shown in Figure 3.47. The magnetic field has constant magnitude inside the toroid whereas in the interior region (say, at point P) and exterior region (say, at point Q), the magnetic field is zero.

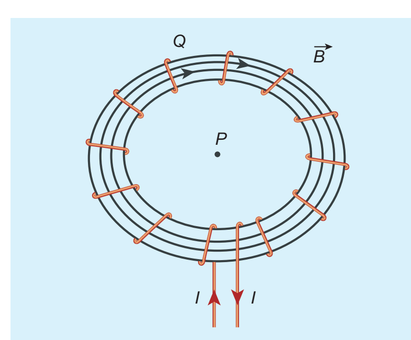


Figure 3.47 Toroid

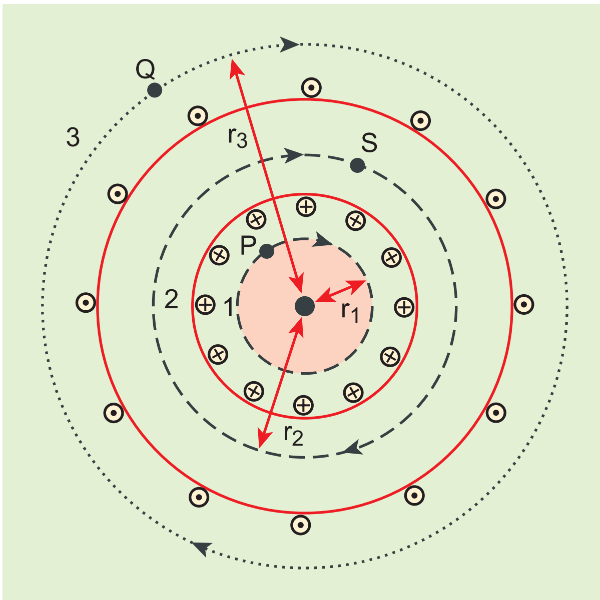


Figure 3.48 Toroid – Amperian loop

(a) Open space interior to the toroid

Let us calculate the magnetic field B_P at point P. We construct an Amperian loop 1 of radius r_1 around the point P as shown in Figure 3.48. For simplicity, we take circular loop so that the length of the loop is its circumference.

$$L_1 = 2\pi r_1$$

Ampère's circuital law for the loop 1 is

$$\oint_{loop1} \vec{B}_P.\vec{dl} = \mu_\circ I_{enclosed}$$

Since, the loop1 encloses no current, $I_{enclosed} = 0$

$$\oint_{loop1} \vec{B}_P.\vec{dl} = 0$$

This is possible only if the magnetic field at point P vanishes i.e.

$$\vec{B}_P = 0$$

(b) Open space exterior to the toroid

Let us calculate the magnetic field B_Q at point Q. We construct an Amperian loop 3 of radius r_3 around the point Q as shown in Figure 3.48. The length of the loop is

$$L_3 = 2\pi r_3$$

Ampère's circuital law for the loop 3 is

$$\oint_{loop3} \vec{B}_Q.\vec{dl} = \mu_\circ I_{enclosed}$$

Since, in each turn of the toroid loop, current coming out of the plane of paper is cancelled by the current going into the plane of paper. Thus, $I_{enclosed} = 0$

$$\oint_{loop3} \vec{B}_Q.\vec{dl} = 0$$

This is possible only if the magnetic field at point Q vanishes i.e.

$$\vec{B}_Q = 0$$

(c) Inside the toroid

Let us calculate the magnetic field B_S at point S by constructing an Amperian loop 2 of radius r_2 around the point S as shown in Figure 3.48. The length of the loop is

$$L_2 = 2\pi r_2$$

Ampere's circuital law for the loop 2 is

$$\oint_{loop2} \vec{B}_S.\vec{dl} = \mu_\circ I_{enclosed}$$

Let I be the current passing through the toroid and N be the number of turns of the toroid, then

$$I_{enclosed} = NI$$

and $$\oint_{loop2} \vec{B}_S.\vec{dl} = \oint_{loop2} Bdl\cos\theta = B2\pi r_2$$

$$\oint_{loop2} \vec{B}_S.\vec{dl} = \mu_\circ NI$$

$$B_S = \mu_\circ \frac{NI}{2\pi r_2}$$

The number of turns per unit length is $n = \frac{N}{2\pi r_2}$, then the magnetic field at point S is

$$B_S = \mu_\circ nI \quad (3.52)$$

3.10 LORENTZ FORCE

When an electric charge q is kept at rest in a magnetic field, no force acts on it. At the same time, if the charge moves in the magnetic field, it experiences a force. This force is different from Coulomb force, studied in unit 1. This force is known as magnetic force. It is given by the equation

$$\vec{F} = q(\vec{v} \times \vec{B}) \quad (3.53)$$

In general, if the charge is moving in both the electric and magnetic fields, the total force experienced by the charge is given by $\vec{F} = q\vec{E}(\vec{v} \times \vec{B})$. It is known as Lorentz force.

3.10.1 Force on a moving charge in a magnetic field

When an electric charge q is moving with velocity $\vec{v}$ in the magnetic field $\vec{B}$, it experiences a force, called magnetic force $\vec{F}_m$. After careful experiments, Lorentz deduced the force experienced by a moving charge in the magnetic field $\vec{F}_m$

$$\vec{F}_m = q(\vec{v} \times \vec{B}) \quad (3.54)$$

In magnitude, $F_m = qvB\sin\theta$ (3.55)

The equations (3.54) and (3.55) imply

1. $\vec{F}_m$ is directly proportional to the magnetic field $\vec{B}$
2. $\vec{F}_m$ is directly proportional to the velocity $\vec{v}$
3. $\vec{F}_m$ is directly proportional to sine of the angle between the velocity and magnetic field
4. $\vec{F}_m$ is directly proportional to the magnitude of the charge q
5. The direction of $\vec{F}_m$ is always perpendicular to $\vec{v}$ and $\vec{B}$ as $\vec{F}_m$ is the cross product of $\vec{v}$ and $\vec{B}$

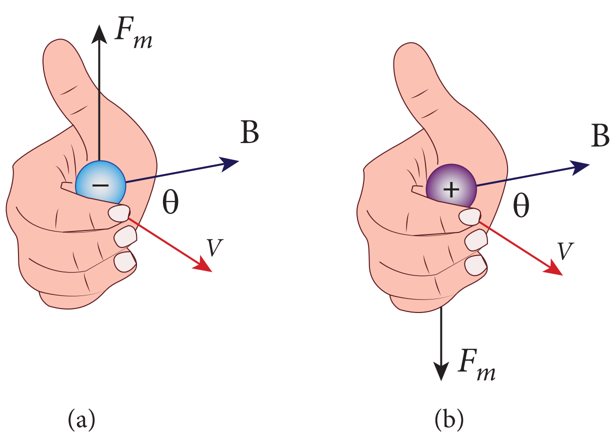


Figure 3.49 Direction of the Lorentz force on (a) positive charge (b) negative charge

7. The direction of $\vec{F}_m$ on negative charge is opposite to the direction of $\vec{F}_m$ on positive charge provided other factors are identical as shown Figure 3.49
8. If velocity $\vec{v}$ of the charge q is along magnetic field $\vec{B}$ then, $\vec{F}_m$ is zero

Definition of tesla

The strength of the magnetic field is one tesla if unit charge moving in it with unit velocity experiences unit force.

$$1\ T = \frac{1\,N\,s}{C\,m} = 1\frac{N}{A\,m} = 1N\,A^{-1}m^{-1}$$

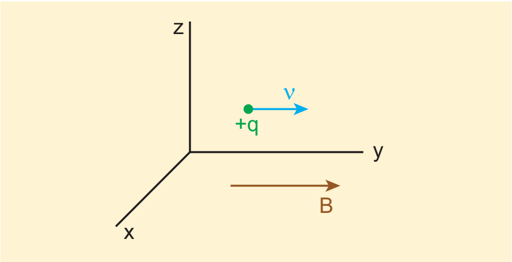
EXAMPLE 3.20

A particle of charge q moves with velocity $\vec{v}$ along positive y - direction in a magnetic field $\vec{B}$. Compute the Lorentz force experienced by the particle (a) when magnetic field is along positive y-direction (b) when magnetic field points in positive z - direction (c) when magnetic field is in zy - plane and making an angle θ with velocity of the particle. Mark the direction of magnetic force in each case.

Solution

Velocity of the particle is $\vec{v} = v\hat{j}$

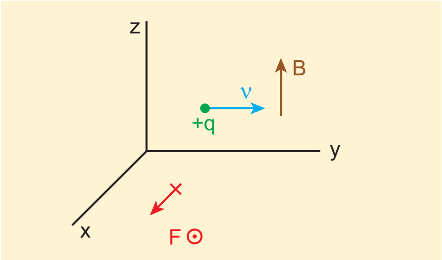
(a) Magnetic field is along positive y - direction, this implies, $\vec{B} = v\hat{j}$



From Lorentz force, $\vec{F}_m = q\left(v\hat{j} \times B\hat{j}\right) = 0$

So, no force acts on the particle when it moves along the direction of magnetic field.

(b) Magnetic field points in positive z - direction, this implies, $\vec{B} = B\hat{k}$

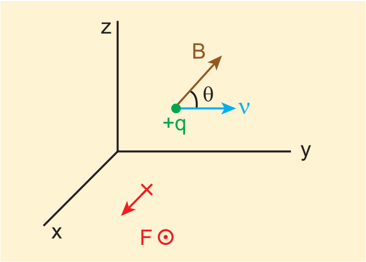


From Lorentz force,

$\vec{F}_m = q\left(v\hat{j} \times B\hat{k}\right) = qvB\,\hat{i}$

Therefore, the magnitude of the Lorentz force is qvB and direction is along positive x - direction.

(c) Magnetic field is in zy - plane and making an angle θ with the velocity of the particle, which implies $\vec{B} = B\cos\theta\,\hat{j} + B\sin\theta\,\hat{k}$



From Lorentz force,

$\vec{F}_m = q\left(v\hat{j}\right)\times(B\cos\theta\hat{j} + B\sin\theta\hat{k})$

$= qvB\sin\theta\,\hat{i}$

EXAMPLE 3.21

Compute the work done and power delivered by the Lorentz force on the particle of charge q moving with velocity $\vec{v}$. Calculate the angle between Lorentz force and velocity of the charged particle and also interpret the result.

Solution

For a charged particle moving on a magnetic field, $\vec{F} = q\left(\vec{v}\times\vec{B}\right)$

The work done by the magnetic field is

$$W = \int \vec{F}.d\vec{r} = \int \vec{F}.\vec{v}dt$$

$$W = q\int\left(\vec{v}\times\vec{B}\right).\vec{v}\,dt = 0$$

Since $\vec{v}\times\vec{B}$ is perpendicular to $\vec{v}$ and hence $\left(\vec{v}\times\vec{B}\right)\cdot\vec{v} = \vec{0}$ This means that Lorentz force do no work on the particle. From work-kinetic energy theorem, (Refer section 4.2.6, XI th standard Volume I)

$$\frac{dW}{dt} = P = 0$$

Since, $\vec{F}.\vec{v} = 0 \Rightarrow \vec{F}$ and $\vec{v}$ are perpendicular to each other. The angle between Lorentz force and velocity of the charged particle is 90°. Thus Lorentz force changes the direction of the velocity but not the magnitude of the velocity. Hence Lorentz force does no work and also does not alter kinetic energy of the particle.

3.10.2 Motion of a charged particle in a uniform magnetic field

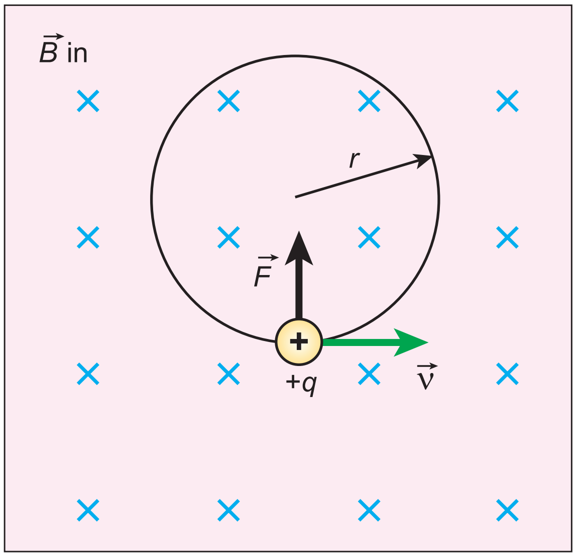


Figure 3.50 Circular motion of a charged particle in a perpendicular uniform magnetic field

Consider a charged particle of charge q having mass m enters into a region of uniform magnetic field $\vec{B}$ with velocity $\vec{v}$ such that velocity is perpendicular to the magnetic field. As soon as the particle enters into the field, Lorentz force acts on it in a direction perpendicular to both magnetic field $\vec{B}$ and velocity $\vec{v}$.

As a result, the charged particle moves in a circular orbit as shown in Figure 3.50.

The Lorentz force on the charged particle is given by

$$\vec{F} = q(\vec{v} \times \vec{B})$$

Since Lorentz force alone acts on the particle, the magnitude of the net force on the particle is

$$\sum_i F_i = F_m = qvB$$

This Lorentz force acts as centripetal force for the particle to execute circular motion. Therefore,

$$qvB = m\frac{v^2}{r}$$

The radius of the circular path is

$$r = \frac{mv}{qB} = \frac{p}{qB} \quad (3.56)$$

where p = mv is the magnitude of the linear momentum of the particle. Let T be the time taken by the particle to finish one complete circular motion, then

$$T = \frac{2\pi r}{v} \quad (3.57)$$

Hence substituting (3.56) in (3.57), we get

$$T = \frac{2\pi m}{qB} \quad (3.58)$$

Equation (3.58) is called the **cyclotron period**. The reciprocal of time period is the frequency *f*, which is

$$f = \frac{1}{T}$$

$$f = \frac{qB}{2\pi m} \quad (3.59)$$

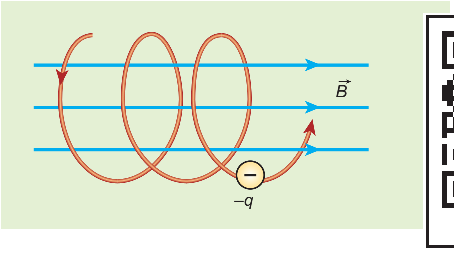
In terms of angular frequency ω,

$$\omega = 2\pi f = \frac{q}{m}B \quad (3.60)$$

Equations (3.59) and (3.60) are called as **cyclotron frequency or gyro-frequency**.

From equations (3.58), (3.59) and (3.60), we infer that time period and frequency depend only on charge-to-mass ratio (specific charge) but not velocity or the radius of the circular path.

If a charged particle moves in a region of uniform magnetic field such that its velocity is not perpendicular to the magnetic field, then the velocity of the particle is split up into two components; one component is parallel to the field while the other perpendicular to the field. The component of velocity parallel to field remains unchanged and the component perpendicular to field keeps changing due to the Lorentz force. Hence the path of the particle is not a circle; it is a helix around the field lines as shown in Figure 3.51.



FFI8SN

Figure 3.51 Helical path of the electron in a uniform magnetic field

For an example, the helical path of an electron when it moves in a magnetic field is shown in Figure 3.52. Inside the particle detector called cloud chamber, the path is made visible by the condensation of water droplets.

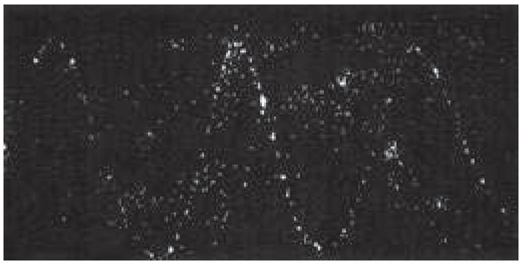


Figure 3.52 Helical path of the electron inside the cloud chamber

EXAMPLE 3.22

An electron moving perpendicular to a uniform magnetic field 0.500 T undergoes circular motion of radius 2.80 mm. What is the speed of electron?

Solution

Charge of an electron q = -1.60×10^{-19} C

$\Rightarrow |q| = 1.60\times10^{-19} C$

Magnitude of magnetic field B = 0.500 T

Mass of the electron, m = 9.11×10^{-31} kg

Radius of the orbit, r = 2.50 mm = 2.50×10^{-3} m

Velocity of the electron, $v = |q|\frac{rB}{m}$

$$v = 1.60\times10^{-19}\times\frac{2.50\times10^{-3}\times0.500}{9.11\times10^{-31}}$$

$$v = 2.195\times10^{8}\, m\, s^{-1}$$

EXAMPLE 3.23

A proton moves in a uniform magnetic field of strength 0.500 T magnetic field is directed along the x-axis. At initial time, $t = 0\ s$, the proton has velocity $\vec{v} = \left(1.95\times10^{5}\hat{i} + 2.00\times10^{5}\hat{k}\right) m\, s^{-1}$. Find

(a) At initial time, what is the acceleration of the proton.

(b) Is the path circular or helical?. If helical, calculate the radius of helical trajectory and also calculate the pitch of the helix (Note: Pitch of the helix is the distance travelled along the helix axis per revolution).

Solution

Magnetic field $\vec{B} = 0.500\hat{i}\ T$

Velocity of the particle

$$\vec{v} = \left(1.95\times10^{5}\hat{i} + 2.00\times10^{5}\hat{k}\right) m s^{-1}$$

Charge of the proton $q = 1.60\times10^{-19} C$

Mass of the proton $m = 1.67\times10^{-27} kg$

(a) The force experienced by the proton is

$$\vec{F} = q\left(\vec{v}\times\vec{B}\right)$$

$$= 1.60\times10^{-19}\times\left(\left(1.95\times10^{5}\hat{i} + 2.00\times10^{5}\hat{k}\right)\times\left(0.500\hat{i}\right)\right)$$

$$\vec{F} = 1.60\times10^{-14} N\,\hat{j}$$

Therefore, from Newton's second law,

$$\vec{a} = \frac{1}{m}\vec{F} = \frac{1}{1.67\times10^{-27}}\left(1.60\times10^{-14}\right)$$

$$= 9.58\times10^{12} m\,s^{-2}$$

(b) Trajectory is helical

Radius of helical path is

$$R = \frac{mv_z}{|q|B} = \frac{1.67\times10^{-27}\times2.00\times10^{5}}{1.60\times10^{-19}\times0.500}$$

$$= 4.175\times10^{-3} m = 4.18 mm$$

Pitch of the helix is the distance travelled along x-axis in a time T, which is P = v_x T

But time,

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{|q|B} = \frac{2\times3.14\times1.67\times10^{-27}}{1.60\times10^{-19}\times0.500}$$

$$= 13.1\times10^{-8} s$$

Hence, pitch of the helix is

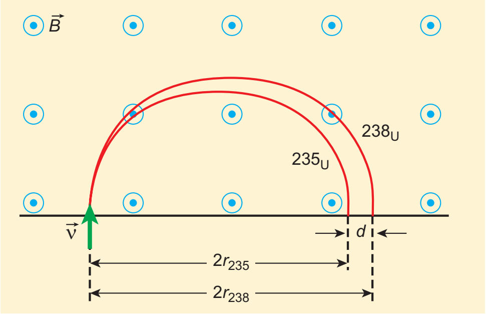
$$P = v_x T = \left(1.95\times10^{5}\right)\left(13.1\times10^{-8}\right)$$

$$= 25.5\times10^{-3} m = 25.5\ mm$$

The proton experiences appreciable acceleration in the magnetic field, hence the pitch of the helix is almost six times greater than the radius of the helix.

EXAMPLE 3.24

Two singly ionized isotopes of uranium ${}^{235}_{92}U$ and ${}^{238}_{92}U$ (isotopes have same atomic number but different mass number) are sent with velocity 1.00 × 10^{5} m s^{-1} into a magnetic field of strength 0.500 T normally. Compute the distance between the two isotopes after they complete a semi-circle. Also compute the time taken by each isotope to complete one semi-circular path. (Given: masses of the isotopes: m_{235} = 3.90 x 10^{-25} kg and m_{238} = 3.95 x 10^{-25} kg)



Solution

Since isotopes are singly ionized, they have equal charge which is equal to the charge of an electron, q = - 1.6 × 10^{-19} C. Mass of uranium ${}^{235}_{92}U$ and ${}^{238}_{92}U$ are 3.90 × 10^{-25} kg and 3.95 × 10^{-25} kg respectively. Magnetic field applied, B = 0.500 T. Velocity of the electron is 1.00 × 10^{5} m s^{-1}, then

(a) the radius of the path of ${}^{235}_{92}U$ is r_{235}

$$r_{235} = \frac{m_{235}v}{|q|B} = \frac{3.90\times10^{-25}\times1.00\times10^{5}}{1.6\times10^{-19}\times0.500}$$

$$= 48.8\times10^{-2} m$$

r_{235} = 48.8 cm

The diameter of the semi-circle due to ${}^{235}_{92}U$ is $d_{235} = 2r_{235}$ = 97.6 cm

The radius of the path of ${}^{238}_{92}U$ is r_{238} then

$$r_{238} = \frac{m_{238}v}{|q|B} = \frac{3.90\times10^{-25}\times1.00\times10^{5}}{1.6\times10^{-19}\times0.500}$$

$$= 49.4\times10^{-2} m$$

r_{238} = 49.4 cm

The diameter of the semi-circle due to ${}^{238}_{92}U$ is $d_{238} = 2r_{238}$ = 98.8 cm

Therefore the separation distance between the isotopes is $\Delta d = d_{238} - d_{235} = 1.2\,\text{cm}$

(b) The time taken by each isotope to complete one semi-circular path are

$$t_{235} = \frac{\text{magnitude of the displacement}}{\text{velocity}}$$

$$= \frac{97.6 \times 10^{-2}}{1.00 \times 10^{5}} = 9.76 \times 10^{-6}\,s = 9.76\ \mu s$$

$$t_{238} = \frac{\text{magnitude of the displacement}}{\text{velocity}}$$

$$= \frac{98.8 \times 10^{-2}}{1.00 \times 10^{5}} = 9.88 \times 10^{-6}\,s = 9.88\ \mu s$$

Note that even though the difference between mass of two isotopes are very small, this arrangement helps us to convert this small difference into an easily measurable distance of separation. This arrangement is known as mass spectrometer. A mass spectrometer is used in many areas in sciences, especially in medicine, in space science, in geology etc. For example, in medicine, anaesthesiologists use it to measure the respiratory gases and biologist use it to determine the reaction mechanisms in photosynthesis.

3.10.3 Motion of a charged particle under crossed electric and magnetic field (velocity selector)

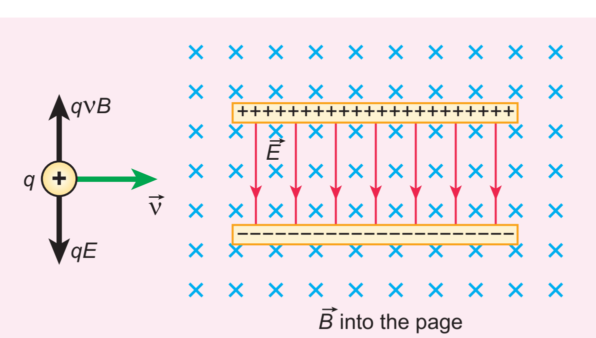


Figure 3.53 Velocity selector

Consider an electric charge q of mass m which enters into a region of uniform magnetic field $\vec{B}$ with velocity $\vec{v}$ such that velocity is not perpendicular to the magnetic field. Then the path of the particle is a helix. The Lorentz force on the charged particle moving in a uniform magnetic field can be balanced by Coulomb force by proper arrangement of electric and magnetic fields.

The Coulomb force acts along the direction of electric field (for a positive charge q) whereas the Lorentz force is perpendicular to the direction of magnetic field. Therefore in order to balance these forces, both electric and magnetic fields must be perpendicular to each other. Such an arrangement of perpendicular electric and magnetic fields are known as cross fields.

For illustration, consider an experimental arrangement as shown in Figure 3.53. In the region of space between parallel plates of a capacitor (which produces uniform electric field), uniform magnetic field is maintained perpendicular to the direction of electric field. Suppose a charged particle enters this space from the left side as shown, the net force on the particle is

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

For a positive charge, the electric force on the charge acts in downward direction whereas the Lorentz force acts upwards. When these two forces balance one another, then

SC53HJ

$$qE = qv_{\circ}B$$

$$\Rightarrow v_{\circ} = \frac{E}{B} \qquad (3.61)$$

This means, for a given magnitude of $\vec{E}$- field and $\vec{B}$- field, the forces act only for the particle moving with particular speed $v_o = \frac{E}{B}$. This speed is independent of mass and charge.

If the charge enters into the crossed fields with velocity v, other than v_o, it results in any of the following possibilities (Table 3.4).

Table 3.4 Deflection based on the velocity – velocity selector

S.No.	Velocity	Deflection
1	$v > v_o$	Charged particle deflects in the direction of Lorentz force
2	$v < v_o$	Charged particle deflects in the direction of Coulomb force
3	$v = v_o$	No deflection and particle moves straight

So by proper choice of electric and magnetic fields, the particle with particular speed can be selected. Such an arrangement of fields is called a **velocity selector**.

Note This principle is used in Bainbridge mass spectrograph to separate the isotopes.

EXAMPLE 3.25

Let E be the electric field of magnitude 6.0×10^6 N C^{-1} and B be the magnetic field magnitude 0.83 T. Suppose an electron is accelerated with a potential of 200 V, will it show zero deflection?. If not, at what potential will it show zero deflection.

***Solution*:**

Electric field, E = 6.0×10^6 N C^{-1} and magnetic field, B = 0.83 T.

Then

$$v = \frac{E}{B} = \frac{6.0 \times 10^6}{0.83} = 7.23 \times 10^6 \, m s^{-1}$$

When an electron goes with this velocity, it shows null deflection. Since the accelerating potential is 200 V, the electron acquires kinetic energy because of this accelerating potential. Hence,

$$\frac{1}{2} mv^2 = eV \Rightarrow v = \sqrt{\frac{eV}{2m}}$$

Since the mass of the electron, $m = 9.1 \times 10^{-31} \, kg$ and charge of an electron, $|q| = e = 1.6 \times 10^{-19} C$. The velocity due to accelerating potential 200 V

$$v_{200} = \sqrt{\frac{2(1.6 \times 10^{-19})(200)}{(9.1 \times 10^{-31})}} = 8.39 \times 10^6 \, m \, s^{-1}$$

Since the speed $v_{200} > v$, the electron is deflected towards direction of Lorentz force. So, in order to have null deflection, the potential, we have to supply is

$$V = \frac{1}{2} \frac{mv^2}{e} = \frac{(9.1 \times 10^{-31}) \times (7.23 \times 10^6)^2}{2 \times (1.6 \times 10^{-19})}$$

$$V = 148.65 \, V$$

3.10.4 Cyclotron

Cyclotron (Figure 3.54) is a device used to accelerate the charged particles to gain large kinetic energy. It is also called as high energy accelerator. It was invented by Lawrence and Livingston in 1934.

Principle

When a charged particle moves normal to the magnetic field, it experiences magnetic Lorentz force.

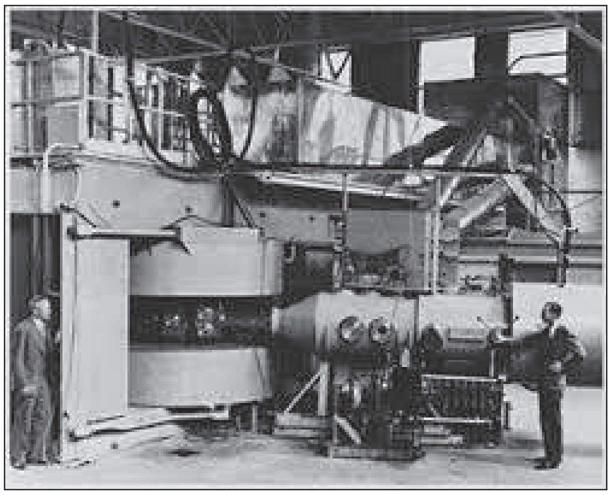


Figure 3.54 Cyclotron invented by Lawrence and Livingston

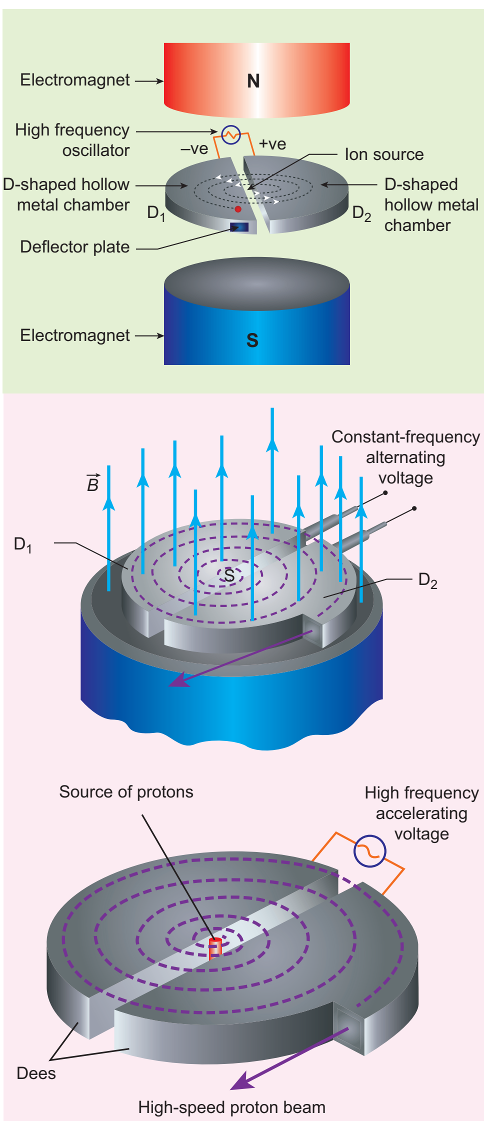


Figure 3.55 construction and working of cyclotron

Construction

The schematic diagram of a cyclotron is shown in Figure 3.55. The particles are allowed to move in between two semi-circular metal containers called Dees (hollow D - shaped objects). Dees are enclosed in an evacuated chamber and it is kept in a region with uniform magnetic field controlled by an electromagnet. The direction of magnetic field is normal to the plane of the Dees. The two Dees are kept separated with a gap and the source S (which ejects the particle to be accelerated) is placed at the center in the gap between the Dees. Dees are connected to high frequency alternating potential difference.

Working

Let us assume that the ion ejected from source S is positively charged. As soon as ion is ejected, it is accelerated towards a Dee (say, Dee – 1) which has negative potential at that time. Since the magnetic field is normal to the plane of the Dees, the ion undergoes circular path. After one semi-circular path in Dee-1, the ion reaches the gap between Dees. At this time, the polarities of the Dees are reversed so that the ion is now accelerated towards Dee-2 with a greater velocity. For this circular motion, the centripetal force of the charged particle q is provided by Lorentz force.

$$\frac{mv^2}{r} = qvB$$

$$\Rightarrow r = \frac{m}{qB}v$$

$$\Rightarrow r \propto v \qquad (3.62)$$

From the equation (3.62), the increase in velocity increases the radius of circular path. This process continues and hence the particle undergoes spiral path of increasing radius. Once it reaches near the edge, it is taken out with the help of deflector plate and allowed to hit the target T.

Very important condition in cyclotron operation is the resonance condition. It happens when the frequency f at which the positive ion circulates in the magnetic field must be equal to the constant frequency of the electrical oscillator f_{osc}

From equation (3.59), we have

$$f_{OSC} = \frac{qB}{2\pi m}$$

The time period of oscillation is

$$T = \frac{2\pi m}{qB}$$

The kinetic energy of the charged particle is

$$KE = \frac{1}{2}mv^2 = \frac{q^2B^2r^2}{2m} \quad (3.63)$$

Limitations of cyclotron

(a) the speed of the ion is limited

(b) electron cannot be accelerated

(c) uncharged particles cannot be accelerated

Note Deutrons (bundles of one proton and one neutron) can be accelerated because it has same charge as that of proton. But neutron (electrically neutral particle) cannot be accelerated by the cyclotron. When a deutron is bombarded with a beryllium target, a beam of high energy neutrons are produced. These high-energy neutrons are sent into the patient's cancerous region to break the bonds in the DNA of the cancer cells (killing the cells). This is used in treatment of fast-neutron cancer therapy.

EXAMPLE 3.26

Suppose a cyclotron is operated to accelerate protons with a magnetic field of strength 1 T. Calculate the frequency in which the electric field between two Dees could be reversed.

Solution

Magnetic field B = 1 T

Mass of the proton, $m_p = 1.67 \times 10^{-27} kg$

Charge of the proton, $q = 1.60 \times 10^{-19} C$

$$f = \frac{qB}{2\pi m_p} = \frac{(1.60 \times 10^{-19})(1)}{2(3.14)(1.67 \times 10^{-27})}$$

$$= 15.3 \times 10^6 Hz = 15.3 MHz$$

3.10.5 Force on a current carrying conductor placed in a magnetic field

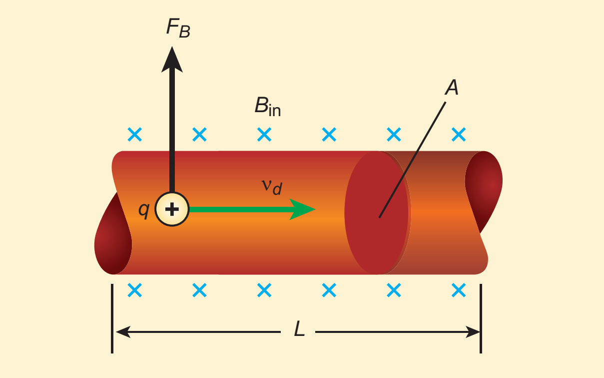


Figure 3.56 Current carrying conductor in a magnetic field

When a current carrying conductor is placed in a magnetic field, the force experienced by the wire is equal to the sum of Lorentz forces on the individual charge carriers in the wire. Consider a small segment of wire of length dl, with cross-sectional area A and current I as shown in Figure 3.56. The free electrons drift opposite to the direction of current. So the relation between current I and magnitude of drift velocity v_d (Refer Unit 2) is

$$I = neAvd \quad (3.64)$$

If the wire is kept in a magnetic field $\vec{B}$, then average force experienced by the charge (here, electron) in the wire is

$$\vec{F} = -e\left(\vec{v}_d \times \vec{B}\right)$$

Let n be the number of free electrons per unit volume, therefore

$$n = \frac{N}{V}$$

where N is the number of free electrons in the small element of volume V = A *dl*.

Hence Lorentz force on the wire of length d*l* is the product of the number of the electrons

(N = nA d*l*) and the force acting on an electron.

$$d\vec{F} = -enAdl\left(\vec{v}_d \times \vec{B}\right)$$

The length d*l* is along the length of the wire and hence the current element in the wire is $I\,d\vec{l} = -enA\vec{v}_d dl$ Therefore the force on the wire is

$$d\vec{F} = \left(I\,d\vec{l} \times \vec{B}\right) \quad (3.65)$$

The force in a straight current carrying conducting wire of length *l* placed in a uniform magnetic field is

$$\vec{F} = \left(I\vec{l} \times \vec{B}\right) \quad (3.66)$$

In magnitude,

$$F = BIl\sin\theta$$

(a) If the conductor is placed along the direction of the magnetic field, the angle between them is θ = 0°. Hence, the force experienced by the conductor is zero.

(b) If the conductor is placed perpendicular to the magnetic field, the angle between them is θ =90° Hence, the force experienced by the conductor is maximum, which is $F = BIl$.

Fleming's left hand rule (mnemonic)

When a current carrying conductor is placed in a magnetic field, the direction of the force experienced by it is given by Fleming's Left Hand Rule (FLHR) as shown in Figure 3.57.

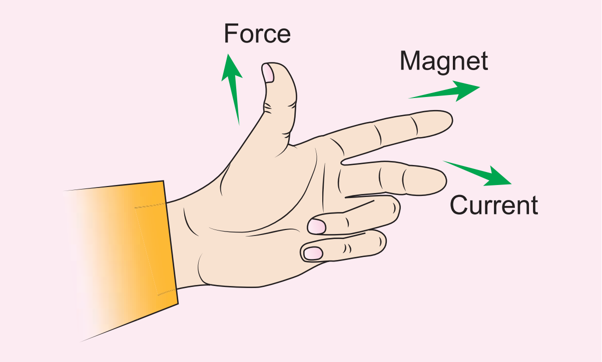
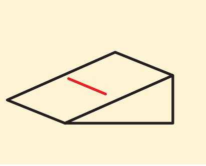


Figure 3.57 Fleming's Left Hand Rule (FLHR)

Stretch forefinger, the middle finger and the thumb of the left hand such that they are in mutually perpendicular directions. If forefinger points the direction of magnetic field, the middle finger points the direction of the electric current, then thumb will point the direction of the force experienced by the conductor.

EXAMPLE 3.27

A metallic rod of linear density is 0.25 kg m^{-1} is lying horizontally on a smooth inclined plane which makes an angle of 45° with the horizontal. The rod is not allowed to slide down by flowing a current through it when a magnetic field of strength 0.25 T is acting on it in the vertical direction. Calculate the electric current flowing in the rod to keep it stationary.



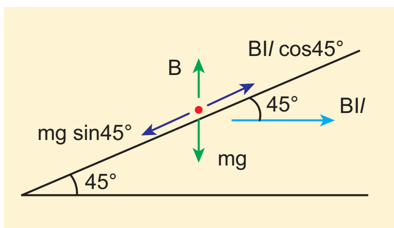
UNIT-3(XII-Physics_Vol-1).indd 184 04-03-2019 10:35:25

Solution

The linear density of the rod i.e., mass per unit length of the rod is 0.25 kg m^{-1}

$$\Rightarrow \frac{m}{l} = 0.25 \text{ kg m}^{-1}$$

Let I be the current flowing in the metallic rod. The direction of electric current is into the paper. The direction of magnetic force IBl is given by Fleming's left hand rule.



For equilibrium,

$$mg\sin 45^\circ = IBl\cos 45^\circ$$

$$\Rightarrow I = \frac{1}{B}\frac{m}{l}g\tan 45^\circ$$

$$= \frac{0.25\,\text{kg m}^{-1}}{0.25\,T} \times 1 \times 9.8\ m\,s^{-2}$$

$$\Rightarrow I = 9.8\,A$$

So, we need to supply current of 9.8 A to keep the metallic rod stationary.

3.10.6 Force between two long parallel current carrying conductors

Two long straight parallel current carrying conductors separated by a distance r are kept in air as shown in Figure 3.58. Let I_1 and I_2 be the electric currents passing through the conductors A and B in same direction (i.e. along z - direction) respectively. The net magnetic field at a distance r due to current I_1 in conductor A is

$$\vec{B}_1 = \frac{\mu_\circ I_1}{2\pi r}(-\hat{i}) = -\frac{\mu_\circ I_1}{2\pi r}\hat{i}$$

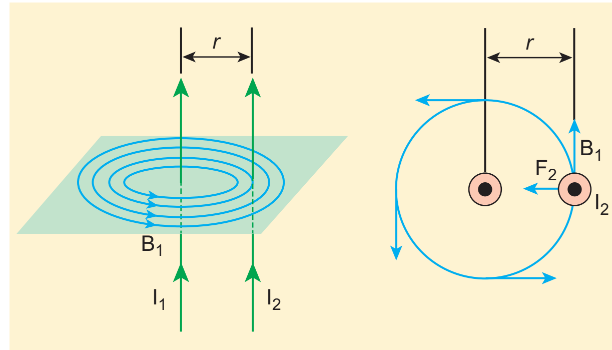


Figure 3.58 Two long straight parallel wires

From thumb rule, the direction of magnetic field is perpendicular to the plane of the paper and inwards (arrow into the page ⊗) i.e. along negative $\hat{i}$ direction.

Let us consider a small elemental length d*l* in conductor B at which the magnetic field $\vec{B}_1$ is present. From equation 3.65, Lorentz force on the element d*l* of conductor B is

$$d\vec{F} = \left(I_2 d\vec{l} \times \vec{B}_1\right) = -I_2 dl \frac{\mu_\circ I_1}{2\pi r}(\hat{k} \times \hat{i})$$

$$= -\frac{\mu_\circ I_1 I_2 dl}{2\pi r}\hat{j}$$

Therefore the force on d*l* of the wire B is directed towards the wire W_1. So the length d*l* is attracted towards the conductor A. The force per unit length of the conductor B due to the wire conductor A is

$$\frac{\vec{F}}{l} = -\frac{\mu_\circ I_1 I_2}{2\pi r}\hat{j}$$

In the same manner, we compute the magnitude of net magnetic induction due to current I_2 (in conductor A) at a distance r in the elemental length *dl* of conductor A is

$$\vec{B}_2 = \frac{\mu_\circ I_2}{2\pi r}\hat{i}$$

From the thumb rule, direction of magnetic field is perpendicular to the plane of the paper and outwards (arrow out of the page ⊙) i.e., along positive $\hat{i}$ direction.

Hence, the magnetic force at element dl of the wire is W_1 is

$$\vec{F} = \left(I_1 \vec{dl} \times \vec{B}_2\right) = I_1 dl \frac{\mu_\circ I_2}{2\pi r}(\hat{k} \times \hat{i})$$

$$= \frac{\mu_\circ I_1 I_2 dl}{2\pi r}\hat{j} \quad (3.67)$$

Therefore the force on dl of conductor A is directed towards the conductor B. So the length dl is attracted towards the conductor B as shown in Figure (3.59).

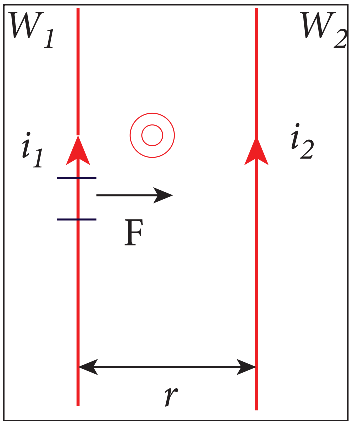


Figure 3.59 Current in both the wire are in the same direction - attracts each other

The force per unit length of the conductor A due to the conductor B is

$$\frac{\vec{F}}{l} = -\frac{\mu_\circ I_1 I_2}{2\pi r}\hat{j}$$

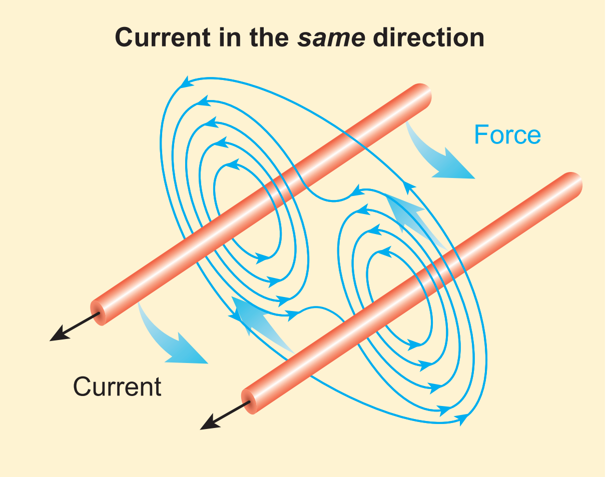


Figure 3.60 Two parallel conductors carrying current in same direction experience an attractive force

Thus the force experienced by two parallel current carrying conductors is attractive if the direction of electric current passing through them is same as shown in Figure 3.60.

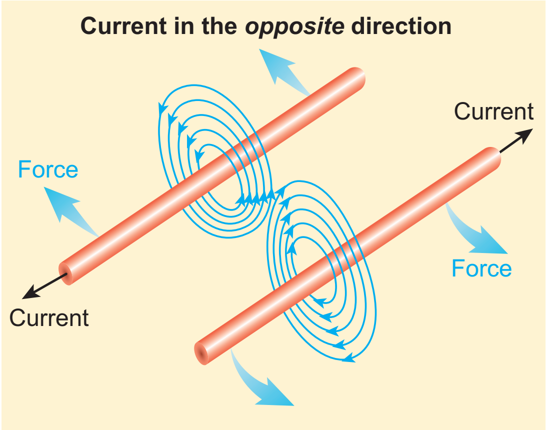


Figure 3.61 Two parallel conductors carrying current in opposite direction experience a repulsive force

Thus the force experienced by two parallel current carrying conductors is repulsive if they carry current in the opposite directions as shown in Figure 3.61.

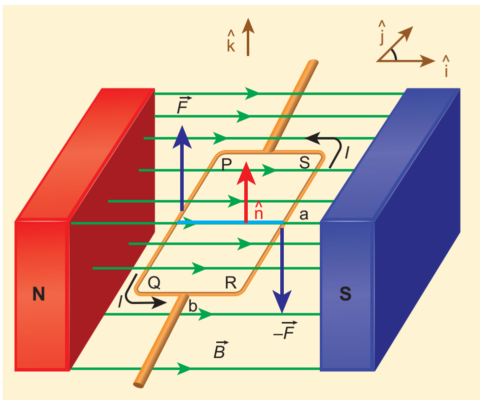
Definition of ampère

One ampère is defined as that current when it is passed through each of the two infinitely long parallel straight conductors kept at a distance of one meter apart in vacuum causes each conductor to experience a force of 2×10^{-7} newton per meter length of conductor.

3.11 TORQUE ON A CURRENT LOOP

The force on a current carrying wire in a magnetic field is responsible for the motor operation.

3.11.1 Expression for torque on a current loop placed in a magnetic field



58QX7F

Figure 3.62 Rectangular coil placed in a magnetic field

Consider a single rectangular loop PQRS kept in a uniform magnetic field $\vec{B}$. Let a and b be the length and breadth of the rectangular loop respectively. Let $\hat{n}$ be the unit vector normal to the plane of the current loop. This unit vector $\hat{n}$ completely describes the orientation of the loop. Let $\vec{B}$ be directed from north pole to south pole of the magnet as shown in Figure 3.62.

When an electric current is sent through the loop, the net force acting is zero but there will be net torque acting on it. For the sake of understanding, we shall consider two configurations of the loop; (i) unit vector $\hat{n}$ pointing perpendicular to the field (ii) unit vector pointing at an angle θ with the field.

(i) when unit vector $\hat{n}$ is perpendicular to the field

In the simple configuration, the unit vector $\hat{n}$ is perpendicular to the field and plane of the loop is lying on xy plane as shown in Figure 3.62. Let the loop be divided into four sections PQ, QR, RS and SP. The Lorentz force on each loop can be calculated as follows:

(a) Force on section PQ

For section PQ, $\vec{l} = -a\,\hat{j}$ and $\vec{B} = B\,\hat{i}$

$$\vec{F}_{PQ} = I\vec{l} \times \vec{B}$$
$$= -IaB(\hat{j} \times \hat{i}) = IaB\,\hat{k}$$

Since the unit vector normal to the plane $\hat{n}$ is along the direction of $\hat{k}$.

(b) The force on section QR

$$\vec{l} = b\vec{i} \text{ and } \vec{B} = B\,\hat{i}$$

$$\vec{F}_{QR} = I\vec{l} \times \vec{B} = IbB(\hat{i} \times \hat{i}) = \vec{0}$$

(c) The force on section RS

$$\vec{l} = a\,\hat{j} \text{ and } \vec{B} = B\,\hat{i}$$

$$\vec{F}_{RS} = I\vec{l} \times \vec{B} = IaB(\hat{j} \times \hat{i}) = -IaB\,\hat{k}$$

Since, the unit vector normal to the plane is along the direction of $-\hat{k}$.

(d) The force on section SP

$$\vec{l} = -b\,\hat{i} \text{ and } \vec{B} = B\,\hat{i}$$

$$\vec{F}_{SP} = I\vec{l} \times \vec{B} = -IbB(\hat{i} \times \hat{i}) = \vec{0}$$

The net force on the rectangular loop is

$$\vec{F}_{net} = \vec{F}_{PQ} + \vec{F}_{QR} + \vec{F}_{RS} + \vec{F}_{SP}$$
$$\vec{F}_{net} = IaB\,\hat{k} + \vec{0} - IaB\,\hat{k} + \vec{0} \Rightarrow \vec{F}_{net} = \vec{0}$$

Hence, the net force on the rectangular loop in this configuration is zero. Now let us calculate the net torque due to these forces about an axis passing through the center

$$\vec{\tau}_{net} = \sum_{i=1}^{4} \vec{\tau}_i = \sum_{i=1}^{4} \vec{r}_i \times \vec{F}_i$$
$$= \left(\frac{b}{2} IaB + 0 + \frac{b}{2} IaB + 0\right)\hat{j}$$
$$\vec{\tau}_{net} = abIB\,\hat{j}$$

Since, A = ab is the area of the rectangular loop PQRS, the net torque for this configuration is

$$\vec{\tau}_{net} = ABI\,\hat{j}$$

When the loop starts rotating due to this torque, the magnetic field $\vec{B}$ is no longer in the plane of the loop. So the above equation is the special case.

When the loop starts rotating about z axis due to this torque, the magnetic field $\vec{B}$ is no longer in the plane of the loop. So the above equation is the special case.

(ii) when unit vector $\hat{n}$ is at an angle θ with the field

In the general case, the unit normal vector $\hat{n}$ and magnetic field $\vec{B}$ is with an angle θ as shown in Figure 3.63.

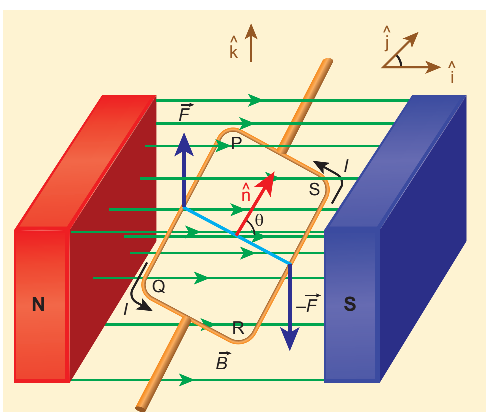


Figure 3.63 Unit vector makes an angle **θ** with the field

(a) The force on section PQ

$$\vec{l} = -a\,\hat{j} \;\; and\, \vec{B} = B\,\hat{i}$$

$$\vec{F}_{PQ} = I\vec{l}\times\vec{B} = -IaB\left(\hat{j}\times\hat{i}\right) = IaB\,\hat{k}$$

Since, the unit vector normal to the plane $\hat{n}$ is along the direction of $\hat{k}$.

(b) The force on section QR

$$\vec{l} = b\cos\left(\frac{\pi}{2}-\theta\right)\check{i} - \sin\left(\frac{\pi}{2}-\theta\right)\check{k}\, and\, \vec{B} = B\,\check{i}$$

$$\vec{F}_{QR} = I\vec{l}\times\vec{B} = -IbB\sin\left(\frac{\pi}{2}-\theta\right)\hat{j}$$

$$\vec{F}_{QR} = -IbB\cos\theta\,\hat{j}$$

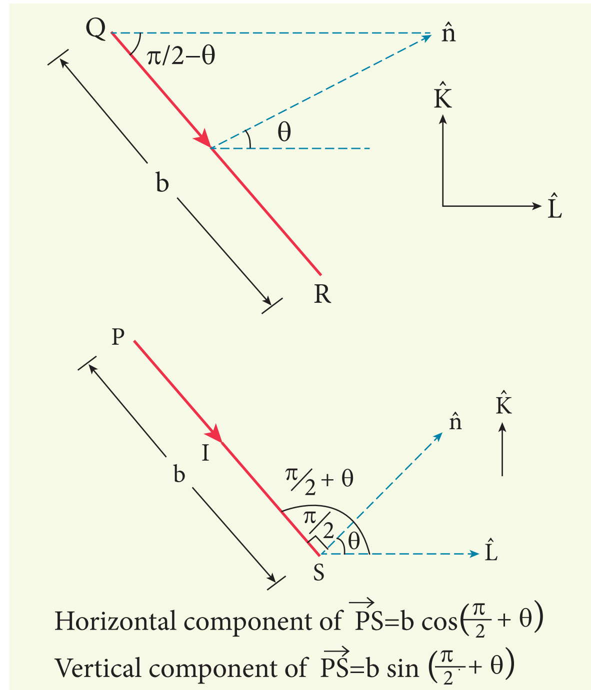


Figure 3.64 Horizontal and vertical component of the sections - (a) QR (b) SP

(c) The force on section RS

$$\vec{l} = a\,\check{j}\, and\, \vec{B} = B\hat{i}$$

$$\vec{F}_{RS} = I\vec{l}\times\vec{B} = IaB\left(\hat{j}\times\hat{i}\right) = -\mathrm{IaB}\,\hat{\mathrm{k}}$$

Since, the unit vector normal to the plane is along the direction of $-\hat{\mathrm{k}}$.

(d) The force on section SP

$$\vec{l} = b\cos\left(\frac{\pi}{2}+\theta\right)\hat{i} + \sin\left(\frac{\pi}{2}+\theta\right)\hat{k}\;\; and\;\; \vec{B} = B\,\hat{i}$$

$$\vec{F}_{SP} = I\vec{l}\times\vec{B} = IbB\sin\left(\frac{\pi}{2}+\theta\right)\hat{j}$$

$$\vec{F}_{SP} = IbB\cos\theta\,\hat{j}$$

The net force on the rectangular loop is

$$\vec{F}_{net} = \vec{F}_{PQ} + \vec{F}_{QR} + \vec{F}_{RS} + \vec{F}_{SP}$$

$$\vec{F}_{net} = IaB\hat{k} - IbB\cos\theta\,\hat{j} - IaB\hat{k} + IbB\cos\theta\,\hat{j}$$

$$\Rightarrow \vec{F}_{net} = \vec{0}$$

Hence, the net force on the rectangular loop in this configuration is also zero. Notice that the force on section QR and SP are not zero here. But, they have equal and opposite effects, but we assume the loop to be rigid, so no deformation. Hence, no torque produced by these two sections.

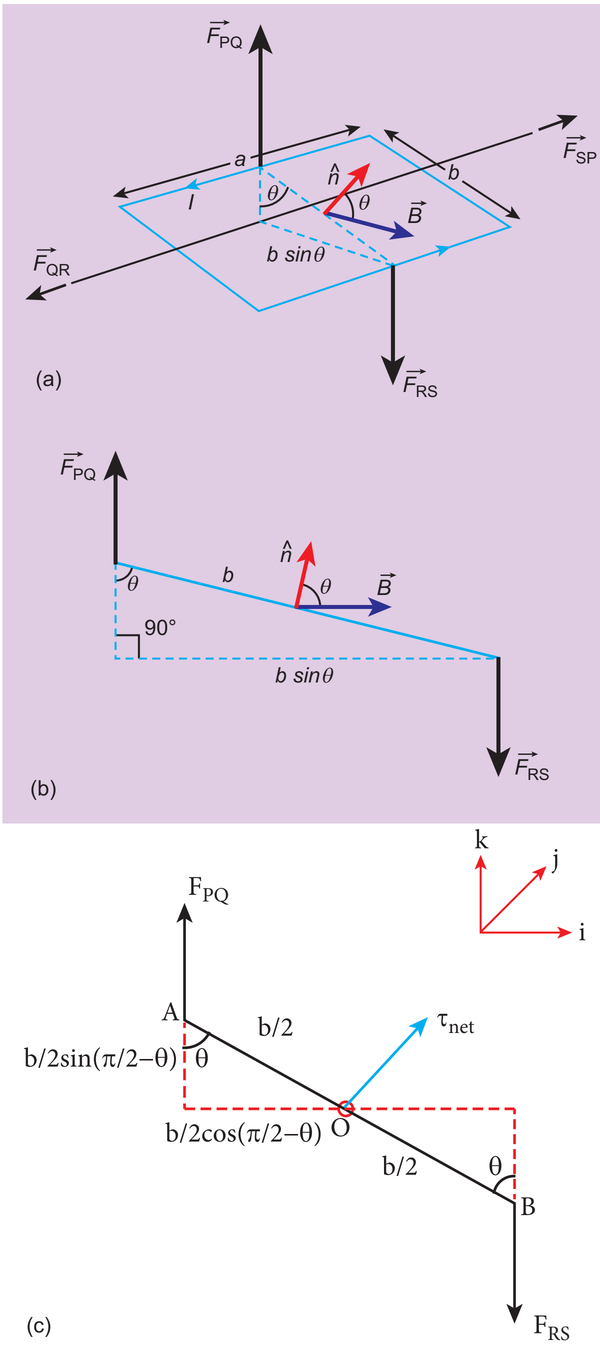


Figure 3.65 Force on the rectangular loop – (a) top view and (b) side view (c) net torque on the loop

Even though the forces PQ and RS also are also equal and opposite, they are not collinear. So these two forces constitute a couple as shown in Figure 3.65 (a). Hence the net torque produced by these two forces about the axis of the rectangular loop is given by

$$\vec{\tau}_{net} = baBI\sin\theta\,\hat{k} = ABI\sin\theta\,\hat{k}$$

From the Figure 3.65 (c),

$$\overline{OA} = \frac{b}{2}\cos\left(\frac{\pi}{2}-\theta\right)\left(-\hat{i}\,\right)+\frac{b}{2}\sin\left(\frac{\pi}{2}-\theta\right)\left(-\hat{k}\,\right)$$

$$= \frac{b}{2}\left(-\sin\theta\,\hat{i} + \cos\theta\,\hat{k}\,\right)$$

$$\overline{OB} = \frac{b}{2}\cos\left(\frac{\pi}{2}-\theta\right)\left(\,\hat{i}\,\right)+\frac{b}{2}\sin\left(\frac{\pi}{2}-\theta\right)\left(-\hat{k}\,\right)$$

$$= \frac{b}{2}\left(-\sin\theta\,\hat{i} + \cos\theta\,\hat{k}\,\right)$$

$$\overline{OA}\times\vec{F}_{PQ} = \left\{\frac{b}{2}\left(-\sin\theta\,\hat{i} + \cos\theta\,\hat{k}\,\right)\right\}\times\left\{IaB\,\hat{k}\,\right\}$$

$$= \frac{1}{2}IabB\sin\theta\,\hat{j}$$

$$\overline{OB}\times\vec{F}_{RS} = \left\{\frac{b}{2}\left(\sin\theta\,\hat{i} + \cos\theta\,\hat{k}\,\right)\right\}\times\left\{-IaB\,\hat{k}\,\right\}$$

$$= \frac{1}{2}IabB\sin\theta\,\hat{j}$$

The net torque $\vec{\tau}_{net} = IabB\sin\theta\,\hat{j}$ (3.68)

Note that the net torque is in the positive y direction which tends to rotate the loop in clockwise direction about the y axis. If the current is passed in the other way (P→S→R→Q→P), then total torque will point in the negative y direction which tends to rotate the loop in anticlockwise direction about y axis.

Another important point is to note that the torque is less in this case compared to earlier case (where the $\hat{n}$ is perpendicular to the magnetic field $\vec{B}$). It is because the perpendicular distance is reduced between the forces $\vec{F}_{PQ}$ and $\vec{F}_{RS}$ in this case.

The equation (3.68) can also be rewritten in terms of magnetic dipole moment $\vec{p}_m = I\vec{A} = I\,ab\,\hat{n}$

$$\vec{\tau}_{net} = \vec{p}\times\vec{B}$$

This is analogous expression for torque experienced by electric dipole in the uniform electric field

$\vec{\tau}_{net} = \vec{p} \times \vec{E}$ which is given in the Unit 1. (Section 1.4.3)

Cases:

(a) When θ = 90°, then the torque on the current loop is maximum which is

$$\vec{\tau}_{net} = abIB\,\hat{j}$$

Note here $\vec{p}_m$ points perpendicular to the magnetic field $\vec{B}$. The torque is maximum in this orientation.

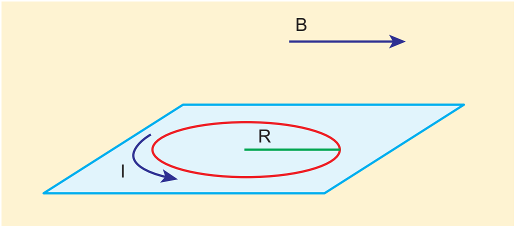
(b) When θ = 0° or 180° then the torque on the current loop is

$$\vec{\tau}_{net} = 0$$

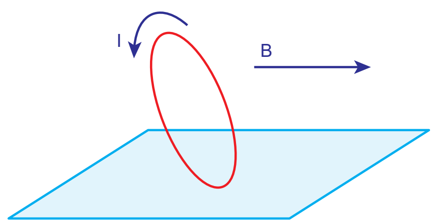
when θ = 0°, $\vec{p}_m$ is parallel to $\vec{B}$ and for θ = 180°, $\vec{p}_m$, is anti - parallel to $\vec{B}$. The torque is zero in these orientations.

EXAMPLE 3.28

Consider a circular wire loop of radius R, mass m kept at rest on a rough surface. Let I be the current flowing through the loop and $\vec{B}$ be the magnetic field acting along horizontal as shown in Figure. Estimate the current I that should be applied so that one edge of the loop is lifted off the surface?



Solution



When the current is passed through the loop, the torque is produced. If the torque acting on the loop is increased then the loop will start to rotate. The loop will start to lift if and only if the magnitude of magnetic torque due to current applied equals to the gravitational torque as shown in Figure

$$\tau_{magnetic} = \tau_{gravitational}$$

$$IAB = mgR$$

But $p_m = IA = I\left(\pi R^2\right)$

$$\pi IR^2 B = mgR$$

$$\Rightarrow I = \frac{mg}{\pi RB}$$

The current estimated using this equation should be applied so that one edge of loop is lifted of the surface.

3.11.2 Moving coil galvanometer

Moving coil galvanometer is a device which is used to indicate the flow of current in an electrical circuit.

Principle When a current carrying loop is placed in a uniform magnetic field it experiences a torque.

Construction

A moving coil galvanometer consists of a rectangular coil PQRS of insulated thin copper wire. The coil contains a large number of turns wound over a light metallic frame. A cylindrical soft-iron core is placed symmetrically inside the coil as shown in Figure 3.66. The rectangular coil is suspended freely between two pole pieces of a horse-shoe magnet.

The upper end of the rectangular coil is attached to one end of fine strip of

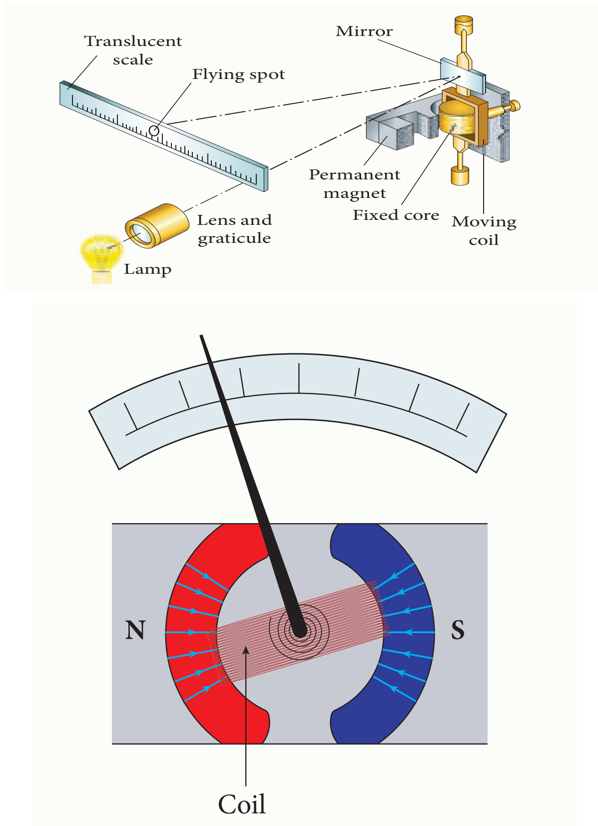


Figure 3.66 Moving coil galvanometer – its parts

phosphor bronze and the lower end of the coil is connected to a hair spring which is also made up of phosphor bronze. In a fine suspension strip W, a small plane mirror is attached in order to measure the deflection of the coil with the help of lamp and scale arrangement. The other end of the mirror is connected to a torsion head T. In order to pass electric current through the galvanometer, the suspension strip W and the spring S are connected to terminals.

Working

Consider a single turn of the rectangular coil PQRS whose length be l and breadth b. PQ = RS = l and QR = SP = b. Let I be the electric current flowing through the rectangular coil PQRS as shown in Figure 3.67. The horse-shoe magnet has hemi - spherical magnetic poles which produces a radial magnetic field. Due to this radial field, the sides QR and SP are always parallel to to the B-field (magnetic field) and experience no force. The sides PQ and RS are always parallel to the B-field and experience force and due to this, torque is produced.

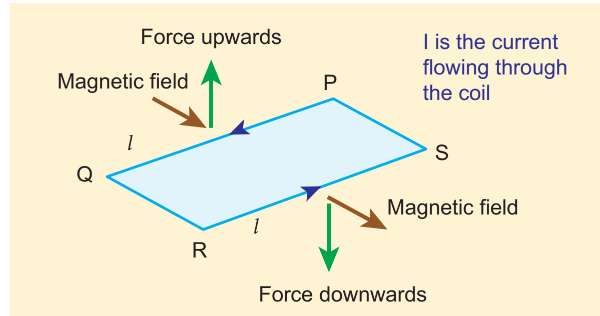


Figure 3.67 Force acting on current carrying coil

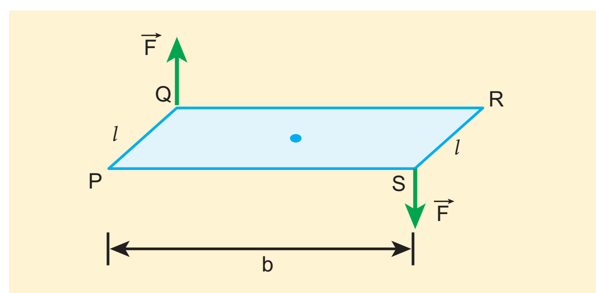


Figure 3.68 Deflection couple

For single turn, the deflection couple as shown in Figure 3.68 is

$$\tau = bF = bBIl = (lb)\,BI = ABI$$

since, area of the coil $A = lb$

For coil with N turns, we get

$$\tau = NABI \qquad (3.69)$$

Due to this deflecting torque, the coil gets twisted and restoring torque (also known as restoring couple) is developed. Hence the magnitude of restoring couple is proportional to the amount of twist θ (Refer Unit 10 of Std. XI Physics).Thus

$$\tau = K\theta \qquad (3.70)$$

where K is the restoring couple per unit twist or torsional constant of the spring.

At equilibrium, the deflection couple is equal to the restoring couple. Therefore by comparing equation (3.69) and (3.70), we get

$$NABI = K\,\theta$$

$$\Rightarrow I = \frac{K}{NAB}\theta \qquad (3.71)$$

(or) $I = G\,\theta$

where, $G = \frac{K}{NAB}$ is called galvanometer constant or current reduction factor of the galvanometer.

Since, suspended moving coil galvanometer is very sensitive, we have to handle with high care while doing experiments. Most of the galvanometer we use are pointer type moving coil galvanometer.

Figure of merit of a galvanometer

It is defined as the current which produces a deflection of one scale division in the galvanometer.

Sensitivity of a galvanometer

The galvanometer is said to be sensitive if it shows large scale deflection even though a small current is passed through it or a small voltage is applied across it.

Current sensitivity: ***It is defined as the deflection produced per unit current flowing through it.***

$$I_S = \frac{\theta}{I} = \frac{NAB}{K} \Rightarrow I_S = \frac{1}{G} \qquad (3.72)$$

The current sensitivity of a galvanometer can be increased

(a) by increasing

(1) the number of turns N

(2) the magnetic induction B

(3) the area of the coil A

(b) by decreasing

the couple per unit twist of the suspension wire k. Phosphor - bronze wire is used as the suspension wire because the couple per unit twist is very small.

Voltage sensitivity: ***It is defined as the deflection produced per unit voltage applied across it.***

$$V_S = \frac{\theta}{V}$$

$$V_S = \frac{\theta}{IR_g} = \frac{NAB}{K\,R_g} \Rightarrow V_S = \frac{1}{GR_g} = \frac{I_S}{R_g} \qquad (3.73)$$

where R_g is the resistance of galvanometer.

EXAMPLE 3.29

The coil of a moving coil galvanometer has 5 turns and each turn has an effective area of 2×10^{-2} m^2. It is suspended in a magnetic field whose strength is 4×10^{-2} Wb m^{-2}. If the torsional constant K of the suspension fibre is 4×10^{-9} N m deg^{-1}.

(a) Find its current sensitivity in degree per micro - ampere.

(b) Calculate the voltage sensitivity of the galvanometer for it to have full scale deflection of 50 divisions for 25 mV.

(c) Compute the resistance of the galvanometer.

Solution

The number of turns of the coil is 5 turns

The area of each coil is 2×10^{-2} m^2

Strength of the magnetic field is

4×10^{-2} Wb m^{-2}

Torsional constant is 4×10^{-9} N m deg^{-1}

(a) Current sensitivity

$$I_S = \frac{NAB}{K} = \frac{5 \times 2 \times 10^{-2} \times 4 \times 10^{-2}}{4 \times 10^{-9}}$$

$= 10^6$ divisions per ampere

$$1\mu A = 1 micro ampere = 10^{-6} ampere$$

Therefore,

$$I_S = 10^6 \frac{div}{A} = 1\frac{div}{10^{-6}A} = 1\frac{div}{\mu A}$$

$$I_S = 1\, div(\mu A)^{-1}$$

(b) Voltage sensitivity

$$V_S = \frac{\theta}{V} = \frac{50 div}{25 mV} = 2\times10^3 div\, V^{-1}$$

(c) The resistance of the galvanometer is

$$R_g = \frac{I_S}{V_S} = \frac{10^6 \frac{div}{A}}{2\times10^3 \frac{div}{V}} = 0.5\times10^3 \frac{V}{A} = 0.5 k\Omega$$

EXAMPLE 3.30

The resistance of a moving coil galvanometer is made twice its original value in order to increase current sensitivity by 50%. Will the voltage sensitivity change? If so, by how much?.

Solution

Yes, voltage sensitivity will change.

Voltage sensitivity is $V_S = \frac{I_S}{R}$

When the resistance is doubled, then new resistance is $R' = 2R$

Increase in current sensitivity is

$$I'_S = \left(1 + \frac{50}{100}\right) I_S = \frac{3}{2} I_S$$

The new voltage sensitivity is

$$V'_S = \frac{\frac{3}{2} I_S}{2R} = \frac{3}{4} V_S$$

Hence the voltage sensitivity decreases. The percentage decrease in voltage sensitivity is

$$\frac{V_S - V'_S}{V_S} \times 100\% = 25\%$$

Conversion of galvanometer into ammeter and voltmeter

A galvanometer is very sensitive instrument to detect the current. It can be easily converted into ammeter and voltmeter.

Galvanometer to an Ammeter

Ammeter is an instrument used to measure current flowing in the electrical circuit. The ammeter must offer low resistance such that it will not change the current passing through it. So ammeter is connected in series to measure the circuit current.

A galvanometer is converted into an ammeter by connecting a low resistance in parallel with the galvanometer. This low resistance is called shunt resistance S. The scale is now calibrated in ampere and the range of ammeter depends on the values of the shunt resistance.

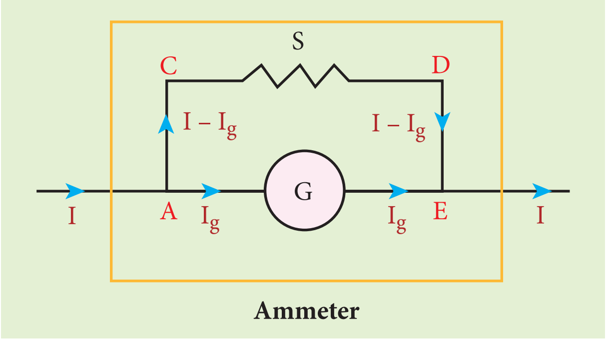


Figure 3.69 Shunt resistance connected in parallel

Let I be the current passing through the circuit as shown in Figure 3.69. When current I reaches the junction A, it divides into two components. Let I_g be the current passing through the galvanometer of resistance R_g through a path AGE and the remaining current $(I - I_g)$ passes along the path ACDE through shunt resistance S. The value of shunt resistance is so adjusted that current I_g produces full scale deflection in the galvanometer. The potential difference across galvanometer is same as the potential difference across shunt resistance.

$$V_{galvanometer} = V_{shunt}$$

$$\Rightarrow I_g R_g = (I - I_g) S$$

$$S = \frac{I_g}{(I - I_g)} R_g \text{ Or}$$

$$I_g = \frac{S}{S + R_g} I \Rightarrow I_g \propto I$$

Since, the deflection in the galvanometer is proportional to the current passing through it.

$$\theta = \frac{1}{G} I_g \Rightarrow \theta \propto I_g \Rightarrow \theta \propto I$$

So, the deflection in the galvanometer measures the current I passing through the circuit (ammeter).

Shunt resistance is connected in parallel to galvanometer. Therefore, resistance of ammeter can be determined by computing the effective resistance, which is

$$\frac{1}{R_{eff}} = \frac{1}{R_g} + \frac{1}{S} \Rightarrow R_{eff} = \frac{R_g S}{R_g + S} = R_a$$

Since, the shunt resistance is a very low resistance and the ratio $\frac{S}{R_g}$ is also small. This means, R_g is also small, i.e., the resistance offered by the ammeter is small. So, when we connect ammeter in series, the ammeter will not change the resistance appreciably and also the current in the circuit. For an ideal ammeter, the resistance must be equal to zero. Hence, the reading in ammeter is always lesser than the actual current in the circuit. Let I_{ideal} be current measured from ideal ammeter and I_{actual} be the actual current measured in the circuit by the ammeter. Then, the percentage error in measuring a current through an ammeter is

$$\frac{\Delta I}{I} \times 100\% = \frac{I_{ideal} - I_{actual}}{I_{actual}} \times 100\%$$

Key points

1. An ammeter is a low resistance instrument and it is always connected in series to the circuit
2. An ideal ammeter has zero resistance
3. In order to increase the range of an ammeter n times, the value of shunt resistance to be connected in parallel is $S = \frac{G}{n-1}$

Galvanometer to a voltmeter

A voltmeter is an instrument used to measure potential difference across any two points in the electrical circuits. It should not draw any current from the circuit otherwise the value of potential difference to be measured will change.

Voltmeter must have high resistance and when it is connected in parallel, it will not draw appreciable current so that it will indicate the true potential difference.

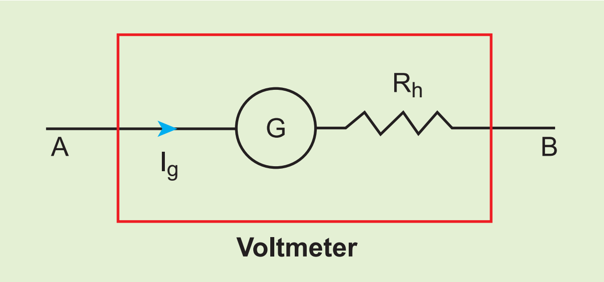


Figure 3.70 Shunt resistance connected in series

A galvanometer is converted into a voltmeter by connecting high resistance R_h in series with galvanometer as shown in Figure 3.74. The scale is now calibrated in volt and the range of voltmeter depends on the values of the resistance connected in series i.e. the value of resistance is so adjusted that only current I_g produces full scale deflection in the galvanometer.

Let R_g be the resistance of galvanometer and I_g be the current with which the galvanometer produces full scale deflection. Since the galvanometer is connected in series with high resistance, the current in the electrical circuit is same as the current passing through the galvanometer.

$$I = I_g$$

$$I = I_g \Rightarrow I_g = \frac{\text{potential difference}}{\text{total resistance}}$$

Since the galvanometer and high resistance are connected in series, the total resistance or effective resistance gives the resistance of voltmeter. The voltmeter resistance is

$$R_v = R_g + R_h$$

Therefore,

$$I_g = \frac{V}{R_g + R_h}$$

$$\Rightarrow R_h = \frac{V}{I_g} - R_g$$

Note that $I_g \propto V$

The deflection in the galvanometer is proportional to current I_g. But current I_g is proportional to the potential difference. Hence the deflection in the galvanometer is proportional to potential difference. Since the resistance of voltmeter is very large, a voltmeter connected in an electrical circuit will draw least current in the circuit. An ideal voltmeter is one which has infinite resistance.

Key points

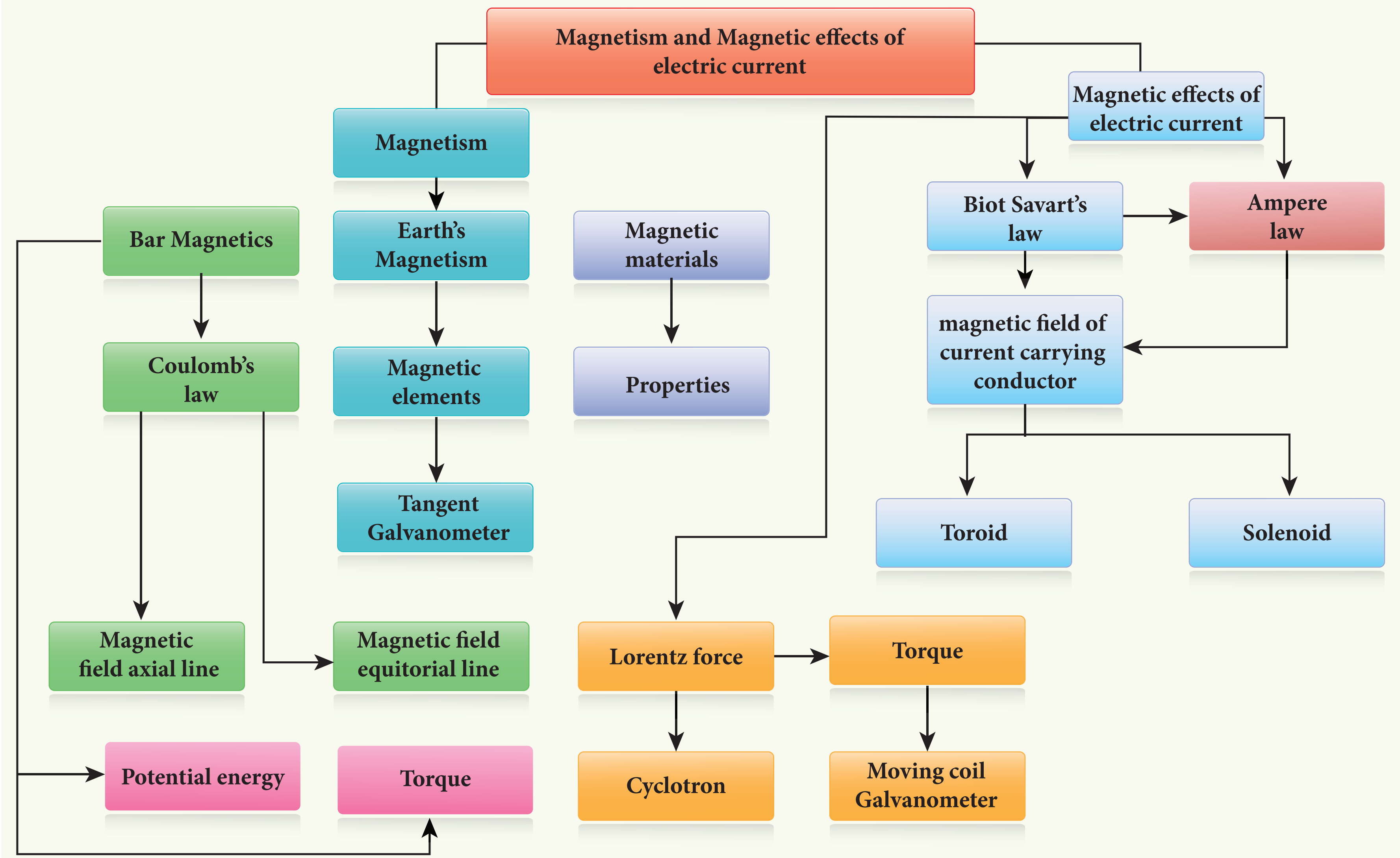
1. Voltmeter is a high resistance instrument and it is always connected in parallel with the circuit element across which the potential difference is to be calculated
2. An ideal voltmeter has infinite resistance
3. In order to increase the range of voltmeter n times the value of resistance to be connected in series with galvanometer is R = (n-1) G

SUMMARY:

- A vertical plane passing through geographic axis is called geographic meridian.
- A vertical plane passing through magnetic axis is called magnetic meridian.
- The angle between magnetic meridian at a point with the geographical meridian is called the declination or magnetic declination.
- The angle subtended by the Earth's total magnetic field $\vec{B}$ with the horizontal direction in the magnetic meridian is called dip or magnetic inclination at that point.
- The magnetic moment is defined as the product of its pole strength and magnetic length. It is a vector quantity, denoted by $\vec{p}_m$.
- The region surrounding magnet where magnetic pole of strength unity experiences a force is known as magnetic field. It is a vector quantity and denoted by $\vec{B}$ Its unit is $N\ A^{-1}\ m^{-1}$.
- The number of magnetic field lines crossing per unit area is called magnetic flux Φ_B. It is a scalar quantity. In SI unit, magnetic flux Φ_B is Weber, symbol Wb.
- Statement of Coulomb's law in magnetism "The force of attraction or repulsion between two magnetic poles is proportional to the product of their pole strengths and inversely proportional to the square of distance between them".
- Magnetic dipole kept in a uniform magnetic field experiences torque.
- Tangent galvanometer is a device used to measure very small currents. It is a moving magnet type galvanometer. Its working is based on tangent law.
- Tangent law is $B = B_H \tan\theta$.
- The magnetic field which is used to magnetize a sample or specimen is called the magnetising field. It is a vector quantity and denoted by $\vec{H}$ and its unit is $A\ m^{-1}$.
- The measure of ability of the material to allow the passage of magnetic lines of force through it is known as magnetic permeability.
- The net magnetic moment per unit volume of material is known as intensity of magnetisation or magnetisation vector or magnetisation.
- Magnetic susceptibility is defined as the ratio of the intensity of magnetisation $(\vec{I})$ induced in the material due to the magnetising field $(\vec{H})$.
- Magnetic materials are classified into three categories: diamagnetic, paramagnetic and ferromagnetic materials.
- The lagging of magnetic induction $\vec{B}$ behind the cyclic variation in magnetising field $\vec{H}$ is defined as "Hysteresis", which means "lagging behind".
- The right hand thumb rule "If we hold the current carrying conductor in our right hand such that the thumb points in the direction of current flow, then the rest of the fingers encircling the wire points in the direction of the magnetic field lines produced".
- Maxwell right hand cork screw rule "If we rotate a screw by a screw driver, then the direction of current is same as the direction in which screw advances, and the direction of rotation of the screw will determine the direction of the magnetic field".

- Ampère's circuital law is $\oint_C \vec{B} \cdot d\vec{l} = \mu_\circ I_{enclosed}$.
- Magnetic field inside the solenoid is $B = \mu_\circ nI$, where n is the number of turns per unit length.
- Magnetic field interior to the toroid is $B = \mu_\circ nI$, where n is the number of turns per unit length.
- Lorentz force is $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$.
- Charged particle moving in a uniform magnetic field will undergo circular motion.
- Fleming's Left Hand Rule: Stretch forefinger, the middle finger and the thumb of the left hand such that they are in mutually perpendicular directions. If we keep the forefinger in the direction of magnetic field, the middle finger in the direction of the electric current, then the thumb points in the direction the force experienced by the conductors.
- One ampere is defined as that current when it is passed through each of the two infinitely long parallel straight conductors kept at a distance of one meter apart in vacuum causes each conductor to experience a force of 2×10^{-7} newton per meter length of the conductor.
- When a current carrying coil is placed in a uniform magnetic field, the net force on it is always zero but net torque is not zero. The magnitude of net torque is τ = NABI sin θ.
- Moving coil galvanometer is an instrument used for the detection and measurement of small currents.
- In moving coil galvanometer, current passing through the galvanometer is directly proportional to the deflection. Mathematically, I = Gθ, where $G = \frac{K}{NAB}$ is called galvanometer constant or current reduction factor of the galvanometer.
- Current sensitivity is defined as the deflection produced per unit current flowing through it, $I_s = \frac{\theta}{I} = \frac{NAB}{K} \Rightarrow I_s = \frac{1}{G}$.
- Voltage sensitivity is defined as the deflection produced per unit voltage which is applied across it, $V_s = \frac{\theta}{V} = \frac{1}{GR_g} = \frac{I_S}{R_g}$, where, R_g is the resistance of galvanometer.
- Ammeter is an instrument used to measure current in an electrical circuit.
- A galvanometer can be converted into an ammeter of given range by connecting a suitable low resistance S called shunt in parallel to the given galvanometer.
- An ideal ammeter has zero resistance.
- Voltmeter is an instrument used to measure potential difference across any element in an electrical circuit.
- A galvanometer can be converted into suitable voltmeter of given range by connecting a suitable resistance R in series with the given galvanometer.
- An ideal voltmeter has infinite resistance.

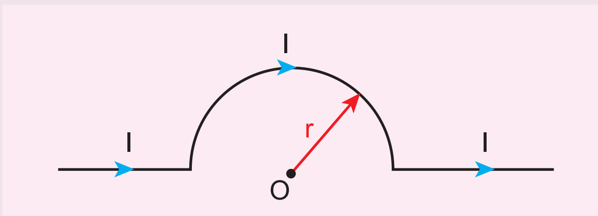
CONCEPT MAP



Magnetism and Magnetic effects of electric current
Magnetism
Earth's Magnetism
Magnetic elements
Tangent Galvanometer
Bar Magnetics
Coulomb's law
Magnetic field axial line
Magnetic field equitorial line
Potential energy
Torque
Magnetic materials
Properties
Magnetic effects of electric current
Biot Savart's law
Ampere law
magnetic field of current carrying conductor
Toroid
Solenoid
Lorentz force
Torque
Cyclotron
Moving coil Galvanometer

I Multiple choice questions

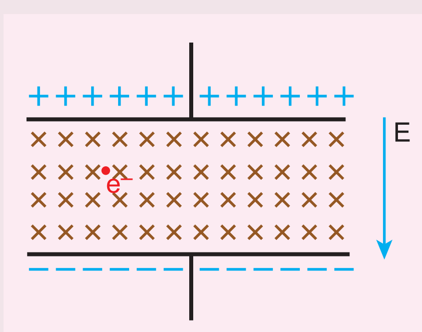
1. The magnetic field at the center O of the following current loop is



(a) $\frac{\mu_\circ I}{4r}\otimes$ (b) $\frac{\mu_\circ I}{4r}\odot$

(c) $\frac{\mu_\circ I}{2r}\otimes$ (d) $\frac{\mu_\circ I}{2r}\odot$

2. An electron moves straight inside a charged parallel plate capacitor of uniform charge density σ. The time taken by the electron to cross the parallel plate capacitor when the plates of the capacitor are kept under constant magnetic field of induction $\vec{B}$ is



(a) $\varepsilon_\circ \frac{elB}{\sigma}$ (b) $\varepsilon_\circ \frac{lB}{\sigma l}$

(c) $\varepsilon_\circ \frac{lB}{e\sigma}$ (d) $\varepsilon_\circ \frac{lB}{\sigma}$

3. The force experienced by a particle having mass m and charge q accelerated through a potential difference V when it is kept under perpendicular magnetic field $\vec{B}$ is

(a) $\sqrt{\frac{2q^3BV}{m}}$ (b) $\sqrt{\frac{q^3B^2V}{2m}}$

(c) $\sqrt{\frac{2q^3B^2V}{m}}$ (d) $\sqrt{\frac{2q^3BV}{m^3}}$

4. A circular coil of radius 5 cm and 50 turns carries a current of 3 ampere. The magnetic dipole moment of the coil is

(a) 1.0 amp – m^2 (b) 1.2 amp – m^2

(c) 0.5 amp – m^2 (d) 0.8 amp – m^2

5. A thin insulated wire forms a plane spiral of N = 100 tight turns carrying a current I = 8 m A (milli ampere). The radii of inside and outside turns are a = 50 mm and b = 100 mm respectively. The magnetic induction at the center of the spiral is

(a) 5 μT (b) 7 μT

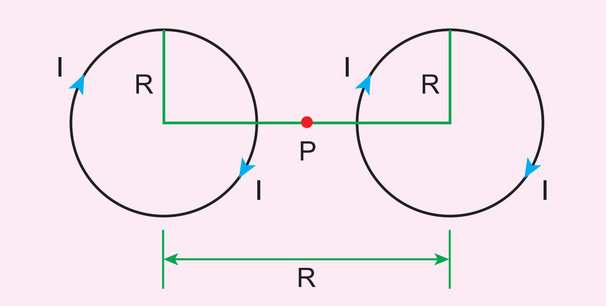
(c) 8 μT (d) 10 μT

6. Three wires of equal lengths are bent in the form of loops. One of the loops is circle, another is a semi-circle and the third one is a square. They are placed in a uniform magnetic field and same electric current is passed through them. Which of the following loop configuration will experience greater torque ?

(a) circle (b) semi-circle

(c) square (d) all of them

7. Two identical coils, each with N turns and radius R are placed coaxially at a distance R as shown in the figure. If I is the current passing through the loops in the same direction, then the magnetic field at a point P which is at exactly at $\frac{R}{2}$ distance between two coils is



(a) $\frac{8N\mu_\circ I}{\sqrt{5}R}$ (b) $\frac{8N\mu_\circ I}{5^{3/2}R}$

(c) $\frac{8N\mu_\circ I}{5R}$ (d) $\frac{4N\mu_\circ I}{\sqrt{5}R}$

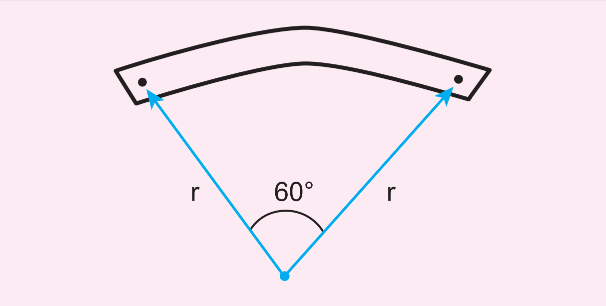
8. A wire of length l carries a current I along the Y direction and magnetic field is given by $\vec{B} = \frac{\beta}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})T$. The magnitude of Lorentz force acting on the wire is

(a) $\sqrt{\frac{2}{\sqrt{3}}}\beta Il$ (b) $\sqrt{\frac{1}{\sqrt{3}}}\beta Il$

(c) $\sqrt{2}\beta Il$ (d) $\sqrt{\frac{1}{2}}\beta Il$

9. A bar magnet of length l and magnetic moment M is bent in the form of an arc as shown in figure. The new magnetic dipole moment will be

(NEET 2014)



(a) M (b) $\frac{3}{\pi}M$

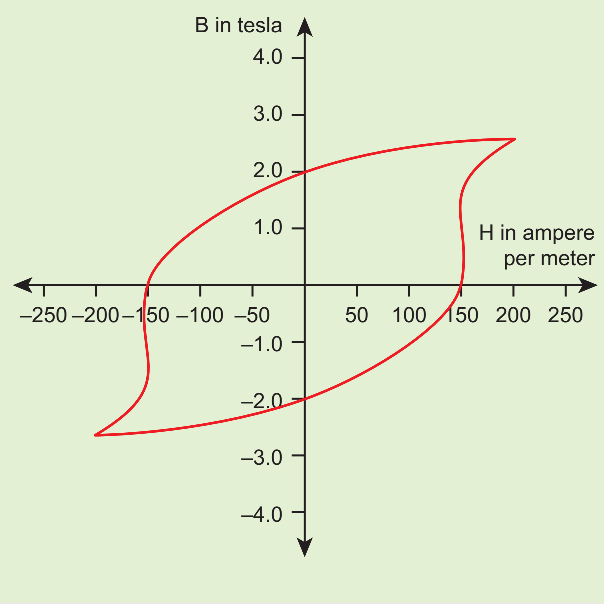
(c) $\frac{2}{\pi}M$ (d) $\frac{1}{2}M$

10. A non-conducting charged ring of charge q, mass m and radius r is rotated with constant angular speed ω. Find the ratio of its magnetic moment with angular momentum is

(a) $\frac{q}{m}$ (b) $\frac{2q}{m}$

(c) $\frac{q}{2m}$ (d) $\frac{q}{4m}$

11. The BH curve for a ferromagnetic material is shown in the figure. The material is placed inside a long solenoid which contains 1000 turns/cm. The current that should be passed in the solenoid to demagnetize the ferromagnet completely is



(a) 1.00 m A (milli ampere) (b) 1.25 mA

(c) 1.50 mA (d) 1.75 mA

12. Two short bar magnets have magnetic moments 1.20 Am^2 and 1.00 Am^2

respectively. They are kept on a horizontal table parallel to each other with their north poles pointing towards the south. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centers is (Horizontal components of Earth's magnetic induction is 3.6×10^{-5} Wb m^{-2})

(NSEP 2000-2001)

(a) 3.60×10^{-5} Wb m^{-2}

(b) 3.5×10^{-5} Wb m^{-2}

(c) 2.56×10^{-4} Wb m^{-2}

(d) 2.2×10^{-4} Wb m^{-2}

13. The vertical component of Earth's magnetic field at a place is equal to the horizontal component. What is the value of angle of dip at this place?

(a) 30^o (b) 45^o

(c) 60^o (d) 90^o

14. A flat dielectric disc of radius R carries an excess charge on its surface. The surface charge density is σ. The disc rotates about an axis perpendicular to its plane passing through the center with angular velocity ω. Find the magnitude of the torque on the disc if it is placed in a uniform magnetic field whose strength is B which is directed perpendicular to the axis of rotation

(a) $\frac{1}{4}\sigma\omega\pi BR$ (b) $\frac{1}{4}\sigma\omega\pi BR^2$

(c) $\frac{1}{4}\sigma\omega\pi BR^3$ (d) $\frac{1}{4}\sigma\omega\pi BR^4$

15. A simple pendulum with charged bob is oscillating with time period T and let θ be the angular displacement. If the uniform magnetic field is switched ON in a direction perpendicular to the plane of oscillation then

(a) time period will decrease but θ will remain constant

(b) time period remain constant but θ will decrease

(c) both T and θ will remain the same

(d) both T and θ will decrease

Answers

1) a	**2)** d	**3)** c	**4)** b	**5)** b
6) a	**7)** b	**8)** a	**9)** b	**10)** c
11) b	**12)** c	**13)** b	**14)** d	**15)** c

II Short answer questions:

1. What is meant by magnetic induction?
2. Define magnetic flux.
3. Define magnetic dipole moment.
4. State Coulomb's inverse law.
5. What is magnetic susceptibility?
6. State Biot-Savart's law.
7. What is magnetic permeability?
8. State Ampere's circuital law.
9. Compare dia, para and ferro-magnetism.
10. What is meant by hysteresis?

III Long answer questions

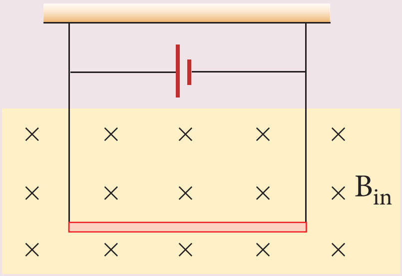
1. Discuss Earth's magnetic field in detail.
2. Deduce the relation for the magnetic induction at a point due to an infinitely long straight conductor carrying current.
3. Obtain a relation for the magnetic induction at a point along the axis of a circular coil carrying current.
4. Compute the torque experienced by a magnetic needle in a uniform magnetic field.
5. Calculate the magnetic induction at a point on the axial line of a bar magnet.
6. Obtain the magnetic induction at a point on the equatorial line of a bar magnet.
7. Find the magnetic induction due to a long straight conductor using Ampere's circuital law.
8. Discuss the working of cyclotron in detail.
9. What is tangent law? Discuss in detail.
10. Explain the principle and working of a moving coil galvanometer.
11. Discuss the conversion of galvanometer into an ammeter and also a voltmeter.
12. Calculate the magnetic field inside and outside of the long solenoid using Ampere's circuital law.

IV. Numerical problems

1. A bar magnet having a magnetic moment $\vec{M}$ is cut into four pieces i.e., first cut in two pieces along the axis of the magnet and each piece is further cut into two pieces. Compute the magnetic moment of each piece.

   Answer $\vec{M}_{new} = \frac{1}{4}\vec{M}$

2. A conductor of linear mass density 0.2 g m^{-1} suspended by two flexible wire as shown in figure. Suppose the tension in the supporting wires is zero when it is kept inside the magnetic field of 1 T whose direction is into the page. Compute the current inside the conductor and also the direction of the current. Assume g = 10 m s^{-2}



   Answer 2 mA

3. A circular coil with cross-sectional area 0.1 cm^2 is kept in a uniform magnetic field of strength 0.2 T. If the current passing in the coil is 3 A and plane of the loop is perpendicular to the direction of magnetic field. Calculate

   (a) total torque on the coil
   (b) total force on the coil
   (c) average force on each electron in the coil due to the magnetic field of the free electron density for the material of the wire is 10^{28} m^{-3}.

Answer (a) zero (b) zero (c) 0.6 x 10^{-23} N

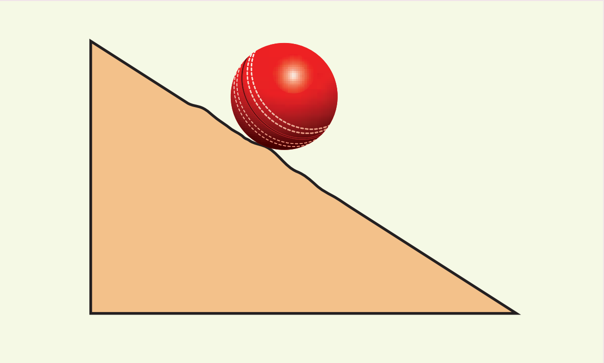
4. A bar magnet is placed in a uniform magnetic field whose strength is 0.8 T. Suppose the bar magnet orient at an angle 30° with the external field experiences a torque of 0.2 N m. Calculate:

   (i) the magnetic moment of the magnet

   (ii) the work done by an applied force in moving it from most stable configuration to the most unstable configuration and also compute the work done by the applied magnetic field in this case.

   Answer (i) 0.5 A m^2 (ii) W = 0.8 J and W_{mag} = -0.8 J

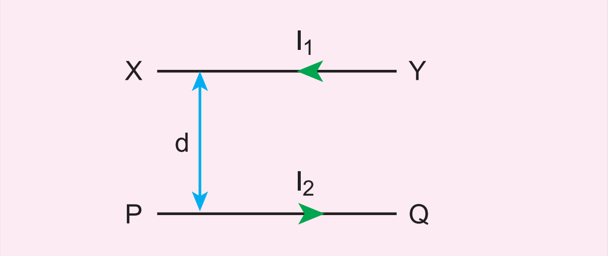
5. A non - conducting sphere has a mass of 100 g and radius 20 cm. A flat compact coil of wire with turns 5 is wrapped tightly around it with each turns concentric with the sphere. This sphere is placed on an inclined plane such that plane of coil is parallel to the inclined plane. A uniform magnetic field of 0.5 T exists in the region in vertically upward direction. Compute the current I required to rest the sphere in equilibrium. Answer $\frac{2}{\pi} A$



6. Calculate the magnetic field at the center of a square loop which carries a current of 1.5 A, length of each loop is 50 cm. Answer 3.4 x 10^{-6} T

7. Show that the magnetic field at any point on the axis of the solenoid having n turns per unit length is $B = \frac{1}{2}\mu_o nI(\cos\theta_1 - \cos\theta_2)$.

8. Let I_1 and I_2 be the steady currents passing through a long horizontal wire XY and PQ respectively. The wire PQ is fixed in horizontal plane and the wire XY be is allowed to move freely in a vertical plane. Let the wire XY is in equilibrium at a height d over the parallel wire PQ as shown in figure.



Show that if the wire XY is slightly displaced and released, it executes Simple Harmonic Motion (SHM). Also, compute the time period of oscillations.

Answer $a_y = -\omega^2 y$ (SHM) and time period

$$T = 2\pi\sqrt{\frac{d}{g}} \text{ in sec}$$

BOOKS FOR REFERENCE

1. H. C. Verma, *Concepts of Physics – Volume 2*, Bharati Bhawan Publisher
2. Halliday, Resnick and Walker, *Fundamentals of Physics*, Wiley Publishers, 10th edition
3. Serway and Jewett, *Physics for scientist and engineers with modern physics*, Brook/Coole publishers, Eighth edition
4. David J. Griffiths, *Introduction to electrodynamics*, Pearson publishers
5. Rita John, *Solid State Physics (Magnetism chapter)*, McGraw Hill Education (India) Pvt. Ltd.
6. Paul Tipler and Gene Mosca, *Physics for scientist and engineers with modern physics*, Sixth edition, W.H. Freeman and Company

ICT CORNER

Magnetism

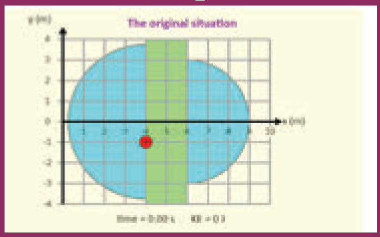
Topic: Cyclotron

In this activity you will be able to visualize and understand the working of cyclotron.

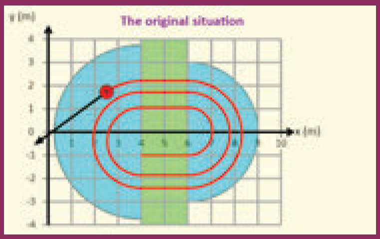
STEPS:

- Open the browser and type ‘physics.bu.edu/~duffy/HTML5/cyclotron.html’ in the address bar.
- Click ‘play’ to release the positively charged particle between the D-shaped sections.
- Observe trajectory of positively charged particle under the magnetic field between D-shaped sections.
- Note the kinetic energy of the particle after some time (say t = 20 s)

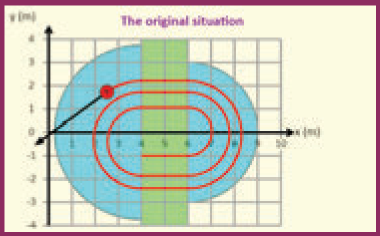
Step1



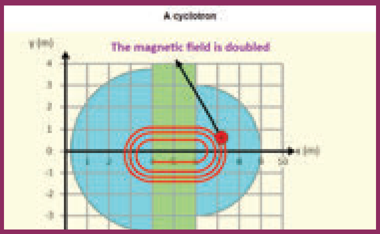
Step2



Step3



Step4



Double the electric and magnetic fields by clicking corresponding buttons and observe the change in kinetic energy for a particular given time t.

URL:

http://physics.bu.edu/~duffy/HTML5/cyclotron.html

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

B263_12_PHYSICS_EM

UNIT 4

ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

"Nature is our kindest friend and best critic in experimental science if we only allow her intimations to fall unbiased on our minds" — Michael Faraday

LEARNING OBJECTIVES

In this unit, the student is exposed to

- the phenomenon of electromagnetic induction
- the application of Lenz's law to find the direction of induced emf
- the concept of Eddy current and its uses
- the phenomenon of self-induction and mutual-induction
- the various methods of producing induced emfs
- the construction and working of AC generators
- the principle of transformers and its role in long distance power communication
- the notion of root mean square value of alternating current
- the idea of phasors and phase relationships in different AC circuits
- the insight about power in an AC circuit and wattless current
- the understanding of energy conservation during LC oscillations

U1YLL7

4.1 ELECTROMAGNETIC INDUCTION

4.1.1 Introduction

In the previous chapter, we have learnt that whenever an electric current flows through a conductor, it produces a magnetic field around it. This was discovered by Christian Oersted. Later, Ampere proved that a current-carrying loop behaves like a bar magnet. These are the magnetic effects produced by the electric current.

Physicists then began to think of the converse effect. Is it possible to produce an electric current with the help of a magnetic field? A series of experiments were conducted to establish the converse effect. These experiments were done by Michael Faraday of UK and Joseph Henry of USA, almost simultaneously and independently. These attempts became successful and led to the discovery of the phenomenon, called Electromagnetic Induction. Michael Faraday is credited with the discovery of electromagnetic induction in 1831.

In this chapter, let us see a few experiments of Faraday, the results and the phenomenon of Electromagnetic Induction. Before that, we will recollect the concept of magnetic flux linked with a surface area.

An anecdote!

Michael Faraday was enormously popular for his lectures as well. In one of his lectures, he demonstrated his experiments which led to the discovery of electromagnetic induction.

At the end of the lecture, one member of the audience approached Faraday and said, "Mr. Faraday, the behaviour of the magnet and the coil of wire was interesting, but what is the use of it?" Faraday answered politely, "Sir, what is the use of a newborn baby?"

Note: We will soon see the greatness of 'that little child' who has now grown as an adult to cater to the energy needs.

4.1.2 Magnetic Flux (Φ_B)

The magnetic flux through an area A in a magnetic field is defined as the number of magnetic field lines passing through that area normally and is given by the equation (Figure 4.1(a)).

$$\Phi_B = \int_A \vec{B} \cdot d\vec{A} = BA\cos\theta \qquad (4.1)$$

where the integral is taken over the area A and θ is the angle between the direction of the magnetic field and the outward normal to the area.

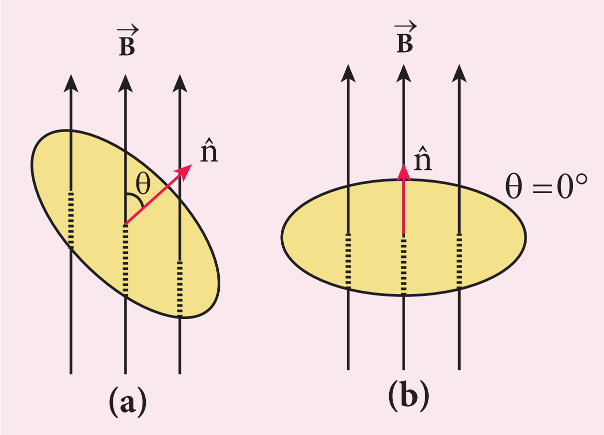


Figure 4.1 Magnetic flux

If the magnetic field $\vec{B}$ is uniform over the area A and is perpendicular to the area as shown in Figure 4.1(b), then the above equation becomes

$$\Phi_B = BA \qquad (4.2)$$

since $\theta = 0°$, $\cos 0° = 1$

The SI unit of magnetic flux is $T\,m^2$. It is also measured in weber or Wb.

$$1\ Wb = 1\ T\,m^2$$

EXAMPLE 4.1

A circular antenna of area 3 m² is installed at a place in Madurai. The plane of the area of antenna is inclined at 47° with the direction of Earth's magnetic field. If the magnitude of Earth's field at that place is 40773.9 nT find the magnetic flux linked with the antenna.

Solution

B = 40773.9 nT; θ = 90° − 47° = 43°; A = 3m²

We know that $\Phi_B = BA\cos\theta$

$= 40,773.9 \times 10^{-9} \times 3 \times \cos 43^o$

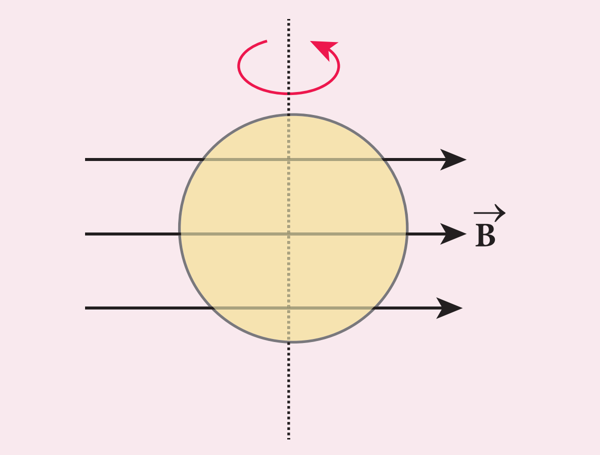
$= 40.7739 \times 10^{-6} \times 3 \times 0.7314$

$= 89.47 \times 10^{-6}$ Wb

$\Phi_B = 89.47\,\mu\text{Wb}$

EXAMPLE 4.2

A circular loop of area $5 \times 10^{-2} m^2$ rotates in a uniform magnetic field of 0.2 T. If the loop rotates about its diameter which is perpendicular to the magnetic field as shown in figure. Find the magnetic flux linked with the loop when its plane is (i) normal to the field (ii) inclined 60° to the field and (iii) parallel to the field.



Solution

$A = 5 \times 10^{-2} m^2; B = 0.2$ T

(i) $\theta = 0^\circ$;

$$\Phi_B = BA\cos\theta = 0.2 \times 5 \times 10^{-2} \times \cos 0^\circ$$

$$\Phi_B = 1 \times 10^{-2} Wb$$

(ii) $\theta = 90^\circ - 60^\circ = 30^\circ$;

$$\Phi_B = BA\cos\theta = 0.2 \times 5 \times 10^{-2} \times \cos 30^\circ$$

$$\Phi_B = 1 \times 10^{-2} \times \frac{\sqrt{3}}{2} = 8.66 \times 10^{-3} Wb$$

(iii) $\theta = 90^\circ$;

$$\Phi_B = BA\cos 90^\circ = 0$$

4.1.3 Faraday's Experiments on Electromagnetic Induction

First Experiment

Consider a closed circuit consisting of a coil *C* of insulated wire and a galvanometer *G* as shown in Figure 4.2(a). The galvanometer does not indicate deflection as there is no electric current in the circuit.

When a bar magnet is inserted into the stationary coil, with its north pole facing the coil, there is a momentary deflection in the galvanometer. This indicates that an electric current is set up in the coil (Figure 4.2(b)). If the magnet is kept stationary inside the coil, the galvanometer does not indicate deflection (Figure 4.2(c)).

The bar magnet is now withdrawn from the coil, the galvanometer again gives a momentary deflection but in the opposite direction. So the electric current flows in opposite direction (Figure 4.2(d)). Now if the magnet is moved faster, it gives a larger deflection due to a greater current in the circuit (Figure 4.2(e))

The bar magnet is reversed i.e., the south pole now faces the coil. When the above experiment is repeated, the deflections are opposite to that obtained in the case of north pole (Figure 4.2(f)).

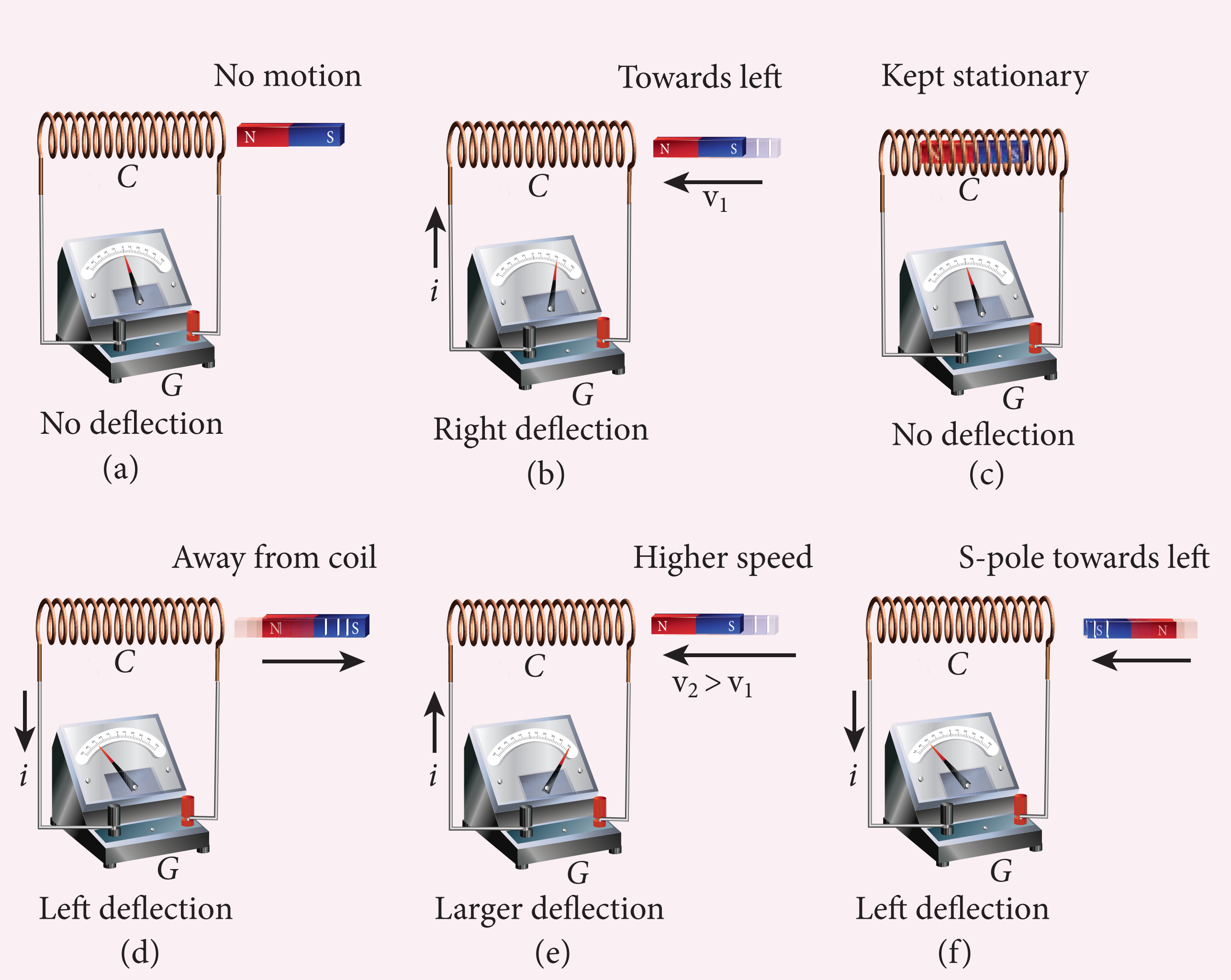


Figure 4.2 Faraday's first experiment

If the magnet is kept stationary and the coil is moved towards or away from the coil, similar results are obtained. It is concluded that whenever there is a relative motion between the coil and the magnet, there is deflection in the galvanometer, indicating the electric current setup in the coil.

Second Experiment

Consider two closed circuits as shown in Figure 4.3(a). The circuit consisting of a coil *P*, a battery *B* and a key *K* is called as primary circuit while the circuit with a coil *S* and a galvanometer *G* is known as secondary circuit. The coils *P* and *S* are kept at rest in close proximity with respect to one another.

If the primary circuit is closed, electric current starts flowing in the primary circuit. At that time, the galvanometer gives a momentary deflection (Figure 4.3(a)).

After that, when the electric current reaches a certain steady value, no deflection is observed in the galvanometer.

Likewise if the primary circuit is broken, the electric current starts decreasing and there is again a sudden deflection but in the opposite direction (Figure 4.3(b)). When the electric current becomes zero, the galvanometer shows no deflection.

From the above observations, it is concluded that whenever the electric current in the primary circuit changes, the galvanometer shows a deflection.

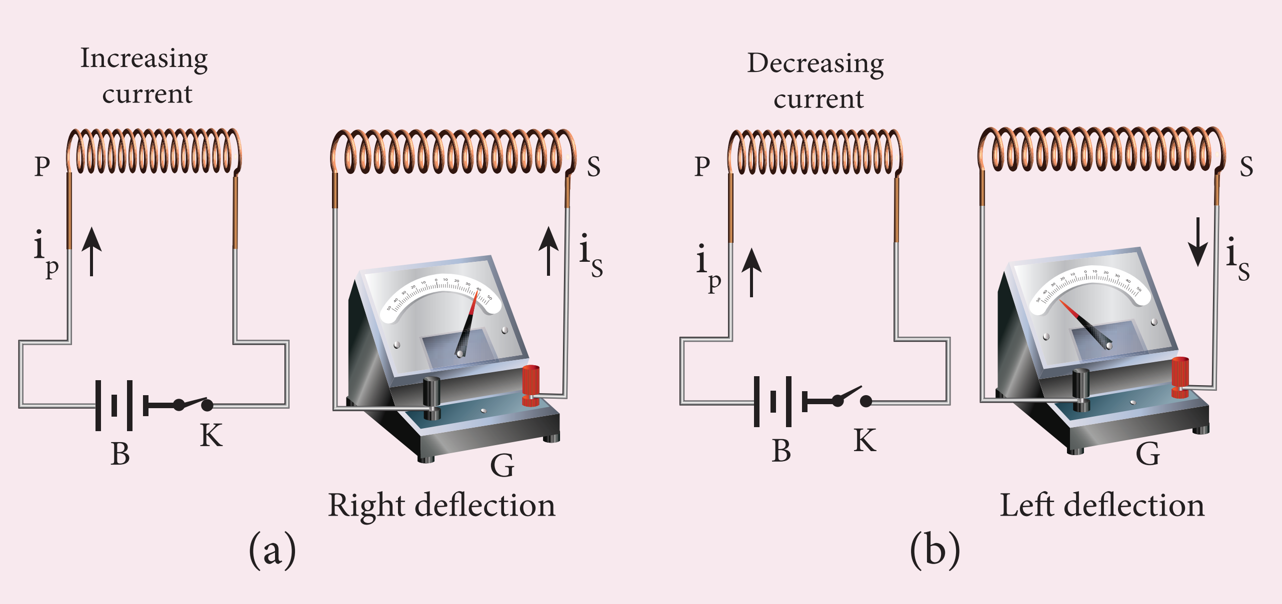


Figure 4.3 Faraday's second experiment

Faraday's Law of Electromagnetic Induction

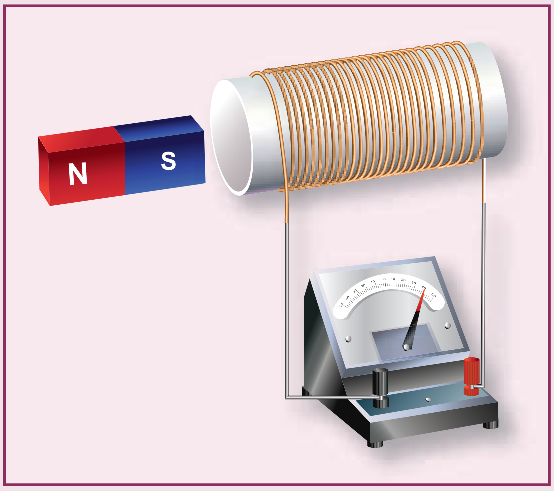
From the results of his experiments, Faraday realized that

> **whenever the magnetic flux linked with a closed coil changes, an emf (electromotive force) is induced and hence an electric current flows in the circuit. This current is called an induced current and the emf giving rise to such current is called an induced emf. This phenomenon is known as electromagnetic induction.**

Based on this idea, Faraday's experiments are understood in the following way. In the first experiment, when a bar magnet is placed close to a coil, some of the magnetic field lines of the bar magnet pass through the coil i.e., the magnetic flux is linked with the coil. When the bar magnet and the coil approach each other, the magnetic flux linked with the coil increases. So this increase in magnetic flux induces an emf and hence a transient

ACTIVITY

Exploring Electromagnetic Induction



Make a circuit containing a coil of insulated wire wound around soft hollow core and a galvanometer as shown in Figure. It is better to use a thin wire for the coil so that we can wind many turns in the available space. Perform the steps described in first experiment of Faraday with the help of a strong bar magnet. Students will get hands-on experience about electromagnetic induction.

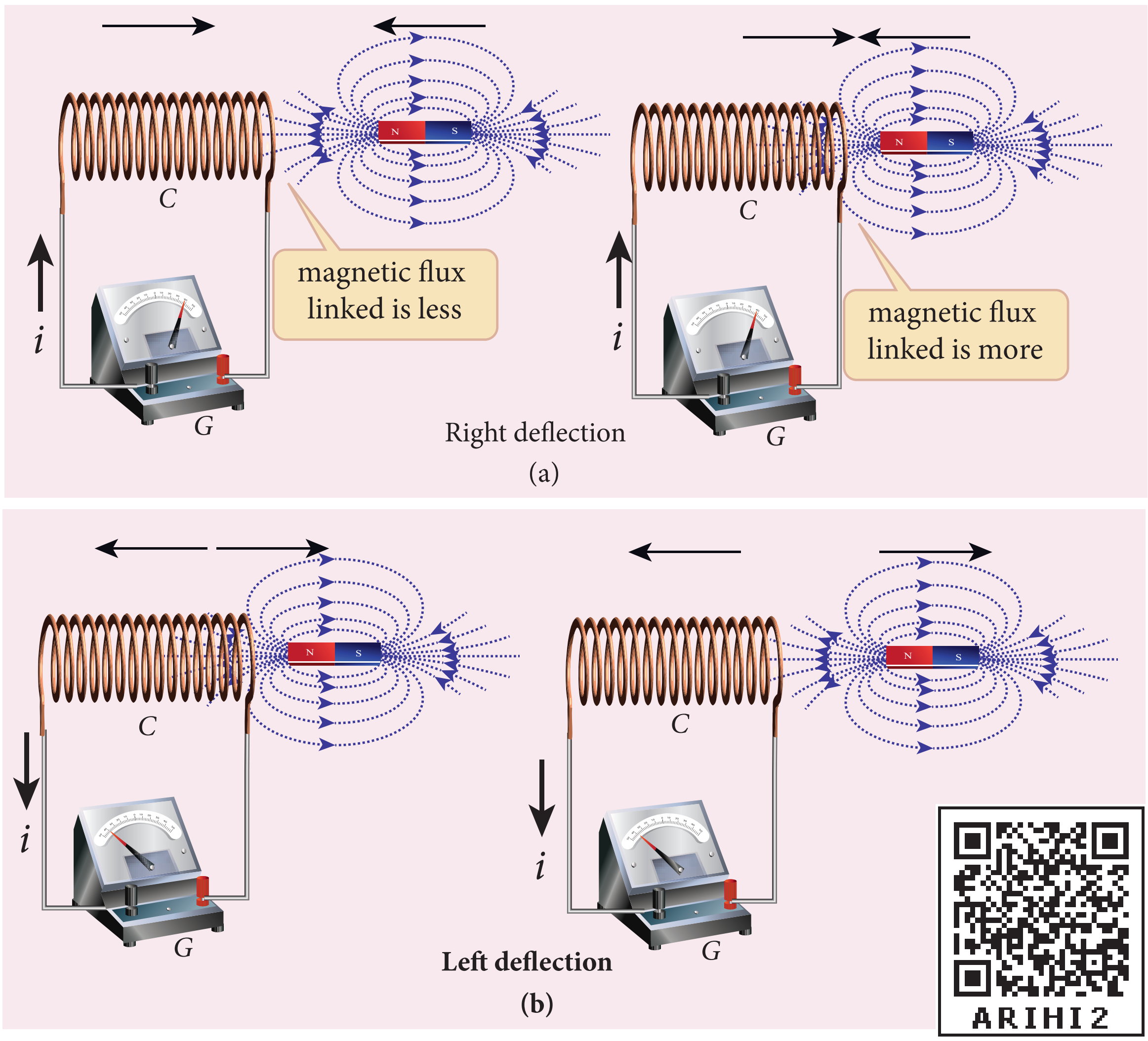


Figure 4.4 Explanation of Faraday's first experiment

electric current flows in the circuit in one direction (Figure 4.4(a)).

At the same time, when they recede away from one another, the magnetic flux linked with the coil decreases. The decrease in magnetic flux again induces an emf in opposite direction and hence an electric current flows in opposite direction (Figure 4.4(b)). So there is deflection in the galvanometer when there is a relative motion between the coil and the magnet.

In the second experiment, when the primary coil *P* carries an electric current, a magnetic field is established around it. The magnetic lines of this field pass through itself and the neighbouring secondary coil *S*.

When the primary circuit is open, no electric current flows in it and hence the magnetic flux linked with the secondary coil is zero (Figure 4.5(a)).

However, when the primary circuit is closed, the increasing current builds up a magnetic field around the primary coil. Therefore, the magnetic flux linked with the secondary coil increases. This increasing flux linked induces a transient electric current in the secondary coil (Figure 4.5(b)). When the electric current in the primary coil reaches a steady value, the magnetic flux linked with the secondary coil does not change and the electric current in the secondary coil will disappear.

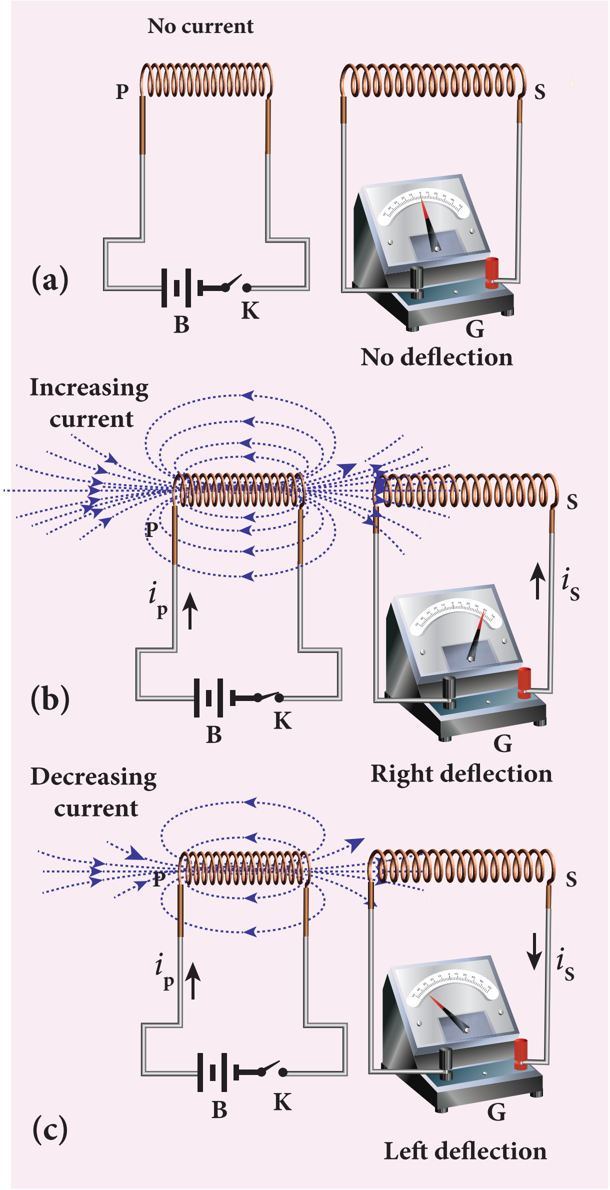


Figure 4.5 Explanation of Faraday's second experiment

Similarly, when the primary circuit is broken, the decreasing primary current induces an electric current in the secondary coil, but in the opposite direction (Figure 4.5(c)). So there is deflection in the galvanometer whenever there is a change in the primary current.

Note The symbol ε is used for permittivity in unit 1 and for emf in this chapter. Reader is advised to use ε in relevant context.

Importance of Electromagnetic Induction!

The application of the phenomenon of Electromagnetic Induction is almost everywhere in the present day life. Right from home appliances to huge factory machineries, from cellphone to computers and internet, from electric guitar to satellite communication, all need electricity for their operation. There is an ever growing demand for electric power.

All these are met with the help of electric generators and transformers which function on electromagnetic induction. The modern, sophisticated human life would not be possible without the discovery of electromagnetic induction.

The conclusions of Faraday's experiments are stated as two laws.

First law

Whenever magnetic flux linked with a closed circuit changes, an emf is induced in the circuit.

Second law

The magnitude of induced emf in a closed circuit is equal to the time rate of change of magnetic flux linked with the circuit.

If the magnetic flux linked with the coil changes by $d\Phi_B$ in a time dt, then the induced emf is given by

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

The negative sign in the above equation gives the direction of the induced current which will be dealt with in the next section.

If a coil consisting of N turns is tightly wound such that each turn covers the same area, then the flux through each turn will be the same. Then total emf induced in the coil is given by

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -\frac{d(N\Phi_B)}{dt} \qquad (4.3)$$

Here, $N\Phi_B$ is called flux linkage, defined as the product of number of turns N of the coil and the magnetic flux linking each turn of the coil Φ_B.

EXAMPLE 4.3

A cylindrical bar magnet is kept along the axis of a circular solenoid. If the magnet is rotated about its axis, find out whether an electric current is induced in the coil.

Solution

The magnetic field of a cylindrical magnet is symmetrical about its axis. As the magnet is rotated along the axis of the solenoid, there is no induced current in the solenoid because the flux linked with the solenoid does not change due to the rotation of the magnet.

EXAMPLE 4.4

A closed coil of 40 turns and of area 200 cm^2, is rotated in a magnetic field of flux density 2 Wb m^{-2}. It rotates from a position where its plane makes an angle of 30° with the field to a position perpendicular to the field in a time 0.2 sec. Find the magnitude of the emf induced in the coil due to its rotation.

Solution

N = 40 turns; B = 2 Wb m^{-2}

A = 200 cm^2 = 200 × 10^{-4} m^2;

Initial flux, $\Phi_i = BA\cos\theta$

$= 2\times 200\times 10^{-4}\times \cos 60^\circ$

since $\theta = 90^\circ - 30^\circ = 60^\circ$

$\Phi_i = 2\times 10^{-2}\,Wb$

Final flux, $\Phi_f = BA\cos\theta$

$= 2\times 200\times 10^{-4}\times \cos 0^\circ$ since $\theta = 0^\circ$

$\Phi_f = 4\times 10^{-2}\,Wb$

Magnitude of the induced emf is

$$\varepsilon = N\frac{d\Phi_B}{dt} = \frac{40\times\left(4\times 10^{-2} - 2\times 10^{-2}\right)}{0.2} = 4V$$

EXAMPLE 4.5

A straight conducting wire is dropped horizontally from a certain height with its length along east – west direction. Will an emf be induced in it? Justify your answer.

Solution

Yes! An emf will be induced in the wire because it moves perpendicular to the horizontal component of Earth's magnetic field.

4.1.4 Lenz's law

A German physicist Heinrich Lenz performed his own experiments on electromagnetic induction and deduced a law to determine the direction of the induced current. This law is known as Lenz's law.

Lenz's law states that the direction of the induced current is such that it always opposes the cause responsible for its production.

Faraday discovered that when magnetic flux linked with a coil changes, an electric current is induced in the circuit. Here the flux change is the cause while the induction of current is the effect. Lenz's law says that the effect always opposes the cause. Therefore, the induced current would flow in a direction so that it could oppose the flux change.

To understand Lenz's law, let us consider two illustrations in which we find the direction of the induced current in a circuit.

Illustration 1

Consider a uniform magnetic field, with its field lines perpendicular to the plane of the paper and pointing inwards. These field lines are represented by crosses (x) as shown in Figure 4.6(a). A rectangular metallic frame *ABCD* is placed in this magnetic field, with its plane perpendicular to the field. The arm *AB* is movable so that it can slide towards right or left.

If the arm AB slides to our right side, the number of field lines (magnetic flux) passing through the frame *ABCD* increases and a current is induced. As suggested by Lenz's law, the induced current opposes this flux increase and it tries to reduce it by producing **another magnetic field pointing outwards i.e., opposite to the existing magnetic field.**

The magnetic lines of this induced field are represented by red-colored circles in the Figure 4.6(b). From the direction of the magnetic field thus produced, the direction of the induced current is found to be anti-clockwise by using right-hand thumb rule.

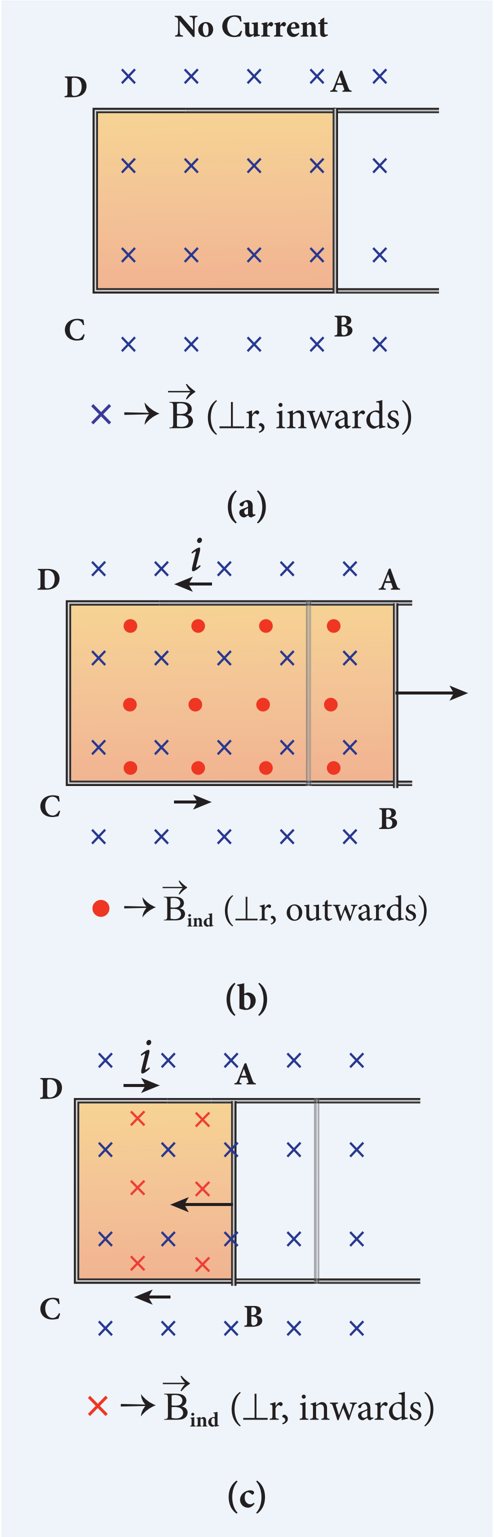


Figure 4.6 First illustration of Lenz's law

The leftward motion of arm AB decreases magnetic flux. The induced current, this time, produces **a magnetic field in the inward direction (red-colored crosses) i.e., in the direction of the existing magnetic field** (Figure 4.6(c)). Therefore, the flux decrease is opposed by the flow of induced current. From this, it is found that induced current flows in clockwise direction.

Illustration 2

Let us move a bar magnet towards the solenoid, with its north pole pointing the solenoid as shown in Figure 4.7(b). This motion increases the magnetic flux of the coil which in turn, induces an electric current. Due to the flow of induced current, the coil becomes a magnetic dipole whose two magnetic poles are on either end of the coil.

In this case, the cause producing the induced current is the movement of the magnet. According to Lenz's law, the induced current should flow in such a way that it opposes the movement of the north pole towards coil. It is possible if the end nearer to the magnet becomes north pole (Figure 4.7(b)). Then it repels the north pole of the bar magnet and opposes the movement of the magnet. Once pole ends are known, the direction of the induced current could be found by using right hand thumb rule.

When the bar magnet is withdrawn, the nearer end becomes south pole which attracts north pole of the bar magnet, opposing the receding motion of the magnet (Figure 4.7(c)).

Thus the direction of the induced current can be found from Lenz's law.

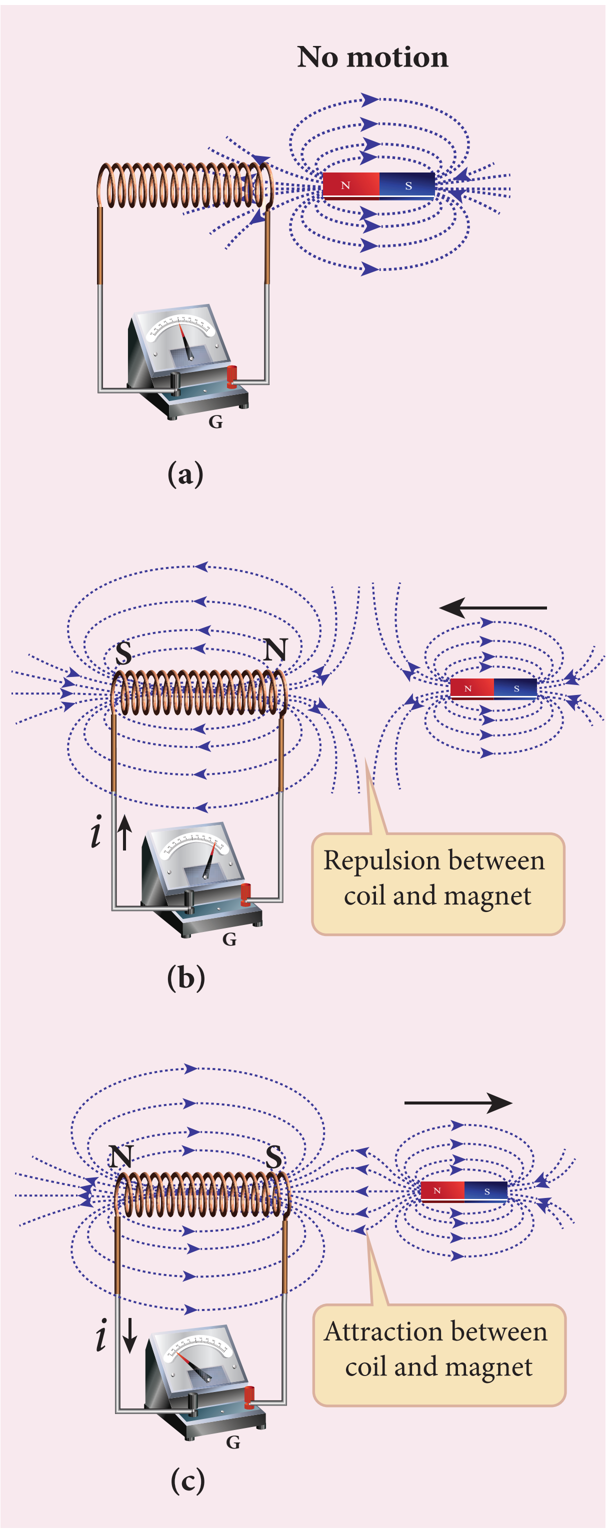
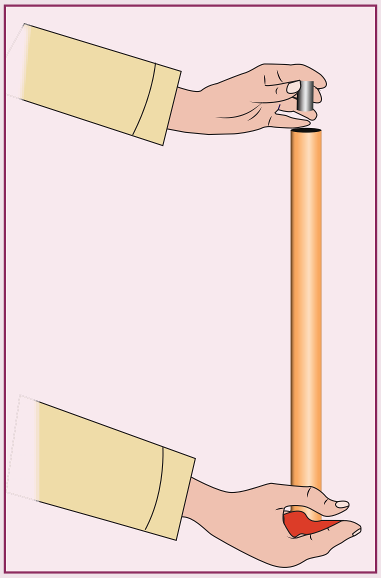


Figure 4.7 Second illustration of Lenz's law

Conservation of energy

The truth of Lenz's law can be established on the basis of the law of conservation of energy. The explanation is as follows: According to Lenz's law, when a magnet is moved either towards or away from a coil, the induced current produced opposes its motion. As a result, there will always be a

Activity



Demonstration of Lenz's law

Take a narrow copper pipe and a strongly magnetized button magnet as shown in figure. Keep the copper pipe vertical and drop the magnet into the pipe. Watch the motion of the magnet and note that magnet has become slower than its free fall. The reason is that an electric current generated by a moving magnet will always *oppose* the original motion of the magnet that produced the current.

resisting force on the moving magnet. Work has to be done by some external agency to move the magnet against this resisting force. Here the mechanical energy of the moving magnet is converted into the electrical energy which in turn, gets converted into Joule heat in the coil i.e., energy is converted from one form to another.

On the contrary to Lenz's law, let us assume that the induced current *helps* the cause responsible for its production. Now when we push the magnet little bit towards the coil, the induced current helps the movement of the magnet towards the coil. Then the magnet starts moving towards the coil without any expense of energy. This, then, becomes a perpetual motion machine. In practice, no such machine is possible. Therefore, the assumption that the induced current *helps* the cause is wrong.

4.1.5 Fleming's right hand rule

When a conductor moves in a magnetic field, the direction of motion of the conductor, the field and the induced current are given by Fleming's right hand rule and is as follows:

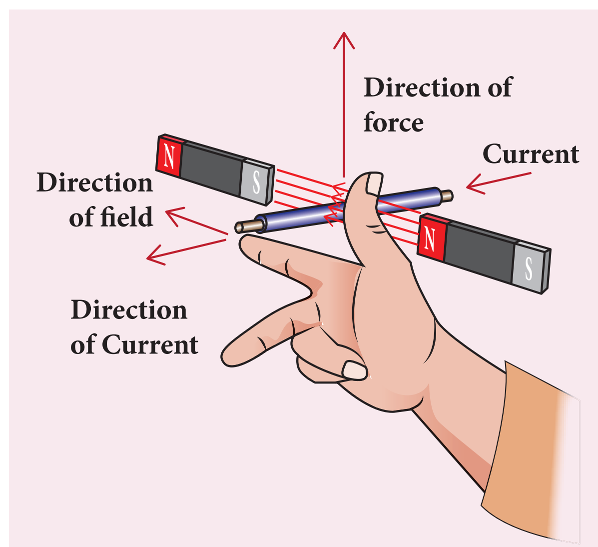


Figure 4.8 Fleming's right hand rule

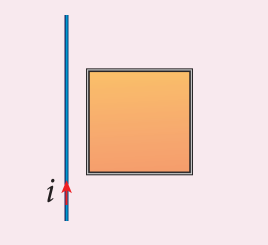
The thumb, index finger and middle finger of right hand are stretched out in mutually perpendicular directions (as shown in Figure 4.8). If the index finger points the direction of the magnetic field and the thumb indicates the direction of motion of the conductor, then the middle finger will indicate the direction of the induced current.

Fleming's right hand rule is also known as generator rule.

UNIT-4(XII-Physics_Vol-1).indd 216 04-03-2019 10:39:00

EXAMPLE 4.6

If the current i flowing in the straight conducting wire as shown in the figure decreases, find out the direction of induced current in the metallic square loop placed near it.

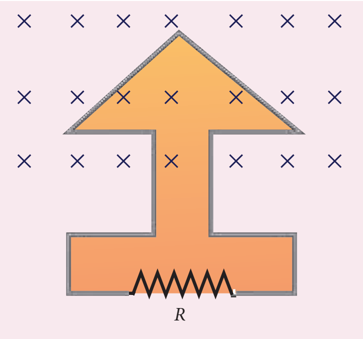


Solution

From right hand rule, the magnetic field by the straight wire is directed into the plane of the square loop perpendicularly and its magnetic flux is decreasing. The decrease in flux is opposed by the current induced in the loop by producing a magnetic field in the same direction as the magnetic field of the wire. Again from right hand rule, for this inward magnetic field, the direction of the induced current in the loop is clockwise.

EXAMPLE 4.7

The magnetic flux passes perpendicular to the plane of the circuit and is directed into the paper. If the magnetic flux varies with respect to time as per the following relation: $\Phi_B = (2t^3 + 3t^2 + 8t + 5)\,mWb$, what is the magnitude of the induced emf in the loop when t = 3 s? Find out the direction of current through the circuit.



Solution

$\Phi_B = (2t^3 + 3t^2 + 8t + 5)\,mWb$; $N = 1$; t = 3 s

(i) $$\varepsilon = \frac{d(N\Phi_B)}{dt}$$

$$= \frac{d}{dt}(2t^3 + 3t^2 + 8t + 5) \times 10^{-3}$$

$$= (6t^2 + 6t + 8) \times 10^{-3}$$

At t = 3 s,

$$\varepsilon = [(6 \times 9) + (6 \times 3) + 8] \times 10^{-3}$$

$$= 80 \times 10^{-3} V \quad = 80\,mV$$

(ii) As time passes, the magnetic flux linked with the loop increases. According to Lenz's law, the direction of the induced current should be in a way so as to oppose the flux increase. So, the induced current flows in such a way to produce a magnetic field opposite to the given field. This magnetic field is perpendicularly outwards. Therefore, the induced current flows in anti-clockwise direction.

4.1.6 Motional emf from Lorentz force

Consider a straight conducting rod AB of length l in a uniform magnetic field $\vec{B}$ which is directed perpendicularly into the plane of the paper as shown in Figure 4.9(a). The length of the rod is normal to the magnetic field. Let the rod move with a constant velocity $\vec{v}$ towards right side.

When the rod moves, the free electrons present in it also move with same velocity $\vec{v}$ in $\vec{B}$. As a result, the Lorentz force acts on free electrons in the direction from B to A and is given by the relation

$$\vec{F}_B = -e\left(\vec{v} \times \vec{B}\right) \quad (4.4)$$

The action of this Lorentz force is to accumulate the free electrons at the end *A*. This accumulation of free electrons produces a potential difference across the rod which in turn establishes an electric field $\vec{E}$ directed along *BA* (Figure 4.9(b)). Due to the electric field $\vec{E}$, the coulomb force starts acting on the free electrons along *AB* and is given by

$$\vec{F}_E = -e\vec{E} \quad (4.5)$$

The magnitude of the electric field $\vec{E}$ keeps on increasing as long as accumulation of electrons at the end *A* continues. The force $\vec{F}_E$ also increases until equilibrium is reached. At equilibrium, the magnetic Lorentz force $\vec{F}_B$ and the coulomb force $\vec{F}_E$ balance each other and no further accumulation of free electrons at the end *A* takes place.

i.e.,
$$\left|\vec{F}_B\right| = \left|\vec{F}_E\right|$$
$$\left|-e\left(\vec{v} \times \vec{B}\right)\right| = \left|-e\vec{E}\right|$$
$$vB \sin 90° = E$$
$$vB = E \quad (4.6)$$

The potential difference between two ends of the rod is

$$V = El$$
$$V = vBl$$

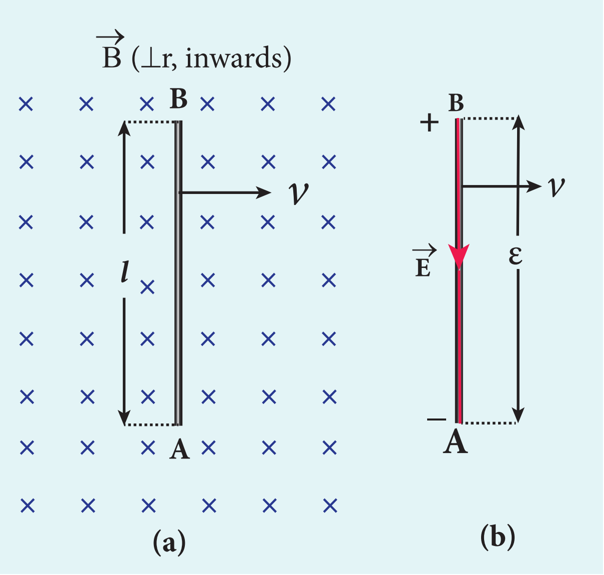


Figure 4.9 Motional emf from Lorentz force

Thus the Lorentz force on the free electrons is responsible to maintain this potential difference and hence produces an emf

$$\varepsilon = Blv \quad (4.7)$$

As this emf is produced due to the movement of the rod, it is often called as motional emf. If the ends A and B are connected by an external circuit of total resistance *R*, then current $i = \frac{\varepsilon}{R} = \frac{Blv}{R}$ flows in it. The direction of the current is found from right-hand thumb rule.

4.1.7 Motional emf from Faraday's law and Energy conservation

Let us consider a rectangular conducting loop of width *l* in a uniform magnetic field $\vec{B}$ which is perpendicular to the plane of the loop and is directed inwards. A part of the loop is in the magnetic field while the remaining part is outside the field as shown in Figure 4.10.

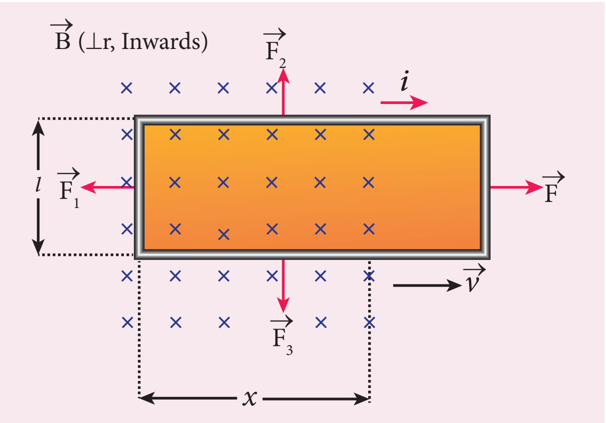


Figure 4.10 Motional emf from Faraday's law

When the loop is pulled with a constant velocity $\vec{v}$ to the right, the area of the portion of the loop within the magnetic field will decrease. Thus, the flux linked with the loop will also decrease. According to Faraday's law, an electric current is induced in the loop which flows in a direction so as to oppose the pull of the loop.

Let x be the length of the loop which is still within the magnetic field, then its area is lx. The magnetic flux linked with the loop is

$$\Phi_B = \int_A \vec{B}\cdot d\vec{A} = BA\cos\theta$$

Here $\theta = 0^\circ$ and $\cos 0^\circ = 1$

$$= BA$$

$$\Phi_B = Blx \quad (4.8)$$

As this magnetic flux decreases due to the movement of the loop, the magnitude of the induced emf is given by

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{d}{dt}(Blx)$$

Here, both B and l are constants. Therefore,

$$\varepsilon = Bl\frac{dx}{dt}$$

$$\varepsilon = Blv \quad (4.9)$$

where $v = \frac{dx}{dt}$ is the velocity of the loop. This emf is known as motional emf since it is produced due to the movement of the loop in the magnetic field.

From Lenz's law, it is found that the induced current flows in clockwise direction. If R is the resistance of the loop, then the induced current is given by

$$i = \frac{\varepsilon}{R}$$

$$i = \frac{Blv}{R} \quad (4.10)$$

Energy conservation

In order to move the loop with a constant velocity $\vec{v}$, a constant force that is equal and opposite to the magnetic force, must be applied. Therefore, mechanical work is done to move the loop. Then the rate of doing work or power is

$$P = \vec{F}\cdot\vec{v} = Fv\cos\theta$$

$$= Fv \qquad \text{Here } \theta = 0^\circ \quad (4.11)$$

Now, let us find the magnetic force acting on the loop due to its movement in the magnetic field. Let three deflecting forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ be acting on the three segments of the loop as shown in Figure 4.10. The general equation of such a deflecting force is given by

$$\vec{F}_d = i\,\vec{l}\times\vec{B}$$

Forces $\vec{F}_2$ *and* $\vec{F}_3$ are equal in magnitude and opposite in direction and cancel each other. Therefore, the force $\vec{F}_1$ alone acts on the left segment of the loop in a direction shown in Figure 4.10 and is given by

$$\vec{F}_1 = i\vec{l}\times\vec{B}$$

$$F = ilB\sin\theta \quad (4.12)$$

Here θ is the angle between $\vec{B}$ and the length vector $\vec{l}$ for the left segment and is 90°

$$\therefore F_1 = ilB\sin 90^\circ = ilB \qquad \text{since } \sin 90^\circ = 1$$

The applied force $\vec{F}$ must be equal to $\vec{F}_1$ in order to just move the loop with a constant velocity $\vec{v}$

$$\therefore \vec{F} = -\vec{F}_1$$

(since $\vec{F}$ and $\vec{F}_1$ are in opposite direction)

Considering only the magnitudes,

$$F = F_1 = ilB$$

Substituting for i from equation (4.10)

$$F = \left(\frac{Blv}{R}\right) lB$$

$$F = \frac{B^2 l^2 v}{R}$$

From equation (4.11), the rate at which the mechanical work is done to pull the loop from the magnetic field or power is given by

$$P = Fv = \left(\frac{B^2 l^2 v}{R}\right) v$$

$$P = \frac{B^2 l^2 v^2}{R} \qquad (4.13)$$

When the induced current flows in the loop, Joule heating takes place. The rate at which thermal energy is dissipated in the loop or power dissipated is

$$P = i^2 R$$

$$P = \left(\frac{Blv}{R}\right)^2 R$$

$$P = \frac{B^2 l^2 v^2}{R} \qquad (4.14)$$

This equation is exactly same as the equation (4.13). **Thus the mechanical work done in moving the loop appears as thermal energy in the loop**.

EXAMPLE 4.8

A conducting rod of length 0.5 m falls freely from the top of a building of height 7.2 m at a place in Chennai where the horizontal component of Earth's magnetic field is 40378.7 *nT*. If the length of the rod is perpendicular to Earth's horizontal magnetic field, find the emf induced across the conductor when the rod is about to touch the ground. [Take g = 10 m s^{-2}]

Solution

l = 0.5 m; h = 7.2 m; u = 0 m s^{-1};
g = 10 m s^{-2}; B_H = 40378.7 *nT*

The final velocity of the rod is

$$v^2 = u^2 + 2gh$$

$$= 0 + (2 \times 10 \times 7.2)$$

$$v^2 = 144$$

$$v = 12\, ms^{-1}$$

Induced emf when the rod is about to touch the ground, $\varepsilon = B_H l v$

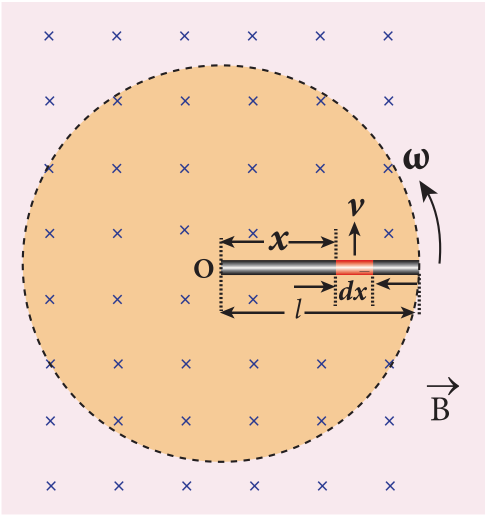
$$= 40{,}378.7 \times 10^{-9} \times 0.5 \times 12$$

$$= 242.27 \times 10^{-6}\, V$$

$$= 242.27\, \mu V$$

EXAMPLE 4.9

A copper rod of length l rotates about one of its ends with an angular velocity ω in a magnetic field B as shown in the figure. The plane of rotation is perpendicular to the field. Find the emf induced between the two ends of the rod.



Solution

Consider a small element of length dx at a distance x from the centre of the circle described by the rod. As this element moves perpendicular to the field with a linear velocity $v = x\omega$, the emf developed in the element dx is

$$d\varepsilon = Bvdx \qquad = B(x\omega)dx$$

This rod is made up of many such elements, moving perpendicular to the field. The emf developed across two ends is

$$\varepsilon = \int d\varepsilon = \int_0^l B\omega x dx = B\omega\left[\frac{x^2}{2}\right]_0^l$$

$$\varepsilon = \frac{1}{2}B\omega l^2$$

4.2 EDDY CURRENTS

According to Faraday's law of electromagnetic induction, an emf is induced in a conductor when the magnetic flux passing through it changes. However, the conductor need not be in the form of a wire or coil.

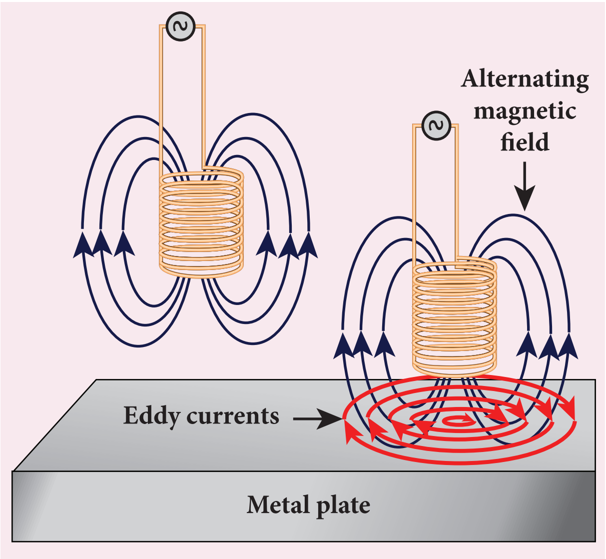


Figure 4.11 Eddy currents

Even for a conductor in the form of a sheet or plate, an emf is induced when magnetic flux linked with it changes. But the difference is that there is no definite loop or path for induced current to flow away. As a result, the induced currents flow in concentric circular paths (Figure 4.11). As these electric currents resemble eddies of water, these are known as Eddy currents. They are also called Foucault currents.

Demonstration

Here is a simple demonstration for the production of eddy currents. Consider a pendulum that can be made to oscillate between the poles of a powerful electromagnet as shown in Figure 4.12(a).

First the electromagnet is switched off, the pendulum is slightly displaced and released. It begins to oscillate and it executes a large number of oscillations before stopping. The air friction is the only damping force.

When the electromagnet is switched on and the disc of the pendulum is made to oscillate, eddy currents are produced in it which will oppose the oscillation. A heavy damping force of eddy currents will bring the pendulum to rest within a few oscillations (Figure 4.12(b)).

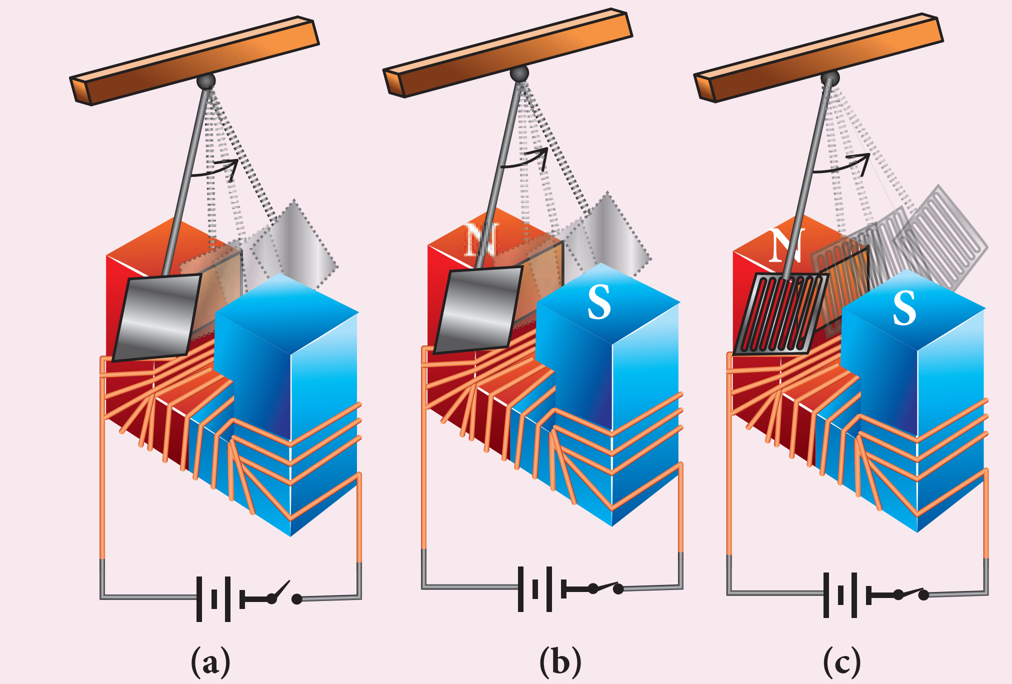


Figure 4.12 Demonstration of eddy currents

However if some slots are cut in the disc as shown in Figure 4.12(c), the eddy currents are reduced. The pendulum now will execute several oscillations before coming to rest. This clearly demonstrates the production of eddy current in the disc of the pendulum.

Drawbacks of Eddy currents

When eddy currents flow in the conductor, a large amount of energy is dissipated in the form of heat. The energy loss due to the flow of eddy current is inevitable but it can be reduced to a greater extent with suitable measures.

The design of transformer core and electric motor armature is crucial in order to minimise the eddy current loss. To reduce these losses, the core of the transformer is made up of thin laminas insulated from one another (Figure 4.13 (a)) while for electric motor the winding is made up of a group of wires insulated from one another (Figure 4.13 (b)). The insulation used does not allow huge eddy currents to flow and hence losses are minimized.

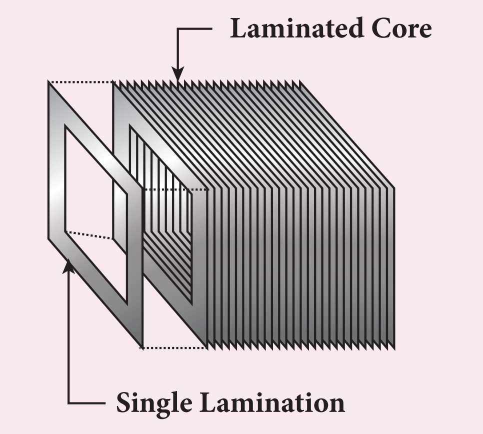


Figure 4.13 (a) Insulated laminas of the core of a transformer

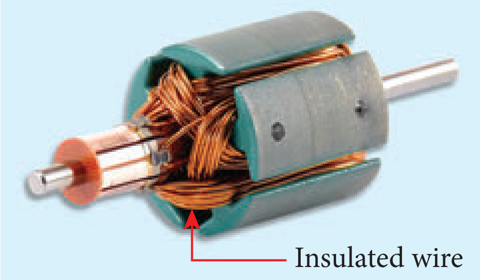
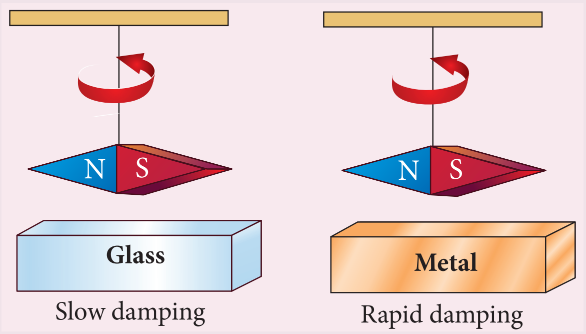


Figure 4.13 (b) Insulated winding of an electric motor

Activity



Make a pendulum with a strong magnet suspended at the lower end of the suspension wire as shown in the first figure. Make it oscillate with a glass plate below it and note the time it takes to come to rest.

Next just place a metallic plate below the oscillating magnet as shown in the second figure and again note the time it takes to stop.

In the second case, the magnet stops soon because eddy currents are produced in the plate which opposes the oscillation of the magnet.

Example

A spherical stone and a spherical metallic ball of same size and mass are dropped from the same height. Which one, a stone or a metal ball, will reach the earth's surface first? Justify your answer. Assume that there is no air friction.

Answer

The stone will reach the earth's surface earlier than the metal ball. The reason is that when the metal ball falls through the magnetic field of earth, the eddy currents are produced in it which opposes its motion. But in the case of stone, no eddy currents are produced and it falls freely.

Application of eddy currents

Though the production of eddy current is undesirable in some cases, it is useful in some other cases. A few of them are

i. Induction stove
ii. Eddy current brake
iii. Eddy current testing
iv. Electromagnetic damping

i. Induction stove

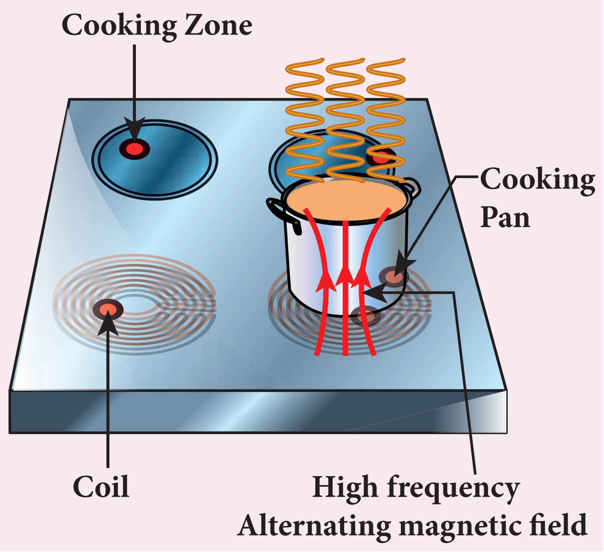


Figure 4.14 Induction stove

Induction stove is used to cook the food quickly and safely with less energy consumption. Below the cooking zone, there is a tightly wound coil of insulated wire. The cooking pan made of suitable material, is placed over the cooking zone. When the stove is switched on, an alternating current flowing in the coil produces high frequency alternating magnetic field which induces very strong eddy currents in the cooking pan. The eddy currents in the pan produce so much of heat due to Joule heating which is used to cook the food.

Note: The frequency of the domestic AC supply is increased from 50–60 Hz to around 20–40 KHz before giving it to the coil in order to produce high frequency alternating magnetic field.

ii. Eddy current brake

This eddy current braking system is generally used in high speed trains and roller coasters. Strong electromagnets are fixed just above the rails. To stop the train, electromagnets are switched on. The magnetic field of these magnets induces eddy currents in the rails which oppose or resist the movement of the train. This is Eddy current linear brake (Figure 4.15(a)).

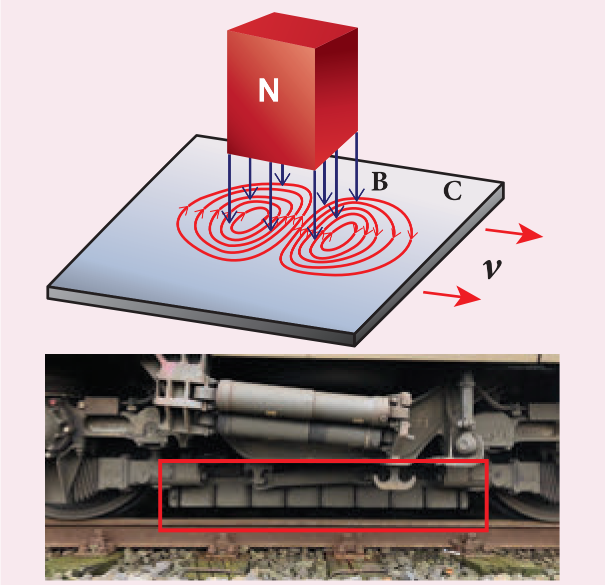


Figure 4.15 (a) Linear Eddy current brake

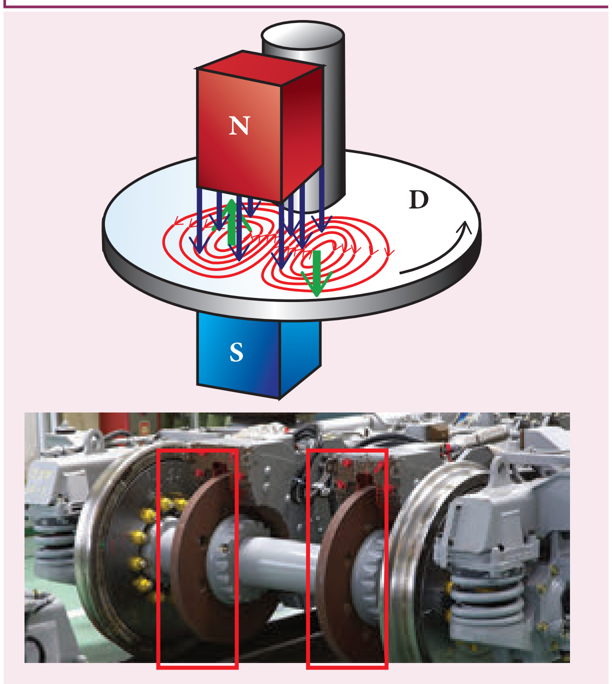


Figure 4.15(b) Circular Eddy current brake

In some cases, the circular disc, connected to the wheel of the train through a common shaft, is made to rotate in between the poles of an electromagnet. When there is a relative motion between the disc and the magnet, eddy currents are induced in the disc which stop the train. This is Eddy current circular brake (Figure 4.15(b))

iii. Eddy current testing

It is one of the simple non–destructive testing methods to find defects like surface cracks, air bubbles present in a specimen. A coil of insulated wire is given an alternating electric current so that it produces an alternating magnetic field. When this coil is brought near the test surface, eddy current is induced in the test surface. The presence of defects causes the change in phase and amplitude of the eddy current that can be detected by some other means.In this way, the defects present in the specimen are identified (Figure 4.16).

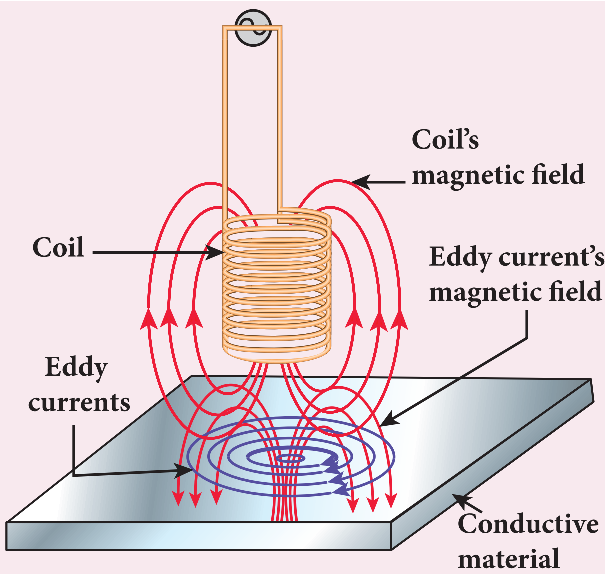


Figure 4.16 Eddy current testing

iv. Electro magnetic damping

The armature of the galvanometer coil is wound on a soft iron cylinder. Once the

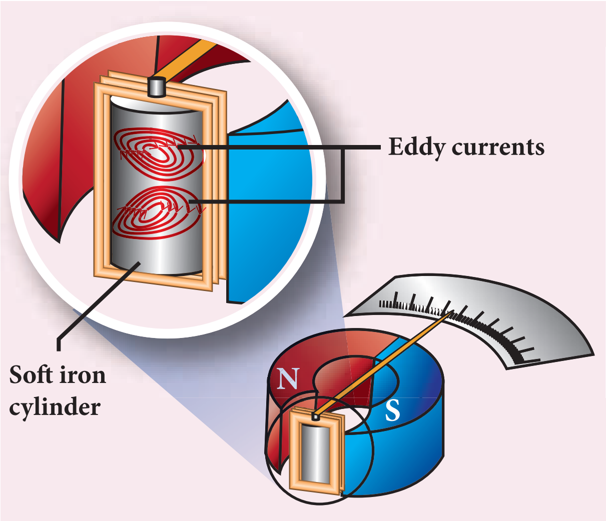


Figure 4.17 Electromagnetic damping

armature is deflected, the relative motion between the soft iron cylinder and the radial magnetic field induces eddy current in the cylinder (Figure 4.17). The damping force due to the flow of eddy current brings the armature to rest immediately and then galvanometer shows a steady deflection. This is called electromagnetic damping.

4.3 SELF–INDUCTION

4.3.1 Introduction

Inductor is a device used to store energy in a magnetic field when an electric current flows through it. The typical examples are coils, solenoids and toroids shown in Figure 4.18.

Inductance is the property of inductors to generate emf due to the change in current flowing through that circuit (self-induction) or a change in current through a neighbouring circuit with which it is magnetically linked (mutual induction). We will study about self-induction and mutual induction in the next sections.

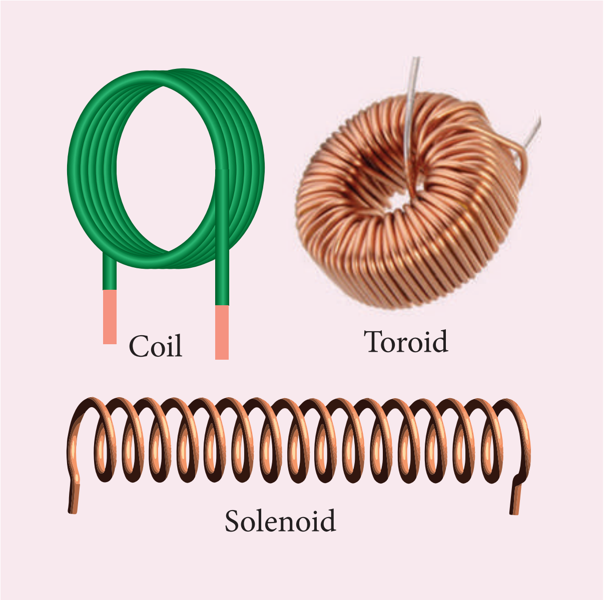


Figure 4.18 Examples for inductor

Self-induction

An electric current flowing through a coil will set up a magnetic field around it. Therefore, the magnetic flux of the magnetic field is linked with that coil itself. If this flux is changed by changing the current, an emf is induced in that same coil (Figure 4.19). This phenomenon is known as self-induction. The emf induced is called self-induced emf.

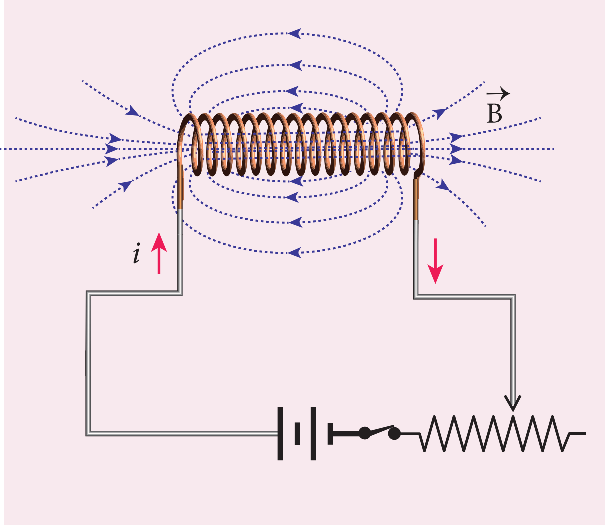


Figure 4.19 Self-Induction

Let Φ_B be the magnetic flux linked with each turn of the coil of N turns, then the total flux linked with the coil $N\Phi_B$ (flux linkage) is proportional to the current i in the coil.

$$N\Phi_B \propto i$$

$$N\Phi_B = Li \qquad (4.15)$$

(or) $$L = \frac{N\Phi_B}{i}$$

The constant of proportionality L is called self-inductance of the coil. It is also referred to as the coefficient of self-induction. If $i = 1A$, then $L = N\Phi_B$. **Self-inductance or simply inductance of a coil is defined as the flux linkage of the coil when 1A current flows through it.**

When the current i changes with time, an emf is induced in it. From Faraday's law of electromagnetic induction, this self-induced emf is given by

$$\varepsilon = -\frac{d(N\Phi_B)}{dt} = -\frac{d(Li)}{dt}$$

(using equation 4.15)

$$\therefore \varepsilon = -L\frac{di}{dt} \qquad (4.16)$$

(or) $$L = \frac{-\varepsilon}{di/dt}$$

The negative sign in the above equation means that the self-induced emf always opposes the change in current with respect to time. If $di/dt = 1\,A\,s^{-1}$, then $L = -\varepsilon$. **Inductance of a coil is also defined as the opposing emf induced in the coil when the rate of change of current through the coil is 1 $A\,s^{-1}$.**

Unit of inductance

Inductance is a scalar and its unit is $Wb\,A^{-1}$ or $V\,s\,A^{-1}$. It is also measured in henry (H).

$1\,H = 1\,Wb\,A^{-1} = 1\,V\,s\,A^{-1}$

The dimensional formula of inductance is $M\,L^2\,T^{-2}A^{-2}$.

If i = 1 A and $N\Phi_B = 1\,Wb\ turns$, then $L = 1H$.

Therefore, **the inductance of the coil is said to be one henry if a current of 1 A produces unit flux linkage in the coil.**

If $di/dt = 1\,A\,s^{-1}$ and $\varepsilon = -1\,V$, then $L = 1H$.

Therefore, **the inductance of the coil is one henry if a current changing at the rate of 1 $A\,s^{-1}$ induces an opposing emf of 1 V in it.**

Physical significance of inductance

We have learnt about inertia in XI standard. In translational motion, mass is a measure of inertia; in the same way, for rotational motion, moment of inertia is a measure of rotational inertia (Refer sections 3.2.1 and 5.4 of XI physics text book). Generally, inertia means opposition to change its state.

The inductance plays the same role in a circuit as mass and moment of inertia play in mechanical motion. When a circuit is switched on, the increasing current induces an emf which opposes the growth of current in a circuit (Figure 4.20(a)). Likewise, when circuit is broken, the decreasing current induces an emf in the reverse direction. This emf now opposes the decay of current (Figure 4.20(b)).

Thus, inductance of the coil opposes any change in current and tries to maintain the original state.

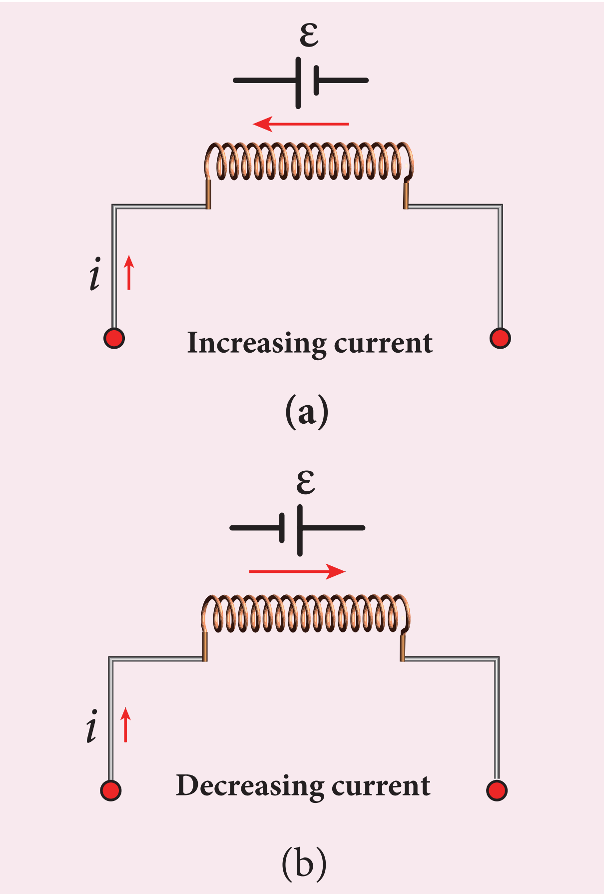


Figure 4.20 Induced emf ε opposes the changing current i

4.3.2 Self-inductance of a long solenoid

Consider a long solenoid of length l and cross-sectional area A. Let n be the number of turns per unit length (or turn density) of the solenoid. When an electric current i is passed through the solenoid, a magnetic field is produced by it which is almost uniform and is directed along the axis of the solenoid as shown in Figure 4.21. The magnetic field at any point inside the solenoid is given by (Refer section 3.9.3)

$$B = \mu_\circ n i$$

As this magnetic field passes through the solenoid, the windings of the solenoid are linked by the field lines. The magnetic flux passing through each turn is

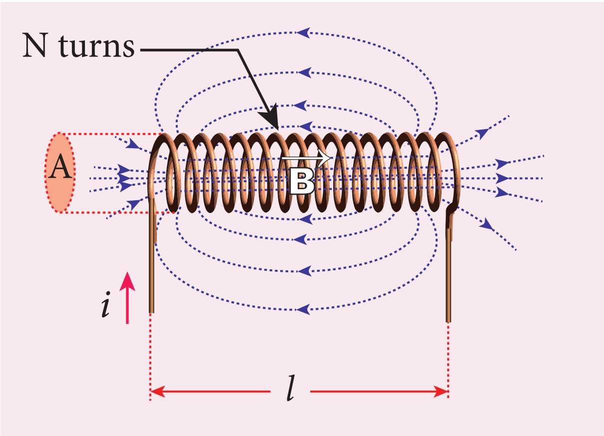


Figure 4.21 Self-inductance of a long solenoid

$$\Phi_B = \int_A \vec{B}.d\vec{A} = BA\cos\theta = BA \text{ since } \theta = 0°$$

$$= (\mu_0 n i)A$$

The total magnetic flux linked or flux linkage of the solenoid with N turns (the total number of turns N is given by $N = n\,l$) is

$$N\Phi_B = (nl)(\mu_0 n i)A$$

$$N\Phi_B = (\mu_\circ n^2 A l)\, i \qquad (4.17)$$

The equation (4.15) is

$$N\Phi_B = Li$$

Comparing equations (4.15) and (4.17), we have

$$L = \mu_\circ n^2 A l$$

From the above equation, it is clear that inductance depends on the geometry of the solenoid (turn density n, cross-sectional area A, length l) and the medium present inside the solenoid. If the solenoid is filled with a dielectric medium of relative permeability μ_r, then

$$L = \mu n^2 A l \;\; or \;\; L = \mu_\circ \mu_r n^2 A l$$

Energy stored in an inductor

Whenever a current is established in the circuit, the inductance opposes the growth of the current. In order to establish a current in the circuit, work is done against this opposition by some external agency. This work done is stored as magnetic potential energy.

Let us assume that electrical resistance of the inductor is negligible and inductor effect alone is considered. The induced emf ε at any instant t is

$$\varepsilon = -L\frac{di}{dt}$$

Let dW be work done in moving a charge dq in a time dt against the opposition, then

$$dW = -\varepsilon dq$$

$$= -\varepsilon i dt \qquad \because dq = idt$$

Substituting for ε from equation (4.16),

$$= -\left(-L\frac{di}{di}\right) idt$$

$$dW = Lidi$$

Total work done in establishing the current i is

$$W = \int dW = \int_0^i Lidi = L\left[\frac{i^2}{2}\right]_o^i$$

$$W = \frac{1}{2}Li^2$$

This work done is stored as magnetic potential energy.

$$\therefore U_B = \frac{1}{2}Li^2 \qquad (4.18)$$

The energy density is the energy stored per unit volume of the space and is given by

$$u_B = \frac{U_B}{Al} \qquad \because \text{Volume of the solenoid} = Al$$

$$u_B = \frac{Li^2}{2Al} = \frac{(\mu_0 n^2 Al)i^2}{2Al} \qquad \because L = \mu_0 n^2 Al$$

$$= \frac{\mu_0 n^2 i^2}{2}$$

$$u_B = \frac{B^2}{2\mu_0} \qquad \because B = \mu_0 ni$$

EXAMPLE 4.10

A solenoid of 500 turns is wound on an iron core of relative permeability 800. The length and radius of the solenoid are 40 cm and 3 cm respectively. Calculate the average emf induced in the solenoid if the current in it changes from 0 to 3 A in 0.4 second.

Solution

N = 500 turns; $\mu_r = 800$;
l = 40 cm = 0.4 m; r = 3 cm = 0.03 m;
di = 3 – 0 = 3 A; dt = 0.4 s

Self inductance,

$$L = \mu n^2 Al \quad \left(\because \mu = \mu_o\mu_r; A = \pi r^2; n = \frac{N}{l}\right)$$

$$= \frac{\mu_o\mu_r N^2 \pi r^2}{l}$$

$$= \frac{4\times 3.14\times 10^{-7}\times 800\times 500^2\times 3.14\times\left(3\times 10^{-2}\right)^2}{0.4}$$

L = 1.77 H

Magnitude of induced emf, $\varepsilon = L\frac{di}{dt}$

$$= \frac{1.77\times 3}{0.4}$$

$$\varepsilon = 13.275V$$

EXAMPLE 4.11

The self-inductance of an air-core solenoid is 4.8 mH. If its core is replaced by iron core, then its self-inductance becomes 1.8 H. Find out the relative permeability of iron.

Solution

$$L_{air} = 4.8 \times 10^{-3} H$$

$$L_{iron} = 1.8 H$$

$$L_{air} = \mu_{\circ} n^2 Al = 4.8 \times 10^{-3} H$$

$$L_{iron} = \mu n^2 Al = \mu_{\circ} \mu_r n^2 Al = 1.8 H$$

$$\therefore \mu_r = \frac{L_{iron}}{L_{air}} = \frac{1.8}{4.8 \times 10^{-3}} = 375$$

4.3.3 Mutual induction

When an electric current passing through a coil changes with time, an emf is induced in the neighbouring coil. This phenomenon is known as mutual induction and the emf is called mutually induced emf.

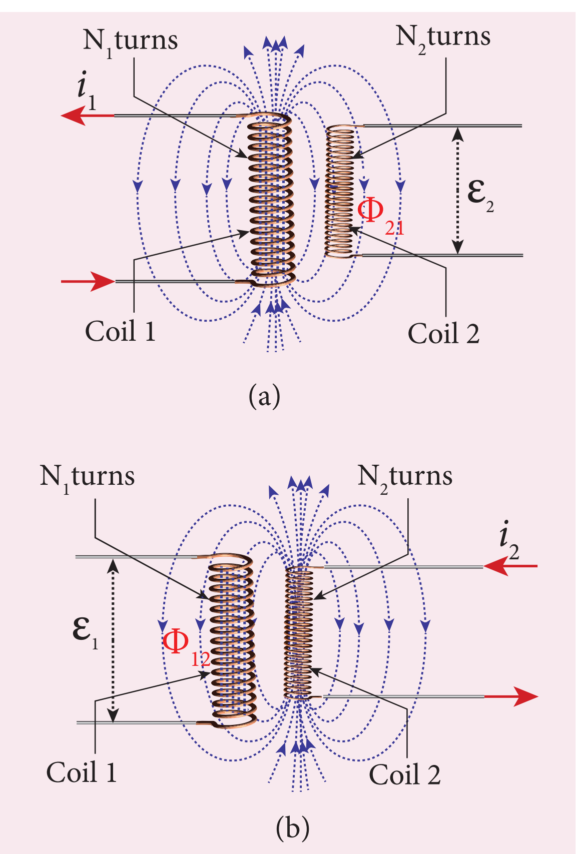


Figure 4.22 Mutual induction

Consider two coils which are placed close to each other. If an electric current i_1 is sent through coil 1, the magnetic field produced by it is also linked with coil 2 as shown in Figure 4.22(a).

Let Φ_{21} be the magnetic flux linked with each turn of the coil 2 of N_2 turns due to coil 1, then the total flux linked with coil 2 $(N_2\Phi_{21})$ is proportional to the current i_1 in the coil 1.

$$N_2 \Phi_{21} \propto i_1$$

$$N_2\Phi_{21} = M_{21}\, i_1 \qquad (4.19)$$

$$\text{(or)} \;\; M_{21} = \frac{N_2\Phi_{21}}{i_1}$$

The constant of proportionality M_{21} is the mutual inductance of the coil 2 with respect to coil 1. It is also called as coefficient of mutual induction. If $i_1 = 1A$, then $M_{21} = N_2\Phi_{21}$. Therefore, **the mutual inductance M_{21} is defined as the flux linkage of the coil 2 when 1A current flows through coil 1.**

When the current i_1 changes with time, an emf ε_2 is induced in coil 2. From Faraday's law of electromagnetic induction, this mutually induced emf ε_2 is given by

$$\varepsilon_2 = -\frac{d(N_2\Phi_{21})}{dt} = -\frac{d(M_{21} i_1)}{dt}$$

$$\varepsilon_2 = -M_{21}\frac{di_1}{dt}$$

$$\text{(or)} \;\; M_{21} = \frac{-\varepsilon_2}{di_1/dt}$$

The negative sign in the above equation shows that the mutually induced emf always opposes the change in current i_1 with respect to time. If $di_1/dt = 1A\,s^{-1}$, then $M_{21} = -\varepsilon_2$.

Mutual inductance M_{21} is also defined as the opposing emf induced in the coil 2 when the rate of change of current through the coil 1 is $1As^{-1}$.

Similarly, if an electric current i_2 through coil 2 changes with time, then emf ε_1 is induced in coil 1. Therefore,

$$M_{12} = \frac{N_1\Phi_{12}}{i_2} \quad \text{and} \quad M_{12} = \frac{-\varepsilon_1}{di_2/dt}$$

where M_{12} is the mutual inductance of the coil 1 with respect to coil 2. It can be shown that for a given pair of coils, the mutual inductance is same.

$$\text{i.e.,} \qquad M_{21} = M_{12} = M$$

In general, the mutual induction between two coils depends on size, shape, the number of turns of the coils, their relative orientation and permeability of the medium.

Unit of mutual-inductance

The unit of mutual inductance is also henry (H).

If $i_1 = 1A$ and $N_2\Phi_{21} = 1\ Wb\ turns$, then $M_{21} = 1H$.

Therefore, **the mutual inductance between two coils is said to be one henry if a current of 1*A* in coil 1 produces unit flux linkage in coil 2.**

If $\frac{di_1}{dt} = 1As^{-1}$ and $\varepsilon_2 = -1\ V$, then $M_{21} = 1H$.

Therefore, **the mutual inductance between two coils is one henry if a current changing at the rate of $1As^{-1}$ in coil 1 induces an opposing emf of 1*V* in coil 2.**

4.3.4 Mutual inductance between two long co-axial solenoids

Consider two long co-axial solenoids of same length *l*. The length of these solenoids is large when compared to their radii so that the magnetic field produced inside the solenoids is uniform and the fringing effect at the ends may be ignored. Let A_1 and A_2 be the area of cross section of the solenoids with A_1 being greater than A_2 as shown in Figure 4.23. The turn density of these solenoids are n_1 and n_2 respectively.

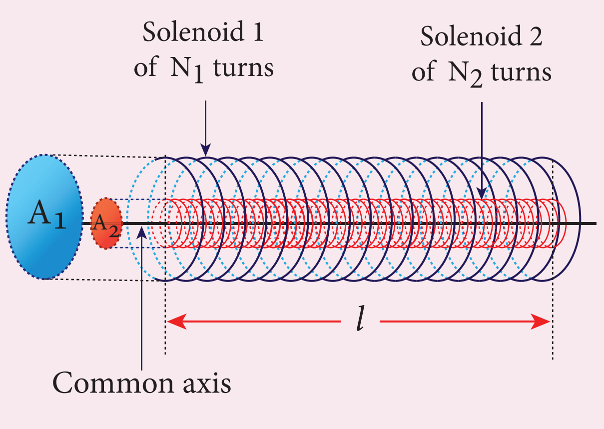


Figure 4.23 Mutual inductance of two long co-axial solenoids

Let i_1 be the current flowing through solenoid 1, then the magnetic field produced inside it is

$$B_1 = \mu_\circ n_1 i_1$$

As the field lines of $\vec{B_1}$ are passing through the area bounded by solenoid 2, the magnetic flux is linked with each turn of solenoid 2 due to solenoid 1 and is given by

$$\Phi_{21} = \int_{A_2} \vec{B_1}.d\vec{A} = B_1 A_2 \qquad \text{since } \theta = 0°$$

$$= (\mu_\circ n_1 i_1) A_2$$

The flux linkage of solenoid 2 with total turns N_2 is

$$N_2\Phi_{21} = (n_2 l)(\mu_\circ n_1 i_1)A_2 \quad \text{since } N_2 = n_2 l$$

$$N_2\Phi_{21} = (\mu_\circ n_1 n_2 A_2 l) i_1 \qquad (4.20)$$

From equation (4.19),

$$N_2\Phi_{21} = M_{21} i_1 \qquad (4.21)$$

Comparing the equations (4.20) and (4.21),

$$M_{21} = \mu_\circ n_1 n_2 A_2 l \qquad (4.22)$$

This gives the expression for mutual inductance M_{21} of the solenoid 2 with respect to solenoid 1. Similarly, we can find mutual inductance M_{12} of solenoid 1 with respect to solenoid 2 as given below.

The magnetic field produced by the solenoid 2 when carrying a current i_2 is

$$B_2 = \mu_\circ n_2 i_2$$

This magnetic field B_2 is uniform inside the solenoid 2 but outside the solenoid 2, it is almost zero. Therefore for solenoid 1, the area A_2 is the effective area over which the magnetic field B_2 is present; not area A_1. Then the magnetic flux Φ_{12} linked with each turn of solenoid 1 due to solenoid 2 is

$$\Phi_{12} = \int_{A_2} \vec{B}_2 \cdot d\vec{A} = B_2 A_2 = (\mu_\circ n_2 i_2) A_2$$

The flux linkage of solenoid 1 with total turns N_1 is

$$N_1\Phi_{12} = (n_1 l)(\mu_o n_2 i_2)A_2 \quad \text{since } N_1 = n_1 l$$

$$N_1\Phi_{12} = (\mu_o n_1 n_2 A_2 l) i_2$$

$$\text{since } N_1\Phi_{12} = M_{12} i_2$$

$$M_{12} i_2 = (\mu_\circ n_1 n_2 A_2 l) i_2$$

Therefore, we get

$$\therefore M_{12} = \mu_\circ n_1 n_2 A_2 l \qquad (4.23)$$

From equation (4.22) and (4.23), we can write

$$M_{12} = M_{21} = M \qquad (4.24)$$

In general, the mutual inductance between two long co-axial solenoids is given by

$$M = \mu_\circ n_1 n_2 A_2 l \qquad (4.25)$$

If a dielectric medium of relative permeability μ_r is present inside the solenoids, then

$$M = \mu n_1 n_2 A_2 l$$

$$\text{(or)} \quad M = \mu_\circ \mu_r n_1 n_2 A_2 l$$

EXAMPLE 4.12

The current flowing in the first coil changes from 2 A to 10 A in 0.4 sec. Find the mutual inductance between two coils if an emf of 60 mV is induced in the second coil. Also determine the induced emf in the second coil if the current in the first coil is changed from 4 A to 16 A in 0.03 sec. Consider only the magnitude of induced emf.

Solution

Case (i):

$di_1 = 10 - 2 = 8$ A; $dt = 0.4$ s;

$\varepsilon_2 = 60 \times 10^{-3} V$

Case (ii):

$di_1 = 16 - 4 = 12$ A; $dt = 0.03$ s

(i) Mutual inductance of the second coil with respect to the first coil

$$M_{21} = \frac{\varepsilon_2}{di_1/dt}$$

$$= \frac{60\times10^{-3}\times0.4}{8}$$

$$M_{21} = 3\times10^{-3} H$$

(ii) Induced emf in the second coil due to the rate of change of current in the first coil is

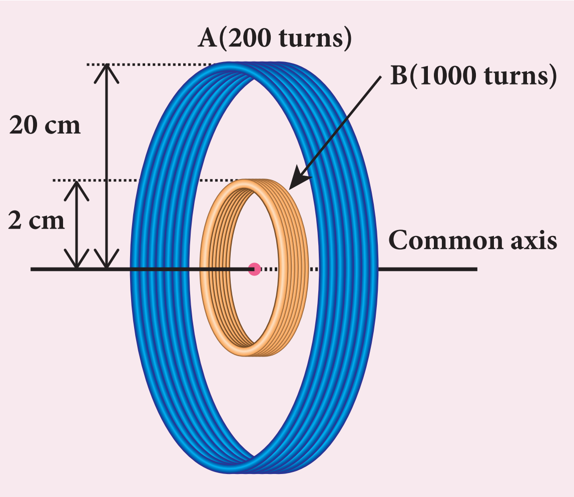
$$\varepsilon_2 = M_{21}\frac{di_1}{dt}$$

$$= \frac{3\times10^{-3}\times12}{0.03}$$

$$\varepsilon_2 = 1.2 V$$

EXAMPLE 4.13

Consider two coplanar, co-axial circular coils *A* and *B* as shown in figure. The radius of coil *A* is 20 cm while that of coil *B* is 2 cm. The number of turns is 200 and 1000 for coils *A* and *B* respectively. Calculate the mutual inductance of coil *B* with respect to coil *A*. If the current in coil *A* changes from 2 A to 6 A in 0.04 sec, determine the induced emf in coil *B* and the rate of change of flux through the coil *B* at that instant.



Solution

N_A = 200 turns; N_B = 1000 turns;
$r_A = 20 \times 10^{-2}$ m; $r_B = 2 \times 10^{-2}$ m;
dt = 0.04 s; di_A = 6−2 = 4A

Let i_A be the current flowing in coil *A*, then the magnetic field B_A at the centre of the circular coil *A* is

$$B_A = \frac{\mu_\circ N_A i_A}{2r_A} = \frac{4\pi\times10^{-7} N_A i_A}{2r_A}$$

$$= \frac{10^{-7}\times2\times3.14\times200}{20\times10^{-2}}\times i_A$$

$$= 6.28\times10^{-4} i_A \ Wbm^{-2}$$

The magnetic flux linkage of coil *B* is

$$N_B\Phi_B = N_B B_A A_B$$

$$= 1000\times6.28\times10^{-4}\times i_A\times3.14\times\left(2\times10^{-2}\right)^2$$

$$= 7.89\times10^{-4} i_A \quad Wb\ turns$$

The mutual inductance of the coil *B* with respect to coil *A* is

$$M_{BA} = \frac{N_B\Phi_B}{i_A} = 7.89\times10^{-4} H$$

Induced emf in coil *B* is

$$\varepsilon_B = -M_{BA}\frac{di_A}{dt}$$

Considering only the magnitude,

$$\varepsilon_B = \frac{7.89\times10^{-4}\times(6-2)}{0.04}$$

$$\varepsilon_B = \frac{7.89\times10^{-4}\times(4)}{4\times10^{-2}}$$

$$\varepsilon_B = 7.89\times10^{-2} V$$

$$\varepsilon_B = 78.9 mV$$

The rate of change of magnetic flux of coil *B* is

$$\frac{d(N_B\Phi_B)}{dt} = \varepsilon_B = 78.9\ mWbs^{-1}$$

4.4 METHODS OF PRODUCING INDUCED EMF

4.4.1 Introduction

Electromotive force is the characteristic of any energy source capable of driving electric charge around a circuit. We have already learnt that it is not actually a force. It is the work done in moving unit electric charge around the circuit. It is measured in $J\,C^{-1}$ or volt.

Some examples of energy source which provide emf are electrochemical cells, thermoelectric devices, solar cells and electrical generators. Of these, electrical generators are most powerful machines. They are used for large scale power generation.

According to Faraday's law of electromagnetic induction, an emf is induced in a circuit when magnetic flux linked with it changes. This emf is called induced emf. The magnitude of the induced emf is given by

$$\varepsilon = \frac{d\Phi_B}{dt} \quad or$$

$$\varepsilon = \frac{d}{dt}(BA\cos\theta) \qquad (4.26)$$

From the above equation, it is clear that induced emf can be produced by changing magnetic flux in any of the following ways.

(i) By changing the magnetic field B

(ii) By changing the area A of the coil and

(iii) By changing the relative orientation θ of the coil with magnetic field

4.4.2 Induction of emf by changing the magnetic field

From Faraday's experiments on electromagnetic induction, it was discovered that an emf is induced in a circuit on changing the magnetic flux of the field through it. The change in flux is brought about by (i) relative motion between the circuit and the magnet (First experiment) (ii) variation in current flowing through the nearby coil (Second experiment).

4.4.3 Induction of emf by changing the area of the coil

Consider a conducting rod of length l moving with a velocity v towards left on a rectangular metallic framework as shown in Figure 4.24. The whole arrangement is placed in a uniform magnetic field $\vec{B}$ whose magnetic lines are perpendicularly directed into the plane of the paper.

As the rod moves from AB to DC in a time dt, the area enclosed by the loop and hence the magnetic flux through the loop decreases.

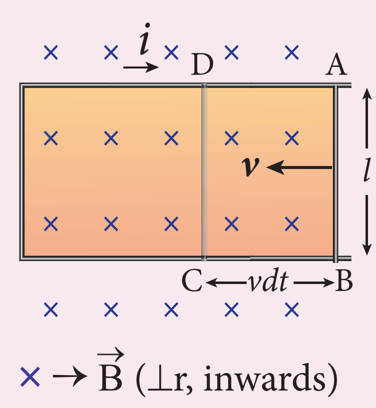


Figure 4.24 Induction of emf by changing the area enclosed by the loop

The change in magnetic flux in time dt is

$$d\Phi_B = B\times \text{ change in area}$$
$$= B\times \text{ Area ABCD}$$
$$= Blvdt \qquad \text{since Area ABCD} = l(vdt)$$
$$\text{(or) } \frac{d\Phi_B}{dt} = Blv$$

As a result of change in flux, an emf is generated in the loop. The magnitude of the induced emf is

$$\varepsilon = \frac{d\Phi_B}{dt}$$
$$\varepsilon = Blv \qquad (4.27)$$

This emf is called **motional emf**. The direction of induced current is found to be clockwise from Fleming's right hand rule.

EXAMPLE 4.14

A circular metal of area 0.03 m^2 rotates in a uniform magnetic field of 0.4 T. The axis of rotation passes through the centre and perpendicular to its plane and is also parallel to the field. If the disc completes 20 revolutions in one second and the resistance of the disc is 4 Ω, calculate the induced emf between the axis and the rim and induced current flowing in the disc.

Solution

$A = 0.03\ m^2$; $B = 0.4\ T$; $f = 20$ rps;
$R = 4\ \Omega$

Area covered in 1 sec = Area of the disc × frequency

$= 0.03 \times 20$
$= 0.6\ m^2$

Induced emf, ε= Rate of change of flux

$$= \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt}$$
$$\varepsilon = \frac{0.4\times 0.6}{1}$$
$$\varepsilon = 0.24\ V$$

Induced current, $i = \frac{\varepsilon}{R} = \frac{0.24}{4} = 0.06\,A$

Note Emf can be induced by changing relative orientation between the coil and the magnetic field. This can be achieved either by rotating a coil in a magnetic field or by rotating a magnetic field within a stationary coil. Here rotating coil type is considered.

4.4.4 Induction of emf by changing relative orientation of the coil with the magnetic field

Consider a rectangular coil of N turns kept in a uniform magnetic field $\vec{B}$ as shown in Figure 4.25(a). The coil rotates in anti-clockwise direction with an angular velocity ω about an axis, perpendicular to the field.

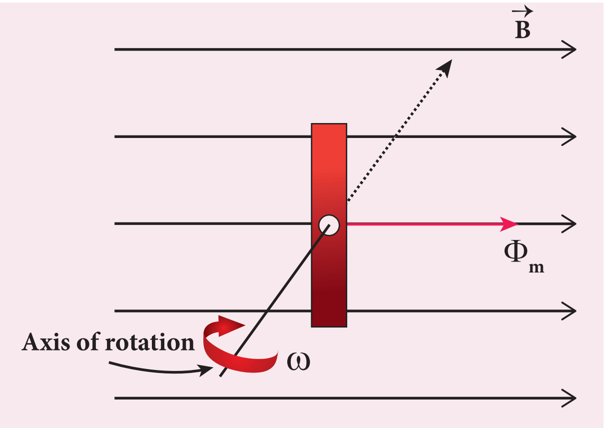


Figure 4.25(a) Top view of the coil with its plane perpendicular to the magnetic field

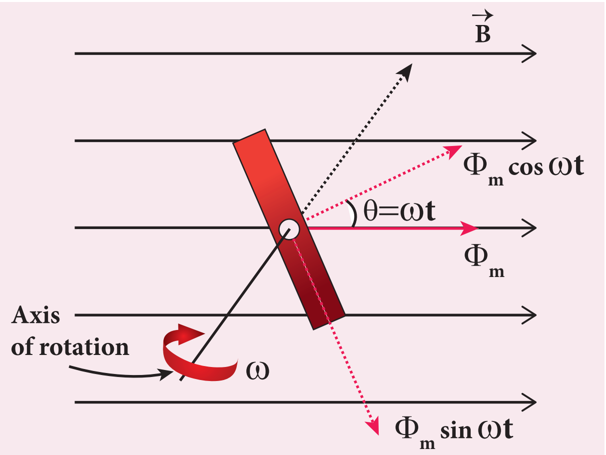


Figure 4.25(b) The coil has rotated through an angle $\theta = \omega t$

At time = 0, the plane of the coil is perpendicular to the field and the flux linked with the coil has its maximum value $\Phi_m = BA$ (where A is the area of the coil).

In a time t seconds, the coil is rotated through an angle θ (= ωt) in anti–clockwise direction. In this position, the flux linked is $\Phi_m \cos\omega t$, a component of Φ_m normal to the plane of the coil (Figure 4.25(b)). The component parallel to the plane ($\Phi_m \sin\omega t$) has no role in electromagnetic induction. Therefore, the flux linkage at this deflected position is

$$N\Phi_B = N\Phi_m \cos\omega t$$

According to Faraday's law, the emf induced at that instant is

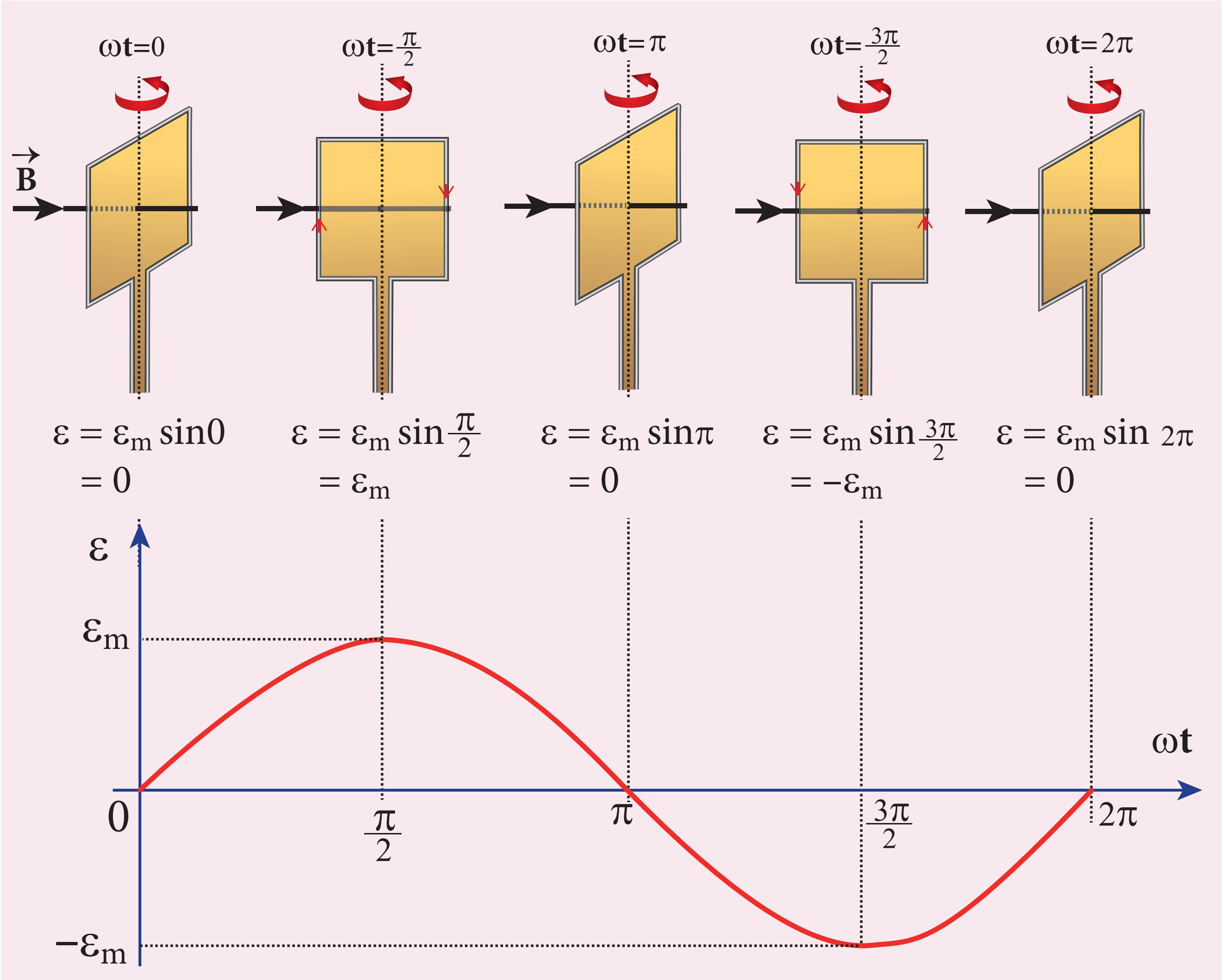


Figure 4.26 Variation of induced emf as a function of ωt

$$\varepsilon = -\frac{d}{dt}(N\Phi_B) = -\frac{d}{dt}(N\Phi_m \cos\omega t)$$

$$= -N\Phi_m(-\sin\omega t)\omega$$

$$= N\Phi_m \omega \sin\omega t$$

When the coil is rotated through 90° from initial position, sin ωt = 1. Then the maximum value of induced emf is

$$\varepsilon_m = N\Phi_m \omega$$

$$\varepsilon_m = NBA\omega \qquad \text{since } \Phi_m = BA$$

Therefore, the value of induced emf at that instant is then given by

$$\varepsilon = \varepsilon_m \sin\omega t \qquad (4.28)$$

It is seen that the induced emf varies as sine function of the time angle *ωt*. The graph between induced emf and time angle for one rotation of coil will be a sine curve (Figure 4.26) and the emf varying in this manner is called sinusoidal emf or alternating emf.

If this alternating voltage is given to a closed circuit, a sinusoidally varying current flows in it. This current is called alternating current and is given by

$$i = I_m \sin\omega t \qquad (4.29)$$

where I_m is the maximum value of induced current.

EXAMPLE 4.15

A rectangular coil of area 70 cm² having 600 turns rotates about an axis perpendicular to a magnetic field of 0.4 Wb m^{-2}. If the coil completes 500 revolutions in a minute, calculate the instantaneous emf when the plane of the coil is (i) perpendicular to the field (ii) parallel to the field and (iii) inclined at 60° with the field.

Solution

$A = 70 \times 10^{-4}\text{m}^2$; $N = 600$ turns

$B = 0.4$ Wbm^{-2}; $f = 500$ rpm

The instantaneous emf is

$$\varepsilon = \varepsilon_m \sin\omega t$$

$$\text{since } \varepsilon_m = N\Phi_m\omega = N(BA)(2\pi f)$$

$$\varepsilon = NBA \times 2\pi f \times \sin\omega t$$

(i) When $\omega t = 0^\circ$,

$$\varepsilon = \varepsilon_m \sin 0 = 0$$

(ii) When $\omega t = 90^\circ$,

$$\varepsilon = \varepsilon_m \sin 90^\circ = NBA \times 2\pi f \times 1$$

$$= 600 \times 0.4 \times 70 \times 10^{-4} \times 2 \times \frac{22}{7} \times \left(\frac{500}{60}\right)$$

$$= 88V$$

(iii) When ωt = 90° – 60° = 30°,

$$\varepsilon = \varepsilon_m \sin 30^\circ = 88 \times \frac{1}{2} = 44\ V$$

4.5 AC GENERATOR

4.5.1 Introduction

AC generator or alternator is an energy conversion device. It converts mechanical energy used to rotate the coil or field magnet into electrical energy. Alternator produces a large scale electrical power for use in homes and industries. AC generator and its components are shown in Figure 4.27.

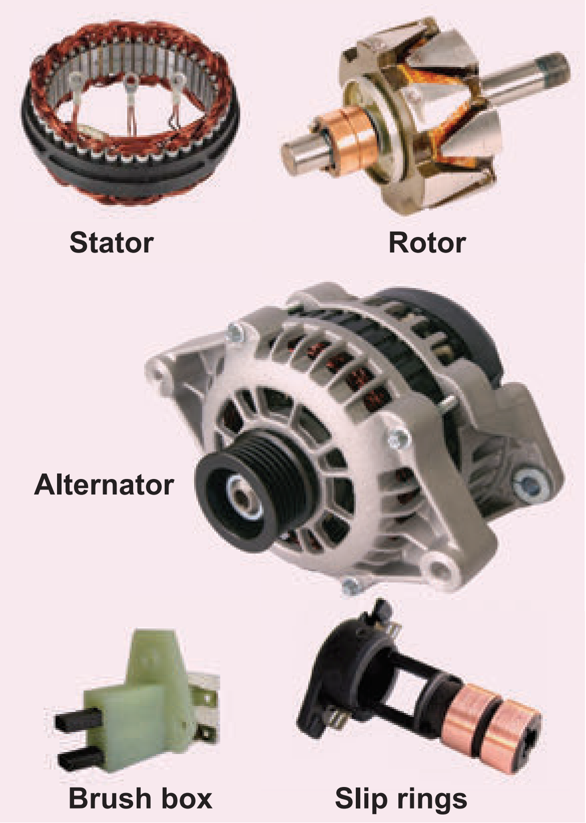


Figure 4.27 AC generator and its components

4.5.2 Principle

Alternators work on the principle of electromagnetic induction. The relative motion between a conductor and a magnetic field changes the magnetic flux linked with the conductor which in turn, induces an emf. The magnitude of the induced emf is given by Faraday's law of electromagnetic induction and its direction by Fleming's right hand rule.

> **Note** Alternating emf is generated by rotating a coil in a magnetic field or by rotating a magnetic field within a stationary coil.
> The first method is used for small AC generators while the second method is employed for large AC generators. The rotating-field method is the one which is mostly used in power stations.

4.5.3 Construction

Alternator consists of two major parts, namely stator and rotor. As their names suggest, stator is stationary while rotor rotates inside the stator. In any standard construction of commercial alternators, the armature winding is mounted on stator and the field magnet on rotor.

The construction details of stator, rotor and various other components involved in them are given below.

i) Stator

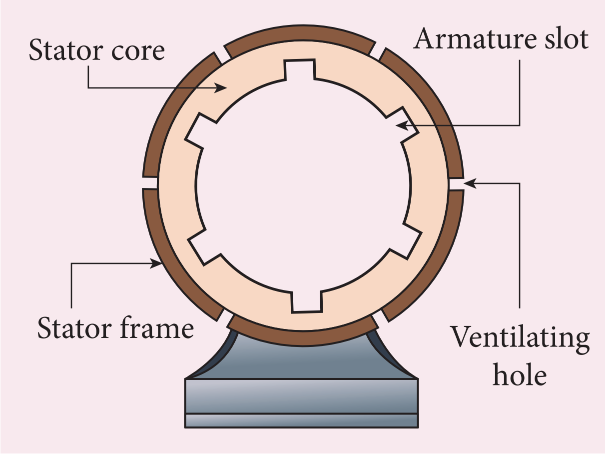


Figure 4.28 Stator and its components

The stationary part which has armature windings mounted in it is called stator. It has three components, namely stator frame, stator core and armature winding.

Stator frame

This is the outer frame used for holding stator core and armature windings in proper position. Stator frame provides best ventilation with the help of holes provided in the frame itself (Figure 4.28).

Stator core

Stator core or armature core is made up of iron or steel alloy. It is a hollow cylinder and is laminated to minimize eddy current loss. The slots are cut on inner surface of the core to accommodate armature windings.

Armature winding

Armature winding is the coil, wound on slots provided in the armature core. One or more than one coil may be employed, depending on the type of alternator.

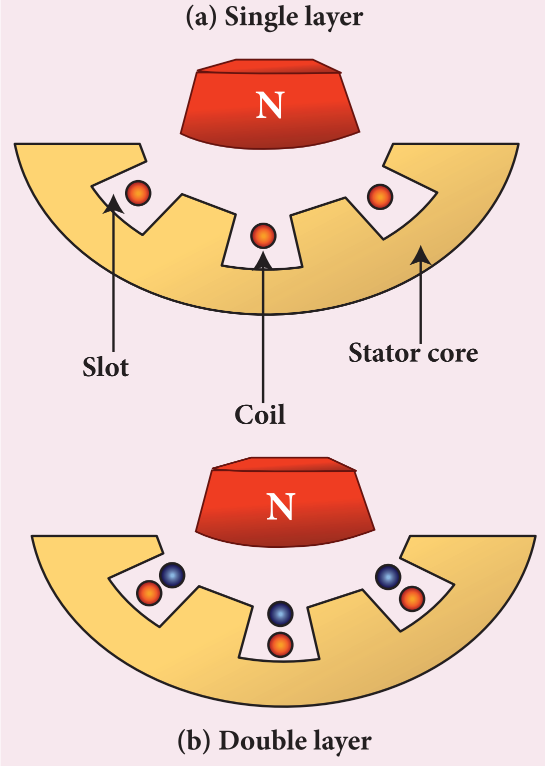


Figure 4.29 Armature windings

Two types of windings are commonly used. They are i) single-layer winding and ii) double-layer winding. In single-layer winding, a slot is occupied by a coil as a single layer. But in double-layer winding, the coils are split into two layers such as top and bottom layers (Figure 4.29).

ii) Rotor

Rotor contains magnetic field windings. The magnetic poles are magnetized by DC source. The ends of field windings are connected to a pair of slip rings, attached to a common shaft about which rotor rotates. Slip rings rotate along with rotor. To maintain connection between the DC source and field windings, two brushes are used which continuously slide over the slip rings.

There are 2 types of rotors used in alternators i) salient pole rotor and ii) cylindrical pole rotor.

MN5C7X

Salient pole rotor

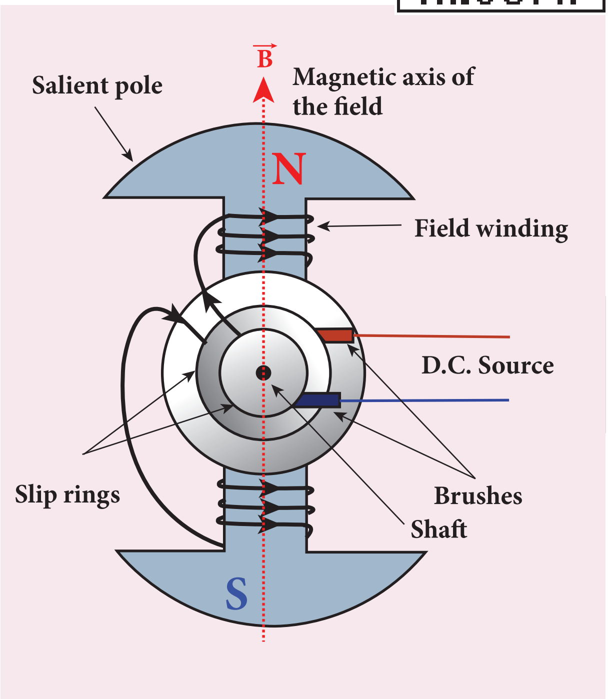


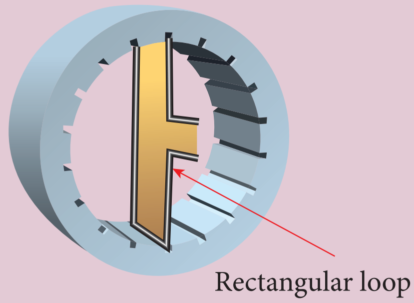
Figure 4.30 Salient 2-pole rotor

Construction of AC generator (Not for examination)

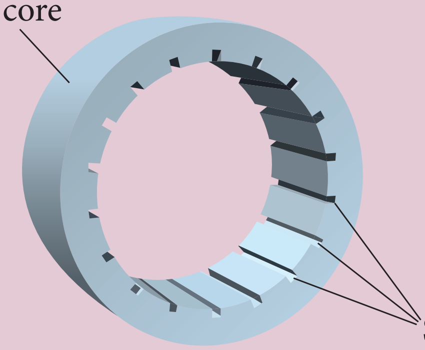
Alternator consists of two major parts, namely stator and rotor. (This box is given for better understanding of constructional details)

i) Stator

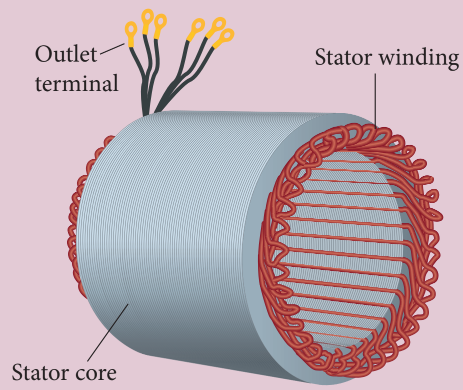
Stator has three components, namely stator frame, stator core and armature winding.



Figure(b): Stator core with rectangular loop



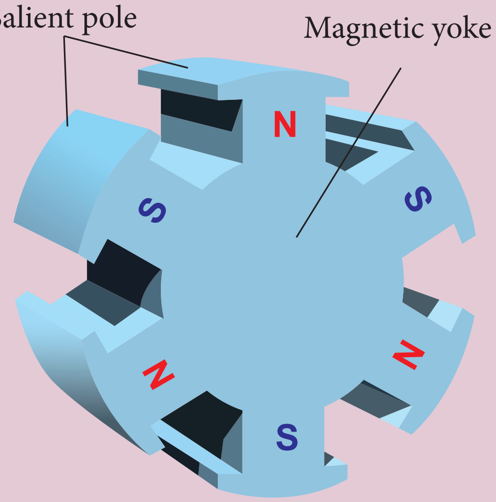
Figure(a): Stator core with empty slots



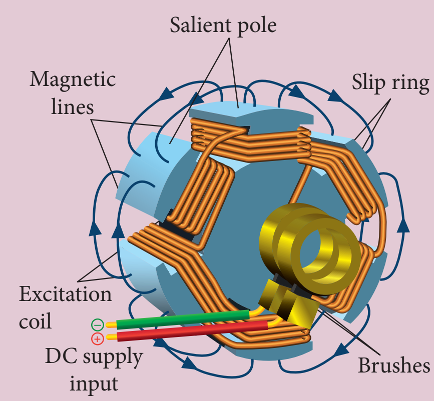
Figure(c): Stator core with armature windings

ii) Rotor

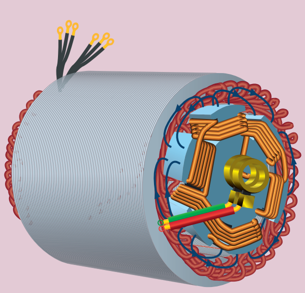
Rotor contains magnetic field windings, slip rings and brushes mounted on the same shaft.



Figure(d): Salient 6-pole rotor



Figure(e): Salient 6-pole rotor with field windings, slip rings and brushes



Figure(f): Stator core and rotor

The word salient means projecting. This rotor has a number of projecting poles having their bases riveted to the rotor. It is mainly used in low-speed alternators. The salient 2-pole rotor is shown in the Figure 4.30.

Cylindrical pole rotor

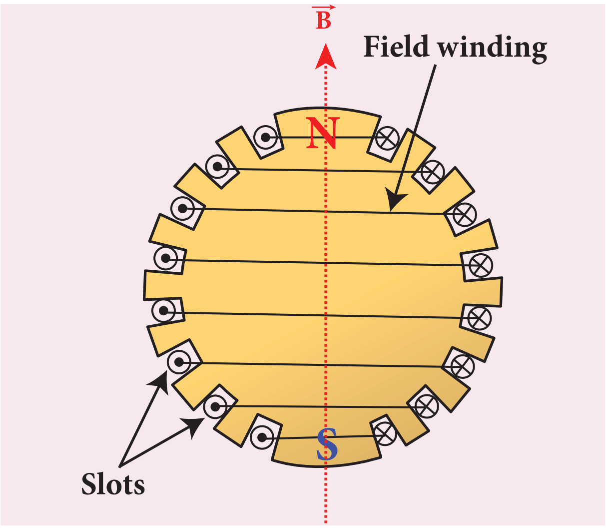


Figure 4.31 Cross-sectional view of cylindrical 2-pole rotor

This rotor consists of a smooth solid cylinder. The slots are cut on the outer surface of the cylinder along its length. It is suitable for very high speed alternators (Figure 4.31).

The frequency of alternating emf induced is directly proportional to the rotor speed. In order to maintain the frequency constant, the rotor must run at a constant speed.

These are standard construction details of alternators. Based on the type of alternator being constructed, the details like number of poles, pole type, number of coils and type of windings would change from one another.

We will discuss the construction and working of two examples, namely single phase and three phase AC generators in the following sections.

4.5.4 Advantages of stationary armature-rotating field alternator

Alternators are generally high current and high voltage machines. The stationary armature-rotating field construction has many advantages. A few of them include:

1) The current is drawn directly from fixed terminals on the stator without the use of brush contacts.
2) The insulation of stationary armature winding is easier.
3) The number of sliding contacts (slip rings) is reduced. Moreover, the sliding contacts are used for low-voltage DC Source.
4) Armature windings can be constructed more rigidly to prevent deformation due to any mechanical stress.

4.5.5 Single phase AC generator

In a single phase AC generator, the armature conductors are connected in series so as to form a single circuit which generates a single-phase alternating emf and hence it is called single-phase alternator.

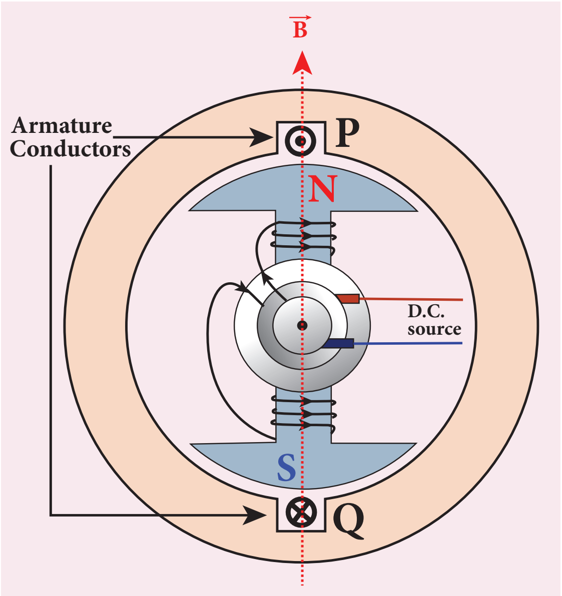


Figure 4.32 Stator core with a rectangular loop and 2-pole rotor

UNIT-4(XII-Physics_Vol-1).indd 240 04-03-2019 10:40:11

The simplified version of a AC generator is discussed here. Consider a stator core consisting of 2 slots in which 2 armature conductors PQ and RS are mounted to form single-turn rectangular loop PQRS as shown in Figure 4.33. Rotor has 2 salient poles with field windings which can be magnetized by means of DC source.

Working

The loop PQRS is stationary and is perpendicular to the plane of the paper. When field windings are excited, magnetic field is produced around it. The direction of magnetic field passing through the armature core is shown in Figure 4.33. Let the field magnet be rotated in clockwise direction by the prime mover. The axis of rotation is perpendicular to the plane of the paper.

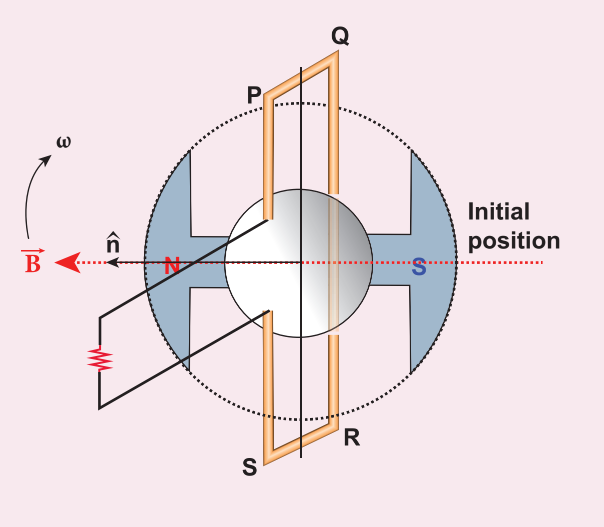


Figure 4.33 The loop PQRS and field magnet in its initial position

Assume that initial position of the field magnet is horizontal. At that instant, the direction of magnetic field is perpendicular to the plane of the loop PQRS. The induced emf is zero (Refer case (iii) of section 4.4). This is represented by origin O in the graph between induced emf and time angle (Figure 4.34).

When field magnet rotates through 90°, magnetic field becomes parallel to PQRS. The induced emfs across PQ and RS would become maximum. Since they are connected in series, emfs are added up and the direction of total induced emf is given by Fleming's right hand rule.

Care has to be taken while applying this rule; the thumb indicates the direction of the motion of the conductor with respect to field. For clockwise rotating poles, the conductor appears to be rotating anti-clockwise. Hence, thumb should point to the left. The direction of the induced emf is at right angles to the plane of the paper. For PQ, it is downwards and for RS upwards. Therefore, the current flows along PQRS. The point A in the graph represents this maximum emf.

For the rotation of 180° from the initial position, the field is again perpendicular to PQRS and the induced emf becomes zero. This is represented by point B.

The field magnet becomes again parallel to PQRS for 270° rotation of field magnet. The induced emf is maximum but the direction is reversed. Thus the current flows along SRQP. This is represented by point C.

On completion of 360°, the induced emf becomes zero and is represented by the point D. From the graph, it is clear that emf induced in PQRS is alternating in nature.

Therefore, when field magnet completes one rotation, induced emf in PQRS finishes one cycle. For this construction, the frequency of the induced emf depends on the speed at which the field magnet rotates.

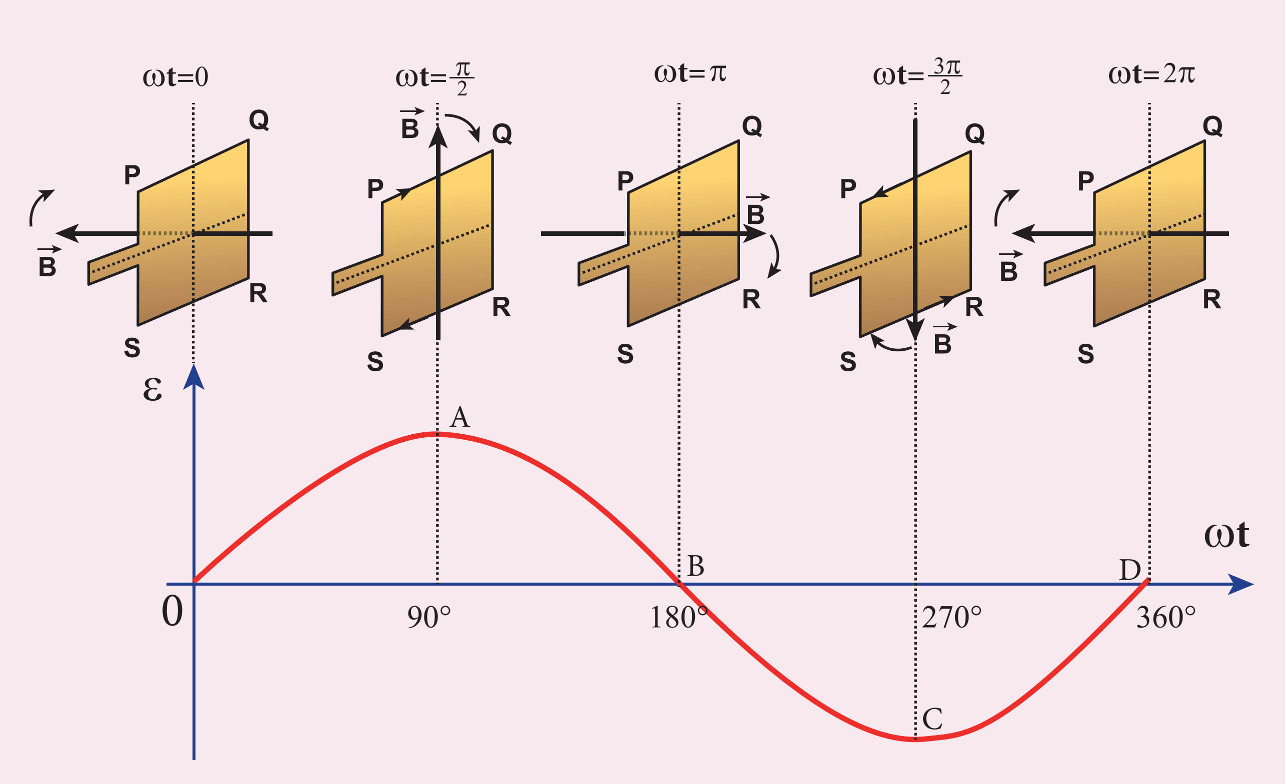
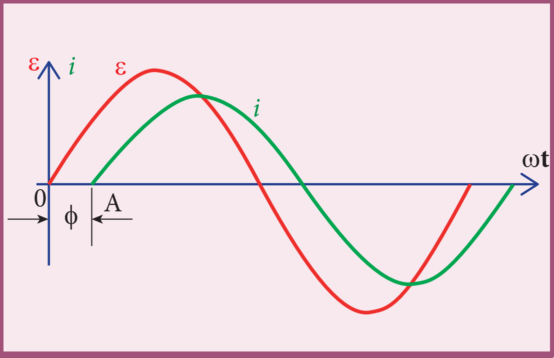


Figure 4.34 Variation of induced emf with respect to time angle

> **Note**
>
> **Phase difference**
>
> If two alternating quantities of same frequency do not pass through a particular point, say zero point, in the same direction at the same instant, they are said to have a phase difference. The angle between zero points is the angle of phase difference.
>
> 
>
>
> For the graph shown above, the phase difference between ε and i is given by OA = ϕ.

4.5.6 Three-phase AC generator

Some AC generators may have more than one coil in the armature core and each coil produces an alternating emf. In these generators, more than one emf is produced. Thus they are called poly-phase generators.

If there are two alternating emfs produced in a generator, it is called two-phase generator. In some AC generators, there are three separate coils, which would give three separate emfs. Hence they are called three-phase AC generators.

In the simplified construction of three-phase AC generator, the armature core has 6 slots, cut on its inner rim. Each slot is 60° away from one another. Six armature conductors are mounted in these slots. The conductors 1 and 4 are joined in series to form coil 1. The conductors 3 and 6 form coil 2 while the conductors 5 and 2 form coil 3. So, these coils are retangular in shape and are 120° apart from one another (Figure 4.35).

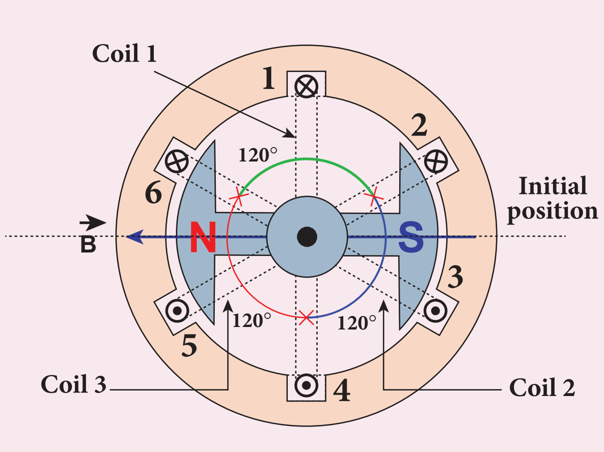


Figure 4.35 Construction of three-phase AC generator

The initial position of the field magnet is horizontal and field direction is perpendicular to the plane of the coil 1. As it is seen in single phase AC generator, when field magnet is rotated from that position in clockwise direction, alternating emf ε_1 in coil 1 begins a cycle from origin O. This is shown in Figure 4.36.

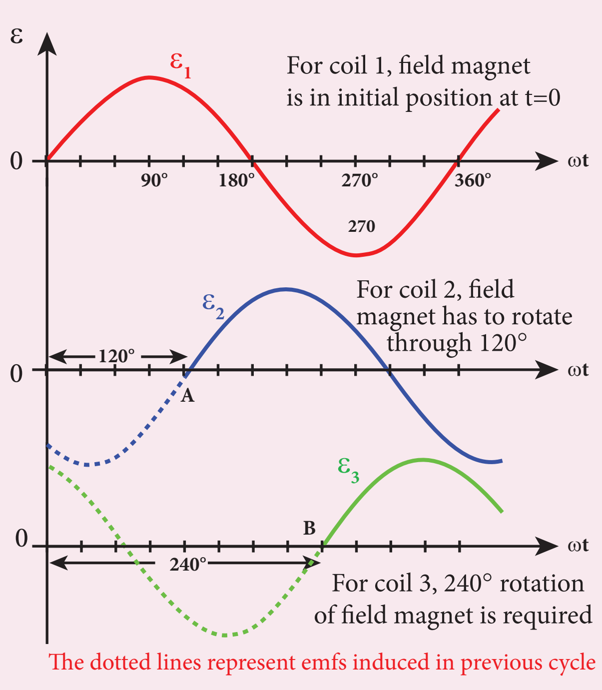


Figure 4.36 Variation of emfs ε_1, ε_2 and ε_3 with time angle.

The corresponding cycle for alternating emf ε_2 in coil 2 starts at point A after field magnet has rotated through 120°. Therefore, the phase difference between ε_1 and ε_2 is 120°. Similarly, emf ε_3 in coil 3 would begin its cycle at point B after 240° rotation of field magnet from initial position. Thus these emfs produced in the three phase AC generator have 120° phase difference between one another.

4.5.7 Advantages of three-phase alternator

Three-phase system has many advantages over single-phase system. Let us see a few of them.

1) For a given dimension of the generator, three-phase machine produces higher power output than a single-phase machine.
2) For the same capacity, three-phase alternator is smaller in size when compared to single phase alternator.
3) Three-phase transmission system is cheaper. A relatively thinner wire is sufficient for transmission of three-phase power.

4.6 TRANSFORMER

Transformer is a stationary device used to transform electrical power from one circuit to another without changing its frequency. The applied alternating voltage is either increased or decreased with corresponding decrease or increase of current in the circuit.

If the transformer converts an alternating current with low voltage into an alternating current with high voltage, it is called step-up transformer. On the contrary, if the transformer converts alternating current with high voltage into an alternating current with low voltage, then it is called step-down transformer.

4.6.1 Construction and working of transformer

Principle

The principle of transformer is the mutual induction between two coils. That is, when an electric current passing through a coil changes with time, an emf is induced in the neighbouring coil.

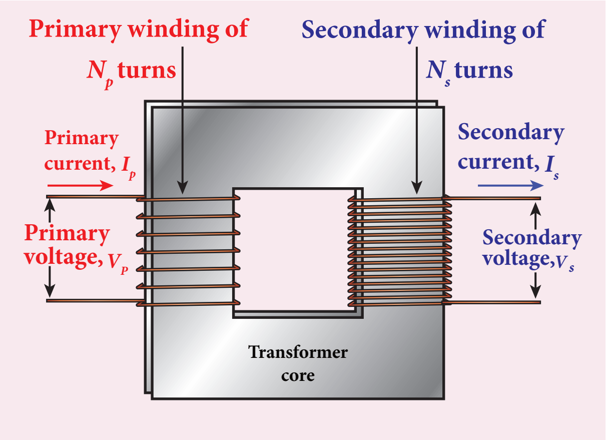


Figure 4.37(a) Construction of transformer

Figure 4.37(b) Roadside transformer

Construction

In the simple construction of transformers, there are two coils of high mutual inductance wound over the same transformer core. The core is generally laminated and is made up of a good magnetic material like silicon steel. Coils are electrically insulated but magnetically linked via transformer core (Figure 4.37).

The coil across which alternating voltage is applied is called primary coil *P* and the coil from which output power is drawn out is called secondary coil *S*. The assembled core and coils are kept in a container which is filled with suitable medium for better insulation and cooling purpose.

Working

If the primary coil is connected to a source of alternating voltage, an alternating magnetic flux is set up in the laminated core. If there is no magnetic flux leakage, then whole of magnetic flux linked with primary coil is also linked with secondary coil. This means that rate at which magnetic flux changes through each turn is same for both primary and secondary coils.

As a result of flux change, emf is induced in both primary and secondary coils. The emf induced in the primary coil ε_p is almost equal and opposite to the applied voltage υ_p and is given by

$$\upsilon_p = \varepsilon_p = -N_p \frac{d\Phi_B}{dt} \qquad (4.30)$$

The frequency of alternating magnetic flux in the core is same as the frequency of the applied voltage. Therefore, induced emf in secondary will also have same frequency as that of applied voltage. The emf induced in the secondary coil ε_s is given by

$$\varepsilon_s = -N_s \frac{d\Phi_B}{dt}$$

where N_p and N_s are the number of turns in the primary and secondary coil respectively. If the secondary circuit is open, then $\varepsilon_s = \upsilon_s$ where υ_s is the voltage across secondary coil.

$$\upsilon_s = \varepsilon_s = -N_s \frac{d\Phi_B}{dt} \quad (4.31)$$

From equations (4.30) and (4.31),

$$\frac{\upsilon_s}{\upsilon_p} = \frac{N_s}{N_p} = K \quad (4.32)$$

This constant K is known as voltage transformation ratio. For an ideal transformer,

Input power $\upsilon_p i_p$ = Output power $\upsilon_s i_s$

where i_p and i_s are the currents in the primary and secondary coil respectively. Therefore,

$$\frac{\upsilon_s}{\upsilon_p} = \frac{N_s}{N_p} = \frac{i_p}{i_s} \quad (4.33)$$

Equation 4.33 is written in terms of amplitude of corresponding quantities,

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s} = K$$

i) If $N_s > N_p (\text{or } K > 1)$, $\therefore$ $V_s > V_p$ and $I_s < I_p$. This is the case of step-up transformer in which voltage is increased and the corresponding current is decreased.

ii) If $N_s < N_p (\text{or } K < 1)$, $\therefore$ $V_s < V_p$ and $I_s > I_p$. This is step-down transformer where voltage is decreased and the current is increased.

Efficiency of a transformer:

The efficiency η of a transformer is defined as the ratio of the useful output power to the input power. Thus

$$\eta = \frac{\text{Output power}}{\text{Input power}} \times 100\% \quad (4.34)$$

Transformers are highly efficient devices having their efficiency in the range of 96 – 99%. Various energy losses in a transformer will not allow them to be 100% efficient.

4.6.2 Energy losses in a transformer

Transformers do not have any moving parts so that its efficiency is much higher than that of rotating machines like generators and motors. But there are many factors which lead to energy loss in a transformer.

i) Core loss or Iron loss

This loss takes place in transformer core. Hysteresis loss (Refer section 3.6) and eddy current loss are known as core loss or Iron loss. When transformer core is magnetized and demagnetized repeatedly by the alternating voltage applied across primary coil, hysteresis takes place due to which some energy is lost in the form of heat. Hysteresis loss is minimized by using steel of high silicon content in making transformer core.

Alternating magnetic flux in the core induces eddy currents in it. Therefore there is energy loss due to the flow of eddy current, called eddy current loss which is minimized by using very thin laminations of transformer core.

ii) Copper loss

Transformer windings have electrical resistance. When an electric current flows through them, some amount of energy is dissipated due to Joule heating. This energy loss is called copper loss which is minimized by using wires of larger diameter.

iii) Flux leakage

Flux leakage happens when the magnetic lines of primary coil are not completely linked with secondary coil. Energy loss due to this flux leakage is minimized by winding coils one over the other.

4.6.3 Advantages of AC in long distance power transmission

Electric power is produced in a large scale at electric power stations with the help of AC generators. These power stations are classified based on the type of fuel used as thermal, hydro electric and nuclear power stations. Most of these stations are located at remote places. Hence the electric power generated is transmitted over long distances through transmission lines to reach towns or cities where it is actually consumed. This process is called power transmission.

But there is a difficulty during power transmission. A sizable fraction of electric power is lost due to Joule heating (i^2R) in the transmission lines which are hundreds of kilometer long. This power loss can be tackled either by reducing current i or by reducing resistance R of the transmission lines. The resistance R can be reduced with thick wires of copper or aluminium. But this increases the cost of production of transmission lines and other related expenses. So this way of reducing power loss is not economically viable.

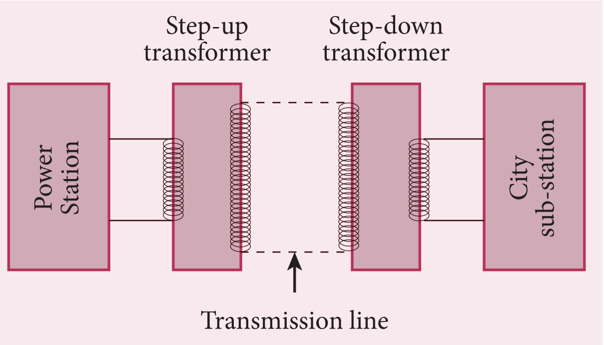


Figure 4.38 Long distance power transmissions

Since power produced is alternating in nature, there is a way out. The most important property of alternating voltage that it can be stepped up and stepped down by using transformers could be exploited in reducing current and thereby reducing power losses to a greater extent.

At the transmitting point, the voltage is increased and the corresponding current is decreased by using step-up transformer (Figure 4.38). Then it is transmitted through transmission lines. This reduced current at high voltage reaches the destination without any appreciable loss. At the receiving point, the voltage is decreased and the current is increased to appropriate values by using step-down transformer and then it is given to consumers. Thus power transmission is done efficiently and economically.

Illustration:

An electric power of 2 MW is transmitted to a place through transmission lines of total resistance, say R = 40 Ω, at two different voltages. One is lower voltage (10 kV) and the other is higher (100 kV). Let us now calculate and compare power losses in these two cases.

Case (i):

$P = 2$ MW; $R = 40\ \Omega$; $V = 10$ kV

Power, $P = VI$

$$\therefore \text{Current, } I = \frac{P}{V}$$

$$= \frac{2\times10^6}{10\times10^3} = 200A$$

Power loss = Heat produced

$$= I^2R = (200)^2 \times 40 = 1.6\times10^6 W$$

$$\%\text{ of power loss} = \frac{1.6\times10^6}{2\times10^6}\times100\%$$

$$= 0.8\times100\% = 80\%$$

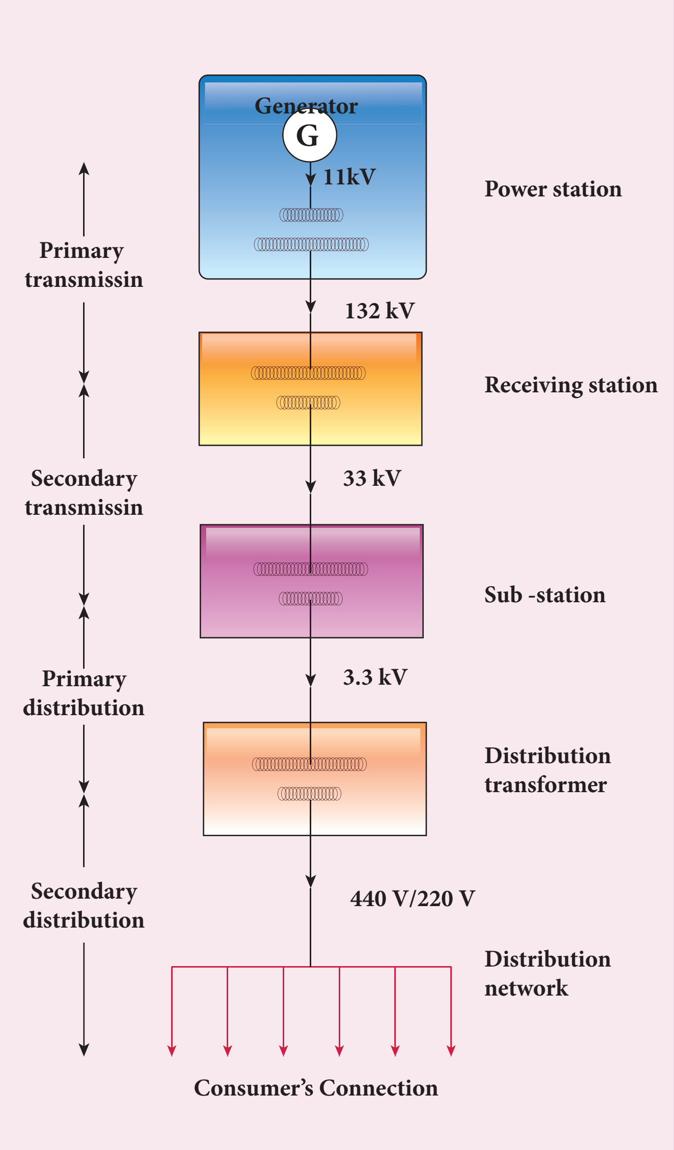
Power system at a glance (Not for Examination)

The generating stations present in a region are interconnected to form a common electrical network and are operated in parallel. This is to ensure uninterrupted power supply to a large number of consumers in the case of failure of any power station or a sudden increase of load beyond the capacity of the generating station.

The various elements such as generating stations, transmission lines, the substations and distributors etc are all tied together for continuous generation and consumption of electric energy.This is called power system. A part of power system consisting of the sub-stations and transmission lines is known as a grid.

In a power system, the transfer of electric power produced to the consumer is carried out in two stages which are further sub-divided into two as given below.

1) Transmission stage
   a) Primary transmission stage
   b) Secondary transmission stage
2) Distribution stage
   a) Primary distribution stage
   b) Secondary transmission stage



and then it is supplied to individual consumers. These two stages of power transmission is presented in a single-line diagram shown in figure. The central system usually generates power at 11 kV which is stepped up to 132 kV and is transmitted through transmission lines. This is known as primary or high-voltage transmission.

This high-voltage power reaches receiving station at the outskirts of the city where it is stepped down to 33 kV and is transmitted as secondary or low-voltage transmission to sub-stations situated within the city limits.

In the primary distribution system, the voltage is reduced from 33 kV to 3.3kV at sub-stations and is given to distribution sub-stations. The voltage is finally brought down to 440V or 230V at distribution sub-station from where secondary distribution is done to factories (440V) and homes (230V) via distribution networks.

Case (ii):

$P = 2$ MW; $R = 40\ \Omega$; $V = 100$ kV

$$\therefore \text{Current, } I = \frac{P}{V}$$

$$= \frac{2\times10^6}{100\times10^3} = 20A$$

Power loss = I^2R

$$= (20)^2 \times 40 = 0.016\times10^6 W$$

$$\% \text{ of power loss} = \frac{0.016\times10^6}{2\times10^6}\times100\%$$

$$= 0.008\times100\% = 0.8\%$$

Thus it is clear that when an electric power is transmitted at higher voltage, the power loss is reduced to a large extent.

EXAMPLE 4.16

An ideal transformer has 460 and 40,000 turns in the primary and secondary coils respectively. Find the voltage developed per turn of the secondary if the transformer is connected to a 230 V AC mains. The secondary is given to a load of resistance $10^4\,\Omega$. Calculate the power delivered to the load.

Solution

$N_P = 460$ turns; $N_S = 40{,}000$ turns

$V_P = 230$ V; $R_S = 10^4\ \Omega$

(i) Secondary voltage,

$$V_S = \frac{V_P N_S}{N_P} = \frac{230\times40{,}000}{460}$$

$$= 20{,}000V$$

$$\text{Secondary voltage per turn, } \frac{V_S}{N_S} = \frac{20{,}000}{40{,}000}$$

$$= 0.5\ V$$

(ii) Power delivered

$$= V_S I_S = \frac{V_S^2}{R_S} = \frac{20{,}000\times20{,}000}{10^4} = 40\ kW$$

EXAMPLE 4.17

An inverter is common electrical device which we use in our homes. When there is no power in our house, inverter gives AC power to run a few electronic appliances like fan or light. An inverter has inbuilt step-up transformer which converts 12 V AC to 240 V AC. The primary coil has 100 turns and the inverter delivers 50 mA to the external circuit. Find the number of turns in the secondary and the primary current.

Solution

$V_p = 12$ V; $V_s = 240$ V

$I_s = 50$mA; $N_p = 100$ turns

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S} = K$$

$$\text{Transformation ratio, } K = \frac{240}{12} = 20$$

The number of turns in the secondary

$$N_S = N_P \times K = 100\times20 = 2000$$

Primary current,

$I_p = K \times I_s = 20 \times 50$ mA $= 1$ A

4.7 ALTERNATING CURRENT

4.7.1 Introduction

In section 4.5, we have seen that when the orientation of the coil with the magnetic field is changed, an alternating emf is induced and hence an alternating current flows in the closed circuit. **An alternating voltage is the voltage which changes polarity at regular intervals of time and the direction of the resulting alternating current also changes accordingly.**

In the Figure 4.39(a), an alternating voltage source is connected to a resistor R in which the upper terminal of the source is positive and lower terminal negative at an instant. Therefore, the current flows in clockwise direction. After a short time, the polarities of the source are reversed so that current now flows in anti-clockwise direction (Figure 4.39(b)). This current which flows in alternate directions in the circuit is called alternating current.

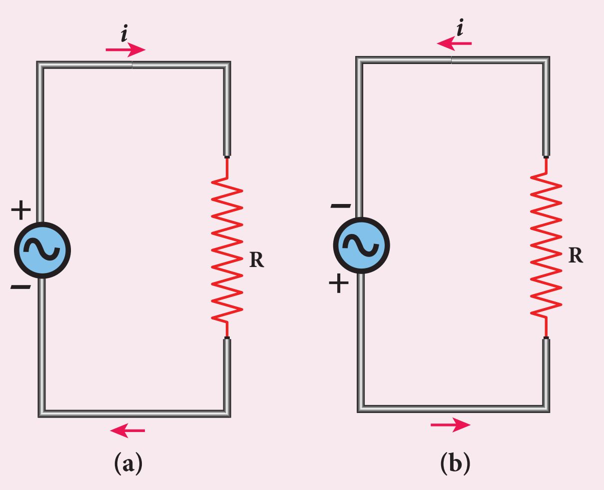


Figure 4.39 Alternating voltage and the corresponding alternating current

Sinusoidal alternating voltage

If the waveform of alternating voltage is a sine wave, then it is known as sinusoidal alternating voltage which is given by the relation.

$$\upsilon = V_m \sin \omega t \quad (4.35)$$

where υ is the instantaneous value of alternating voltage; V_m is the maximum value (amplitude) and ω is the angular frequency of the alternating voltage. When sinusoidal alternating voltage is applied to a closed circuit, the resulting alternating current is also sinusoidal in nature and its relation is

$$i = I_m \sin \omega t \quad (4.36)$$

where I_m is the maximum value (amplitude) of the alternating current. The direction of sinusoidal voltage or current is reversed after every half-cycle and its magnitude is also changing continuously as shown in Figure 4.40.

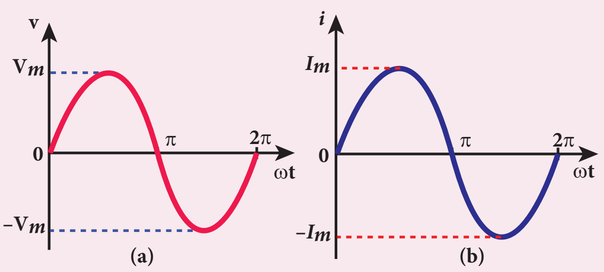


Figure 4.40 (a) Sinusoidal alternating voltage (b) Sinusoidal alternating current

Note Interestingly, sine waves are very common in nature. The periodic motions like waves in water, swinging of pendulum are associated with sine waves. Thus sine wave seems to be nature's standard. Also refer unit 11 of XI physics text book.

4.7.1 Mean or Average value of AC

The current and voltage in a DC system remain constant over a period of time so that there is no problem in specifying their magnitudes. However, an alternating current or voltage varies from time to time. Then a question arises how to express the magnitude of an alternating current or voltage. Though there are many ways of expressing it, we limit our discussion with two ways, namely mean value and RMS (Root Mean Square) value of AC.

Mean or Average value of AC

We have learnt that the magnitude of an alternating current in a circuit changes from one instant to other instant and its direction also reverses for every half cycle. During positive half cycle, current is taken as positive and during negative cycle it is negative. Therefore mean or average value of symmetrical alternating current over one complete cycle is zero.

Therefore the average or mean value is measured over one half of a cycle. These electrical terms, average current and average voltage can be used in both AC and DC circuit analysis and calculations.

The average value of alternating current is defined as the average of all values of current over a positive half-cycle or negative half-cycle.

The instantaneous value of sinusoidal alternating current is given by the equation $i = I_m \sin\omega t$ or $i = I_m \sin\theta$ (where $\theta = \omega t$) whose graphical representation is given in Figure 4.41.

The sum of all currents over a half-cycle is given by area of positive half-cycle (or negative half-cycle). Therefore,

$$I_{av} = \frac{\text{Area of positive half-cycle (or negative half-cycle)}}{\text{Base length of half-cycle}} \quad (4.37)$$

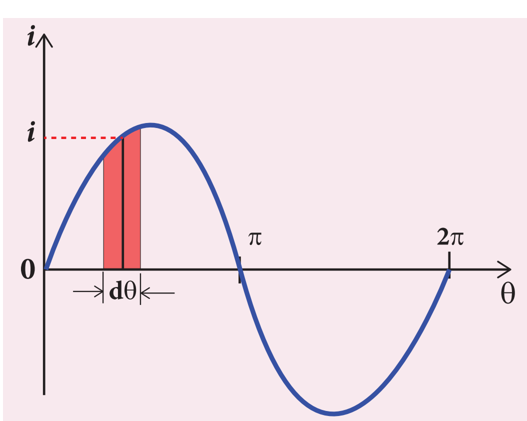


Figure 4.41 Sine wave of an alternating current

Consider an elementary strip of thickness $d\theta$ in the positive half-cycle of the current wave (Figure 4.41). Let i be the mid-ordinate of that strip.

Area of the elementary strip = $i\, d\theta$

Area of positive half-cycle

$$= \int_0^{\pi} i d\theta = \int_0^{\pi} I_m \sin\theta d\theta$$

$$= I_m \left[-\cos\theta\right]_0^{\pi} = -I_m \left[\cos\pi - \cos 0\right] = 2I_m$$

Substituting this in equation (4.37), we get (The base length of half-cycle is π)

Average value of AC, $I_{av} = \frac{2I_m}{\pi}$

$$I_{av} = 0.637\, I_m \quad (4.38)$$

Hence the average value of AC is 0.637 times the maximum value I_m of the alternating current. For negative half-cycle, $I_{av} = -0.637\, I_m$.

Note

For example, if we consider n currents in a half-cycle of AC, namely $i_1, i_2, \ldots i_n$, then average value is given by

$$I_{av} = \frac{\text{Sum of all currents over half-cycle}}{\text{Number of currents}}$$

$$I_{av} = \frac{i_1 + i_2 + \ldots + i_n}{n}$$

4.7.2 RMS value of AC

The term RMS refers to time-varying sinusoidal currents and voltages and not used in DC systems.

The root mean square value of an alternating current is defined as the square root of the mean of the squares of all currents over one cycle. It is denoted by

I_{RMS}. For alternating voltages, the RMS value is given by V_{RMS}.

The alternating current $i = I_m \sin\omega t$ or $i = I_m \sin\theta$, is represented graphically in Figure 4.42. The corresponding squared current wave is also shown by the dotted lines.

The sum of the squares of all currents over one cycle is given by the area of one cycle of squared wave. Therefore,

$$I_{RMS} = \sqrt{\frac{\text{Area of one cycle of squared wave}}{\text{Base length of one cycle}}} \quad (4.39)$$

An elementary area of thickness $d\theta$ is considered in the first half-cycle of the squared current wave as shown in Figure 4.42. Let i^2 be the mid-ordinate of the element.

Area of the element = $i^2 d\theta$

Area of one cycle of squared wave $= \int_0^{2\pi} i^2 d\theta$

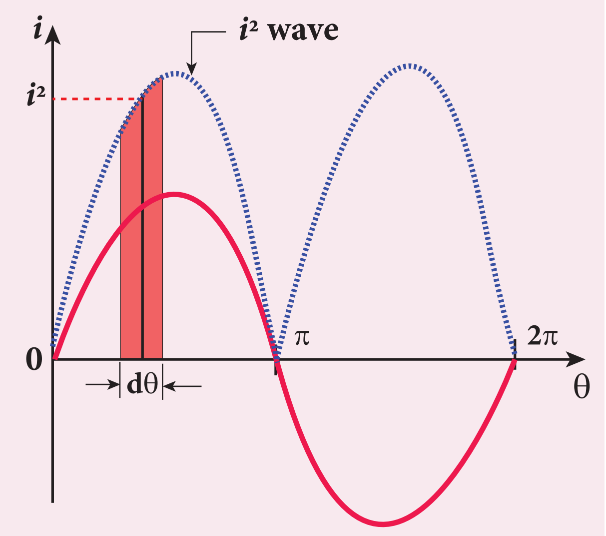


Figure 4.42 Squared wave of AC

$$= \int_0^{2\pi} I_m^2 \sin^2\theta d\theta = I_m^2 \int_0^{2\pi} \sin^2\theta d\theta \quad (4.40)$$

$$= I_m^2 \int_0^{2\pi} \left[\frac{1-\cos 2\theta}{2}\right] d\theta$$

$$\text{since } \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \frac{I_m^2}{2}\left[\int_0^{2\pi} d\theta - \int_0^{2\pi} \cos 2\theta d\theta\right]$$

$$= \frac{I_m^2}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{I_m^2}{2}\left[\left(2\pi - \frac{\sin 2\times 2\pi}{2}\right) - \left(0 - \frac{\sin 0}{2}\right)\right]$$

$$= \frac{I_m^2}{2} \times 2\pi = I_m^2 \pi \qquad [\because \sin 0 = \sin 4\pi = 0]$$

Substituting this in equation (4.39), we get

$$I_{RMS} = \sqrt{\frac{I_m^2 \pi}{2\pi}} = \frac{I_m}{\sqrt{2}}$$ [Base length of one cycle is 2π]

$$I_{rms} = 0.707\, I_m \quad (4.41)$$

Thus we find that for a symmetrical sinusoidal current rms value of current is 70.7 % of its peak value.

Similarly for alternating voltage, it can be shown that

$$V_{rms} = 0.707\, V_m \quad (4.42)$$

Note

RMS value of alternating current is also called effective value and is represented as I_{eff}. It is used to compare RMS current of AC to an equivalent steady current.

RMS value is also defined as that value of the steady current which when flowing through a given circuit for a given time produces the same amount of heat as produced by the alternating current when flowing through the same circuit for the same time. The effective value of an alternating voltage is represented by V_{eff}.

Note For example, if we consider n currents in one cycle of AC, namely $i_1, i_2, \ldots i_n$, then RMS value is given by

$$I_{RMS} = \sqrt{\frac{\text{Sum of squares of all currents over one cycle}}{\text{Number of currents}}}$$

$$I_{RMS} = \sqrt{\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n}}$$

DO YOU KNOW? For common household appliances, the voltage rating and current rating are generally specified in terms of their RMS value. The domestic AC supply is 230V, 50 Hz. It is the RMS or effective value. Its peak value will be $V_m = \sqrt{2}\ V_{rms} = \sqrt{2} \times 230 = 325V$.

EXAMPLE 4.18

Write down the equation for a sinusoidal voltage of 50 Hz and its peak value is 20 V. Draw the corresponding voltage versus time graph.

Solution

$f = 50\,Hz$; $V_m = 20\ V$

Instantaneous voltage, $\upsilon = V_m \sin \omega t$

$= V_m \sin 2\pi \nu t$

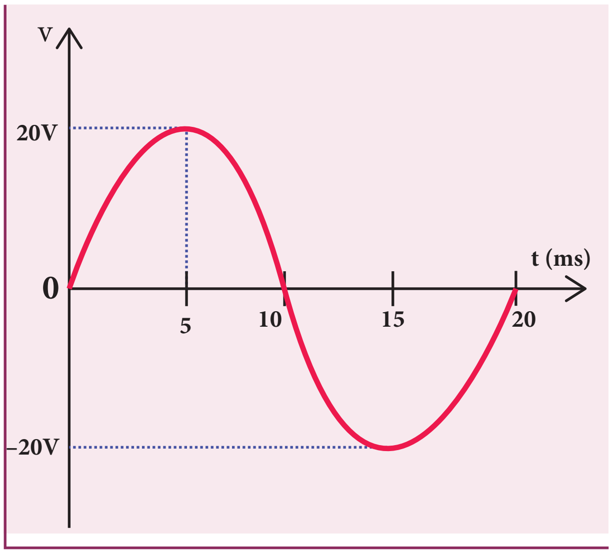
$= 20 \sin(2\pi \times 50)\,t = 20 \sin(100 \times 3.14)\,t$

$\upsilon = 20 \sin 314\,t$

Time for one cycle, $T = \frac{1}{f} = \frac{1}{50} = 0.02\,s$

$= 20 \times 10^{-3}\,s \quad = 20\,ms$

The wave form is given below.



EXAMPLE 4.19

The equation for an alternating current is given by $i = 77 \sin 314t$. Find the peak value, frequency, time period and instantaneous value at t = 2 ms.

Solution

$i = 77 \sin 314t$; $t = 2\text{ ms} = 2 \times 10^{-3}\text{ s}$

The general equation of an alternating current is $i = I_m \sin \omega t$. On comparsion,

(i) Peak value, $I_m = 77\,A$

(ii) Frequency, $f = \frac{\omega}{2\pi} = \frac{314}{2 \times 3.14} = 50\,Hz$

(iii) Time period, $T = \frac{1}{f} = \frac{1}{50} = 0.02\,s$

(iv) At t = 2 m s,

Instantaneous value,

$i = 77 \sin(314 \times 2 \times 10^{-3})$

$i = 45.24\,A$

Phasor and phasor diagram

Phasor

A sinusoidal alternating voltage (or current) can be represented by a vector which rotates about the origin in anti-clockwise direction at a constant angular velocity ω. Such a rotating vector is called a phasor. A phasor is drawn in such a way that

- the length of the line segment equals the peak value V_m (or I_m) of the alternating voltage (or current)
- its angular velocity ω is equal to the angular frequency of the alternating voltage (or current)
- the projection of phasor on any vertical axis gives the instantaneous value of the alternating voltage (or current)
- the angle between the phasor and the axis of reference (positive x-axis) indicates the phase of the alternating voltage (or current).

The notion of phasors is introduced to analyse phase relationship between voltage and current in different AC circuits.

Phasor diagram

The diagram which shows various phasors and their phase relations is called phasor diagram. Consider a sinusoidal alternating voltage $\upsilon = V_m \sin \omega t$ applied to a circuit. This voltage can be represented by a phasor, namely $\overrightarrow{OA}$ as shown in Figure 4.43.

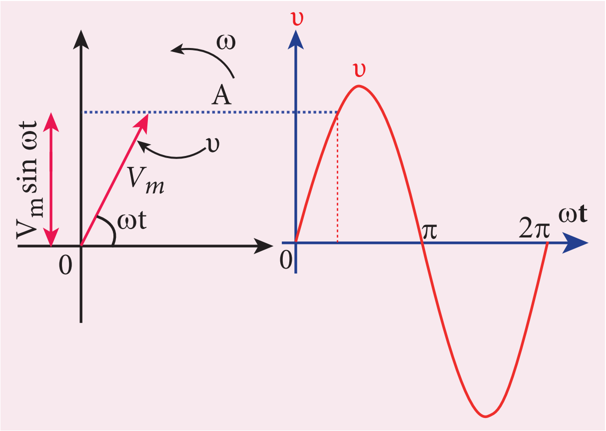


Figure 4.43 Phasor diagram for an alternating voltage $\upsilon = V_m \sin \omega t$

Here the length of $\overrightarrow{OA}$ equals the peak value (V_m), the angle it makes with x-axis gives the phase (ωt) of the applied voltage. Its projection on y-axis provides the instantaneous value ($V_m \sin \omega t$) at that instant.

When $\overrightarrow{OA}$ rotates about O with angular velocity ω in anti-clockwise direction, the

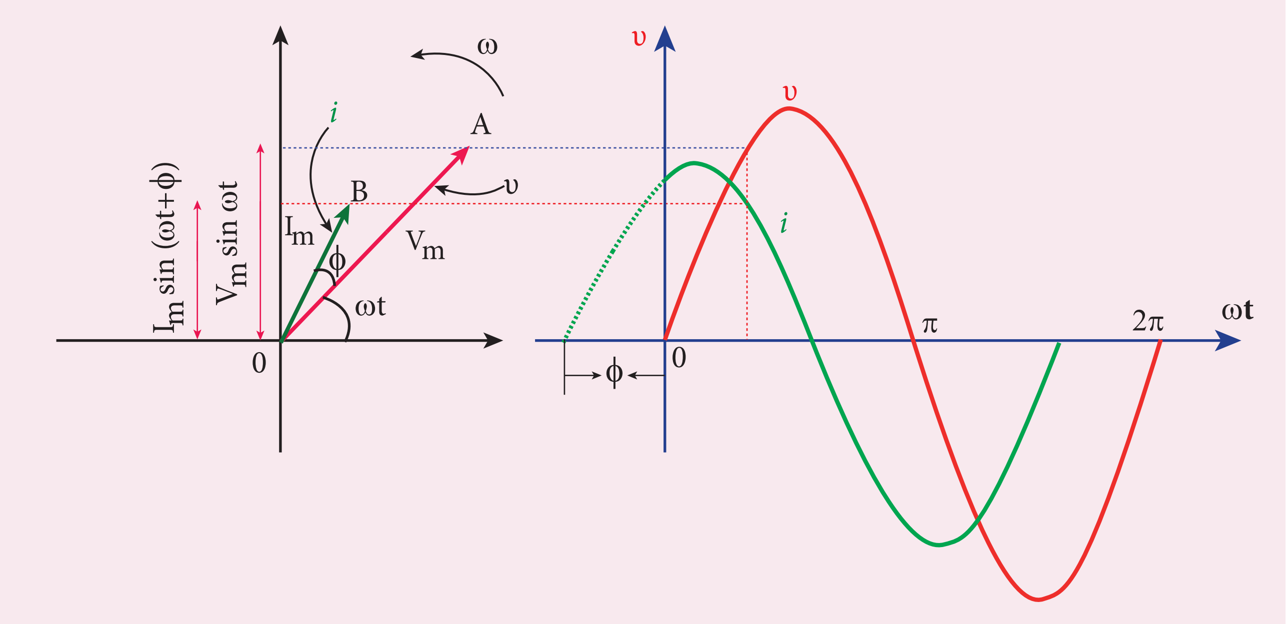


Figure 4.44 Phasor diagram and wave diagram say that i leads υ by ϕ

waveform of the voltage is generated. For one full rotation of $\overrightarrow{OA}$, one cycle of voltage is produced.

The alternating current in the same circuit may be given by the relation $i = I_m \sin(\omega t + \phi)$ which is represented by another phasor $\overrightarrow{OB}$. Here ϕ is the phase angle between voltage and current. In this case, the current leads the voltage by phase angle ϕ which is shown in Figure 4.44. If the current lags behind the voltage, then we write $i = I_m \sin(\omega t - \phi)$.

4.7.3 AC circuit containing pure resistor

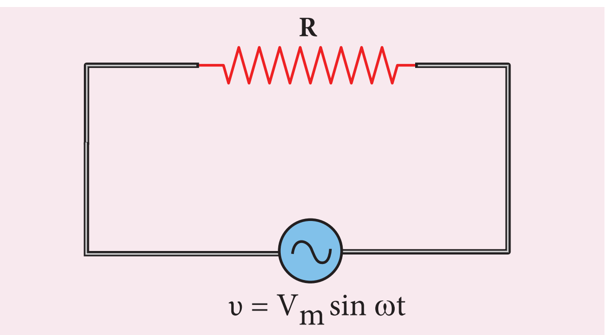


Figure 4.45 AC circuit with resistance

Consider a circuit containing a pure resistor of resistance R connected across an alternating voltage source (Figure 4.45). The instantaneous value of the alternating voltage is given by

$$\upsilon = V_m \sin \omega t \quad (4.43)$$

An alternating current i flowing in the circuit due to this voltage, develops a potential drop across R and is given by

$$V_R = iR \quad (4.44)$$

Kirchoff's loop rule (Refer section 2.4) states that the algebraic sum of potential differences in a closed circuit is zero. For this resistive circuit,

$$\upsilon - V_R = 0$$

From equation (4.43) and (4.44),

$$V_m \sin \omega t = iR$$

$$i = \frac{V_m}{R} \sin \omega t$$

$$i = I_m \sin \omega t \quad (4.45)$$

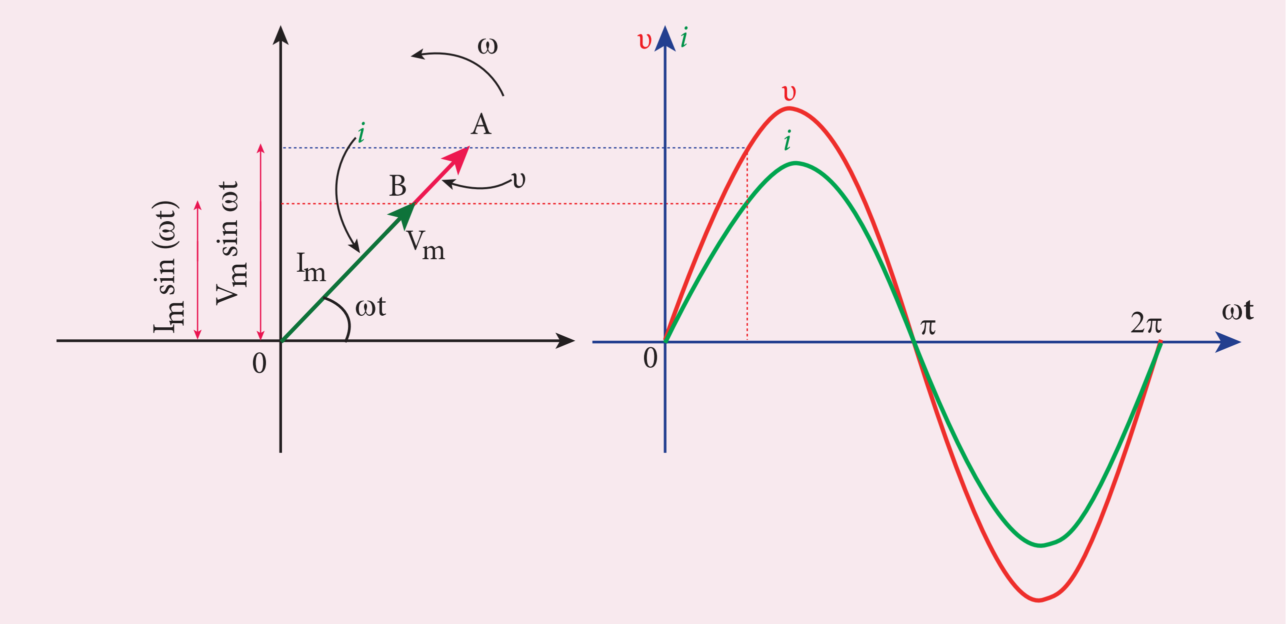


Figure 4.46 Phasor diagram and wave diagram for AC circuit with R

where $\frac{V_m}{R} = I_m$, the peak value of alternating current in the circuit. From equations (4.43) and (4.45), it is clear that the applied voltage and the current are in phase with each other in a resistive circuit. It means that they reach their maxima and minima simultaneously. This is indicated in the phasor diagram (Figure 4.46). The wave diagram also depicts that current is in phase with the applied voltage (Figure 4.46).

4.7.4 AC circuit containing only an inductor

Consider a circuit containing a pure inductor of inductance L connected across an alternating voltage source (Figure 4.47). The alternating voltage is given by the equation.

$$\upsilon = V_m \sin \omega t \qquad (4.46)$$

The alternating current flowing through the inductor induces a self-induced emf or back emf in the circuit. The back emf is given by

$$\text{Back emf}, \varepsilon = -L\frac{di}{dt}$$

By applying Kirchoff's loop rule to the purely inductive circuit, we get

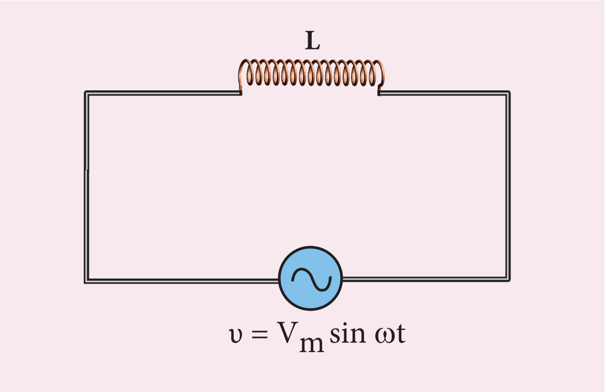


Figure 4.47 AC circuit with inductance

$$\upsilon + \varepsilon = 0$$

$$V_m \sin \omega t = L\frac{di}{dt}$$

$$di = \frac{V_m}{L}\sin \omega t \; dt$$

Integrating both sides, we get

$$i = \frac{V_m}{L}\int \sin \omega t \, dt$$

$$i = \frac{V_m}{L\omega}\left(-\cos \omega t\right) + \text{constant}$$

The integration constant in the above equation is independent of time. Since the voltage in the circuit has only time dependent part, we can set the time independent part in the current (integration constant) into zero.

$$\begin{pmatrix} \cos \omega t = \sin\left(\pi/_2 - \omega t\right) \\ -\sin\left(\pi/_2 - \omega t\right) = \sin\left(\omega t - \pi/_2\right) \end{pmatrix}$$

$$i = \frac{V_m}{\omega L}\sin\left(\omega t - \pi/_2\right)$$

$$\text{or} \quad i = I_m \sin\left(\omega t - \pi/_2\right) \qquad (4.47)$$

where $\frac{V_m}{\omega L} = I_m$, the peak value of the alternating current in the circuit. From equation (4.46) and (4.47), it is evident that current lags behind the applied voltage by $\pi/_2$ in an inductive circuit. This fact is depicted in the phasor diagram. In the wave diagram also, it is seen that current lags the voltage by 90° (Figure 4.48).

Inductive reactance X_L

The peak value of current I_m is given by $I_m = \frac{V_m}{\omega L}$. Let us compare this equation with $I_m = \frac{V_m}{R}$ from resistive circuit. The quantity ωL plays the same role as the resistance in resistive circuit. This is the resistance offered by the inductor, called inductive reactance (X_L). It is measured in ohm.

$$X_L = \omega L$$

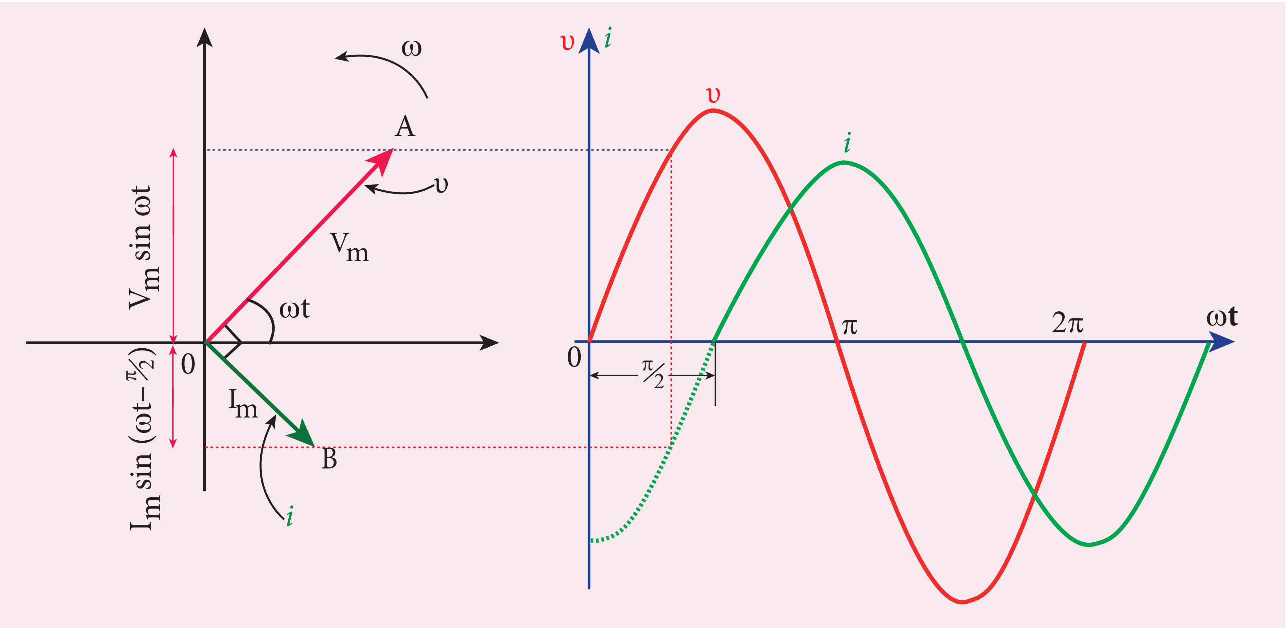


Figure 4.48 Phasor diagram and wave diagram for AC circuit with L

An inductor blocks AC but it allows DC. Why? and How?

An inductor L is a closely wound helical coil. The steady DC current flowing through L produces uniform magnetic field around it and the magnetic flux linked remains constant. Therefore there is no self-induction and self-induced emf (back emf). Since inductor behaves like a resistor, DC flows through an inductor.

The AC flowing through L produces time-varying magnetic field which in turn induces self-induced emf (back emf). This back emf, according to Lenz's law, opposes any change in the current. Since AC varies both in magnitude and direction, its flow is opposed in L. For an ideal inductor of zero ohmic resistance, the back emf is equal and opposite to the applied emf. Therefore L blocks AC.

The inductive reactance (X_L) varies directly as the frequency.

$$X_L = 2\pi f L \qquad (4.48)$$

where f is the frequency of the alternating current. For a steady current, $f = 0$. Therefore, $X_L = 0$. Thus an ideal inductor offers no resistance to steady DC current.

4.7.5 AC circuit containing only a capacitor

Consider a circuit containing a capacitor of capacitance C connected across an alternating voltage source (Figure 4.49). The alternating voltage is given by

$$\upsilon = V_m \sin \omega t \qquad (4.49)$$

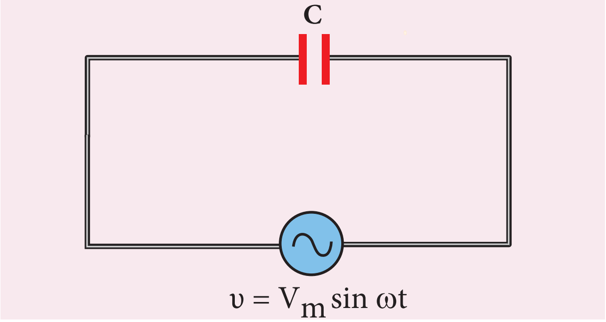


Figure 4.49 AC circuit with capacitance

Let q be the instantaneous charge on the capacitor. The emf across the capacitor at that instant is $\frac{q}{C}$. According to Kirchoff's loop rule,

$$\upsilon - \frac{q}{C} = 0$$

$$q = CV_m \sin \omega t$$

By the definition of current,

$$i = \frac{dq}{dt} = \frac{d}{dt}\left(CV_m \sin \omega t\right)$$

$$= CV_m \frac{d}{dt}\left(\sin \omega t\right)$$

$$= CV_m \, \omega \cos \omega t$$

$$\text{or} \quad i = \frac{V_m}{1/C\omega} \sin\left(\omega t + \pi/2\right)$$

Instantaneous value of current,

$$i = I_m \sin\left(\omega t + \pi/2\right) \qquad (4.50)$$

where $\frac{V_m}{1/C\omega} = I_m$, the peak value of the alternating current. From equations (4.49) and (4.50), it is clear that current leads the applied voltage by $\pi/2$ in a capacitive circuit. This is shown pictorially in Figure 4.50. The wave diagram for a capacitive circuit also shows that the current leads the applied voltage by 90°.

Capacitive reactance X_C

The peak value of current I_m is given by $I_m = \frac{V_m}{1/C\omega}$. Let us compare this equation with $I_m = \frac{V_m}{R}$ from resistive circuit. The quantity $1/C\omega$ plays the same role as the resistance R in resistive circuit. This is the resistance offered by the capacitor, called capacitive reactance (X_C). It measured in ohm.

$$X_c = \frac{1}{\omega C} \qquad (4.51)$$

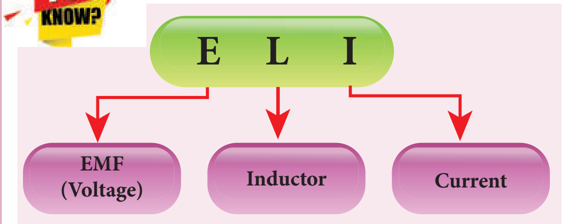
The capacitive reactance (X_C) varies inversely as the frequency. For a steady current, $f = 0$.

$$\therefore X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{0} = \infty$$

Thus a capacitive circuit offers infinite resistance to the steady current. So that steady current cannot flow through the capacitor.

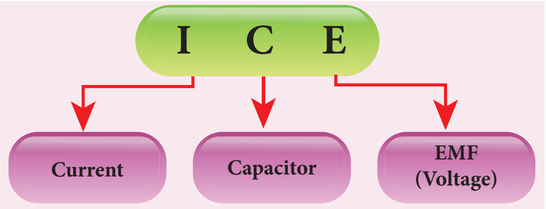
DO YOU KNOW?

What is ELI?



ELI is an acronym which means that EMF (voltage) leads the current in an inductive circuit.

What is ICE?



ICE is an acronym which means that the current leads the EMF (voltage) current in a capacitive circuit.

It is easier to may remember the results of AC circuits with the mnemonic 'ELI the ICE man'.

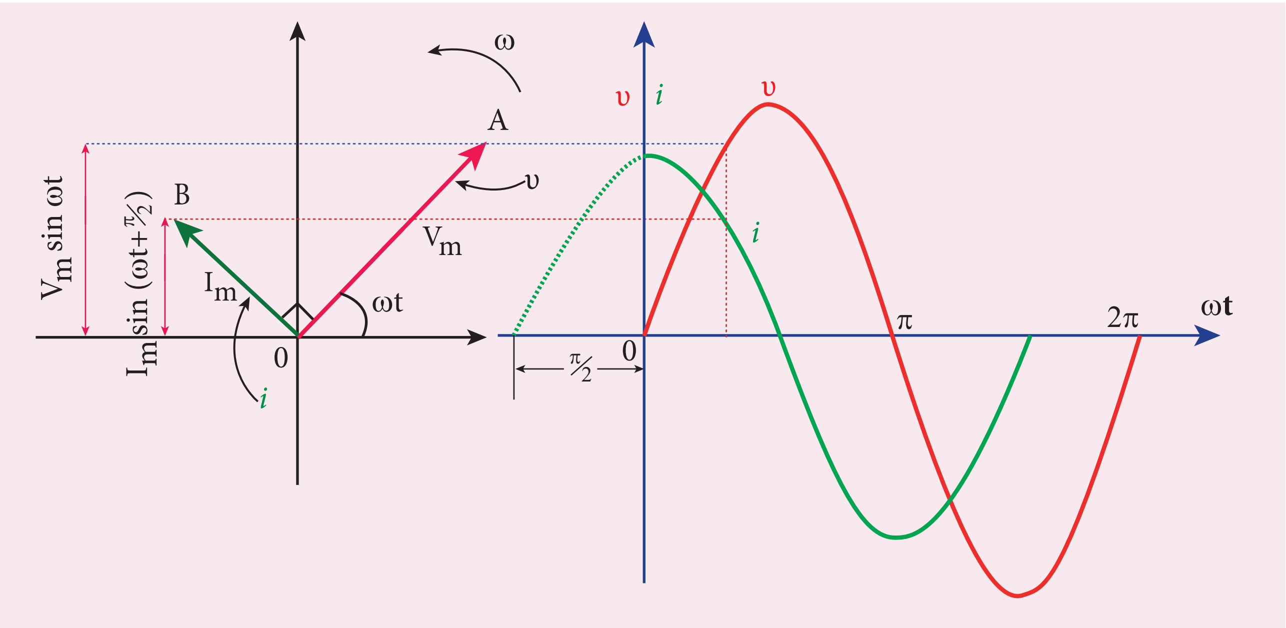


Figure 4.50 Phasor diagram and wave diagram for AC circuit with C

EXAMPLE 4.20

A 400 mH coil of negligible resistance is connected to an AC circuit in which an effective current of 6 mA is flowing. Find out the voltage across the coil if the frequency is 1000 Hz.

Solution

$L = 400 \times 10^{-3}$ H; $I_{eff} = 6 \times 10^{-3}$A

$f = 1000$ Hz

Inductive reactance, $X_L = L\omega = L \times 2\pi f$

$= 2 \times 3.14 \times 1000 \times 0.4$

$= 2512\ \Omega$

Voltage across L,

$V = I\,X_L = 6 \times 10^{-3} \times\ 2512$

$V = 15.072\,V\,(RMS)$

EXAMPLE 4.21

A capacitor of capacitance $\frac{10^2}{\pi}$ μF is connected across a 220 V, 50 Hz A.C. mains. Calculate the capacitive reactance, RMS value of current and write down the equations of voltage and current.

Solution

$C = \frac{10^2}{\pi} \times 10^{-6} F,\ V_{RMS} = 220\ V;\ f = 50 Hz$

(i) Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$= \frac{1}{2 \times \pi \times 50 \times \frac{10^{-4}}{\pi}} = 100\,\Omega$$

(ii) RMS value of current,

$$I_{RMS} = \frac{V_{RMS}}{X_C} = \frac{220}{100} = 2.2\,A$$

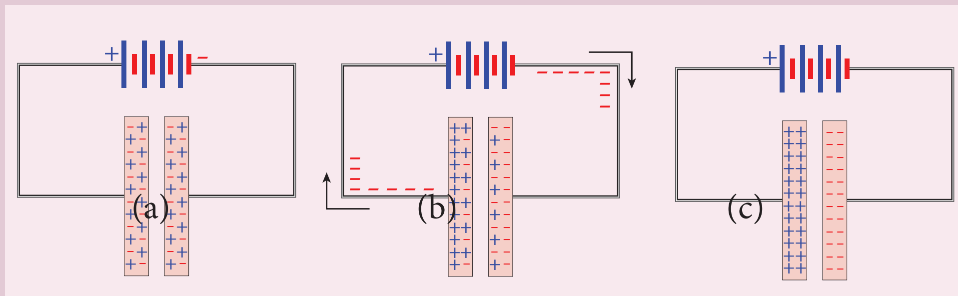
(iii) $V_m = 220 \times \sqrt{2} = 311\ V$

A capacitor blocks DC but it allows AC. Why? and How? (Not for examination)

Capacitors have two parallel metallic plates placed close to each other and there is a gap between plates. Whenever a source of voltage (either DC voltage or AC voltage) is connected across a capacitor C, the electrons from the source will reach the plate and stop. They cannot jump across the gap between plates to continue its flow in the circuit. Therefore the electrons flowing in one direction (i.e. DC) cannot pass through the capacitor. But the electrons from AC source seem to flow through C. Let us see what really happens!

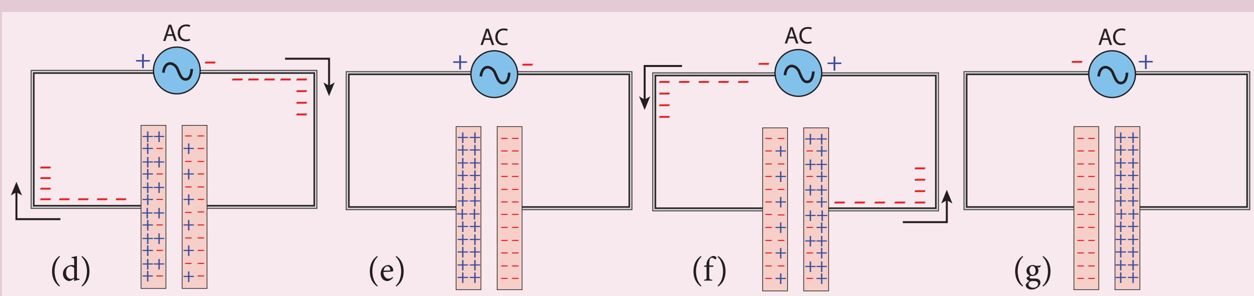
DC cannot flow through a capacitor:

Consider a parallel plate capacitor whose plates are uncharged (same amount of positive and negative charges). A DC source (battery) is connected across C as shown in Figure (a).



As soon as battery is connected, electrons start to flow from the negative terminal and are accumulated at the right plate, making it negative. Due to this negative potential, the electrons present in the nearby left plate are repelled and are moved towards positive terminal of the battery. When electrons leave the left plate, it becomes positively charged. This process is known as charging. The direction of flow of electrons is shown by arrows.

The charging of the plates continues till the level of the battery. Once C is fully charged and current will stop. At this time, we say that capacitor is blocking DC Figure (c).



AC flows (?!) through a capacitor:

Now an AC source is connected across C. At an instant, the right side of the source is at negative potential, then the electrons flow from negative terminal to the right plate and from left plate to the positive terminal as shown in Figure (d) but no electron crosses the gap between the plates. These electron-flows are represented by arrows. thus, the charging of the plates takes place and the plates become fully charged (Figure (e)).

After a short time, the polarities of AC source are reversed and the right side of the source is now positive. The electrons which were accumulated in the right plate start to flow to the positive terminal and the electrons from negative terminal flow to the left plate to neutralize the positive charges stored in it. As a result, the net charges present in the plates begin to decrease and this is called discharging. These electron-flows are represented by arrows as shown in Figure (f). Once the charges are exhausted, C will be charged again but with reversed polarities as shown in Figure (g).

Thus the electrons flow in one direction while charging the capacitor and its direction is reversed while discharging (the conventional current is also opposite in both cases). Though electrons flow in the circuit, no electron crosses the gap between the plates. In this way, AC flows through a capacitor.

$$I_m = 2.2 \times \sqrt{2} = 3.1A$$

Therefore,

$$\upsilon = 311 \sin 314t$$

$$i = 3.1 \sin\left(314t + \pi/2\right)$$

4.7.6 AC circuit containing a resistor, an inductor and a capacitor in series – Series RLC circuit

Consider a circuit containing a resistor of resistance R, a inductor of inductance L and a capacitor of capacitance C connected across an alternating voltage source (Figure 4.51). The applied alternating voltage is given by the equation.

$$\upsilon = V_m \sin \omega t \qquad (4.52)$$

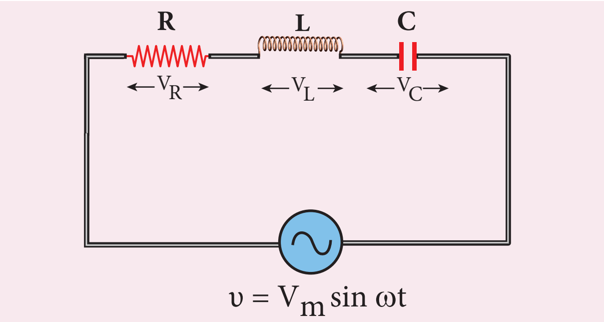


Figure 4.51 AC circuit containing *R*, *L* and *C*

Let i be the resulting circuit current in the circuit at that instant. As a result, the voltage is developed across *R*, *L* and *C*.

We know that voltage across R (V_R) is in phase with i, voltage across L (V_L) leads i by $\pi/2$ and voltage across C (V_C) lags i by $\pi/2$.

The phasor diagram is drawn with current as the reference phasor. The current is represented by the phasor $\overrightarrow{OI}$, V_R by $\overrightarrow{OA}$; V_L by $\overrightarrow{OB}$ and V_C by $\overrightarrow{OC}$ as shown in Figure 4.52.

The length of these phasors are

$OI = I_m$, $OA = I_mR$, $OB = I_mX_L$; $OC = I_mX_C$

The circuit is either effectively inductive or capacitive or resistive that depends on the value of V_L or V_C. Let us assume that $V_L > V_C$ so that net voltage drop across L-C combination is $V_L - V_C$ which is represented by a phasor $\overrightarrow{AD}$.

By parallelogram law, the diagonal $\overrightarrow{OE}$ gives the resultant voltage υ of V_R and $(V_L - V_C)$ and its length OE is equal to V_m. Therefore,

$$V_m^2 = V_R^2 + (V_L - V_C)^2$$

$$= \sqrt{(I_mR)^2 + (I_mX_L - I_mX_C)^2}$$

$$= I_m\sqrt{R^2 + (X_L - X_C)^2}$$

$$\text{or} \quad I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}} \qquad (4.53)$$

$$\text{or} \quad I_m = \frac{V_m}{Z}$$

$$\text{where} \quad Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad (4.54)$$

Z is called impedance of the circuit which refers to the effective opposition to the circuit current by the series *RLC* circuit. The voltage triangle and impedance triangle are given in the Figure 4.53.

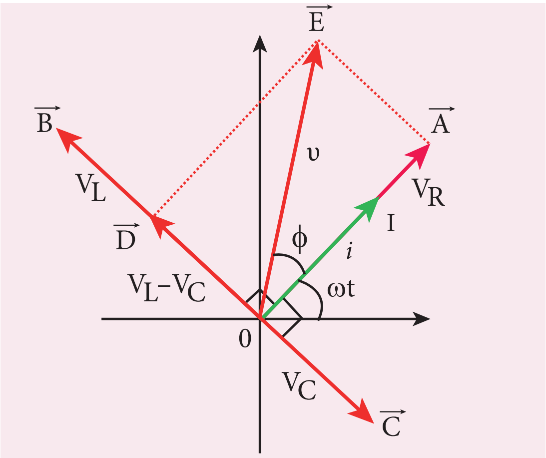


Figure 4.52 Phasor diagram for a series *RLC* – circuit when $V_L > V_C$

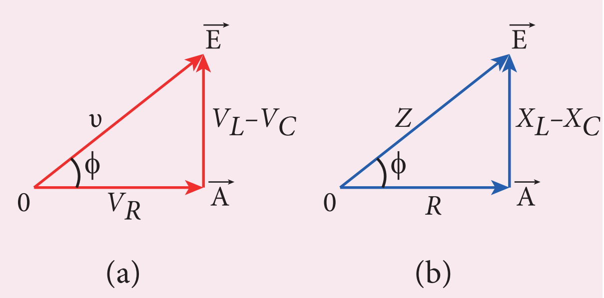


Figure 4.53 Voltage and impedance triangle when $X_L > X_C$

From phasor diagram, the phase angle between v and i is found out from the following relation

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} \quad (4.55)$$

Special cases

(i) If $X_L > X_C$, $(X_L - X_C)$ is positive and phase angle ϕ is also positive. It means that the applied voltage leads the current by ϕ (or current lags behind voltage by ϕ). The circuit is inductive.

$$\therefore \upsilon = V_m \sin\omega t;\ i = I_m \sin(\omega t - \phi)$$

(ii) If $X_L < X_C$, $(X_L - X_C)$ is negative and ϕ is also negative. Therefore current leads voltage by ϕ and the circuit is capacitive.

$$\therefore \upsilon = V_m \sin\omega t;\ i = I_m \sin(\omega t + \phi)$$

(iii) If $X_L = X_C$, ϕ is zero. Therefore current and voltage are in the same phase and the circuit is resistive.

$$\therefore \upsilon = V_m \sin\omega t;\quad i = I_m \sin\omega t$$

4.7.7 Resonance in series RLC Circuit

When the frequency of the applied alternating source (ω_r) is equal to the natural frequency $\left[1/\sqrt{LC} \right]$ of the RLC circuit, the current in the circuit reaches its maximum value. Then the circuit is said to be in electrical resonance. The frequency at which resonance takes place is called resonant frequency.

Resonant angular frequency, $\omega_r = \frac{1}{\sqrt{LC}}$

$$\text{or} \quad f_r = \frac{1}{2\pi\sqrt{LC}} \quad (4.56)$$

At series resonance,

$$\omega_r = \frac{1}{\sqrt{LC}} \quad \text{or} \quad \omega_r^2 = \frac{1}{LC}$$

$$\omega_r L = \frac{1}{\omega_r C} \quad \text{or}$$

$$X_L = X_C \quad (4.57)$$

Table 4.1 Summary of results of AC circuits

Type of Impedance	Value of Impedance	Phase angle of current with voltage	Power factor
Resistance	R	0°	1
Inductance	$X_L = \omega L$	90° lag	0
Capacitance	$X_C = 1/\omega C$	90° lead	0
R- L - C	$\sqrt{R^2 + \left(\omega L - 1/\omega C\right)^2}$	Between 0° and 90° lag or lead	Between 0 and 1

Since X_L and X_C are frequency dependent, the resonance condition $(X_L = X_C)$ can be achieved by the varying the frequency of the applied voltage.

Effects of series resonance

When series resonance occurs, the impedance of the circuit is minimum and is equal to the resistance of the circuit. As a result of this, the current in the circuit becomes maximum. This is shown in the resonance curve drawn between current and frequency (Figure 4.54).

At resonance, the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R \quad \text{since } X_L = X_C$$

Therefore, the current in the circuit is

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$I_m = \frac{V_m}{R} \qquad (4.58)$$

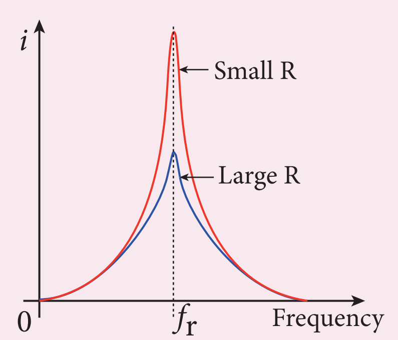


Figure 4.54 Resonance curve

The maximum current at series resonance is limited by the resistance of the circuit. For smaller resistance, larger current with sharper curve is obtained and vice versa.

Applications of series RLC resonant circuit

RLC circuits have many applications like filter circuits, oscillators, voltage multipliers etc. An important use of series RLC resonant circuits is in the tuning circuits of radio and TV systems. The signals from many broadcasting stations at different frequencies are available in the air. To receive the signal of a particular station, tuning is done.

The tuning is commonly achieved by varying capacitance of a parallel plate variable capacitor, thereby changing the resonant frequency of the circuit. When resonant frequency is nearly equal to the frequency of the signal of the particular station, the amplitude of the current in the circuit is maximum. Thus the signal of that station alone is received.

Note The phenomenon of electrical resonance is possible when the circuit contains both *L* and *C*. Only then the voltage across *L* and *C* cancel one another when V_L and V_C are 180° out of phase and the circuit becomes purely resistive. This implies that resonance will not occur in a *RL* and *RC* circuits.

4.7.8 Quality factor or Q–factor

The current in the series *RLC* circuit becomes maximum at resonance. Due to the increase in current, the voltage across *L* and *C* are also increased. This magnification of voltages at series resonance is termed as Q–factor.

It is defined as the ratio of voltage across L or C to the applied voltage.

$$\text{Q-factor} = \frac{\text{Voltage across L or C}}{\text{Applied voltage}}$$

At resonance, the circuit is purely resistive. Therefore, the applied voltage is equal to the voltage across R.

$$\text{Q-factor} = \frac{I_m X_L}{I_m R} = \frac{X_L}{R}$$

$$\text{Q-factor} = \frac{\omega_r L}{R} \quad (4.59)$$

$$\text{Q-factor} = \frac{L}{R\sqrt{LC}} \quad \text{since } \omega_r = \frac{1}{\sqrt{LC}}$$

$$\text{Q-factor} = \frac{1}{R}\sqrt{\frac{L}{C}} \quad (4.60)$$

The physical meaning is that Q–factor indicates the number of times the voltage across L or C is greater than the applied voltage at resonance.

EXAMPLE 4.22

Find the impedance of a series RLC circuit if the inductive reactance, capacitive reactance and resistance are 184 Ω, 144 Ω and 30 Ω respectively. Also calculate the phase angle between voltage and current.

Solution

$X_L = 184\ \Omega$; $X_C = 144\ \Omega$

$R = 30\ \Omega$

(i) The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{30^2 + (184 - 144)^2}$$

$$= \sqrt{900 + 1600}$$

Impedance, $Z = 50\ \Omega$

(ii) Phase angle is

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$= \frac{184 - 144}{30} = 1.33$$

$$\phi = 53.1^\circ$$

Since the phase angle is positive, voltage leads current by 53.1° for this inductive circuit.

EXAMPLE 4.23

A 500 µH inductor, $\frac{80}{\pi^2} pF$ capacitor and a 628 Ω resistor are connected to form a series RLC circuit. Calculate the resonant frequency and Q-factor of this circuit at resonance.

Solution

$$L = 500\times10^{-6}\text{H};\ C = \frac{80}{\pi^2}\times10^{-12} F;\ R = 628\Omega$$

(i) Resonant frequency is

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{500\times10^{-6}\times\frac{80}{\pi^2}\times10^{-12}}}$$

$$= \frac{1}{2\sqrt{40{,}000\times10^{-18}}}$$

$$= \frac{10{,}000\times10^3}{4}$$

$$f_r = 2500\ KHz$$

(ii) Q–factor

$$= \frac{\omega_r L}{R} = \frac{2\times3.14\times2500\times10^3\times500\times10^{-6}}{628}$$

$$Q = 12.5$$

EXAMPLE 4.24

Find the instantaneous value of alternating voltage $\upsilon = 10\sin(3\pi \times 10^4 t)$ volt at i) 0 s ii) 50 μs iii) 75 μs.

Solution

The given equation is $\upsilon = 10\sin\left(3\pi \times 10^4 t\right)$

(i) At t = 0 s,

$$\upsilon = 10\sin 0 = 0\ V$$

(ii) At t = 50 μs,

$$\upsilon = 10\sin\left(3\pi \times 10^4 \times 50 \times 10^{-6}\right)$$
$$= 10\sin\left(150\pi \times 10^{-2}\right)$$
$$= 10\sin(4.71\ rad)$$
$$= 10 \times -0.99$$
$$= -9.9\ V$$

(iii) At t = 75 μs,

$$\upsilon = 10\sin\left(3\pi \times 10^4 \times 75 \times 10^{-6}\right)$$
$$= 10\sin\left(225\pi \times 10^{-2}\right)$$
$$= 10\sin(7.071\ rad)$$
$$= 10 \times 0.709$$
$$= 7.09\ V$$

EXAMPLE 4.25

The current in an inductive circuit is given by 0.3 sin (200t – 40°) A. Write the equation for the voltage across it if the inductance is 40 mH.

Solution

$L = 40 \times 10^{-3}$ H; $i = 0.1 \sin(200t - 40^\circ)$

$X_L = \omega L = 200 \times 40 \times 10^{-3} = 8\ \Omega$

$V_m = I_m X_L = 0.3 \times 8 = 2.4$ V

In an inductive circuit, the voltage leads the current by 90° Therefore,

$$\upsilon = V_m \sin\left(\omega t + 90^\circ\right)$$
$$\upsilon = 2.4\sin\left(200t - 40^\circ + 90^\circ\right)$$
$$\upsilon = 2.4\sin\left(200t + 50^\circ\right) volt$$

4.8 POWER IN AC CIRCUITS

4.8.1 Introduction of power in AC circuits

Power of a circuit is defined as the rate of consumption of electric energy in that circuit. It is given by the product of the voltage and current. In an AC circuit, the voltage and current vary continuously with time. Let us first calculate the power at an instant and then it is averaged over a complete cycle.

The alternating voltage and alternating current in the series *RLC* circuit at an instant are given by

$$\upsilon = V_m \sin\omega t \quad \text{and} \quad i = I_m \sin\left(\omega t + \phi\right)$$

where ϕ is the phase angle between υ and i. The instantaneous power is then written as

$$P = \upsilon\, i$$
$$= V_m I_m \sin\omega t \sin\left(\omega t + \phi\right)$$
$$= V_m I_m \sin\omega t\left[\sin\omega t\cos\phi - \cos\omega t\sin\phi\right]$$
$$P = V_m I_m\left[\cos\phi\sin^2\omega t - \sin\omega t\cos\omega t\sin\phi\right] \quad (4.61)$$

Here the average of $\sin^2 \omega t$ over a cycle is $\frac{1}{2}$ and that of $\sin \omega t \cos \omega t$ is zero. Substituting these values, we obtain average power over a cycle.

$$P_{av} = V_m I_m \cos\phi \times \frac{1}{2}$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi$$

$$P_{av} = V_{RMS}\ I_{RMS} \cos\phi \qquad (4.62)$$

where $V_{RMS} I_{RMS}$ is called apparent power and cos ϕ is power factor. The average power of an AC circuit is also known as the true power of the circuit.

Special Cases

(i) For a purely resistive circuit, the phase angle between voltage and current is zero and cos ϕ = 1.

$\therefore P_{av} = V_{RMS}\ I_{RMS}$

(ii) For a purely inductive or capacitive circuit, the phase angle is $\pm \frac{\pi}{2}$ and $\cos\left(\pm \frac{\pi}{2}\right) = 0$.

$\therefore P_{av} = 0$

(iii) For series RLC circuit, the phase angle $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

$\therefore P_{av} = V_{RMS}\ I_{RMS}\ \cos\phi$

(iv) For series RLC circuit at resonance, the phase angle is zero and $\cos\phi = 1$.

$\therefore P_{av} = V_{RMS}\ I_{RMS}$

4.8.2 Wattless current

Consider an AC circuit in which there is a phase angle of ϕ between V_{RMS} and I_{RMS} and voltage is assumed to be leading the current by ϕ as shown in the phasor diagram (Figure 4.55).

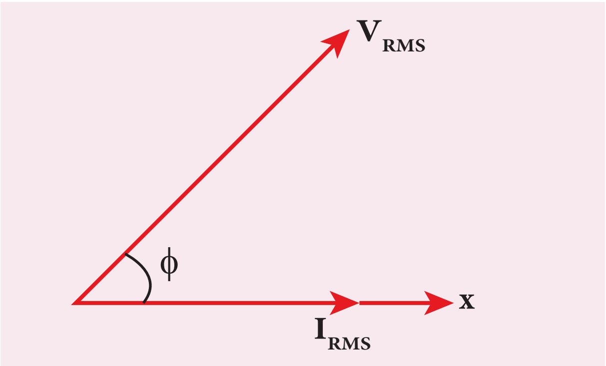


Figure 4.55 V_{RMS} leads I_{RMS} by ϕ

Now, I_{RMS} is resolved into two perpendicular components namely, $I_{RMS} \cos\phi$ along V_{RMS} and $I_{RMS} \sin\phi$ perpendicular to V_{RMS} as shown in Figure 4.56.

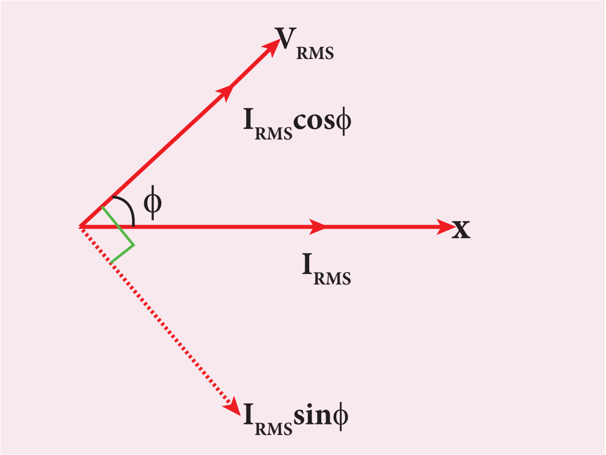


Figure 4.56 The components of I_{RMS}

(i) The component of current $(I_{RMS} \cos\phi)$ which is in phase with the voltage is called active component. The power consumed by this current $= V_{RMS} I_{RMS} \cos\phi$. So that it is also known as 'Wattful' current.

(ii) The other component $(I_{RMS} \sin\phi)$ which has a phase angle of $\frac{\pi}{2}$ with the voltage is called reactive component. The power consumed is zero. So that it is also known as 'Wattless' current.

The current in an AC circuit is said to be wattless current if the power consumed by it is zero. This wattless current happens in a purely inductive or capacitive circuit.

4.8.3 Power factor

The power factor of a circuit is defined in one of the following ways:

(i) **Power factor = cos ϕ = cosine of the angle of lead or lag**

(ii) **Power factor =** $\frac{R}{Z} = \frac{\text{Resistance}}{\text{Impedance}}$

(iii) **Power factor =** $\frac{VI\cos\phi}{VI}$

$$= \frac{\text{True power}}{\text{Apparent power}}$$

Some examples for power factors:

(i) Power factor = cos 0° = 1 for a pure resistive circuit because the phase angle ϕ between voltage and current is zero.

(ii) Power factor $= \cos\left(\pm \pi/2\right) = 0$ for a purely inductive or capacitive circuit because the phase angle ϕ between voltage and current is $\pm \pi/2$.

(iii) Power factor lies between 0 and 1 for a circuit having R, L and C in varying proportions.

4.8.4 Advantages and disadvantages of AC over DC

There are many advantages and disadvantages of AC system over DC system.

Advantages:

(i) The generation of AC is cheaper than that of DC.

(ii) When AC is supplied at higher voltages, the transmission losses are small compared to DC transmission.

(iii) AC can easily be converted into DC with the help of rectifiers.

Disadvantages:

(i) Alternating voltages cannot be used for certain applications e.g. charging of batteries, electroplating, electric traction etc.

(ii) At high voltages, it is more dangerous to work with AC than DC.

EXAMPLE 4.26

A series RLC circuit which resonates at 400 kHz has 80 μH inductor, 2000 pF capacitor and 50 Ω resistor. Calculate (i) Q-factor of the circuit (ii) the new value of capacitance when the value of inductance is doubled and (iii) the new Q-factor.

Solution

$L = 80 \times 10^{-6}$H; $C = 2000 \times 10^{-12}$ F

$R = 50\ \Omega$; $f_r = 400 \times 10^3$Hz

(i) Q-factor, $Q_1 = \frac{1}{R}\sqrt{\frac{L}{C}}$

$$= \frac{1}{50}\sqrt{\frac{80\times 10^{-6}}{200\times 10^{-12}}} = 4$$

(ii) When $L_2 = 2$ L

$= 2 \times 80 \times 10^{-6}$ H $= 160 \times 10^{-6}$ H,

$$C_2 = \frac{1}{4\pi^2 f_r^2 L_2}$$

$$= \frac{1}{4\times 3.14^2 \times \left(400\times 10^3\right)^2 \times 160\times 10^{-6}}$$

$$\simeq 1000\times 10^{-12} F$$

$$C_2 \simeq 1000\, pF$$

(iii) $Q_2 = \frac{1}{R}\sqrt{\frac{L_2}{C_2}} = \frac{1}{50}\sqrt{\frac{160\times 10^{-6}}{1000\times 10^{-12}}}$

$$= \frac{1}{50}\sqrt{\frac{16\times 10^{-5}}{10^{-9}}} = \frac{4\times 10^2}{50} = 8$$

EXAMPLE 4.27

A capacitor of capacitance $\frac{10^{-4}}{\pi}F$, an inductor of inductance $\frac{2}{\pi}H$ and a resistor of resistance 100 Ω are connected to form a series *RLC* circuit. When an AC supply of 220 V, 50 Hz is applied to the circuit, determine (i) the impedance of the circuit (ii) the peak value of current flowing in the circuit (iii) the power factor of the circuit and (iv) the power factor of the circuit at resonance.

Solution

$$L = \frac{2}{\pi}H; C = \frac{10^{-4}}{\pi}F; R = 100\Omega$$
$$V_{RMS} = 220\ V; f = 50\,Hz$$

$$X_L = 2\pi fL = 2\pi \times 50 \times {}^{2}\!/_{\pi} = 200\Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times \frac{10^{-4}}{\pi}} = 100\Omega$$

(i) Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{100^2 + (200-100)^2} = 141.4\ \Omega$$

(ii) Peak value of current,

$$I_m = \frac{V_m}{Z} = \frac{\sqrt{2}\,V_{RMS}}{Z}$$
$$= \frac{\sqrt{2}\times 220}{141.4} = 2.2\text{ A}$$

(iii) Power factor of the circuit,

$$\cos\phi = \frac{R}{Z} = \frac{100}{141.4} = 0.707$$

(iv) Power factor at resonance

$$\cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$$

4.9 OSCILLATION IN LC CIRCUITS

4.9.1 Energy conversion during LC oscillations

We have learnt that energy can be stored in both inductors and capacitors (Refer sections 1.8.2 and 4.3.2). In inductors, the energy is stored in the form of magnetic field while in capacitors; it is stored as the electric field.

Whenever energy is given to a circuit containing a pure inductor of inductance *L* and a capacitor of capacitance *C*, the energy oscillates back and forth between the magnetic field of the inductor and the electric field of the capacitor. Thus the electrical oscillations of definite frequency are generated. These oscillations are called *LC* oscillations.

Generation of LC oscillations

Let us assume that the capacitor is fully charged with maximum charge Q_m at the initial stage. So that the energy stored in the capacitor is maximum and is given by $U_E = {Q_m^2}/{2C}$. As there is no current in the inductor, the energy stored in it is zero i.e., $U_B = 0$. Therefore, the total energy is wholly electrical. This is shown in Figure 4.57(a).

The capacitor now begins to discharge through the inductor that establishes current *i* in clockwise direction. This current produces a magnetic field around the inductor and the energy stored in the inductor is given by $U_B = {Li^2}/{2}$. As the charge in the capacitor decreases, the energy stored in it also decreases and is given by

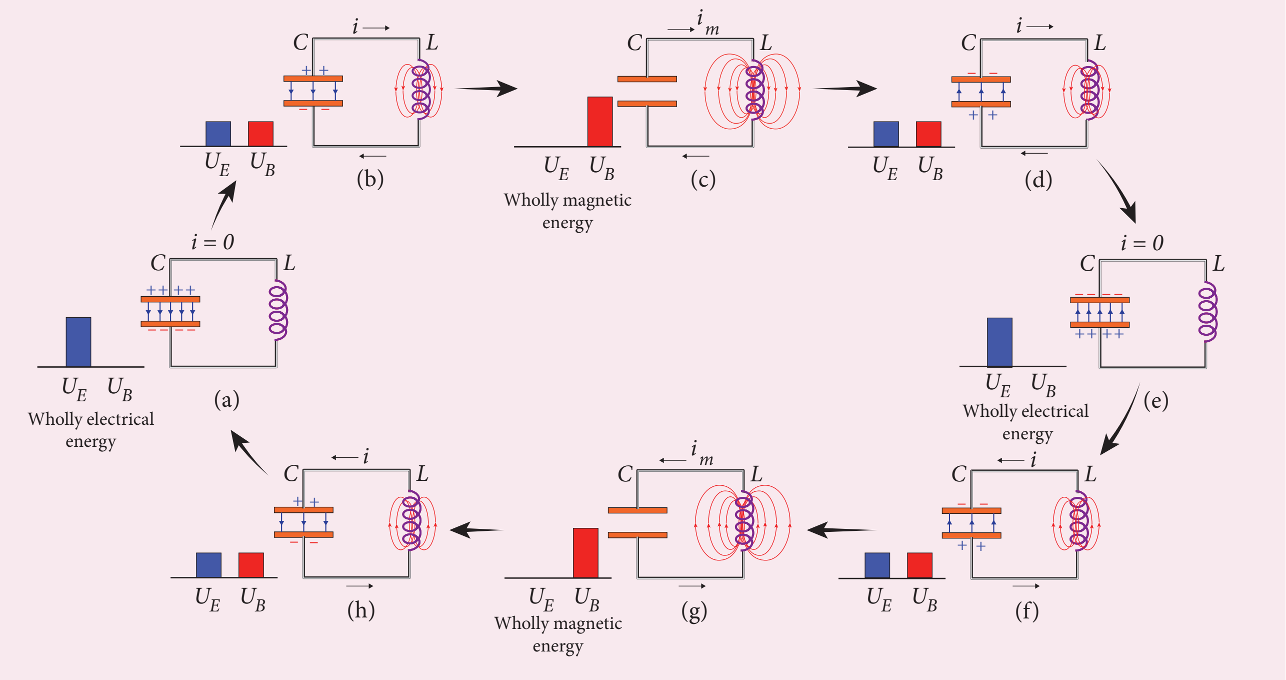


Figure 4.57 LC oscillations

$U_E = q^2/2C$. Thus there is a transfer of some part of energy from the capacitor to the inductor. At that instant, the total energy is the sum of electrical and magnetic energies (Figure 4.57(b)).

When the charges in the capacitor are exhausted, its energy becomes zero i.e., $U_E = 0$. The energy is fully transferred to the magnetic field of the inductor and its energy is maximum. This maximum energy is given by $U_B = LI_m^2/2$ where I_m is the maximum current flowing in the circuit. The total energy is wholly magnetic (Figure 4.57(c)).

Even though the charge in the capacitor is zero, the current will continue to flow in the same direction because the inductor will not allow it to stop immediately. The current is made to flow with decreasing magnitude by the collapsing magnetic field of the inductor. As a result of this, the capacitor begins to charge in the opposite direction. A part of the energy is transferred from the inductor back to the capacitor. The total energy is the sum of the electrical and magnetic energies (Figure 4.57(d)).

When the current in the circuit reduces to zero, the capacitor becomes fully charged in the opposite direction. The energy stored in the capacitor becomes maximum. Since the current is zero, the energy stored in the inductor is zero. The total energy is wholly electrical (Figure 4.57(e)).

The state of the circuit is similar to the initial state but the difference is that the capacitor is charged in opposite direction. The capacitor then starts to discharge through the inductor with anti-clockwise current. The total energy is the sum of the electrical and magnetic energies (Figure 4.57(f)).

As already explained, the processes are repeated in opposite direction (Figure 4.57(g) and (h)). Finally, the circuit returns to the initial state (Figure 4.57(a)). Thus, when the circuit goes through these stages, an alternating current flows in the circuit. As this process is repeated again and again, the electrical oscillations of definite frequency are generated. These are known as *LC* oscillations.

In the ideal *LC* circuit, there is no loss of energy. Therefore, the oscillations will continue indefinitely. Such oscillations are called undamped oscillations.

Note But in practice, the Joule heating and radiation of electromagnetic waves from the circuit decrease the energy of the system. Therefore, the oscillations become damped oscillations.

4.9.2 Conservation of energy in LC oscillations

During *LC* oscillations in *LC* circuits, the energy of the system oscillates between the electric field of the capacitor and the magnetic field of the inductor. Although, these two forms of energy vary with time, the total energy remains constant. It means that *LC* oscillations take place in accordance with the law of conservation of energy.

$$\text{Total energy}, U = U_E + U_B = \frac{q^2}{2C} + \frac{1}{2}Li^2$$

Let us consider 3 different stages of *LC* oscillations and calculate the total energy of the system.

Case (i) When the charge in the capacitor, $q = Q_m$ and the current through the inductor, $i = 0$, the total energy is given by

$$U = \frac{Q_m^2}{2C} + 0 = \frac{Q_m^2}{2C} \qquad (4.63)$$

The total energy is wholly electrical.

Case (ii) When charge = 0 ; current = I_m, the total energy is

$$U = 0 + \frac{1}{2}LI_m^2 = \frac{1}{2}LI_m^2$$

$$= \frac{L}{2} \times \left(\frac{Q_m^2}{LC}\right) \quad \text{since } I_m = Q_m\omega = \frac{Q_m}{\sqrt{LC}}$$

$$= \frac{Q_m^2}{2C} \qquad (4.64)$$

The total energy is wholly magnetic.

Case (iii) When charge = q; current = i, the total energy is

$$U = \frac{q^2}{2C} + \frac{1}{2}Li^2$$

Since $q = Q_m \cos\omega t,\ i = -\frac{dq}{dt} = Q_m \omega \sin\omega t$.

The negative sign in current indicates that the charge in the capacitor decreases with time.

$$U = \frac{Q_m^2 \cos^2 \omega t}{2C} + \frac{L\omega^2 Q_m^2 \sin^2 \omega t}{2}$$

$$= \frac{Q_m^2 \cos^2 \omega t}{2C} + \frac{LQ_m^2 \sin^2 \omega t}{2LC} \quad \text{since } \omega^2 = \frac{1}{LC}$$

$$= \frac{Q_m^2}{2C}\left(\cos^2 \omega t + \sin^2 \omega t\right)$$

$$U = \frac{Q_m^2}{2C} \qquad (4.65)$$

From above three cases, it is clear that the total energy of the system remains constant.

4.9.3 Analogies between LC oscillations and simple harmonic oscillations

(i) Qualitative treatment

The electromagnetic oscillations of *LC* system can be compared with the mechanical oscillations of a spring-mass system.

There are two forms of energy involved in *LC* oscillations. One is electrical energy

Table 4.3 Energy in two oscillatory systems

LC oscillator		Spring-mass system	
Element	**Energy**	**Element**	**Energy**
Capacitor	Electrical Energy $= \frac{1}{2}\left(\frac{1}{C}\right)q^2$	Spring	Potential energy $= \frac{1}{2}k\,x^2$
Inductor	Magnetic energy $= \frac{1}{2}Li^2 \quad i = \frac{dq}{dt}$	Mass	Kinetic energy $= \frac{1}{2}mv^2 \quad v = \frac{dx}{dt}$

of the charged capacitor; the other magnetic energy of the inductor carrying current.

Likewise, the mechanical energy of the spring-mass system exists in two forms; the potential energy of the compressed or extended spring and the kinetic energy of the mass. The Table 4.3 lists these two pairs of energy.

By examining the Table 4.3, the analogies between the various quantities can be understood and these correspondences are given in the Table 4.4.

The angular frequency of oscillations of a spring-mass is given by (Refer equation 10.22 of section 10.4.1 of XI physics text book).

$$\omega = \sqrt{\frac{k}{m}}$$

From Table 4.4, $k \to {}^{1}\!/_{C}$ and $m \to L$. Therefore, the angular frequency of LC oscillations is given by

$$\omega = \frac{1}{\sqrt{LC}} \qquad (4.66)$$

(ii) Quantitative treatment

The mechanical energy of the spring-mass system is given by

$$E = \frac{1}{2}mv^2 + \frac{1}{2}k\,x^2 \qquad (4.67)$$

The energy E remains constant for varying values of x and v. Differentiating E with respect to time, we get

Table 4.4 Analogies between electrical and mechanical quantities

Electrical system	Mechanical system
Charge q	Displacement x
Current $i = \frac{dq}{dt}$	Velocity $v = \frac{dx}{dt}$
Inductance L	Mass m
Reciprocal of capacitance $\frac{1}{C}$	Force constant k
Electrical energy $= \frac{1}{2}\left(\frac{1}{C}\right)q^2$	Potential energy $= \frac{1}{2}k\,x^2$
Magnetic energy $= \frac{1}{2}Li^2$	Kinetic energy $= \frac{1}{2}mv^2$
Electromagnetic energy $U = \frac{1}{2}\left(\frac{1}{C}\right)q^2 + \frac{1}{2}Li^2$	Mechanical energy $E = \frac{1}{2}k\,x^2 + \frac{1}{2}mv^2$

$$\frac{dE}{dt} = \frac{1}{2}m\left(2v\frac{dv}{dt}\right) + \frac{1}{2}k\left(2x\frac{dx}{dt}\right) = 0$$

$$\text{or} \quad m\frac{d^2x}{dt^2} + kx = 0 \qquad (4.68)$$

$$\text{since } \frac{dx}{dt} = v \;\; and \;\; \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

This is the differential equation of the oscillations of the spring-mass system. The general solution of equation (4.68) is of the form

$$x(t) = X_m \cos(\omega t + \phi) \qquad (4.69)$$

where X_m is the maximum value of $x(t)$, ω the angular frequency and ϕ the phase constant.

Similarly, the electromagnetic energy of the *LC* system is given by

$$U = \frac{1}{2}Li^2 + \frac{1}{2}\left(\frac{1}{C}\right)q^2 = \text{constant} \qquad (4.70)$$

Differentiating *U* with respect to time, we get

$$\frac{dU}{dt} = \frac{1}{2}L\left(2i\frac{di}{dt}\right) + \frac{1}{2C}\left(2q\frac{dq}{dt}\right) = 0$$

$$\text{or} \quad L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0 \qquad (4.71)$$

$$\text{since } i = \frac{dq}{dt} \text{ and } \frac{di}{dt} = \frac{d^2q}{dt^2}$$

The general solution of equation (4.71) is of the form

$$q(t) = Q_m \cos(\omega t + \phi) \qquad (4.72)$$

where Q_m is the maximum value of $q(t)$, ω the angular frequency and ϕ the phase constant.

Current in the LC circuit

The current flowing in the *LC* circuit is obtained by differentiating *q(t)* with respect to time.

$$i(t) = \frac{dq}{dt} = \frac{d}{dt}\left[Q_m \cos(\omega t + \phi)\right]$$

$$= -Q_m \omega \sin(\omega t + \phi) \qquad \text{since } I_m = Q_m \omega$$

$$\text{or} \quad i(t) = -I_m \sin(\omega t + \phi) \qquad (4.73)$$

The equation (4.73) clearly shows that current varies as a function of time *t*. In fact, it is a sinusoidally varying alternating current with angular frequency ω.

Angular frequency of LC oscillations

By differentiating equation (4.72) twice, we get

$$\frac{d^2q}{dt} = -Q_m \omega^2 \cos(\omega t + \phi) \qquad (4.74)$$

Substituting equations (4.72) and (4.74) in equation (4.71), we obtain

$$L\left[-Q_m \omega^2 \cos(\omega t + \phi)\right] + \frac{1}{C}Q_m \cos(\omega t + \phi) = 0$$

Rearranging the terms, the angular frequency of *LC* oscillations is given by

$$\omega = \frac{1}{\sqrt{LC}} \qquad (4.75)$$

This equation is the same as that obtained from qualitative analogy.

Oscillations of electrical and magnetic energy

The electrical energy of the LC oscillator is

$$U_E = \frac{q^2}{2C} = \frac{Q_m^2}{2C}\cos^2(\omega t + \phi) \qquad (4.76)$$

The magnetic energy is

$$U_B = \frac{1}{2}Li^2 = \frac{Q_m^2}{2C}\sin^2(\omega t + \phi) \qquad (4.77)$$

If the two energies are plotted with an assumption of $\phi = 0$, we obtain Figure 4.58.

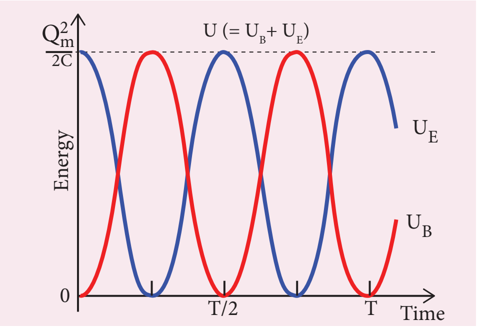


Figure 4.58 The variation of U_E and U_B as a function of time

From the graph, it can be noted that

(i) At any instant $U_E + U_B = \frac{Q_m^2}{2C}$ =constant

(ii) The maximum values of U_E and U_B are both $\frac{Q_m^2}{2C}$.

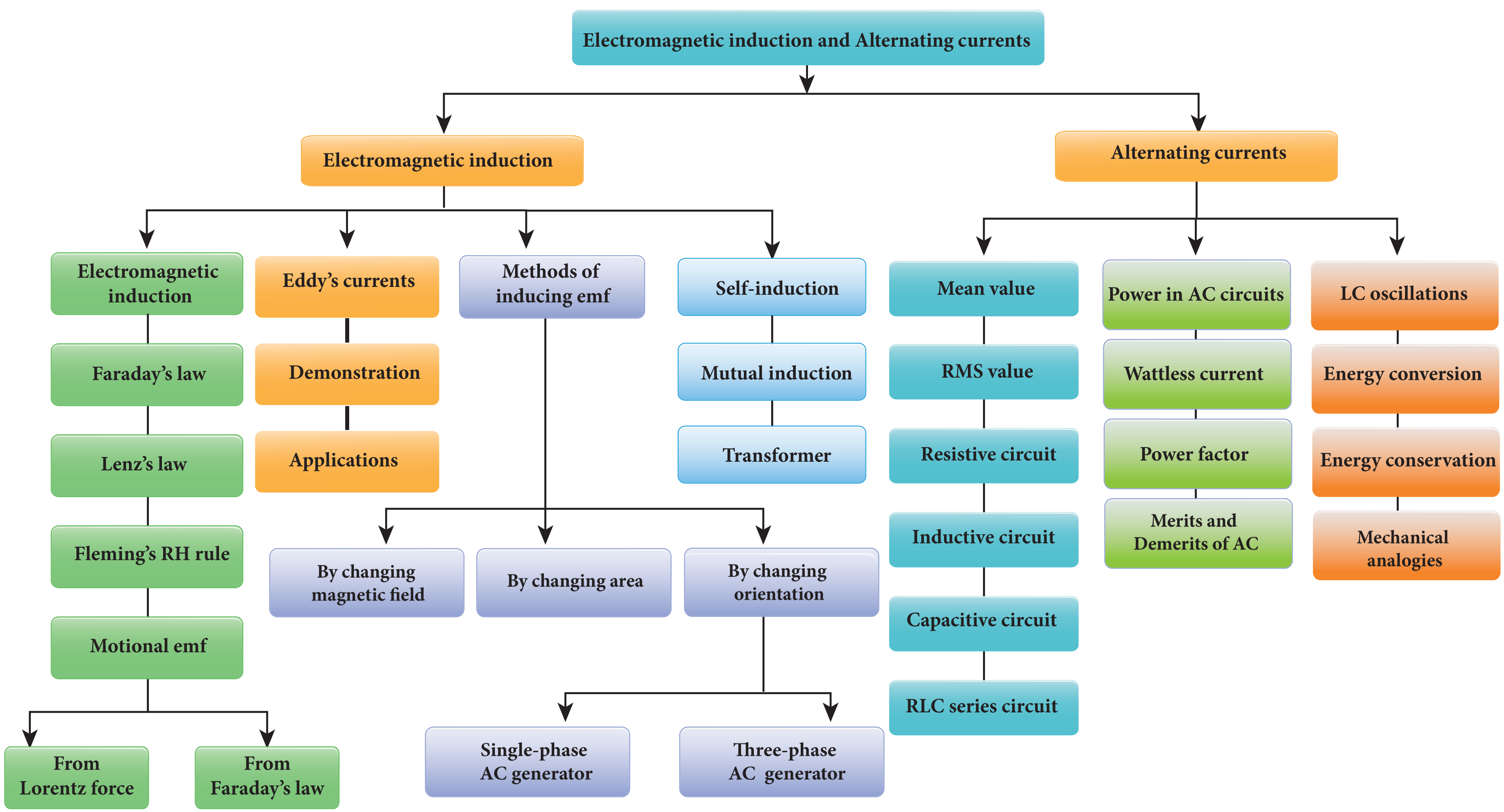
(iii) When U_E is Maximum, U_B is zero and vice versa.

SUMMARY

- Whenever the magnetic flux linked with a closed coil changes, an emf is induced and hence an electric current flows in the circuit. This phenomenon is known as electromagnetic induction.
- Faraday's first law states that whenever magnetic flux linked with a closed circuit changes, an emf is induced in the circuit.
- Faraday's second law states that the magnitude of induced emf in a closed circuit is equal to the time rate of change of magnetic flux linked with the circuit.
- Lenz's law states that the direction of the induced current is such that it always opposes the cause responsible for its production.
- Lenz's law is established on the basis of the law of conservation of energy.
- Fleming's right hand rule states that if the index finger points the direction of the magnetic field and the thumb indicates the direction of motion of the conductor, then the middle finger will indicate the direction of the induced current.
- Even for a conductor in the form of a sheet or a plate, an emf is induced when magnetic flux linked with it changes. The induced currents flow in concentric circular paths called Eddy currents or Foucault currents.
- Inductor is a device used to store energy in a magnetic field when an electric current flows through it.
- If the flux linked with the coil is changed, an emf is induced in that same coil. This phenomenon is known as self-induction. The emf induced is called self-induced emf.
- When an electric current passing through a coil changes with time, an emf is induced in the neighbouring coil. This phenomenon is known as mutual induction and the emf is called mutually induced emf.
- AC generator or alternator is an energy conversion device. It converts mechanical energy used to rotate the coil or field magnet into electrical energy.
- In some AC generators, there are three separate coils, which would give three separate emfs. Hence they are called three-phase AC generators.
- Transformer is a stationary device used to transform AC electric power from one circuit to another without changing its frequency.
- The efficiency of a transformer is defined as the ratio of the useful output power to the input power.
- An alternating voltage is a voltage which changes polarity at regular intervals of time and the resulting alternating current changes direction accordingly.
- The average value of alternating current is defined as the average of all values of current over a positive half-circle or negative half-circle.
- The root mean square value or effective value of an alternating current is defined as the square root of the mean of the squares of all currents over one cycle.

- A sinusoidal alternating voltage (or current) can be represented by a vector which rotates about the origin in anti-clockwise direction at a constant angular velocity. Such a rotating vector is called a phasor.
- When the frequency of the applied alternating source is equal to the natural frequency of the RLC circuit, the current in the circuit reaches its maximum value. Then the circuit is said to be in electrical resonance.
- The magnification of voltages at series resonance is termed as Q–factor.
- Power of a circuit is defined as the rate of consumption of electric energy in that circuit. It depends on the components of the circuit.
- Whenever energy is given to a LC circuit, the electrical oscillations of definite frequency are generated. These oscillations are called LC oscillations.
- During LC oscillations, the total energy remains constant. It means that LC oscillations take place in accordance with the law of conservation of energy.

CONCEPT MAP



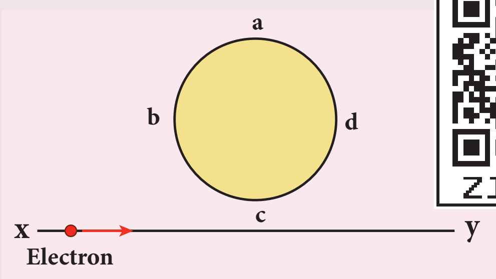
Electromagnetic induction and Alternating currents
Electromagnetic induction
Alternating currents
Electromagnetic induction
Faraday's law
Lenz's law
Fleming's RH rule
Motional emf
From Lorentz force
From Faraday's law
Eddy's currents
Demonstration
Applications
Methods of inducing emf
By changing magnetic field
By changing area
By changing orientation
Single-phase AC generator
Three-phase AC generator
Self-induction
Mutual induction
Transformer
Mean value
RMS value
Resistive circuit
Inductive circuit
Capacitive circuit
RLC series circuit
Power in AC circuits
Wattless current
Power factor
Merits and Demerits of AC
LC oscillations
Energy conversion
Energy conservation
Mechanical analogies

EVALUATION

I Multiple Choice Questions

1. An electron moves on a straight line path XY as shown in the figure. The coil *abcd* is adjacent to the path of the electron. What will be the direction of current, if any, induced in the coil?

(NEET – 2015)



ZJR6WT

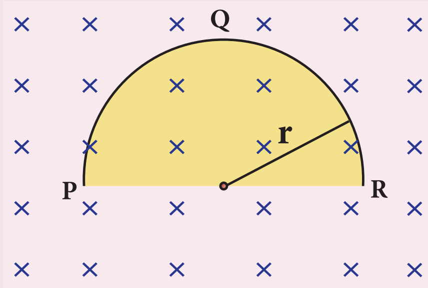
(a) The current will reverse its direction as the electron goes past the coil

(b) No current will be induced

(c) abcd

(d) adcb

2. A thin semi-circular conducting ring (PQR) of radius r is falling with its plane vertical in a horizontal magnetic field B, as shown in the figure.



The potential difference developed across the ring when its speed v, is

(NEET 2014)

(a) Zero

(b) $\frac{Bv\pi r^2}{2}$ and P is at higher potential

(c) πrBv and R is at higher potential

(d) $2rBv$ and R is at higher potential

3. The flux linked with a coil at any instant t is given by $\Phi_B = 10t^2 - 50t + 250$. The induced emf at t = 3s is

(a) $-190\ V$ (b) $-10\ V$

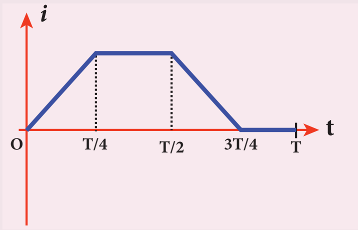
(c) $10\ V$ (d) $190\ V$

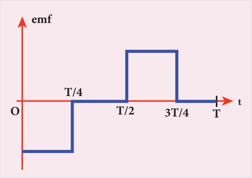
4. When the current changes from +2A to −2A in 0.05 s, an emf of 8 V is induced in a coil. The co-efficient of self-induction of the coil is

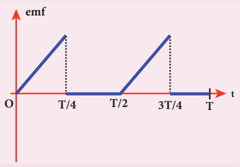
(a) 0.2 H (b) 0.4 H

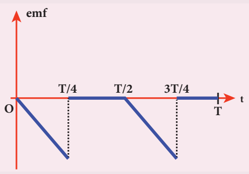
(c) 0.8 H (d) 0.1 H

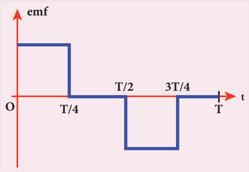
5. The current *i* flowing in a coil varies with time as shown in the figure. The variation of induced emf with time would be (NEET – 2011)



(a)


(b)


(c)


(d)


6. A circular coil with a cross-sectional area of 4 cm^2 has 10 turns. It is placed at the centre of a long solenoid that has 15 turns/cm and a cross-sectional area of 10 cm^2. The axis of the coil coincides with the axis of the solenoid. What is their mutual inductance?

(a) 7.54 μH (b) 8.54 μH

(c) 9.54 μH (d) 10.54 μH

7. In a transformer, the number of turns in the primary and the secondary are 410 and 1230 respectively. If the current in primary is 6A, then that in the secondary coil is

(a) 2 A (b) 18 A

(c) 12 A (d) 1 A

8. A step-down transformer reduces the supply voltage from 220 V to 11 V and increase the current from 6 A to 100 A. Then its efficiency is

(a) 1.2 (b) 0.83

(c) 0.12 (d) 0.9

9. In an electrical circuit, *R, L, C* and AC voltage source are all connected in series. When *L* is removed from the circuit, the phase difference between the voltage and current in the circuit is $\pi/3$. Instead, if C is removed from the circuit, the phase difference is again $\pi/3$. The power factor of the circuit is

(NEET 2012)

(a) $1/2$ (b) $1/\sqrt{2}$

(c) 1 (d) $\sqrt{3}/2$

10. In a series RL circuit, the resistance and inductive reactance are the same. Then the phase difference between the voltage and current in the circuit is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{6}$ (d) zero

11. In a series resonant *RLC* circuit, the voltage across 100 Ω resistor is 40 V. The resonant frequency ω is 250 rad/s. If the value of C is 4 μF, then the voltage across L is

(a) 600 V (b) 4000 V

(c) 400V (d) 1 V

12. An inductor 20 *mH*, a capacitor 50 *μF* and a resistor 40 Ω are connected in series across a source of emf v = 10 sin 340 t. The power loss in AC circuit is

(a) 0.76 W (b) 0.89 W

(c) 0.46 W (d) 0.67 W

13. The instantaneous values of alternating current and voltage in a circuit are

$i = \frac{1}{\sqrt{2}}\sin(100\pi t)$ A and

$v = \frac{1}{\sqrt{2}}\sin\left(100\pi t + \frac{\pi}{3}\right)$ V.

The average power in watts consumed in the circuit is (IIT Main 2012)

(a) $\frac{1}{4}$ (b) $\frac{\sqrt{3}}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{8}$

14. In an oscillating LC circuit, the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic fields is

(a) $\frac{Q}{2}$ (b) $\frac{Q}{\sqrt{3}}$

(c) $\frac{Q}{\sqrt{2}}$ (d) Q

15. $\frac{20}{\pi^2}H$ inductor is connected to a capacitor of capacitance C. The value of C in order to impart maximum power at 50 Hz is

(a) 50 μF (b) 0.5 μF

(c) 500 μF (d) 5 μF

Answers

1) a	2) d	3) b	4) d	5) a
6) a	7) a	8) b	9) c	10) a
11) c	12) c	13) d	14) c	15) d

II Short Answer Questions

1. What is meant by electromagnetic induction?
4. State Faraday's laws of electromagnetic induction.
5. State Lenz's law.
6. State Fleming's right hand rule.
7. How is Eddy current produced? How do they flow in a conductor?
8. Mention the ways of producing induced emf.
9. What for an inductor is used? Give some examples.
10. What do you mean by self-induction?
11. What is meant by mutual induction?
12. Give the principle of AC generator.
13. List out the advantages of stationary armature-rotating field system of AC generator.
14. What are step-up and step-down transformers?
15. Define average value of an alternating current.
16. How will you define RMS value of an alternating current?
17. What are phasors?
18. Define electric resonance.
19. What do you mean by resonant frequency?
20. How will you define Q-factor?
21. What is meant by wattles current?
22. Give any one definition of power factor.
23. What are LC oscillations?

III Long Answer Questions

1. Establish the fact that the relative motion between the coil and the magnet induces an emf in the coil of a closed circuit.
2. Give an illustration of determining direction of induced current by using Lenz's law.
3. Show that Lenz's law is in accordance with the law of conservation of energy.
4. Obtain an expression for motional emf from Lorentz force.
5. Using Faraday's law of electromagnetic induction, derive an equation for motional emf.
6. Give the uses of Foucault current.
7. Define self-inductance of a coil interms of (i) magnetic flux and (ii) induced emf.
8. How will you define the unit of inductance?
9. What do you understand by self-inductance of a coil? Give its physical significance.
10. Assuming that the length of the solenoid is large when compared to

its diameter, find the equation for its inductance.

11. An inductor of inductance L carries an electric current i. How much energy is stored while establishing the current in it?

12. Show that the mutual inductance between a pair of coils is same ($M_{12} = M_{21}$).

13. How will you induce an emf by changing the area enclosed by the coil?

14. Show mathematically that the rotation of a coil in a magnetic field over one rotation induces an alternating emf of one cycle.

15. Elaborate the standard construction details of AC generator.

16. Explain the working of a single-phase AC generator with necessary diagram.

17. How are the three different emfs generated in a three-phase AC generator? Show the graphical representation of these three emfs.

18. Explain the construction and working of transformer.

19. Mention the various energy losses in a transformer.

20. Give the advantage of AC in long distance power transmission with an example.

21. Find out the phase relationship between voltage and current in a pure inductive circuit.

22. Derive an expression for phase angle between the applied voltage and current in a series RLC circuit.

23. Define inductive and capacitive reactance. Give their units.

24. Obtain an expression for average power of AC over a cycle. Discuss its special cases.

25. Show that the total energy is conserved during LC oscillations.

26. Prove that energy is conserved during electromagnetic induction.

27. Compare the electromagnetic oscillations of LC circuit with the mechanical oscillations of block-spring system to find the expression for angular frequency of LC oscillators mathematically.

IV. Numerical problems

1. A square coil of side 30 cm with 500 turns is kept in a uniform magnetic field of 0.4 T. The plane of the coil is inclined at an angle of 30° to the field. Calculate the magnetic flux through the coil. (Ans: 9 Wb)

2. A straight metal wire crosses a magnetic field of flux 4 mWb in a time 0.4 s. Find the magnitude of the emf induced in the wire. (Ans: 10 mV)

3. The magnetic flux passing through a coil perpendicular to its plane is a function of time and is given by $\Phi_B = (2t^3 + 4t^2 + 8t + 8)\ Wb$. If the resistance of the coil is 5 Ω, determine the induced current through the coil at a time t = 3 second. (Ans: 17.2 A)

4. A closely wound coil of radius 0.02 m is placed perpendicular to the magnetic field. When the magnetic field is changed from 8000 T to 2000 T in 6 s, an emf of 44 V is induced. Calculate the number of turns in the coil. (Ans: 35 turns)

5. A rectangular coil of area 6 cm^2 having 3500 turns is kept in a uniform magnetic field of 0.4 T. Initially, the plane of the coil is perpendicular to the field and is then rotated through an angle of 180^o. If the resistance of the coil is 35 Ω, find the amount of charge flowing through the coil.

(Ans: 48×10^{-3} C)

6. An induced current of 2.5 mA flows through a single conductor of resistance 100 Ω. Find out the rate at which the magnetic flux is cut by the conductor.

(Ans: 250 $mWbs^{-1}$)

7. A fan of metal blades of length 0.4 m rotates normal to a magnetic field of $4\times10^{-3}T$. If the induced emf between the centre and edge of the blade is 0.02 V, determine the rate of rotation of the blade.

(Ans: 9.95 revolutions/second)

8. A bicycle wheel with metal spokes of 1 m long rotates in Earth's magnetic field. The plane of the wheel is perpendicular to the horizontal component of Earth's field of $4\times10^{-5}T$. If the emf induced across the spokes is 31.4 mV, calculate the rate of revolution of the wheel.

(Ans: 250 revolutions/second)

9. Determine the self-inductance of 4000 turn air-core solenoid of length 2m and diameter 0.04 m. (Ans: 12.62 mH)

10. A coil of 200 turns carries a current of 4 A. If the magnetic flux through the coil is 6 x 10^{-5} Wb, find the magnetic energy stored in the medium surrounding the coil. (Ans: 0.024 J)

11. A 50 cm long solenoid has 400 turns per cm. The diameter of the solenoid is 0.04 m. Find the magnetic flux of a turn when it carries a current of 1 A.

(Ans: 1.26 Wb)

12. A coil of 200 turns carries a current of 0.4 A. If the magnetic flux of 4 mWb is linked with the coil, find the inductance of the coil. (Ans: 2H)

13. Two air core solenoids have the same length of 80 cm and same cross–sectional area 5 cm^2. Find the mutual inductance between them if the number of turns in the first coil is 1200 turns and that in the second coil is 400 turns. (Ans: 0.38 mH)

14. A long solenoid having 400 turns per cm carries a current 2A. A 100 turn coil of cross-sectional area 4 cm^2 is placed co-axially inside the solenoid so that the coil is in the field produced by the solenoid. Find the emf induced in the coil if the current through the solenoid reverses its direction in 0.04 sec.

(Ans: 0.20 V)

15. A 200 turn coil of radius 2 cm is placed co-axially within a long solenoid of 3 cm radius. If the turn density of the solenoid is 90 turns per cm, then calculate mutual inductance of the coil.

(Ans: 2.84 mH)

16. The solenoids S_1 and S_2 are wound on an iron-core of relative permeability 900. The area of their cross-section and their length are the same and are 4 cm^2 and 0.04 m respectively. If the number of turns in S_1 is 200 and that in S_2 is 800, calculate the mutual inductance between the coils. The current in solenoid 1 is increased form 2A to 8A

in 0.04 second. Calculate the induced emf in solenoid 2.

(Ans: 1.81H; 271.5 V)

17. A step-down transformer connected to main supply of 220 V is made to operate 11V,88W lamp. Calculate (i) Transformation ratio and (ii) Current in the primary.

(Ans: 1/20 and 0.4A)

18. A 200V/120V step-down transformer of 90% efficiency is connected to an induction stove of resistance 40 Ω. Find the current drawn by the primary of the transformer.

(Ans: 2A)

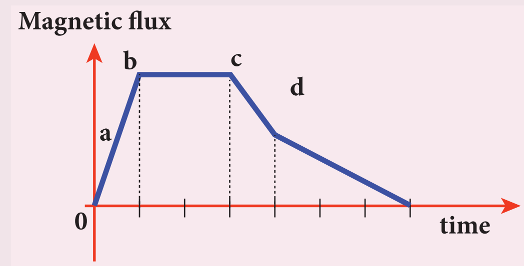
19. The 300 turn primary of a transformer has resistance 0.82 Ω and the resistance of its secondary of 1200 turns is 6.2 Ω. Find the voltage across the primary if the power output from the secondary at 1600V is 32 kW. Calculate the power losses in both coils when the transformer efficiency is 80%.

(Ans: 8.2 kW and 2.48 kW)

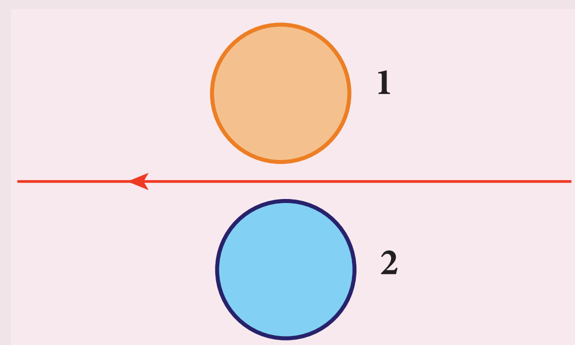
20. Calculate the instantaneous value at 60^o, average value and RMS value of an alternating current whose peak value is 20 A. (Ans: 17.32A, 12.74A, 14.14 A)

IV Conceptual Questions

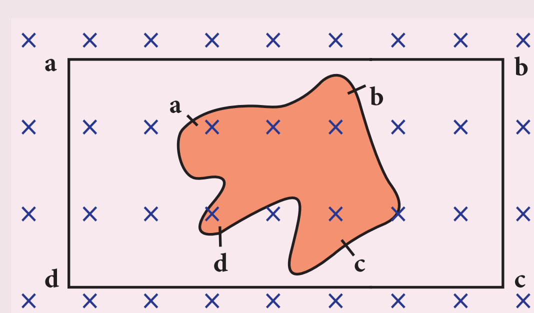
1. A graph between the magnitude of the magnetic flux linked with a closed loop and time is given in the figure. Arrange the regions of the graph in ascending order of the magnitude of induced emf in the loop.



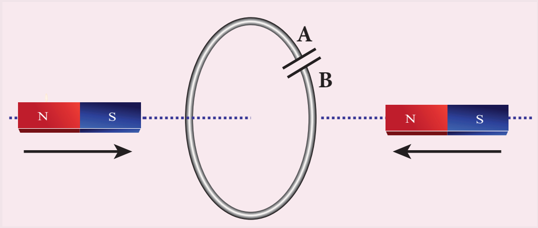
2. Using Lenz's law, predict the direction of induced current in conducting rings 1 and 2 when current in the wire is steadily decreasing.



3. A flexible metallic loop abcd in the shape of a square is kept in a magnetic field with its plane perpendicular to the field. The magnetic field is directed into the paper normally. Find the direction of the induced current when the square loop is crushed into an irregular shape as shown in the figure.



4. Predict the polarity of the capacitor in a closed circular loop when two bar magnets are moved as shown in the figure.



5. In series LC circuit, the voltages across L and C are 180° out of phase. Is it correct? Explain.
6. When does power factor of a series RLC circuit become maximum?
7. Draw graphs showing the distribution of charge in a capacitor and current through an inductor during LC oscillations with respect to time. Assume that the charge in the capacitor is maximum initially.

BOOK FOR REFERENCES

1. H.C.Verma, Concepts of Physics, Volume 1 and 2, Bharathi Bhawan publishers.
2. Halliday, Resnick and Walker, Principles of Physics, Wiley publishers.
3. D.C.Tayal, Electricity and Magnetism, Himalaya Publishing House.
4. K.K.Tewari, Electricity and Magnetism with Electronics, S.Chand Publishers.
5. B.L.Theraja and A.K.Theraja, A text book of Electrical Technology, Volume 1 and 2, S.Chand publishers.

ICT CORNER

Electromagnetic induction and alternating current

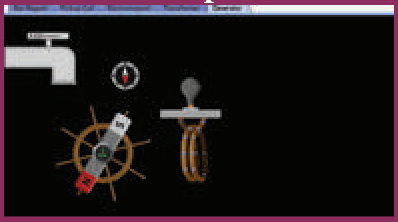
In this activity you will be able to
(1) understand electromagnetic induction.
(2) verify Faraday's laws in virtual lab.

Topic: Faraday's electromagnetic lab

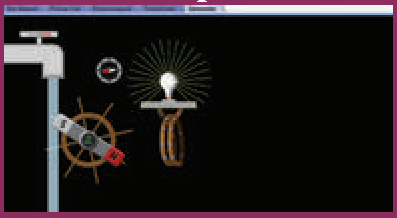
STEPS:

- Open the browser and type "phet.colorado.edu" in the address bar. Click play with simulation tab. Search Faraday's electromagnetic lab in the search box.
- Select 'pick coil' tab. Move the magnet through the coil. Note what happens when the magnetic field linked with the coil changes. Change the loop area, flux change and observe the intensity of current with the help of glowing bulb.
- Select 'Electromagnet' tab, Change the current flowing through the coil and observe the change in magnetic flux generated.
- Select 'Generator' tab. Observe induced emf in the coil if you change the angular velocity of the coil.

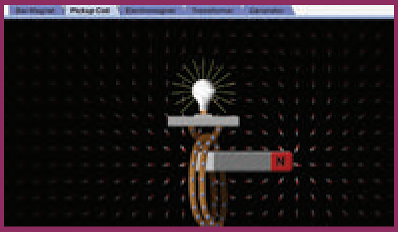
Step1



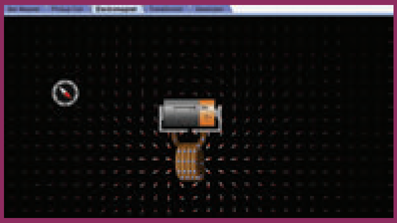
Step2



Step3



Step4



B263_12_PHYSICS_EM

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

UNIT 5

ELECTROMAGNETIC WAVES

"One scientific epoch ended and another began with James Clerk Maxwell"
– Albert Einstein

LEARNING OBJECTIVES

In this unit, the student is exposed to

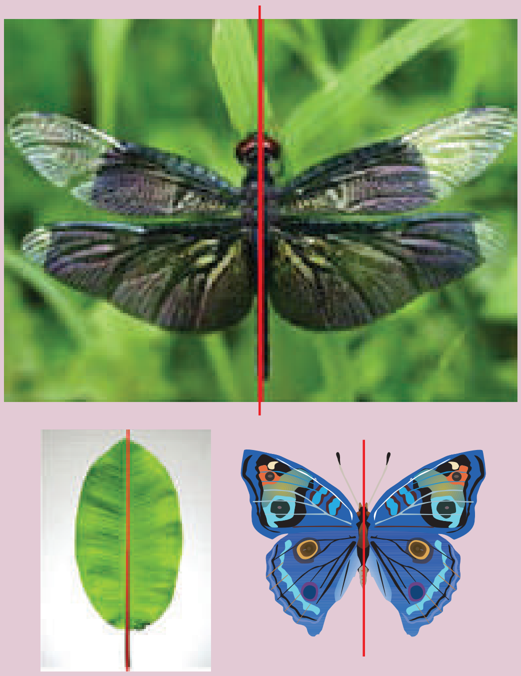
- the displacement current
- Maxwell's correction to Ampere's circuital law
- Maxwell's equation in integral form
- production and properties of electromagnetic waves – Hertz's experiment
- sources of electromagnetic waves
- electromagnetic spectrum

5.1 INTRODUCTION

Figure 5.1 Visible spectrum – rainbow and lightning

We see the world around us through light. Light from the Sun is one of the sources of energy without which we human beings cannot survive in this planet. Light plays crucial role in understanding the structure and properties of various things from atom to universe. Without light, even our eyes cannot see objects. What is light?. This puzzle made many physicists sleepless until middle of 19th century. Earlier, many scientists thought that optics and electromagnetism are two different branches of physics. But from the work of James Clerk Maxwell, who actually enlightened the concept of light from his theoretical prediction is that light is an electromagnetic wave which moves with the speed equal to 3×10^8 m/s (in free space or vacuum). Later, it was confirmed that light is just only small portion of electromagnetic spectrum, which ranges from gamma rays to radio waves.

In the unit 4, we studied that time varying magnetic field produces an electric field (Faraday's law of electromagnetic induction). Maxwell strongly believed that nature must possess symmetry and he asked the following question, "when the time varying magnetic field produces an electric field, why not the time varying electric field produce a magnetic field?"



Symmetry in nature

Later he proved that indeed it exists, which is often known as Maxwell's law of induction. In 1888, H. Hertz experimentally verified Maxwell's predication and hence, this understanding resulted in new technological invention, especially in wireless communication, LASER (Light Amplification by Stimulated Emission of Radiation) technology, RADAR (**Ra**dio **D**etection **A**nd **R**anging), etc.

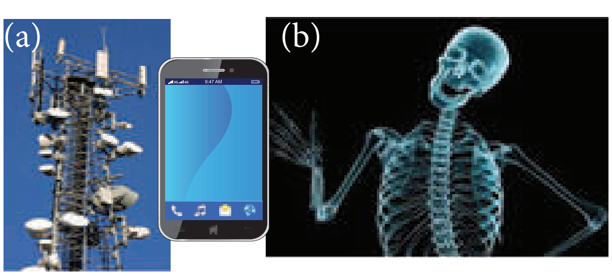


Figure 5.2 (a) cell phone and cell phone tower (b) X-ray radiograph of a human being

In today's digital world, cell phones (Figure 5.2 (a)) have greater influence in our day to day life. It is a faster and more effective mode of transferring information from one place to another. It works on the basis that light is an electromagnetic wave. In hospitals, the location of bone fracture can be detected using X-rays as shown in Figure 5.2 (b), which is also an electromagnetic wave. For cooking microwave oven is used. The microwave is also an electromagnetic wave. There are plenty of applications of electromagnetic waves in engineering, medical (example LASER surgery, etc), defence (example, RADAR signals) and also in fundamental scientific research. In this unit, basics of electromagnetic waves are covered.

5.1.1 Displacement current and Maxwell's correction to Ampere's circuital law

In unit 4, we studied Faraday's law of electromagnetic induction which states that the change in magnetic field produces an electric field. Mathematically

$$\oint_l \vec{E}\cdot\vec{dl} = -\frac{\partial}{\partial t}\Phi_B = -\frac{\partial}{\partial t}\oint_S \vec{B}\cdot\vec{dS} \qquad (5.1)$$

$$\underbrace{\oint_l \vec{E}\cdot\vec{dl}}_{\text{Electric field induced along a closed loop}} = \underbrace{-\frac{\partial}{\partial t}\Phi_B}_{\text{Variation of magnetic flux with time}} = \underbrace{-\frac{\partial}{\partial t}\oint_S \vec{B}\cdot\vec{dS}}_{\text{Changing magnetic flux } \Phi_B \text{ in the region enclosed by the loop}}$$

where Φ_B is the magnetic flux and $\frac{\partial}{\partial t}$ is the partial derivative with respect to time. Equation (5.1) means that the electric field $\vec{E}$ is induced along a closed loop by the changing magnetic flux Φ_B in the region encircled by the loop. Now the question asked by James Clerk Maxwell is 'Is converse of this statement true?' Answer is 'yes'. He showed that the change in electric field also produces magnetic field which is

$$\oint_l \vec{B}\cdot\vec{dl} = -\frac{\partial}{\partial t}\Phi_E = -\frac{\partial}{\partial t}\oint_S \vec{B}\cdot\vec{dS} \quad (5.2)$$

$$\underbrace{\oint_l \vec{E}\cdot\vec{dl}}_{\text{Magnetic field induced along a closed loop}} = \underbrace{-\frac{\partial}{\partial t}\Phi_E}_{\text{Variation of electric flux with time}} = \underbrace{-\frac{\partial}{\partial t}\oint_S \vec{B}\cdot\vec{dS}}_{\text{Changing electric flux } \Phi_E \text{ in the region enclosed by the loop}}$$

where Φ_B is the electric flux. This is known as Maxwell's law of induction, which explains that the magnetic field $\vec{B}$ induced along a closed loop by the changing electric flux Φ_E in the region encircled by that loop. This in turn, explains the existence of radio waves, gamma rays, infrared rays, etc.

In order to understand how the changing electric field produces magnetic field, let us consider a situation of 'charging a parallel plate capacitor' shown in Figure 5.3 Assume that the medium in between the capacitor plates is a non-conducting medium.

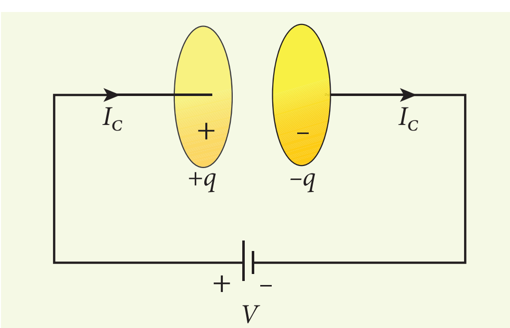


Figure 5.3 charging of capacitor

The electric current passing through the wire is the conduction current I_C. This current generates magnetic field around the wire (refer Unit 3) connected across the capacitor. Therefore, when a magnetic needle is kept near the wire, deflection is observed. In order to compute the strength of magnetic field at a point, we use Ampere's circuital law (from Unit 3) which states that 'the line integral of the magnetic field $\vec{B}$ around any closed loop is equal to μ_o times the net current I threading through the area enclosed by the loop'. Ampere's law in equation form is

$$\oint_l \vec{B}\cdot d\vec{l} = \mu_o I(t) \quad (5.3)$$

where μ_o is the permeability of free space.

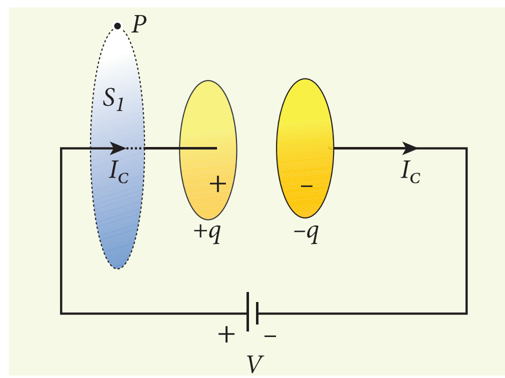


Figure 5.4 Applying Ampere's circuital law - loop enclosing surface

To calculate the magnetic field at a point P near the wire as shown in Figure 5.4, let us draw an amperian loop (circular loop) which encloses the surface S_1 (circular surface). Therefore, using Ampere's circuital law (equation 5.3), we get

$$\oint_{S_1} \vec{B}\cdot d\vec{l} = \mu_0 I_C \quad (5.4)$$

where I_c is the conduction current.

Suppose the same loop is enclosed by balloon shaped surface S_2 as shown in Figure 5.5. This means that the boundaries of two surfaces S_1 and S_2 are same but shape of the enclosing surfaces are different (first surface (S_1) is circular in shape and second one is balloon shaped surface (S_2)). As the Ampere's law applied for a given closed loop does not depend on shape of the enclosing surface, the integrals will give the same answer. But by applying Ampere's circuital law (equation 5.3), we get

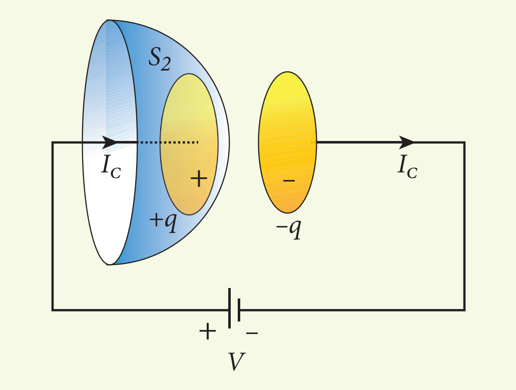


Figure 5.5 Applying Ampere's circuital law - loop enclosing surface S2

$$\oint_{\text{enclosing } S_2} \vec{B}\cdot d\vec{l} = 0 \qquad (5.5)$$

The right hand side of equation is zero because the surface S_2 no where touches the wire carrying conduction current and further, there is no current in between the plates of the capacitor (there is a discontinuity). So the magnetic field at a point P is zero. Hence there is an inconsistency between equation (5.4) and equation (5.5). J. C.Maxwell resolved this inconsistency as follows:

Due to external source (battery or cell), the capacitor gets charged up because of current flowing through the capacitor. This produces an increasing electric field between the capacitor plates. So, there must be a current associated with the changing electric field in between the capacitor plates. In other words, the time varying electric flux (or time varying electric field) existing between the plates of the capacitor also produces a current known as displacement current.

Note

Displacement current

The name stuck because Maxwell named it. The word displacement is poorly chosen because nothing is being displaced here.

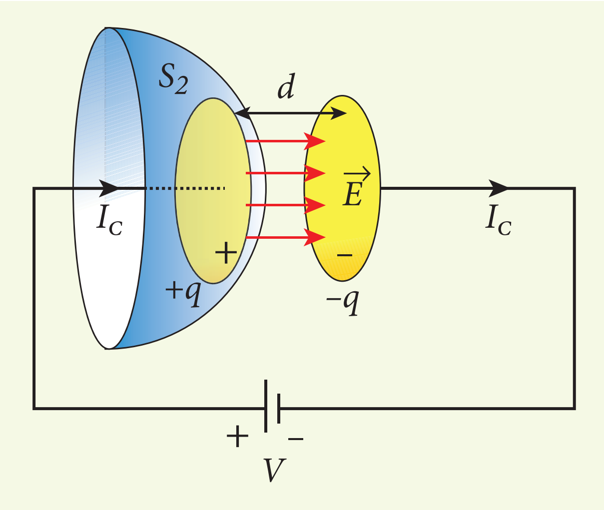


Figure 5.6 Applying Gauss's law between the plates of the capacitor

From Gauss's law (refer Unit 1), the electric flux between the plates of the capacitor (Figure 5.6) is

$$\Phi_E = \oiint \vec{E}\cdot d\vec{A} = EA = \frac{q}{\varepsilon_0}$$

where A is the area of the plates of capacitor.

The change in electric flux is

$$\frac{d\Phi_E}{dt} = \frac{1}{\varepsilon_\circ}\frac{dq}{dt} \Rightarrow \frac{dq}{dt} = I_d = \varepsilon_\circ \frac{d\Phi_E}{dt}$$

where I_d is known as displacement current. The **displacement current can be defined as the current which comes into play in the region in which the electric field and the electric flux are changing with time**. In other words, whenever the change in electric field takes place, displacement current is produced. Maxwell modified Ampere's law as

$$\oint \vec{B}\cdot d\vec{s} = \mu_\circ I = \mu_\circ (I_C + I_d) \qquad (5.6)$$

where $I = I_c + I_d$ which means the total current enclosed by the surface is sum of conduction current and displacement current. When a constant current is applied, displacement current $I_d = 0$ and hence $I_c = I$. Between the plates, the conduction current $I_c = 0$ and hence $I_d = I$.

EXAMPLE 5.1

Consider a parallel plate capacitor which is maintained at potential of 200 V. If the separation distance between the plates of the capacitor and area of the plates are 1 mm and 20 cm². Calculate the displacement current for the time in μs.

Solution

Potential difference between the plates of the capacitor, V = 200 V

The distance between the plates,

$$d = 1\ \text{mm} = 1 \times 10^{-3}\ \text{m}$$

Area of the plates of the capacitor,

$$A = 20\ \text{cm}^2 = 20 \times 10^{-4}\ \text{m}^2$$

Time is given in micro-second, $\mu s = 10^{-6}$ s

Displacement current

$$I_d = \varepsilon_\circ \frac{d\Phi_B}{dt} \Rightarrow I_d = \varepsilon_\circ \frac{EA}{t}$$

But electric field, $E = \frac{V}{d}$

Therefore,

$$I = \frac{V}{d} I_d = \varepsilon_\circ \frac{VA}{td} = 8.85 \times 10^{-12} \times \frac{200 \times 20 \times 10^{-4}}{10^{-6} \times 1 \times 10^{-3}}$$

$$= 35400 \times 10^{-7} = 3.5\ mA$$

5.1.3 Maxwell's equations in integral form

Electrodynamics can be summarized into four basic equations, known as Maxwell's equations. These equations are analogous to Newton's equations in mechanics. Maxwell's equations completely explain the behaviour of charges, currents and properties of electric and magnetic fields. These equations can be written in integral form (or integration form) or derivative form (or differentiation form). The differential form of Maxwell's equation is beyond higher secondary level because we need to learn additional mathematical operations like curl of vector fields and divergence of vector fields. So we focus here only in integral form of Maxwell's equations:

1. First equation is nothing but the Gauss's law. It relates the net electric flux to net electric charge enclosed in a surface. Mathematically, it is expressed as

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_\circ} \quad \text{(Gauss's law)} \quad (5.7)$$

   where $\vec{E}$ is the electric field and $Q_{enclosed}$ is the charge enclosed. This equation is true for both discrete or continuous distribution of charges. It also indicates that the electric field lines start from positive charge and terminate at negative charge. This implies that the electric field lines do not form a continuous closed path. In other words, it means that isolated positive charge or negative charge can exist.

2. Second equation has no name. But this law is similar to Gauss's law in electrostatics. So this law can also be called as Gauss's law in magnetism. The

surface integral of magnetic field over a closed surface is zero. Mathematically,

$$\oint \vec{B}\cdot d\vec{A} = 0 \qquad \text{(No name)} \qquad (5.8)$$

where $\vec{B}$ is the magnetic field. This equation implies that the magnetic lines of force form a continuous closed path. In other words, it means that no isolated magnetic monopole exists.

3. Third equation is Faraday's law of electromagnetic induction. This law relates electric field with the changing magnetic flux which is mathematically written as

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\Phi_B \text{(Faraday's law)} \qquad (5.9)$$

where $\vec{E}$ is the electric field. This equation implies that the line integral of the electric field around any closed path is equal to the rate of change of magnetic flux through the closed path bounded by the surface. Our modern technological revolution is due to Faraday's laws of electromagnetic induction. The electrical energy supplied to our houses from electricity board by using Faraday's law of induction.

4. Fourth equation is modified Ampere's circuital law. This is also known as Ampere – Maxwell's law. This law relates the magnetic field around any closed path to the conduction current and displacement current through that path.

$$\oint \vec{B}.\vec{dl} = \mu_0 I_{enclosed} + \mu_0\varepsilon_0 \frac{d}{dt}\int_s \vec{E}.\vec{dA} \qquad (5.10)$$

(Ampere-Maxwell's law)

where $\vec{B}$ is the magnetic field. This equation shows that both conduction and also displacement current produces magnetic field. These four equations are known as Maxwell's equations in electrodynamics. This equation ensures the existence of electromagnetic waves. The entire communication system in the world depends on electromagnetic waves. In fact our understanding of stars, galaxy, planets etc come by analysing the electromagnetic waves emitted by these astronomical objects.

5.2 ELECTROMAGNETIC WAVES

Electromagnetic waves are non-mechanical waves which move with speed equals to the speed of light (in vacuum). It is a transverse wave. In the following subsections, we discuss the production of electromagnetic waves and its properties, sources of electromagnetic waves and also classification of electromagnetic spectrum.

5.2.1 Production and properties of electromagnetic waves - Hertz experiment

Maxwell's prediction was experimentally confirmed by Heinrich Rudolf Hertz (Figure 5.7 (a)) in 1888. The experimental set up used is shown in Figure 5.7 (b).

It consists of two metal electrodes which are made of small spherical metals as shown in Figure 5.7. These are connected to larger spheres and the ends of them are connected to induction coil with very large number of turns. This is to produce very high electromotive force (emf). Since the

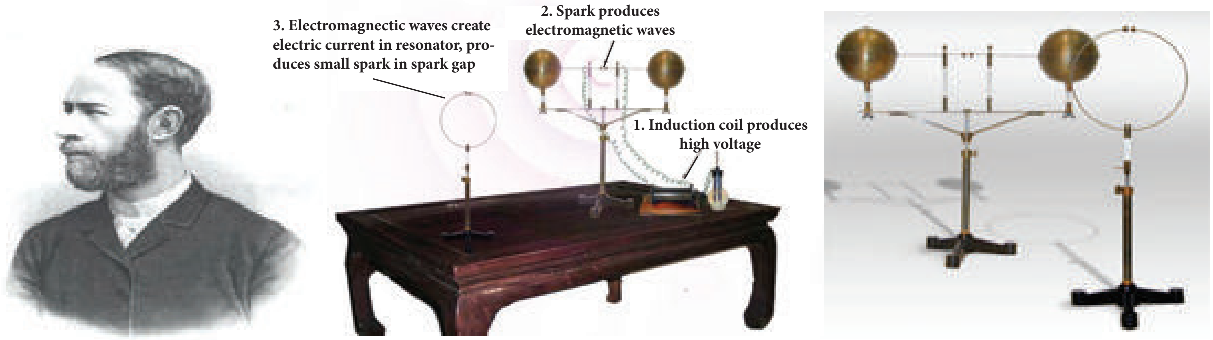


Figure 5.7 Hertz experiment (a) Heinrich Rudolf Hertz (1857 – 1894) (b) Hertz apparatus

coil is maintained at very high potential, air between the electrodes gets ionized and spark (spark means discharge of electricity) is produced. The gap between electrode (ring type – not completely closed and has a small gap in between) kept at a distance also gets spark. This implies that the energy is transmitted from electrode to the receiver (ring electrode) as a wave, known as electromagnetic waves. If the receiver is rotated by 90° - then no spark is observed by the receiver. This confirms that electromagnetic waves are transverse waves as predicted by Maxwell. Hertz detected radio waves and also computed the speed of radio waves which is equal to the speed of light (3×10^8 m s^{-1}).

Properties of electromagnetic waves

1. Electromagnetic waves are produced by any accelerated charge.
2. Electromagnetic waves do not require any medium for propagation. So electromagnetic wave is a non-mechanical wave.
3. Electromagnetic waves are transverse in nature. This means that the oscillating electric field vector, oscillating magnetic field vector and propagation vector (gives direction of propagation) are mutually perpendicular to each other. The electric and magnetic fields are in the y and z directions respectively and the direction of propagation is along x direction. This is shown in Figure 5.8.
4. Electromagnetic waves travel with speed which is equal to the speed of light in vacuum or free space, $c = \frac{1}{\sqrt{\varepsilon_\circ \mu_\circ}} = 3 \times 10^8 \, \mathrm{m\,s^{-1}}$, where ε_o is the permittivity of free space or vacuum and μ_o is the permeability of free space or vacuum (refer Unit 1 for permittivity and Unit 3 for permeability).
5. In a medium with permittivity ε and permeability μ, the speed of electromagnetic wave is less than speed in free space or vacuum, that is, $v < c$. In a medium of refractive index,
$$\mu = \frac{c}{v} = \frac{\frac{1}{\sqrt{\varepsilon_\circ \mu_\circ}}}{\frac{1}{\sqrt{\varepsilon \mu}}} \Rightarrow \mu = \sqrt{\varepsilon_r \mu_r}$$, where ε_r is the relative permittivity of the medium (also known as dielectric constant) and μ_r is the relative permeability of the medium.
6. Electromagnetic waves are not deflected by electric field or magnetic field.

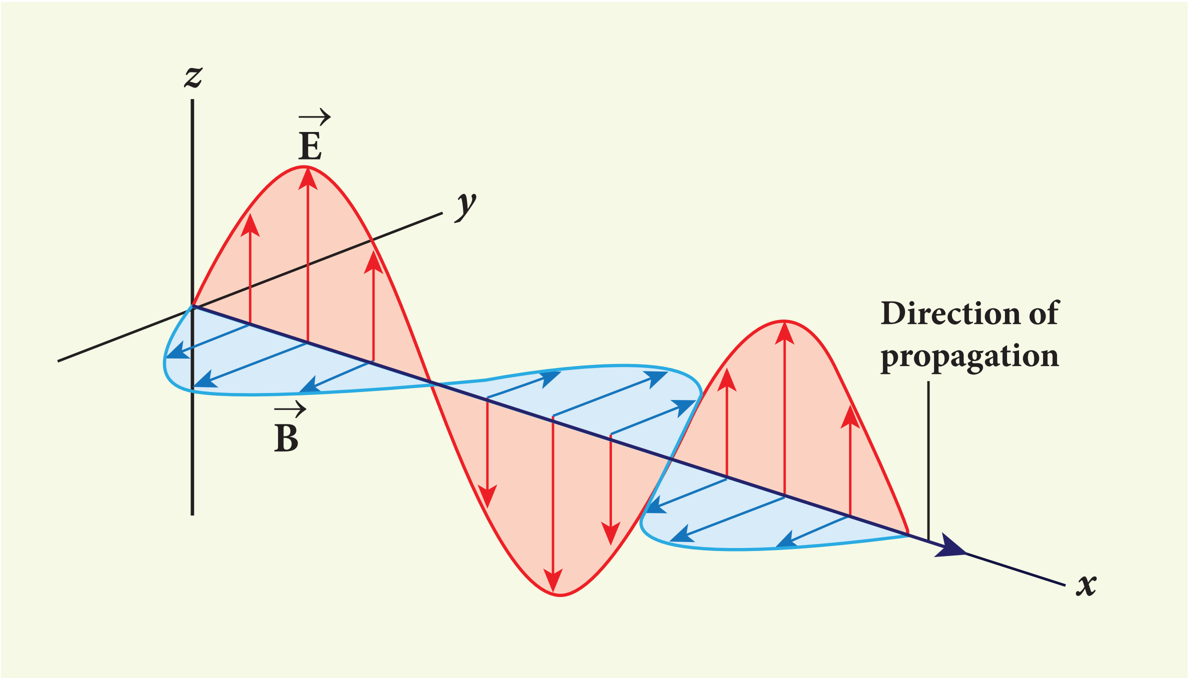


Figure 5.8 Electromagnetic waves – transverse wave

7. Electromagnetic waves can show interference, diffraction and can also be polarized.
8. The energy density (energy per unit volume) associated with an electromagnetic wave propagating in vacuum or free space is

$$u = \frac{1}{2}\varepsilon_{\circ} E^2 + \frac{1}{2\mu_{\circ}} B^2$$

where, $\frac{1}{2}\varepsilon_{\circ} E^2 = u_E$ is the energy density in an electric field and $\frac{1}{2\mu_0} B^2 = u_B$ is the energy density in a magnetic field.

Since, $E = Bc \Rightarrow u_B = u_E$.

The energy density of the electromagnetic wave is

$$u = \varepsilon_0 E^2 = \frac{1}{\mu_0} B^2$$

9. The average energy density for electromagnetic wave,

$$\langle u \rangle = \frac{1}{2}\varepsilon_{\circ} E^2 = \frac{1}{2}\frac{1}{\mu_{\circ}} B^2.$$

10. The energy crossing per unit area per unit time and perpendicular to the direction of propagation of electromagnetic wave is called the intensity.

Intensity, $I = \langle u \rangle c$ or

$$I = \frac{\text{total electromagnetic energy (U)}}{\text{Surface area (A)} \times \text{time (t)}}$$

$$= \frac{\text{Power (P)}}{\text{Surface area (A)}}$$

Note

For a point source, $I = \frac{P}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$

For a line source, $I \propto \frac{1}{r}$

For a plane source, I is independent of r

11. Like other waves, electromagnetic waves also carry energy and momentum. For the electromagnetic wave of energy U propagating with speed c has linear momentum which

is given by $= \frac{\text{Energy}}{\text{speed}} = \frac{U}{c}$. The force exerted by an electromagnetic wave on unit area of a surface is called radiation pressure.

12. If the electromagnetic wave incident on a material surface is completely absorbed, then the energy delivered is U and momentum imparted on the surface is $p = \frac{U}{c}$.
13. If the incident electromagnetic wave of energy U is totally reflected from the surface, then the momentum delivered to the surface is $\Delta p = \frac{U}{c} - \left(-\frac{U}{c}\right) = 2\frac{U}{c}$.
14. The rate of flow of energy crossing a unit area is known as pointing vector for electromagnetic waves, which is $\vec{S} = \frac{1}{\mu_\circ}\left(\vec{E}\times\vec{B}\right) = c^2\varepsilon_\circ\left(\vec{E}\times\vec{B}\right)$. The unit for pointing vector is W m^{-2}. The pointing vector at any point gives the direction of energy transport from that point.
15. Electromagnetic waves carries not only energy and momentum but also angular momentum.

EXAMPLE 5.2

The relative magnetic permeability of the medium is 2.5 and the relative electrical permittivity of the medium is 2.25. Compute the refractive index of the medium.

Solution

Dielectric constant (relative permeability of the medium) is $\varepsilon_r = 2.25$

Magnetic permeability is $\mu_r = 2.5$

Refractive index of the medium,

$$n = \sqrt{\varepsilon_r \mu_r} = \sqrt{2.25 \times 2.5} = 2.37$$

5.2.2 Sources of electromagnetic waves

PCYVBK

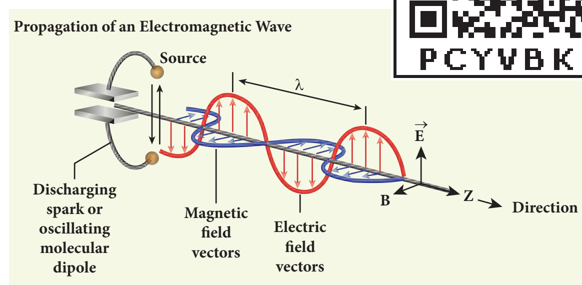


Figure 5.9 Oscillating charges - sources of electromagnetic waves

Any stationary source charge produces only electric field (refer Unit 1). When the charge moves with uniform velocity, it produces steady current which gives rise to magnetic field (not time dependent, only space dependent) around the conductor in which charge flows. If the charged particle accelerates, in addition to electric field it also produces magnetic field. Both electric and magnetic fields are time varying fields. Since the electromagnetic waves are transverse waves, the direction of propagation of electromagnetic waves is perpendicular to the plane containing electric and magnetic field vectors.

Any oscillatory motion is also an accelerating motion, so, when the charge oscillates (oscillating molecular dipole) about their mean position as shown in Figure 5.9, it produces electromagnetic waves.

Suppose the electromagnetic field in free space propagates along z direction, and if the electric field vector points along y axis then the magnetic field vector will be mutually perpendicular to both electric field and the propagation vector direction, which means

$$E_y = E_\circ \sin\left(kz - \omega t\right)$$

$$B_x = B_\circ \sin\left(kz - \omega t\right)$$

where, E_o and B_o are amplitude of oscillating electric and magnetic field, k is a wave number, ω is the angular frequency of the wave and $\hat{k}$ (unit vector, here it is called propagation vector) denotes the direction of propagation of electromagnetic wave.

Note that both electric field and magnetic field oscillate with a frequency (frequency of electromagnetic wave) which is equal to the frequency of the source (here, oscillating charge is the source for the production of electromagnetic waves). In free space or in vacuum, the ratio between E_o and B_o is equal to the speed of electromagnetic wave, which is equal to speed of light c.

$$c = \frac{E_o}{B_o}$$

In any medium, the ratio of E_o and B_o is equal to the speed of electromagnetic wave in that medium, mathematically, it can be written as

$$v = \frac{E_o}{B_o} < c$$

Further, the energy of electromagnetic waves comes from the energy of the oscillating charge.

EXAMPLE 5.3

Compute the speed of the electromagnetic wave in a medium if the amplitude of electric and magnetic fields are 3×10^4 N C^{-1} and 2×10^{-4} T, respectively.

Solution

The amplitude of the electric field, $E_o = 3 \times 10^4$ N C^{-1}

The amplitude of the magnetic field, $B_o = 2 \times 10^{-4}$ T. Therefore, speed of the electromagnetic wave in a medium is

$$v = \frac{3\times10^4}{2\times10^{-4}} = 1.5\times10^8\, ms^{-1}$$

5.2.3 Electromagnetic spectrum

ELECTROMAGNETIC SPECTRUM

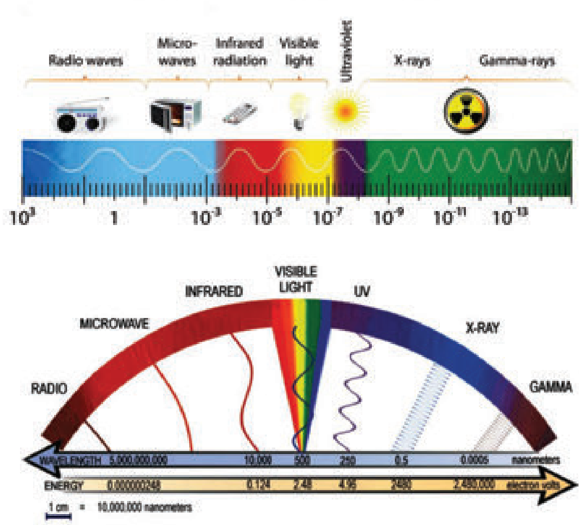


Figure 5.10 Electromagnetic spectrum -

Electromagnetic spectrum is an orderly distribution of electromagnetic waves in terms of wavelength or frequency as shown in Figure 5.10.

Radio waves

It is produced by oscillators in electric circuits. The wavelength range is 1×10^{-1} m to 1×10^4 m and frequency range is 3×10^9 Hz to 3×10^4 Hz. It obeys reflection and diffraction. It is used in radio and television communication systems and also in cellular phones to transmit voice communication in the ultra high frequency band.

Microwaves

It is produced by electromagnetic oscillators in electric circuits. The wavelength range is 1×10^{-3} m to 3×10^{-1} m and frequency range is 3×10^{11} Hz to 1×10^9 Hz. It obeys reflection and polarization. It is used in radar system for aircraft navigation, speed of the vehicle, microwave oven for cooking and very long distance wireless communication through satellites.

Measuring the speed of light using the microwave oven

Nowadays the microwave oven is very commonly used to heat the food items. Micro waves of wavelengths 1 mm to 30 cm are produced in these ovens. Such waves form the standing waves between the interior walls of the oven. It is interesting to note that the speed of light can be measured using micro wave oven.

(a) (b) (c) (d)

We studied about the standing waves in XI physics, Volume 2, Unit 11. The standing waves have nodes and antinodes at fixed points. At node point, the amplitude of the wave is zero and at antinodes point, the amplitude is maximum. In other words, the maximal energy of microwaves is located at antinode points. When we keep some food items like chappathi or choclate (after removing the rotating platform) inside the oven, we can notice that at antinode locations, chappathi will be burnt more than other locations. It is shown in the Figure (c) and (d). The distance between two successive burnt spots will give the wavelength of microwave. The frequency of microwave is printed in the panel of oven. By knowing wavelength and frequency of microwaves, using the formula $v\lambda = c$, we can calculate the speed of light c.

Infrared radiation

It is produced from hot bodies (also known as heat waves) and also when the molecules undergo rotational and vibrational transitions. The wavelength range is 8×10^{-7} m to 5×10^{-3} m and frequency range are 4×10^{14} Hz to 6×10^{10} Hz. It provides electrical energy to satellites by means of solar cells. It is used to produce dehydrated fruits, in green houses to keep the plants warm, heat therapy for muscular pain or sprain, TV remote as a signal carrier, to look through haze fog or mist and used in night vision or infrared photography.

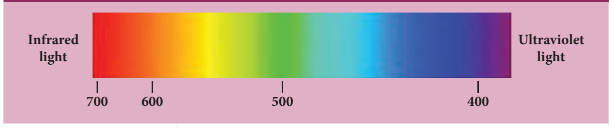
Visible light

It is produced by incandescent bodies and also it is radiated by excited atoms in gases. The wavelength range is 4×10^{-7} m to 7×10^{-7} m and frequency range are 7×10^{14} Hz to 4×10^{14} Hz. It obeys the laws of reflection, refraction, interference, diffraction, polarization, photo-electric effect and photographic action. It can be used to study the structure of molecules, arrangement of electrons in external shells of atoms and sensation of our eyes.

Ultraviolet radiation

It is produced by Sun, arc and ionized gases. The wavelength range is 6×10^{-10} m to 4×10^{-7} m and frequency range are 5×10^{17} Hz to 7×10^{14} Hz. It has less penetrating power. It can be absorbed by atmospheric ozone and harmful to human body. It is used to destroy bacteria, sterilizing the surgical instruments, burglar alarm, detect the invisible writing, finger prints and also in the study of molecular structure.

Table 5.1 visible region, frequency and wavelength of different types of radiation



Type of Radiation	Frequency Range (Hz)	Wavelength Range
gamma-rays	10^{20}-10^{24}	$<10^{-12}$m
x-rays	10^{17}-10^{20}	1 nm - 1pm
ultraviolet	10^{15}-10^{17}	400 nm - 1nm
visible	4 - 7.5 × 10^{14}	750 nm - 400nm
near-infrared	1 × 10^{14}-4 × 10^{14}	2.5 μm - 750nm
infrared	10^{13}-10^{14}	25 μm - 2.5μm
microwaves	3 × 10^{11}-10^{13}	1 mm - 25 μm
radio waves	< 3 × 10^{11}	> 1 mm

X-rays

It is produced when there is a sudden deceleration of high speed electrons at high-atomic number target, and also by electronic transitions among the innermost orbits of atoms. The wavelength range 10^{-13} m to 10^{-8} m and frequency range are 3 × 10^{21} Hz to 1 × 10^{16} Hz. X-rays have more penetrating power than ultraviolet radiation. X-rays are used extensively in studying structures of inner atomic electron shells and crystal structures. It is used in detecting fractures, diseased organs, formation of bones and stones, observing the progress of healing bones. Further, in a finished metal product, it is used to detect faults, cracks, flaws and holes.

Gamma rays

It is produced by transitions of atomic nuclei and decay of certain elementary particles. They produce chemical reactions on photographic plates, fluorescence, ionisation, diffraction. The wavelength range is 1 × 10^{-14} m to 1 × 10^{-10} m and frequency range are 3 × 10^{22} Hz to 3 × 10^{18} Hz. Gamma rays have high penetrating power than X-rays and ultraviolet radiations; it has no charge but harmful to human body. Gamma rays provide information about the structure of atomic nuclei. It is used in radio therapy for the treatment of cancer and tumour, in food industry to kill pathogenic microorganism.

EXAMPLE 5.4

A magnetron in a microwave oven emits electromagnetic waves (em waves) with frequency f = 2450 MHz. What magnetic field strength is required for electrons to move in circular paths with this frequency?.

Solution

Frequency of the electromagnetic waves given is f = 2450 MHz

The corresponding angular frequency is

$\omega = 2\pi f = 2 \text{ x } 3.14 \text{ x } 2450 \text{ x } 10^6$

$= 15{,}386 \text{ x } 10^6$ Hz

$= 1.54 \times 10^{10}\ s^{-1}$

The magnetic field $B = \dfrac{m_e \omega}{|q|}$

Mass of the electron, $m_e = 9.22 \text{ x } 10^{-31}$ kg

Charge of the electron

$$q = -1.60 \times 10^{-19} C \Rightarrow |q| = 1.60 \times 10^{-19} C$$

$$B = \frac{\left(9.22 \times 10^{-31}\right)\left(1.54 \times 10^{10}\right)}{\left(1.60 \times 10^{-19}\right)} = 8.87425 \times 10^{-2} T$$

B = 0.0887 T

This magnetic field can be easily produced with a permanent magnet. So, electromagnetic waves of frequency 2450 MHz can be used for heating and cooking food because they are strongly absorbed by water molecules.

5.3 TYPES OF SPECTRUM-EMISSION AND ABSORPTION SPECTRUM-FRAUNHOFER LINES

29KQ1G

Blackbody radiation curves

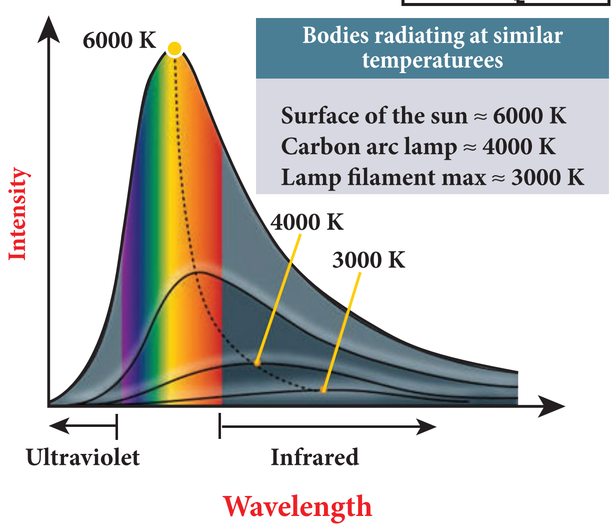


Figure 5.11 Black body radiation spectrum – variation with temperature

When an object burns, it emits colours. That is, it emits electromagnetic radiation which depends on temperature. If the object becomes hot then it glows in red colour. If the temperature of the object is further increased then it glows in reddish-orange colour and becomes white when it is hottest. The spectrum in Figure 5.11 usually called as black body spectrum (refer plus one volume two Unit 8). It is a continuous frequency (or wavelength) curve and depends on the body's temperature.

Suppose we allow a beam of white light to pass through the prism as shown in Figure 5.12, it is split into its seven constituent colours which can be viewed on the screen as continuous spectrum. This phenomenon is known as dispersion of light and the definite

Figure 5.12 White light passed through prism – dispersion

pattern of colours obtained on the screen after dispersion is called as spectrum. The plural for spectrum is spectra. The spectra can be broadly classified into two catagories:

(a) Emission spectra

When the spectrum of self luminous source is taken, we get emission spectrum. Each source has its own characteristic emission spectrum. The emission spectrum can be divided into three types:

(i) Continuous emission spectra (or continuous spectra)

Figure 5.13 continuous emission spectra

If the light from incandescent lamp (filament bulb) is allowed to pass through prism (simplest spectroscope), it splits into seven colours. Thus, it consists of wavelengths containing all the visible colours ranging from violet to red (Figure 5.13). Examples: spectrum obtained from carbon arc, incandescent solids, liquids gives continuous spectra.

(ii) Line emission spectrum (or line spectrum):

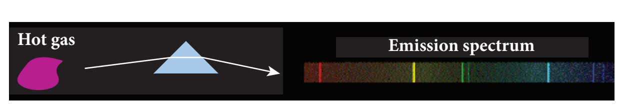


Figure 5.14 line emission spectra

Suppose light from hot gas is allowed to pass through prism, line spectrum is observed (Figure 5.14). Line spectra are also known as discontinuous spectra. The line spectra are sharp lines of definite wavelengths or frequencies. Such spectra arise due to excited atoms of elements. These lines are the characteristics of the element which means it is different for different elements. Examples: spectra of atomic hydrogen, helium, etc.

(3) Band emission spectrum (or band spectrum)

Band spectrum consists of several number of very closely spaced spectral lines which overlapped together forming specific bands which are separated by dark spaces, known as band spectra. This spectrum has a sharp edge at one end and fades out at the other end. Such spectra arise when the molecules are excited. Band spectrum is the characteristic of the molecule hence, the structure of the molecules can be studied using their band spectra. Examples, spectra of hydrogen gas, ammonia gas in the discharge tube etc.

(b) Absorption spectra

When light is allowed to pass through a medium or an absorbing substance then the spectrum obtained is known as absorption spectrum. It is the characteristic of absorbing substance. Absorption spectrum is classified into three types:

(i) Continuous absorption spectrum

When the light is passed through a medium, it is dispersed by the prism, we get continuous absorption spectrum. For instance, when we pass white light through a blue glass plate, it absorbs everything except blue. This is an example of continuous absorption spectrum.

(ii) Line absorption spectrum

Figure 5.15 line absorption spectra

When light from the incandescent lamp is passed through cold gas (medium), the spectrum obtained through the dispersion due to prism is line absorption spectrum (Figure 5.15). Similarly, if the light from the carbon arc is made to pass through sodium vapour, a continuous spectrum of carbon arc with two dark lines in the yellow region of sodium vapour is obtained.

(iii) Band absorption spectrum

When the white light is passed through the iodine vapour, dark bands on continuous bright background is obtained. This type of band is also obtained when white light is passed through diluted solution of blood or chlorophyll or through certain solutions of organic and inorganic compounds.

Fraunhofer lines

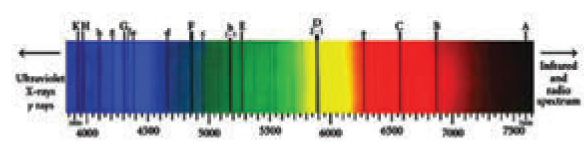


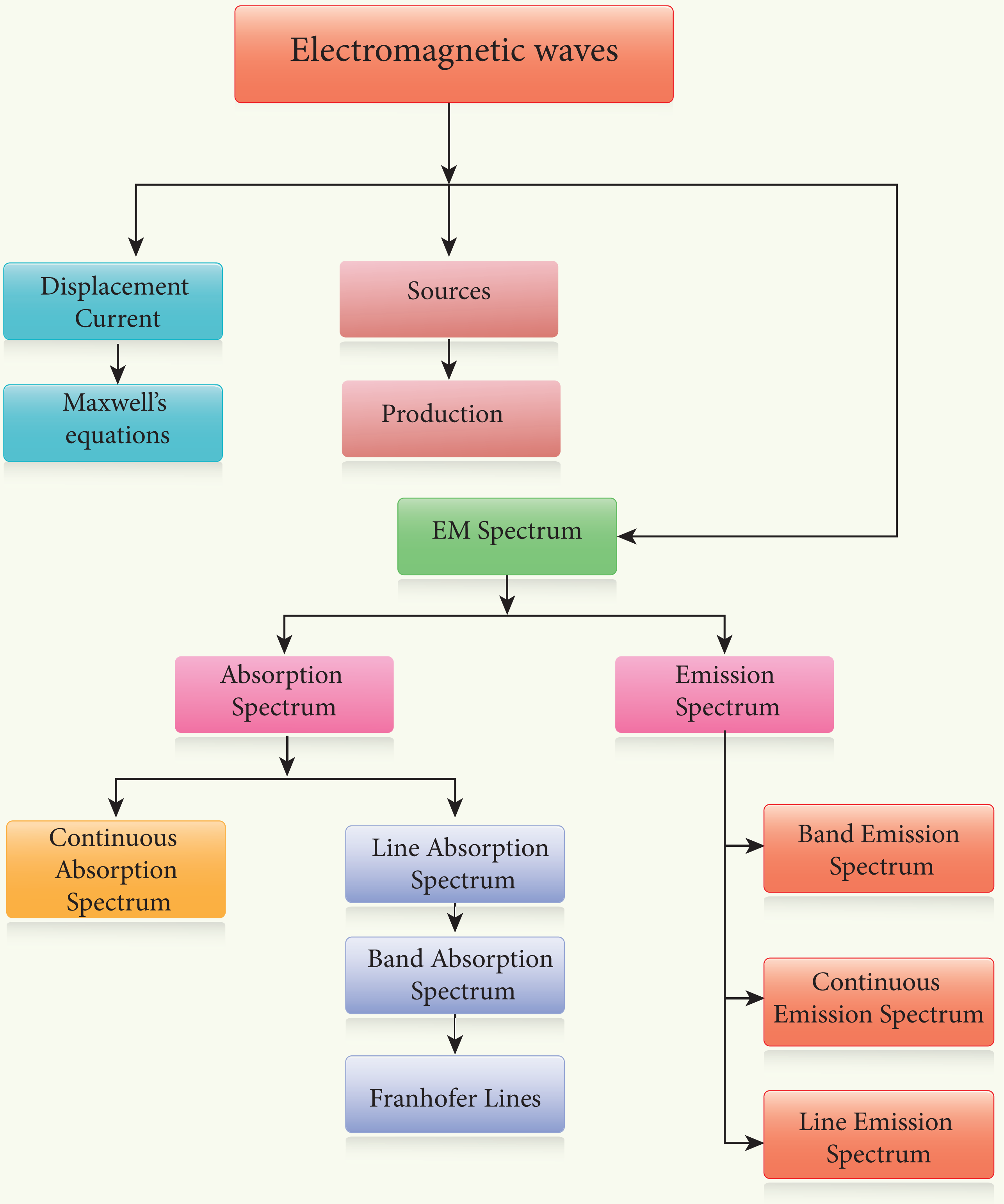
Figure 5.16 Solar spectrum - Fraunhofer lines

When the spectrum obtained from the Sun is examined, it consists of large number of dark lines (line absorption spectrum). These dark lines in the solar spectrum are known as Fraunhofer lines (Figure 5.16). The absorption spectra for various materials are compared with the Fraunhofer lines in the solar spectrum, which helps in identifying elements present in the Sun's atmosphere.

SUMMARY

- Displacement current can be defined as 'the current which comes into play in the region in which the electric field and the electric flux are changing with time'
- Maxwell modified Ampere's law as
  $\oint \vec{B} \cdot d\vec{S} = \mu_o I = \mu_o (I_c + I_d)$
- An electromagnetic wave is radiated by an accelerated charge which propagates through space as coupled electric and magnetic fields, oscillating perpendicular to each other and to the direction of propagation of the wave
- Electromagnetic wave is a transverse wave. They are non-mechanical wave and do not require any medium for propagation
- The instantaneous magnitude of the electric and magnetic field vectors in electromagnetic wave are related by E = Bc
- Electromagnetic waves are transverse in nature. This means that the oscillating electric field vector, oscillating magnetic field vector and propagation vector are (gives direction of propagation) mutually perpendicular to each other
- Electromagnetic waves can show interference, diffraction and also can be polarized
- The average energy density $\langle u \rangle = 2u_e = 2u_m = \varepsilon_\circ E^2 = \frac{1}{\mu_\circ} B^2$
- The energy crossing per unit area per unit time and perpendicular to the direction of propagation of electromagnetic wave is called the intensity, which is $I = \langle u \rangle c$
- If the electromagnetic wave incident on a material surface is completely absorbed, then the energy delivered is U and momentum imparted on the surface is $p = \frac{U}{c}$
- If the incident electromagnetic wave of energy U is totally reflected from the surface, then the momentum delivered to the surface is $\Delta p = \frac{U}{c} - \left(-\frac{U}{c}\right) = 2\frac{U}{c}$
- The rate of flow of energy crossing a unit area is known as poynting vector for electromagnetic waves, which is $\vec{S} = \frac{1}{\mu_\circ}(\vec{E} \times \vec{B}) = c^2 \varepsilon_\circ (\vec{E} \times \vec{B})$.
- Electromagnetic waves carry not only energy and momentum but also angular momentum.
- Types of spectrum – emission and absorption
- When the spectrum of self luminous source is taken, we get emission spectrum. Each source has its own characteristic emission spectrum. The emission spectrum can be divided into three types: continuous, line and band.
- The spectrum obtained from the Sun is examined, it consists of large number of dark lines (line absorption spectrum). These dark lines in the solar spectrum are known as Fraunhofer lines.

CONCEPT MAP


Electromagnetic waves
Displacement Current
Maxwell's equations
Sources
Production
EM Spectrum
Absorption Spectrum
Emission Spectrum
Continuous Absorption Spectrum
Line Absorption Spectrum
Band Absorption Spectrum
Franhofer Lines
Band Emission Spectrum
Continuous Emission Spectrum
Line Emission Spectrum

EVALUATION

I Multiple choice questions

1. The dimension of $\frac{1}{\mu_o \varepsilon_\circ}$ is

(a) $[L\,T^{-1}]$ (b) $[L^2\,T^{-2}]$

(c) $[L^{-1}\,T]$ (d) $[L^{-2}\,T^2]$

2. If the amplitude of the magnetic field is 3×10^{-6} T, then amplitude of the electric field for a electromagnetic waves is

(a) 100 V m^{-1} (b) 300 V m^{-1}

(c) 600 V m^{-1} (d) 900 V m^{-1}

3. Which of the following electromagnetic radiation is used for viewing objects through fog

(a) microwave (b) gamma rays

(c) X- rays (d) infrared

4. Which of the following are false for electromagnetic waves

(a) transverse

(b) mechanical waves

(c) longitudinal

(d) produced by accelerating charges

5. Consider an oscillator which has a charged particle and oscillates about its mean position with a frequency of 300 MHz. The wavelength of electromagnetic waves produced by this oscillator is

(a) 1 m (b) 10 m

(c) 100 m (d) 1000 m

6. The electric and the magnetic field, associated with an electromagnetic wave, propagating along X axis can be represented by

(a) $\vec{E} = E_\circ \hat{j}$ and $\vec{B} = B_\circ \hat{k}$

(b) $\vec{E} = E_\circ \hat{k}$ and $\vec{B} = B_\circ \hat{j}$

(c) $\vec{E} = E_\circ \hat{i}$ and $\vec{B} = B_\circ \hat{j}$

(d) $\vec{E} = E_\circ \hat{j}$ and $\vec{B} = B_\circ \hat{i}$

7. In an electromagnetic wave in free space the rms value of the electric field is 3 V m^{-1}. The peak value of the magnetic field is

(a) 1.414×10^{-8} T (b) 1.0×10^{-8} T

(c) 2.828×10^{-8} T (d) 2.0×10^{-8} T

8. During the propagation of electromagnetic waves in a medium:

(a) electric energy density is double of the magnetic energy density

(b) electric energy density is half of the magnetic energy density

(c) electric energy density is equal to the magnetic energy density

(d) both electric and magnetic energy densities are zero

9. If the magnetic monopole exists, then which of the Maxwell's equation to be modified?.

(a) $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_\circ}$

(b) $\oint \vec{E} \cdot d\vec{A} = 0$

(c) $\oint \vec{E} \cdot d\vec{A} = \mu_\circ I_{enclosed} + \mu_\circ \epsilon_\circ \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$

(d) $\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \Phi_B$

10. A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is

(a) $\frac{E}{c}$ (b) $2\frac{E}{c}$

(c) Ec (d) $\frac{E}{c^2}$

11. Which of the following is an electromagnetic wave?

(a) α - rays (b) β - rays

(c) γ - rays (d) all of them

12. Which one of them is used to produce a propagating electromagnetic wave?.

(a) an accelerating charge

(b) a charge moving at constant velocity

(c) a stationary charge

(d) an uncharged particle

13. Let $E = E_o \sin[10^6 x - \omega t]$ be the electric field of plane electromagnetic wave, the value of ω is

(a) 0.3×10^{-14} rad s^{-1}

(b) 3×10^{-14} rad s^{-1}

(c) 0.3×10^{14} rad s^{-1}

(d) 3×10^{14} rad s^{-1}

14. Which of the following is NOT true for electromagnetic waves?.

(a) it transport energy

(b) it transport momentum

(c) it transport angular momentum

(d) in vacuum, it travels with different speeds which depend on their frequency

15. The electric and magnetic fields of an electromagnetic wave are

(a) in phase and perpendicular to each other

(b) out of phase and not perpendicular to each other

(c) in phase and not perpendicular to each other

(d) out of phase and perpendicular to each other

Answers

1) b	**2)** d	**3)** d	**4)** c	**5)** a
6) b	**7)** a	**8)** c	**9)** b	**10)** b
11) c	**12)** a	**13)** d	**14)** d	**15)** a

II Short answer questions

1. What is displacement current?

2. What are electromagnetic waves?

3. Write down the integral form of modified Ampere's circuital law.

4. Explain the concept of intensity of electromagnetic waves.

5. What is meant by Fraunhofer lines?

III Long answer questions

1. Write down Maxwell equations in integral form.

2. Write short notes on (a) microwave (b) X-ray (c) radio waves (d) visible spectrum

3. Discuss briefly the experiment conducted by Hertz to produce and detect electromagnetic spectrum.

4. Explain the Maxwell's modification of Ampere's circuital law.

5. Write down the properties of electromagnetic waves.

6. Discuss the source of electromagnetic waves.

7. What is emission spectra?. Give their types.

8. What is absorption spectra?. Give their types.

IV Numerical problems

1. Consider a parallel plate capacitor whose plates are closely spaced. Let R be the radius of the plates and the current in the wire connected to the plates is 5 A, calculate the displacement current through the surface passing between the plates by directly calculating the rate of change of flux of electric field through the surface.

   Answer: $I_d = I_c = 5$ A

2. A transmitter consists of LC circuit with an inductance of 1 μH and a capacitance of 1 μF. What is the wavelength of the electromagnetic waves it emits?

   Answer: 18.84×10^{-6} m

3. A pulse of light of duration 10^{-6} s is absorbed completely by a small object initially at rest. If the power of the pulse is 60×10^{-3} W, calculate the final momentum of the object.

   Answer: 20×10^{-17} kg m s^{-1}

4. Let an electromagnetic wave propagate along the x direction, the magnetic field oscillates at a frequency of 10^{10} Hz and has an amplitude of 10^{-5} T, acting along the y - direction. Then, compute the wavelength of the wave. Also write down the expression for electric field in this case.

   Answer: $\lambda = 3 \times 10^{-18}$ m and

   $$\vec{E}(x,t) = 3\times10^{3} \sin\left(2.09\times10^{18} x - 6.28\times10^{10} t\right)\hat{i} \;\; NC^{-1}$$

5. If the relative permeability and relative permittivity of the medium is 1.0 and 2.25, respectively. Find the speed of the electromagnetic wave in this medium.

   Answer: v = 2 m s^{-1}

BOOKS FOR REFERENCE:

1. H. C. Verma, *Concepts of Physics – Volume 2*, Bharati Bhawan Publisher
2. Halliday, Resnick and Walker, *Fundamentals of Physics*, Wiley Publishers, 10th edition
3. Serway and Jewett, *Physics for scientist and engineers with modern physics*, Brook/Coole publishers, Eighth edition
4. David J. Griffiths, *Introduction to electrodynamics*, Pearson publishers
5. Paul Tipler and Gene Mosca, *Physics for scientist and engineers with modern physics*, Sixth edition, W.H.Freeman and Company

ICT CORNER

Electromagnetic waves

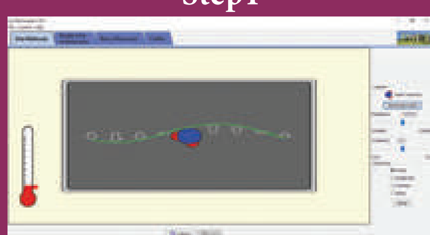
In this activity you will be able to how do microwaves heat food?

Physics of microwaves and heating food

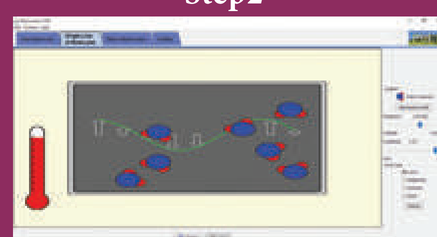
STEPS:

- Open the browser and type “phet.colorado.edu/en/simulation/microwaves” in the address bar. Run the simulation.
- select ‘one molecule’ tab. Turn on the microwave using the button in the right control panel. The arrows indicate the strength and direction of the force that would be exerted by the micro wave on the water molecules present in food. Observe the response of water molecule in response to this force?
- Observe how do microwaves heat food by rotating water molecule?
- Change amplitude and frequency of microwave and discuss how fast the water molecules are rotating?

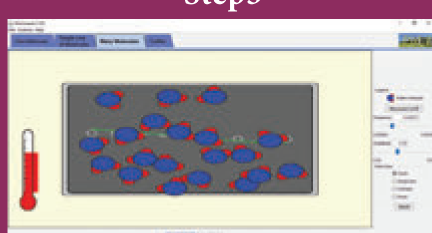
Step1



Step2



Step3



Step4

Discuss the relationship between rotating speed of the molecule with cooking time.

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

B263_12_PHYSICS_EM

UNIT-5(XII-Physics_Vol-1).indd 302 04-03-2019 11:05:56

Higher Secondary Second Year

PHYSICS

PRACTICAL

LIST OF EXPERIMENTS

1. Determination of the specific resistance of the material of the given coil using metre bridge.
2. Determination of the value of the horizontal component of the Earth's magnetic field using tangent galvanometer.
3. Determination of the magnetic field at a point on the axis of a circular coil.
4. Determination of the refractive index of the material of the prism by finding angle of prism and angle of minimum deviation using spectrometer.
5. Determination of the wavelength of a composite light by normal incidence method using diffraction grating and spectrometer (The number of lines per metre length of the grating is given).
6. Investigation of the voltage-current (V-I) characteristics of PN junction diode.
7. Investigation of the voltage-current (V-I) characteristics of Zener diode.
8. Investigation of the static characteristics of a NPN Junction transistor in common emitter configuration.
9. Verification of the truth table of the basic logic gates using integrated circuits.
10. Verification of De Morgan's theorems using integrated circuits.

R2TEF8

1. SPECIFIC RESISTANCE OF THE MATERIAL OF THE COIL USING METRE BRIDGE

AIM

To determine the specific resistance of the material of the given coil using metre bridge.

APPARATUS REQUIRED

Meter bridge, galvanometer, key, resistance box, connecting wires, Lechlanche cell, jockey and high resistance.

FORMULA

$$\rho = \frac{X\pi r^2}{L} \ (\Omega m)$$

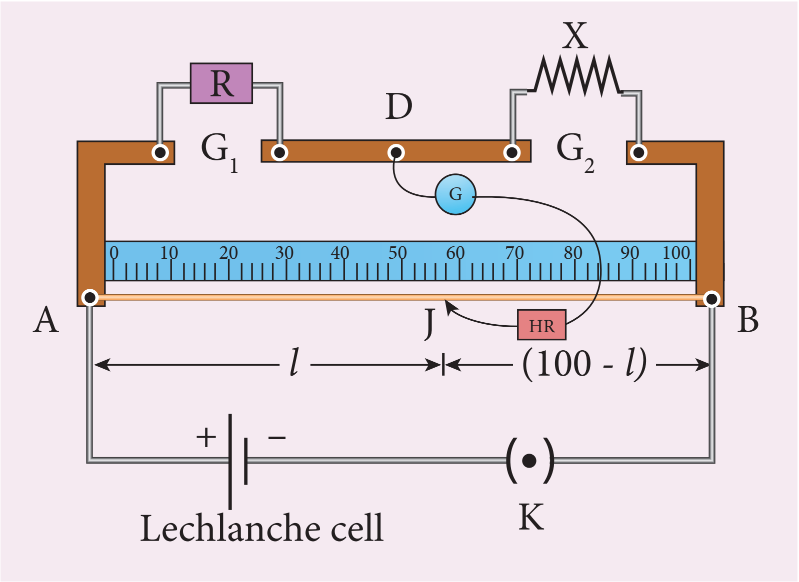
where, $X \rightarrow$ Resistance of the given coil (Ω)

$R \rightarrow$ Known resistance (Ω)

$L \rightarrow$ Length of the coil (m)

$r \rightarrow$ Radius of the wire (m)

CIRCUIT DIAGRAM



PROCEDURE

- A resistance box *R* is connected in the left gap and the unknown resistance X in the right gap.
- A Lechlanche cell is connected across the wire of length 1 m through a key.
- A sensitive galvanometer *G* is connected between the central strip and the jockey through a high resistance (HR).
- With a suitable resistance included in the resistance box, the circuit is switched on.
- To check the circuit connections, the jockey is pressed near one end of the wire, say A. The galvanometer will show deflection in one direction. When the jockey is pressed near the other end of the wire B, the galvanometer will show deflection in the opposite directions. This ensures that the circuit connections are correct.

- By moving the jockey over the wire, the point on the wire at which the galvanometer shows null deflection i.e., balancing point J is found.
- The balancing length $AJ = l$ is noted.
- The unknown resistance X_1 is found using the formula $X_1 = \frac{R(100-l)}{l}$.
- The experiment is repeated for different values of R.
- The same procedure is repeated after interchanging R and X.
- The unknown resistance X_2 is found using the formula $X_2 = \frac{Rl}{(100-l)}$.
- The experiment is repeated for same values of R as before.
- The resistance of the given coil is found from the mean value of X_1 and X_2.
- The radius of the wire r is found using screw gauge.
- The length of the coil L is measured using meter scale.
- From the values of X, r and L, the specific resistance of the material of the wire is determined.

OBSERVATION

length of the coil L = ________________________ cm.

Table 1 To find the resistance of the given coil

S.No.	Resistance R (Ω)	Before interchanging		After interchanging		Mean
		Balancing length l (cm)	$X_1 = \frac{R(100-l)}{l}$ (Ω)	Balancing length l (cm)	$X_2 = \frac{Rl}{(100-l)}$ (Ω)	$X = \frac{X_1 + X_2}{2}$ (Ω)
1						
2						
3						

Mean resistance, X = -----------Ω

Table 2 To find the radius of the wire

Zero error =

Zero correction =

LC = 0.01 mm

Sl.No.	PSR (mm)	HSC (div.)	Total Reading = PSR + (HSC × LC) (mm)	Corrected Reading = TR ± ZC (mm)
1				
2				
3				
4				
5				
6				

Mean diameter 2r = ... cm

Radius of the wire r = ... cm

r = ... m

CALCULATION

(i) $\rho = \frac{X\pi r^2}{L} =$

RESULT

The specific resistance of the material of the given coil = ______________________ (Ωm)

Note:

i) To check the circuit connections:

The meter bridge wire is touched near one end (say, end A) with jockey, galvanometer shows a deflection in any one direction. Now the other end (say, end B) is touched. If the galvanometer shows a deflection in the opposite direction, then the circuit connections are correct.

ii) The usage of high resistance (HR):

The galvanometer is a very sensitive device. If any high current flows through the galvanometer, its coil gets damaged. Therefore in order to protect the galvanometer, a high resistance (HR) is used. When HR is connected in series with the galvanometer, the current through it is reduced so that the galvanometer is protected. But the balancing length is not accurate.

iii) To find the accurate balancing length:

The HR is first included in the circuit (that is, the plug key in HR is removed), the approximate balancing length is found. Now HR is excluded in the circuit (that is, the plug key in HR is closed), then the accurate balancing length is found.

2. HORIZONTAL COMPONENT OF EARTH'S MAGNETIC FIELD USING TANGENT GALVANOMETER

AIM

To determine the horizontal component of the Earth's magnetic field using tangent galvanometer.

APPARATUS REQUIRED

Tangent galvanometer (TG), commutator, battery, rheostat, ammeter, key and connecting wires.

FORMULA

$$B_H = \frac{\mu_0 n k}{2r} \text{ (Tesla)}$$

$$k = \frac{I}{\tan\theta} \ (A)$$

where,
- B_H → Horizontal component of the Earth's magnetic field (T)
- μ_0 → Permeability of free space ($4\pi \times 10^{-7}$ H m^{-1})
- n → Number of turns of TG in the circuit (No unit)
- k → Reduction factor of TG (A)
- r → Radius of the coil (m)

CIRCUIT DIAGRAM

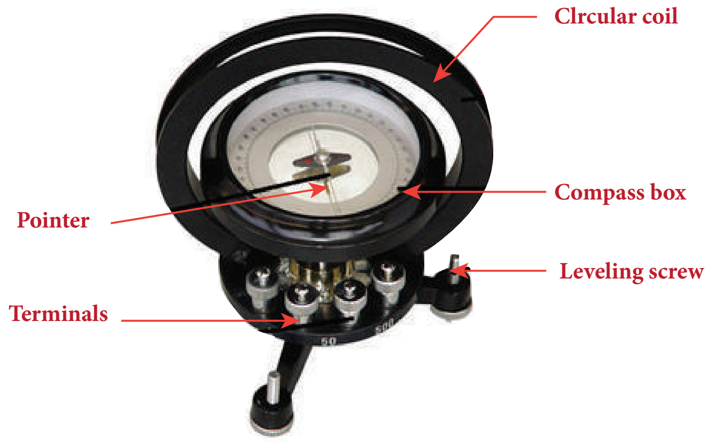
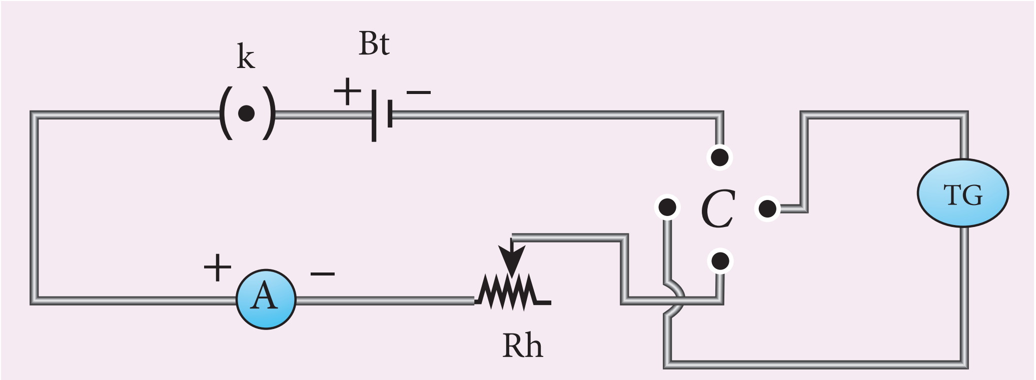


Figure (a) Tangent Galvanometer

Figure (b) Number of turns



(c) Circuit diagram

PROCEDURE

- The preliminary adjustments are carried out as follows.
  a. The leveling screws at the base of TG are adjusted so that the circular turn table is horizontal and the plane of the circular coil is vertical.
  b. The circular coil is rotated so that its plane is in the magnetic meridian i.e., along the north-south direction.
  c. The compass box alone is rotated till the aluminium pointer reads $0° - 0°$.
- The connections are made as shown in Figure (c).
- The number of turns *n* is selected and the circuit is switched on.
- The range of current through TG is chosen in such a way that the deflection of the aluminium pointer lies between $30° - 60°$.
- A suitable current is allowed to pass through the circuit, the deflections θ_1 and θ_2 are noted from two ends of the aluminium pointer.
- Now the direction of current is reversed using commutator C, the deflections θ_3 and θ_4 in the opposite direction are noted.
- The mean value θ of θ_1, θ_2, θ_3 and θ_4 is calculated and tabulated.
- The reduction factor k is calculated for each case and it is found that k is a constant.
- The experiment is repeated for various values of current and the readings are noted and tabulated.
- The radius of the circular coil is found by measuring the circumference of the coil using a thread around the coil.
- From the values of *r*, *n* and *k*, the horizontal component of Earth's magnetic field is determined.

Commutator:

It is a kind of switch employed in electrical circuits, electric motors and electric generators. It is used to reverse the direction of current in the circuit.

OBSERVATION

Number of turns of the coil n =

Circumference of the coil ($2\pi r$) =

Radius of the coil r =

S.No	Current I (A)	Deflection in TG (degree)				Mean θ (degree)	$k = \frac{I}{\tan\theta}$
		θ_1	θ_2	θ_3	θ_4		
1							
2							
3							
4							
						Mean	

CALCULATION

$$B_H = \frac{\mu_0 n k}{2r} =$$

RESULT

The horizontal component of Earth's magnetic field is found to be ________

Note:

i) The magnetic materials and magnets present in the vicinity of TG should be removed.

ii) The readings from the ends of the aluminium pointer should be taken without parallax error.

iii) The deflections of TG is restricted between 30° and 60°. It is because, the TG is most sensi tive for deflection around 45° and is least sensitive around 0° and 90°. We know that

$$I = k \tan\theta$$

$$\text{or } dI = k \sec^2\theta \, d\theta$$

$$\frac{d\theta}{dI} = \frac{\sin 2\theta}{2I}$$

For given current, sensitivity $\frac{d\theta}{dI}$ maximum for $\sin 2\theta = 1$ or $\theta = 45°$

3. MAGNETIC FIELD ALONG THE AXIS OF A CIRCULAR COIL-DETERMINATION OF B_H

AIM

To determine the horizontal component of Earth's magnetic field using current carrying circular coil and deflection magnetometer.

APPARATUS REQUIRED

Circular coil apparatus, compass box, rheostat, battery or power supply, ammeter, commutator, key and connecting wires.

FORMULA

$$B_H = \frac{\mu_0 n I r^2}{2\left(r^2 + x^2\right)^{3/2}} \left(\frac{1}{\tan\theta}\right) \text{ (Tesla)}$$

where, B_H → Horizontal component of Earth's magnetic field (T)

μ_o → Permeability of the free space ($4\pi \times 10^{-7}$ H m^{-1})

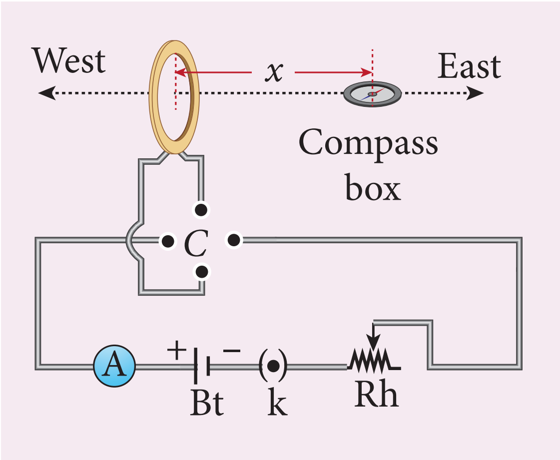
n → Number of turns included in the circuit (No unit)

I → Current flowing through the coil (A)

r → Radius of the circular coil (m)

x → Distance between center of compass box and centre of the coil (m)

CIRCUIT DIAGRAM



PROCEDURE

- The preliminary adjustments are carried out as follows.
  i. The leveling screws are adjusted so that the circular coil is vertical.
  ii. The wooden bench is adjusted to be along the magnetic east-west direction i.e., along aluminium pointer.
  iii. The circular coil is rotated so that its plane is in magnetic meridian i.e., along the north-south direction.
  iv. A compass box is placed with its centre coinciding with the axis of the coil.
  v. The compass box alone is rotated till the aluminium pointer reads $0° - 0°$
- Electrical connections are made as shown in the circuit diagram.
- The compass box is placed along its axis, with its centre at a distance x from the centre of the coil on one side.

- A suitable current (1A) is passed through the coil by adjusting rheostat so that the deflection of the aluminium pointer lies between 30º and 60º.
- The value of the current I is noted from ammeter.
- Two readings θ_1 and θ_2 corresponding to two ends of the pointer are noted.
- Now the direction of the current is reversed using commutator, two more readings θ_3 and θ_4 are noted.
- Now the compass box is taken to the other side and is kept at the same distance *x*.
- Four more readings θ_5, θ_6, θ_7 and θ_8 are taken as done before.
- These eight readings and their average value are tabulated.
- The experiment is repeated for another value of current, say 1.5 A by keeping the compass box at the same distance *x*.
- The radius of the circular coil is found by measuring the circumference of the coil using a thread around the coil.
- The number of turns n of the coil is noted.
- From the values of *n, r, x* and $I/\tan\theta$, the horizontal component of Earth's magnetic field is now found using the formula.

OBSERVATION

Number of turns in the coil *n*=

Circumference of the coil (2πr) =

Radius of the coil r =

To find horizontal component of the Earth's magnetic field

S.No	Distance x (cm)	Current I (A)	Deflection for eastern side				Deflection for western side				Mean θ	$\frac{I}{\tan\theta}$
			θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8		
											Mean	

CALCULATION

$$B_H = \frac{\mu_0 n I r^2}{2\left(r^2 + x^2\right)^{3/2}}\left(\frac{1}{\tan\theta}\right) =$$

RESULT

Horizontal component of the Earth's magnetic field at a place = __________T

4. REFRACTIVE INDEX OF THE MATERIAL OF THE PRISM

AIM To determine the refractive index of the material of a prism using spectrometer.

APPARATUS REQUIRED Spectrometer, prism, prism clamp, sodium vapour lamp, spirit level.

FORMULA

$$\mu = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)} \quad \text{(No unit)}$$

where, μ → Refractive index of the material of the prism (No unit)

A → Angle of the prism (degree)

D → Angle of minimum deviation (degree)

DIAGRAMS

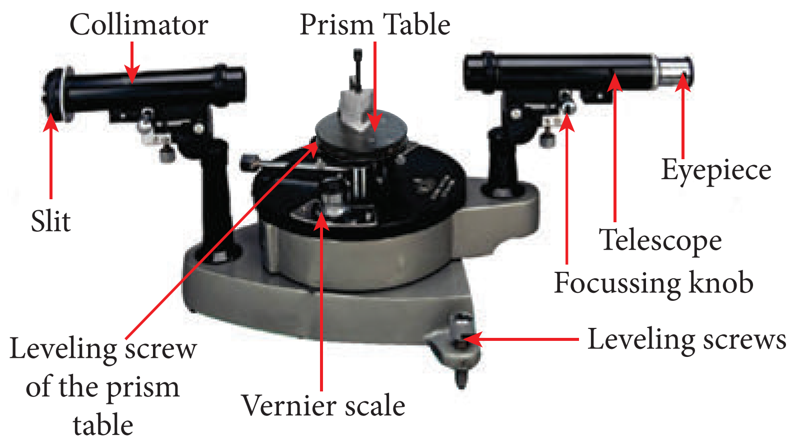


Figure (a) Angle of the prism

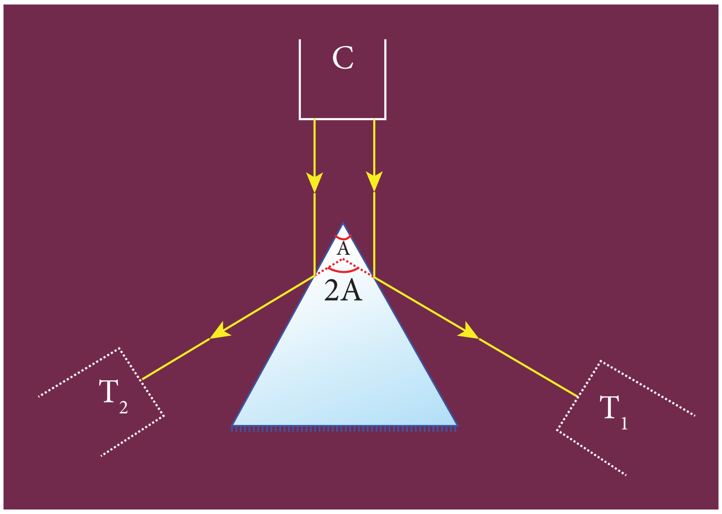


Figure (b) Angle of the prism

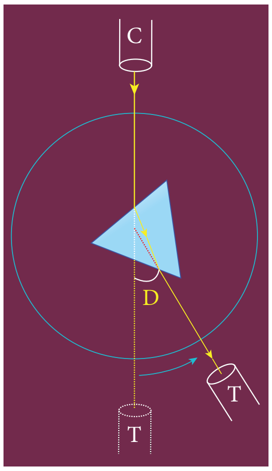


Figure (c) Angle of minimum deviation

PROCEDURE

1) Initial adjustments of the spectrometer

- Eye-piece: The eye-piece of the telescope is adjusted so that the cross-wires are seen clearly.
- Slit: The slit of the collimator is adjusted such that it is very thin and vertical.
- Base of the spectrometer: The base of the spectrometer is adjusted to be horizontal using leveling screws.
- Telescope: The telescope is turned towards a distant object and is adjusted till the clear inverted image of the distant object is seen. Now the telescope is adjusted to receive parallel rays.
- Collimator: The telescope is brought in line with the collimator. Collimator is adjusted until a clear image of the slit is seen in the telescope. Now the collimator gives parallel rays.
- Prism table: Using a spirit level, the prism table is adjusted to be horizontal with the three leveling screws provided in the prism table.

2) Determination of angle of the prism (A)

- The slit is illuminated by yellow light from sodium vapour lamp.
- The given equilateral prism is placed on the prism table in such a way that refracting edge of the prism is facing the collimator.
- The light emerging from the collimator is incident on both reflecting faces of the prism and is reflected.
- The telescope is rotated towards left to obtain reflected image of the slit from face 1 of the prism and is fixed.
- Using tangential screws, the telescope is adjusted until the vertical cross-wire coincides with the reflected image of the slit.
- The main scale reading and vernier coincidence are noted from both vernier scales.
- The telescope is now rotated towards right to obtain the reflected image from face 2 of the prism. As before, the readings are taken.
- The difference between the two readings gives 2A from which the angle of the prism A is calculated.

3) Determination of angle of minimum deviation (D)

- The prism table is rotated such that the light emerging from the collimator is incident on one of the refracting faces of the prism, gets refracted and emerges out from the other refracting face.
- The telescope is turned to view the refracted image.
- Looking through the telescope, the prism table is rotated in such a direction that the image moves towards the direct ray.

- At one particular position, the refracted ray begins to retrace its path. The position where the refracted image returns is the position of minimum deviation.
- The telescope is fixed in this position and is adjusted until the vertical cross-wire coincides with the refracted image of the slit.
- The readings are taken from both vernier scales.
- The prism is now removed and the telescope is rotated to obtain the direct ray image and the readings are taken.
- The readings are tabulated and the difference between these two readings gives the angle of minimum deviation D.
- From the values of A and D, the refractive index of the material of the glass prism is determined.

Least count

1 MSD = 30′

Number of vernier scale divisions = 30

For spectrometer, 30 vernier scale divisions will cover 29 main scale divisions.

∴ 30 VSD = 29 MSD

Or 1 VSD = 29 / 30 MSD

Least count (LC) = 1 MSD – 1 VSD

= 1 / 30 MSD

= 1′

OBSERVATION

Table 1 To find the angle of the prism (A)

Image	Vernier A (Degree)			Vernier B (Degree)		
	MSR	VSC	TR	MSR	VSC	TR
Reflected image from face 1						
Reflected image from face 2						
Difference 2A						

Mean 2A =

Mean A =

Table 2 To find the angle of minimum deviation (D)

Image	Vernier A (Degree)			Vernier B (Degree)		
	MSR	VSC	TR	MSR	VSC	TR
Refracted image						
Direct image						
Difference D						

Mean D =

RESULT

1. Angle of the Prism (A) = ... (degree)
2. Angle of the minimum deviation of the prism (D) =............ (degree)
3. Refractive index of the material of the Prism (μ) =............ (No unit)

Note:

i) Once initial adjustments are done, spectrometer should not be disturbed.

ii) Total reading TR = MSR + (VSC × LC)

Where

MSR → Main Scale Reading

VSC → Vernier Scale Coincidence

LC → Least count (= 1′)

5. WAVELENGTH OF THE CONSTITUENT COLOURS OF A COMPOSITE LIGHT USING DIFFRACTION GRATING AND SPECTROMETER

AIM

To find the wavelength of the constituent colours of a composite light using diffraction grating and spectrometer.

APPARATUS REQUIRED

Spectrometer, mercury vapour lamp, diffraction grating, grating table, and spirit level.

FORMULA

$$\lambda = \frac{\sin\theta}{nN} \text{ Å}$$

where, λ → Wavelength of the constituent colours of a composite light (Å)

N → Number of lines per metre length of the given grating (No unit) (the value of N for the grating is given)

n → Order of the diffraction (No unit)

θ → Angle of diffraction (degree)

DIAGRAMS

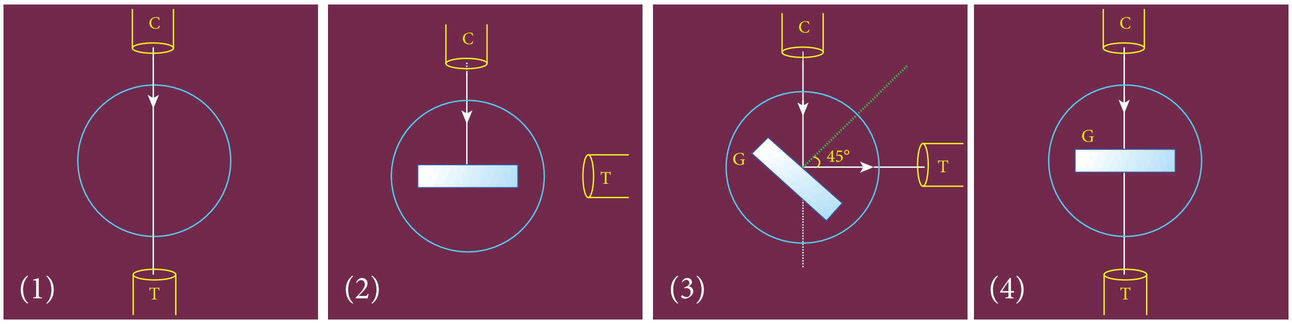


Figure (a) Normal incidence

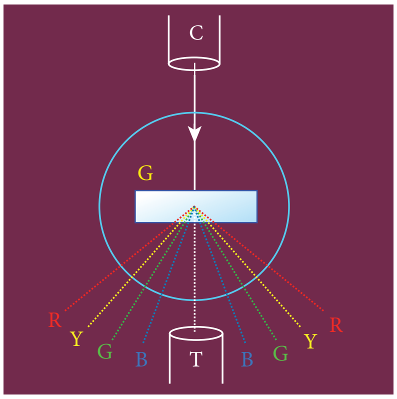


Figure (b) Angle of diffraction

PROCEDURE

1) **Initial adjustments of the spectrometer**

- Eye-piece: The eye-piece of the telescope is adjusted so that the cross-wires are seen clearly.
- Slit: The slit of the collimator is adjusted such that it is very thin and vertical.
- Base of the spectrometer: The base of the spectrometer is adjusted to be horizontal using leveling screws.

- Telescope: The telescope is turned towards a distant object and is adjusted till the clear image of the distant object is seen. Now the telescope is adjusted to receive parallel rays.
- Collimator: The telescope is brought in line with the collimator. Collimator is adjusted until a clear image of the slit is seen in the telescope. Now the collimator gives parallel rays.
- Grating table: Using a spirit level, the grating table is adjusted to be horizontal with the three leveling screws provided in the grating table.

2) Adjustment of the grating for normal incidence

- The slit is illuminated with a composite light (white light) from mercury vapour lamp.
- The telescope is brought in line with the collimator. The vertical cross-wire is made to coincide with the image of the slit (Figure (a)1).
- The vernier disc alone is rotated till the vernier scale reads 0^0 - 180^0 and is fixed. This is the reading for the direct ray.
- The telescope is then rotated (anti-clockwise) through an angle of 90^0 and fixed (Figure (a)2).
- Now the plane transmission grating is mounted on the grating table.
- The grating table alone is rotated so that the light reflected from the grating coincides with vertical cross-wire of the telescope. The reflected image is white in colour (Figure (a)3).
- Now the vernier disc is released. The vernier disc along with grating table is rotated through an angle of 45^0 in the appropriate direction such that the light from the collimator is incident normally on the grating (Figure (a)4).

3) Determination of wave length of the constituent colours of the mercury spectrum

- The telescope is released and is brought in line with the collimator to receive central direct image. This undispersed image is white in colour.
- The diffracted images of the slit are observed on either side of the direct image.
- The diffracted image consists of the prominent colours of mercury spectrum in increasing order of wavelength.
- The telescope is turned to any one side (say left) of direct image to observe first order diffracted image.
- The vertical cross-wire is made to coincide with the prominent spectral lines (violet, blue, yellow and red) and the readings of both vernier scales for each case are noted.
- Now the telescope is rotated to the right side of the direct image and the first order image is observed.
- The vertical cross-wire is made to coincide with the same prominent spectral lines and the readings of both vernier scales for each case are again noted.
- The readings are tabulated.
- The difference between these two readings gives the value of 2θ for the particular spectral line.
- The number of lines per metre length of the given grating N is noted from the grating.
- From the values of N, n and θ, the wave length of the prominent colours of the mercury light is determined using the given formula.

OBSERVATION

To find the wave length of prominent colours of the mercury spectrum

Colour of Light	Diffracted Ray Reading (Degree)												Difference 2θ (Degree)			θ (Degree)
	Left						Right									
	Vernier A			Vernier B			Vernier A			Vernier B						
	MSR	VSC	TR	MSR	VSC	TR	MSR	VSC	TR	MSR	VSC	TR	VER A	VER B	Mean	
Blue																
Green																
Yellow																
Red																

CALCULATION

(i) For blue, $\lambda = \frac{\sin\theta}{nN}$,

(ii) For green, $\lambda = \frac{\sin\theta}{nN}$

(iii) For yellow, $\lambda = \frac{\sin\theta}{nN}$,

(iv) For red, $\lambda = \frac{\sin\theta}{nN}$

RESULT

1. The wavelength of blue line = ----------------m
2. The wavelength of green line = ------------------m
3. The wavelength of yellow line = ----------------m
4. The wavelength of red line = ----------------m

Note:

i) Once initial adjustments are done, spectrometer should not be disturbed.

ii) Total reading TR = MSR + (VSC × LC)

Where

MSR → Main Scale Reading

VSC → Vernier Scale Coincidence

LC → Least count (= 1′)

6. VOLTAGE-CURRENT CHARACTERISTICS OF A PN JUNCTION DIODE

AIM

To draw the voltage-current (V- I) characteristics of the PN junction diode and to determine its knee voltage and forward resistance.

APPARATUS REQUIRED

PN junction diode (IN4007), variable DC power supply, milli-ammeter, micro-ammeter, voltmeter, resistance and connecting wires.

FORMULA

$$R_F = \frac{\Delta V_F}{\Delta I_F} \; (\Omega)$$

where, $R_F \rightarrow$ Forward resistance of the diode (Ω)

$\Delta V_F \rightarrow$ The change in forward voltage (volt)

$\Delta I_F \rightarrow$ The change in forward current (mA)

CIRCUIT DIAGRAM

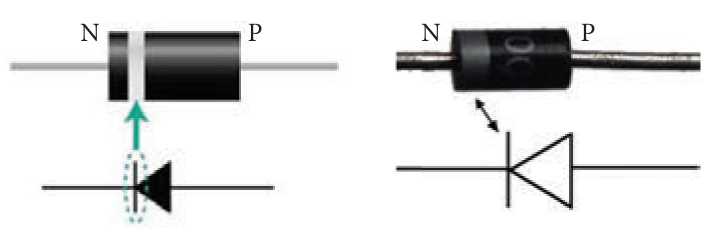


Figure (a) PN junction diode and its symbol (Silver ring denotes the negative terminal of the diode)

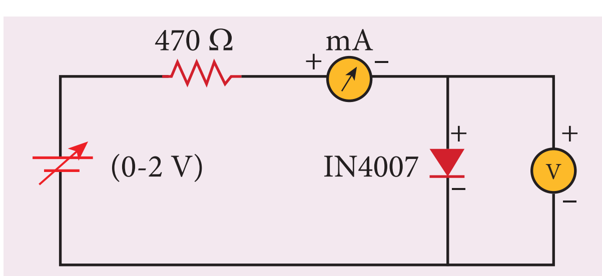


Figure (b) PN junction diode in forward bias

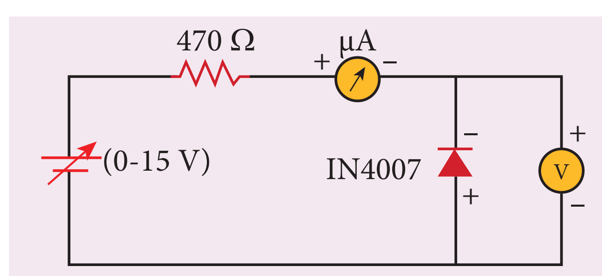


Figure (c) PN junction diode in reverse bias

Precaution

Care should be taken to connect the terminals of ammeter, voltmeter, dc power supply and the PN junction diode with right polarity.

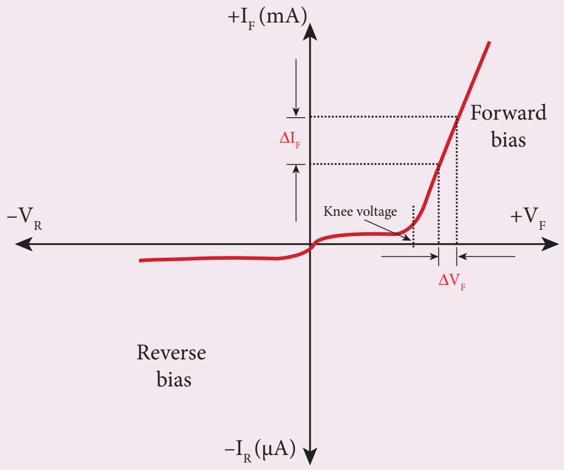
PROCEDURE

i) Forward bias characteristics

- In the forward bias, the P- region of the diode is connected to the positive terminal and N-region to the negative terminal of the DC power supply.
- The connections are given as per the circuit diagram.
- The voltage across the diode can be varied with the help of the variable DC power supply.
- The forward voltage (V_F) across the diode is increased from 0.1 V in steps of 0.1 V up to 0.8|V and the forward current (I_F) through the diode is noted from the milli-ammeter. The readings are tabulated.
- The forward voltage V_F and the forward current I_F are taken as positive.
- A graph is drawn taking the forward voltage (V_F) along the x-axis and the forward current (I_F) along the y-axis.
- The voltage corresponding to the dotted line in the forward characteristics gives the knee voltage or threshold voltage or turn-on voltage of the diode.
- The slope in the linear portion of the forward characteristics is calculated. The reciprocal of the slope gives the forward resistance of the diode.

ii) Reverse bias characteristics

- In the reverse bias, the polarity of the DC power supply is reversed so that the P- region of the diode is connected to the negative terminal and N-region to the positive terminal of the DC power supply
- The connections are made as given in the circuit diagram.
- The voltage across the diode can be varied with the help of the variable DC power supply.
- The reverse voltage (V_R) across the diode is increased from 1 V in steps of 1 V up to 5 V and the reverse current (I_R) through the diode is noted from the micro-ammeter. The readings are tabulated.
- The reverse voltage V_R and reverse current I_R are taken as negative.
- A graph is drawn taking the reverse bias voltage (V_R) along negative x-axis and the reverse bias current (I_R) along negative y-axis.



Slope = $\frac{\Delta I_F}{\Delta V_F}$

Forward resistance = $\frac{1}{\text{slope}}$

OBSERVATION

Table 1 Forward bias characteristic curve

S.No.	Forward bias voltage V_F (volt)	Forward bias current I_F (mA)

Table 2 Reverse bias characteristic curve

S.No.	Reverse bias voltage V_R (volt)	Reverse bias current I_R (μA)

CALCULATION

(i) Forward resistance R_F=

(ii) knee voltage =

RESULT

The V-I characteristics of the PN junction diode are studied.

i) Knee voltage of the PN junction diode =……………..V

ii) Forward resistance of the diode =……………..Ω

Practical Tips

- The DC power supply voltage should be increased only up to the specified range in the forward (0 – 2V) and reverse (0 – 15V) directions. Forward bias offers very low resistance and hence an external resistance of 470Ω is connected as a safety measure.
- The voltage applied beyond this limit may damage the resistance or the diode.
- In the forward bias, the current flow will be almost zero till it crosses the junction potential or knee voltage (approximately 0.7 V). Once knee voltage is crossed, the current increases with the applied voltage.
- The diode voltage in the forward direction should be increased in steps of 0.1 V to a maximum of 0.8 V after the threshold voltage to calculate the forward resistance.
- The diode voltage in the reverse direction is increased in steps of 1 V to a maximum of 5 V. The current must be measured using micro-ammeter as the strength of current in the reverse direction is very less. This is due to the flow of the minority charge carriers called the leakage current.

7. VOLTAGE-CURRENT CHARACTERISTICS OF A ZENER DIODE

AIM

To draw the voltage-current (V-I) characteristic curves of a Zener diode and to determine its knee voltage, forward resistance and reverse breakdown voltage.

APPARATUS REQUIRED

Zener diode IZ5.6V, variable dc power supply (0 – 15V), milli ammeter, volt meter, 470 Ω resistance, and connecting wires.

FORMULA

$$R_F = \frac{\Delta V_F}{\Delta I_F}\ (\Omega)$$

where, $R_F \rightarrow$ Forward resistance of the diode (Ω)

$\Delta V_F \rightarrow$ The change in forward voltage (volt)

$\Delta I_F \rightarrow$ The change in forward current (mA)

CIRCUIT DIAGRAM

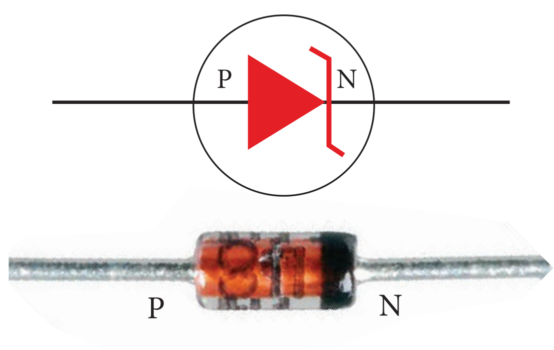


Figure (a) Zener diode and its symbol (The black colour ring denotes the negative terminal of the Zener diode)

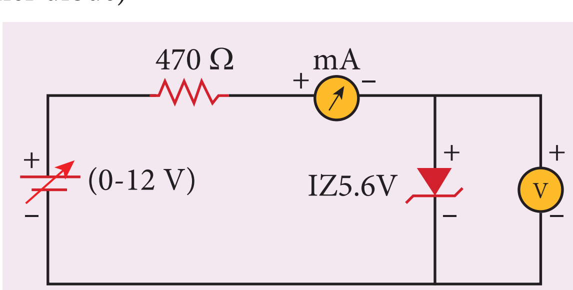


Figure (b) Zener diode in forward bias

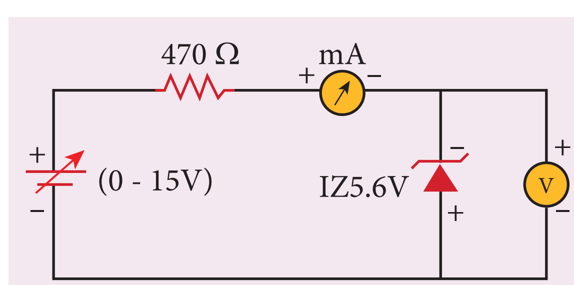


Figure (c) Zener diode in reverse bias

Precaution

Care should be taken to connect the terminals of ammeter, voltmeter, dc power supply and the Zener diode with right polarity.

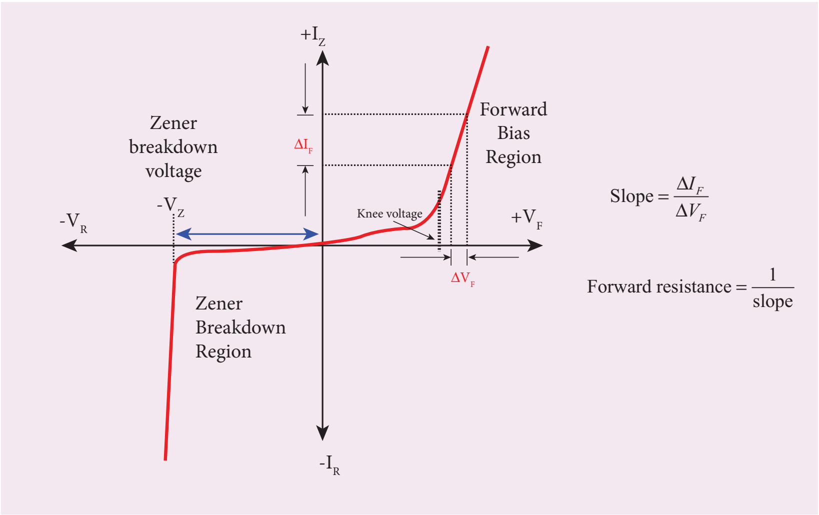
PROCEDURE

i) Forward bias characteristics

- In the forward bias, the P- region of the diode is connected to the positive terminal and N-region to the negative terminal of the DC power supply.
- The connections are given as per the circuit diagram.
- The voltage across the diode can be varied with the help of the variable DC power supply.
- The forward voltage (V_F) across the diode is increased from 0.1V in steps of 0.1V up to 0.8V and the forward current (I_F) through the diode is noted from the milli-ammeter. The readings are tabulated.
- The forward voltage and the forward current are taken as positive.
- A graph is drawn taking the forward voltage along the x-axis and the forward current along the y-axis.
- The voltage corresponding to the dotted line in the forward characteristics gives the knee voltage or threshold voltage or turn-on voltage of the diode.
- The slope in the linear portion of the forward characteristics is calculated. The reciprocal of the slope gives the forward resistance of the diode.

ii) Reverse bias characteristics

- In the reverse bias, the polarity of the DC power supply is reversed so that the P- region of the diode is connected to the negative terminal and N-region to the positive terminal of the DC power supply
- The connections are made as given in the circuit diagram.
- The voltage across the diode can be varied with the help of the variable DC power supply.
- The reverse voltage (V_R) across the diode is increased from 0.5V in steps of 0.5V up to 6V and the reverse current (I_R) through the diode is noted from the milli-ammeter. The readings are tabulated.
- Initially, the voltage is increased in steps of 0.5V. When the breakdown region is approximately reached, then the input voltage may be raised in steps of, say 0.1V to find the breakdown voltage.
- The reverse voltage and reverse current are taken as negative.
- A graph is drawn taking the reverse bias voltage along negative x-axis and the reverse bias current along negative y-axis.
- In the reverse bias, Zener breakdown occurs at a particular voltage called Zener voltage V_Z (~5.6 to 5.8V) and a large amount of current flows through the diode which is the characteristics of a Zener diode.
- The breakdown voltage of the Zener diode is determined from the graph as shown.



OBSERVATION

Table 1 Forward bias characteristic curve

S.No.	Forward bias voltage V_F (volt)	Forward bias current I_F (mA)

Table 2 Reverse bias characteristic curve

S.No.	Reverse bias voltage V_R (volt)	Reverse bias current I_R (mA)

CALCULATION

(i) Forward resistance R_F=

(ii) knee voltage =

(iii) The breakdown voltage of the Zener diode V_Z= ----V

RESULT

The V-I characteristics of the Zener diode are studied.

(i) Forward resistance R_F=

(ii) knee voltage =

(iii) The breakdown voltage of the Zener diode V_Z= ----V

Practical Tips

- The DC power supply voltage should to be increased only up to the specified range in the forward (0 – 2 V) and reverse (0 – 15 V) directions.
- The voltage applied beyond this limit may damage the resistor or the diode.
- Zener diode functions like an ordinary PN junction diode in the forward direction. Hence the forward characteristic is the same for both PN junction diode and Zener diode. Therefore, knee voltage and forward resistance can be determined as explained in the previous experiment.
- Unlike ordinary PN junction diode, the reverse current in Zener diode is measured using milli-ammeter due to the large flow of current.

8 CHARACTERISTICS OF A NPN-JUNCTION TRANSISTOR IN COMMON EMITTER CONFIGURATION

AIM

To study the characteristics and to determine the current gain of a NPN junction transistor in common emitter configuration.

APPARATUS REQUIRED

Transistor - BC 548/BC107, bread board, micro ammeter, milli ammeter, voltmeters, variable DC power supply and connecting wires.

FORMULA

$$r_i = \left[\frac{\Delta V_{BE}}{\Delta I_B}\right]_{V_{CE}} (\Omega), \quad r_o = \left[\frac{\Delta V_{CE}}{\Delta I_C}\right]_{I_B} (\Omega), \quad \beta = \left[\frac{\Delta I_C}{\Delta I_B}\right]_{V_{CE}} \text{ (No unit)}$$

Where, r_i → Input impedance (Ω)

ΔV_{BE} → The change in base-emitter voltage (volt)

ΔI_B → The change in base current (μA)

r_o → Output impedance (Ω)

ΔV_{CE} → The change in collector-emitter voltage (volt)

ΔI_C → The change in collector current (mA)

β → Current gain of the transistor (No unit)

CIRCUIT DIAGRAM

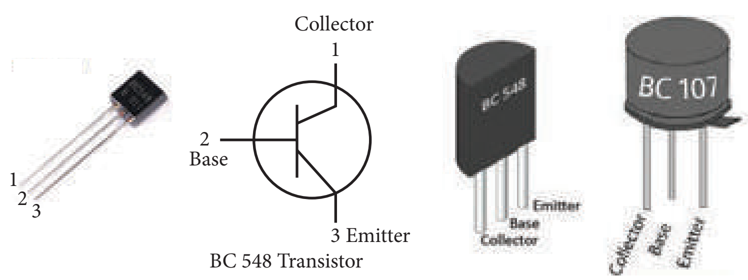


Figure (a) NPN - Junction transistor and its symbol (Transistor is held with the flat surface facing us)

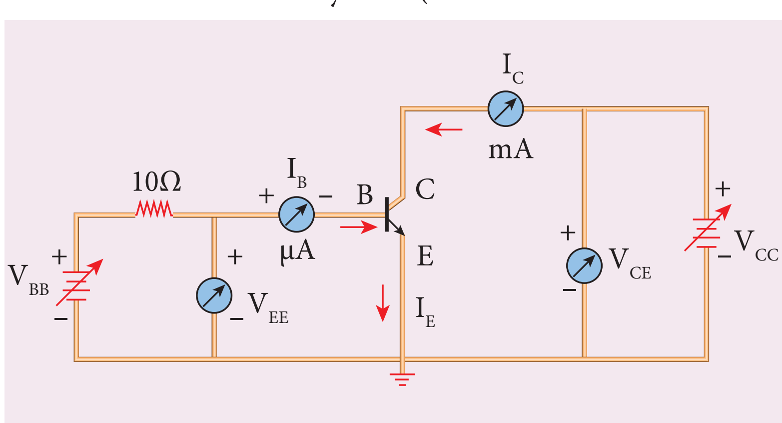


Figure (b) NPN junction transistor in CE configuration

> **Note**
>
> A resistor is connected in series with the base to prevent excess current flowing into the base.

Precautions

- Care should be taken to connect the terminals of ammeters, voltmeters, and dc power supplies with right polarity.
- The collector and emitter terminals of the transistor must not be interchanged.

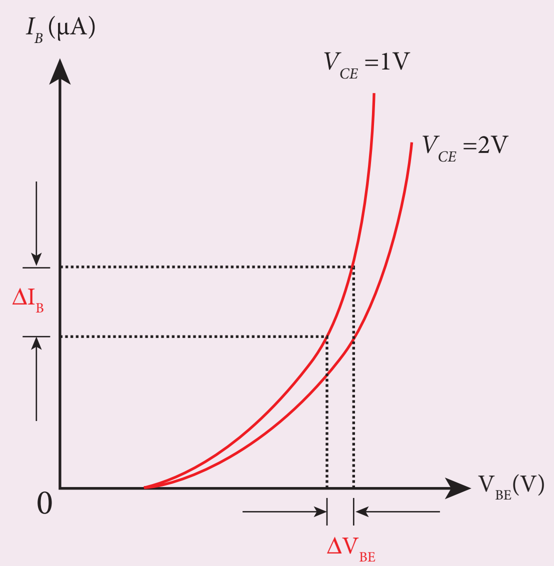
PROCEDURE

- The connections are given as shown in the diagram.
- The current and voltage at the input and output regions can be varied by adjusting the DC power supply.

(i) Input characteristic curve: V_{BE} vs I_B (V_{CE} constant)

- The collector-emitter voltage V_{CE} is kept constant.
- The base-emitter voltage V_{BE} is varied in steps of 0.1V and the corresponding base current (I_B) is noted. The readings are taken till V_{CE} reaches a constant value.
- The same procedure is repeated for different values of V_{CE}. The readings are tabulated.
- A graph is plotted by taking V_{BE} along x-axis and I_B along y-axis for both the values of V_{CE}.
- The curves thus obtained are called the input characteristics of a transistor.
- The reciprocal of the slope of these curves gives the input impedance of the transistor.

S. No	V_{CE} = 1V		V_{CE} = 2V	
	V_{BE} (V)	I_B (μA)	V_{BE} (V)	I_B (μA)



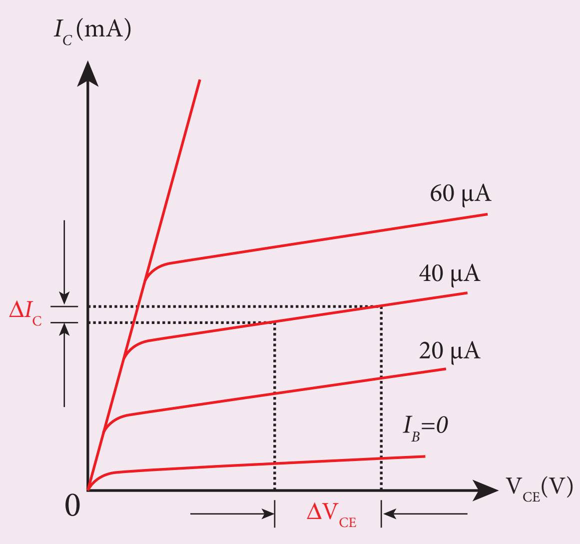
$$\text{Slope} = \frac{\Delta I_B}{\Delta V_{BE}}$$

$$\text{Input impedance} = \frac{1}{\text{slope}}$$

(ii) Output characteristic curve: V_{CE} vs I_C (I_B constant)

- The base current I_B is kept constant.
- V_{CE} is varied in steps of 1V and the corresponding collector current I_C is noted. The readings are taken till the collector current becomes almost constant.
- Initially I_B is kept at 0 mA and the corresponding collector current is noted. This current is the reverse saturation current I_{CEO}.
- The experiment is repeated for various values of I_B. The readings are tabulated.
- A graph is drawn by taking V_{CE} along x-axis and I_C along y-axis for various values of I_B.
- The set of curves thus obtained is called the output characteristics of a transistor.
- The reciprocal of the slope of the curve gives output impedance of the transistor.

S. No	I_B =20 μA		I_B =40 μA	
	V_{CE} (V)	I_C (mA)	V_{CE} (V)	I_C (mA)



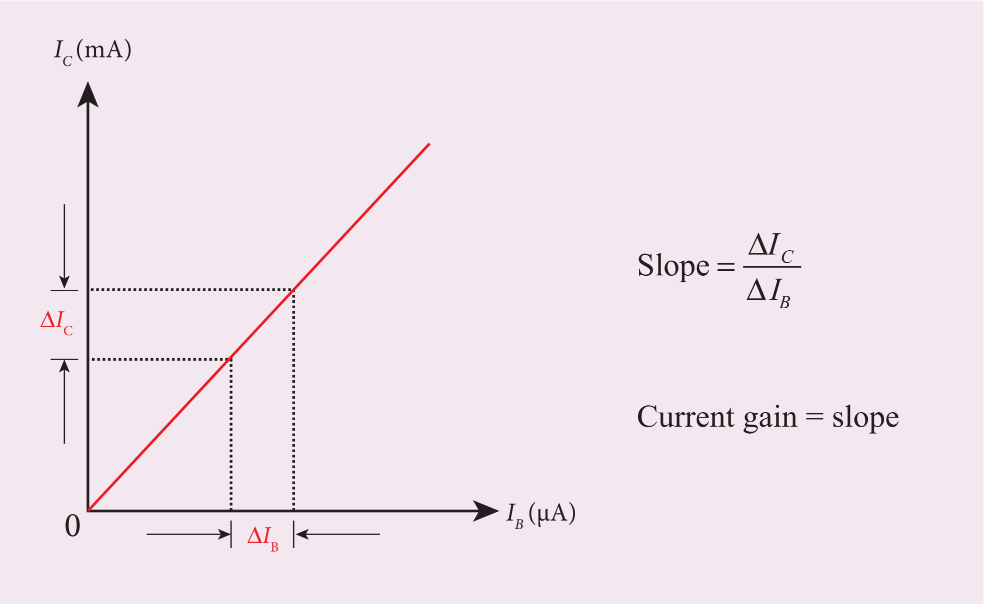
$$\text{Slope} = \frac{\Delta I_C}{\Delta V_{CE}}$$

$$\text{Output impedance} = \frac{1}{\text{slope}}$$

(iii) Transfer characteristic curve: I_B vs I_C (V_{CE} constant)

- The collector-emitter voltage V_{CE} is kept constant.
- The base current I_B is varied in steps of 10 μA and the corresponding collector current I_C is noted.
- This is repeated by changing the value of V_{CE}. The readings are tabulated.
- The transfer characteristics is a plot between the input current IB along x-axis and the output current I_C along y-axis keeping V_{CE} constant.
- The slope of the transfer characteristics plot gives the current gain β can be calculated.

S.No	V_{CE} =1V		V_{CE} =2V	
	I_B (μA)	I_C (mA)	I_B (μA)	I_C (mA)



RESULT

i) The input, output and transfer characteristics of the NPN junction in common emitter mode are drawn.

ii) (a) Input impedance = _________Ω

(b) Output impedance = _________Ω

(c) Current gain β = ____(no unit)

9. VERIFICATION OF TRUTH TABLES OF LOGIC GATES USING INTEGRATED CIRCUITS

AIM To verify the truth tables of AND, OR, NOT, EX-OR, NAND and NOR gates using integrated circuits

COMPONENTS REQUIRED AND gate (IC 7408), NOT gate (IC 7404), OR gate (IC 7432), NAND gate (IC 7400), NOR gate (IC 7402), X-OR gate (IC 7486), Power supply, Digital IC trainer kit, connecting wires.

BOOLEAN EXPRESSIONS

(i) AND gate $Y = A.B$

(ii) OR gate $Y = A+B$

(iii) NOT gate $Y = \overline{A}$

(iv) Ex OR gate $Y = \overline{A}B + A\overline{B}$

(v) NAND gate $Y = \overline{A.B}$

(vi) NOR gate $Y = \overline{A+B}$

CIRCUIT DIAGRAM

Pin Identification

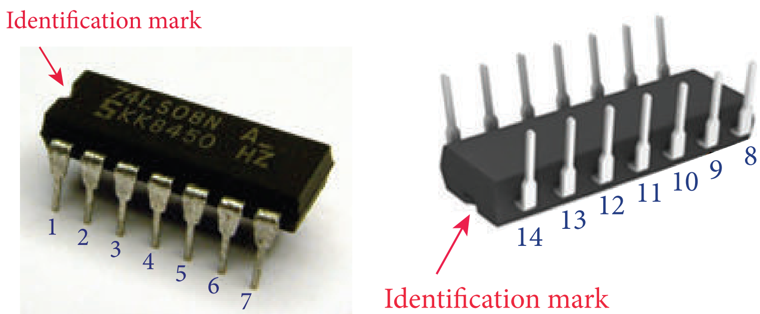
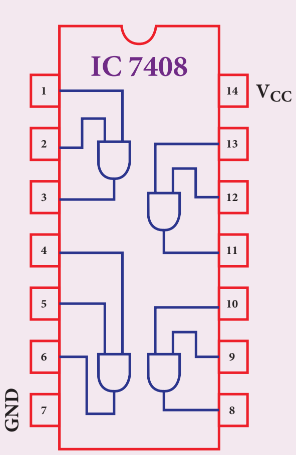


Figure (a) Integrated circuit

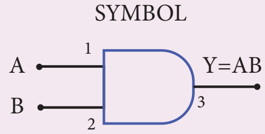
> **Note:**
>
> The chip must be inserted in the bread board in such a way that the identification mark should be on our left side. In this position, pin numbers are counted as marked in the picture above. Pin identification is the same for all chips that are mentioned below.

AND Gate:

PIN DIAGRAM-IC 7408



SYMBOL

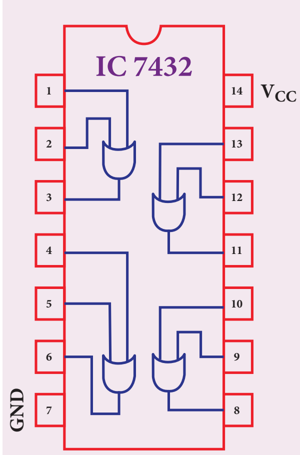


TRUTH TABLE

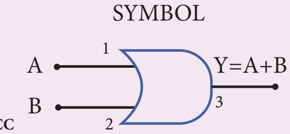
A	B	Y=AB
0	0	0
0	1	0
1	0	0
1	1	1

OR Gate:

PIN DIAGRAM-IC 7432



SYMBOL

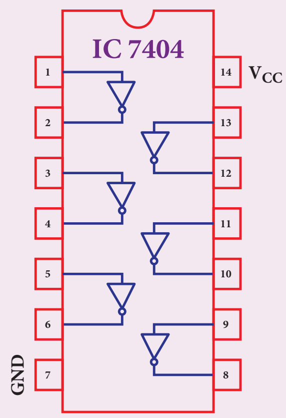


TRUTH TABLE

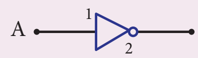
A	B	Y=A+B
0	0	0
0	1	1
1	0	1
1	1	1

NOT Gate:

PIN DIAGRAM-IC 7404



SYMBOL

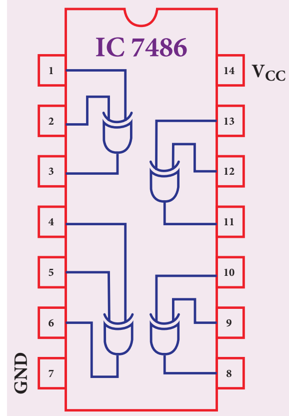


TRUTH TABLE

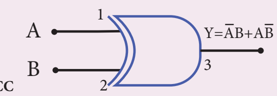
A	$Y=\bar{A}$
0	1
1	0

X-OR Gate :

PIN DIAGRAM-IC 7486



SYMBOL

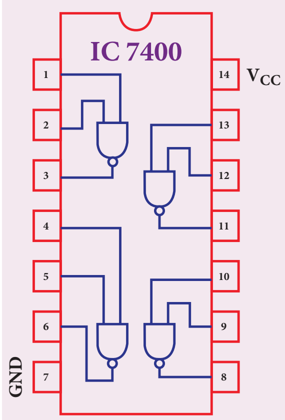


TRUTH TABLE

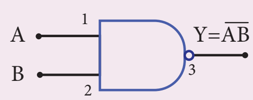
A	B	$Y=\bar{A}B+A\bar{B}$
0	0	0
0	1	1
1	0	1
1	1	0

NAND Gate:

PIN DIAGRAM-IC 7400



SYMBOL

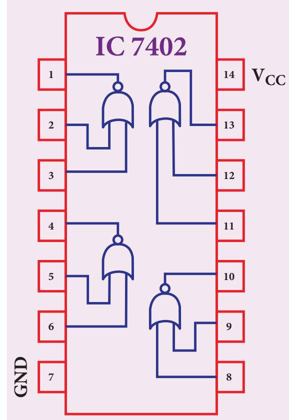


TRUTH TABLE

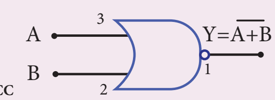
A	B	$Y=\overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

NOR Gate:

PIN DIAGRAM-IC 7402



SYMBOL



TRUTH TABLE

A	B	$Y=\overline{A+B}$
0	0	1
0	1	1
1	0	1
1	1	0

PROCEDURE

- To verify the truth table of a logic gate, the suitable IC is taken and the connections are given using the circuit diagram.
- For all the ICs, 5V is applied to the pin 14 while the pin 7 is connected to the ground.
- The logical inputs of the truth table are applied and the corresponding output is noted.
- Similarly the output is noted for all other combinations of inputs.
- In this way, the truth table of a logic gate is verified.

RESULT

The truth table of logic gates AND, OR, NOT, Ex-OR, NAND and NOR using integrated circuits is verified.

Precautions

(i) V_{CC} and ground pins must not be interchanged while making connections. Otherwise the chip will be damaged. (ii) The pin configuration for NOR gate is different from other gates

10. VERIFICATION OF DE MORGAN'S THEOREMS

AIM: To verify De Morgan's first and second theorems.

COMPONENTS REQUIRED: Power Supply (0 – 5V), IC 7400, 7408, 7432, 7404, and 7402, Digital IC trainer kit, connecting wires.

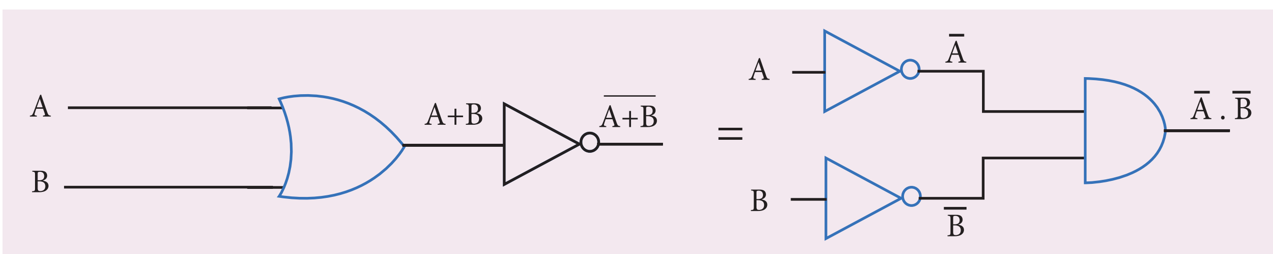
FORMULA

De Morgan's first theorem $\overline{A+B} = \overline{A}.\overline{B}$

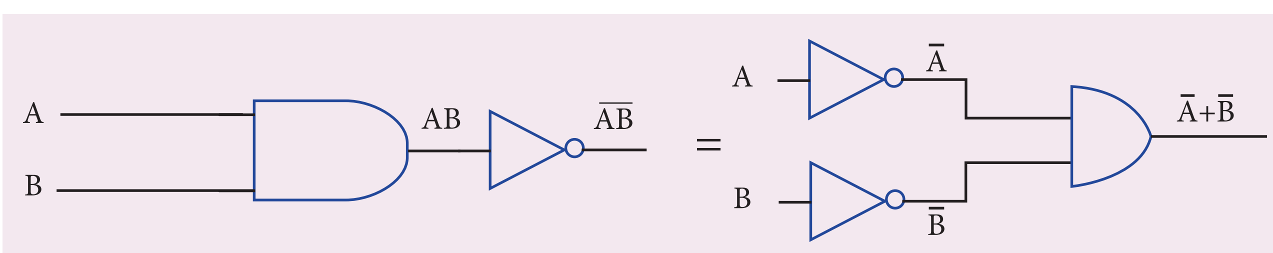
De Morgan's second theorem $\overline{A.B} = \overline{A}+\overline{B}$

CIRCUIT DIAGRAM:

De Morgan's first theorem



De Morgan's second theorem



PROCEDURE:

i) Verification of De Morgan's first theorem

- The connections are made for LHS $\left[\overline{A+B}\right]$ of the theorem as shown in the circuit diagram using appropriate ICs.
- The output is noted and tabulated for all combinations of logical inputs of the truth table.
- The same procedure is repeated for RHS $\left[\overline{A}.\overline{B}\right]$ of the theorem.
- From the truth table, it can be shown that $\overline{A+B} = \overline{A}.\overline{B}$.

ii) Verification of De Morgan's second theorem

- The connections are made for LHS $\left[\overline{A.B}\right]$ of the theorem as shown in the circuit diagram using appropriate ICs.
- The output is noted and tabulated for all combinations of logical inputs of the truth table.
- The same procedure is repeated for RHS $\left[\overline{A}+\overline{B}\right]$ of the theorem.
- From the truth table, it can be shown that $\overline{A.B}=\overline{A}+\overline{B}$.

OBSERVATION

De-Morgan's first theorem

Truth Table

A	B	$\overline{A+B}$	$\overline{A}.\overline{B}$
0	0		
0	1		
1	0		
1	1		

De-Morgan's second theorem

Truth Table

A	B	$\overline{A.B}$	$\overline{A}+\overline{B}$
0	0		
0	1		
1	0		
1	1		

RESULT

De Morgan's first and second theorems are verified.

Note

The pin diagram for IC 7408, IC 7432 and IC 7404 can be taken from previous experiment

Precautions

V_{CC} and ground pins must not be interchanged while making connections. Otherwise the chip will be damaged.

For the ICs used, 5V is applied to the pin 14 while the pin 7 is connected to the ground.

SUGGESTED QUESTIONS FOR THE PRACTICAL EXAMINATION

1. Determine the resistance of a given wire using metre bridge. Also find the radius of the wire using screw gauge and hence determine the specific resistance of the material of the wire. Take at least 4 readings.

2. Determine the value of the horizontal component of the Earth's magnetic field, using tangent galvanometer. Take at least 4 readings.

3. Determine the value of the horizontal component of the Earth's magnetic field using the magnetic field produced along the axial line of the carrying-current circular coil. Take at least 2 readings.

4. Using the spectrometer, measure the angle of the given prism and angle of minimum deviation. Hence calculate the refractive index of the material of the prism.

5. Adjust the grating for normal incidence using the spectrometer. Determine the wavelength of green, blue, yellow and red lines of mercury spectrum (The number of lines per metre length of the grating can be noted from the grating).

6. Draw the V-I characteristics of PN junction diode and determine its forward resistance and knee voltage from forward characteristics.

7. Draw the V-I characteristics of Zener diode and determine its forward resistance and knee voltage from forward characteristics. Also find break down voltage of the Zener diode from reverse characteristics.

8. Draw the input and transfer characteristic curves of the given NPN junction transistor in CE mode. Find the input impedance from input characteristics and current gain from transfer characteristics.

9. Draw the output and transfer characteristic curves of the given NPN junction transistor in CE mode. Find the output impedance from output characteristics and current gain from transfer characteristics.

10. Verify the truth table of logic gates AND, NOT, Ex-OR and NOR gates using integrated circuits.

11. Verify the truth table of logic gates OR, NOT, Ex-OR and NOR gates using integrated circuits.

12. Verify De Morgan's first and second theorems.

7SDAC3

Solved examples

UVRFM7

Competitive Exam corner

GLOSSARY
கலைச்சொற்கள்

JNZ51Y

1.	Absorption spectra	-	உட்கவர் நிறமாலை
2.	Armature	-	சுருள் தொகுப்பு
3.	Axial symmetry	-	அச்சுச் சமச்சீர்
4.	Average current	-	சராசரி மின்னோட்டம்
5.	Blackbody radiation	-	கரும்பொருள் கதிர்வீச்சு
6.	Charge	-	மின்னூட்டம்
7.	Continuous charge distribution	-	தொடர் மின்னூட்டப் பரவல்
8.	Conventional current	-	மரபு மின்னோட்டம்
9.	Conservation of charges	-	மின்னூட்டம் மாறாத் தன்மை
10.	Capacitor	-	மின்தேக்கி
11.	Corona discharge	-	ஒளிவட்ட மின்னிறக்கம் அல்லது சிதறொளி மின்னிறக்கம்
12.	Capacitance	-	மின்தேக்குத்திறன்
13.	Coercivity	-	காந்த நீக்குத்திறன்
14.	Current density	-	மின்னோட்ட அடர்த்தி
15.	Conductivity	-	மின்கடத்து எண்
16.	Configuration	-	நிலை அமைப்பு
17.	Conduction current	-	கடத்து மின்னோட்டம்
18.	Carbon Resistor	-	கார்பன் மின்தடை
19.	Current sensitivity	-	மின்னோட்ட உணர்வுநுட்பம்
20.	Dielectrics	-	மின்காப்புகள்
21.	Displacement current	-	இடப்பெயர்ச்சி மின்னோட்டம்
22.	Declination angle	-	காந்த ஒதுக்கக்கோணம்
23.	Dielectric strength	-	மின்காப்பு வலிமை
24.	Drift velocity	-	இழுப்பு திசைவேகம்
25.	Dielectric constant	-	மின்காப்பு மாறிலி
26.	Eddy current	-	சுழல் மின்னோட்டம்
27.	Electromagnetic damping	-	மின்காந்தத் தணிப்பு
28.	Electronic devices	-	மின்னணு சாதனங்கள்
29.	Electrostatics	-	நிலை மின்னியல்
30.	Electric charge	-	மின்னூட்டம்,மின்துகள்
31.	Electric field	-	மின்புலம்
32.	Electric dipole	-	மின்னிருமுனை (மின் இருமுனை)
33.	Equivalent capacitance	-	தொகுபயன் மின்தேக்குத்திறன் அல்லது இணை மின்தேக்குத்திறன்

34.	Electrostatic induction	-	நிலைமின் தூண்டல்
35.	Electrostatic potential energy	-	நிலை மின்னழுத்த ஆற்றல்
36.	Electric flux	-	மின்பாயம்
37.	Equi-potential surface	-	சமமின்னழுத்தப் பரப்பு
38.	Electrostatic equilibrium	-	நிலை மின் சமநிலை
39.	Electrostatic shielding	-	நிலை மின் தடுப்புறை
40.	Energy density	-	ஆற்றல் அடர்த்தி
41.	Electrostatic potential	-	நிலை மின்னழுத்தம்
42.	Electric battery	-	மின்கலத் தொகுப்பு
43.	Emission spectra	-	வெளியிடு நிறமாலை
44.	Equivalent Resistance	-	தொகுபயன் மின்தடை
45.	Flux leakage	-	பாயக்கசிவு
46.	Figure of merit of a galvanometer	-	கால்வானா மீட்டரின் தர ஒப்பீட்டு எண்
47.	Finite value	-	வரம்பிற்குட்பட்ட மதிப்பு
48.	Free electrons	-	கட்டுறா எலக்ட்ரான்கள்
49.	Horizontal component of the Earth's magnetic field	-	புவி காந்தப்புலத்தின் கிடைத்தளக்கூறு
50.	Hysteresis	-	காந்தத் தயக்கம்
51.	Helical path	-	சுருள்பாதை
52.	superposition principle	-	மேற்பொருந்தல் தத்துவம்
53.	Insulators	-	காப்பான்கள்
54.	Inverter	-	மின் புரட்டி
55.	Inductance	-	மின்தூண்டல் எண்
56.	Inductor	-	மின்தூண்டி
57.	Inclination angle	-	காந்தச் சரிவுக்கோணம்
58.	Intensity of magnetization	-	காந்தமாக்கும் செறிவு
59.	Impedance	-	மின்மறுப்பு
60.	Linear charge density	-	மின்னூட்ட நீள் அடர்த்தி
61.	Lighthing bolt	-	மின்னல் வெட்டு
62.	Laminated Core	-	மென்தகட்டு உள்ளகம்
63.	Lightning conductor	-	மின்னல் கடத்தி
64.	Mutual-induction	-	பரிமாற்று மின்தூண்டல்
65.	Metallic conductor	-	உலோகக் கடத்தி
66.	Moving Coil galvanometer	-	இயங்குசுருள் கால்வானா மீட்டர்
67.	Magnetic meridian	-	காந்த துருவத்தளம்
68.	Magnetic domain	-	காந்தப் பெருங்கூறு, காந்தக் களம்
69.	Magnetic induction	-	காந்தத்தூண்டல்
70.	Magnetising field	-	காந்தமாக்கு புலம்
71.	Magnetic Flux	-	காந்தப்பாயம்
72.	Magnetic susceptibility	-	காந்த ஏற்புத்திறன்
73.	Magnetic permeability	-	காந்த உட்புகுதிறன்
74.	Magnetic flux	-	காந்தப்பாயம்
75.	Magnetic dipole moment	-	காந்த இருமுனை திருப்புத்திறன்
76.	Magnetic declination	-	காந்த ஒதுக்கம்

77. Magnetic dip or inclination - காந்தச் சரிவு
78. Non ohmic conductor - ஓம் விதிக்கு உட்படாத கடத்தி
79. Propagation vector - பரவும் வெக்டர்
80. Phasor - கட்ட வெக்டர்
81. Power factor - திறன் காரணி
82. Potential difference - மின்னழுத்த வேறுபாடு
83. Permittivity - விடுதிறன்
84. Quantization - குவாண்டமாக்கல் அல்லது துளிமமாக்கல்
85. Resonance - ஒத்ததிர்வு
86. Rotor - சுழலி
87. Relative permeability - ஒப்புமை உட்புகுதிறன்
89. Resistors in series - மின்தடைகள் தொடரிணைப்பு
90. Retentivity - காந்தப்பற்றுத்திறன்
91. Surface charge density - மின்னூட்டப் பரப்படர்த்தி
92. Slip rings - நழுவு வளையங்கள்
93. Series and parallel - தொடரிணைப்பு, பக்கவிணைப்பு
94. Self-induction - தன்மின்தூண்டல்
95. Successive collisions - அடுத்தடுத்த மோதல்கள்
96. Stator - நிலையான பகுதி
97. Superconductors - மீக்கடத்திகள்
98. Semiconductor - குறை கடத்தி
99. Solar spectrum - சூரிய நிறமாலை
100. Shunt resistance - இணை மின்தடை
101. Solenoid - வரிச்சுருள்
102. Temperature coefficient of Resistivity - வெப்பநிலை மின்தடை எண்
103. Toroid - வட்ட வரிச்சுருள்
104. Torsional constant - முறுக்குக் கோணம்
105. Transverse wave - குறுக்கலை
106. Torsion balance - முறுக்குத் தராசு
107. Transformer - மின்மாற்றி
108. Thermistor - வெப்பமாறு மின்தடை
109. Voltage sensitivity - மின்னழுத்த உணர்வுநுட்பம்
110. Volume element - பருமக் கூறு
111. Wave diagram - அலை வரைபடம்
112. Wattful current - முழுத்திறன் மின்னோட்டம்
113. Wattless current - சுழித்திறன் மின்னோட்டம்
114. Winding - கம்பிச் சுற்று

State Council of Educational Research and Training, Chennai-6

Higher Secondary Physics

List of Subject Experts in Textbook Writing

Domain Expert & Mentor

Prof. Rita John
Professor and Head
Department of Theoretical physics,
University of madras, Chennai.

Reviewers

Dr. V. Mani
Principal Scientist F, Head (C-MET)
Department of Electronics & Information Technology, Hyderabad,
Government of India.

Prof. P. Ravindran
Department of Physics,
School of Basic and Applied Sciences,
Central University of Tamil Nadu, Thiruvarur.

Dr. Rajeev shesha Joshi
Assistant Professor,
School of Physical Sciences,
Central University of Karnataka.

Content Writers

Mr. C. Joseph Prabagar
Assistant Professor,
PG and Research Department of Physics,
Loyola College (Autonomous), Chennai – 34

Dr. S. S. Naina Mohammed
Assistant Professor,
PG and Research Department of Physics,
Government Arts College
Udumalpet, Tiruppur District.

Dr. P.Balamurugan
Assistant Professor,
PG and Research Department of Physics,
Government Arts College for Men (Autonomous)
Nandanam, Chennai.

Dr.C. Ramachandra raja
Associate Professor of Physics
Government Arts College,
Kumbakonam.

Tamil Translators

Mr.E.Elangovan
Head Master,
Government Hr Sec School
Ramanayakanpettai,
Vellore District.

Dr.K.Vasudevan
PGT, Government ADW.Hr.Sec.School
Kalangani
Namakkal District.

Mr.V.Balamurugan
PG Assistant in Physics,
Government Hr.Sec.School,
Kadambattur,
Tiruvallur District

Mr. S. Ravishankar
PG Assistant in Physics,
S.R.M. Hr. Sec. School,
Ambattur,
Tiruvallur District

Content Readers

Dr.K.Rajarajan
Associate Professor of Physics,
R.V.Government Arts College,
Chengalpattu

Dr.M.Jeyarani Malliga
Associate Professor of Physics,
Bharathi Women's College
Chennai.

Dr.K.Rajakumari
Assistant Professor of Physics,
Queen Mary's College,
Chennai.

Mrs. Bhavani
Assistant Professor of Physics,
Ethiraj College for women,
Chennai.

SCERT Subject Coordinators

Mrs. P.Nantha
Senior Lecturer
SCERT, Chennai – 06.

Mrs.D.Shanmuga Sundari
Graduate Assistant
P.U.M.School, Melathulukkankulam
Kariyapatti Union, Virudhunagar District

Mr.G.Arulraja
Graduate Assistant
MKV Government Boys Hr.Sec.School
Arani, Thiruvallur District.

ICT Coordinator

Mr.G.Bergin
PG Assistant (Physics)
GBHSS, Sayalkudi,
Ramanathapuram District.

QR CODE Management Team

R. Jaganathan
SGT, PUMS - Ganesapuram,
Polur , Thiruvannamalai.

S. Albert Valavan Babu
B.T, G.H.S, Perumal Kovil,
Paramakudi, Ramanathapuram.

A. Devi Jesintha
B.T, G.H.S, N.M. Kovil, Vellore.

Typist

Mrs. D. Kavitha
SCERT

Art and Design Team

Layout & QC
M. Asker Ali

Illustration
K. Sasi Kumar
S. Durga Devi

Cover Design
Kathir Arumugam

Co-ordination
Ramesh Munisamy

This book has been printed on 80 G.S.M.
Elegant Maplitho paper.
Printed by offset at:

NOTE

NOTE

NOTE

NOTE

NOTE