UNIT 3

MAGNETISM AND MAGNETIC EFFECTS OF ELECTRIC CURRENT

"The magnetic force is animate, or imitates a soul; in many respects it surpasses the human soul while it is united to an organic body" – William Gilbert

Learning Objectives

In this unit, the student is exposed to

3IWE3S

- Earth's magnetic field and magnetic elements
- Basic property of magnets
- Statement of Coulomb inverse square law of magnetism
- Magnetic dipole
- Magnetic induction at a point due to axial line and equatorial line
- Torque acting on a bar magnet in a uniform magnetic field
- Potential energy of a bar magnet placed in a uniform magnetic field
- Tangent law and tangent Galvanometer
- Magnetic properties – permeability, susceptibility etc
- Classification of magnetic materials – dia, para and ferro magnetic materials
- Concept of Hysteresis
- Magnetic effects of electric current – long straight conductor and circular coil
- Right hand thumb rule and Maxwell's right hand cork screw rule
- Biot-Savart's law – applications
- Current loop as a magnetic dipole
- Magnetic dipole moment of revolving electron
- Ampère's circuital law – applications
- Solenoid and toroid
- Lorentz force – charged particle moving in an electromagnetic field
- Cyclotron
- Force on a current carrying conductor in a magnetic field
- Force between two long parallel current carrying conductor
- Torque on a current loop in a magnetic field
- Moving coil Galvanometer

3.1 INTRODUCTION TO MAGNETISM

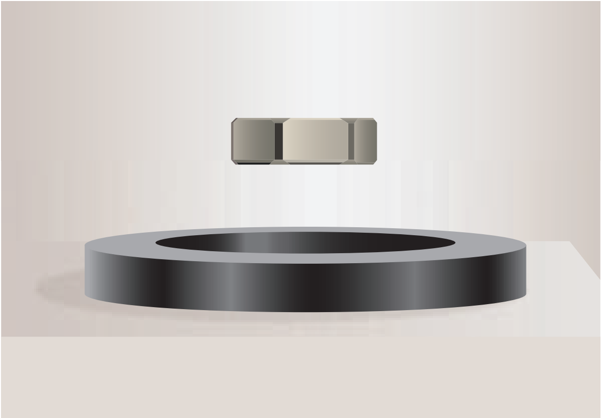


Figure 3.1: Magnetic levitation

Magnets! no doubt, its behaviour will attract everyone (see Figure 3.1). The world enjoys its benefits, to lead a modern luxurious life. The study of magnets fascinated scientists around our globe for many centuries and even now, door for research on magnets is still open.

Many birds and animals have magnetic sense in their eyes using Earth's magnetic field for navigation.

Magnetic sensing in eyes - for Zebrafinches bird, due to protein cryptochromes Cry4 present in retina, it uses Earth magnetic field for navigation

Magnetism is everywhere from tiny particles like electrons to the entire universe. Historically the word 'magnetism' was derived from iron ore magnetite ($Fe_3 O_4$). In olden days, magnets were used as magnetic compass for navigation, magnetic therapy for treatment and also used in magic shows.

In modern days, most of the things we use in our daily life contain magnets (Figure 3.2). Motors, cycle dynamo, loudspeakers, magnetic tapes used in audio and video recording, mobile phones, head phones, CD, pen-drive, hard disc of laptop, refrigerator door, generator are a few examples.

Earlier, both electricity and magnetism were thought to be two independent branches in physics. In 1820, H.C. Oersted observed the deflection of magnetic compass needle kept near a current carrying wire. This unified the two different branches, electricity and magnetism as a single subject 'electromagnetism' in physics.

In this unit, basics of magnets and their properties are given. Later, how a current carrying conductor (here only steady current, not time-varying current is considered) behaves like a magnet is presented.

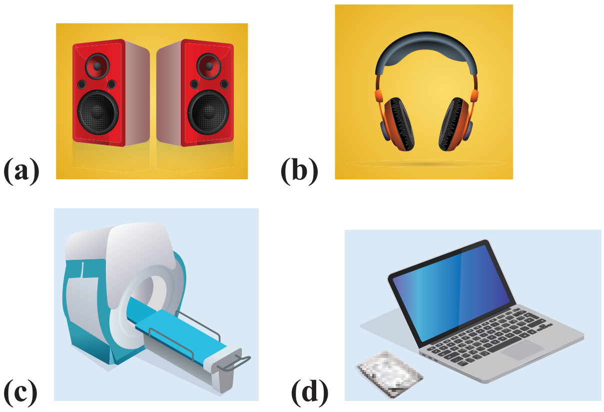


Figure 3.2 Uses of magnets in modern world – (a) speakers (b) head phones (c) MRI scan (d) Hard disc of laptop

3.1.1 Earth's magnetic field and magnetic elements

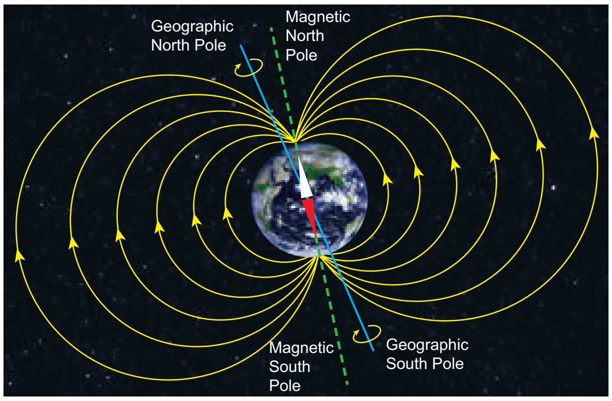


Figure 3.3 Earth's magnetic field

From the activities performed in lower classes, we have noticed that the needle in a magnetic compass or freely suspended magnet comes to rest in a position which is approximately along the geographical north-south direction of the Earth. William Gilbert in 1600 proposed that Earth itself behaves like a gigantic powerful bar magnet. But this theory is not successful because the temperature inside the Earth is very high and so it will not be possible for a magnet to retain its magnetism.

Gover suggested that the Earth's magnetic field is due to hot rays coming out from the Sun. These rays will heat up the air near equatorial region. Once air becomes hotter, it rises above and will move towards northern and southern hemispheres and get electrified. This may be responsible to magnetize the ferromagnetic materials near the Earth's surface. Till date, so many theories have been proposed. But none of the theory completely explains the cause for the Earth's magnetism.

The north pole of magnetic compass needle is attracted towards the magnetic south pole of the Earth which is near the geographic north pole (Figure 3.3). Similarly, the south pole of magnetic compass needle is attracted towards the geographic north pole of the Earth which is near magnetic north-pole. **The branch of physics which deals with the Earth's magnetic field is called Geomagnetism or Terrestrial magnetism.**

There are three quantities required to specify the magnetic field of the Earth on its surface, which are often called as the elements of the Earth's magnetic field. They are

(a) magnetic declination (D)
(b) magnetic dip or inclination (I)
(c) the horizontal component of the Earth's magnetic field (B_H)

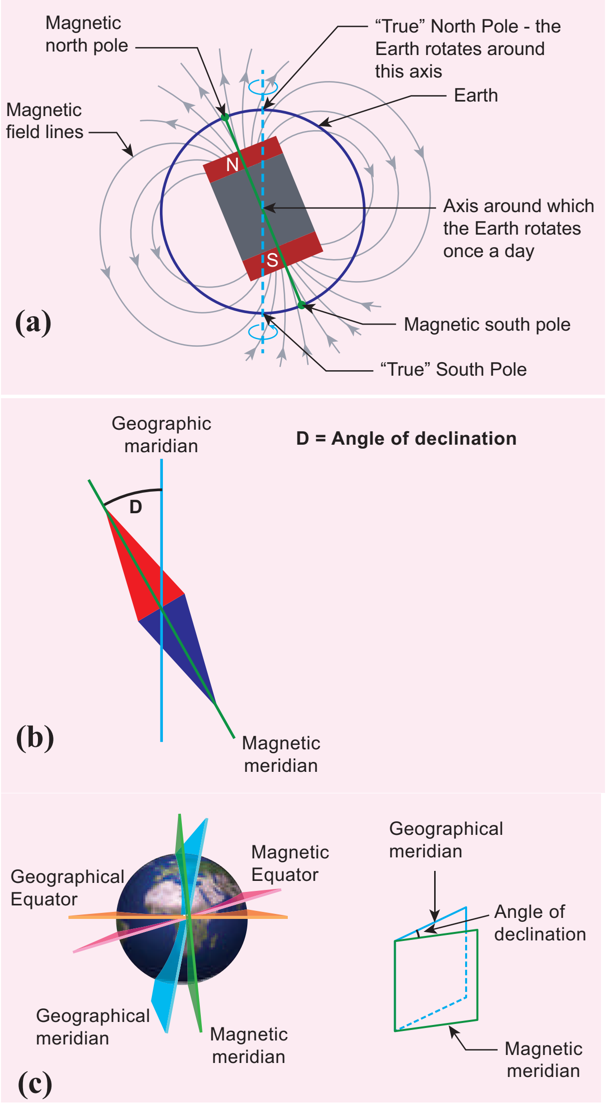


Figure 3.4 Declination angle

Day and night occur because Earth spins about an axis called geographic axis. A vertical plane passing through the geographic axis is called geographic meridian and a great circle perpendicular to Earth's geographic axis is called geographic equator.

The straight line which connects magnetic poles of Earth is known as magnetic axis. A vertical plane passing through magnetic axis is called magnetic meridian and a great circle perpendicular to Earth's magnetic axis is called magnetic equator.

When a magnetic needle is freely suspended, the alignment of the magnet does not exactly lie along the geographic meridian as shown in Figure 3.4. **The angle between magnetic meridian at a point and geographical meridian is called the *declination or magnetic declination (D)*.** At higher latitudes, the declination is greater whereas near the equator, the declination is smaller. In India, declination angle is very small and for Chennai, magnetic declination angle is -1° 8' (which is negative (west)).

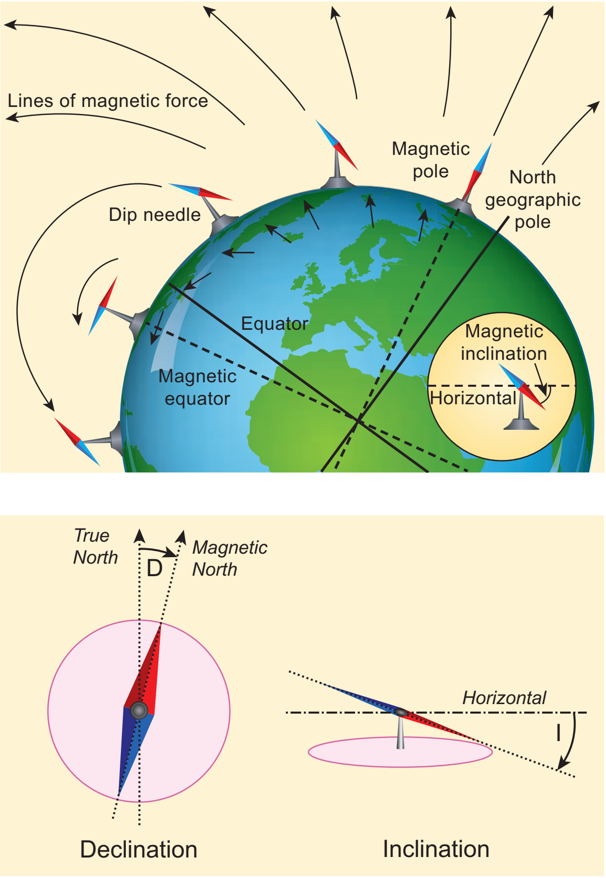


Figure 3.5 Inclination angle

The angle subtended by the Earth's total magnetic field $\vec{B}$ with the horizontal direction in the magnetic meridian is called dip or magnetic inclination (I) at that point (Figure 3.5). For Chennai, inclination angle is 14° 16'. **The component of Earth's magnetic field along the horizontal direction in the magnetic meridian is called horizontal component of Earth's magnetic field, denoted by B_H.**

Let B_E be the net Earth's magnetic field at a point P on the surface of the Earth. B_E can be resolved into two perpendicular components.

Horizontal component $B_H = B_E \cos I$ (3.1)

Vertical component $B_V = B_E \sin I$ (3.2)

Dividing equation (3.2) and (3.1), we get

$$\tan I = \frac{B_V}{B_H} \quad (3.3)$$

(i) At magnetic equator

The Earth's magnetic field is parallel to the surface of the Earth (i.e., horizontal) which implies that the needle of magnetic compass rests horizontally at an angle of dip, I = 0° as shown in figure 3.6.

$B_H = B_E$

$B_V = 0$

This implies that the horizontal component is maximum at equator and vertical component is zero at equator.

(ii) At magnetic poles

The Earth's magnetic field is perpendicular to the surface of the Earth (i.e., vertical) which implies that the needle

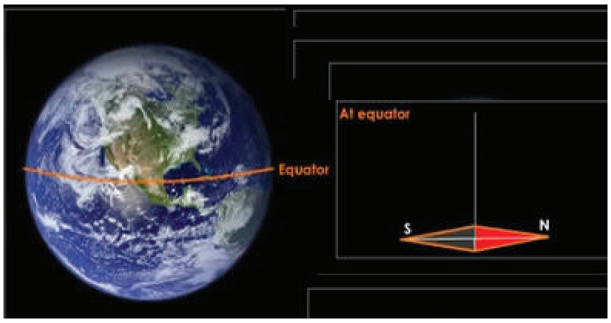


Figure 3.6: Needle of magnetic compass rests horizontally at an angle of dip – at magnetic equator

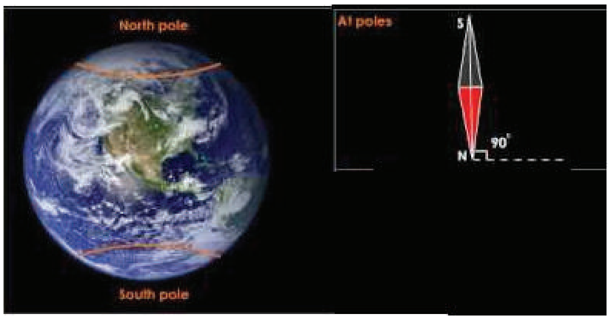


Figure 3.7: Needle of magnetic compass rests vertically at an angle of dip – at magnetic poles

of magnetic compass rests vertically at an angle of dip, I = 90° as shown in Figure 3.7. Hence,

$B_H = 0$

$B_V = B_E$

This implies that the vertical component is maximum at poles and horizontal component is zero at poles.

EXAMPLE 3.1

The horizontal component and vertical components of Earth's magnetic field at a place are 0.15 *G* and 0.26 *G* respectively. Calculate the angle of dip and resultant magnetic field.

Solution:

$B_H = 0.15$ G and $B_V = 0.26$ G

$$\tan I = \frac{0.26}{0.15} \Rightarrow I = \tan^{-1}(1.732) = 60^\circ$$

The resultant magnetic field of the Earth is

$$B = \sqrt{B_H^2 + B_V^2} = 0.3G$$

3.1.2 Basic properties of magnets

Some basic terminologies and properties used in describing bar magnet.

(a) Magnetic dipole moment

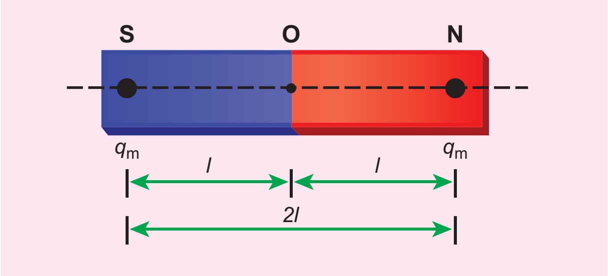


Figure 3.8 **A bar magnet**

Consider a bar magnet as shown in Figure 3.8. Let q_m be the pole strength (it is also called as magnetic charge) of the magnetic pole and let l be the distance between the geometrical center of bar magnet O and one end of the pole. **The magnetic dipole moment is defined as the product of its pole strength and magnetic length**. It is a vector quantity, denoted by $\vec{p}_m$.

$$\vec{p}_m = q_m \vec{d} \qquad (3.4)$$

where $\vec{d}$ is the vector drawn from south pole to north pole and its magnitude $|\vec{d}| = 2l$.

The magnitude of magnetic dipole moment is $p_m = 2q_m l$

DO YOU KNOW?

Aurora Borealis and Aurora Australis

People living at high latitude regions (near Arctic or Antarctic) might experience dazzling coloured natural lights across the night sky. This ethereal display on the sky is known as aurora borealis (northern lights) or

aurora australis (southern lights). These lights are often called as polar lights. The lights are seen above the magnetic poles of the northern and southern hemispheres. They are called as "Aurora borealis" in the north and "Aurora australis" in the south. This occurs as a result of interaction between the gaseous particles in the Earth's atmosphere with highly charged particles released from the Sun's atmosphere through solar wind. These particles emit light due to collision and variations in colour are due to the type of the gas particles that take part in the collisions. A pale yellowish – green colour is produced when the ionized oxygen takes part in the collision and a blue or purplish – red aurora is produced due to ionized nitrogen molecules.

The SI unit of magnetic moment is A m^2. Note that the direction of magnetic moment is from South pole to North pole.

(b) Magnetic field

Magnetic field is the region or space around every magnet within which its influence can be felt by keeping another magnet in that region. The magnetic field $\vec{B}$ at a point is defined as a force experienced by the bar magnet of unit pole strength.

$$\vec{B} = \frac{1}{q_m}\vec{F} \qquad (3.5)$$

Its unit is N A^{-1} m^{-1}.

(c) Types of magnets

Magnets are classified into natural magnets and artificial magnets. For example, iron, cobalt, nickel, etc. are natural magnets. Strengths of natural magnets are very weak and the shapes of the magnet are irregular. Artificial magnets are made by us in order to have desired shape and strength. If the magnet is in the form of rectangular shape or cylindrical shape, then it is known as bar magnet.

Properties of magnet

The following are the properties of bar magnet (Figure 3.9)

1. A freely suspended bar magnet will always point along the north-south direction.
2. A magnet attracts another magnet or magnetic substances towards itself. The attractive force is maximum near the end of the bar magnet. When a bar magnet is dipped into iron filling, they cling to the ends of the magnet.

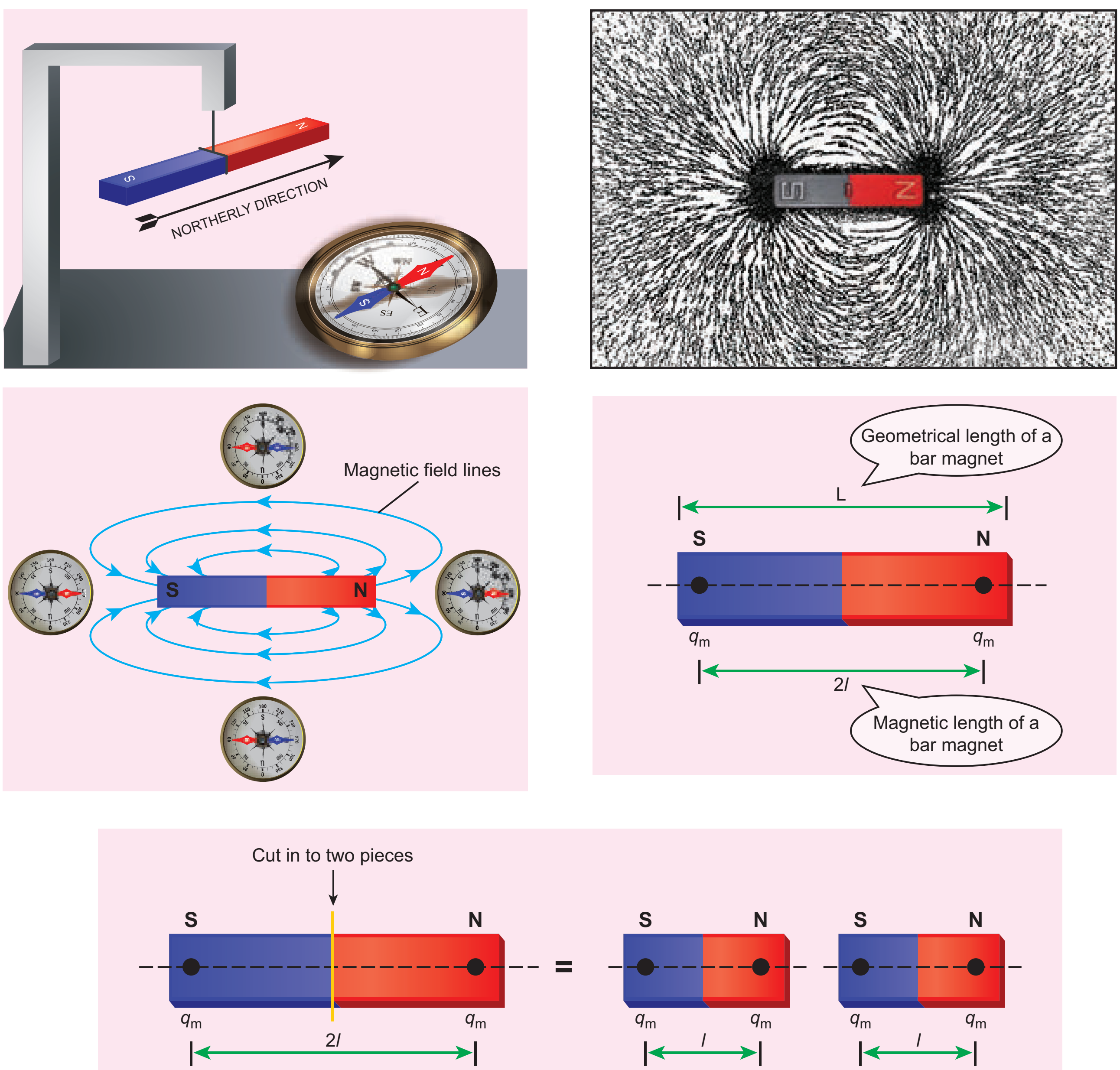


Figure 3.9 Properties of bar magnet

3. When a magnet is broken into pieces, each piece behaves like a magnet with poles at its ends.
4. Two poles of a magnet have pole strength equal to one another.
5. The length of the bar magnet is called geometrical length and the length between two magnetic poles in a bar magnet is called magnetic length. Magnetic length is always slightly smaller than geometrical length. The ratio of magnetic length and geometrical length is $\frac{5}{6}$.

$$\frac{\textit{Magnetic length}}{\textit{Geometrical length}} = \frac{5}{6} = 0.833$$

EXAMPLE 3.2

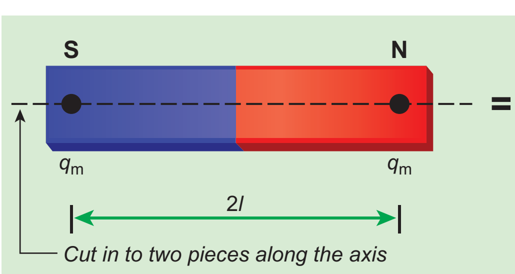
Let the magnetic moment of a bar magnet be $\vec{p}_m$ whose magnetic length is d = $2l$ and pole strength is q_m. Compute the magnetic moment of the bar magnet when it is cut into two pieces

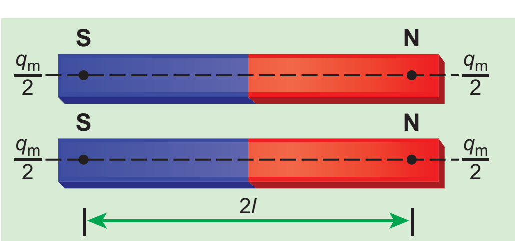
(a) along its length

(b) perpendicular to its length.

Solution

(a) a bar magnet cut into two pieces along its length:





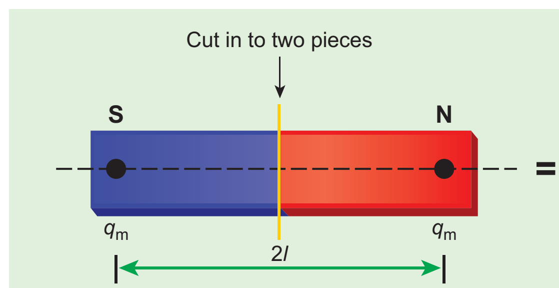
When the bar magnet is cut along the axis into two pieces, new magnetic pole strength is $q'_m = \frac{q_m}{2}$ but magnetic length does not change. So, the magnetic moment is

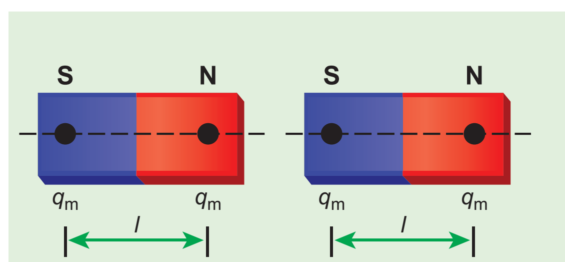
$$p'_m = q'_m 2l$$

$$p'_m = \frac{q_m}{2} 2l = \frac{1}{2}(q_m 2l) = \frac{1}{2} p_m$$

In vector notation, $\vec{p}'_m = \frac{1}{2}\vec{p}_m$

(b) a bar magnet cut into two pieces perpendicular to the axis:





When the bar magnet is cut perpendicular to the axis into two pieces, magnetic pole strength will not change but magnetic length will be halved. So the magnetic moment is

$$p'_m = q_m \times \frac{1}{2}(2l) = \frac{1}{2}(q_m \cdot 2l) = \frac{1}{2} p_m$$

In vector notation $\vec{p}'_m = \frac{1}{2}\vec{p}_m$

Note

(i) Pole strength is a scalar quantity with dimension $[M^0LT^0A]$. Its SI unit is $N\,T^{-1}$ (newton per tesla) or A m (ampere-metre).

(ii) Like positive and negative charges in electrostatics, north pole of a magnet experiences a force in the direction of magnetic field while south pole of a magnet experiences force opposite to the magnetic field.

(iii) Pole strength depends on the nature of materials of the magnet, area of cross-section and the state of magnetization.

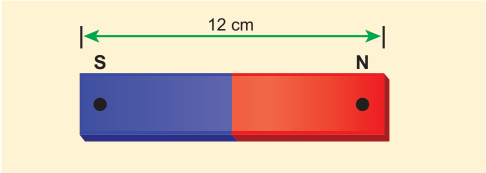
(iv) If a magnet is cut into two equal halves along the length then pole strength is reduced to half.

(v) If a magnet is cut into two equal halves perpendicular to the length, then pole strength remains same.

(vi) If a magnet is cut into two pieces, we will not get separate north and south poles. Instead, we get two magnets. In other words, isolated monopole does not exist in nature.

EXAMPLE 3.3

Compute the magnetic length of a uniform bar magnet if the geometrical length of the magnet is 12 cm. Mark the positions of magnetic pole points.



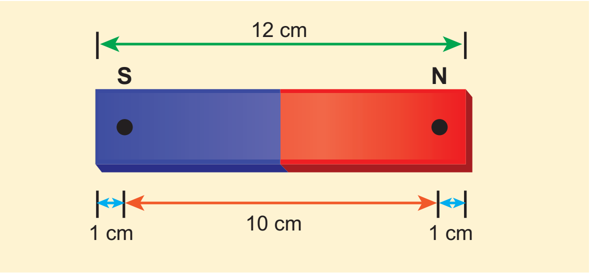
Solution

Geometrical length of the bar magnet is 12 cm

$$Magnetic\ length = \frac{5}{6}\times(geometrical\ length)$$

$$=\frac{5}{6}\times 12 = 10\,cm$$

In this figure, the dot implies the pole points.



Magnetic field lines

1. Magnetic field lines are continuous closed curves. The direction of magnetic field lines is from North pole to South pole outside the magnet (Figure 3.10) and South pole to North pole inside the magnet.

2. The direction of magnetic field at any point on the curve is known by drawing tangent to the magnetic line of force at that point. In the Figure No. 3.10 (b), the tangent drawn at points P, Q and R gives the direction of magnetic field $\vec{B}$ at that point.

3. Magnetic field lines never intersect each other. Otherwise, the magnetic compass needle would point towards two directions, which is not possible.

4. The degree of closeness of the field lines determines the relative strength of the magnetic field. The magnetic field is strong where magnetic field lines crowd and weak where magnetic field lines thin out.

(d) Magnetic flux

The number of magnetic field lines crossing per unit area is called magnetic flux Φ_B. Mathematically, the magnetic flux through a surface of area A in a uniform magnetic field is defined as

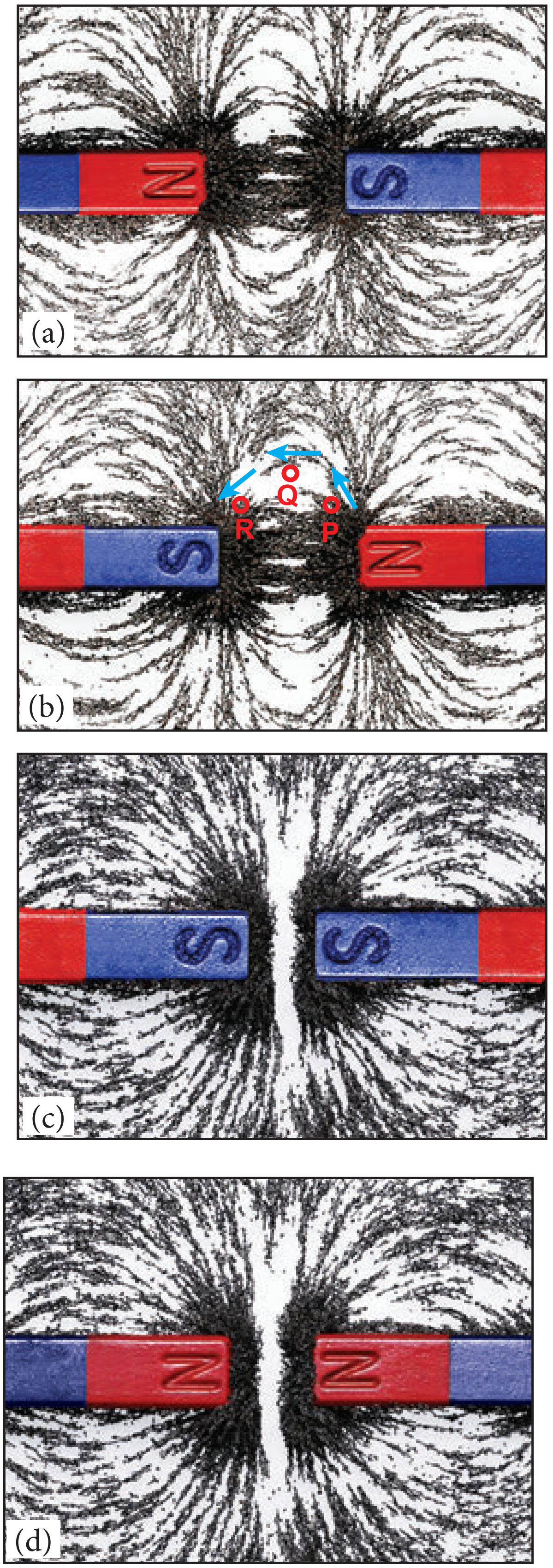


Figure 3.10 Properties of magnetic field lines– unlike poles attracts each other shown in picture (a) and (b) like poles repel each other-shown in picture (c) and (d)

$$\Phi_B = \vec{B}.\vec{A} = BA\cos\theta = B_{\perp}A \qquad (3.6)$$

where θ is the angle between $\vec{B}$ and $\vec{A}$ as shown in Figure 3.11.

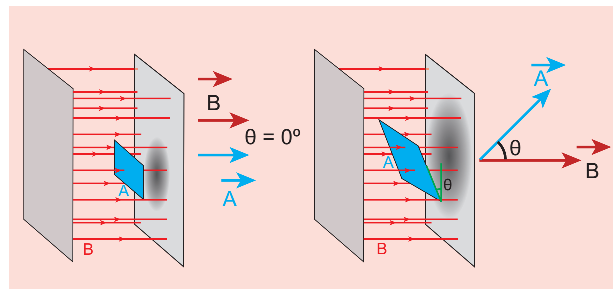


Figure 3.11 Magnetic flux

Special cases

(a) When $\vec{B}$ is normal to the surface i.e., $\theta = 0^{\circ}$, the magnetic flux is $\Phi_B = BA$ (maximum).

(b) When $\vec{B}$ is parallel to the surface i.e., $\theta = 90^{\circ}$, the magnetic flux is $\Phi_B = 0$.

Suppose the magnetic field is not uniform over the surface, the equation (3.6) can be written as

$$\Phi_B = \int \vec{B}.d\vec{A}$$

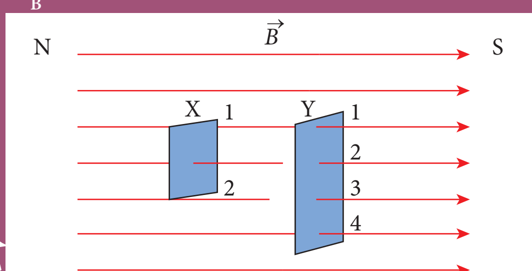
Magnetic flux is a scalar quantity. The SI unit for magnetic flux is weber, which is denoted by symbol Wb. Dimensional formula for magnetic flux is $\left[ML^2T^{-2}A^{-1}\right]$. The CGS unit of magnetic flux is Maxwell.

$$1 \text{ weber} = 10^8 \text{ maxwell}$$

The *magnetic flux density* can also be defined as the number of magnetic field lines crossing unit area kept normal to the direction of line of force. Its unit is Wb m^{-2} or tesla.

Note Here the integral is taken over area.

Let X and Y be two planar strips whose orientation is such that the direction of area vector of planar strips is parallel to the direction of the magnetic field $\vec{B}$ as shown in figure. The number of magnetic field lines passing through area of the strip X is two. Therefore, the flux passing through area X is $\Phi_B = 2$ Wb. Similarly, the number of magnetic field lines passing through area of strip Y is $\Phi_B = 4$ Wb.



(e) Uniform magnetic field and Non-uniform magnetic field

Uniform magnetic field

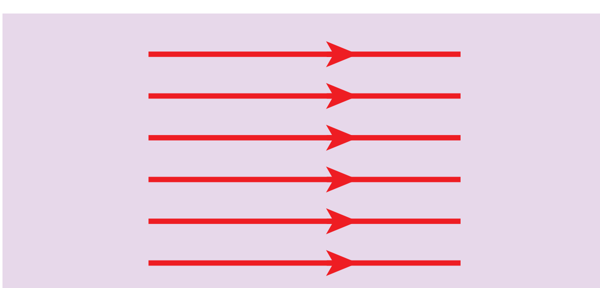


Figure 3.12 Uniform magnetic field

Magnetic field is said to be uniform if it has same magnitude and direction at all the points in a given region. Example, locally Earth's magnetic field is uniform.

The magnetic field of Earth has same value over the entire area of your school!

Non-uniform magnetic field

Magnetic field is said to be non-uniform if the magnitude or direction or both varies at all its points. Example: magnetic field of a bar magnet

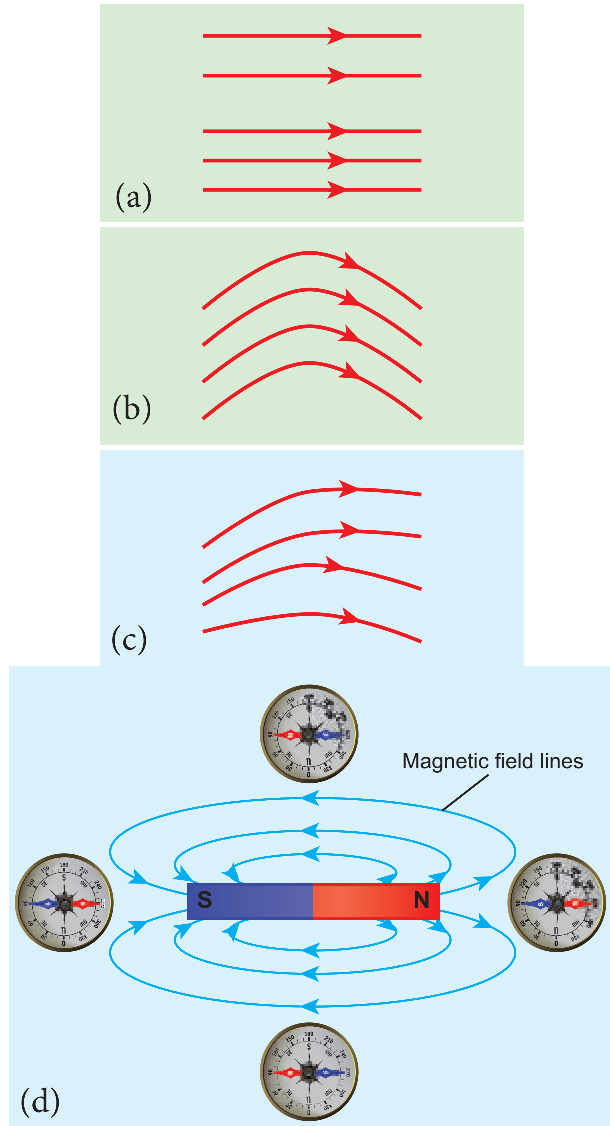
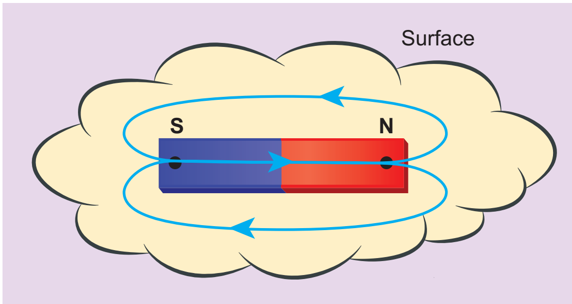


Figure 3.13 Non-uniform magnetic field – (a) direction is constant (b) direction is not a constant (c) both magnitude and direction are not constant (d) magnetic field of a bar magnet

EXAMPLE 3.4

Calculate the magnetic flux coming out from the surface containing magnetic dipole (say, a bar magnet) as shown in figure.



Solution

Magnetic dipole is kept, the total flux emanating from the closed surface S is zero. So,

$$\Phi_B = \oint \vec{B}.d\vec{A} = 0$$

Here the integral is taken over closed surface. Since no isolated magnetic pole (called magnetic monopole) exists, this integral is always zero,

$$\oint \vec{B}.d\vec{A} = 0$$

This is similar to Gauss's law in electrostatics. (Refer unit 1)

3.2 COULOMB'S INVERSE SQUARE LAW OF MAGNETISM

Consider two bar magnets A and B as shown in Figure 3.14.

When the north pole of magnet A and the north pole of magnet B or the south pole of magnet A and the south pole of magnet B are brought closer, they repel each other. On the other hand, when the north pole of magnet A and the south pole of magnet B or the south pole of magnet A and the north pole of magnet B are brought closer, their poles attract each other. This looks similar to Coulomb's law for static charges studied

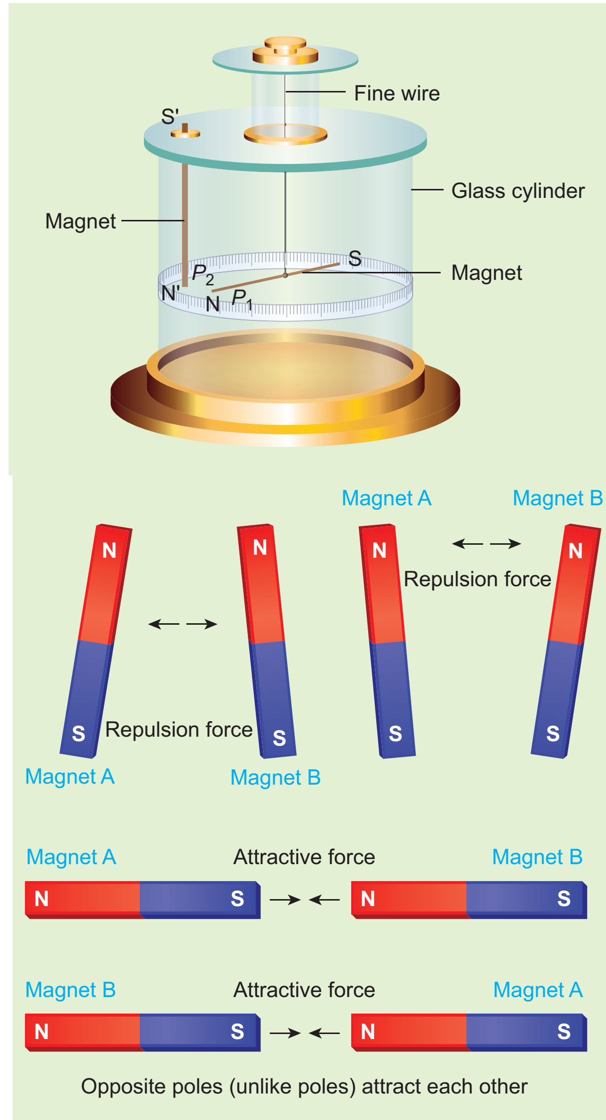


Figure 3.14: Magnetic poles behave like electric charges – like poles repel and unlike poles attract

in Unit I (opposite charges attract and like charges repel each other). So analogous to Coulomb's law in electrostatics, (Refer unit 1) we can state Coulomb's law for magnetism (Figure 3.15) as follows:

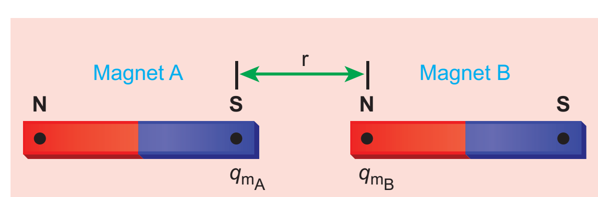


Figure 3.15 Coulomb's law – force between two magnetic pole strength

The force of attraction or repulsion between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them.

Mathematically, we can write

$$\vec{F} \propto \frac{q_{m_A} q_{m_B}}{r^2}\hat{r}$$

where m_A and m_B are pole strengths of two poles and *r* is the distance between two magnetic poles.

$$\vec{F} = k\frac{q_{m_A} q_{m_B}}{r^2}\hat{r} \qquad (3.7)$$

In magnitude, $$\vec{F} = k\frac{q_{m_A} q_{m_B}}{r^2} \qquad (3.8)$$

where k is a proportionality constant whose value depends on the surrounding medium. In S.I. unit, the value of k for free space is $k = \frac{\mu_\circ}{4\pi} \approx 10^{-7}\, H\, m^{-1}$, where μ_o is the absolute permeability of free space (air or vacuum).

EXAMPLE 3.5

The repulsive force between two magnetic poles in air is 9×10^{-3} N. If the two poles are equal in strength and are separated by a distance of 10 cm, calculate the pole strength of each pole.

***Solution*:**

The force between two poles are given by

$$\vec{F} = k\frac{q_{m_A} q_{m_B}}{r^2}\hat{r}$$

The magnitude of the force is

$$F = k\frac{q_{m_A} q_{m_B}}{r^2}$$

Given : $F = 9 \times 10^{-3}$ N, $r = 10$ cm $= 10 \times 10^{-2}$ m

Therefore,

$$9\times10^{-3} = 10^{-7} \times \frac{q_m^2}{\left(10\times10^{-2}\right)^2} \Rightarrow q_m = 30\, N\, T^{-1}$$

3.2.1 Magnetic field at a point along the axial line of the magnetic dipole (bar magnet)

Consider a bar magnet NS as shown in Figure 3.16. Let N be the North Pole and S be the south pole of the bar magnet, each of pole strength q_m and separated by a distance of 2*l*. The magnetic field at a point C (lies along the axis of the magnet) at a distance from the geometrical center O of the bar magnet can be computed by keeping unit north pole (q_{mc} = 1 A m) at C. The force experienced by the unit north pole at C due to pole strength can be computed using Coulomb's law of magnetism as follows:

The force of repulsion between north pole of the bar magnet and unit north pole at point C (in free space) is

$$\vec{F}_N = \frac{\mu_\circ}{4\pi}\frac{q_m}{(r-l)^2}\hat{i} \qquad (3.9)$$

where r – *l* is the distance between north pole of the bar magnet and unit north pole at C.

The force of attraction between South Pole of the bar magnet and unit North Pole at point C (in free space) is

$$\vec{F}_S = -\frac{\mu_\circ}{4\pi}\frac{q_m}{(r+l)^2}\hat{i} \qquad (3.10)$$

where r + *l* is the distance between south pole of the bar magnet and unit north pole at C.

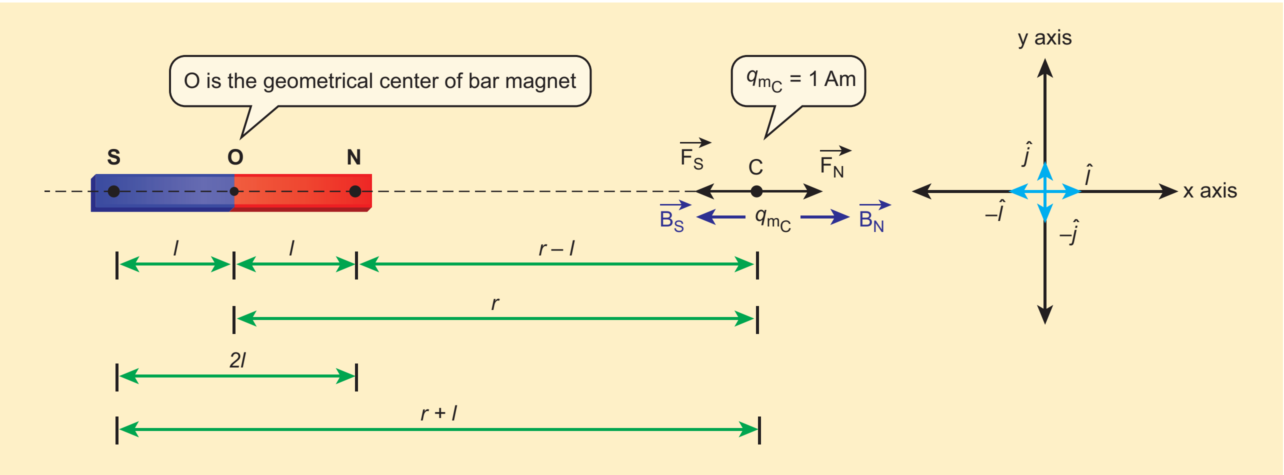


Figure 3.16 Magnetic field at a point along the axial line due to magnetic dipole

From equation (3.9) and (3.10), the net force at point C is $\vec{F} = \vec{F}_N + \vec{F}_S$. From definition, this net force is the magnetic field due to magnetic dipole at a point C $\left(\vec{F} = \vec{B}\right)$

$$\vec{B} = \frac{\mu_\circ}{4\pi}\frac{q_m}{(r-l)^2}\hat{i} + \left(-\frac{\mu_\circ}{4\pi}\frac{q_m}{(r+l)^2}\hat{i}\right)$$

$$\vec{B} = \frac{\mu_\circ q_m}{4\pi}\left(\frac{1}{\left(r-l\right)^2} - \frac{1}{\left(r+l\right)^2}\right)\hat{i}$$

$$\vec{B} = \frac{\mu_\circ 2r}{4\pi}\left(\frac{q_m \cdot (2l)}{\left(r^2-l^2\right)^2}\right)\hat{i} \tag{3.11}$$

Since, magnitude of magnetic dipole moment is $|\vec{p}_m| = p_m = q_m \cdot 2l$ the magnetic field at a point C equation (3.11) can be written as

$$\vec{B}_{axial} = \frac{\mu_\circ}{4\pi}\left(\frac{2rp_m}{\left(r^2-l^2\right)^2}\right)\hat{i} \tag{3.12}$$

If the distance between two poles in a bar magnet are small (looks like short magnet) compared to the distance between geometrical centre O of bar magnet and the location of point C i.e., r >>l then,

$$(r^2-l^2)^2 \approx r^4 \tag{3.13}$$

Therefore, using equation (3.13) in equation (3.12), we get

$$\vec{B}_{axial} = \frac{\mu_\circ}{4\pi}\left(\frac{2p_m}{r^3}\right)\hat{i} = \frac{\mu_\circ}{4\pi}\frac{2}{r^3}\vec{p}_m \tag{3.14}$$

where $\vec{p}_m = p_m\hat{i}$.

3.2.2. Magnetic field at a point along the equatorial line due to a magnetic dipole (bar magnet)

Consider a bar magnet NS as shown in Figure 3.17. Let N be the north pole and S be the south pole of the bar magnet, each with pole strength q_m and separated by a distance of 2l. The magnetic field at a point C (lies along the equatorial line) at a distance r from the geometrical center O of the bar magnet can be computed by keeping unit north pole (q_{mC} = 1 A m) at C. The force experienced by the unit north pole at C due to pole strength N-S can be computed using Coulomb's law of magnetism as follows:

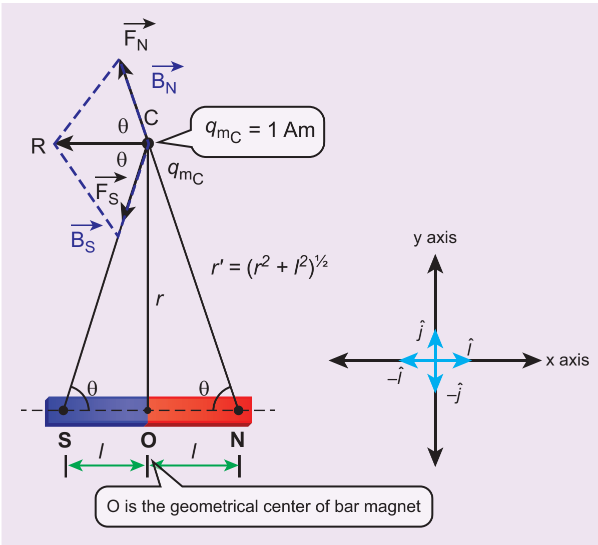


Figure 3.17 Magnetic field at a point along the equatorial line due to a magnetic dipole

The force of repulsion between North Pole of the bar magnet and unit north pole at point C (in free space) is

$$\vec{F}_N = -F_N \cos\theta \hat{i} + F_N \sin\theta \hat{j} \qquad (3.15)$$

where $F_N = \frac{\mu_\circ}{4\pi} \frac{q_m}{r'^2}$

The force of attraction (in free space) between south pole of the bar magnet and unit north pole at point C is (Figure 3.18) is

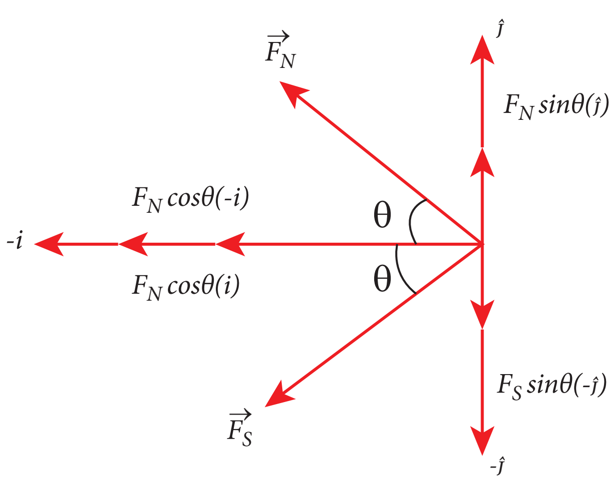


Figure 3.18 Components of force

$$\vec{F}_S = -F_S \cos\theta \hat{i} - F_S \sin\theta \hat{j} \qquad (3.16)$$

where, $F_S = \frac{\mu_\circ}{4\pi} \frac{q_m}{r'^2}$

From equation (3.15) and equation (3.16), the net force at point C is $\vec{F} = \vec{F}_N + \vec{F}_S$. This net force is equal to the magnetic field at the point C.

$$\vec{B} = -(F_N + F_S)\cos\theta \hat{i}$$

Since, $F_N = F_S$

$$\vec{B} = -\frac{2\mu_\circ}{4\pi} \frac{q_m}{r'^2} \cos\theta \hat{i} = -\frac{2\mu_\circ}{4\pi} \frac{q_m}{(r^2 + l^2)} \cos\theta \hat{i} \qquad (3.17)$$

In a right angle triangle NOC as shown in the Figure 3.17

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{l}{r'} = \frac{l}{(r^2 + l^2)^{\frac{1}{2}}} \qquad (3.18)$$

Substituting equation (3.18) in equation (3.17) we get

$$\vec{B} = -\frac{\mu_\circ}{4\pi} \frac{q_m \times (2l)}{(r^2 + l^2)^{\frac{3}{2}}} \hat{i} \qquad (3.19)$$

Since, magnitude of magnetic dipole moment is $|\vec{p}_m| = p_m = q_m \cdot 2l$ and substituting in equation (3.19), the magnetic field at a point C is

$$\vec{B}_{equatorial} = -\frac{\mu_\circ}{4\pi} \frac{p_m}{(r^2 + l^2)^{\frac{3}{2}}} \hat{i} \qquad (3.20)$$

If the distance between two poles in a bar magnet are small (looks like short magnet) when compared to the distance between geometrical center O of bar magnet and the location of point C i.e., r >>*l*, then,

$$(r^2 + l^2)^{\frac{3}{2}} \approx r^3 \qquad (3.21)$$

Therefore, using equation (3.21) in equation (3.20), we get

$$\vec{B}_{equatorial} = -\frac{\mu_\circ}{4\pi}\frac{p_m}{r^3}\hat{i}$$

Since $p_m\hat{i} = \vec{p}_m$, In general, the magnetic field at equatorial point is given by

$$\vec{B}_{equatorial} = -\frac{\mu_\circ}{4\pi}\frac{\vec{p}_m}{r^3} \qquad (3.22)$$

Note that magnitude of B_{axial} is twice that of magnitude of $B_{equatorial}$ and the direction of B_{axial} and $B_{equatorial}$ are opposite.

EXAMPLE 3.6

A short bar magnet has a magnetic moment of 0.5 J T^{-1}. Calculate magnitude and direction of the magnetic field produced by the bar magnet which is kept at a distance of 0.1 m from the center of the bar magnet along (a) axial line of the bar magnet and (b) normal bisector of the bar magnet.

Solution

Given magnetic moment 0.5 J T^{-1} and distance r = 0.1 m

(a) When the point lies on the axial line of the bar magnet, the magnetic field for short magnet is given by

$$\vec{B}_{axial} = \frac{\mu_\circ}{4\pi}\left(\frac{2p_m}{r^3}\right)\hat{i}$$

$$\vec{B}_{axial} = 10^{-7} \times \left(\frac{2\times 0.5}{(0.1)^3}\right)\hat{i} = 1\times 10^{-4}\,T\,\hat{i}$$

Hence, the magnitude of the magnetic field along axial is B_{axial} = 1 x 10^{-4} T and direction is towards South to North.

(b) When the point lies on the normal bisector (equatorial) line of the bar magnet, the magnetic field for short magnet is given by

$$\vec{B}_{equatorial} = -\frac{\mu_\circ}{4\pi}\frac{p_m}{r^3}\hat{i}$$

$$\vec{B}_{equatorial} = -10^{-7}\left(\frac{0.5}{(0.1)^3}\right)\hat{i} = -0.5\times 10^{-4}\,T\,\hat{i}$$

Hence, the magnitude of the magnetic field along axial is $B_{equatorial}$ = 0.5 x 10^{-4} T and direction is towards North to South.

Note that magnitude of B_{axial} is twice that of magnitude of $B_{equatorial}$ and the direction of B_{axial} and $B_{equatorial}$ are opposite.

3.3 TORQUE ACTING ON A BAR MAGNET IN UNIFORM MAGNETIC FIELD

Consider a magnet of length $2l$ of pole strength q_m kept in a uniform magnetic field $\vec{B}$ as shown in Figure 3.19. Each pole experiences a force of magnitude $q_m B$ but acts in opposite direction. Therefore, the net force exerted on the magnet is zero, so that there is no translatory motion. These two forces constitute a couple (about midpoint of bar magnet) which will rotate and try to align in the direction of the magnetic field $\vec{B}$.

The force experienced by north pole,

$$\vec{F}_N = q_m\vec{B} \qquad (3.23)$$

The force experienced by south pole,

$$\vec{F}_S = -q_m\vec{B} \qquad (3.24)$$

Adding equations (3.23) and (3.24), we get the net force acting on the dipole as

$$\vec{F} = \vec{F}_N + \vec{F}_S = \vec{0}$$

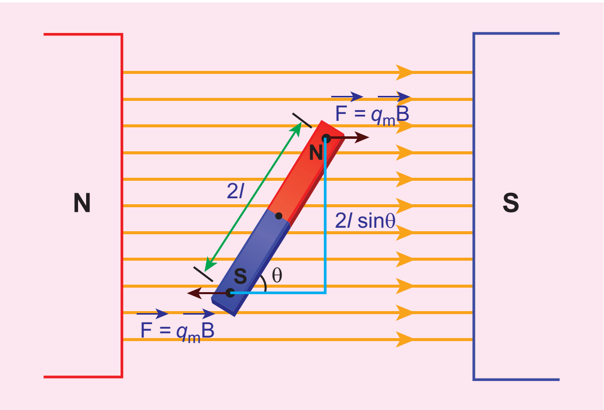


Figure 3.19 Magnetic dipole kept in a uniform magnetic field

This implies, that the net force acting on the dipole is zero, but forms a couple which tends to rotate the bar magnet clockwise (here) in order to align it along $\vec{B}$.

The moment of force or torque experienced by north and south pole about point O is

$$\vec{\tau} = \overrightarrow{ON} \times \vec{F}_N + \overrightarrow{OS} \times \vec{F}_S$$

$$\vec{\tau} = \overrightarrow{ON} \times q_m\vec{B} + \overrightarrow{OS} \times \left(-q_m\vec{B}\right)$$

By using right hand cork screw rule, we conclude that the total torque is pointing into the paper. Since the magnitudes $\left|\overrightarrow{ON}\right| = \left|\overrightarrow{OS}\right| = l$ and $\left|q_m\vec{B}\right| = \left|-q_m\vec{B}\right|$, the magnitude of total torque about point O

$$\tau = l \times q_m B\sin\theta + l \times q_m B\sin\theta$$

$$\tau = 2l \times q_m B\sin\theta$$

$$\tau = p_m B\sin\theta \qquad (\therefore q_m \times 2l = p_m)$$

In vector notation, $\vec{\tau} = \vec{p}_m \times \vec{B}$ (3.25)

EXAMPLE 3.7

Show the time period of oscillation when a bar magnet is kept in a uniform magnetic field is $T = 2\pi\sqrt{\frac{1}{p_m B}}$ in second, where I represents moment of inertia of the bar magnet, p_m is the magnetic moment and is the magnetic field.

Solution

The magnitude of deflecting torque (the torque which makes the object rotate) acting on the bar magnet which will tend to align the bar magnet parallel to the direction of the uniform magnetic field $\vec{B}$ is

$$|\vec{\tau}| = p_m B\sin\theta$$

The magnitude of restoring torque acting on the bar magnet can be written as

$$|\vec{\tau}| = I\frac{d^2\theta}{dt^2}$$

Under equilibrium conditions, both magnitude of deflecting torque and restoring torque will be equal but act in the opposite directions, which means

$$I\frac{d^2\theta}{dt^2} = -p_m B\sin\theta$$

DO YOU KNOW?

(a) Why a freely suspended bar magnet in your laboratory experiences only torque (rotational motion) but not any translatory motion even though Earth has non-uniform magnetic field?

It is because Earth's magnetic field is locally(physics laboratory)uniform.

(b) Suppose we keep a freely suspended bar magnet in a non-uniform magnetic field. What will happen?

It will undergo translatory motion (net force) and rotational motion (torque).

The negative sign implies that both are in opposite directions. The above equation can be written as

$$\frac{d^2\theta}{dt^2} = -\frac{p_m B}{I}\sin\theta$$

This is non-linear second order homogeneous differential equation. In order to make it linear, we use small angle approximation as we did in XI volume II (Unit 10 – oscillations, Refer section 10.4.4) i.e., $\sin\theta \approx \theta$, we get

$$\frac{d^2\theta}{dt^2} = -\frac{p_m B}{I}\,\theta$$

This linear second order homogeneous differential equation is a Simple Harmonic differential equation. Therefore,

> Comparing with Simple Harmonic Motion (SHM) differential equation
>
> $$\frac{d^2x}{dt^2} = -\omega^2 x$$
>
> where ω is the angular frequency of the oscillation.

$$\omega^2 = \frac{p_m B}{I} \Rightarrow \omega = \sqrt{\frac{p_m B}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{p_m B}}$$

$$T = 2\pi\sqrt{\frac{I}{p_m B_H}} \text{ in second}$$

where, B_H is the horizontal component of Earth's magnetic field.

3.3.1. Potential energy of a bar magnet in a uniform magnetic field

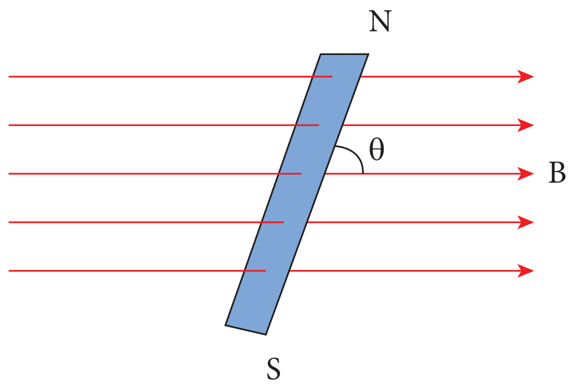


Figure 3.20: A bar magnet (magnetic dipole) in a uniform magnetic field

When a bar magnet (magnetic dipole) of dipole moment $\vec{p}_m$ is held at an angle θ with the direction of a uniform magnetic field $\vec{B}$, as shown in Figure 3.20 the magnitude of the torque acting on the dipole is

$$|\vec{\tau}_B| = |\vec{p}_m||\vec{B}|\sin\theta$$

If the dipole is rotated through a very small angular displacement dθ against the torque τ_B at constant angular velocity, then the work done by external torque $(\vec{\tau}_{ext})$ for this small angular displacement is given by

$$dW = |\vec{\tau}_{ext}|\,d\theta$$

The bar magnet has to be moved at constant angular velocity, which implies that $|\vec{\tau}_B| = |\vec{\tau}_{ext}|$

$$dW = p_m B\sin\theta\, d\theta$$

Total work done in rotating the dipole from θ′ to θ is

$$W = \int_{\theta'}^{\theta} \tau d\theta = \int_{\theta'}^{\theta} p_m B\sin\theta\, d\theta = p_m B\left[-\cos\theta\, d\theta\right]_{\theta'}^{\theta}$$

$$W = -p_m B(\cos\theta - \cos\theta')$$

This work done is stored as potential energy in bar magnet at an angle θ when it is rotated from θ′ to θ and it can be written as

$$U = -p_m B(\cos\theta - \cos\theta') \qquad (3.26)$$

In fact, the equation (3.26) gives the difference in potential energy between the angular positions θ′ and θ. We can choose the reference point θ′ = 90°, so that second term in the equation becomes zero and the equation (3.26) can be written as

$$U = -p_m B\,(\cos\theta) \qquad (3.27)$$

The potential energy stored in a bar magnet in a uniform magnetic field is given by

$$U = -\vec{p}_m \cdot \vec{B} \qquad (3.28)$$

Case 1

(i) If θ = 0°, then

$$U = -p_m B\,(\cos 0^\circ) = -p_m\,\mathrm{B}$$

(ii) If θ = 180°, then

$$U = -p_m B\,(\cos 180^\circ) = p_m\,\mathrm{B}$$

We can infer from the above two results, the potential energy of the bar magnet is minimum when it is aligned along the external magnetic field and maximum when the bar magnet is aligned anti-parallel to external magnetic field.

EXAMPLE 3.8

Consider a magnetic dipole which on switching ON external magnetic field orient only in two possible ways i.e., one along the direction of the magnetic field (parallel to the field) and another anti-parallel to magnetic field. Compute the energy for the possible orientation. Sketch the graph.

Solution

Let $\vec{p}_m$ be the dipole and before switching ON the external magnetic field, there is no orientation. Therefore, the energy U = 0.

As soon as external magnetic field is switched ON, the magnetic dipole orient parallel (θ = 0°) to the magnetic field with energy,

$$U_{parallel} = U_{minimum} = -p_m B \cos 0$$

$$U_{parallel} = -p_m B$$

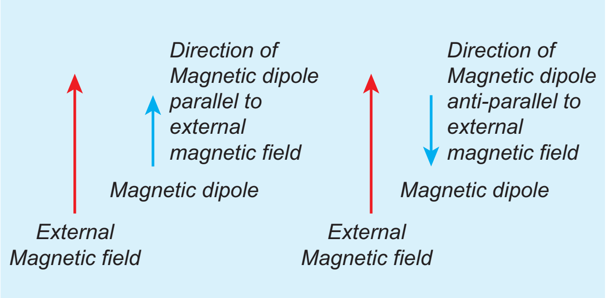
since cos 0° = 1

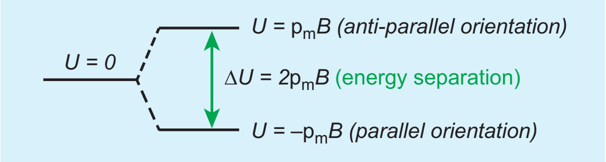
Otherwise, the magnetic dipole orients anti-parallel (θ = 180°) to the magnetic field with energy,

$$U_{anti-parallel} = U_{maximum} = -p_m B \cos 180$$

$$\Rightarrow U_{anti-parallel} = p_m B$$

since cos 180° = -1





3.3.2 Tangent law and Tangent Galvanometer

Tangent Galvanometer (Figure 3.21) is a device used to measure very small currents. It is a moving magnet type galvanometer. Its working is based on tangent law.

Figure 3.21 Tangent Galvanometer

Tangent law

When a magnetic needle or magnet is freely suspended in two mutually perpendicular uniform magnetic fields, it will come to rest in the direction of the resultant of the two fields.

Let B be the magnetic field produced by passing current through the coil of the tangent Galvanometer and B_H be the horizontal component of earth's magnetic field. Under the action of two magnetic fields, the needle comes to rest making angle θ with B_H, such that

$$B = B_H \tan\theta \qquad (3.29)$$

Construction

Tangent Galvanometer (TG) consists of copper coil wounded on a non-magnetic circular frame. The frame is made up of brass or wood which is mounted vertically on a horizontal base table (turn table) with three levelling screws as shown in Figure 3.22. The TG is provided with two or more coils of different number of turns. Most of the equipment we use in laboratory consists of 2 turns, 5 turns and 50 turns which are of different thickness and are used for measuring currents of different strengths.

At the center of turn table, a small upright projection is seen on which compass box (also known as magnetometre box) is placed. Compass box consists of a small magnetic needle which is pivoted at the center, such that arrangement shows the center of both magnetic needle and circular coil exactly coincide. A thin aluminium pointer is attached to the magnetic needle normally and moves over circular scale. The circular scale is divided into four quadrants and graduated in degrees which are used to measure the deflection of aluminium pointer on a circular degree scale. In order to avoid parallax error in measurement, a mirror is placed below the aluminium pointer.

Figure 3.22 Tangent Galvanometer and its parts

Precautions

1. All the nearby magnets and magnetic materials are kept away from the instrument.
2. Using spirit level, the levelling screws at the base are adjustedso that the small

magnetic needle is exactly horizontal and also coil (mounted on the frame) is exactly vertical.

3. The plane of the coil is kept parallel to the small magnetic needle by rotating the coil about its vertical axis. So, the coil remains in magnetic meridian.

Figure 3.23 Compass box

4. The compass box (as shown in Figure 3.23) is rotated such that the pointer reads 0° – 0°

Theory

The circuit connection for Tangent Galvanometer (TG) experiment is shown in Figure 3.24. When no current is passed through the coil, the small magnetic needle lies along horizontal component of Earth's magnetic field. When the circuit is switched ON, the electric current will pass through the circular coil and produce magnetic field. The magnetic field produced due to the circulatory electric current is discussed **(in section 3.8.3).** Now there are two fields which are acting mutually perpendicular to each other. They are:

(1) the magnetic field (B) due to the electric current in the coil acting normal to the plane of the coil.

(2) the horizontal component of Earth's magnetic field (B_H)

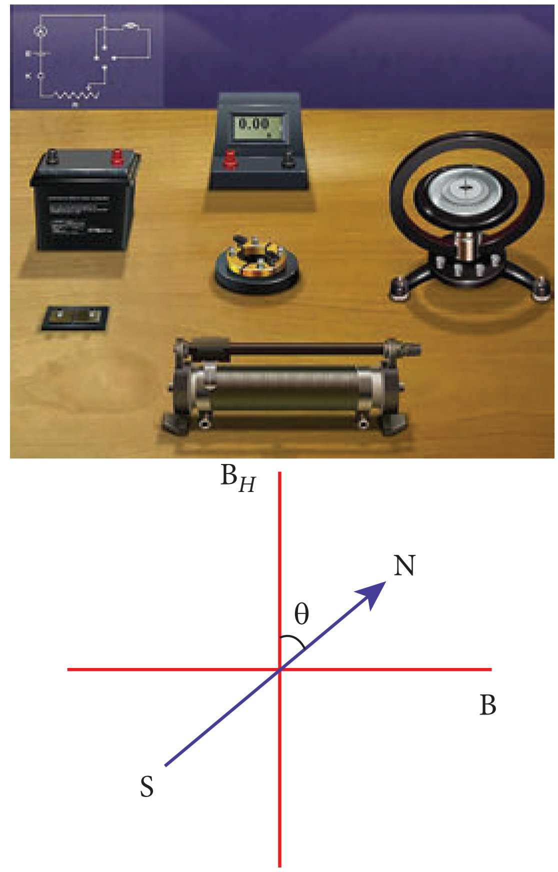


Figure 3.24 (a) circuit connection (b) resultant position of pivoted needle

Because of these crossed fields, the pivoted magnetic needle deflects through an angle θ. From tangent law (equation 3.29),

$$B = B_H \tan\theta$$

When an electric current is passed through a circular coil of radius R having N turns, the magnitude of magnetic field at the center is

$$B = \mu_\circ \frac{NI}{2R} \qquad (3.30)$$

From equation (3.29) and equation (3.30), we get

$$\mu_\circ \frac{NI}{2R} = B_H \tan\theta$$

The horizontal component of Earth's magnetic field can be determined as

$$B_H = \mu_\circ \frac{NI}{2R}\frac{1}{\tan\theta} \text{ in tesla} \qquad (3.31)$$

Note

1. The current in circuit can be calculated from I = K tan θ, where K is called reduction factor of tangent Galvanometer, where

$$K = \frac{2RB_H}{\mu_\circ N}$$

2. Sensitivity measures the change in the deflection produced by a unit current, mathematically

$$\frac{d\theta}{dI} = \frac{1}{K\left(1+\frac{I^2}{K^2}\right)}$$

3. The tangent Galvanometer is most sensitive at a deflection of 45°. Generally the deflection is taken between 30° and 60°.

EXAMPLE 3.9

A coil of a tangent galvanometer of diametre 0.24 m has 100 turns. If the horizontal component of Earth's magnetic field is 25×10^{-6} T then, calculate the current which gives a deflection of 60°.

Solution

The diameter of the coil is 0.24 m. Therefore, radius of the coil is 0.12 m.

Number of turns is 100 turns.

Earth's magnetic field is 25×10^{-6} T

Deflection is

$$\theta = 60^\circ \Rightarrow \tan 60^\circ = \sqrt{3} = 1.732$$

$$I = \frac{2RB_H}{\mu_\circ N}\tan\theta$$

$$= \frac{2\times 0.12\times 25\times 10^{-6}}{4\times 10^{-7}\times 3.14\times 100}\times 1.732 = 0.82\times 10^{-1}\,A.$$

$$I = 0.082\ A$$

3.4 MAGNETIC PROPERTIES

All the materials we use are not magnetic materials. Further, all the magnetic materials will not behave identically. So, in order to differentiate one magnetic material from another, we need to know some basic parameters. They are:

(a) Magnetising field

The magnetic field which is used to magnetize a sample or specimen is called the magnetising field. Magnetising field is a vector quantity and it denoted by $\vec{H}$ and its unit is A m^{-1}.

(b) Magnetic permeability

The magnetic permeability can be defined as **the measure of ability of the material to allow the passage of magnetic field lines through it or measure of the capacity of the substance to take magnetisation or the degree of penetration of magnetic field through the substance.**

In free space, the permeability (or absolute permeability) is denoted by μ_0 and for any medium it is denoted by μ.**The relative permeability μ_r is defined as the ratio between absolute permeability of the medium to the permeability of free space.**

$$\mu_r = \frac{\mu}{\mu_\circ} \qquad (3.32)$$

Relative permeability is a dimensionless number and has no units. For free space (air or vacuum), the relative permeability is unity i.e., μ_r = 1. In isotropic medium, μ is a scalar but for non-isotropic medium, μ is a tensor.

DO YOU KNOW?

Physical quantity	Component	Direction
Scalar	1 Component	No direction (no unit vector)
Vector	Each Component	1 direction (one unit vector)
Tensor	Each Component	More than one direction (more than one unit vectors)

Physical quantity	Component	Rank
Scalar	1 Component with zero direction	Zero
Vector	Each Component has one direction	One
Tensor of rank two	Each Component associated with two directions	Two
Tensor of rank three	Each Component associated with three directions	Three
Tensor of rank n	Each Component associated with n directions	n

(c) Intensity of magnetisation

Any bulk material (any object of finite size) contains a large number of atoms. Each atom consists of electrons which undergo orbital motion. Due to orbital motion, electron has magnetic moment which is a vector quantity. In general, these magnetic moments orient randomly, therefore, the net magnetic moment is zero per unit volume of the material.

When such a material is kept in an external magnetic field, atomic dipoles are created and hence, it will try to align partially or fully along the direction of external field. The net magnetic moment per unit volume of the material is known **as intensity of magnetisation or magnetisation vector or magnetisation.** It is a vector quantity. Mathematically,

$$\vec{M} = \frac{magnetic\,moment}{volume} = \frac{1}{V}\vec{p}_m \qquad (3.33)$$

The SI unit of intensity of magnetisation is ampere metre^{-1}. For a bar magnet of pole strength q_m, length $2l$ and area of cross-section A, the magnetic moment of the bar magnet is $\vec{p}_m = q_m \vec{2l}$ and volume of the bar magnet is $V = A\left|\vec{2l}\right| = 2l\,A$. The intensity of magnetisation for a bar magnet is

$$\vec{M} = \frac{magnetic\,moment}{volume} = \frac{q_m \vec{2l}}{2l\,A} \qquad (3.34)$$

In magnitude, equation (3.34) is

$$\left|\vec{M}\right| = M = \frac{q_m \times 2l}{2l \times A} \Rightarrow M = \frac{q_m}{A}$$

This means, **for a bar magnet the intensity of magnetisation can be defined as the pole strength per unit area (face area).**

(d) Magnetic induction or total magnetic field

When a substance like soft iron bar is placed in an uniform magnetising field $\vec{H}$, it becomes a magnet, which means that the substance gets magnetised. The magnetic induction (**total magnetic field**) inside the specimen $\vec{B}$ is equal to the sum of the magnetic field $\vec{B}_\circ$ produced in vacuum due to the magnetising field and the magnetic field $\vec{B}_m$ due to the induced magnetisation of the substance.

$$\vec{B} = \vec{B}_o + \vec{B}_m = \mu_\circ \vec{H} + \mu_\circ \vec{I}$$

$$\Rightarrow \vec{B} = \vec{B}_o + \vec{B}_m = \mu_\circ(\vec{H} + \vec{I}) \qquad (3.35)$$

(e) Magnetic susceptibility

When a substance is kept in a magnetising field $\vec{H}$, magnetic susceptibility gives information about how a material respond to the external (applied) magnetic field. In other words, the magnetic susceptibility measures, how easily and how strongly a material can be magnetised. It is defined as the ratio of the intensity of magnetisation $(\vec{M})$ induced in the material due to the magnetising field $(\vec{H})$

$$\chi_m = \frac{|\vec{M}|}{|\vec{H}|} \qquad (3.36)$$

It is a dimensionless quantity. For an isotropic medium, susceptibility is a scalar but for non-isotropic medium, susceptibility is a tensor. Magnetic susceptibility for some of the isotropic substances is given in Table 3.1.

Table 3.1 Magnetic susceptibility for various materials

Material	Magnetic susceptibility (χ_m)
Aluminium	2.3×10^{-5}
Copper	-0.98×10^{-5}
Diamond	-2.2×10^{-5}
Gold	-3.6×10^{-5}
Mercury	-3.2×10^{-5}
Silver	-2.6×10^{-5}
Titanium	7.06×10^{-5}
Tungsten	6.8×10^{-5}
Carbon dioxide (1 atm)	-2.3×10^{-9}
Oxygen (1 atm)	2090×10^{-9}

EXAMPLE 3.10

Compute the intensity of magnetisation of the bar magnet whose mass, magnetic moment and density are 200 g, 2 A m^2 and 8 g cm^{-3}, respectively.

Solution

Density of the magnet is

$$Density = \frac{Mass}{Volume} \Rightarrow Volume = \frac{Mass}{Density}$$

$$Volume = \frac{200 \times 10^{-3}\, kg}{\left(8 \times 10^{-3}\, kg\right) \times 10^{6}\, m^{-3}} = 25 \times 10^{-6}\, m^{3}$$

Magnitude of magnetic moment $p_m = 2\, A m^2$

Intensity of magnetization,

$$I = \frac{Magnetic\, moment}{Volume} = \frac{2}{25 \times 10^{-6}}$$

$$M = 0.8 \times 10^{5}\, A m^{-1}$$

EXAMPLE 3.11

Using the relation $\vec{B} = \mu_{\circ}(\vec{H} + \vec{M})$, show that $\chi_m = \mu_r - 1$.

Solution

$\vec{B} = \mu_{\circ}(\vec{H} + \vec{M})$,

But from equation (3.36), in vector form,

$\vec{M} = \chi_m \vec{H}$

Hence, $\quad \vec{B} = \mu_{\circ}(\chi_m + 1)\vec{H} \Rightarrow \vec{B} = \mu\vec{H}$

where, $\quad \mu = \mu_{\circ}(\chi_m + 1) \Rightarrow \chi_m + 1 = \frac{\mu}{\mu_{\circ}} = \mu_r$

$$\Rightarrow \chi_m = \mu_r - 1$$

EXAMPLE 3.12

Two materials X and Y are magnetised, whose intensity of magnetisation are 500 A m^{-1} and 2000 A m^{-1}, respectively. If the magnetising field is 1000 A m^{-1}, then which one among these materials can be easily magnetized?.

Solution

The susceptibility of material X is

$$\chi_{m,X} = \frac{|\vec{M}|}{|\vec{H}|} = \frac{500}{1000} = 0.5$$

The susceptibility of material Y is

$$\chi_{m,Y} = \frac{|\vec{M}|}{|\vec{H}|} = \frac{2000}{1000} = 2$$

Since, susceptibility of material Y is greater than that of material X, material Y can be easily magnetized than X.

3.5 CLASSIFICATION OF MAGNETIC MATERIALS

The magnetic materials are generally classified into three types based on the behaviour of materials in a magnetising field. They are diamagnetic, paramagnetic and ferromagnetic materials which are dealt with in this section.

(a) Diamagnetic materials

The orbital motion of electrons around the nucleus produces a magnetic field perpendicular to the plane of the orbit. Thus each electron orbit has finite orbital magnetic dipole moment. Since the orbital planes are oriented in random manner, the vector sum of magnetic moments is zero and there is no resultant magnetic moment for each atom.

In the presence of an external magnetic field, some electrons are speeded up and some are slowed down. The electrons whose moments were anti-parallel are speeded up according to Lenz's law and this produces an induced magnetic moment in a direction opposite to the field. The induced moment disappears as soon as the external field is removed.

When placed in a non-uniform magnetic field, the interaction between induced magnetic moment and the external field creates a force which tends to move the material from stronger part to weaker part of the external field. It means that diamagnetic material is repelled by the field.

This action is called diamagnetic action and such materials are known as diamagnetic materials. Examples: Bismuth, Copper and Water etc.

The properties of diamagnetic materials are

i) Magnetic susceptibility is negative.
ii) Relative permeability is slightly less than unity.
iii) The magnetic field lines are repelled or expelled by diamagnetic materials when placed in a magnetic field.
iv) Susceptibility is nearly temperature independent.

Note Superconductors are perfect diamagnetic materials. The expulsion of magnetic flux from a superconductor during its transition to the superconducting state is known as Meissner effect. (see figure 3.25)

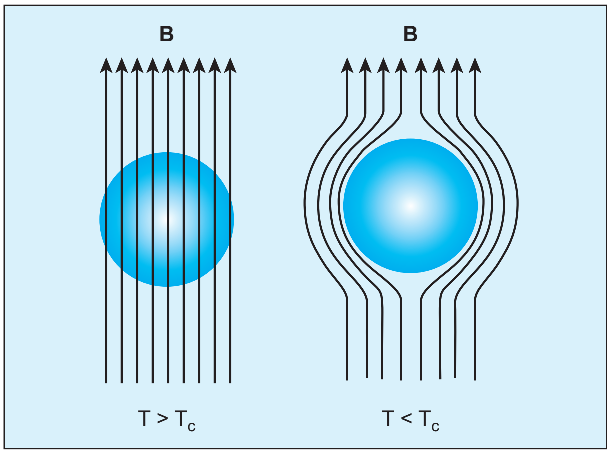


Figure 3.25 Meissner effect – superconductors behaves like perfect diamagnetic materials below transition temperature T_C.

Magnetic levitated train

Magnetic levitated train is also called as Maglev train. This train floats above few centimetre from the guideway because of electromagnet used. Maglev train does not need wheels and also achieve greater speed. The basic mechanism of working of Maglev train involves two sets of magnets. One set is used to repel which makes train to float above the track and another set is used to move the floating train ahead at very great speed. These trains are quieter, smoother and environmental friendly compared conventional trains and have potential for moving with much higher speeds with technology in future.

(b) Paramagnetic materials

In some magnetic materials, each atom or molecule has net magnetic dipole moment which is the vector sum of orbital and spin magnetic moments of electrons. Due to the random orientation of these magnetic moments, the net magnetic moment of the materials is zero.

In the presence of an external magnetic field, the torque acting on the atomic dipoles will align them in the field direction. As a result, there is net magnetic dipole moment induced in the direction of the applied field. The induced dipole moment is present as long as the external field exists.

When placed in a non-uniform magnetic field, the paramagnetic materials will have a tendency to move from weaker to stronger part of the field. Materials which exhibit weak magnetism in the direction of the applied field are known as paramagnetic materials. Examples: Aluminium, Platinum and chromium etc.

The properties of paramagnetic materials are:

i) Magnetic susceptibility is positive and small.
ii) Relative permeability is greater than unity.
iii) The magnetic field lines are attracted into the paramagnetic materials when placed in a magnetic field.
iv) Susceptibility is inversely proportional to temperature.

Curie's law

When temperature is increased, thermal vibration will upset the alignment of magnetic dipole moments. Therefore, the magnetic susceptibility decreases with increase in temperature. In many cases, the susceptibility of the materials is

$$\chi_m \propto \frac{1}{T} \quad \text{or} \quad \chi_m = \frac{C}{T}$$

This relation is called Curie's law. Here C is called Curie constant and temperature T is in kelvin. The graph drawn between magnetic susceptibility and temperature is shown in Figure 3.26, which is a rectangular hyperbola.

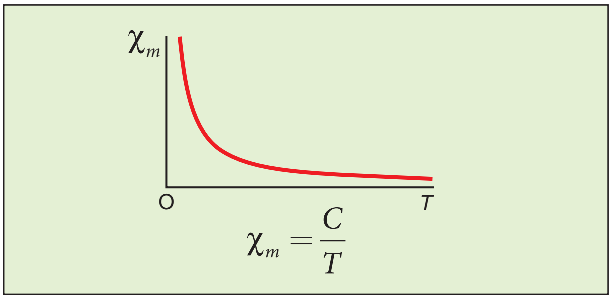


Figure 3.26 Curie's law – susceptibility vs temperature

(c) Ferromagnetic materials

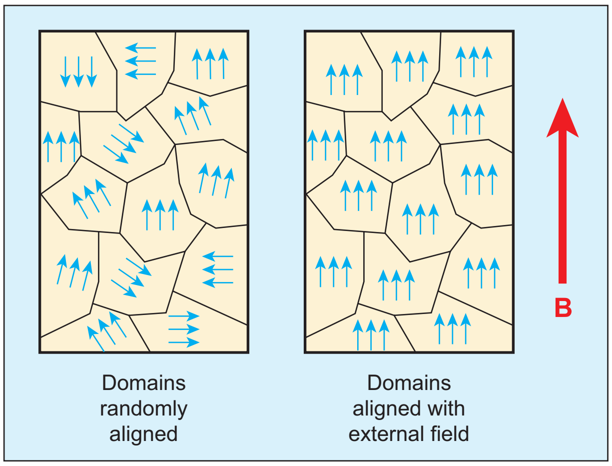


Figure 3.27 magnetic domains – ferromagnetic materials

An atom or a molecule in a ferromagnetic material possesses net magnetic dipole moment as in a paramagnetic material. A ferromagnetic material is made up of smaller regions, called ferromagnetic domain (Figure 3.27). Within each domain, the magnetic moments are spontaneously aligned in a direction. This alignment is caused by strong interaction arising from electron spin which depends on the inter-atomic distance. Each domain has net magnetisation in a direction. However the direction of magnetisation varies from domain to domain and thus net magnetisation of the specimen is zero.

In the presence of external magnetic field, two processes take place

(1) the domains having magnetic moments parallel to the field grow in size

(2) the other domains (not parallel to field) are rotated so that they are aligned with the field.

As a result of these mechanisms, there is a strong net magnetisation of the material in the direction of the applied field (Figure 3.28).

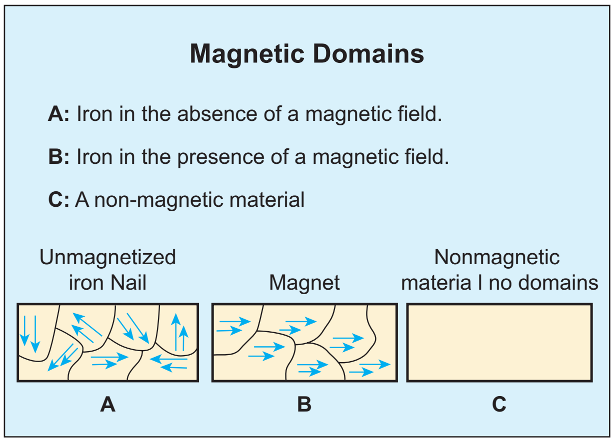


Figure 3.28 Processes of domain magnetization

When placed in a non-uniform magnetic field, the ferromagnetic materials will have a strong tendency to move from weaker to stronger part of the field. Materials which exhibit strong magnetism in the direction of applied field are called ferromagnetic materials. Examples: Iron, Nickel and Cobalt.

The properties of ferromagnetic materials are:

i) Magnetic susceptibility is positive and large.

ii) Relative permeability is large.

iii) The magnetic field lines are strongly attracted into the ferromagnetic materials when placed in a magnetic field.

iv) Susceptibility is inversely proportional to temperature.

Curie-Weiss law

As temperature increases, the ferromagnetism decreases due to the increased thermal agitation of the atomic dipoles. At a particular temperature, ferromagnetic material becomes paramagnetic. This temperature is known as Curie temperature T_C. The susceptibility of the material above the Curie temperature is given by

$$\chi_m = \frac{C}{T - T_C}$$

This relation is called Curie-Weiss law. The constant C is called Curie constant and temperature T is in kelvin. A plot of magnetic susceptibility with temperature is as shown in Figure 3.29.

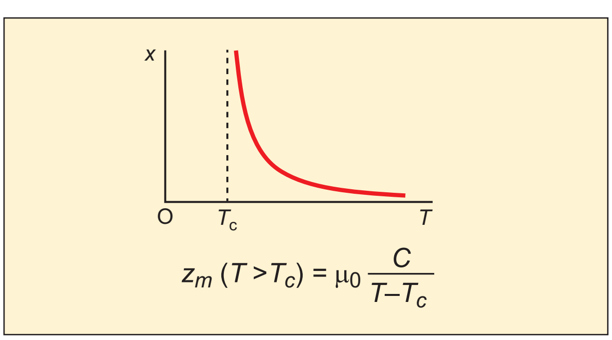
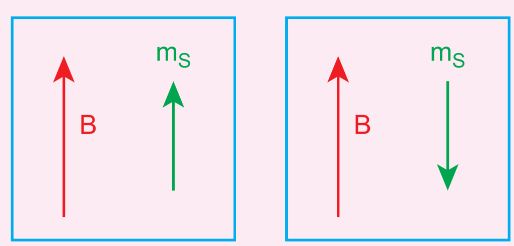


Figure 3.29 Curie-Weiss law – Susceptibility vs temperature

Spin

DO YOU KNOW?

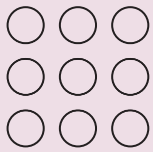
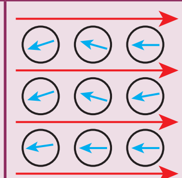
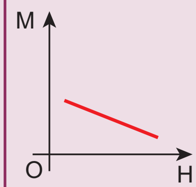
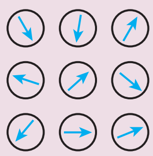
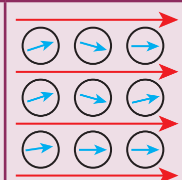
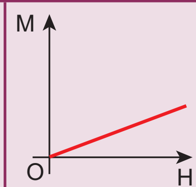
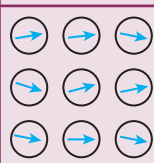
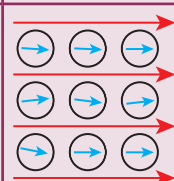
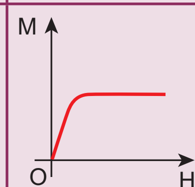
Like mass and charge for particles, spin is also another important attribute for an elementary particle. Spin is a quantum mechanical phenomenon (this is discussed in Volume 2) which is responsible for magnetic properties of the material. Spin in quantum mechanics is entirely different from spin we encounter in classical mechanics. Spin in quantum mechanics does not mean rotation; it is intrinsic angular momentum which does not have classical analogue. For historical reason, the name spin is retained. Spin of a particle takes only positive values but the orientation of the spin vector takes plus or minus values in an external magnetic field. For an example, electron has spin $s = \frac{1}{2}$. In the presence of magnetic field, the spin will orient either parallel or anti-parallel to the direction of magnetic field.



Spin is parallel to the magnetic field direction (Spin up)

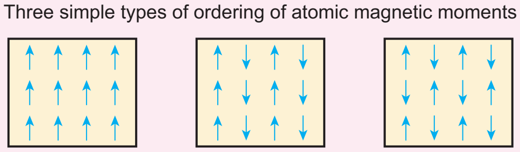
Spin is anti-parallel to the magnetic field direction (Spin down)

This implies that the magnetic spin m_s takes two values for an electron, such as $m_s = \frac{1}{2}$ (spin up) and $m_s = -\frac{1}{2}$ (spin down). Spin for proton and neutron is $s = \frac{1}{2}$. For a photon is spin s = 1.

Type of magnetism	Magnetising field is absent (H = 0)	Magnetising field is present ($H \neq 0$)	Magnetisation of the material	Susceptibility	Relative permeability
Diamagnetism	 (Zero magnetic moment)	 (Aligned opposite to the field)		Negative	Less than unity
Paramagnetism	 (Net magnetic moment but random alignment)	 (Aligned with the field)		Positive and small	Greater than unity
Ferromagnetism	 (Net magnetic moment in a domain but random alignment of domains)	 (Aligned with the field)		Positive and large	Very large

DO YOU KNOW?

Three simple types of ordering of atomic magnetic moments



Ferromagnetic (Adjacent magnetic moments are aligned) (a)

Antiferromagnetic (Adjacent magnetic moments are antiparallel and of equal magnitude) (b)

Ferrimagnetic (Adjacent magnetic moments are antiparallel and of unequal magnitude) (c)

3.6 HYSTERESIS

When a ferromagnetic material is kept in a magnetising field, the material gets magnetised by induction. An important characteristic of ferromagnetic material is that the variation of magnetic induction $\vec{B}$ with magnetising field $\vec{H}$ is not linear. It means that the ratio $\frac{B}{H} = \mu$ is not a constant. Let us study this behaviour in detail.

A ferromagnetic material (example, Iron) is magnetised slowly by a magnetising field $\vec{H}$. The magnetic induction $\vec{B}$ of the material increases from point A with the magnitude of the magnetising field and then attains a saturated level. This response of the material is depicted by the path AC as shown in Figure 3.30. Saturation magnetization is defined as the maximum point up to which the material can be magnetised by applying the magnetising field.

If the magnetising field is now reduced, the magnetic induction also decreases but does not retrace the original path CA. It takes different path CD. When the magnetising field is zero, the magnetic induction is not zero and it has positive value. This implies that some

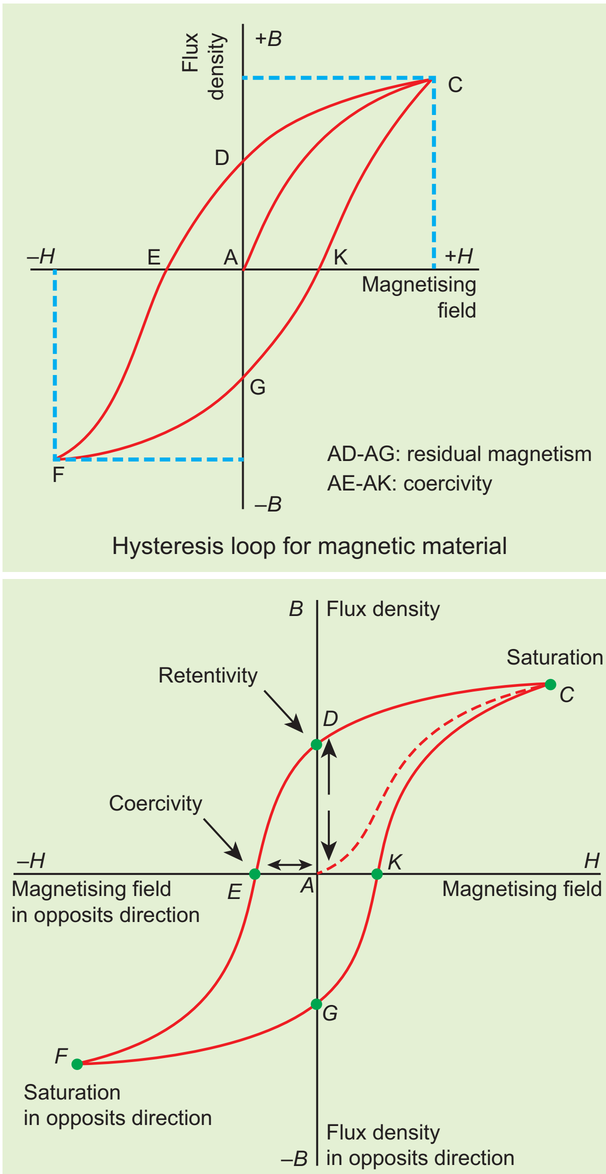


Figure 3.30 Hysteresis – plot for B vs H

magnetism is left in the specimen even when H = 0. The **residual magnetism AD present in the specimen is called remanence or retentivity. It is defined as the ability of the materials to retain the magnetism in them even magnetising field vanishes.**

In order to demagnetise the material, the magnetising field is gradually increased in the reverse direction. Now the magnetic induction decreases along DE and becomes zero at E. The magnetising field AE in the reverse direction is required to bring residual magnetism to zero. **The magnitude of the reverse magnetising field for which the residual magnetism of the material vanishes is called its coercivity.**

Further increase of $\vec{H}$ in the reverse direction, the magnetic induction increases along EF until it reaches saturation at F in the reverse direction. If magnetising field is decreased and then increased with direction reversed, the magnetic induction traces the path FGKC. This closed curve ACDEFGKC is called hysteresis loop and it represents a cycle of magnetisation.

In the entire cycle, the magnetic induction B lags behind the magnetising field H. This **phenomenon of lagging of magnetic induction behind the magnetising field is called hysteresis.** Hysteresis means 'lagging behind'.

Hysteresis loss

During the magnetisation of the specimen through a cycle, there is loss of energy in the form of heat. This loss is attributed to the rotation and orientation of molecular magnets in various directions. It is found that the energy lost (or dissipated) per unit volume of the material when it is carried through one cycle of magnetisation is equal to the area of the hysteresis loop. Thus, the loss of energy for a complete cycle is ΔE,

$$\Delta E = \oint \vec{H}.d\vec{B}$$

where $\vec{B}$ is in ampere – metre^2 and $\vec{H}$ is in ampere per meter. The loss in energy is measured in joules.

Hard and soft magnetic materials

Based on the shape and size of the hysteresis loop, ferromagnetic materials are classified as soft magnetic materials with smaller area and hard magnetic materials with larger area. The comparison of the hysteresis loops for two magnetic materials is shown in Figure 3.31. Properties of soft and hard magnetic materials are compared in Table 3.2.

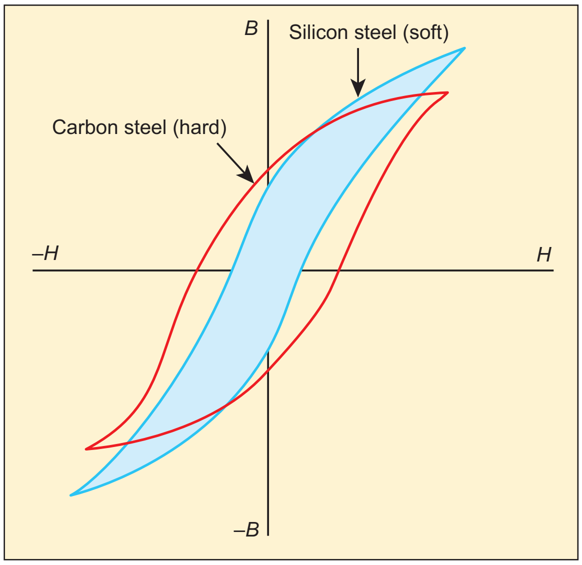


Figure 3.31 Comparison of two ferromagnetic materials – hysteresis loop

Applications of hysteresis loop

The significance of hysteresis loop is that it provides information such as retentivity, coercivity, permeability, susceptibility and energy loss during one cycle of magnetisation for each ferromagnetic material. Therefore, the study of hysteresis loop will help us in selecting proper and suitable material for a given purpose. Some examples:

i) Permanent magnets:

The materials with high retentivity, high coercivity and high permeability are suitable for making permanent magnets.
Examples: Steel and Alnico

ii) Electromagnets:

The materials with high initial permeability, low retentivity, low coercivity and thin hysteresis loop with smaller area are preferred to make electromagnets.

Table 3.2 Difference between soft and hard ferromagnetic materials

S.No.	Properties	Soft ferromagnetic materials	Hard ferromagnetic materials
1	When external field is removed	Magnetisation disappears	Magnetisation persists
2	Area of the loop	Small	Large
3	Retentivity	Low	High
4	Coercivity	Low	High
5	Susceptibility and magnetic permeability	High	Low
6	Hysteresis loss	Less	More
7	Uses	Solenoid core, transformer core and electromagnets	Permanent magnets
8	Examples	Soft iron, Mumetal, Stalloy etc.	Steel, Alnico, Lodestone etc.

Examples: Soft iron and Mumetal (Nickel Iron alloy).

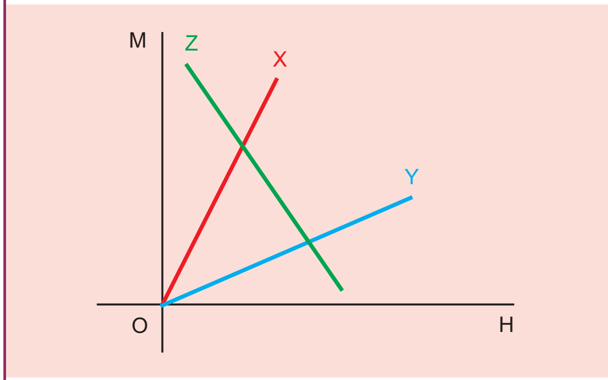
iii) Core of the transformer:

The materials with high initial permeability, large magnetic induction and thin hysteresis loop with smaller area are needed to design transformer cores.

Examples: Soft iron

EXAMPLE 3.13

The following figure shows the variation of intensity of magnetisation with the applied magnetic field intensity for three magnetic materials X, Y and Z. Identify the materials X,Y and Z.



Solution

The slope of M-H graph measures the magnetic susceptibility, which is

$$\chi_m = \frac{M}{H}$$

Material X: Slope is positive and larger value. So, it is a ferromagnetic material.

Material Y: Slope is positive and lesser value than X. So, it could be a paramagnetic material.

Material Z: Slope is negative and hence, it is a diamagnetic material.

3.3 MAGNETIC EFFECTS OF CURRENT

3.7.1 Oersted experiment

In 1820, Hans Christian Oersted while preparing for his lecture in physics noticed that electric current passing through a wire deflects the nearby magnetic compass. By proper investigation, he observed that the deflection of magnetic compass is due to the change in magnetic field produced around current carrying conductor (Figure 3.32). When the direction of current is reversed, the magnetic compass deflects in opposite direction. This lead to the development of the theory 'electromagnetism' which unifies the two branches in physics, namely electricity and magnetism.

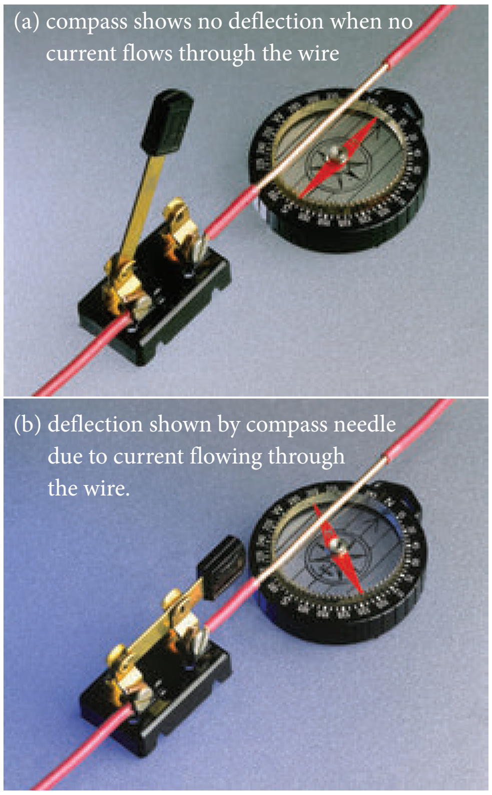
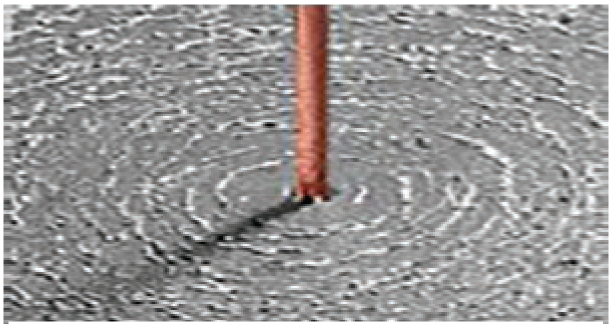


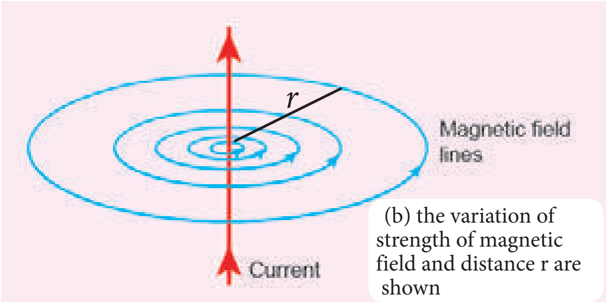
Figure 3.32 Oersted's experiment - current carrying wire and deflection of magnetic needle

3.7.2 Magnetic field around a straight current carrying conductor and circular loop

(a) Current carrying straight conductor:



(a) the photograph of magnetic field lines curling around the conductor carrying current



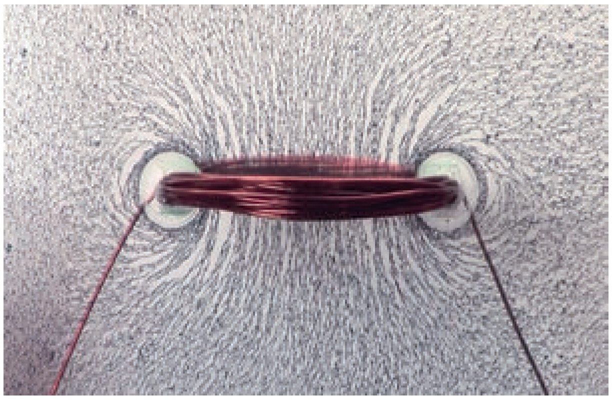
(b) the variation of strength of magnetic field and distance r are shown

Figure 3.33 Magnetic field lines around straight, long wire carrying current

Suppose we keep a magnetic compass near a current carrying straight conductor, then the needle of the magnetic compass experiences a torque and deflects to align in the direction of the magnetic field at that point. Tracing out the direction shown by magnetic compass, we can draw the magnetic field lines at a distance. For a straight current carrying conductor, the nature of magnetic field is like concentric circles having their center at the axis of the conductor as shown in Figure 3.33 (a).

The direction of circular magnetic field lines will be clockwise or anticlockwise depending on the direction of current in the conductor. If the strength (or magnitude) of the current is increased then the density of the magnetic field will also increase. The strength of the magnetic field (B) decreases as the distance (r) from the conductor increases are shown in Figure 3.33 (b).

(b) Circular coil carrying current



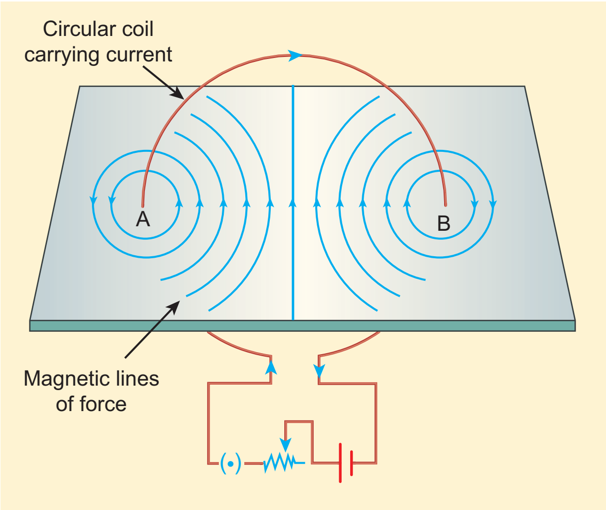


Figure 3.34 The magnetic field lines curling around the circular coil carrying current.

Suppose we keep a magnetic compass near a current carrying circular conductor, then the needle of the magnetic compass experiences a torque and deflects to align in the direction of the magnetic field at that point. We can notice that at the points A and B in the vicinity of the coil, the magnetic field lines are circular. The magnetic field lines are nearly parallel to each other near the center of the loop, indicating that the field present near the center of the coil is almost uniform (Figure 3.34).

The strength of the magnetic field is increased if either the current in the coil or the number of turns or both are increased. The polarity (north pole or south pole) depends on the direction of current in the loop.

3.7.3 Right hand thumb rule

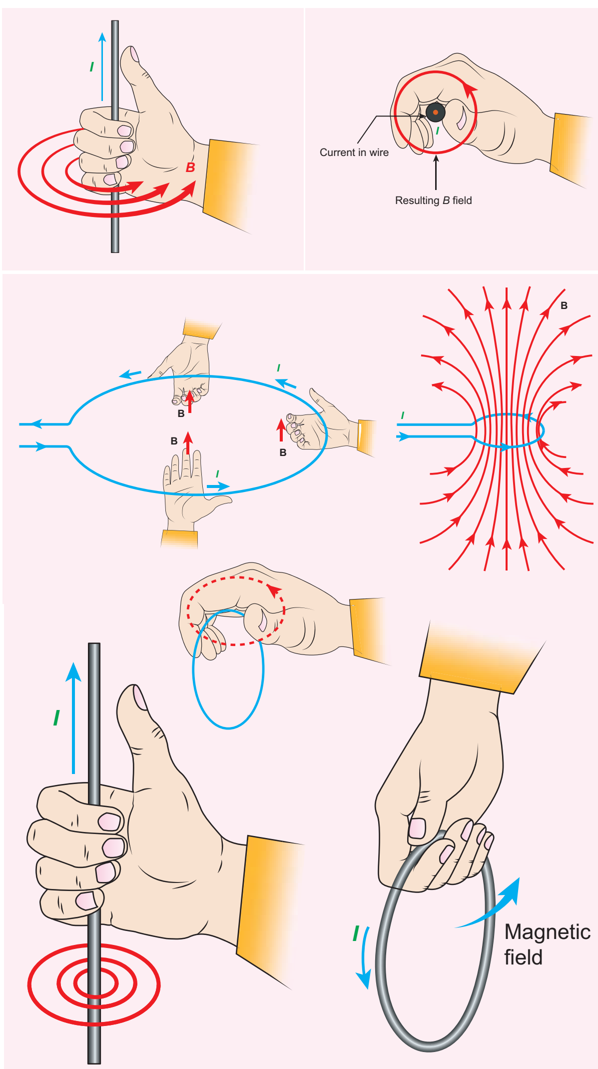


Figure 3.35 Right hand rule – straight conductor and circular loop

The right hand rule is a mnemonic to find the direction of magnetic field when the direction of current in a conductor is known.

If we hold the current carrying conductor in our right hand such that the thumb points in the direction of current flow, then the fingers encircling the wire points in the direction of the magnetic field lines produced.

The Figure 3.35 shows the right hand rule for current carrying straight conductor and circular coil.

Note

Mnemonic means that it is a special word or a collection of words used to help a person to remember something.

3.7.4 Maxwell's right hand cork screw rule

This rule is used to determine the direction of the magnetic field. If we rotate a right-handed screw using a screw driver, then the direction of current is same as the direction in which screw advances and the direction of rotation of the screw gives the direction of the magnetic field. (Figure 3.36)

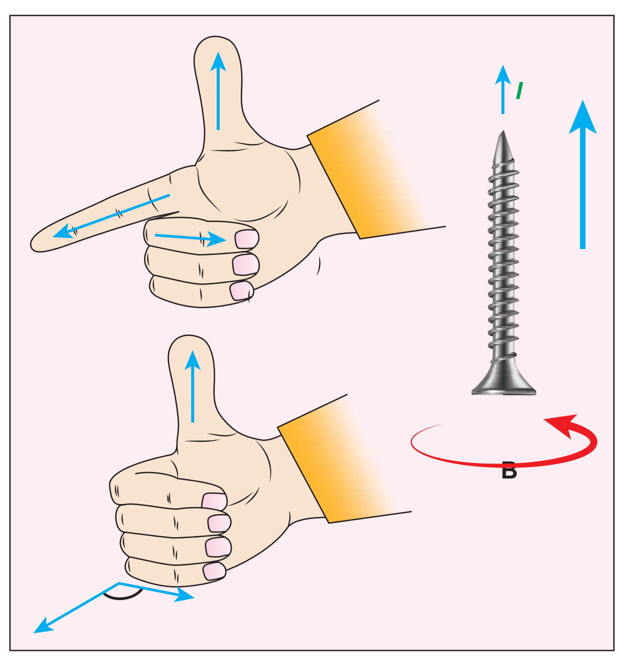
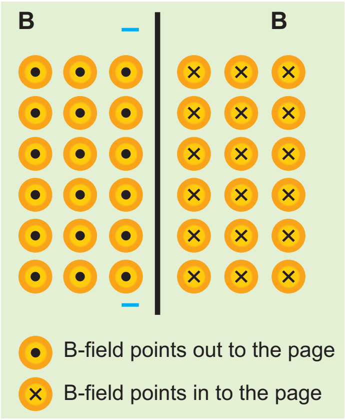


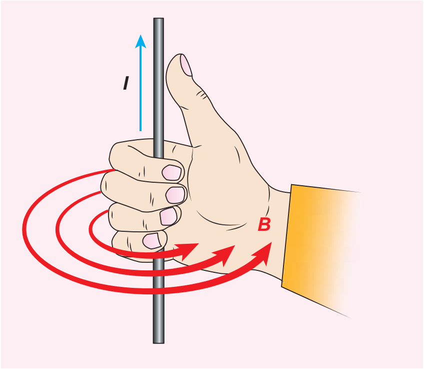
Figure 3.36 Maxwell's right hand cork screw rule

EXAMPLE 3.14

The magnetic field shown in the figure is due to the current carrying wire. In which direction does the current flow in the wire?.



Solution



Using right hand rule, current flows upwards.

3.8 BIOT - SAVART LAW

Soon after the Oersted's discovery, both Jean-Baptiste Biot and Felix Savart in 1819 did quantitative experiments on the force experienced by a magnet kept near current carrying wire and arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the magnetic field. This is true for any shape of the conductor.

3.8.1 Definition and explanation of Biot- Savart law

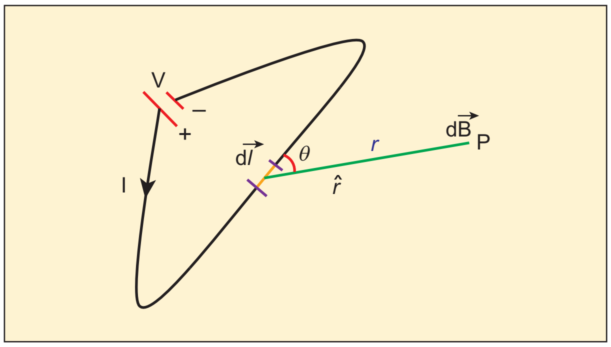


Figure 3.37 Magnetic field at a point P due to current carrying conductor

Biot and Savart experimentally observed that the magnitude of magnetic field $d\vec{B}$ at a point P (Figure 3.37) at a distance r from the small elemental length taken on a conductor carrying current varies

(i) directly as the strength of the current I

(ii) directly as the magnitude of the length element $d\vec{l}$

(iii) directly as the sine of the angle (say,θ) between $d\vec{l}$ and $\hat{r}$.

(iv) inversely as the square of the distance between the point P and length element $d\vec{l}$.

This is expressed as

$$dB \propto \frac{Idl}{r^2}\sin\theta$$

$$dB = k\frac{I\,dl}{r^2}\sin\theta$$

where $k = \frac{\mu_\circ}{4\pi}$ in SI units and k = 1 in CGS units. In vector notation,

$$d\vec{B} = \frac{\mu_\circ}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} \qquad (3.37)$$

Here vector $d\vec{B}$ is perpendicular to both I $d\vec{l}$ (pointing the direction of current flow) and the unit vector $\hat{r}$ directed from $d\vec{l}$ toward point P (Figure 3.38).

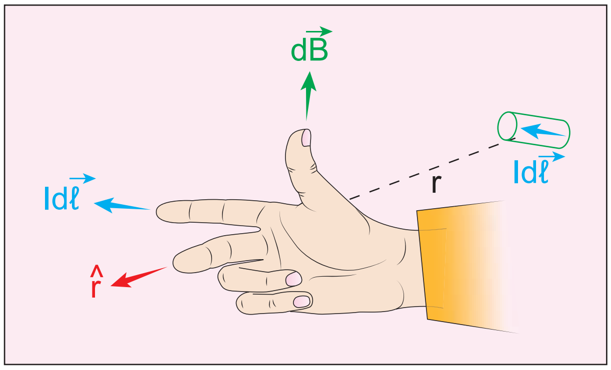


Figure 3.38 The direction of magnetic field using right hand rule

The equation (3.37) is used to compute the magnetic field only due to a small elemental length dl of the conductor. The net magnetic field at P due to the conductor is obtained from principle of superposition by considering the contribution from all current elements $I d\vec{l}$. Hence integrating equation (3.37), we get

$$\vec{B} = \int d\vec{B} = \frac{\mu_\circ I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2} \qquad (3.38)$$

where the integral is taken over the entire current distribution.

Cases

1. If the point P lies on the conductor, then $\theta = 0^\circ$. Therefore, $d\vec{B}$ is zero.
2. If the point lies perpendicular to the conductor, then $\theta = 90^\circ$. Therefore, $d\vec{B}$ is maximum and is given by $d\vec{B} = \frac{I\,dl}{r^2}\hat{n}$ where $\hat{n}$ is the unit vector perpendicular to both $I d\vec{l}$ and $\hat{r}$

Note Electric current is not a vector quantity. It is a scalar quantity. But electric current in a conductor has direction of flow. Therefore, the electric current flowing in a small elemental conductor can be taken as vector quantity i.e. $I\,d\vec{l}$

Similarities between Coulomb's law and Biot-Savort's law

Electric and magnetic fields

- obey inverse square law, so they are long range fields.
- obey the principle of superposition and are linear with respect to source. In magnitude,

$$E \propto q$$

$$B \propto Idl$$

Difference between Coulomb's law and Biot-Savort's law

S. No.	Electric field	Magnetic field
1	Produced by a scalar source i.e., an electric charge q	Produced by a vector source i.e., current element $I\,d\vec{l}$
2	It is directed along the position vector joining the source and the point at which the field is calculated	It is directed perpendicular to the position vector $\vec{r}$ and the current element $I\,d\vec{l}$
3	Does not depend on angle	Depends on the angle between the position vector $\vec{r}$ and the current element $I\,d\vec{l}$

Note that the exponent of charge q (source) and exponent of electric field E is unity. Similarly, the exponent of current element Idl (source) and exponent of magnetic field B is unity. In other words, electric field $\vec{E}$ is proportional only to charge (source) and not on higher powers of charge $\left(q^2, q^3, etc\right)$. Similarly, magnetic field $\vec{B}$ is proportional to current element $Id\vec{l}$ (source) and not on square or cube or higher powers of current element. The cause and effect have linear relationship.

3.8.2 Magnetic field due to long straight conductor carrying current

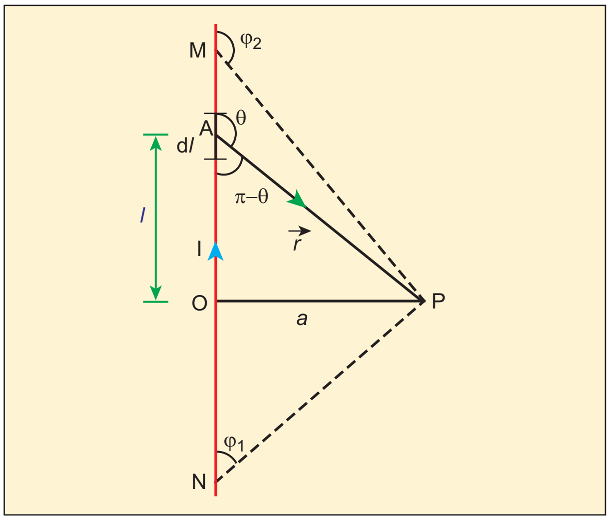


Figure 3.39 Magnetic field due to a long straight current carrying conductor

Consider a long straight wire NM with current I flowing from N to M as shown in Figure 3.39. Let P be the point at a distance a from point O. Consider an element of length dl of the wire at a distance l from point O and $\vec{r}$ be the vector joining the element dl with the point P. Let θ be the angle between $\vec{dl}$ and $\vec{r}$. Then, the magnetic field at P due to the element is

$$d\vec{B} = \frac{\mu_\circ I}{4\pi}\frac{d\vec{l}}{r^2}\sin\theta\begin{pmatrix}\textit{unit vector perpendicular}\\ \textit{to } d\vec{l} \textit{ and } \vec{r}\end{pmatrix}$$

The direction of the field is perpendicular to the plane of the paper and going into it. This can be determined by taking the cross product between two vectors $\vec{dl}$ and $\vec{r}$ (let it be $\hat{n}$). The net magnetic field can be determined by integrating equation (3.38) with proper limits.

From the Figure 3.39, in a right angle triangle PAO,

$$\tan(\pi - \theta) = \frac{a}{l}$$

$$l = -\frac{a}{\tan\theta} \quad \text{(since tan } (\pi\text{- }\theta) = \text{- tan } \theta\text{)}$$

$$l = -a\cot\theta \quad and \quad r = a\,cosec\,\theta$$

Differentiating,

$$dl = a\,cosec^2\theta\,d\theta$$

$$d\vec{B} = \frac{\mu_\circ I}{4\pi}\frac{\left(a\,cosec^2\theta\,d\theta\right)}{\left(a\,cosec\,\theta\right)^2}\sin\theta\,d\theta\,\hat{n}$$

$$d\vec{B} = \frac{\mu_\circ I}{4\pi}\frac{\left(a\,cosec^2\theta\,d\theta\right)}{a^2\,cosec^2\,\theta}\sin\theta\,d\theta\,\hat{n}$$

$$= \frac{\mu_\circ I}{4\pi a}\sin\theta\,d\theta\,\hat{n}$$

This is the magnetic field at a point P due to the current in small elemental length. Note that we have expressed the magnetic field OP in terms of angular coordinate i.e. θ. Therefore, the net magnetic field at the point P which can be obtained by integrating $d\vec{B}$ by varying the angle from $\theta = \varphi_1$ to $\theta = \varphi_2$ is

$$\vec{B} = \frac{\mu_\circ I}{4\pi a}\int_{\varphi_1}^{\varphi_2}\sin\theta\,d\theta\,\hat{n} = \frac{\mu_\circ I}{4\pi a}(\cos\varphi_1 - \cos\varphi_2)\,\hat{n}$$

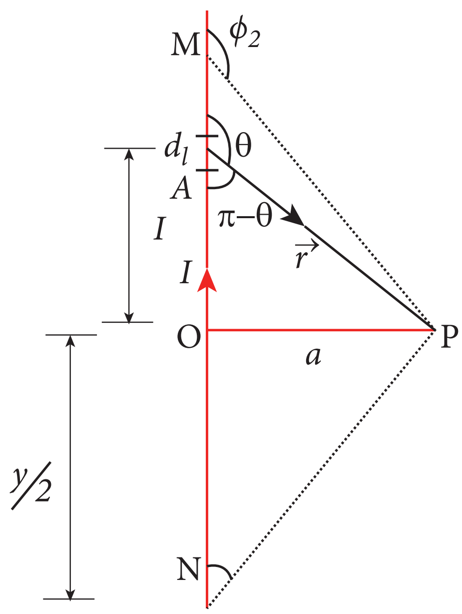
For a an infinitely long straight wire, $\varphi_1 = 0$ and $\varphi_2 = \pi$, the magnetic field is

$$\vec{B} = \frac{\mu_\circ I}{2\pi a}\hat{n} \qquad (3.39)$$

Note that here $\hat{n}$ represents the unit vector from the point O to P.

EXAMPLE 3.15

Calculate the magnetic field at a point P which is perpendicular bisector to current carrying straight wire as shown in figure.



Solution

Let the length MN = y and the point P is on its perpendicular bisector. Let O be the point on the conductor as shown in figure.

Therefore, $OM = ON = \frac{y}{2}$, then

$$\cos\varphi_1 = \frac{\frac{y}{2}}{\sqrt{\frac{y^2}{4}+a^2}} = \frac{\textit{adjacent length}}{\textit{hypotenuse length}}$$

$$= \frac{ON}{PN} = -\frac{\frac{y}{2}}{\sqrt{\frac{y^2}{4}+a^2}} = -\frac{y}{\sqrt{y^2+4a^2}}$$

$$\cos\varphi_2 = \frac{\textit{adjacent length}}{\textit{hypotenuse length}} = \frac{OM}{PM}$$

$$= -\frac{\frac{y}{2}}{\sqrt{\frac{y^2}{4}+a^2}} = -\frac{y}{\sqrt{y^2+4a^2}}$$

Hence,

$$\vec{B} = \frac{\mu_\circ I}{4\pi a}\frac{2y}{\sqrt{y^2+4a^2}}\hat{n}$$

For long straight wire, $y \to \infty$,

$$\vec{B} = \frac{\mu_\circ I}{2\pi a}\hat{n}$$

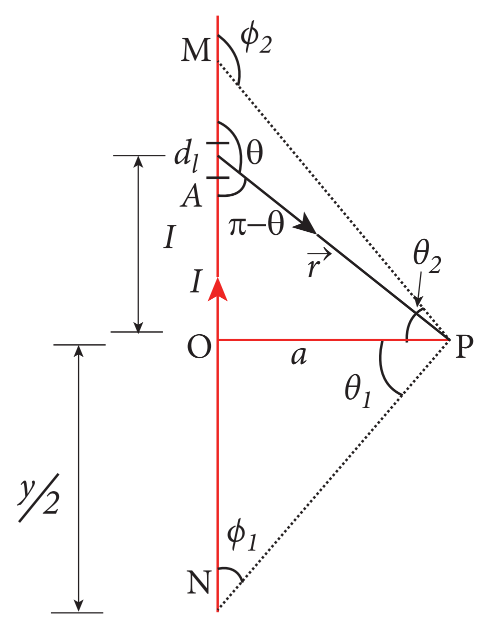
The result obtained is same as we obtained in equation (3.39).

EXAMPLE 3.16

Show that for a straight conductor, the magnetic field

$$\vec{B} = \frac{\mu_\circ I}{4\pi a}(\cos\varphi_1 - \cos\varphi_2)\hat{n}$$

$$= \frac{\mu_\circ I}{4\pi a}(\sin\theta_1 + \sin\theta_2)\hat{n}$$



Solution:

In a right angle triangle OPN, let the angle $\angle OPN = \theta_1$ which implies, $\varphi_1 = \frac{\pi}{2} - \theta_1$

and also in a right angle triangle OPM, $\angle OPM = \theta_2$ which implies, $\varphi_2 = \frac{\pi}{2} + \theta_2$

Hence,

$$\vec{B} = \frac{\mu_\circ I}{4\pi a}\left(\cos\left(\frac{\pi}{2} - \theta_1\right) - \cos\left(\frac{\pi}{2} + \theta_2\right)\right)\hat{n}$$

$$= \frac{\mu_\circ I}{4\pi a}(\sin\theta_1 + \sin\theta_2)\hat{n}$$

3.8.3 Magnetic field produced along the axis of the current carrying circular coil

Consider a current carrying circular loop of radius R and let I be the current flowing through the wire in the direction as shown in Figure 3.40. The magnetic field at a point P on the axis of the circular coil at a distance z from its center of the coil O. It is computed by taking two diametrically opposite line elements of the coil each of length $d\vec{l}$ at C and D. Let $\vec{r}$ be the vector joining the current element (I $d\vec{l}$) at C to the point P.

$$PC = PD = r = \sqrt{R^2 + Z^2} \text{ and}$$

$$\text{angle } \angle CPO = \angle DPO = \theta$$

According to Biot-Savart's law, the magnetic field at P due to the current element $I\, d\vec{l}$ is

$$d\vec{B} = \frac{\mu_\circ}{4\pi}\frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

The magnitude of magnetic field due to current element $I\, d\vec{l}$ at C and D are equal because of equal distance from the coil. The magnetic field $d\vec{B}$ due to each current element $I\, d\vec{l}$ is resolved into two components; dB sin θ along y - direction and dB cos θ along z - direction. Horizontal components of each current element cancels out while the vertical components (dB cos θ $\hat{k}$) alone contribute to total magnetic field at the point P.

If we integrate $d\vec{l}$ around the loop, $d\vec{B}$ sweeps out a cone as shown in Figure 3.40, then the net magnetic field $\vec{B}$ at point P is

$$\vec{\mathrm{B}} = \int d\vec{B} = \int dB \cos\theta\, \hat{k}$$

$$\vec{\mathrm{B}} = \frac{\mu_\circ I}{4\pi}\int \frac{dl}{r^2}\cos\theta\, \hat{k}$$

But $\cos\theta = \frac{R}{\left(R^2 + Z^2\right)^{\frac{1}{2}}}$, using Pythagorous theorem $r^2 = R^2 + Z^2$ and integrating line element from 0 to 2πR, we get

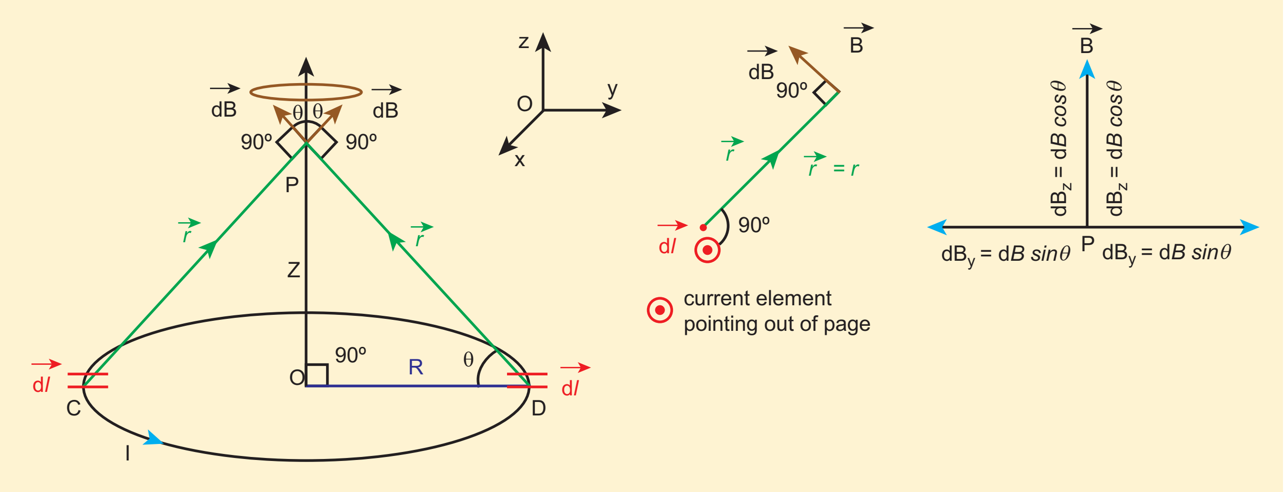


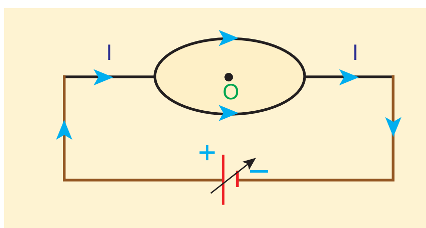
Figure 3.40 Current carrying circular loop using Biot-Savart's law

$$\vec{B} = \frac{\mu_\circ I}{2\pi}\frac{R^2}{\left(R^2+Z^2\right)^{\frac{3}{2}}}\hat{k} \qquad (3.40)$$

Note that the magnetic field $\vec{B}$ points along the direction from the point O to P. Suppose if the current flows in clockwise direction, then magnetic field points in the direction from the point P to O.

EXAMPLE 3.17

What is the magnetic field at the center of the loop shown in figure?



Solution

The magnetic field due to current in the upper hemisphere and lower hemisphere of the circular coil are equal in magnitude but opposite in direction. Hence, the net magnetic field at the center of the loop (at point O) is zero $\vec{B} = \vec{0}$.

3.8.4 Current loop as a magnetic dipole

The magnetic field from the centre of a circular loop of radius R along the axis is given by

$$\vec{B} = \frac{\mu_\circ I}{2}\frac{R^2}{\left(R^2+z^2\right)^{\frac{3}{2}}}\hat{k}$$

At larger distance Z >> R, therefore $R^2+Z^2 \approx Z^2$, we have

$$\vec{B} = \frac{\mu_\circ I}{2}\frac{R^2}{Z^3}\hat{k} \qquad (3.41)$$

Let A be the area of the circular loop A = π R^2. So rewriting the equation (3.41) in terms of area of the loop, we have

$$\vec{B} = \frac{\mu_\circ I}{2\pi}\frac{A}{Z^3}\hat{k}$$

(multiply and divide by 2)

$$\vec{B} = \frac{\mu_\circ}{4\pi}\frac{2IA}{Z^3}\hat{k} \qquad (3.42)$$

Comparing equation (3.42) with equation (3.14) dimensionally, we get

$$p_m = I\,A$$

where p_m is called magnetic dipole moment. In vector notation,

$$\vec{p}_m = I\vec{A} \qquad (3.43)$$

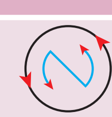
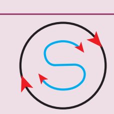
This implies that a current carrying circular loop behaves as a magnetic dipole of magnetic moment $\vec{p}_m$. So, **the magnetic dipole moment of any current loop is equal to the product of the current and area of the loop.**

Right hand thumb rule

In order to determine the direction of magnetic moment, we use right hand thumb rule (mnemonic) which states that

If we curl the fingers of right hand in the direction of current in the loop, then the stretched thumb gives the direction of the magnetic moment associated with the loop.

Table 3.3 End rule – polarity with direction of current in circular loop

Current in circular loop	Polarity	Picture
Anti-clockwise current	North Pole	 Anti-clockwise current Polarity: North Pole
Clockwise current	South Pole	 Clockwise current Polarity: South Pole

3.8.5 Magnetic dipole moment of revolving electron

Suppose an electron undergoes circular motion around the nucleus as shown in Figure 3.41. The circulating electron in a loop is like current in a circular loop (since flow of charge is current). The magnetic dipole moment due to current carrying circular loop is

$$\vec{\mu}_L = I\vec{A} \qquad (3.44)$$

In magnitude,

$$\mu_L = IA$$

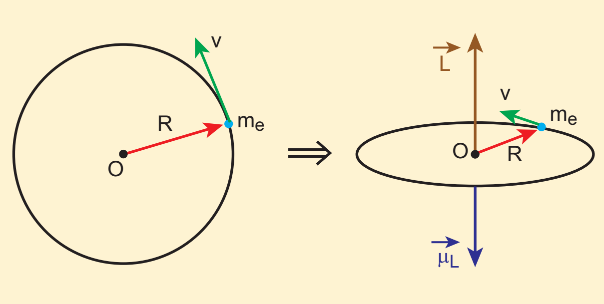


Figure 3.41 (a) Electron revolving in a circular orbit (b) Direction of magnetic dipole moment vector and orbital angular momentum vector are opposite

If T is the time period of an electron, the current due to circular motion of the electron is

$$I = \frac{-e}{T} \qquad (3.45)$$

where $-e$ is the charge of an electron. If R is the radius of the circular orbit and v is the velocity of the electron in the circular orbit, then

$$T = \frac{2\pi R}{v} \qquad (3.46)$$

Using equation (3.45) and equation (3.46) in equation (3.44), we get

$$\mu_L = -\frac{e}{\frac{2\pi R}{v}}\pi R^2 = -\frac{evR}{2} \qquad (3.47)$$

where A = πR^2 is the area of the circular loop. By definition, angular momentum of the electron about O is

$$\vec{L} = \vec{R} \times \vec{p}$$

In magnitude,

$$L = Rp = mvR \qquad (3.48)$$

Using equation (3.47) and equation (3.48), we get

$$\frac{\mu_L}{L} = -\frac{\frac{evR}{2}}{mvR} = -\frac{e}{2m} \Rightarrow \vec{\mu}_L = -\frac{e}{2m}\vec{L} \qquad (3.49)$$

The negative sign indicates that the magnetic moment and angular momentum are in opposite direction.

In magnitude,

$$\frac{\mu_L}{L} = \frac{e}{2m} = \frac{1.60\times10^{-19}}{2\times9.11\times10^{-31}} = 0.0878\times10^{12}$$

$$\frac{\mu_L}{L} = 8.78\times10^{10}\,C\,kg^{-1} = \text{constant}$$

The ratio $\frac{\mu_L}{L}$ is a constant and also known as gyro-magnetic ratio $\left(\frac{e}{2m}\right)$. It must be noted that the gyro-magnetic ratio is a constant of proportionality which connects angular momentum of the electron and the magnetic moment of the electron.

According to Neil's Bohr quantization rule, the angular momentum of an electron moving in a stationary orbit is quantized, which means,

$$L = n\hbar = n\frac{h}{2\pi}$$

where, h is the Planck's constant (h = 6.63 x 10^{-34} J s) and number n takes natural numbers

(i.e., n = 1,2,3,....). Hence,

$$\mu_L = \frac{e}{2m}L = n\frac{eh}{4\pi m}Am^2$$

$$\mu_L = n\frac{(1.60\times10^{-19})h}{4\pi m}Am^2$$

$$= n\frac{(1.60\times10^{-19})(6.63\times10^{-34})}{4\times3.14\times(9.11\times10^{-31})}$$

$$\mu_L = n\times9.27\times10^{-24}Am^2$$

The minimum magnetic moment can be obtained by substituting n = 1,

$$\mu_L = 9.27\times10^{-24}Am^2 = 9.27\times10^{-24}JT^{-1}$$
$$= (\mu_L)_{\min} = \mu_B$$

where, $\mu_B = \frac{eh}{4\pi m} = 9.27\times10^{-24}Am^2$ is called Bohr magneton. This is a convenient unit with which one can measure atomic magnetic moments.

Note: Bohr quantization rule will be discussed in unit 8 of second volume

3.9 AMPÈRE'S CIRCUITAL LAW

Ampère's circuital law is used to calculate magnetic field at a point whenever there is a symmetry in the problem. This is similar to Gauss's law in electrostatics. These are powerful methods whenever there is symmetry in the problem.

3.9.1 Definition and explanation of Ampère's circuital law

Ampère's law: The line integral of magnetic field over a closed loop is μ_0 times net current enclosed by the loop.

$$\oint_C \vec{B}.\vec{dl} = \mu_\circ I_{enclosed} \qquad (3.50)$$

where $I_{enclosed}$ is the net current linked by the closed loop C. Note that the line integral does not depend on the shape of the path or the position of the conductor with the magnetic field.

Note

Line integral means integral over a line or curve, symbol used is $\int$.

Closed line integral means integral over a closed curve (or line), symbol is $\oint$ or $\oint_c$

3.9.2 Magnetic field due to the current carrying wire of infinite length using Ampère's law

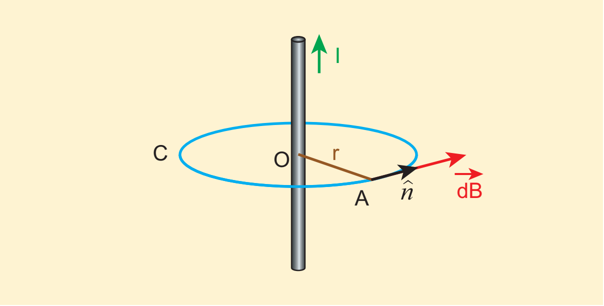


Figure 3.42 Ampèrian loop for current carrying straight wire

Consider a straight conductor of infinite length carrying current I and the direction of magnetic field lines is shown in Figure 3.42. Since the wire is geometrically cylindrical in shape and symmetrical about its axis, we construct an Ampèrian loop in the form of a circular shape at a distance *r* from the centre of the conductor as shown in Figure 3.42. From the Ampère's law, we get

$$\oint_C \vec{B}.d\vec{l} = \mu_\circ I$$

where $d\vec{l}$ is the line element along the amperian loop (tangent to the circular loop). Hence, the angle between magnetic field vector and line element is zero. Therefore,

$$\oint_C B\,dl = \mu_\circ I$$

where I is the current enclosed by the Ampèrian loop. Due to the symmetry, the magnitude of the magnetic field is uniform over the Ampèrian loop, we can take B out of the integration.

$$B\oint_C dl = \mu_\circ I$$

For a circular loop, the circumference is 2πr, which implies,

$$B\int_0^{2\pi r} dl = \mu_\circ I$$

$$\vec{B}\cdot 2\pi r = \mu_\circ I$$

$$B = \frac{\mu_\circ I}{2\pi r}$$

In vector form, the magnetic field is

$$\vec{B} = \frac{\mu_\circ I}{2\pi r}\hat{n}$$

where $\hat{n}$ is the unit vector along the tangent to the Ampèrian loop as shown in the Figure 3.42. This perfectly agrees with the result obtained from Biot-Savart's law as given in equation (3.39).

EXAMPLE 3.18

Compute the magnitude of the magnetic field of a long, straight wire carrying a current of 1 A at distance of 1m from it. Compare it with Earth's magnetic field.

Solution

Given that 1 = 1 A and radius r = 1 m

$$B_{straightwire} = \frac{\mu_\circ I}{2\pi r} = \frac{4\pi\times10^{-7}\times1}{2\pi\times1} = 2\times10^{-7}\,T$$

But the Earth's magnetic field is $B_{Earth} \sim 10^{-5}\,T$

So, $B_{straightwire}$ is one hundred times smaller than B_{Earth}.

Solenoid

A solenoid is a long coil of wire closely wound in the form of helix as shown in Figure 3.43. When electric current is passed through the solenoid, the magnetic field is produced. The magnetic field of the solenoid is due to the superposition of magnetic fields of each turn of the solenoid. The direction of magnetic field due to solenoid is given by right hand palm-rule (mnemonic).

Inside the solenoid, the magnetic field is nearly uniform and parallel to its axis whereas, outside the solenoid the field is negligibly small. Based on the direction of the current, one end of the solenoid behaves like North Pole and the other end behaves like South Pole.

The current carrying solenoid is held in right hand. If the fingers curl in the direction of current, then extended thumb gives the direction of magnetic field of current carrying solenoid. It is shown in Figure 3.44. Hence, the magnetic field of a

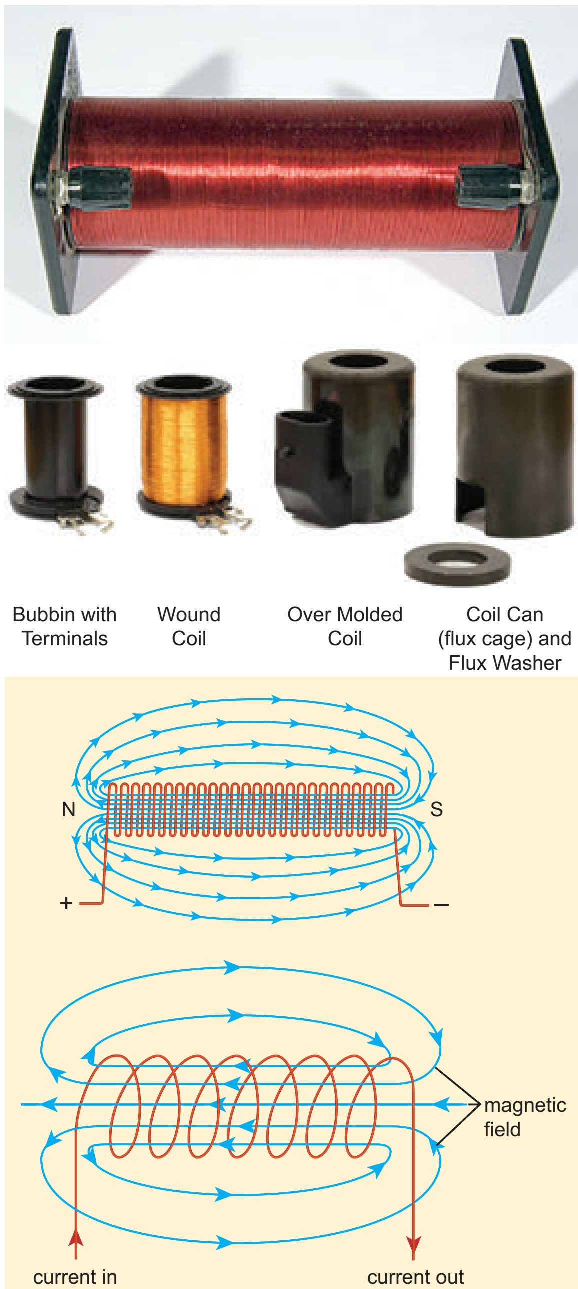


Figure 3.43 Solenoid

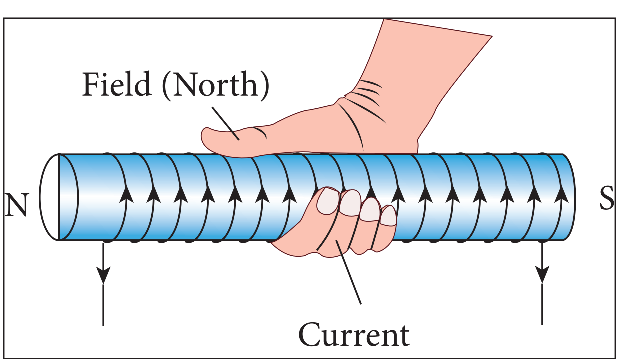


Figure 3.44 The direction of magnetic field of solenoid

solenoid looks like the magnetic field of a bar magnet.

The solenoid is assumed to be long which means that the length of the solenoid is large when compared to its diameter. The winding need not to be always circular, it can also be in other shapes. We consider here only circularly wound solenoid as shown in Figure 3.45.

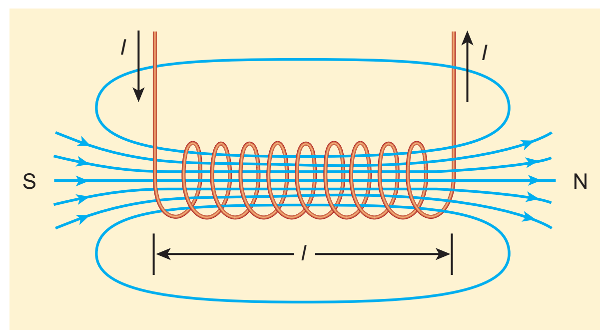


Figure 3.45 Solenoid as a bar magnet

3.9.3 Magnetic field due to a long current carrying solenoid

Consider a solenoid of length L having N turns. The diametre of the solenoid is assumed to be much smaller when compared to its length and the coil is wound very closely.

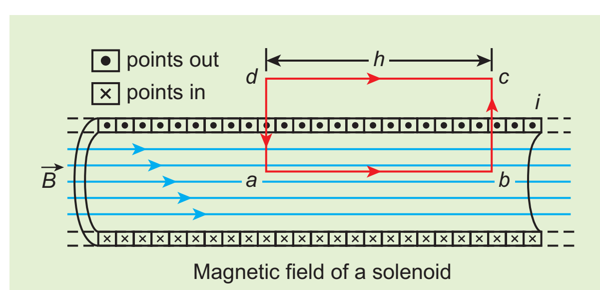


Figure 3.46 Amperian loop for solenoid

In order to calculate the magnetic field at any point inside the solenoid, we use Ampere's circuital law. Consider a rectangular loop abcd as shown in Figure 3.46. Then from Ampère's circuital law,

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_\circ I_{enclosed}$$

$$= \mu_\circ \times (total\,current\,enclosed\,by\,Amperian\,loop)$$

The left hand side of the equation is

$$\oint_C \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}.d\vec{l} + \int_b^c \vec{B}.d\vec{l} + \int_c^d \vec{B}.d\vec{l} + \int_d^a \vec{B}.d\vec{l}$$

Since the elemental lengths along bc and da are perpendicular to the magnetic field which is along the axis of the solenoid, the integrals

$$\int_b^c \vec{B}.d\vec{l} = \int_b^c |\vec{B}|\,|d\vec{l}|\cos 90^\circ = 0$$

$$\int_d^a \vec{B}.d\vec{l} = 0$$

Since the magnetic field outside the solenoid is zero, the integral $\int_c^d \vec{B}.d\vec{l} = 0$

For the path along ab, the integral is

$$\int_a^b \vec{B}.d\vec{l} = B\int_a^b dl\cos 0^\circ = B\int_a^b dl$$

where the length of the loop ab as shown in the Figure 3.46 is h. But the choice of length of the loop ab is arbitrary. We can take very large loop such that it is equal to the length of the solenoid L. Therefore the integral is

$$\int_a^b \vec{B}.d\vec{l} = BL$$

Let N I be the current passing through the solenoid of N turns, then

$$\int_a^b \vec{B}.d\vec{l} = BL = \mu_\circ NI \Rightarrow B = \mu_\circ \frac{NI}{L}$$

The number of turns per unit length is given by $\frac{N}{L} = n$, Then

$$B = \mu_\circ \frac{nLI}{L} = \mu_\circ nI \qquad (3.51)$$

Since n is a constant for a given solenoid and μ_0 is also constant. For a fixed current I, the magnetic field inside the solenoid is also a constant.

Note

Solenoid can be used as electromagnets. It produces strong magnetic field that can be turned ON or OFF. This is not possible in case of permanent magnet. Further the strength of the magnetic field can be increased by keeping iron bar inside the solenoid. This is because the magnetic field of the solenoid magnetizes the iron bar and hence the net magnetic field is the sum of magnetic field of the solenoid and magnetic field of magnetised iron. Because of these properties, solenoids are useful in designing variety of electrical appliances.

EXAMPLE 3.19

Calculate the magnetic field inside a solenoid, when

(a) the length of the solenoid becomes twice and fixed number of turns

(b) both the length of the solenoid and number of turns are double

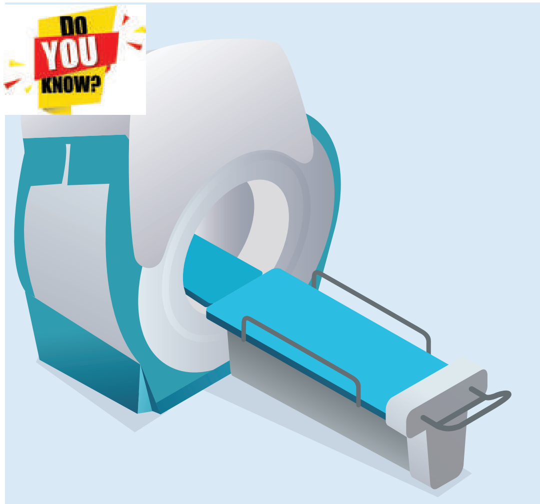
(c) the number of turns becomes twice for the fixed length of the solenoid

Compare the results.

Solution

The magnetic field of a solenoid (inside) is

$$B_{L,N} = \mu_\circ \frac{NI}{L}$$



MRI is Magnetic Resonance Imaging which helps the physicians to diagonise or monitor treatment for a variety of abnormal conditions happening within the head, chest, abdomen and pelvis. It is a non invasive medical test. The patient is placed in a circular opening (actually interior of a solenoid which is made up of superconducting wire) and large current is sent through the superconducting wire to produce a strong magnetic field. So, it uses more powerful magnet, radio frequency pulses and a computer to produce pictures of organs which helps the physicians to examine various parts of the body.

(a) length of the solenoid becomes twice and fixed number of turns

L→2L (length becomes twice)

N→N (number of turns are fixed)

The magnetic field is

$$B_{2L,N} = \mu_\circ \frac{NI}{2L} = \frac{1}{2} B_{L,N}$$

(b) both the length of the solenoid and number of turns are double

L→2L (length becomes twice)

N→2N (number of turns becomes twice)

The magnetic field is

$$B_{2L,2N} = \mu_\circ \frac{2NI}{2L} = B_{L,N}$$

(c) the number of turns becomes twice but for the fixed length of the solenoid

L→L (length is fixed)

N→2N (number of turns becomes twice)

The magnetic field is

$$B_{L,2N} = \mu_\circ \frac{2NI}{L} = 2B_{L,N}$$

From the above results,

$$B_{L,2N} > B_{2L,2N} > B_{2L,N}$$

Thus, strength of the magnetic field is increased when we pack more loops into the same length for a given current.

3.9.5 Toroid

A solenoid is bent in such a way its ends are joined together to form a closed ring shape, is called a toroid which is shown in Figure 3.47. The magnetic field has constant magnitude inside the toroid whereas in the interior region (say, at point P) and exterior region (say, at point Q), the magnetic field is zero.

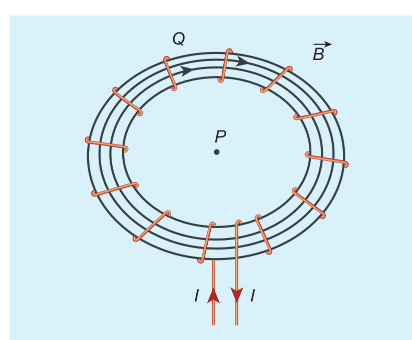


Figure 3.47 Toroid

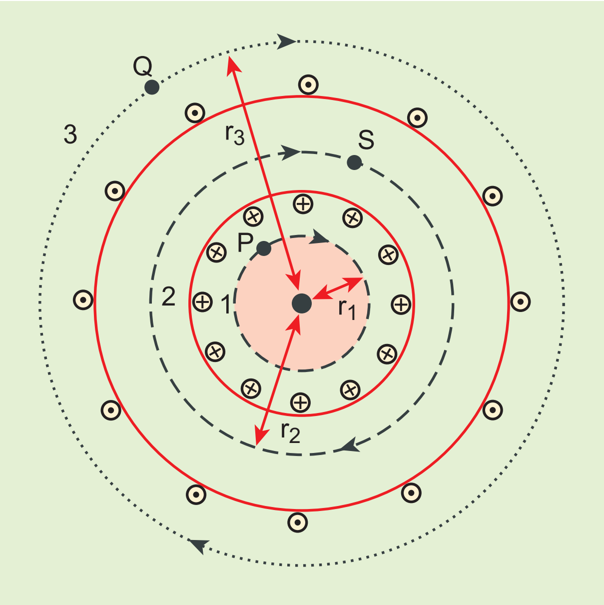


Figure 3.48 Toroid – Amperian loop

(a) Open space interior to the toroid

Let us calculate the magnetic field B_P at point P. We construct an Amperian loop 1 of radius r_1 around the point P as shown in Figure 3.48. For simplicity, we take circular loop so that the length of the loop is its circumference.

$$L_1 = 2\pi r_1$$

Ampère's circuital law for the loop 1 is

$$\oint_{loop1} \vec{B}_P . \vec{dl} = \mu_\circ I_{enclosed}$$

Since, the loop1 encloses no current, $I_{enclosed} = 0$

$$\oint_{loop1} \vec{B}_P . \vec{dl} = 0$$

This is possible only if the magnetic field at point P vanishes i.e.

$$\vec{B}_P = 0$$

(b) Open space exterior to the toroid

Let us calculate the magnetic field B_Q at point Q. We construct an Amperian loop 3 of radius r_3 around the point Q as shown in Figure 3.48. The length of the loop is

$$L_3 = 2\pi r_3$$

Ampère's circuital law for the loop 3 is

$$\oint_{loop3} \vec{B}_Q . \vec{dl} = \mu_\circ I_{enclosed}$$

Since, in each turn of the toroid loop, current coming out of the plane of paper is cancelled by the current going into the plane of paper. Thus, $I_{enclosed} = 0$

$$\oint_{loop3} \vec{B}_Q . \vec{dl} = 0$$

This is possible only if the magnetic field at point Q vanishes i.e.

$$\vec{B}_Q = 0$$

(c) Inside the toroid

Let us calculate the magnetic field B_S at point S by constructing an Amperian loop 2 of radius r_2 around the point S as shown in Figure 3.48. The length of the loop is

$$L_2 = 2\pi r_2$$

Ampere's circuital law for the loop 2 is

$$\oint_{loop2} \vec{B}_S . \vec{dl} = \mu_\circ I_{enclosed}$$

Let I be the current passing through the toroid and N be the number of turns of the toroid, then

$$I_{enclosed} = NI$$

and $$\oint_{loop2} \vec{B}_S . \vec{dl} = \oint_{loop2} Bdl\cos\theta = B2\pi r_2$$

$$\oint_{loop2} \vec{B}_S . \vec{dl} = \mu_\circ NI$$

$$B_S = \mu_\circ \frac{NI}{2\pi r_2}$$

The number of turns per unit length is $n = \frac{N}{2\pi r_2}$, then the magnetic field at point S is

$$B_S = \mu_\circ nI \qquad (3.52)$$

3.10 LORENTZ FORCE

When an electric charge q is kept at rest in a magnetic field, no force acts on it. At the same time, if the charge moves in the magnetic field, it experiences a force. This force is different from Coulomb force, studied in unit 1. This force is known as magnetic force. It is given by the equation

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right) \qquad (3.53)$$

In general, if the charge is moving in both the electric and magnetic fields, the total force experienced by the charge is given by $\vec{F} = q\vec{E}\left(\vec{v} \times \vec{B}\right)$. It is known as Lorentz force.

3.10.1 Force on a moving charge in a magnetic field

When an electric charge q is moving with velocity $\vec{v}$ in the magnetic field $\vec{B}$, it experiences a force, called magnetic force $\vec{F}_m$. After careful experiments, Lorentz deduced the force experienced by a moving charge in the magnetic field $\vec{F}_m$

$$\vec{F}_m = q\left(\vec{v} \times \vec{B}\right) \qquad (3.54)$$

In magnitude, $F_m = qvB\sin\theta \qquad (3.55)$

The equations (3.54) and (3.55) imply

1. $\vec{F}_m$ is directly proportional to the magnetic field $\vec{B}$
2. $\vec{F}_m$ is directly proportional to the velocity $\vec{v}$
3. $\vec{F}_m$ is directly proportional to sine of the angle between the velocity and magnetic field
4. $\vec{F}_m$ is directly proportional to the magnitude of the charge q
5. The direction of $\vec{F}_m$ is always perpendicular to $\vec{v}$ and $\vec{B}$ as $\vec{F}_m$ is the cross product of $\vec{v}$ and $\vec{B}$

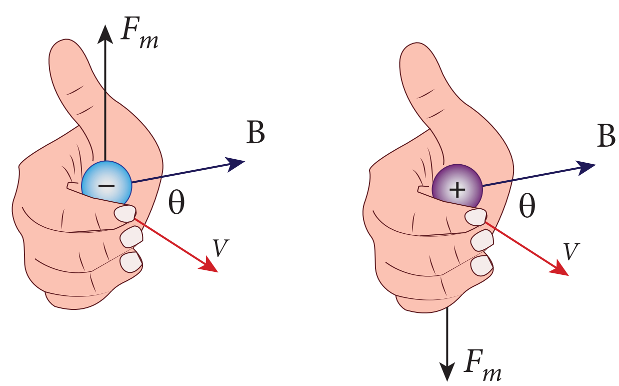


Figure 3.49 Direction of the Lorentz force on (a) positive charge (b) negative charge

7. The direction of $\vec{F}_m$ on negative charge is opposite to the direction of $\vec{F}_m$ on positive charge provided other factors are identical as shown Figure 3.49
8. If velocity $\vec{v}$ of the charge q is along magnetic field $\vec{B}$ then, $\vec{F}_m$ is zero

Definition of tesla

The strength of the magnetic field is one tesla if unit charge moving in it with unit velocity experiences unit force.

$$1\ T = \frac{1\,N\,s}{C\,m} = 1\frac{N}{A\,m} = 1N\,A^{-1}m^{-1}$$

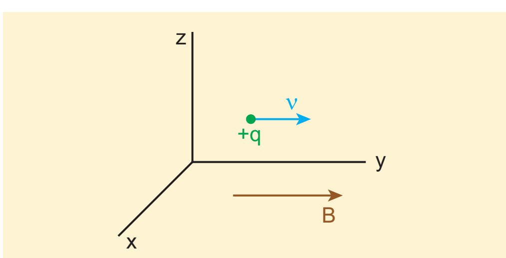
EXAMPLE 3.20

A particle of charge q moves with velocity $\vec{v}$ along positive y - direction in a magnetic field $\vec{B}$. Compute the Lorentz force experienced by the particle (a) when magnetic field is along positive y-direction (b) when magnetic field points in positive z - direction (c) when magnetic field is in zy - plane and making an angle θ with velocity of the particle. Mark the direction of magnetic force in each case.

Solution

Velocity of the particle is $\vec{v} = v\,\hat{j}$

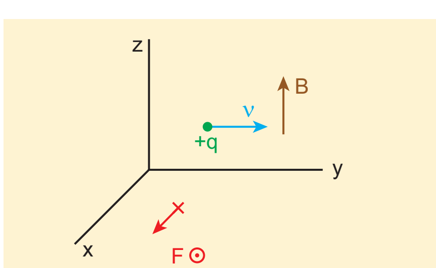
(a) Magnetic field is along positive y - direction, this implies, $\vec{B} = v\,\hat{j}$



From Lorentz force, $\vec{F}_m = q\left(v\,\hat{j} \times B\,\hat{j}\right) = 0$

So, no force acts on the particle when it moves along the direction of magnetic field.

(b) Magnetic field points in positive z - direction, this implies, $\vec{B} = B\,\hat{k}$

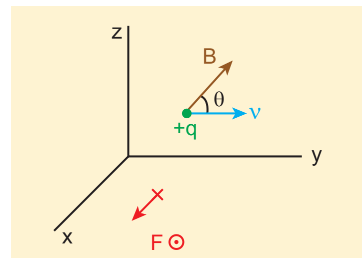


From Lorentz force,

$\vec{F}_m = q\left(v\hat{j} \times B\hat{k}\right) = qvB\,\hat{i}$

Therefore, the magnitude of the Lorentz force is qvB and direction is along positive x - direction.

(c) Magnetic field is in zy - plane and making an angle θ with the velocity of the particle, which implies $\vec{B} = B\cos\theta\,\hat{j} + B\sin\theta\,\hat{k}$



From Lorentz force,

$\vec{F}_m = q\left(v\hat{j}\right) \times (B\cos\theta\hat{j} + B\sin\theta\hat{k})$

$= \mathrm{qvB}\sin\theta\,\hat{\mathrm{i}}$

EXAMPLE 3.21

Compute the work done and power delivered by the Lorentz force on the particle of charge q moving with velocity $\vec{v}$. Calculate the angle between Lorentz force and velocity of the charged particle and also interpret the result.

Solution

For a charged particle moving on a magnetic field, $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$

The work done by the magnetic field is

$$W = \int \vec{F}.d\vec{r} = \int \vec{F}.\vec{v}dt$$

$$W = q\int \left(\vec{v} \times \vec{B}\right).\vec{v}\,dt = 0$$

Since $\vec{v} \times \vec{B}$ is perpendicular to $\vec{v}$ and hence $\left(\vec{v} \times \vec{B}\right) \cdot \vec{v} = \vec{0}$ This means that Lorentz force do no work on the particle. From work-kinetic energy theorem, (Refer section 4.2.6, XI th standard Volume I)

$$\frac{dW}{dt} = P = 0$$

Since, $\vec{F}.\vec{v}=0 \Rightarrow \vec{F}$ and $\vec{v}$ are perpendicular to each other. The angle between Lorentz force and velocity of the charged particle is 90°. Thus Lorentz force changes the direction of the velocity but not the magnitude of the velocity. Hence Lorentz force does no work and also does not alter kinetic energy of the particle.

3.10.2 Motion of a charged particle in a uniform magnetic field

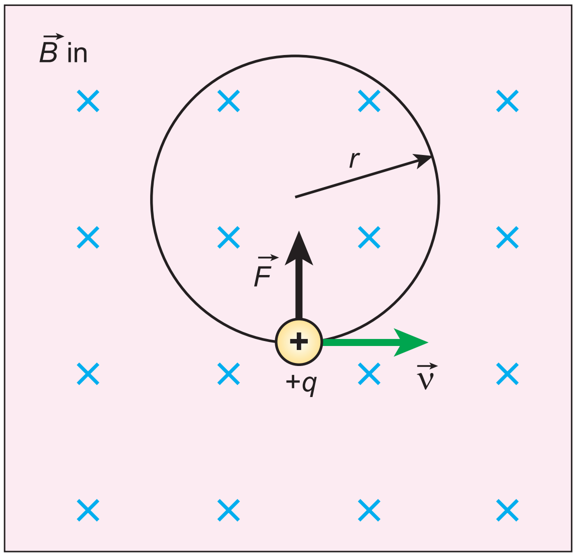


Figure 3.50 Circular motion of a charged particle in a perpendicular uniform magnetic field

Consider a charged particle of charge q having mass m enters into a region of uniform magnetic field $\vec{B}$ with velocity $\vec{v}$ such that velocity is perpendicular to the magnetic field. As soon as the particle enters into the field, Lorentz force acts on it in a direction perpendicular to both magnetic field $\vec{B}$ and velocity $\vec{v}$.

As a result, the charged particle moves in a circular orbit as shown in Figure 3.50.

The Lorentz force on the charged particle is given by

$$\vec{F} = q(\vec{v} \times \vec{B})$$

Since Lorentz force alone acts on the particle, the magnitude of the net force on the particle is

$$\sum_i F_i = F_m = qvB$$

This Lorentz force acts as centripetal force for the particle to execute circular motion. Therefore,

$$qvB = m\frac{v^2}{r}$$

The radius of the circular path is

$$r = \frac{mv}{qB} = \frac{p}{qB} \qquad (3.56)$$

where p = mv is the magnitude of the linear momentum of the particle. Let T be the time taken by the particle to finish one complete circular motion, then

$$T = \frac{2\pi r}{v} \qquad (3.57)$$

Hence substituting (3.56) in (3.57), we get

$$T = \frac{2\pi m}{qB} \qquad (3.58)$$

Equation (3.58) is called the **cyclotron period**. The reciprocal of time period is the frequency *f*, which is

$$f = \frac{1}{T}$$

$$f = \frac{qB}{2\pi m} \qquad (3.59)$$

In terms of angular frequency ω,

$$\omega = 2\pi f = \frac{q}{m}B \qquad (3.60)$$

Equations (3.59) and (3.60) are called as **cyclotron frequency or gyro-frequency**.

From equations (3.58), (3.59) and (3.60), we infer that time period and frequency depend only on charge-to-mass ratio (specific charge) but not velocity or the radius of the circular path.

If a charged particle moves in a region of uniform magnetic field such that its velocity is not perpendicular to the magnetic field, then the velocity of the particle is split up into two components; one component is parallel to the field while the other perpendicular to the field. The component of velocity parallel to field remains unchanged and the component perpendicular to field keeps changing due to the Lorentz force. Hence the path of the particle is not a circle; it is a helix around the field lines as shown in Figure 3.51.

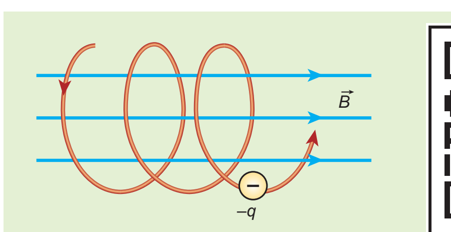


Figure 3.51 Helical path of the electron in a uniform magnetic field

For an example, the helical path of an electron when it moves in a magnetic field is shown in Figure 3.52. Inside the particle detector called cloud chamber, the path is made visible by the condensation of water droplets.

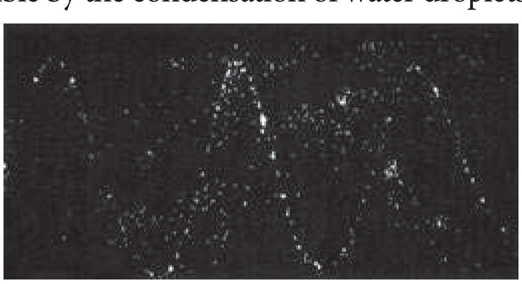


Figure 3.52 Helical path of the electron inside the cloud chamber

EXAMPLE 3.22

An electron moving perpendicular to a uniform magnetic field 0.500 T undergoes circular motion of radius 2.80 mm. What is the speed of electron?

Solution

Charge of an electron q = -1.60 × 10^{-19} C

$\Rightarrow |q| = 1.60\times10^{-19}\,C$

Magnitude of magnetic field B = 0.500 T

Mass of the electron, m = 9.11 × 10^{-31} kg

Radius of the orbit, r = 2.50 mm = 2.50 × 10^{-3} m

Velocity of the electron, $v = |q|\dfrac{rB}{m}$

$$v = 1.60\times10^{-19}\times\frac{2.50\times10^{-3}\times0.500}{9.11\times10^{-31}}$$

$$v = 2.195\times10^{8}\,m\,s^{-1}$$

EXAMPLE 3.23

A proton moves in a uniform magnetic field of strength 0.500 T magnetic field is directed along the x-axis. At initial time, $t = 0\,s$, the proton has velocity $\vec{v} = \left(1.95\times10^{5}\,\hat{i} + 2.00\times10^{5}\,\hat{k}\right)m\,s^{-1}$. Find

(a) At initial time, what is the acceleration of the proton.

(b) Is the path circular or helical?. If helical, calculate the radius of helical trajectory and also calculate the pitch of the helix (Note: Pitch of the helix is the distance travelled along the helix axis per revolution).

Solution

Magnetic field $\vec{B} = 0.500\hat{i}\,T$

Velocity of the particle

$$\vec{v}=\left(1.95\times10^{5}\hat{i}\ +2.00\times10^{5}\hat{k}\ \right)ms^{-1}$$

Charge of the proton $q=1.60\times10^{-19}C$

Mass of the proton $m=1.67\times10^{-27}kg$

(a) The force experienced by the proton is

$$\vec{F}=q\left(\vec{v}\times\vec{B}\right)$$

$$=1.60\times10^{-19}\times\left(\left(1.95\times10^{5}\hat{i}\ +2.00\times10^{5}\hat{k}\ \right)\times\left(0.500\hat{i}\ \right)\right)$$

$$\vec{F}=1.60\times10^{-14}N\,\hat{j}$$

Therefore, from Newton's second law,

$$\vec{a}=\frac{1}{m}\vec{F}=\frac{1}{1.67\times10^{-27}}\left(1.60\times10^{-14}\right)$$

$$=9.58\times10^{12}m\,s^{-2}$$

(b) Trajectory is helical

Radius of helical path is

$$R=\frac{mv_z}{|q|B}=\frac{1.67\times10^{-27}\times2.00\times10^{5}}{1.60\times10^{-19}\times0.500}$$

$$=4.175\times10^{-3}m=4.18mm$$

Pitch of the helix is the distance travelled along x-axis in a time T, which is P = v_x T

But time,

$$T=\frac{2\pi}{\omega}=\frac{2\pi m}{|q|B}=\frac{2\times3.14\times1.67\times10^{-27}}{1.60\times10^{-19}\times0.500}$$

$$=13.1\times10^{-8}s$$

Hence, pitch of the helix is

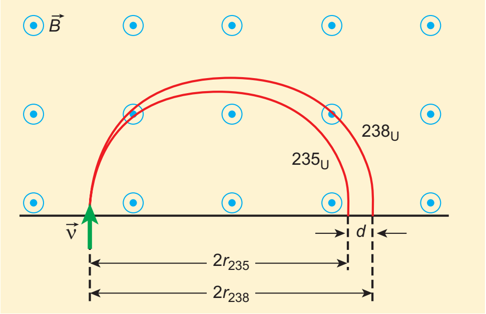
$$P=v_xT=\left(1.95\times10^{5}\right)\left(13.1\times10^{-8}\right)$$

$$=25.5\times10^{-3}m=25.5\ mm$$

The proton experiences appreciable acceleration in the magnetic field, hence the pitch of the helix is almost six times greater than the radius of the helix.

EXAMPLE 3.24

Two singly ionized isotopes of uranium ${}^{235}_{92}U$ and ${}^{238}_{92}U$ (isotopes have same atomic number but different mass number) are sent with velocity 1.00×10^{5} m s^{-1} into a magnetic field of strength 0.500 T normally. Compute the distance between the two isotopes after they complete a semi-circle. Also compute the time taken by each isotope to complete one semi-circular path. (Given: masses of the isotopes: m_{235} = 3.90 x 10^{-25} kg and m_{238} = 3.95 x 10^{-25} kg)



Solution

Since isotopes are singly ionized, they have equal charge which is equal to the charge of an electron, q = - 1.6 × 10^{-19} C. Mass of uranium ${}^{235}_{92}U$ and ${}^{238}_{92}U$ are 3.90 × 10^{-25} kg and 3.95 × 10^{-25} kg respectively. Magnetic field applied, B = 0.500 T. Velocity of the electron is 1.00 × 10^{5} m s^{-1}, then

(a) the radius of the path of ${}^{235}_{92}U$ is r_{235}

$$r_{235}=\frac{m_{235}v}{|q|B}=\frac{3.90\times10^{-25}\times1.00\times10^{5}}{1.6\times10^{-19}\times0.500}$$

$$=48.8\times10^{-2}m$$

r_{235} = 48.8 cm

The diameter of the semi-circle due to ${}^{235}_{92}U$ is d_{235} = $2r_{235}$ = 97.6 cm

The radius of the path of ${}^{238}_{92}U$ is r_{238} then

$$r_{238}=\frac{m_{238}v}{|q|B}=\frac{3.90\times10^{-25}\times1.00\times10^{5}}{1.6\times10^{-19}\times0.500}$$

$$=49.4\times10^{-2}m$$

r_{238} = 49.4 cm

The diameter of the semi-circle due to ${}^{238}_{92}U$ is d_{238} = $2r_{238}$ = 98.8 cm

Therefore the separation distance between the isotopes is $\Delta d = d_{238} - d_{235} = 1.2\,\text{cm}$

(b) The time taken by each isotope to complete one semi-circular path are

$$t_{235} = \frac{\text{magnitude of the displacement}}{\text{velocity}}$$

$$= \frac{97.6\times10^{-2}}{1.00\times10^{5}} = 9.76\times10^{-6}\,s = 9.76\ \mu s$$

$$t_{238} = \frac{\text{magnitude of the displacement}}{\text{velocity}}$$

$$= \frac{98.8\times10^{-2}}{1.00\times10^{5}} = 9.88\times10^{-6}\,s = 9.88\ \mu s$$

Note that even though the difference between mass of two isotopes are very small, this arrangement helps us to convert this small difference into an easily measurable distance of separation. This arrangement is known as mass spectrometer. A mass spectrometer is used in many areas in sciences, especially in medicine, in space science, in geology etc. For example, in medicine, anaesthesiologists use it to measure the respiratory gases and biologist use it to determine the reaction mechanisms in photosynthesis.

3.10.3 Motion of a charged particle under crossed electric and magnetic field (velocity selector)

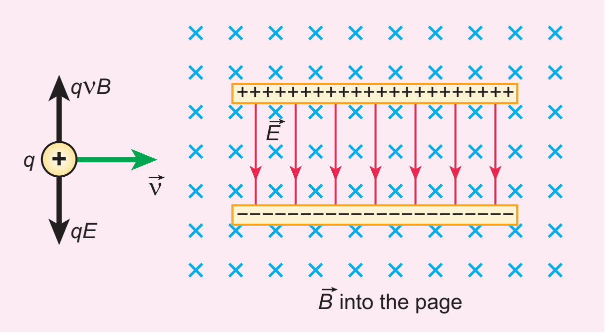


Figure 3.53 **Velocity selector**

Consider an electric charge q of mass m which enters into a region of uniform magnetic field $\vec{B}$ with velocity $\vec{v}$ such that velocity is not perpendicular to the magnetic field. Then the path of the particle is a helix. The Lorentz force on the charged particle moving in a uniform magnetic field can be balanced by Coulomb force by proper arrangement of electric and magnetic fields.

The Coulomb force acts along the direction of electric field (for a positive charge q) whereas the Lorentz force is perpendicular to the direction of magnetic field. Therefore in order to balance these forces, both electric and magnetic fields must be perpendicular to each other. Such an arrangement of perpendicular electric and magnetic fields are known as cross fields.

For illustration, consider an experimental arrangement as shown in Figure 3.53. In the region of space between parallel plates of a capacitor (which produces uniform electric field), uniform magnetic field is maintained perpendicular to the direction of electric field. Suppose a charged particle enters this space from the left side as shown, the net force on the particle is

$$\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

For a positive charge, the electric force on the charge acts in downward direction whereas the Lorentz force acts upwards. When these two forces balance one another, then

$$qE = qv_{\circ}B$$

$$\Rightarrow v_{\circ} = \frac{E}{B} \tag{3.61}$$

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This means, for a given magnitude of $\vec{E}$- field and $\vec{B}$- field, the forces act only for the particle moving with particular speed $v_o = \frac{E}{B}$. This speed is independent of mass and charge.

If the charge enters into the crossed fields with velocity v, other than v_o, it results in any of the following possibilities (Table 3.4).

Table 3.4 Deflection based on the velocity – velocity selector

S.No.	Velocity	Deflection
1	$v > v_o$	Charged particle deflects in the direction of Lorentz force
2	$v < v_o$	Charged particle deflects in the direction of Coulomb force
3	$v = v_o$	No deflection and particle moves straight

So by proper choice of electric and magnetic fields, the particle with particular speed can be selected. Such an arrangement of fields is called a **velocity selector**.

Note This principle is used in Bainbridge mass spectrograph to separate the isotopes.

EXAMPLE 3.25

Let E be the electric field of magnitude 6.0×10^6 N C^{-1} and B be the magnetic field magnitude 0.83 T. Suppose an electron is accelerated with a potential of 200 V, will it show zero deflection?. If not, at what potential will it show zero deflection.

***Solution*:**

Electric field, E = 6.0×10^6 N C^{-1} and magnetic field, B = 0.83 T.

Then

$$v = \frac{E}{B} = \frac{6.0\times10^6}{0.83} = 7.23\times10^6\, ms^{-1}$$

When an electron goes with this velocity, it shows null deflection. Since the accelerating potential is 200 V, the electron acquires kinetic energy because of this accelerating potential. Hence,

$$\frac{1}{2}mv^2 = eV \Rightarrow v = \sqrt{\frac{eV}{2m}}$$

Since the mass of the electron, $m = 9.1\times10^{-31}\,kg$ and charge of an electron, $|q| = e = 1.6\times10^{-19}\,C$. The velocity due to accelerating potential 200 V

$$v_{200} = \sqrt{\frac{2\left(1.6\times10^{-19}\right)(200)}{\left(9.1\times10^{-31}\right)}} = 8.39\times10^6\, m\, s^{-1}$$

Since the speed $v_{200} > v$, the electron is deflected towards direction of Lorentz force. So, in order to have null deflection, the potential, we have to supply is

$$V = \frac{1}{2}\frac{mv^2}{e} = \frac{\left(9.1\times10^{-31}\right)\times\left(7.23\times10^6\right)^2}{2\times\left(1.6\times10^{-19}\right)}$$

$$V = 148.65\ V$$

3.10.4 Cyclotron

Cyclotron (Figure 3.54) is a device used to accelerate the charged particles to gain large kinetic energy. It is also called as high energy accelerator. It was invented by Lawrence and Livingston in 1934.

Principle

When a charged particle moves normal to the magnetic field, it experiences magnetic Lorentz force.

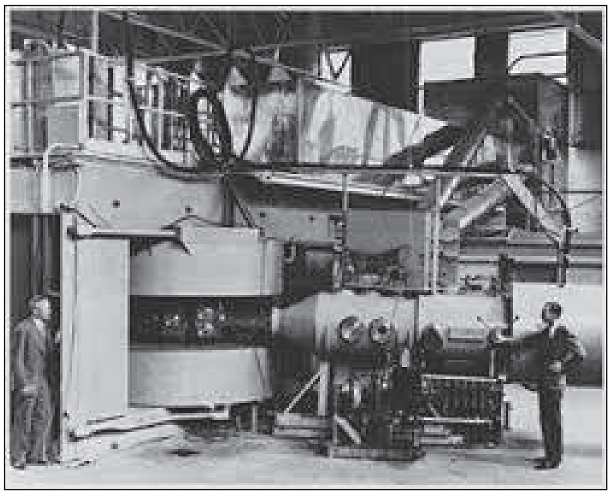


Figure 3.54 **Cyclotron invented by Lawrence and Livingston**

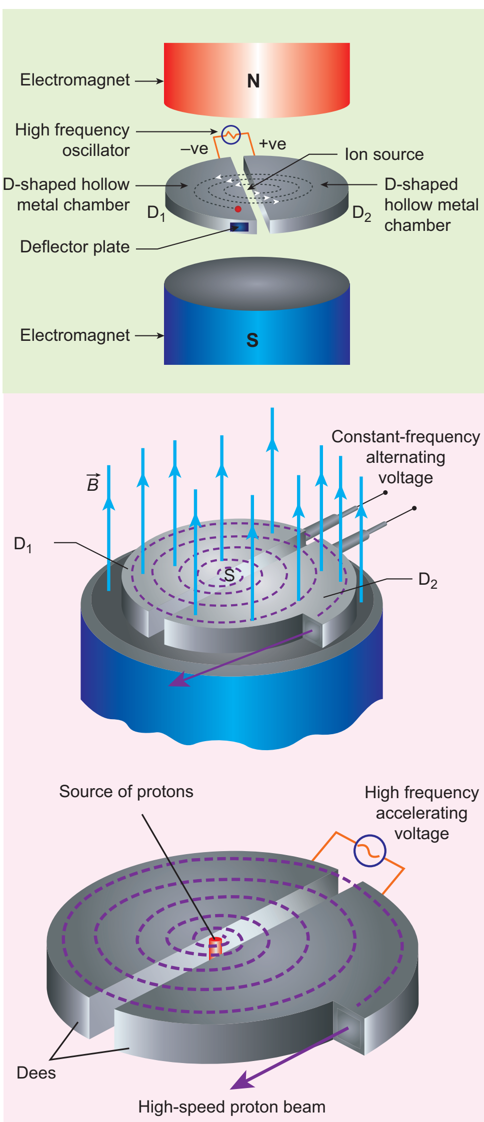


Figure 3.55 construction and working of cyclotron

Construction

The schematic diagram of a cyclotron is shown in Figure 3.55. The particles are allowed to move in between two semi-circular metal containers called Dees (hollow D - shaped objects). Dees are enclosed in an evacuated chamber and it is kept in a region with uniform magnetic field controlled by an electromagnet. The direction of magnetic field is normal to the plane of the Dees. The two Dees are kept separated with a gap and the source S (which ejects the particle to be accelerated) is placed at the center in the gap between the Dees. Dees are connected to high frequency alternating potential difference.

Working

Let us assume that the ion ejected from source S is positively charged. As soon as ion is ejected, it is accelerated towards a Dee (say, Dee – 1) which has negative potential at that time. Since the magnetic field is normal to the plane of the Dees, the ion undergoes circular path. After one semi-circular path in Dee-1, the ion reaches the gap between Dees. At this time, the polarities of the Dees are reversed so that the ion is now accelerated towards Dee-2 with a greater velocity. For this circular motion, the centripetal force of the charged particle q is provided by Lorentz force.

$$\frac{mv^2}{r} = qvB$$

$$\Rightarrow r = \frac{m}{qB}v$$

$$\Rightarrow r \propto v \qquad (3.62)$$

From the equation (3.62), the increase in velocity increases the radius of circular path. This process continues and hence the particle undergoes spiral path of increasing radius. Once it reaches near the edge, it is taken out with the help of deflector plate and allowed to hit the target T.

Very important condition in cyclotron operation is the resonance condition. It happens when the frequency f at which the positive ion circulates in the magnetic field must be equal to the constant frequency of the electrical oscillator f_{osc}

From equation (3.59), we have

$$f_{OSC} = \frac{qB}{2\pi m}$$

The time period of oscillation is

$$T = \frac{2\pi m}{qB}$$

The kinetic energy of the charged particle is

$$KE = \frac{1}{2}mv^2 = \frac{q^2B^2r^2}{2m} \quad (3.63)$$

Limitations of cyclotron

(a) the speed of the ion is limited

(b) electron cannot be accelerated

(c) uncharged particles cannot be accelerated

Note Deutrons (bundles of one proton and one neutron) can be accelerated because it has same charge as that of proton. But neutron (electrically neutral particle) cannot be accelerated by the cyclotron. When a deutron is bombarded with a beryllium target, a beam of high energy neutrons are produced. These high-energy neutrons are sent into the patient's cancerous region to break the bonds in the DNA of the cancer cells (killing the cells). This is used in treatment of fast-neutron cancer therapy.

EXAMPLE 3.26

Suppose a cyclotron is operated to accelerate protons with a magnetic field of strength 1 T. Calculate the frequency in which the electric field between two Dees could be reversed.

Solution

Magnetic field B = 1 T

Mass of the proton, $m_p = 1.67 \times 10^{-27}\, kg$

Charge of the proton, $q = 1.60 \times 10^{-19}\, C$

$$f = \frac{qB}{2\pi m_p} = \frac{(1.60 \times 10^{-19})(1)}{2(3.14)(1.67 \times 10^{-27})}$$

$$= 15.3 \times 10^6\, Hz = 15.3\, MHz$$

3.10.5 Force on a current carrying conductor placed in a magnetic field

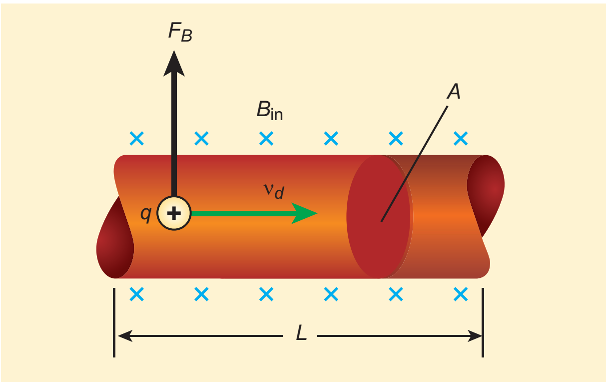


Figure 3.56 Current carrying conductor in a magnetic field

When a current carrying conductor is placed in a magnetic field, the force experienced by the wire is equal to the sum of Lorentz forces on the individual charge carriers in the wire. Consider a small segment of wire of length dl, with cross-sectional area A and current I as shown in Figure 3.56. The free electrons drift opposite to the direction of current. So the relation between current I and magnitude of drift velocity v_d (Refer Unit 2) is

$$I = neAv_d \quad (3.64)$$

If the wire is kept in a magnetic field $\vec{B}$, then average force experienced by the charge (here, electron) in the wire is

$$\vec{F} = -e\left(\vec{v}_d \times \vec{B}\right)$$

Let n be the number of free electrons per unit volume, therefore

$$n = \frac{N}{V}$$

where N is the number of free electrons in the small element of volume V = A *dl*.

Hence Lorentz force on the wire of length d*l* is the product of the number of the electrons

(N = nA d*l*) and the force acting on an electron.

$$d\vec{F} = -enAdl\left(\vec{v}_d \times \vec{B}\right)$$

The length d*l* is along the length of the wire and hence the current element in the wire is $I\,d\vec{l} = -enA\vec{v}_d dl$ Therefore the force on the wire is

$$d\vec{F} = \left(I\,d\vec{l} \times \vec{B}\right) \qquad (3.65)$$

The force in a straight current carrying conducting wire of length l placed in a uniform magnetic field is

$$\vec{F} = \left(I\vec{l} \times \vec{B}\right) \qquad (3.66)$$

In magnitude,

$$F = BIl\sin\theta$$

(a) If the conductor is placed along the direction of the magnetic field, the angle between them is θ = 0°. Hence, the force experienced by the conductor is zero.

(b) If the conductor is placed perpendicular to the magnetic field, the angle between them is θ =90° Hence, the force experienced by the conductor is maximum, which is $F = BIl$.

Fleming's left hand rule (mnemonic)

When a current carrying conductor is placed in a magnetic field, the direction of the force experienced by it is given by Fleming's Left Hand Rule (FLHR) as shown in Figure 3.57.

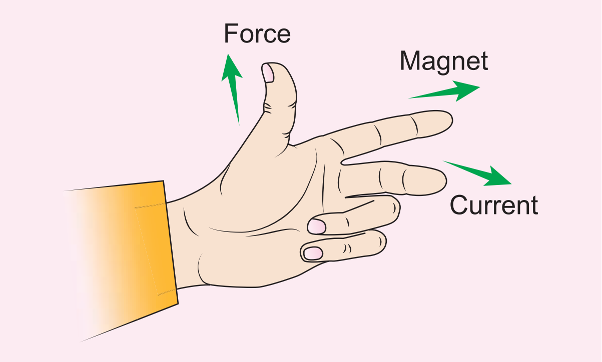
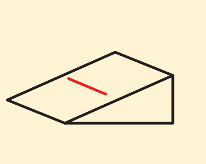


Figure 3.57 Fleming's Left Hand Rule (FLHR)

Stretch forefinger, the middle finger and the thumb of the left hand such that they are in mutually perpendicular directions. If forefinger points the direction of magnetic field, the middle finger points the direction of the electric current, then thumb will point the direction of the force experienced by the conductor.

EXAMPLE 3.27

A metallic rod of linear density is 0.25 kg m^{-1} is lying horizontally on a smooth inclined plane which makes an angle of 45° with the horizontal. The rod is not allowed to slide down by flowing a current through it when a magnetic field of strength 0.25 T is acting on it in the vertical direction. Calculate the electric current flowing in the rod to keep it stationary.

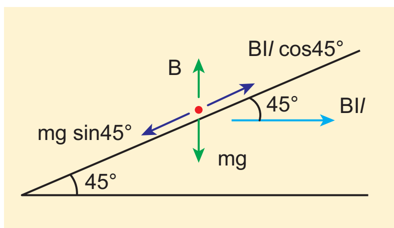


Solution

The linear density of the rod i.e., mass per unit length of the rod is 0.25 kg m^{-1}

$$\Rightarrow \frac{m}{l} = 0.25 \ \text{kg m}^{-1}$$

Let I be the current flowing in the metallic rod. The direction of electric current is into the paper. The direction of magnetic force IBl is given by Fleming's left hand rule.



For equilibrium,

$$mg\sin 45^{\circ} = IBl\cos 45^{\circ}$$

$$\Rightarrow I = \frac{1}{B}\frac{m}{l}g\tan 45^{\circ}$$

$$= \frac{0.25\,\text{kg m}^{-1}}{0.25\,T} \times 1 \times 9.8\ m\,s^{-2}$$

$$\Rightarrow I = 9.8\,A$$

So, we need to supply current of 9.8 A to keep the metallic rod stationary.

3.10.6 Force between two long parallel current carrying conductors

Two long straight parallel current carrying conductors separated by a distance r are kept in air as shown in Figure 3.58. Let I_1 and I_2 be the electric currents passing through the conductors A and B in same direction (i.e. along z - direction) respectively. The net magnetic field at a distance r due to current I_1 in conductor A is

$$\vec{B}_1 = \frac{\mu_\circ I_1}{2\pi r}(-\hat{i}) = -\frac{\mu_\circ I_1}{2\pi r}\hat{i}$$

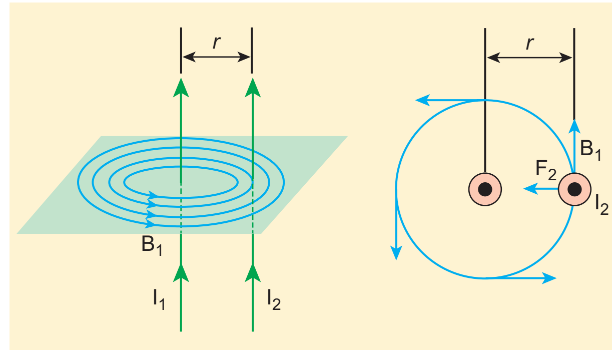


Figure 3.58 Two long straight parallel wires

From thumb rule, the direction of magnetic field is perpendicular to the plane of the paper and inwards (arrow into the page ⊗) i.e. along negative $\hat{i}$ direction.

Let us consider a small elemental length dl in conductor B at which the magnetic field $\vec{B}_1$ is present. From equation 3.65, Lorentz force on the element dl of conductor B is

$$d\vec{F} = \left(I_2 d\vec{l} \times \vec{B}_1\right) = -I_2 dl \frac{\mu_\circ I_1}{2\pi r}(\hat{k} \times \hat{i})$$

$$= -\frac{\mu_\circ I_1 I_2 dl}{2\pi r}\hat{j}$$

Therefore the force on dl of the wire B is directed towards the wire W_1. So the length dl is attracted towards the conductor A. The force per unit length of the conductor B due to the wire conductor A is

$$\frac{\vec{F}}{l} = -\frac{\mu_\circ I_1 I_2}{2\pi r}\hat{j}$$

In the same manner, we compute the magnitude of net magnetic induction due to current I_2 (in conductor A) at a distance r in the elemental length dl of conductor A is

$$\vec{B}_2 = \frac{\mu_\circ I_2}{2\pi r}\hat{i}$$

From the thumb rule, direction of magnetic field is perpendicular to the plane of the paper and outwards (arrow out of the page ⊙) i.e., along positive $\hat{i}$ direction.

Hence, the magnetic force at element dl of the wire is W_1 is

$$\vec{F} = \left(I_1 d\vec{l} \times \vec{B}_2\right) = I_1 dl \frac{\mu_\circ I_2}{2\pi r}(\hat{k} \times \hat{i})$$
$$= \frac{\mu_\circ I_1 I_2 dl}{2\pi r}\hat{j} \qquad (3.67)$$

Therefore the force on dl of conductor A is directed towards the conductor B. So the length dl is attracted towards the conductor B as shown in Figure (3.59).

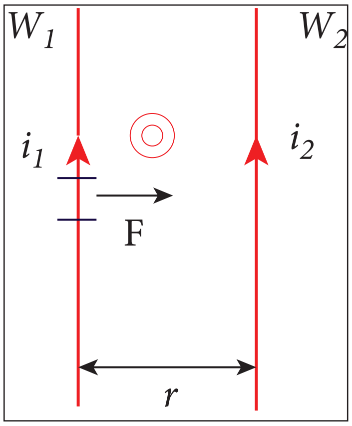


Figure 3.59 Current in both the wire are in the same direction - attracts each other

The force per unit length of the conductor A due to the conductor B is

$$\frac{\vec{F}}{l} = -\frac{\mu_\circ I_1 I_2}{2\pi r}\hat{j}$$

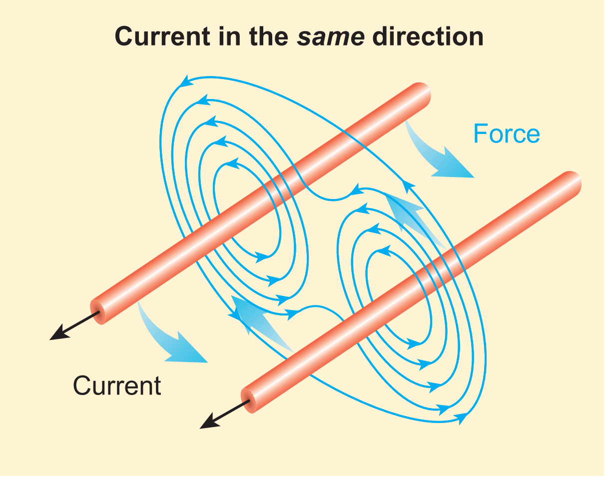


Figure 3.60 Two parallel conductors carrying current in same direction experience an attractive force

Thus the force experienced by two parallel current carrying conductors is attractive if the direction of electric current passing through them is same as shown in Figure 3.60.

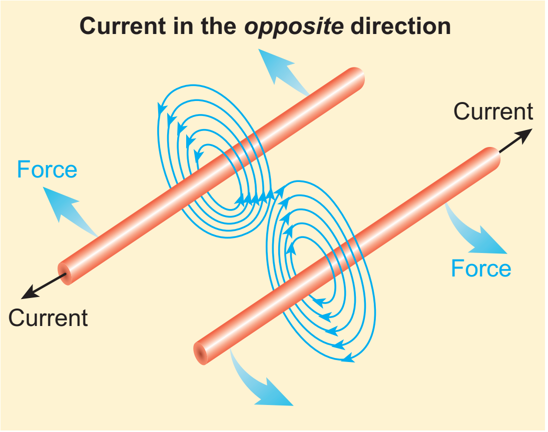


Figure 3.61 Two parallel conductors carrying current in opposite direction experience a repulsive force

Thus the force experienced by two parallel current carrying conductors is repulsive if they carry current in the opposite directions as shown in Figure 3.61.

Definition of ampère

One ampère is defined as that current when it is passed through each of the two infinitely long parallel straight conductors kept at a distance of one meter apart in vacuum causes each conductor to experience a force of 2×10^{-7} newton per meter length of conductor.

3.11 TORQUE ON A CURRENT LOOP

The force on a current carrying wire in a magnetic field is responsible for the motor operation.

3.11.1 Expression for torque on a current loop placed in a magnetic field

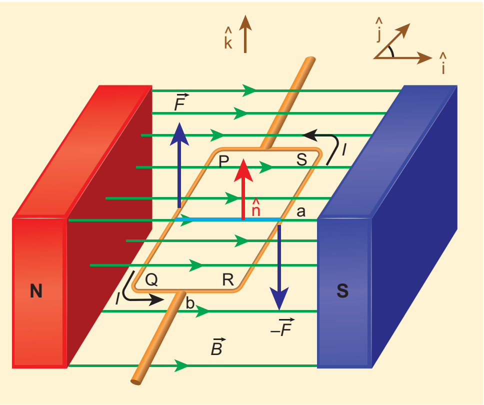


Figure 3.62 Rectangular coil placed in a magnetic field

Consider a single rectangular loop PQRS kept in a uniform magnetic field $\vec{B}$. Let a and b be the length and breadth of the rectangular loop respectively. Let $\hat{n}$ be the unit vector normal to the plane of the current loop. This unit vector $\hat{n}$ completely describes the orientation of the loop. Let $\vec{B}$ be directed from north pole to south pole of the magnet as shown in Figure 3.62.

When an electric current is sent through the loop, the net force acting is zero but there will be net torque acting on it. For the sake of understanding, we shall consider two configurations of the loop; (i) unit vector $\hat{n}$ pointing perpendicular to the field (ii) unit vector pointing at an angle θ with the field.

(i) when unit vector $\hat{n}$ is perpendicular to the field

In the simple configuration, the unit vector $\hat{n}$ is perpendicular to the field and plane of the loop is lying on xy plane as shown in Figure 3.62. Let the loop be divided into four sections PQ, QR, RS and SP. The Lorentz force on each loop can be calculated as follows:

(a) Force on section PQ

For section PQ, $\vec{l} = -a\,\hat{j}$ and $\vec{B} = B\,\hat{i}$

$$\vec{F}_{PQ} = I\vec{l} \times \vec{B}$$
$$= -I\,a\,B(\hat{j} \times \hat{i}) = I\,a\,B\,\hat{k}$$

Since the unit vector normal to the plane $\hat{n}$ is along the direction of $\hat{k}$.

(b) The force on section QR

$\vec{l} = b\vec{i}$ and $\vec{B} = B\,\hat{i}$

$$\vec{F}_{QR} = I\vec{l} \times \vec{B} = I\,b\,B(\hat{i} \times \hat{i}) = \vec{0}$$

(c) The force on section RS

$\vec{l} = a\,\hat{j}$ and $\vec{B} = B\,\hat{i}$

$$\vec{F}_{RS} = I\vec{l} \times \vec{B} = I\,a\,B(\hat{j} \times \hat{i}) = -IaB\,\hat{k}$$

Since, the unit vector normal to the plane is along the direction of $-\hat{k}$.

(d) The force on section SP

$\vec{l} = -b\,\hat{i}$ and $\vec{B} = B\,\hat{i}$

$$\vec{F}_{SP} = I\vec{l} \times \vec{B} = -I\,b\,B(\hat{i} \times \hat{i}) = \vec{0}$$

The net force on the rectangular loop is

$$\vec{F}_{net} = \vec{F}_{PQ} + \vec{F}_{QR} + \vec{F}_{RS} + \vec{F}_{SP}$$
$$\vec{F}_{net} = IaB\,\hat{k} + \vec{0} - IaB\,\hat{k} + \vec{0} \Rightarrow \vec{F}_{net} = \vec{0}$$

Hence, the net force on the rectangular loop in this configuration is zero. Now let us calculate the net torque due to these forces about an axis passing through the center

$$\vec{\tau}_{net} = \sum_{i=1}^{4} \vec{\tau}_i = \sum_{i=1}^{4} \vec{r}_i \times \vec{F}_i$$
$$= \left(\frac{b}{2} IaB + 0 + \frac{b}{2} IaB + 0\right)\hat{j}$$
$$\vec{\tau}_{net} = abIB\,\hat{j}$$

Since, A = ab is the area of the rectangular loop PQRS, the net torque for this configuration is

$$\vec{\tau}_{net} = ABI\,\hat{j}$$

When the loop starts rotating due to this torque, the magnetic field $\vec{B}$ is no longer in the plane of the loop. So the above equation is the special case.

When the loop starts rotating about z axis due to this torque, the magnetic field $\vec{B}$ is no longer in the plane of the loop. So the above equation is the special case.

(ii) when unit vector $\hat{n}$ is at an angle θ with the field

In the general case, the unit normal vector $\hat{n}$ and magnetic field $\vec{B}$ is with an angle θ as shown in Figure 3.63.

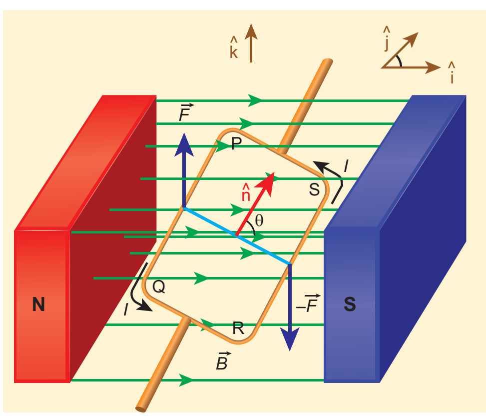


Figure 3.63 Unit vector makes an angle **θ** with the field

(a) The force on section PQ

$$\vec{l} = -a\hat{j} \;\; and\, \vec{B} = B\hat{i}$$

$$\vec{F}_{PQ} = I\vec{l} \times \vec{B} = -IaB(\hat{j} \times \hat{i}) = IaB\,\hat{k}$$

Since, the unit vector normal to the plane $\hat{n}$ is along the direction of $\hat{k}$.

(b) The force on section QR

$$\vec{l} = b\cos\left(\frac{\pi}{2}-\theta\right)\hat{i} - \sin\left(\frac{\pi}{2}-\theta\right)\hat{k}\, and\, \vec{B} = B\,\hat{i}$$

$$\vec{F}_{QR} = I\vec{l} \times \vec{B} = -IbB\sin\left(\frac{\pi}{2}-\theta\right)\hat{j}$$

$$\vec{F}_{QR} = -IbB\cos\theta\,\hat{j}$$

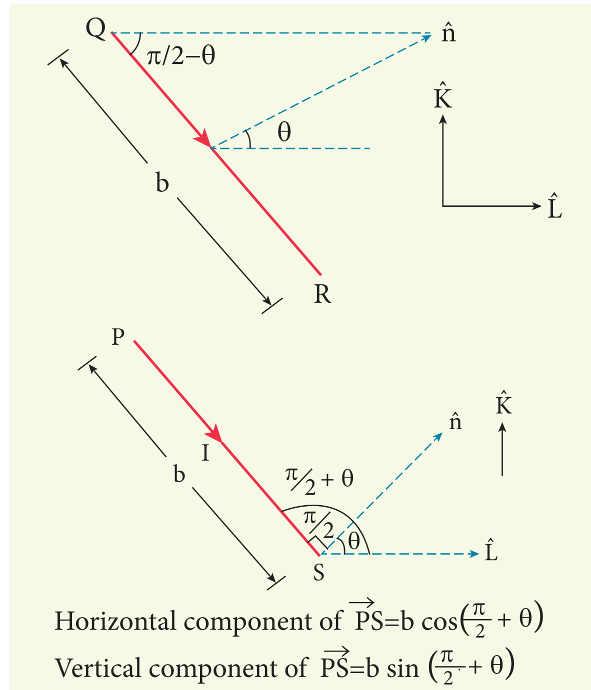


Figure 3.64 Horizontal and vertical component of the sections - (a) QR (b) SP

(c) The force on section RS

$$\vec{l} = a\hat{j}\, and\, \vec{B} = B\hat{i}$$

$$\vec{F}_{RS} = I\vec{l} \times \vec{B} = IaB(\hat{j} \times \hat{i}) = -\mathrm{IaB}\,\hat{k}$$

Since, the unit vector normal to the plane is along the direction of $-\hat{k}$.

(d) The force on section SP

$$\vec{l} = b\cos\left(\frac{\pi}{2}+\theta\right)\hat{i} + \sin\left(\frac{\pi}{2}+\theta\right)\hat{k} \;\; and \;\; \vec{B} = B\,\hat{i}$$

$$\vec{F}_{SP} = I\vec{l} \times \vec{B} = IbB\sin\left(\frac{\pi}{2}+\theta\right)\hat{j}$$

$$\vec{F}_{SP} = IbB\cos\theta\,\hat{j}$$

The net force on the rectangular loop is

$$\vec{F}_{net} = \vec{F}_{PQ} + \vec{F}_{QR} + \vec{F}_{RS} + \vec{F}_{SP}$$

$$\vec{F}_{net} = IaB\hat{k} - IbB\cos\theta\,\hat{j} - IaB\hat{k} + IbB\cos\theta\,\hat{j}$$

$$\Rightarrow \vec{F}_{net} = \vec{0}$$

Hence, the net force on the rectangular loop in this configuration is also zero. Notice that the force on section QR and SP are not zero here. But, they have equal and opposite effects, but we assume the loop to be rigid, so no deformation. Hence, no torque produced by these two sections.

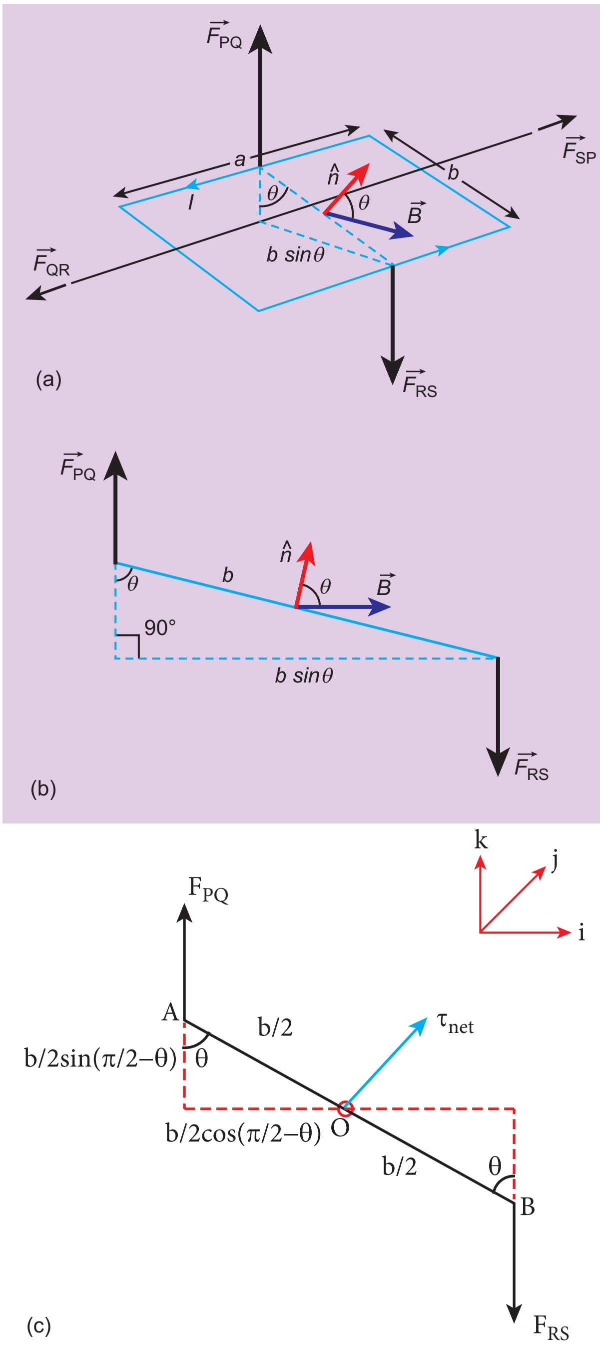


Figure 3.65 Force on the rectangular loop – (a) top view and (b) side view (c) net torque on the loop

Even though the forces PQ and RS also are also equal and opposite, they are not collinear. So these two forces constitute a couple as shown in Figure 3.65 (a). Hence the net torque produced by these two forces about the axis of the rectangular loop is given by

$$\vec{\tau}_{net} = baBI\sin\theta\,\hat{k} = ABI\sin\theta\,\hat{k}$$

From the Figure 3.65 (c),

$$\overrightarrow{OA} = \frac{b}{2}\cos\left(\frac{\pi}{2}-\theta\right)(-\hat{i}) + \frac{b}{2}\sin\left(\frac{\pi}{2}-\theta\right)(-\hat{k})$$

$$= \frac{b}{2}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{k})$$

$$\overrightarrow{OB} = \frac{b}{2}\cos\left(\frac{\pi}{2}-\theta\right)(\hat{i}) + \frac{b}{2}\sin\left(\frac{\pi}{2}-\theta\right)(-\hat{k})$$

$$= \frac{b}{2}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{k})$$

$$\overrightarrow{OA} \times \vec{F}_{PQ} = \left\{\frac{b}{2}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{k})\right\} \times \{IaB\,\hat{k}\}$$

$$= \frac{1}{2}IabB\sin\theta\,\hat{j}$$

$$\overrightarrow{OB} \times \vec{F}_{RS} = \left\{\frac{b}{2}(\sin\theta\,\hat{i} + \cos\theta\,\hat{k})\right\} \times \{-IaB\,\hat{k}\}$$

$$= \frac{1}{2}IabB\sin\theta\,\hat{j}$$

The net torque $\vec{\tau}_{net} = IabB\sin\theta\,\hat{j}$

(3.68)

Note that the net torque is in the positive y direction which tends to rotate the loop in clockwise direction about the y axis. If the current is passed in the other way (P→S→R→Q→P), then total torque will point in the negative y direction which tends to rotate the loop in anticlockwise direction about y axis.

Another important point is to note that the torque is less in this case compared to earlier case (where the $\hat{n}$ is perpendicular to the magnetic field $\vec{B}$). It is because the perpendicular distance is reduced between the forces $\vec{F}_{PQ}$ and $\vec{F}_{RS}$ in this case.

The equation (3.68) can also be rewritten in terms of magnetic dipole moment $\vec{p}_m = I\vec{A} = Iab\,\hat{n}$

$$\vec{\tau}_{net} = \vec{p} \times \vec{B}$$

This is analogous expression for torque experienced by electric dipole in the uniform electric field

$\vec{\tau}_{net} = \vec{p} \times \vec{E}$ which is given in the Unit 1. (Section 1.4.3)

Cases:

(a) When $\theta = 90^\circ$, then the torque on the current loop is maximum which is

$$\vec{\tau}_{net} = abIB\,\hat{j}$$

Note here $\vec{p}_m$ points perpendicular to the magnetic field $\vec{B}$. The torque is maximum in this orientation.

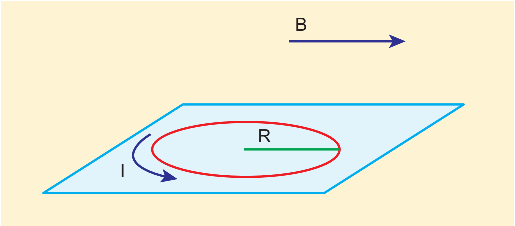
(b) When $\theta = 0^\circ$ or 180° then the torque on the current loop is

$$\vec{\tau}_{net} = 0$$

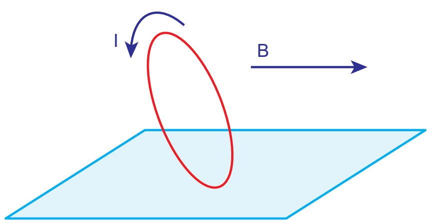
when $\theta = 0^\circ$, $\vec{p}_m$ is parallel to $\vec{B}$ and for $\theta = 180^\circ$, $\vec{p}_m$, is anti - parallel to $\vec{B}$. The torque is zero in these orientations.

EXAMPLE 3.28

Consider a circular wire loop of radius R, mass m kept at rest on a rough surface. Let I be the current flowing through the loop and $\vec{B}$ be the magnetic field acting along horizontal as shown in Figure. Estimate the current I that should be applied so that one edge of the loop is lifted off the surface?



Solution



When the current is passed through the loop, the torque is produced. If the torque acting on the loop is increased then the loop will start to rotate. The loop will start to lift if and only if the magnitude of magnetic torque due to current applied equals to the gravitational torque as shown in Figure

$$\tau_{magnetic} = \tau_{gravitational}$$

$$IAB = mgR$$

But $p_m = IA = I\left(\pi R^2\right)$

$$\pi IR^2 B = mgR$$

$$\Rightarrow I = \frac{mg}{\pi RB}$$

The current estimated using this equation should be applied so that one edge of loop is lifted of the surface.

3.11.2 Moving coil galvanometer

Moving coil galvanometer is a device which is used to indicate the flow of current in an electrical circuit.

Principle When a current carrying loop is placed in a uniform magnetic field it experiences a torque.

Construction

A moving coil galvanometer consists of a rectangular coil PQRS of insulated thin copper wire. The coil contains a large number of turns wound over a light metallic frame. A cylindrical soft-iron core is placed symmetrically inside the coil as shown in Figure 3.66. The rectangular coil is suspended freely between two pole pieces of a horse-shoe magnet.

The upper end of the rectangular coil is attached to one end of fine strip of

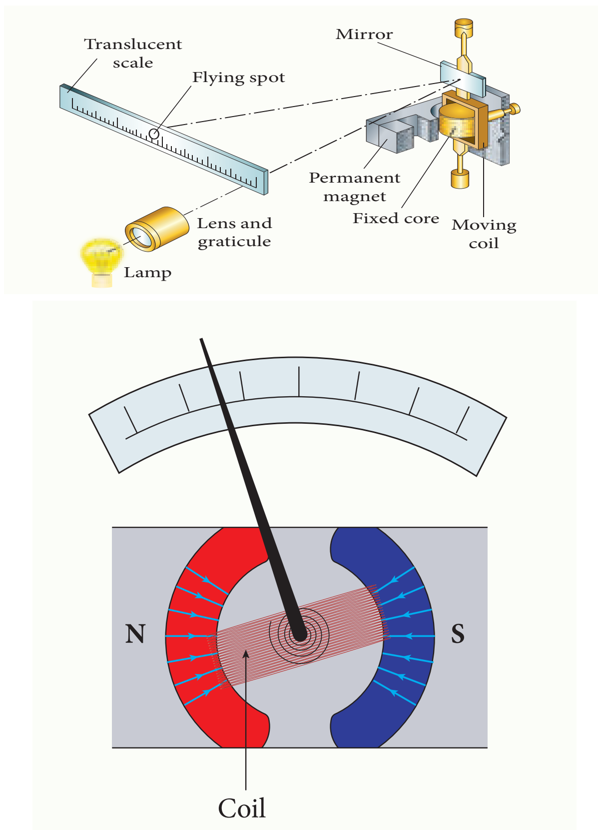


Figure 3.66 Moving coil galvanometer – its parts

phosphor bronze and the lower end of the coil is connected to a hair spring which is also made up of phosphor bronze. In a fine suspension strip W, a small plane mirror is attached in order to measure the deflection of the coil with the help of lamp and scale arrangement. The other end of the mirror is connected to a torsion head T. In order to pass electric current through the galvanometer, the suspension strip W and the spring S are connected to terminals.

Working

Consider a single turn of the rectangular coil PQRS whose length be l and breadth b. PQ = RS = l and QR = SP = b. Let I be the electric current flowing through the rectangular coil PQRS as shown in Figure 3.67. The horse-shoe magnet has hemi - spherical magnetic poles which produces a

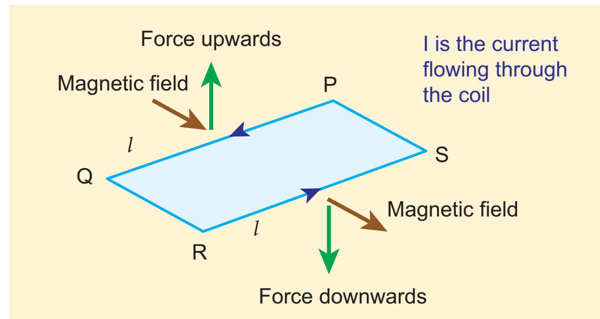


Figure 3.67 Force acting on current carrying coil

radial magnetic field. Due to this radial field, the sides QR and SP are always parallel to to the B-field (magnetic field) and experience no force. The sides PQ and RS are always parallel to the B-field and experience force and due to this, torque is produced.

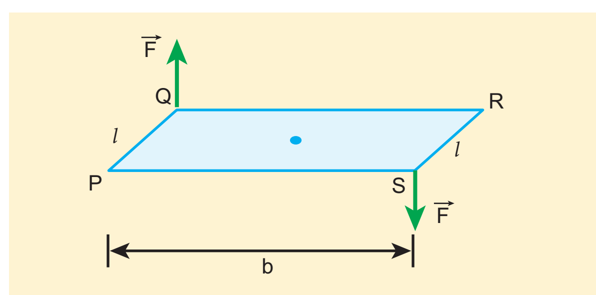


Figure 3.68 Deflection couple

For single turn, the deflection couple as shown in Figure 3.68 is

$$\tau = bF = bBIl = (lb)\,BI = ABI$$

since, area of the coil $A = lb$

For coil with N turns, we get

$$\tau = NABI \qquad (3.69)$$

Due to this deflecting torque, the coil gets twisted and restoring torque (also known as restoring couple) is developed. Hence the magnitude of restoring couple is proportional to the amount of twist θ (Refer Unit 10 of Std. XI Physics).Thus

$$\tau = K\theta \qquad (3.70)$$

where K is the restoring couple per unit twist or torsional constant of the spring.

At equilibrium, the deflection couple is equal to the restoring couple. Therefore by comparing equation (3.69) and (3.70), we get

$$NABI = K\,\theta$$

$$\Rightarrow I = \frac{K}{NAB}\theta \qquad (3.71)$$

(or) $I = G\,\theta$

where, $G = \dfrac{K}{NAB}$ is called galvanometer constant or current reduction factor of the galvanometer.

Since, suspended moving coil galvanometer is very sensitive, we have to handle with high care while doing experiments. Most of the galvanometer we use are pointer type moving coil galvanometer.

Figure of merit of a galvanometer

It is defined as the current which produces a deflection of one scale division in the galvanometer.

Sensitivity of a galvanometer

The galvanometer is said to be sensitive if it shows large scale deflection even though a small current is passed through it or a small voltage is applied across it.

Current sensitivity: ***It is defined as the deflection produced per unit current flowing through it.***

$$I_S = \frac{\theta}{I} = \frac{NAB}{K} \Rightarrow I_S = \frac{1}{G} \qquad (3.72)$$

The current sensitivity of a galvanometer can be increased

(a) by increasing

(1) the number of turns N

(2) the magnetic induction B

(3) the area of the coil A

(b) by decreasing

the couple per unit twist of the suspension wire k. Phosphor - bronze wire is used as the suspension wire because the couple per unit twist is very small.

Voltage sensitivity: ***It is defined as the deflection produced per unit voltage applied across it.***

$$V_S = \frac{\theta}{V}$$

$$V_S = \frac{\theta}{IR_g} = \frac{NAB}{K R_g} \Rightarrow V_S = \frac{1}{GR_g} = \frac{I_S}{R_g} \qquad (3.73)$$

where R_g is the resistance of galvanometer.

EXAMPLE 3.29

The coil of a moving coil galvanometer has 5 turns and each turn has an effective area of 2×10^{-2} m^2. It is suspended in a magnetic field whose strength is 4×10^{-2} Wb m^{-2}. If the torsional constant K of the suspension fibre is 4×10^{-9} N m deg^{-1}.

(a) Find its current sensitivity in degree per micro - ampere.

(b) Calculate the voltage sensitivity of the galvanometer for it to have full scale deflection of 50 divisions for 25 mV.

(c) Compute the resistance of the galvanometer.

Solution

The number of turns of the coil is 5 turns

The area of each coil is 2×10^{-2} m^2

Strength of the magnetic field is 4×10^{-2} Wb m^{-2}

Torsional constant is 4×10^{-9} N m deg^{-1}

(a) Current sensitivity

$$I_S = \frac{NAB}{K} = \frac{5\times2\times10^{-2}\times4\times10^{-2}}{4\times10^{-9}}$$

$= 10^6$ divisions per ampere

$$1\mu A = 1 micro\,ampere = 10^{-6} ampere$$

Therefore,

$$I_S = 10^6 \frac{div}{A} = 1\frac{div}{10^{-6}A} = 1\frac{div}{\mu A}$$

$$I_S = 1\,div(\mu A)^{-1}$$

(b) Voltage sensitivity

$$V_S = \frac{\theta}{V} = \frac{50\,div}{25\,mV} = 2\times10^3\,div\,V^{-1}$$

(c) The resistance of the galvanometer is

$$R_g = \frac{I_S}{V_S} = \frac{10^6\frac{div}{A}}{2\times10^3\frac{div}{V}} = 0.5\times10^3\frac{V}{A} = 0.5k\Omega$$

EXAMPLE 3.30

The resistance of a moving coil galvanometer is made twice its original value in order to increase current sensitivity by 50%. Will the voltage sensitivity change? If so, by how much?.

Solution

Yes, voltage sensitivity will change.

Voltage sensitivity is $V_S = \frac{I_S}{R}$

When the resistance is doubled, then new resistance is $R' = 2R$

Increase in current sensitivity is

$$I_S' = \left(1+\frac{50}{100}\right)I_S = \frac{3}{2}I_S$$

The new voltage sensitivity is

$$V_S' = \frac{\frac{3}{2}I_S}{2R} = \frac{3}{4}V_S$$

Hence the voltage sensitivity decreases. The percentage decrease in voltage sensitivity is

$$\frac{V_S - V_S'}{V_S}\times100\% = 25\%$$

Conversion of galvanometer into ammeter and voltmeter

A galvanometer is very sensitive instrument to detect the current. It can be easily converted into ammeter and voltmeter.

Galvanometer to an Ammeter

Ammeter is an instrument used to measure current flowing in the electrical circuit. The ammeter must offer low resistance such that it will not change the current passing through it. So ammeter is connected in series to measure the circuit current.

A galvanometer is converted into an ammeter by connecting a low resistance in parallel with the galvanometer. This low resistance is called shunt resistance S. The scale is now calibrated in ampere and the range of ammeter depends on the values of the shunt resistance.

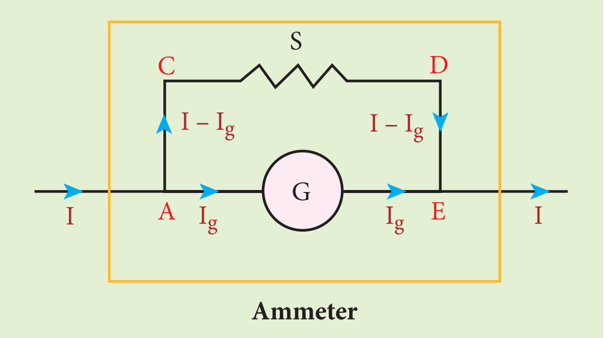


Figure 3.69 Shunt resistance connected in parallel

Let I be the current passing through the circuit as shown in Figure 3.69. When current I reaches the junction A, it divides into two components. Let I_g be the current passing through the galvanometer of resistance R_g through a path AGE and the remaining current $(I - I_g)$ passes along the path ACDE through shunt resistance S. The value of shunt resistance is so adjusted that current I_g produces full scale deflection in the galvanometer. The potential difference across galvanometer is same as the potential difference across shunt resistance.

$$V_{galvanometer} = V_{shunt}$$

$$\Rightarrow I_g R_g = \left(I - I_g\right) S$$

$$S = \frac{I_g}{\left(I - I_g\right)} R_g \text{ Or}$$

$$I_g = \frac{S}{S + R_g} I \Rightarrow I_g \propto I$$

Since, the deflection in the galvanometer is proportional to the current passing through it.

$$\theta = \frac{1}{G} I_g \Rightarrow \theta \propto I_g \Rightarrow \theta \propto I$$

So, the deflection in the galvanometer measures the current I passing through the circuit (ammeter).

Shunt resistance is connected in parallel to galvanometer. Therefore, resistance of ammeter can be determined by computing the effective resistance, which is

$$\frac{1}{R_{eff}} = \frac{1}{R_g} + \frac{1}{S} \Rightarrow R_{eff} = \frac{R_g S}{R_g + S} = R_a$$

Since, the shunt resistance is a very low resistance and the ratio $\frac{S}{R_g}$ is also small. This means, R_g is also small, i.e., the resistance offered by the ammeter is small. So, when we connect ammeter in series, the ammeter will not change the resistance appreciably and also the current in the circuit. For an ideal ammeter, the resistance must be equal to zero. Hence, the reading in ammeter is always lesser than the actual current in the circuit. Let I_{ideal} be current measured from ideal ammeter and I_{actual} be the actual current measured in the circuit by the ammeter. Then, the percentage error in measuring a current through an ammeter is

$$\frac{\Delta I}{I} \times 100\% = \frac{I_{ideal} - I_{actual}}{I_{actual}} \times 100\%$$

Key points

1. An ammeter is a low resistance instrument and it is always connected in series to the circuit
2. An ideal ammeter has zero resistance
3. In order to increase the range of an ammeter n times, the value of shunt resistance to be connected in parallel is $S = \frac{G}{n-1}$

Galvanometer to a voltmeter

A voltmeter is an instrument used to measure potential difference across any two points in the electrical circuits. It should not draw any current from the circuit otherwise the value of potential difference to be measured will change.

Voltmeter must have high resistance and when it is connected in parallel, it will not draw appreciable current so that it will indicate the true potential difference.

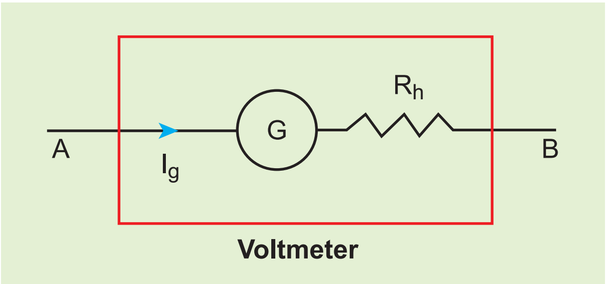


Figure 3.70 Shunt resistance connected in series

A galvanometer is converted into a voltmeter by connecting high resistance R_h in series with galvanometer as shown in Figure 3.74. The scale is now calibrated in volt and the range of voltmeter depends on the values of the resistance connected in series i.e. the value of resistance is so adjusted that only current I_g produces full scale deflection in the galvanometer.

Let R_g be the resistance of galvanometer and I_g be the current with which the galvanometer produces full scale deflection. Since the galvanometer is connected in series with high resistance, the current in the electrical circuit is same as the current passing through the galvanometer.

$$I = I_g$$

$$I = I_g \Rightarrow I_g = \frac{\text{potential difference}}{\text{total resistance}}$$

Since the galvanometer and high resistance are connected in series, the total resistance or effective resistance gives the resistance of voltmeter. The voltmeter resistance is

$$R_v = R_g + R_h$$

Therefore,

$$I_g = \frac{V}{R_g + R_h}$$

$$\Rightarrow R_h = \frac{V}{I_g} - R_g$$

Note that $I_g \propto V$

The deflection in the galvanometer is proportional to current I_g. But current I_g is proportional to the potential difference. Hence the deflection in the galvanometer is proportional to potential difference. Since the resistance of voltmeter is very large, a voltmeter connected in an electrical circuit will draw least current in the circuit. An ideal voltmeter is one which has infinite resistance.

Key points

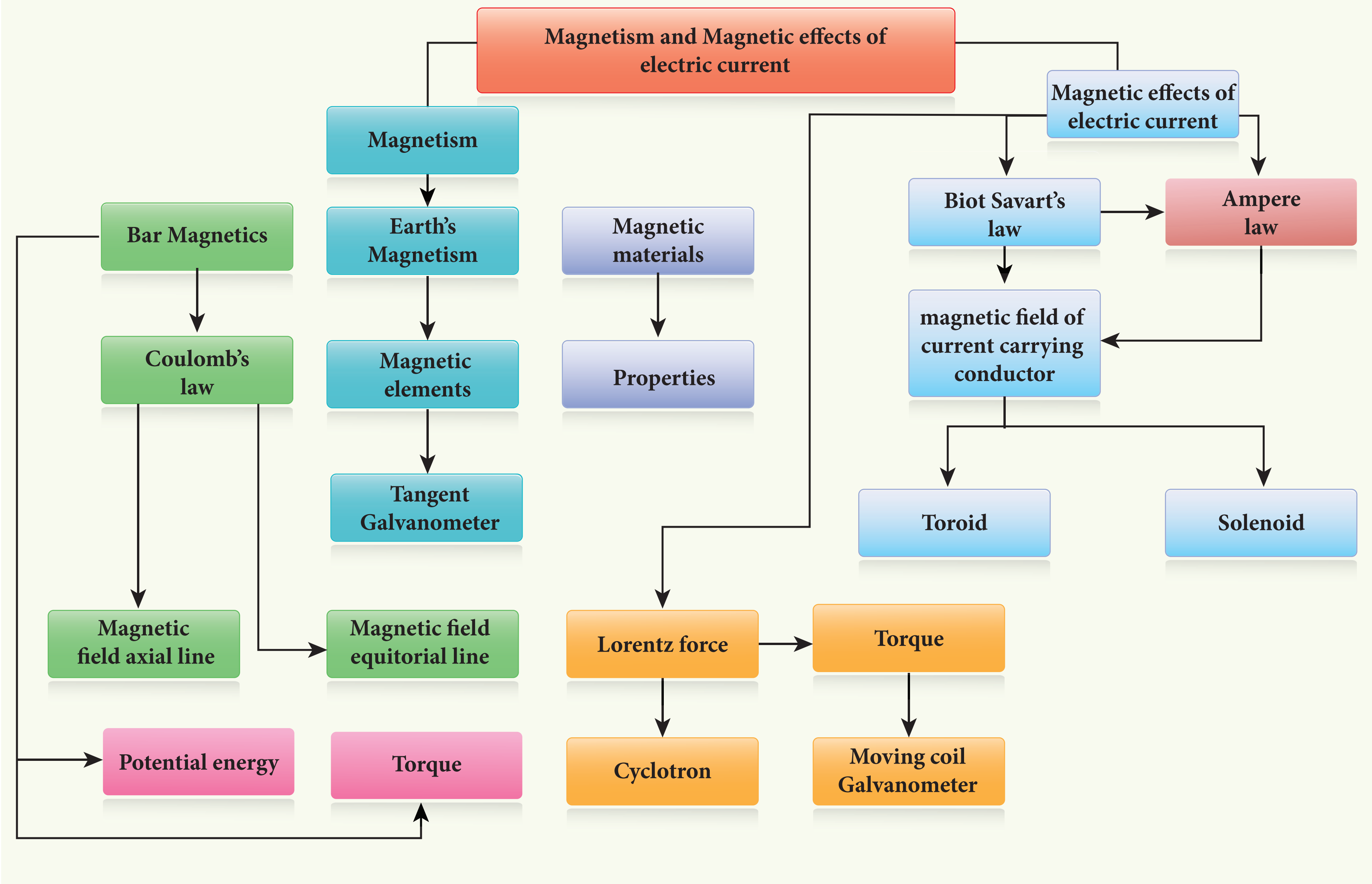
1. Voltmeter is a high resistance instrument and it is always connected in parallel with the circuit element across which the potential difference is to be calculated
2. An ideal voltmeter has infinite resistance
3. In order to increase the range of voltmeter n times the value of resistance to be connected in series with galvanometer is R = (n-1) G

SUMMARY:

- A vertical plane passing through geographic axis is called geographic meridian.
- A vertical plane passing through magnetic axis is called magnetic meridian.
- The angle between magnetic meridian at a point with the geographical meridian is called the declination or magnetic declination.
- The angle subtended by the Earth's total magnetic field $\vec{B}$ with the horizontal direction in the magnetic meridian is called dip or magnetic inclination at that point.
- The magnetic moment is defined as the product of its pole strength and magnetic length. It is a vector quantity, denoted by $\vec{p}_m$.
- The region surrounding magnet where magnetic pole of strength unity experiences a force is known as magnetic field. It is a vector quantity and denoted by $\vec{B}$ Its unit is $N\,A^{-1}\,m^{-1}$.
- The number of magnetic field lines crossing per unit area is called magnetic flux Φ_B. It is a scalar quantity. In SI unit, magnetic flux Φ_B is Weber, symbol Wb.
- Statement of Coulomb's law in magnetism "The force of attraction or repulsion between two magnetic poles is proportional to the product of their pole strengths and inversely proportional to the square of distance between them".
- Magnetic dipole kept in a uniform magnetic field experiences torque.
- Tangent galvanometer is a device used to measure very small currents. It is a moving magnet type galvanometer. Its working is based on tangent law.
- Tangent law is $B = B_H \tan\theta$.
- The magnetic field which is used to magnetize a sample or specimen is called the magnetising field. It is a vector quantity and denoted by $\vec{H}$ and its unit is $A\,m^{-1}$.
- The measure of ability of the material to allow the passage of magnetic lines of force through it is known as magnetic permeability.
- The net magnetic moment per unit volume of material is known as intensity of magnetisation or magnetisation vector or magnetisation.
- Magnetic susceptibility is defined as the ratio of the intensity of magnetisation $(\vec{I})$ induced in the material due to the magnetising field $(\vec{H})$.
- Magnetic materials are classified into three categories: diamagnetic, paramagnetic and ferromagnetic materials.
- The lagging of magnetic induction $\vec{B}$ behind the cyclic variation in magnetising field $\vec{H}$ is defined as "Hysteresis", which means "lagging behind".
- The right hand thumb rule "If we hold the current carrying conductor in our right hand such that the thumb points in the direction of current flow, then the rest of the fingers encircling the wire points in the direction of the magnetic field lines produced".
- Maxwell right hand cork screw rule "If we rotate a screw by a screw driver, then the direction of current is same as the direction in which screw advances, and the direction of rotation of the screw will determine the direction of the magnetic field".

- Ampère's circuital law is $\oint_C \vec{B}\cdot d\vec{l} = \mu_\circ I_{enclosed}$.
- Magnetic field inside the solenoid is $B = \mu_\circ nI$, where n is the number of turns per unit length.
- Magnetic field interior to the toroid is $B = \mu_\circ nI$, where n is the number of turns per unit length.
- Lorentz force is $\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$.
- Charged particle moving in a uniform magnetic field will undergo circular motion.
- Fleming's Left Hand Rule: Stretch forefinger, the middle finger and the thumb of the left hand such that they are in mutually perpendicular directions. If we keep the forefinger in the direction of magnetic field, the middle finger in the direction of the electric current, then the thumb points in the direction the force experienced by the conductors.
- One ampere is defined as that current when it is passed through each of the two infinitely long parallel straight conductors kept at a distance of one meter apart in vacuum causes each conductor to experience a force of 2×10^{-7} newton per meter length of the conductor.
- When a current carrying coil is placed in a uniform magnetic field, the net force on it is always zero but net torque is not zero. The magnitude of net torque is τ = NABI sin θ.
- Moving coil galvanometer is an instrument used for the detection and measurement of small currents.
- In moving coil galvanometer, current passing through the galvanometer is directly proportional to the deflection. Mathematically, I = Gθ, where $G = \frac{K}{NAB}$ is called galvanometer constant or current reduction factor of the galvanometer.
- Current sensitivity is defined as the deflection produced per unit current flowing through it, $I_s = \frac{\theta}{I} = \frac{NAB}{K} \Rightarrow I_s = \frac{1}{G}$.
- Voltage sensitivity is defined as the deflection produced per unit voltage which is applied across it, $V_s = \frac{\theta}{V} = \frac{1}{GR_g} = \frac{I_S}{R_g}$, where, R_g is the resistance of galvanometer.
- Ammeter is an instrument used to measure current in an electrical circuit.
- A galvanometer can be converted into an ammeter of given range by connecting a suitable low resistance S called shunt in parallel to the given galvanometer.
- An ideal ammeter has zero resistance.
- Voltmeter is an instrument used to measure potential difference across any element in an electrical circuit.
- A galvanometer can be converted into suitable voltmeter of given range by connecting a suitable resistance R in series with the given galvanometer.
- An ideal voltmeter has infinite resistance.

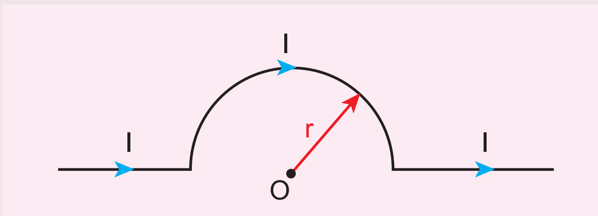
CONCEPT MAP



Magnetism and Magnetic effects of electric current
Magnetism
Earth's Magnetism
Magnetic elements
Tangent Galvanometer
Bar Magnetics
Coulomb's law
Magnetic field axial line
Magnetic field equitorial line
Potential energy
Torque
Magnetic materials
Properties
Magnetic effects of electric current
Biot Savart's law
Ampere law
magnetic field of current carrying conductor
Toroid
Solenoid
Lorentz force
Torque
Cyclotron
Moving coil Galvanometer

I Multiple choice questions

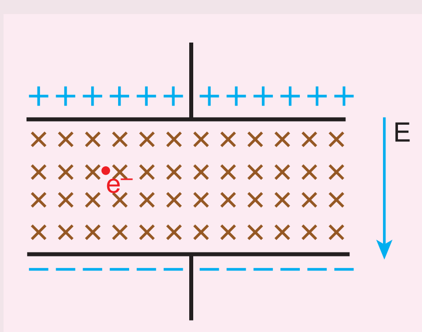
1. The magnetic field at the center O of the following current loop is



(a) $\frac{\mu_\circ I}{4r}\otimes$ (b) $\frac{\mu_\circ I}{4r}\odot$

(c) $\frac{\mu_\circ I}{2r}\otimes$ (d) $\frac{\mu_\circ I}{2r}\odot$

2. An electron moves straight inside a charged parallel plate capacitor of uniform charge density σ. The time taken by the electron to cross the parallel plate capacitor when the plates of the capacitor are kept under constant magnetic field of induction $\vec{B}$ is



H5CRWB

(a) $\varepsilon_\circ \frac{elB}{\sigma}$ (b) $\varepsilon_\circ \frac{lB}{\sigma l}$

(c) $\varepsilon_\circ \frac{lB}{e\sigma}$ (d) $\varepsilon_\circ \frac{lB}{\sigma}$

3. The force experienced by a particle having mass m and charge q accelerated through a potential difference V when it is kept under perpendicular magnetic field $\vec{B}$ is

(a) $\sqrt{\frac{2q^3BV}{m}}$ (b) $\sqrt{\frac{q^3B^2V}{2m}}$

(c) $\sqrt{\frac{2q^3B^2V}{m}}$ (d) $\sqrt{\frac{2q^3BV}{m^3}}$

4. A circular coil of radius 5 cm and 50 turns carries a current of 3 ampere. The magnetic dipole moment of the coil is

(a) 1.0 amp – m^2 (b) 1.2 amp – m^2

(c) 0.5 amp – m^2 (d) 0.8 amp – m^2

5. A thin insulated wire forms a plane spiral of N = 100 tight turns carrying a current I = 8 m A (milli ampere). The radii of inside and outside turns are a = 50 mm and b = 100 mm respectively. The magnetic induction at the center of the spiral is

(a) 5 μT (b) 7 μT

(c) 8 μT (d) 10 μT

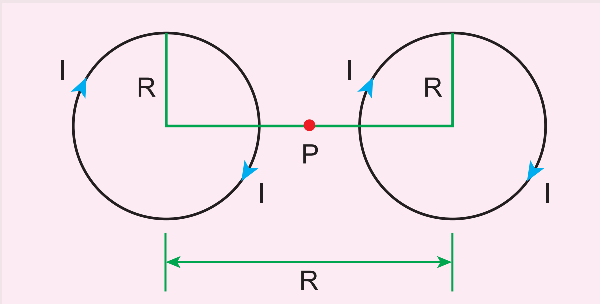
6. Three wires of equal lengths are bent in the form of loops. One of the loops is circle, another is a semi-circle and the third one is a square. They are placed in a uniform magnetic field and same electric current is passed through them. Which of the following loop configuration will experience greater torque ?

(a) circle (b) semi-circle

(c) square (d) all of them

7. Two identical coils, each with N turns and radius R are placed coaxially at a distance R as shown in the figure. If I is the current passing through the loops in the same direction, then the

magnetic field at a point P which is at exactly at $\frac{R}{2}$ distance between two coils is



(a) $\frac{8N\mu_\circ I}{\sqrt{5}R}$ (b) $\frac{8N\mu_\circ I}{5^{3/2}R}$

(c) $\frac{8N\mu_\circ I}{5R}$ (d) $\frac{4N\mu_\circ I}{\sqrt{5}R}$

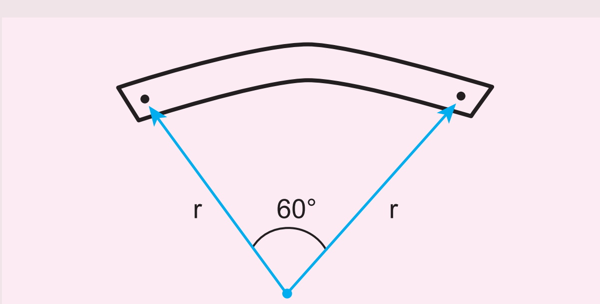
8. A wire of length l carries a current I along the Y direction and magnetic field is given by $\vec{B} = \frac{\beta}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})T$. The magnitude of Lorentz force acting on the wire is

(a) $\sqrt{\frac{2}{\sqrt{3}}}\beta Il$ (b) $\sqrt{\frac{1}{\sqrt{3}}}\beta Il$

(c) $\sqrt{2}\beta Il$ (d) $\sqrt{\frac{1}{2}}\beta Il$

9. A bar magnet of length l and magnetic moment M is bent in the form of an arc as shown in figure. The new magnetic dipole moment will be

(NEET 2014)



(a) M (b) $\frac{3}{\pi}M$

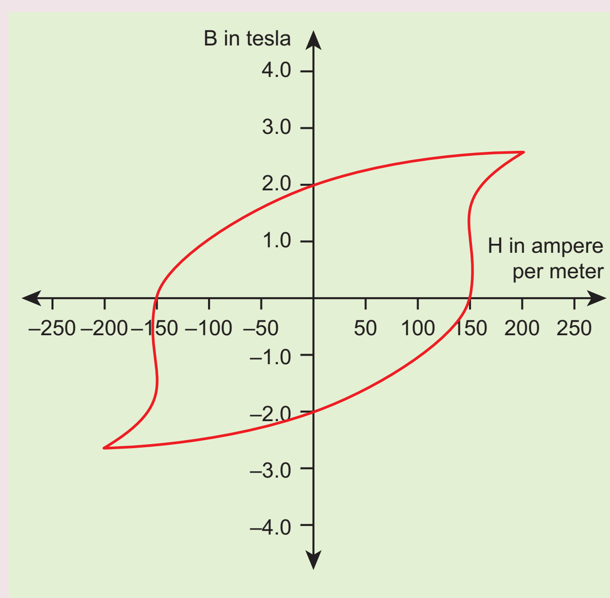
(c) $\frac{2}{\pi}M$ (d) $\frac{1}{2}M$

10. A non-conducting charged ring of charge q, mass m and radius r is rotated with constant angular speed ω. Find the ratio of its magnetic moment with angular momentum is

(a) $\frac{q}{m}$ (b) $\frac{2q}{m}$

(c) $\frac{q}{2m}$ (d) $\frac{q}{4m}$

11. The BH curve for a ferromagnetic material is shown in the figure. The material is placed inside a long solenoid which contains 1000 turns/cm. The current that should be passed in the solenoid to demagnetize the ferromagnet completely is



(a) 1.00 m A (milli ampere) (b) 1.25 mA

(c) 1.50 mA (d) 1.75 mA

12. Two short bar magnets have magnetic moments 1.20 Am^2 and 1.00 Am^2

respectively. They are kept on a horizontal table parallel to each other with their north poles pointing towards the south. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centers is (Horizontal components of Earth's magnetic induction is 3.6×10^{-5} Wb m^{-2})

(NSEP 2000-2001)

(a) 3.60×10^{-5} Wb m^{-2}

(b) 3.5×10^{-5} Wb m^{-2}

(c) 2.56×10^{-4} Wb m^{-2}

(d) 2.2×10^{-4} Wb m^{-2}

13. The vertical component of Earth's magnetic field at a place is equal to the horizontal component. What is the value of angle of dip at this place?

(a) 30° (b) 45°

(c) 60° (d) 90°

14. A flat dielectric disc of radius R carries an excess charge on its surface. The surface charge density is σ. The disc rotates about an axis perpendicular to its plane passing through the center with angular velocity ω. Find the magnitude of the torque on the disc if it is placed in a uniform magnetic field whose strength is B which is directed perpendicular to the axis of rotation

(a) $\frac{1}{4}\sigma\omega\pi BR$ (b) $\frac{1}{4}\sigma\omega\pi BR^2$

(c) $\frac{1}{4}\sigma\omega\pi BR^3$ (d) $\frac{1}{4}\sigma\omega\pi BR^4$

15. A simple pendulum with charged bob is oscillating with time period T and let θ be the angular displacement. If the uniform magnetic field is switched ON in a direction perpendicular to the plane of oscillation then

(a) time period will decrease but θ will remain constant

(b) time period remain constant but θ will decrease

(c) both T and θ will remain the same

(d) both T and θ will decrease

Answers

1) a	**2)** d	**3)** c	**4)** b	**5)** b
6) a	**7)** b	**8)** a	**9)** b	**10)** c
11) b	**12)** c	**13)** b	**14)** d	**15)** c

II Short answer questions:

1. What is meant by magnetic induction?
2. Define magnetic flux.
3. Define magnetic dipole moment.
4. State Coulomb's inverse law.
5. What is magnetic susceptibility?
6. State Biot-Savart's law.
7. What is magnetic permeability?
8. State Ampere's circuital law.
9. Compare dia, para and ferro-magnetism.
10. What is meant by hysteresis?

III Long answer questions

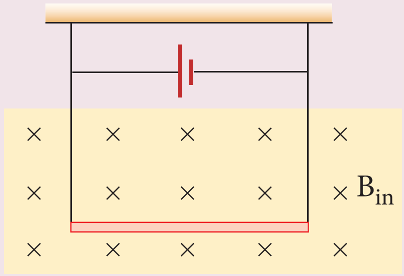
1. Discuss Earth's magnetic field in detail.
2. Deduce the relation for the magnetic induction at a point due to an infinitely long straight conductor carrying current.
3. Obtain a relation for the magnetic induction at a point along the axis of a circular coil carrying current.
4. Compute the torque experienced by a magnetic needle in a uniform magnetic field.
5. Calculate the magnetic induction at a point on the axial line of a bar magnet.
6. Obtain the magnetic induction at a point on the equatorial line of a bar magnet.
7. Find the magnetic induction due to a long straight conductor using Ampere's circuital law.
8. Discuss the working of cyclotron in detail.
9. What is tangent law? Discuss in detail.
10. Explain the principle and working of a moving coil galvanometer.
11. Discuss the conversion of galvanometer into an ammeter and also a voltmeter.
12. Calculate the magnetic field inside and outside of the long solenoid using Ampere's circuital law.

IV. Numerical problems

1. A bar magnet having a magnetic moment $\vec{M}$ is cut into four pieces i.e., first cut in two pieces along the axis of the magnet and each piece is further cut into two pieces. Compute the magnetic moment of each piece.

   Answer $\vec{M}_{new} = \frac{1}{4}\vec{M}$

2. A conductor of linear mass density 0.2 g m^{-1} suspended by two flexible wire as shown in figure. Suppose the tension in the supporting wires is zero when it is kept inside the magnetic field of 1 T whose direction is into the page. Compute the current inside the conductor and also the direction of the current. Assume g = 10 m s^{-2}



   Answer 2 mA

3. A circular coil with cross-sectional area 0.1 cm^2 is kept in a uniform magnetic field of strength 0.2 T. If the current passing in the coil is 3 A and plane of the loop is perpendicular to the direction of magnetic field. Calculate

   (a) total torque on the coil
   (b) total force on the coil
   (c) average force on each electron in the coil due to the magnetic field of the free electron density for the material of the wire is 10^{28} m^{-3}.

Answer (a) zero (b) zero (c) 0.6 x 10^{-23} N

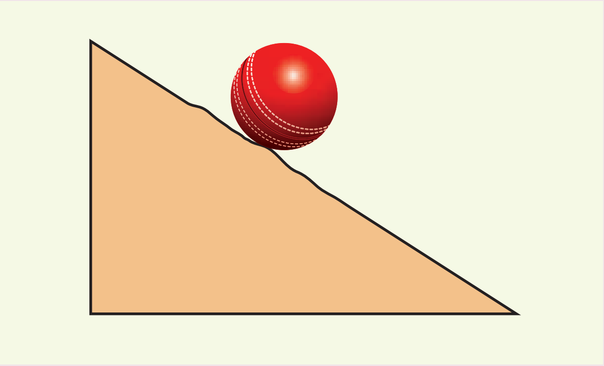
4. A bar magnet is placed in a uniform magnetic field whose strength is 0.8 T. Suppose the bar magnet orient at an angle 30° with the external field experiences a torque of 0.2 N m. Calculate:

(i) the magnetic moment of the magnet

(ii) the work done by an applied force in moving it from most stable configuration to the most unstable configuration and also compute the work done by the applied magnetic field in this case.

Answer (i) 0.5 A m^2 (ii) W = 0.8 J and W_{mag} = -0.8 J

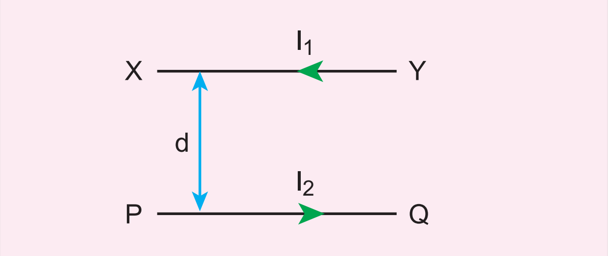
5. A non - conducting sphere has a mass of 100 g and radius 20 cm. A flat compact coil of wire with turns 5 is wrapped tightly around it with each turns concentric with the sphere. This sphere is placed on an inclined plane such that plane of coil is parallel to the inclined plane. A uniform magnetic field of 0.5 T exists in the region in vertically upward direction. Compute the current I required to rest the sphere in equilibrium. Answer $\frac{2}{\pi} A$



6. Calculate the magnetic field at the center of a square loop which carries a current of 1.5 A, length of each loop is 50 cm. Answer 3.4 x 10^{-6} T

7. Show that the magnetic field at any point on the axis of the solenoid having n turns per unit length is $B = \frac{1}{2}\mu_o nI(\cos\theta_1 - \cos\theta_2)$.

8. Let I_1 and I_2 be the steady currents passing through a long horizontal wire XY and PQ respectively. The wire PQ is fixed in horizontal plane and the wire XY be is allowed to move freely in a vertical plane. Let the wire XY is in equilibrium at a height d over the parallel wire PQ as shown in figure.



Show that if the wire XY is slightly displaced and released, it executes Simple Harmonic Motion (SHM). Also, compute the time period of oscillations.

Answer $a_y = -\omega^2 y$ (SHM) and time period

$$T = 2\pi\sqrt{\frac{d}{g}} \text{ in sec}$$

BOOKS FOR REFERENCE

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4. David J. Griffiths, *Introduction to electrodynamics*, Pearson publishers
5. Rita John, *Solid State Physics (Magnetism chapter)*, McGraw Hill Education (India) Pvt. Ltd.
6. Paul Tipler and Gene Mosca, *Physics for scientist and engineers with modern physics*, Sixth edition, W.H. Freeman and Company

ICT CORNER

Magnetism

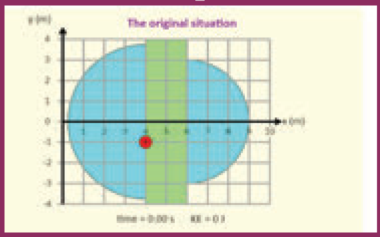
Topic: Cyclotron

In this activity you will be able to visualize and understand the working of cyclotron.

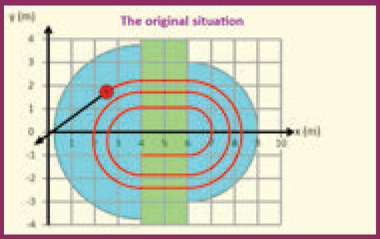
STEPS:

- Open the browser and type ‘physics.bu.edu/~duffy/HTML5/cyclotron.html’ in the address bar.
- Click ‘play’ to release the positively charged particle between the D-shaped sections.
- Observe trajectory of positively charged particle under the magnetic field between D-shaped sections.
- Note the kinetic energy of the particle after some time (say t = 20 s)

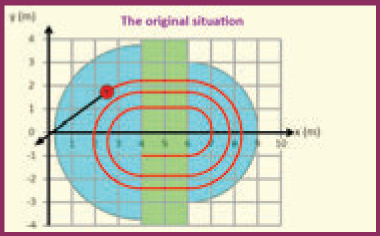
Step1



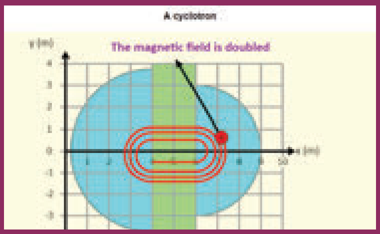
Step2



Step3



Step4



Double the electric and magnetic fields by clicking corresponding buttons and observe the change in kinetic energy for a particular given time t.

URL:

http://physics.bu.edu/~duffy/HTML5/cyclotron.html

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

B263_12_PHYSICS_EM