UNIT 4

ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

"Nature is our kindest friend and best critic in experimental science if we only allow her intimations to fall unbiased on our minds" — Michael Faraday

LEARNING OBJECTIVES

In this unit, the student is exposed to

- the phenomenon of electromagnetic induction
- the application of Lenz's law to find the direction of induced emf
- the concept of Eddy current and its uses
- the phenomenon of self-induction and mutual-induction
- the various methods of producing induced emfs
- the construction and working of AC generators
- the principle of transformers and its role in long distance power communication
- the notion of root mean square value of alternating current
- the idea of phasors and phase relationships in different AC circuits
- the insight about power in an AC circuit and wattless current
- the understanding of energy conservation during LC oscillations

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4.1 ELECTROMAGNETIC INDUCTION

4.1.1 Introduction

In the previous chapter, we have learnt that whenever an electric current flows through a conductor, it produces a magnetic field around it. This was discovered by Christian Oersted. Later, Ampere proved that a current-carrying loop behaves like a bar magnet. These are the magnetic effects produced by the electric current.

Physicists then began to think of the converse effect. Is it possible to produce an electric current with the help of a magnetic field? A series of experiments were conducted to establish the converse effect. These experiments were done by Michael Faraday of UK and Joseph Henry of USA, almost simultaneously and independently. These attempts became successful and led to the discovery of the phenomenon, called Electromagnetic Induction. Michael Faraday is credited with the discovery of electromagnetic induction in 1831.

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In this chapter, let us see a few experiments of Faraday, the results and the phenomenon of Electromagnetic Induction. Before that, we will recollect the concept of magnetic flux linked with a surface area.

An anecdote!

Michael Faraday was enormously popular for his lectures as well. In one of his lectures, he demonstrated his experiments which led to the discovery of electromagnetic induction.

At the end of the lecture, one member of the audience approached Faraday and said, "Mr. Faraday, the behaviour of the magnet and the coil of wire was interesting, but what is the use of it?" Faraday answered politely, "Sir, what is the use of a newborn baby?"

Note: We will soon see the greatness of 'that little child' who has now grown as an adult to cater to the energy needs.

4.1.2 Magnetic Flux (Φ_B)

The magnetic flux through an area A in a magnetic field is defined as the number of magnetic field lines passing through that area normally and is given by the equation (Figure 4.1(a)).

$$\Phi_B = \int_A \vec{B} \cdot d\vec{A} = BA \cos\theta \qquad (4.1)$$

where the integral is taken over the area A and θ is the angle between the direction of the magnetic field and the outward normal to the area.

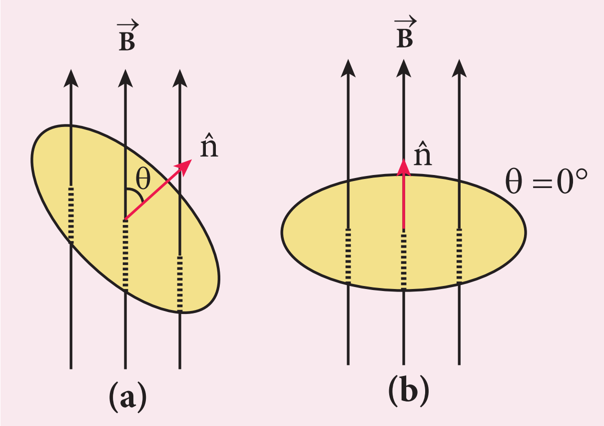


Figure 4.1 Magnetic flux

If the magnetic field $\vec{B}$ is uniform over the area A and is perpendicular to the area as shown in Figure 4.1(b), then the above equation becomes

$$\Phi_B = BA \qquad (4.2)$$

since $\theta = 0°$, $\cos 0° = 1$

The SI unit of magnetic flux is $T\,m^2$. It is also measured in weber or Wb.

$$1\ Wb = 1\ T\,m^2$$

EXAMPLE 4.1

A circular antenna of area 3 m² is installed at a place in Madurai. The plane of the area of antenna is inclined at 47° with the direction of Earth's magnetic field. If the magnitude of Earth's field at that place is 40773.9 nT find the magnetic flux linked with the antenna.

Solution

B = 40773.9 nT; θ = 90° – 47° = 43°; $A = 3\text{m}^2$

We know that $\Phi_B = BA\cos\theta$

$= 40,773.9 \times 10^{-9} \times 3 \times \cos 43^{o}$

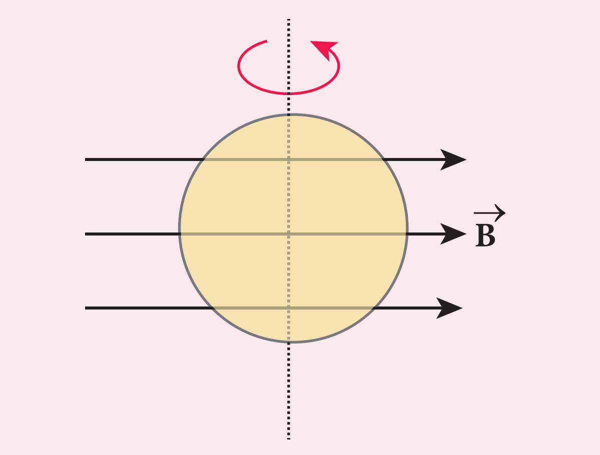
$= 40.7739 \times 10^{-6} \times 3 \times 0.7314$

$= 89.47 \times 10^{-6}$ Wb

$\Phi_B = 89.47\,\mu\text{Wb}$

EXAMPLE 4.2

A circular loop of area $5 \times 10^{-2} m^2$ rotates in a uniform magnetic field of 0.2 T. If the loop rotates about its diameter which is perpendicular to the magnetic field as shown in figure. Find the magnetic flux linked with the loop when its plane is (i) normal to the field (ii) inclined 60° to the field and (iii) parallel to the field.



Solution

$A = 5 \times 10^{-2} m^2$; $B = 0.2$ T

(i) $\theta = 0^{\circ}$;

$$\Phi_B = BA\cos\theta = 0.2 \times 5 \times 10^{-2} \times \cos 0^{\circ}$$

$$\Phi_B = 1 \times 10^{-2}\,Wb$$

(ii) $\theta = 90^{\circ} - 60^{\circ} = 30^{\circ}$;

$$\Phi_B = BA\cos\theta = 0.2 \times 5 \times 10^{-2} \times \cos 30^{\circ}$$

$$\Phi_B = 1 \times 10^{-2} \times \frac{\sqrt{3}}{2} = 8.66 \times 10^{-3}\,Wb$$

(iii) $\theta = 90^{\circ}$;

$$\Phi_B = BA\cos 90^{\circ} = 0$$

4.1.3 Faraday's Experiments on Electromagnetic Induction

First Experiment

Consider a closed circuit consisting of a coil *C* of insulated wire and a galvanometer *G* as shown in Figure 4.2(a). The galvanometer does not indicate deflection as there is no electric current in the circuit.

When a bar magnet is inserted into the stationary coil, with its north pole facing the coil, there is a momentary deflection in the galvanometer. This indicates that an electric current is set up in the coil (Figure 4.2(b)). If the magnet is kept stationary inside the coil, the galvanometer does not indicate deflection (Figure 4.2(c)).

The bar magnet is now withdrawn from the coil, the galvanometer again gives a momentary deflection but in the opposite direction. So the electric current flows in opposite direction (Figure 4.2(d)). Now if the magnet is moved faster, it gives a larger deflection due to a greater current in the circuit (Figure 4.2(e))

The bar magnet is reversed i.e., the south pole now faces the coil. When the above experiment is repeated, the deflections are opposite to that obtained in the case of north pole (Figure 4.2(f)).

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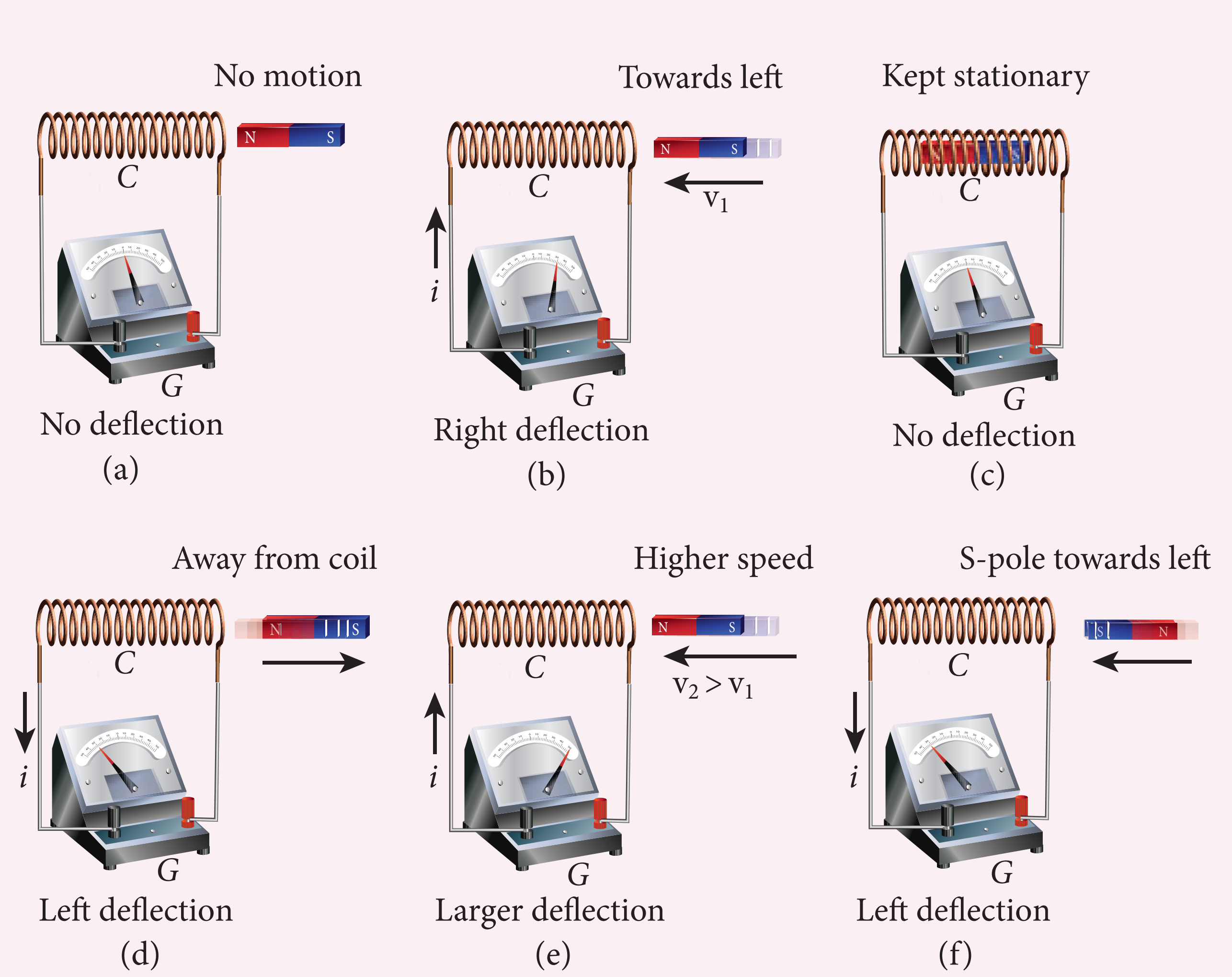

Figure 4.2 Faraday's first experiment

If the magnet is kept stationary and the coil is moved towards or away from the coil, similar results are obtained. It is concluded that whenever there is a relative motion between the coil and the magnet, there is deflection in the galvanometer, indicating the electric current setup in the coil.

Second Experiment

Consider two closed circuits as shown in Figure 4.3(a). The circuit consisting of a coil *P*, a battery *B* and a key *K* is called as primary circuit while the circuit with a coil *S* and a galvanometer *G* is known as secondary circuit. The coils *P* and *S* are kept at rest in close proximity with respect to one another.

If the primary circuit is closed, electric current starts flowing in the primary circuit. At that time, the galvanometer gives a momentary deflection (Figure 4.3(a)).

After that, when the electric current reaches a certain steady value, no deflection is observed in the galvanometer.

Likewise if the primary circuit is broken, the electric current starts decreasing and there is again a sudden deflection but in the opposite direction (Figure 4.3(b)). When the electric current becomes zero, the galvanometer shows no deflection.

From the above observations, it is concluded that whenever the electric current in the primary circuit changes, the galvanometer shows a deflection.

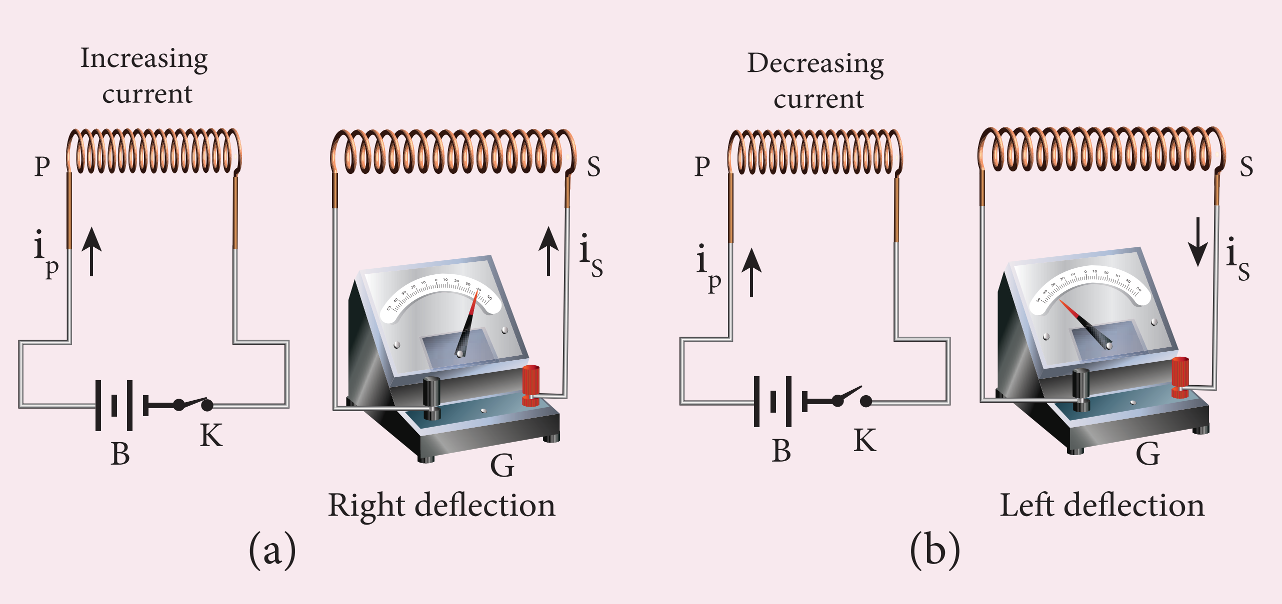

Figure 4.3 Faraday's second experiment

Faraday's Law of Electromagnetic Induction

From the results of his experiments, Faraday realized that

> **whenever the magnetic flux linked with a closed coil changes, an emf (electromotive force) is induced and hence an electric current flows in the circuit. This current is called an induced current and the emf giving rise to such current is called an induced emf. This phenomenon is known as electromagnetic induction.**

Based on this idea, Faraday's experiments are understood in the following way. In the first experiment, when a bar magnet is placed close to a coil, some of the magnetic field lines of the bar magnet pass through the coil i.e., the magnetic flux is linked with the coil. When the bar magnet and the coil approach each other, the magnetic flux linked with the coil increases. So this increase in magnetic flux induces an emf and hence a transient

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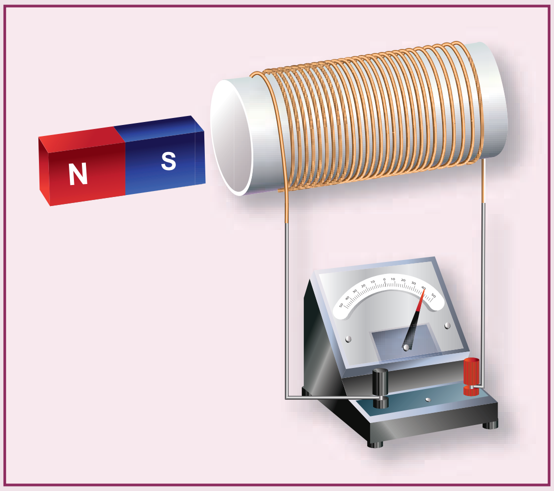
Exploring Electromagnetic Induction



Make a circuit containing a coil of insulated wire wound around soft hollow core and a galvanometer as shown in Figure. It is better to use a thin wire for the coil so that we can wind many turns in the available space. Perform the steps described in first experiment of Faraday with the help of a strong bar magnet. Students will get hands-on experience about electromagnetic induction.

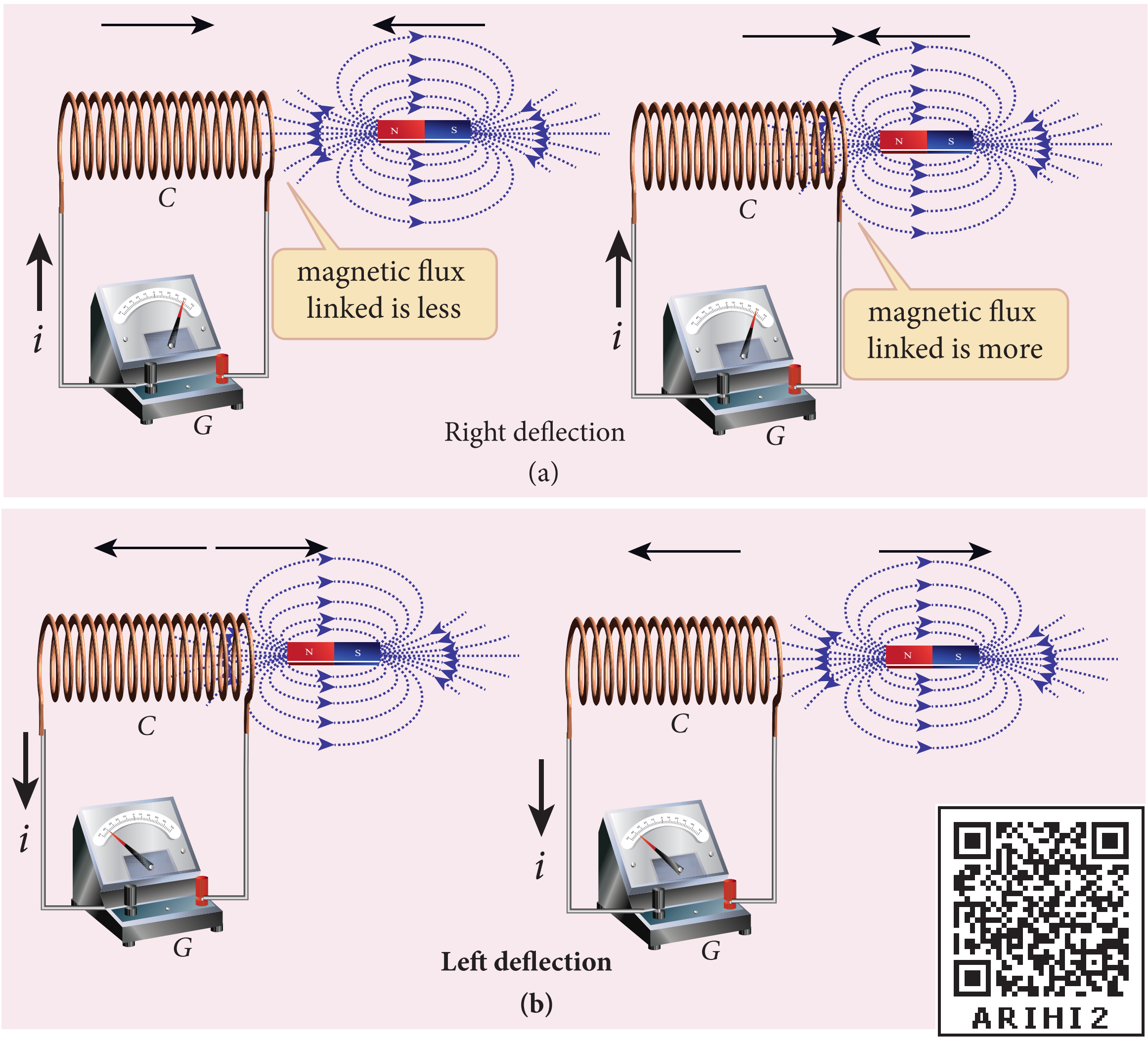


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Figure 4.4 Explanation of Faraday's first experiment

electric current flows in the circuit in one direction (Figure 4.4(a)).

At the same time, when they recede away from one another, the magnetic flux linked with the coil decreases. The decrease in magnetic flux again induces an emf in opposite direction and hence an electric current flows in opposite direction (Figure 4.4(b)). So there is deflection in the galvanometer when there is a relative motion between the coil and the magnet.

In the second experiment, when the primary coil *P* carries an electric current, a magnetic field is established around it. The magnetic lines of this field pass through itself and the neighbouring secondary coil *S*.

When the primary circuit is open, no electric current flows in it and hence the magnetic flux linked with the secondary coil is zero (Figure 4.5(a)).

However, when the primary circuit is closed, the increasing current builds up a magnetic field around the primary coil. Therefore, the magnetic flux linked with the secondary coil increases. This increasing flux linked induces a transient electric current in the secondary coil (Figure 4.5(b)). When the electric current in the primary coil reaches a steady value, the magnetic flux linked with the secondary coil does not change and the electric current in the secondary coil will disappear.

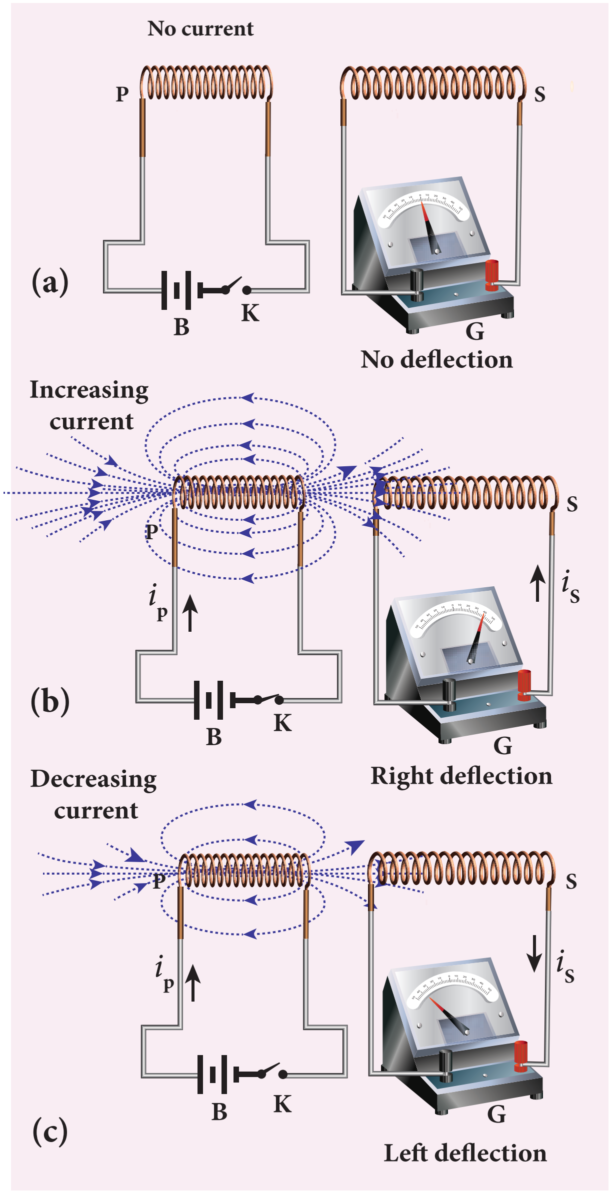

Figure 4.5 Explanation of Faraday's second experiment

Similarly, when the primary circuit is broken, the decreasing primary current induces an electric current in the secondary coil, but in the opposite direction (Figure 4.5(c)). So there is deflection in the galvanometer whenever there is a change in the primary current.

Note The symbol ε is used for permittivity in unit 1 and for emf in this chapter. Reader is advised to use ε in relevant context.

Importance of Electromagnetic Induction!

The application of the phenomenon of Electromagnetic Induction is almost everywhere in the present day life. Right from home appliances to huge factory machineries, from cellphone to computers and internet, from electric guitar to satellite communication, all need electricity for their operation. There is an ever growing demand for electric power.

All these are met with the help of electric generators and transformers which function on electromagnetic induction. The modern, sophisticated human life would not be possible without the discovery of electromagnetic induction.

The conclusions of Faraday's experiments are stated as two laws.

First law

Whenever magnetic flux linked with a closed circuit changes, an emf is induced in the circuit.

Second law

The magnitude of induced emf in a closed circuit is equal to the time rate of change of magnetic flux linked with the circuit.

If the magnetic flux linked with the coil changes by $d\Phi_B$ in a time dt, then the induced emf is given by

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

The negative sign in the above equation gives the direction of the induced current which will be dealt with in the next section.

If a coil consisting of N turns is tightly wound such that each turn covers the same area, then the flux through each turn will be the same. Then total emf induced in the coil is given by

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -\frac{d(N\Phi_B)}{dt} \quad (4.3)$$

Here, $N\Phi_B$ is called flux linkage, defined as the product of number of turns N of the coil and the magnetic flux linking each turn of the coil Φ_B.

EXAMPLE 4.3

A cylindrical bar magnet is kept along the axis of a circular solenoid. If the magnet is rotated about its axis, find out whether an electric current is induced in the coil.

Solution

The magnetic field of a cylindrical magnet is symmetrical about its axis. As the magnet is rotated along the axis of the solenoid, there is no induced current in the solenoid because the flux linked with the solenoid does not change due to the rotation of the magnet.

EXAMPLE 4.4

A closed coil of 40 turns and of area 200 cm^2, is rotated in a magnetic field of flux density 2 Wb m^{-2}. It rotates from a position where its plane makes an angle of 30° with the field to a position perpendicular to the field in a time 0.2 sec. Find the magnitude of the emf induced in the coil due to its rotation.

Solution

N = 40 turns; B = 2 Wb m^{-2}

A = 200 cm^2 = 200 × 10^{-4} m^2;

Initial flux, $\Phi_i = BA\cos\theta$

$$= 2\times 200\times 10^{-4}\times\cos 60^\circ$$

since $\theta = 90^\circ - 30^\circ = 60^\circ$

$$\Phi_i = 2\times 10^{-2}\,Wb$$

Final flux, $\Phi_f = BA\cos\theta$

$$= 2\times 200\times 10^{-4}\times\cos 0^\circ \qquad \text{since } \theta = 0^\circ$$

$$\Phi_f = 4\times 10^{-2}\,Wb$$

Magnitude of the induced emf is

$$\varepsilon = N\frac{d\Phi_B}{dt}$$

$$= \frac{40\times\left(4\times 10^{-2} - 2\times 10^{-2}\right)}{0.2} = 4V$$

EXAMPLE 4.5

A straight conducting wire is dropped horizontally from a certain height with its length along east – west direction. Will an emf be induced in it? Justify your answer.

Solution

Yes! An emf will be induced in the wire because it moves perpendicular to the horizontal component of Earth's magnetic field.

4.1.4 Lenz's law

A German physicist Heinrich Lenz performed his own experiments on electromagnetic induction and deduced a

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law to determine the direction of the induced current. This law is known as Lenz's law.

Lenz's law states that the direction of the induced current is such that it always opposes the cause responsible for its production.

Faraday discovered that when magnetic flux linked with a coil changes, an electric current is induced in the circuit. Here the flux change is the cause while the induction of current is the effect. Lenz's law says that the effect always opposes the cause. Therefore, the induced current would flow in a direction so that it could oppose the flux change.

To understand Lenz's law, let us consider two illustrations in which we find the direction of the induced current in a circuit.

Illustration 1

Consider a uniform magnetic field, with its field lines perpendicular to the plane of the paper and pointing inwards. These field lines are represented by crosses (x) as shown in Figure 4.6(a). A rectangular metallic frame *ABCD* is placed in this magnetic field, with its plane perpendicular to the field. The arm *AB* is movable so that it can slide towards right or left.

If the arm AB slides to our right side, the number of field lines (magnetic flux) passing through the frame *ABCD* increases and a current is induced. As suggested by Lenz's law, the induced current opposes this flux increase and it tries to reduce it by producing **another magnetic field pointing outwards i.e., opposite to the existing magnetic field.**

The magnetic lines of this induced field are represented by red-colored circles in the Figure 4.6(b). From the direction of the magnetic field thus produced, the direction of the induced current is found to be anti-clockwise by using right-hand thumb rule.

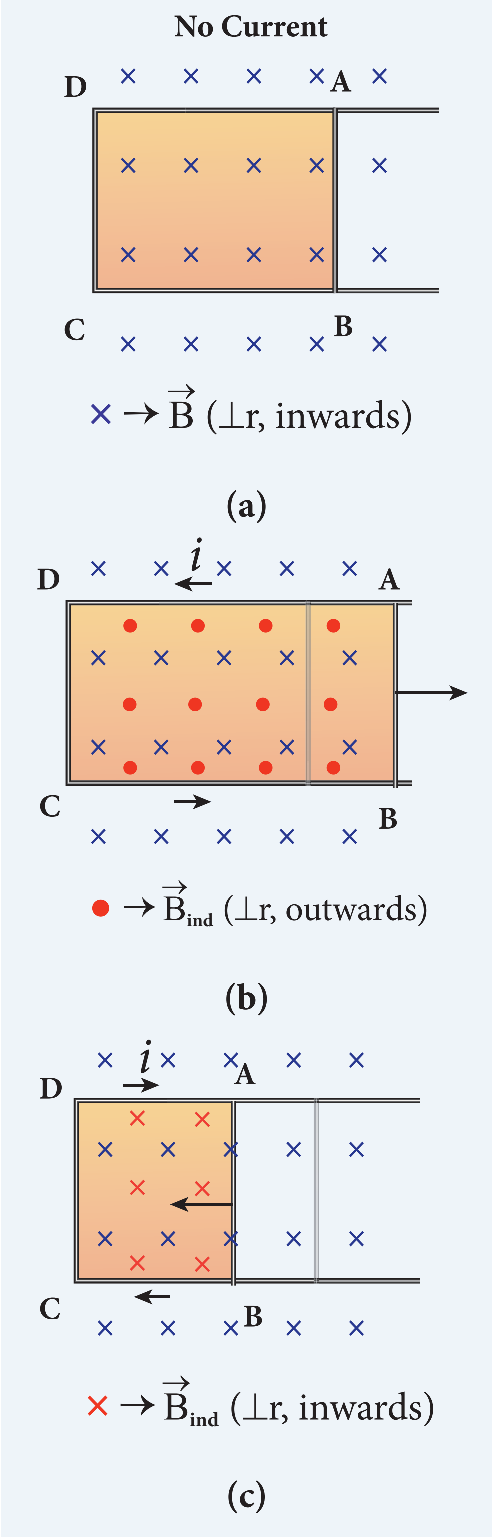

Figure 4.6 First illustration of Lenz's law

The leftward motion of arm *AB* decreases magnetic flux. The induced current, this time, produces **a magnetic field in the inward direction (red-colored crosses) i.e., in the direction of the existing magnetic field** (Figure 4.6(c)). Therefore, the flux decrease is opposed by the flow of induced current. From this, it is found that induced current flows in clockwise direction.

Illustration 2

Let us move a bar magnet towards the solenoid, with its north pole pointing the solenoid as shown in Figure 4.7(b). This motion increases the magnetic flux of the coil which in turn, induces an electric current. Due to the flow of induced current, the coil becomes a magnetic dipole whose two magnetic poles are on either end of the coil.

In this case, the cause producing the induced current is the movement of the magnet. According to Lenz's law, the induced current should flow in such a way that it opposes the movement of the north pole towards coil. It is possible if the end nearer to the magnet becomes north pole (Figure 4.7(b)). Then it repels the north pole of the bar magnet and opposes the movement of the magnet. Once pole ends are known, the direction of the induced current could be found by using right hand thumb rule.

When the bar magnet is withdrawn, the nearer end becomes south pole which attracts north pole of the bar magnet, opposing the receding motion of the magnet (Figure 4.7(c)).

Thus the direction of the induced current can be found from Lenz's law.

Conservation of energy

The truth of Lenz's law can be established on the basis of the law of conservation

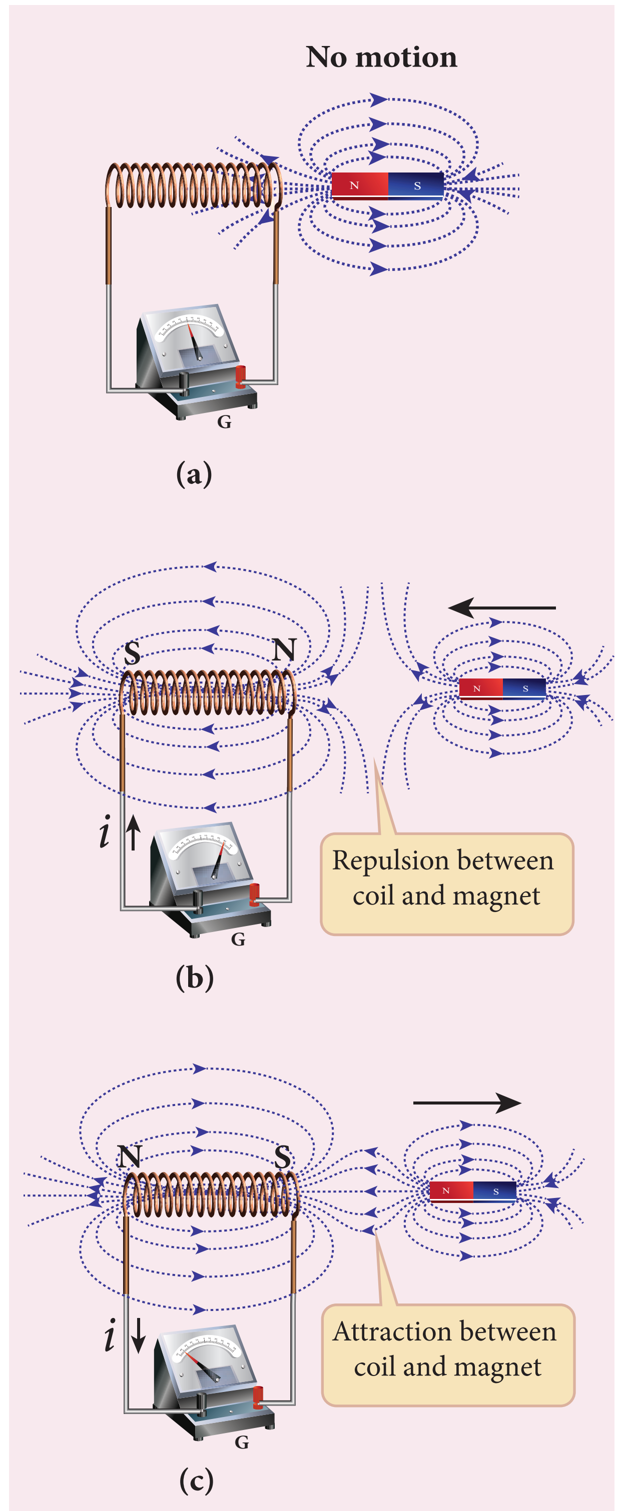

Figure 4.7 Second illustration of Lenz's law

of energy. The explanation is as follows: According to Lenz's law, when a magnet is moved either towards or away from a coil, the induced current produced opposes its motion. As a result, there will always be a

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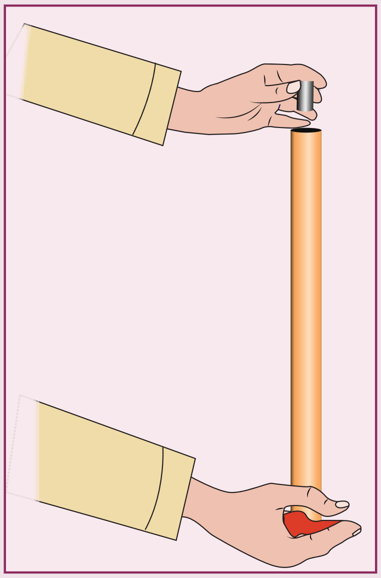


Demonstration of Lenz's law

Take a narrow copper pipe and a strongly magnetized button magnet as shown in figure. Keep the copper pipe vertical and drop the magnet into the pipe. Watch the motion of the magnet and note that magnet has become slower than its free fall. The reason is that an electric current generated by a moving magnet will always *oppose* the original motion of the magnet that produced the current.

resisting force on the moving magnet. Work has to be done by some external agency to move the magnet against this resisting force. Here the mechanical energy of the moving magnet is converted into the electrical energy which in turn, gets converted into Joule heat in the coil i.e., energy is converted from one form to another.

On the contrary to Lenz's law, let us assume that the induced current *helps* the cause responsible for its production. Now when we push the magnet little bit towards the coil, the induced current helps the movement of the magnet towards the coil. Then the magnet starts moving towards the coil without any expense of energy. This, then, becomes a perpetual motion machine. In practice, no such machine is possible. Therefore, the assumption that the induced current *helps* the cause is wrong.

4.1.5 Fleming's right hand rule

When a conductor moves in a magnetic field, the direction of motion of the conductor, the field and the induced current are given by Fleming's right hand rule and is as follows:

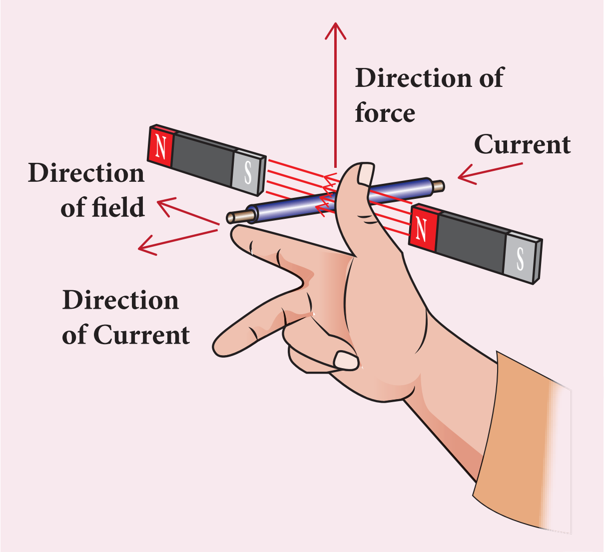


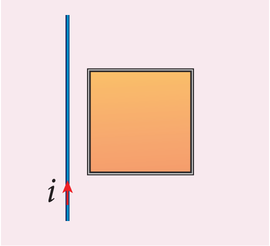
Figure 4.8 Fleming's right hand rule

The thumb, index finger and middle finger of right hand are stretched out in mutually perpendicular directions (as shown in Figure 4.8). If the index finger points the direction of the magnetic field and the thumb indicates the direction of motion of the conductor, then the middle finger will indicate the direction of the induced current.

Fleming's right hand rule is also known as generator rule.

EXAMPLE 4.6

If the current i flowing in the straight conducting wire as shown in the figure decreases, find out the direction of induced current in the metallic square loop placed near it.

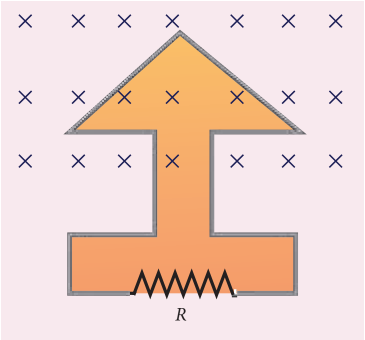


Solution

From right hand rule, the magnetic field by the straight wire is directed into the plane of the square loop perpendicularly and its magnetic flux is decreasing. The decrease in flux is opposed by the current induced in the loop by producing a magnetic field in the same direction as the magnetic field of the wire. Again from right hand rule, for this inward magnetic field, the direction of the induced current in the loop is clockwise.

EXAMPLE 4.7

The magnetic flux passes perpendicular to the plane of the circuit and is directed into the paper. If the magnetic flux varies with respect to time as per the following relation: $\Phi_B = \left(2t^3 + 3t^2 + 8t + 5\right) mWb$, what is the magnitude of the induced emf in the loop when t = 3 s? Find out the direction of current through the circuit.



Solution

$\Phi_B = \left(2t^3 + 3t^2 + 8t + 5\right) mWb$; $N = 1$; t = 3 s

(i) $$\varepsilon = \frac{d\left(N\Phi_B\right)}{dt}$$

$$= \frac{d}{dt}\left(2t^3 + 3t^2 + 8t + 5\right)\times 10^{-3}$$

$$= \left(6t^2 + 6t + 8\right)\times 10^{-3}$$

At t = 3 s,

$$\varepsilon = \left[\left(6\times 9\right) + \left(6\times 3\right) + 8\right]\times 10^{-3}$$

$$= 80\times 10^{-3} V \quad = 80\, mV$$

(ii) As time passes, the magnetic flux linked with the loop increases. According to Lenz's law, the direction of the induced current should be in a way so as to oppose the flux increase. So, the induced current flows in such a way to produce a magnetic field opposite to the given field. This magnetic field is perpendicularly outwards. Therefore, the induced current flows in anti-clockwise direction.

4.1.6 Motional emf from Lorentz force

Consider a straight conducting rod AB of length l in a uniform magnetic field $\vec{B}$ which is directed perpendicularly into the plane of the paper as shown in Figure 4.9(a). The length of the rod is normal to the magnetic field. Let the rod move with a constant velocity $\vec{v}$ towards right side.

When the rod moves, the free electrons present in it also move with same velocity $\vec{v}$ in $\vec{B}$. As a result, the Lorentz force acts on free electrons in the direction from B to A and is given by the relation

$$\vec{F}_B = -e(\vec{v} \times \vec{B}) \quad (4.4)$$

The action of this Lorentz force is to accumulate the free electrons at the end *A*. This accumulation of free electrons produces a potential difference across the rod which in turn establishes an electric field $\vec{E}$ directed along *BA* (Figure 4.9(b)). Due to the electric field $\vec{E}$, the coulomb force starts acting on the free electrons along *AB* and is given by

$$\vec{F}_E = -e\vec{E} \quad (4.5)$$

The magnitude of the electric field $\vec{E}$ keeps on increasing as long as accumulation of electrons at the end *A* continues. The force $\vec{F}_E$ also increases until equilibrium is reached. At equilibrium, the magnetic Lorentz force $\vec{F}_B$ and the coulomb force $\vec{F}_E$ balance each other and no further accumulation of free electrons at the end *A* takes place.

i.e.,
$$|\vec{F}_B| = |\vec{F}_E|$$
$$|-e\,(\vec{v} \times \vec{B})| = |-e\vec{E}|$$
$$vB \sin 90° = E$$
$$vB = E \quad (4.6)$$

The potential difference between two ends of the rod is

$$V = El$$
$$V = vBl$$

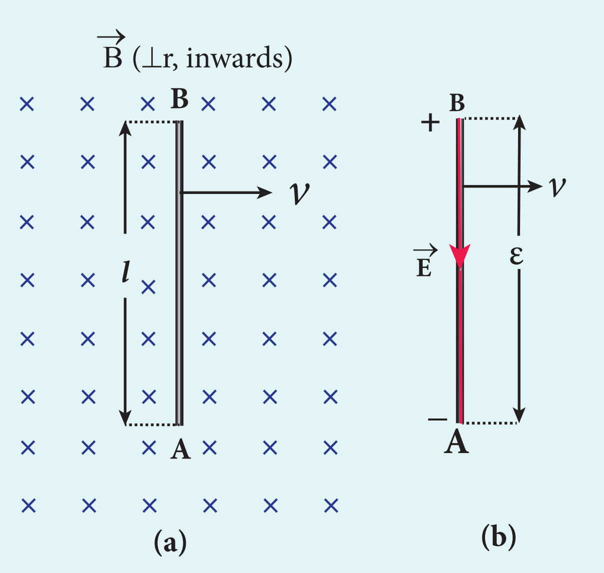


Figure 4.9 Motional emf from Lorentz force

Thus the Lorentz force on the free electrons is responsible to maintain this potential difference and hence produces an emf

$$\varepsilon = Blv \quad (4.7)$$

As this emf is produced due to the movement of the rod, it is often called as motional emf. If the ends A and B are connected by an external circuit of total resistance *R*, then current $i = \frac{\varepsilon}{R} = \frac{Blv}{R}$ flows in it. The direction of the current is found from right-hand thumb rule.

4.1.7 Motional emf from Faraday's law and Energy conservation

Let us consider a rectangular conducting loop of width l in a uniform magnetic field $\vec{B}$ which is perpendicular to the plane of the loop and is directed inwards. A part of the loop is in the magnetic field while the remaining part is outside the field as shown in Figure 4.10.

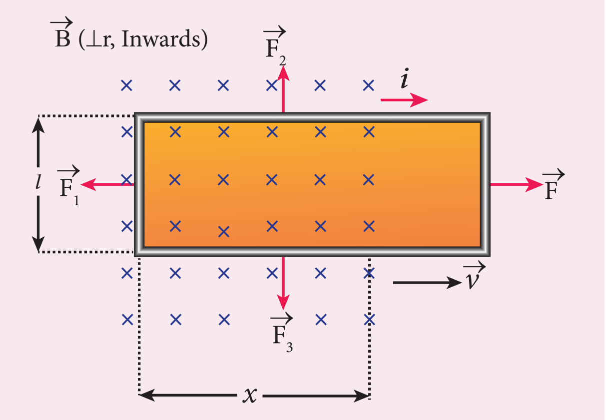


Figure 4.10 Motional emf from Faraday's law

When the loop is pulled with a constant velocity $\vec{v}$ to the right, the area of the portion of the loop within the magnetic field will decrease. Thus, the flux linked with the loop will also decrease. According to Faraday's law, an electric current is induced in the loop which flows in a direction so as to oppose the pull of the loop.

Let x be the length of the loop which is still within the magnetic field, then its area is lx. The magnetic flux linked with the loop is

$$\Phi_B = \int_A \vec{B}\cdot d\vec{A} = BA\cos\theta$$

Here $\theta = 0^\circ$ and $\cos 0^\circ = 1$

$$= BA$$

$$\Phi_B = Blx \qquad (4.8)$$

As this magnetic flux decreases due to the movement of the loop, the magnitude of the induced emf is given by

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{d}{dt}(Blx)$$

Here, both B and l are constants. Therefore,

$$\varepsilon = Bl\frac{dx}{dt}$$

$$\varepsilon = Blv \qquad (4.9)$$

where $v = \frac{dx}{dt}$ is the velocity of the loop. This emf is known as motional emf since it is produced due to the movement of the loop in the magnetic field.

From Lenz's law, it is found that the induced current flows in clockwise direction. If R is the resistance of the loop, then the induced current is given by

$$i = \frac{\varepsilon}{R}$$

$$i = \frac{Blv}{R} \qquad (4.10)$$

Energy conservation

In order to move the loop with a constant velocity $\vec{v}$, a constant force that is equal and opposite to the magnetic force, must be applied. Therefore, mechanical work is done to move the loop. Then the rate of doing work or power is

$$P = \vec{F}\cdot\vec{v} = Fv\cos\theta$$

$$= Fv \qquad \text{Here } \theta = 0^\circ \qquad (4.11)$$

Now, let us find the magnetic force acting on the loop due to its movement in the magnetic field. Let three deflecting forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ be acting on the three segments of the loop as shown in Figure 4.10. The general equation of such a deflecting force is given by

$$\vec{F}_d = i\,\vec{l}\times\vec{B}$$

Forces $\vec{F}_2$ *and* $\vec{F}_3$ are equal in magnitude and opposite in direction and cancel each other. Therefore, the force $\vec{F}_1$ alone acts on the left segment of the loop in a direction shown in Figure 4.10 and is given by

$$\vec{F}_1 = i\vec{l}\times\vec{B}$$

$$F = ilB\sin\theta \qquad (4.12)$$

Here θ is the angle between $\vec{B}$ and the length vector $\vec{l}$ for the left segment and is 90°

$$\therefore F_1 = ilB\sin 90^\circ = ilB \qquad \text{since } \sin 90^\circ = 1$$

The applied force $\vec{F}$ must be equal to $\vec{F}_1$ in order to just move the loop with a constant velocity $\vec{v}$

$$\therefore \vec{F} = -\vec{F}_1$$

(since $\vec{F}$ and $\vec{F}_1$ are in opposite direction)

Considering only the magnitudes,

$$F = F_1 = ilB$$

Substituting for i from equation (4.10)

$$F = \left(\frac{Blv}{R}\right) lB$$

$$F = \frac{B^2l^2v}{R}$$

From equation (4.11), the rate at which the mechanical work is done to pull the loop from the magnetic field or power is given by

$$P = Fv = \left(\frac{B^2l^2v}{R}\right)v$$

$$P = \frac{B^2l^2v^2}{R} \qquad (4.13)$$

When the induced current flows in the loop, Joule heating takes place. The rate at which thermal energy is dissipated in the loop or power dissipated is

$$P = i^2R$$

$$P = \left(\frac{Blv}{R}\right)^2 R$$

$$P = \frac{B^2l^2v^2}{R} \qquad (4.14)$$

This equation is exactly same as the equation (4.13). **Thus the mechanical work done in moving the loop appears as thermal energy in the loop**.

EXAMPLE 4.8

A conducting rod of length 0.5 m falls freely from the top of a building of height 7.2 m at a place in Chennai where the horizontal component of Earth's magnetic field is 40378.7 *nT*. If the length of the rod is perpendicular to Earth's horizontal magnetic field, find the emf induced across the conductor when the rod is about to touch the ground. [Take g = 10 m s^{-2}]

Solution

$l = 0.5$ m; $h = 7.2$ m; $u = 0$ m s^{-1};
$g = 10$ m s^{-2}; $B_H = 40378.7\ nT$

The final velocity of the rod is

$$v^2 = u^2 + 2gh$$

$$= 0 + (2 \times 10 \times 7.2)$$

$$v^2 = 144$$

$$v = 12\,ms^{-1}$$

Induced emf when the rod is about to touch the ground, $\varepsilon = B_H lv$

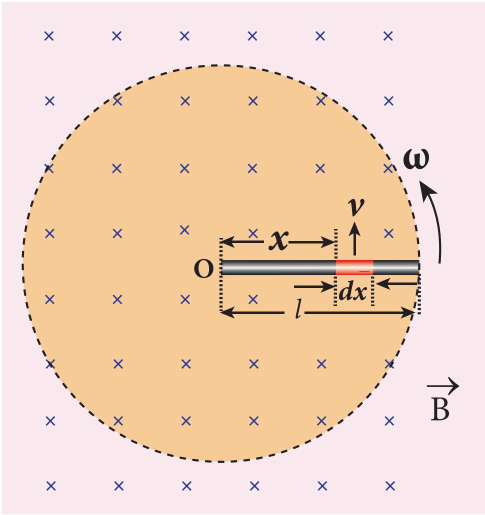
$$= 40{,}378.7 \times 10^{-9} \times 0.5 \times 12$$

$$= 242.27 \times 10^{-6}\,V$$

$$= 242.27\,\mu V$$

EXAMPLE 4.9

A copper rod of length l rotates about one of its ends with an angular velocity ω in a magnetic field B as shown in the figure. The plane of rotation is perpendicular to the field. Find the emf induced between the two ends of the rod.



Solution

Consider a small element of length dx at a distance x from the centre of the circle described by the rod. As this element moves perpendicular to the field with a linear velocity $v = x\omega$, the emf developed in the element dx is

$$d\varepsilon = Bvdx \qquad = B(x\omega)dx$$

This rod is made up of many such elements, moving perpendicular to the field. The emf developed across two ends is

$$\varepsilon = \int d\varepsilon = \int_0^l B\omega x dx \;= B\omega\left[\frac{x^2}{2}\right]_0^l$$

$$\varepsilon = \frac{1}{2}B\omega l^2$$

4.2 EDDY CURRENTS

According to Faraday's law of electromagnetic induction, an emf is induced in a conductor when the magnetic flux passing through it changes. However, the conductor need not be in the form of a wire or coil.

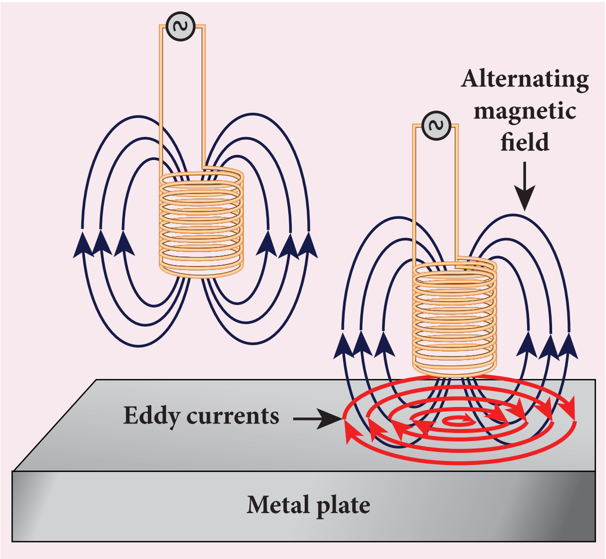


Figure 4.11 Eddy currents

Even for a conductor in the form of a sheet or plate, an emf is induced when magnetic flux linked with it changes. But the difference is that there is no definite loop or path for induced current to flow away. As a result, the induced currents flow in concentric circular paths (Figure 4.11). As these electric currents resemble eddies of water, these are known as Eddy currents. They are also called Foucault currents.

Demonstration

Here is a simple demonstration for the production of eddy currents. Consider a pendulum that can be made to oscillate between the poles of a powerful electromagnet as shown in Figure 4.12(a).

First the electromagnet is switched off, the pendulum is slightly displaced and released. It begins to oscillate and it executes a large number of oscillations before stopping. The air friction is the only damping force.

When the electromagnet is switched on and the disc of the pendulum is made to oscillate, eddy currents are produced in it which will oppose the oscillation. A heavy damping force of eddy currents will bring the pendulum to rest within a few oscillations (Figure 4.12(b)).

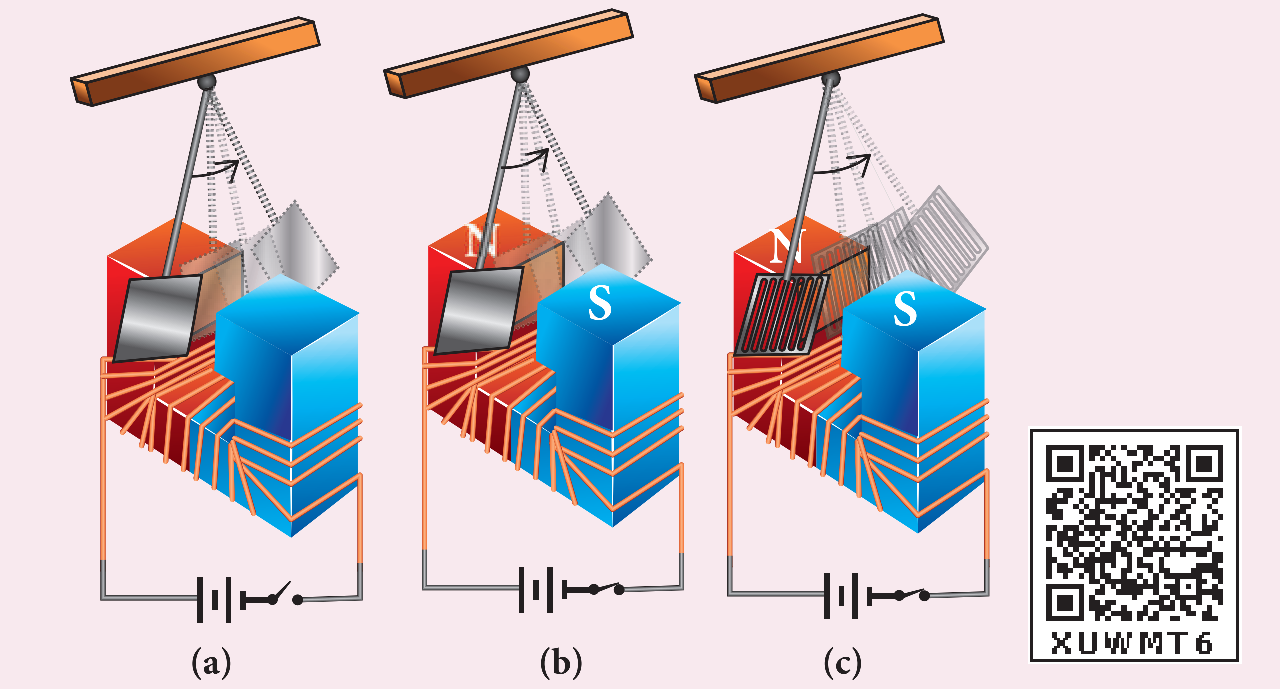


Figure 4.12 Demonstration of eddy currents

However if some slots are cut in the disc as shown in Figure 4.12(c), the eddy currents are reduced. The pendulum now will execute several oscillations before coming to rest. This clearly demonstrates the production of eddy current in the disc of the pendulum.

Drawbacks of Eddy currents

When eddy currents flow in the conductor, a large amount of energy is dissipated in the form of heat. The energy loss due to the flow of eddy current is inevitable but it can be reduced to a greater extent with suitable measures.

The design of transformer core and electric motor armature is crucial in order to minimise the eddy current loss. To reduce these losses, the core of the transformer is made up of thin laminas insulated from one another (Figure 4.13 (a)) while for electric motor the winding is made up of a group of wires insulated from one another (Figure 4.13 (b)). The insulation used does not allow huge eddy currents to flow and hence losses are minimized.

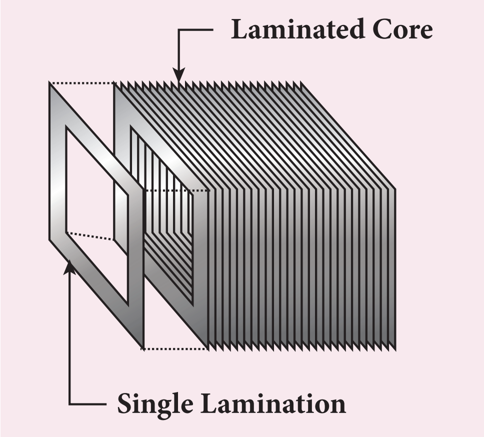


Figure 4.13 (a) Insulated laminas of the core of a transformer

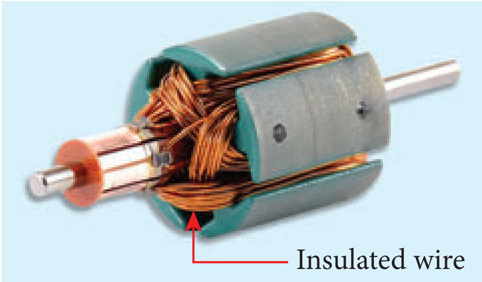
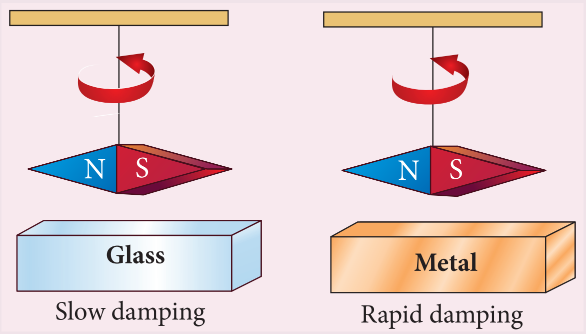


Figure 4.13 (b) Insulated winding of an electric motor

Activity



Make a pendulum with a strong magnet suspended at the lower end of the suspension wire as shown in the first figure. Make it oscillate with a glass plate below it and note the time it takes to come to rest.

Next just place a metallic plate below the oscillating magnet as shown in the second figure and again note the time it takes to stop.

In the second case, the magnet stops soon because eddy currents are produced in the plate which opposes the oscillation of the magnet.

Example

A spherical stone and a spherical metallic ball of same size and mass are dropped from the same height. Which one, a stone or a metal ball, will reach the earth's surface first? Justify your answer. Assume that there is no air friction.

Answer

The stone will reach the earth's surface earlier than the metal ball. The reason is that when the metal ball falls through the magnetic field of earth, the eddy currents are produced in it which opposes its motion. But in the case of stone, no eddy currents are produced and it falls freely.

Application of eddy currents

Though the production of eddy current is undesirable in some cases, it is useful in some other cases. A few of them are

i. Induction stove
ii. Eddy current brake
iii. Eddy current testing
iv. Electromagnetic damping

i. Induction stove

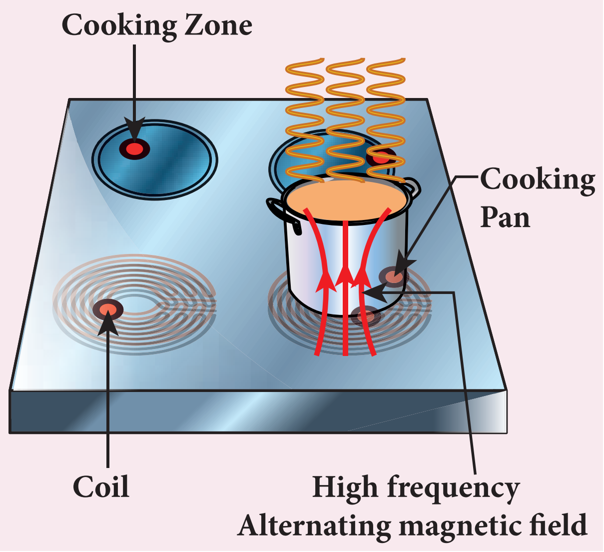


Figure 4.14 Induction stove

Induction stove is used to cook the food quickly and safely with less energy consumption. Below the cooking zone, there is a tightly wound coil of insulated wire. The cooking pan made of suitable material, is placed over the cooking zone. When the stove is switched on, an alternating current flowing in the coil produces high frequency alternating magnetic field which induces very strong eddy currents in the cooking pan. The eddy currents in the pan produce so much of heat due to Joule heating which is used to cook the food.

Note: The frequency of the domestic AC supply is increased from 50–60 Hz to around 20–40 KHz before giving it to the coil in order to produce high frequency alternating magnetic field.

ii. Eddy current brake

This eddy current braking system is generally used in high speed trains and roller coasters. Strong electromagnets are fixed just above the rails. To stop the train, electromagnets are switched on. The magnetic field of these magnets induces eddy currents in the rails which oppose or resist the movement of the train. This is Eddy current linear brake (Figure 4.15(a)).

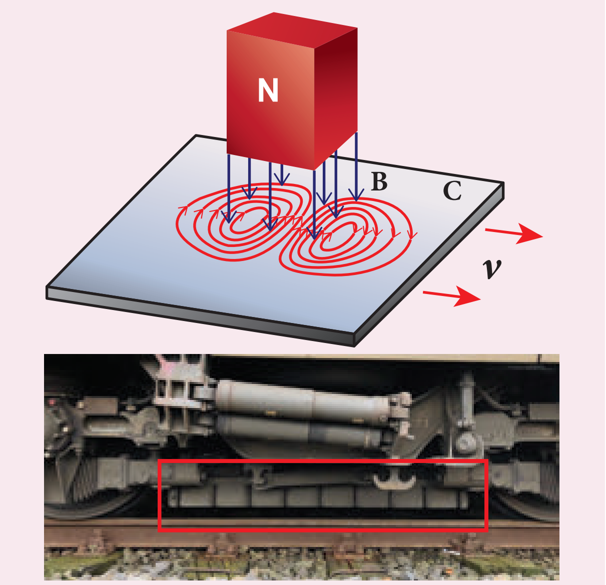


Figure 4.15 (a) Linear Eddy current brake

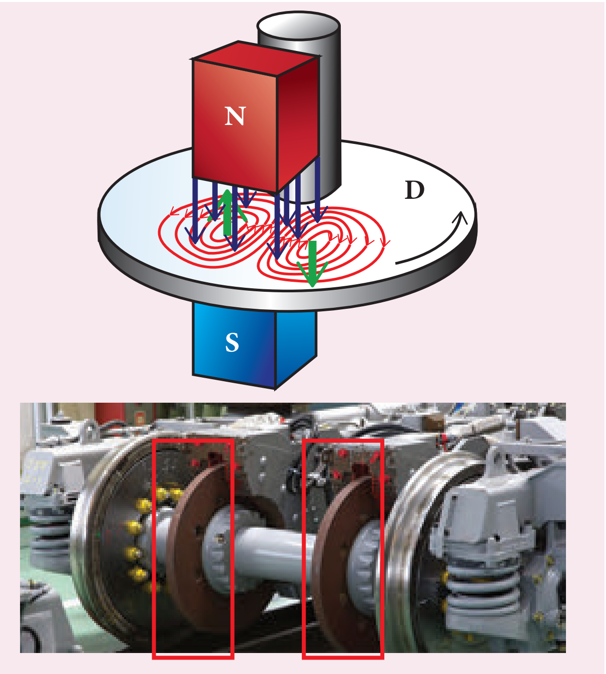


Figure 4.15(b) Circular Eddy current brake

In some cases, the circular disc, connected to the wheel of the train through a common shaft, is made to rotate in between the poles of an electromagnet. When there is a relative motion between the disc and the magnet, eddy currents are induced in the disc which stop the train. This is Eddy current circular brake (Figure 4.15(b))

iii. Eddy current testing

It is one of the simple non–destructive testing methods to find defects like surface cracks, air bubbles present in a specimen. A coil of insulated wire is given an alternating electric current so that it produces an alternating magnetic field. When this coil is brought near the test surface, eddy current is induced in the test surface. The presence of defects causes the change in phase and amplitude of the eddy current that can be detected by some other means.In this way, the defects present in the specimen are identified (Figure 4.16).

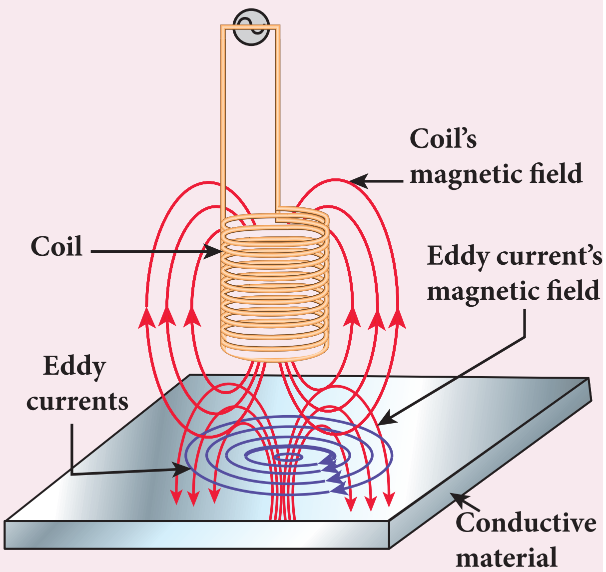


Figure 4.16 Eddy current testing

iv. Electro magnetic damping

The armature of the galvanometer coil is wound on a soft iron cylinder. Once the

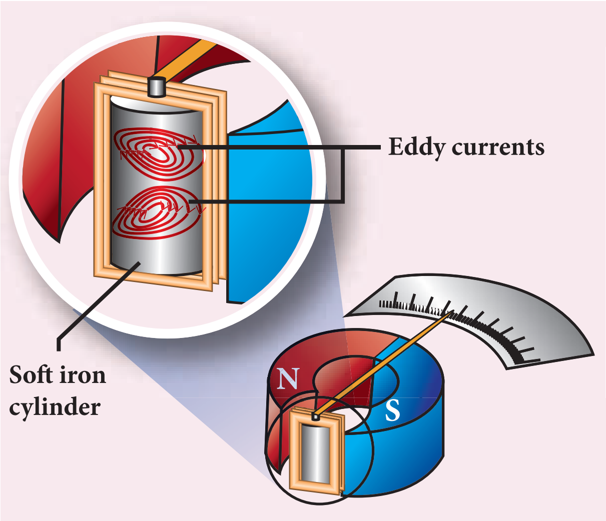


Figure 4.17 Electromagnetic damping

armature is deflected, the relative motion between the soft iron cylinder and the radial magnetic field induces eddy current in the cylinder (Figure 4.17). The damping force due to the flow of eddy current brings the armature to rest immediately and then galvanometer shows a steady deflection. This is called electromagnetic damping.

4.3 SELF–INDUCTION

4.3.1 Introduction

Inductor is a device used to store energy in a magnetic field when an electric current flows through it. The typical examples are coils, solenoids and toroids shown in Figure 4.18.

Inductance is the property of inductors to generate emf due to the change in current flowing through that circuit (self-induction) or a change in current through a neighbouring circuit with which it is magnetically linked (mutual induction). We will study about self-induction and mutual induction in the next sections.

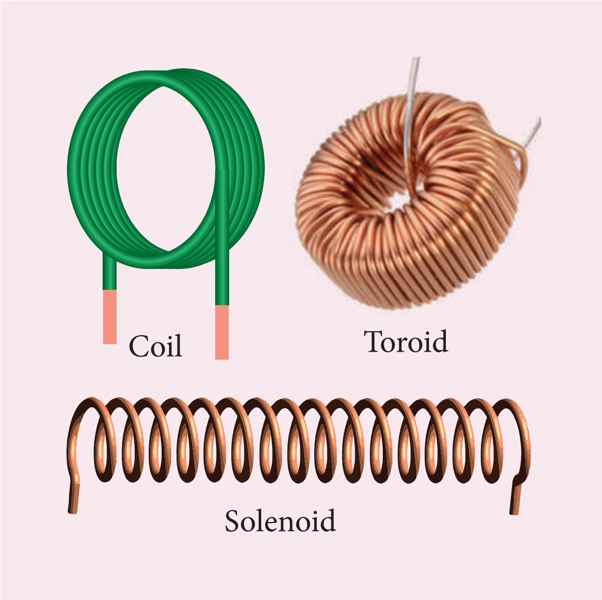


Figure 4.18 Examples for inductor

Self-induction

An electric current flowing through a coil will set up a magnetic field around it. Therefore, the magnetic flux of the magnetic field is linked with that coil itself. If this flux is changed by changing the current, an emf is induced in that same coil (Figure 4.19). This phenomenon is known as self-induction. The emf induced is called self-induced emf.

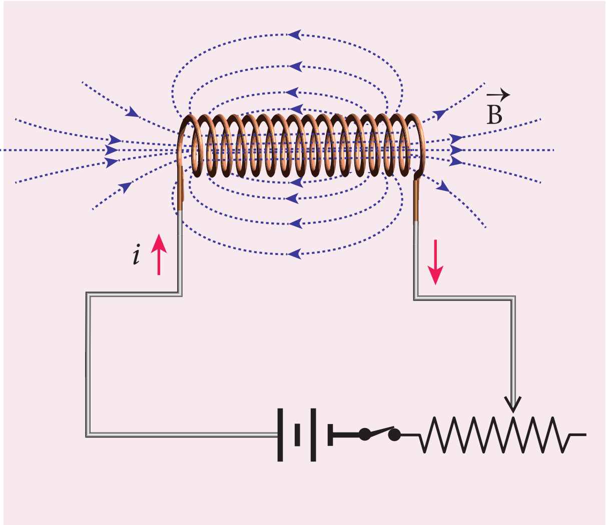


Figure 4.19 Self-Induction

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Let Φ_B be the magnetic flux linked with each turn of the coil of N turns, then the total flux linked with the coil $N\Phi_B$ (flux linkage) is proportional to the current *i* in the coil.

$$N\Phi_B \propto i$$

$$N\Phi_B = Li \qquad (4.15)$$

$$\text{(or)} \quad L = \frac{N\Phi_B}{i}$$

The constant of proportionality L is called self-inductance of the coil. It is also referred to as the coefficient of self-induction. If $i = 1A$, then $L = N\Phi_B$. **Self-inductance or simply inductance of a coil is defined as the flux linkage of the coil when 1A current flows through it.**

When the current *i* changes with time, an emf is induced in it. From Faraday's law of electromagnetic induction, this self-induced emf is given by

$$\varepsilon = -\frac{d(N\Phi_B)}{dt} = -\frac{d(Li)}{dt}$$

(using equation 4.15)

$$\therefore \varepsilon = -L\frac{di}{dt} \qquad (4.16)$$

$$\text{(or)} \quad L = \frac{-\varepsilon}{di/dt}$$

The negative sign in the above equation means that the self-induced emf always opposes the change in current with respect to time. If $di/dt = 1\,A\,s^{-1}$, then $L = -\varepsilon$. **Inductance of a coil is also defined as the opposing emf induced in the coil when the rate of change of current through the coil is 1 $A\,s^{-1}$.**

Unit of inductance

Inductance is a scalar and its unit is $Wb\,A^{-1}$ or $V\,s\,A^{-1}$. It is also measured in henry (H).

$1\,H = 1\,Wb\,A^{-1} = 1\,V\,s\,A^{-1}$

The dimensional formula of inductance is $M\,L^2\,T^{-2}A^{-2}$.

If i = 1 A and $N\Phi_B = 1$ *Wb turns*, then $L = 1H$.

Therefore, **the inductance of the coil is said to be one henry if a current of 1 *A* produces unit flux linkage in the coil.**

If $di/dt = 1\,A\,s^{-1}$ and $\varepsilon = -1\,V$, then $L = 1H$.

Therefore, **the inductance of the coil is one henry if a current changing at the rate of 1 $A\,s^{-1}$ induces an opposing emf of 1 *V* in it.**

Physical significance of inductance

We have learnt about inertia in XI standard. In translational motion, mass is a measure of inertia; in the same way, for rotational motion, moment of inertia is a measure of rotational inertia (Refer sections 3.2.1 and 5.4 of XI physics text book). Generally, inertia means opposition to change its state.

The inductance plays the same role in a circuit as mass and moment of inertia play in mechanical motion. When a circuit is switched on, the increasing current induces an emf which opposes the growth of current in a circuit (Figure 4.20(a)). Likewise, when circuit is broken, the decreasing current induces an emf in the reverse direction. This emf now opposes the decay of current (Figure 4.20(b)).

Thus, inductance of the coil opposes any change in current and tries to maintain the original state.

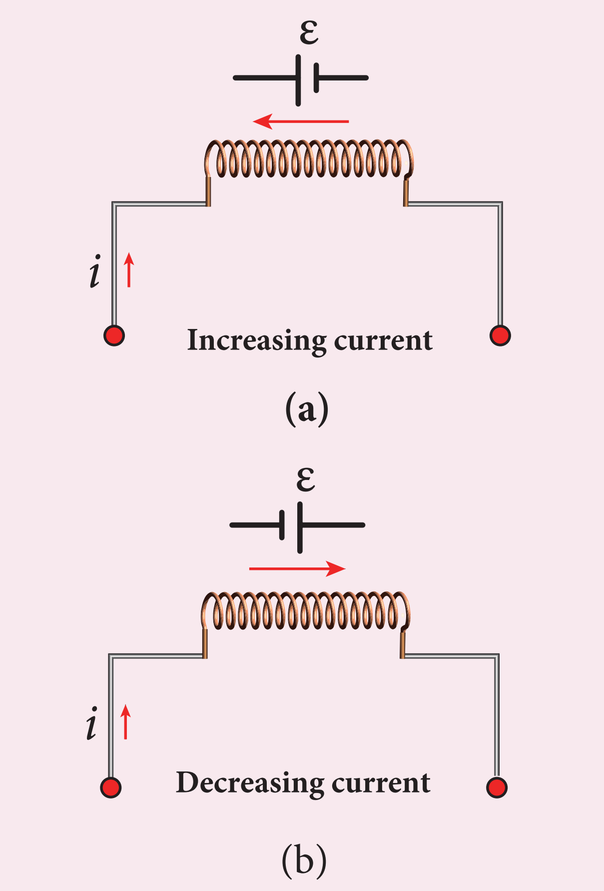

Figure 4.20 Induced emf ε opposes the changing current i

4.3.2 Self-inductance of a long solenoid

Consider a long solenoid of length l and cross-sectional area A. Let n be the number of turns per unit length (or turn density) of the solenoid. When an electric current i is passed through the solenoid, a magnetic field is produced by it which is almost uniform and is directed along the axis of the solenoid as shown in Figure 4.21. The magnetic field at any point inside the solenoid is given by (Refer section 3.9.3)

$$B = \mu_\circ ni$$

As this magnetic field passes through the solenoid, the windings of the solenoid are linked by the field lines. The magnetic flux passing through each turn is

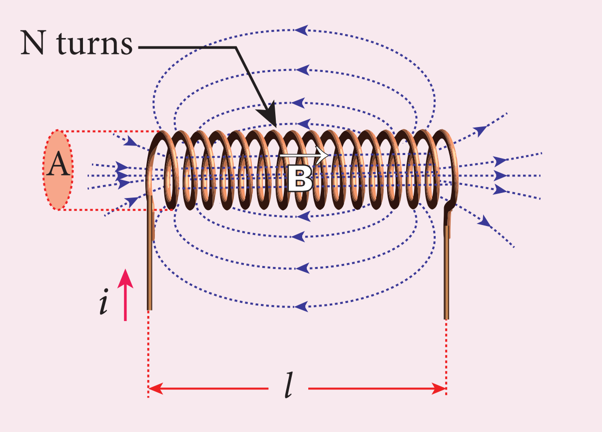

Figure 4.21 Self-inductance of a long solenoid

$$\Phi_B = \int_A \vec{B}.d\vec{A} = BA\cos\theta = BA \text{ since } \theta = 0°$$

$$= (\mu_0 ni)A$$

The total magnetic flux linked or flux linkage of the solenoid with N turns (the total number of turns N is given by $N = n\,l$) is

$$N\Phi_B = (nl)(\mu_0 ni)\text{A}$$

$$N\Phi_B = (\mu_\circ n^2 Al)\,i \qquad (4.17)$$

The equation (4.15) is

$$N\Phi_B = Li$$

Comparing equations (4.15) and (4.17), we have

$$L = \mu_\circ n^2 Al$$

From the above equation, it is clear that inductance depends on the geometry of the solenoid (turn density n, cross-sectional area A, length l) and the medium present inside the solenoid. If the solenoid is filled with a dielectric medium of relative permeability μ_r, then

$$L = \mu n^2 Al \;\; or \;\; L = \mu_\circ \mu_r n^2 Al$$

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Energy stored in an inductor

Whenever a current is established in the circuit, the inductance opposes the growth of the current. In order to establish a current in the circuit, work is done against this opposition by some external agency. This work done is stored as magnetic potential energy.

Let us assume that electrical resistance of the inductor is negligible and inductor effect alone is considered. The induced emf ε at any instant t is

$$\varepsilon = -L\frac{di}{dt}$$

Let dW be work done in moving a charge dq in a time dt against the opposition, then

$$dW = -\varepsilon dq$$

$$= -\varepsilon i dt \qquad \because dq = idt$$

Substituting for ε from equation (4.16),

$$= -\left(-L\frac{di}{dt}\right)idt$$

$$dW = Lidi$$

Total work done in establishing the current i is

$$W = \int dW = \int_0^i Lidi = L\left[\frac{i^2}{2}\right]_o^i$$

$$W = \frac{1}{2}Li^2$$

This work done is stored as magnetic potential energy.

$$\therefore U_B = \frac{1}{2}Li^2 \qquad (4.18)$$

The energy density is the energy stored per unit volume of the space and is given by

$$u_B = \frac{U_B}{Al} \qquad \because \text{ Volume of the solenoid} = Al$$

$$u_B = \frac{Li^2}{2Al} = \frac{(\mu_0 n^2 Al)i^2}{2Al} \qquad \because L = \mu_0 n^2 Al$$

$$= \frac{\mu_0 n^2 i^2}{2}$$

$$u_B = \frac{B^2}{2\mu_0} \qquad \because B = \mu_0 ni$$

EXAMPLE 4.10

A solenoid of 500 turns is wound on an iron core of relative permeability 800. The length and radius of the solenoid are 40 cm and 3 cm respectively. Calculate the average emf induced in the solenoid if the current in it changes from 0 to 3 A in 0.4 second.

Solution

N = 500 turns; $\mu_r = 800$;

l = 40 cm = 0.4 m; r = 3 cm = 0.03 m;

di = 3 – 0 = 3 A; dt = 0.4 s

Self inductance,

$$L = \mu n^2 Al \quad \left(\because \mu = \mu_o\mu_r; A = \pi r^2; n = \frac{N}{l}\right)$$

$$= \frac{\mu_o\mu_r N^2 \pi r^2}{l}$$

$$= \frac{4\times 3.14\times 10^{-7}\times 800\times 500^2\times 3.14\times (3\times 10^{-2})^2}{0.4}$$

L = 1.77 H

Magnitude of induced emf, $\varepsilon = L\frac{di}{dt}$

$$= \frac{1.77\times 3}{0.4}$$

$$\varepsilon = 13.275V$$

EXAMPLE 4.11

The self-inductance of an air-core solenoid is 4.8 mH. If its core is replaced by iron core, then its self-inductance becomes 1.8 H. Find out the relative permeability of iron.

Solution

$$L_{air} = 4.8\times10^{-3}H$$

$$L_{iron} = 1.8H$$

$$L_{air} = \mu_\circ n^2 Al = 4.8\times10^{-3}H$$

$$L_{iron} = \mu n^2 Al = \mu_\circ\mu_r n^2 Al = 1.8H$$

$$\therefore \mu_r = \frac{L_{iron}}{L_{air}} = \frac{1.8}{4.8\times10^{-3}} = 375$$

4.3.3 Mutual induction

When an electric current passing through a coil changes with time, an emf is induced in the neighbouring coil. This phenomenon is known as mutual induction and the emf is called mutually induced emf.

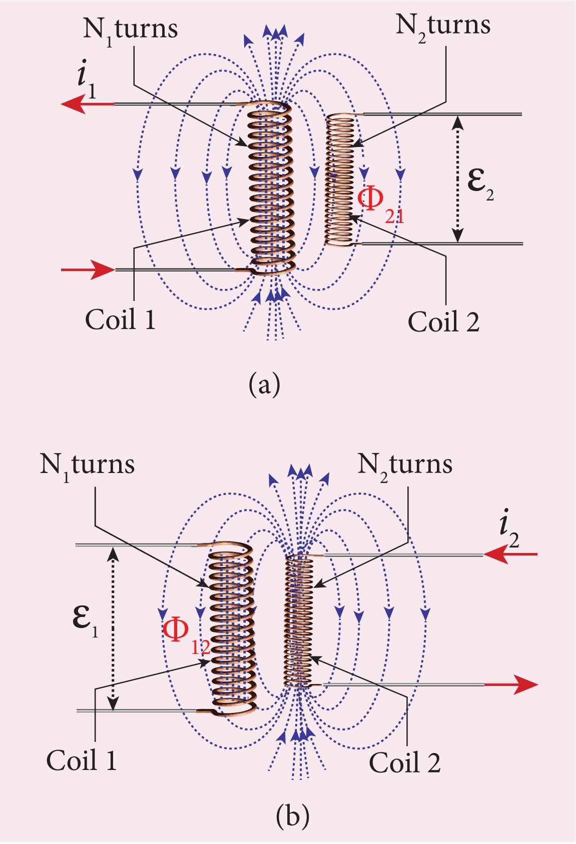


Figure 4.22 Mutual induction

Consider two coils which are placed close to each other. If an electric current i_1 is sent through coil 1, the magnetic field produced by it is also linked with coil 2 as shown in Figure 4.22(a).

Let Φ_{21} be the magnetic flux linked with each turn of the coil 2 of N_2 turns due to coil 1, then the total flux linked with coil 2 $(N_2\Phi_{21})$ is proportional to the current i_1 in the coil 1.

$$N_2\Phi_{21} \propto i_1$$

$$N_2\Phi_{21} = M_{21}\, i_1 \qquad (4.19)$$

$$\text{(or)}\; M_{21} = \frac{N_2\Phi_{21}}{i_1}$$

The constant of proportionality M_{21} is the mutual inductance of the coil 2 with respect to coil 1. It is also called as coefficient of mutual induction. If $i_1 = 1A$, then $M_{21} = N_2\Phi_{21}$. Therefore, **the mutual inductance M_{21} is defined as the flux linkage of the coil 2 when 1A current flows through coil 1.**

When the current i_1 changes with time, an emf ε_2 is induced in coil 2. From Faraday's law of electromagnetic induction, this mutually induced emf ε_2 is given by

$$\varepsilon_2 = -\frac{d(N_2\Phi_{21})}{dt} = -\frac{d(M_{21}i_1)}{dt}$$

$$\varepsilon_2 = -M_{21}\frac{di_1}{dt}$$

$$\text{(or)}\; M_{21} = \frac{-\varepsilon_2}{di_1/dt}$$

The negative sign in the above equation shows that the mutually induced emf always opposes the change in current i_1 with respect to time. If $di_1/dt = 1A\,s^{-1}$, then $M_{21} = -\varepsilon_2$.

Mutual inductance M_{21} is also defined as the opposing emf induced in the coil 2 when the rate of change of current through the coil 1 is $1As^{-1}$.

Similarly, if an electric current i_2 through coil 2 changes with time, then emf ε_1 is induced in coil 1. Therefore,

$$M_{12} = \frac{N_1\Phi_{12}}{i_2} \quad \text{and} \quad M_{12} = \frac{-\varepsilon_1}{di_2/dt}$$

where M_{12} is the mutual inductance of the coil 1 with respect to coil 2. It can be shown that for a given pair of coils, the mutual inductance is same.

$$\text{i.e.,} \quad M_{21} = M_{12} = M$$

In general, the mutual induction between two coils depends on size, shape, the number of turns of the coils, their relative orientation and permeability of the medium.

Unit of mutual-inductance

The unit of mutual inductance is also henry (H).

If $i_1 = 1A$ and $N_2\Phi_{21} = 1\ Wb\ turns$, then $M_{21} = 1H$.

Therefore, **the mutual inductance between two coils is said to be one henry if a current of 1A in coil 1 produces unit flux linkage in coil 2.**

If $\frac{di_1}{dt} = 1As^{-1}$ and $\varepsilon_2 = -1\ V$, then $M_{21} = 1H$.

Therefore, **the mutual inductance between two coils is one henry if a current changing at the rate of $1As^{-1}$ in coil 1 induces an opposing emf of 1V in coil 2.**

4.3.4 Mutual inductance between two long co-axial solenoids

Consider two long co-axial solenoids of same length l. The length of these solenoids is large when compared to their radii so that the magnetic field produced inside the solenoids is uniform and the fringing effect at the ends may be ignored. Let A_1 and A_2 be the area of cross section of the solenoids with A_1 being greater than A_2 as shown in Figure 4.23. The turn density of these solenoids are n_1 and n_2 respectively.

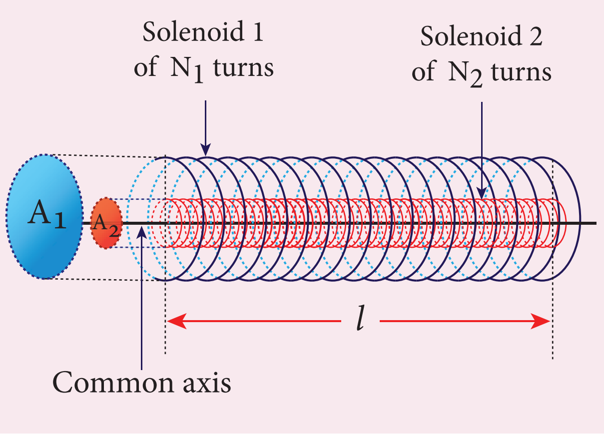


Figure 4.23 Mutual inductance of two long co-axial solenoids

Let i_1 be the current flowing through solenoid 1, then the magnetic field produced inside it is

$$B_1 = \mu_\circ n_1 i_1$$

As the field lines of $\vec{B_1}$ are passing through the area bounded by solenoid 2, the magnetic flux is linked with each turn of solenoid 2 due to solenoid 1 and is given by

$$\Phi_{21} = \int_{A_2} \vec{B_1}.d\vec{A} = B_1A_2 \qquad \text{since } \theta = 0°$$

$$= (\mu_\circ n_1 i_1) A_2$$

The flux linkage of solenoid 2 with total turns N_2 is

$$N_2\Phi_{21} = (n_2 l)(\mu_\circ n_1 i_1) A_2 \quad \text{since } N_2 = n_2 l$$

$$N_2\Phi_{21} = (\mu_\circ n_1 n_2 A_2 l) i_1 \tag{4.20}$$

From equation (4.19),

$$N_2\Phi_{21} = M_{21} i_1 \tag{4.21}$$

Comparing the equations (4.20) and (4.21),

$$M_{21} = \mu_\circ n_1 n_2 A_2 l \tag{4.22}$$

This gives the expression for mutual inductance M_{21} of the solenoid 2 with respect to solenoid 1. Similarly, we can find mutual inductance M_{12} of solenoid 1 with respect to solenoid 2 as given below.

The magnetic field produced by the solenoid 2 when carrying a current i_2 is

$$B_2 = \mu_\circ n_2 i_2$$

This magnetic field B_2 is uniform inside the solenoid 2 but outside the solenoid 2, it is almost zero. Therefore for solenoid 1, the area A_2 is the effective area over which the magnetic field B_2 is present; not area A_1. Then the magnetic flux Φ_{12} linked with each turn of solenoid 1 due to solenoid 2 is

$$\Phi_{12} = \int_{A_2} \vec{B}_2 \cdot d\vec{A} = B_2 A_2 = (\mu_\circ n_2 i_2) A_2$$

The flux linkage of solenoid 1 with total turns N_1 is

$$N_1\Phi_{12} = (n_1 l)(\mu_o n_2 i_2) A_2 \quad \text{since } N_1 = n_1 l$$

$$N_1\Phi_{12} = (\mu_o n_1 n_2 A_2 l) i_2$$

$$\text{since } N_1\Phi_{12} = M_{12} i_2$$

$$M_{12} i_2 = (\mu_\circ n_1 n_2 A_2 l) i_2$$

Therefore, we get

$$\therefore M_{12} = \mu_\circ n_1 n_2 A_2 l \tag{4.23}$$

From equation (4.22) and (4.23), we can write

$$M_{12} = M_{21} = M \tag{4.24}$$

In general, the mutual inductance between two long co-axial solenoids is given by

$$M = \mu_\circ n_1 n_2 A_2 l \tag{4.25}$$

If a dielectric medium of relative permeability μ_r is present inside the solenoids, then

$$M = \mu n_1 n_2 A_2 l$$

$$\text{(or)} \quad M = \mu_\circ \mu_r n_1 n_2 A_2 l$$

EXAMPLE 4.12

The current flowing in the first coil changes from 2 A to 10 A in 0.4 sec. Find the mutual inductance between two coils if an emf of 60 mV is induced in the second coil. Also determine the induced emf in the second coil if the current in the first coil is changed from 4 A to 16 A in 0.03 sec. Consider only the magnitude of induced emf.

Solution

Case (i):

$di_1 = 10 - 2 = 8$ A; $dt = 0.4$ s;

$\varepsilon_2 = 60 \times 10^{-3} V$

Case (ii):

$di_1 = 16 - 4 = 12$ A; $dt = 0.03$ s

(i) Mutual inductance of the second coil with respect to the first coil

$$M_{21} = \frac{\varepsilon_2}{di_1/dt}$$

$$= \frac{60\times10^{-3}\times0.4}{8}$$

$$M_{21} = 3\times10^{-3}H$$

(ii) Induced emf in the second coil due to the rate of change of current in the first coil is

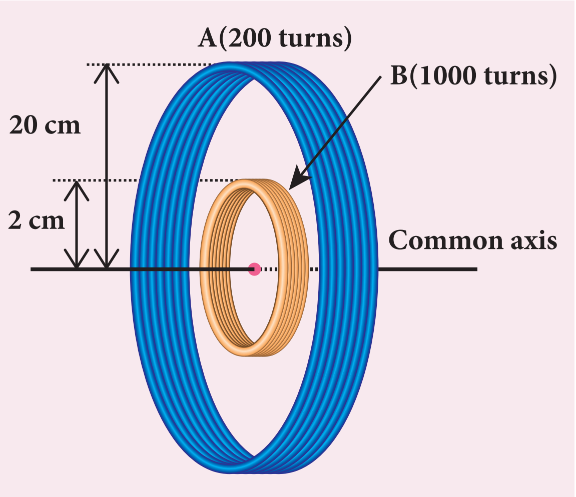
$$\varepsilon_2 = M_{21}\frac{di_1}{dt}$$

$$= \frac{3\times10^{-3}\times12}{0.03}$$

$$\varepsilon_2 = 1.2V$$

EXAMPLE 4.13

Consider two coplanar, co-axial circular coils *A* and *B* as shown in figure. The radius of coil *A* is 20 cm while that of coil *B* is 2 cm. The number of turns is 200 and 1000 for coils *A* and *B* respectively. Calculate the mutual inductance of coil *B* with respect to coil *A*. If the current in coil *A* changes from 2 A to 6 A in 0.04 sec, determine the induced emf in coil *B* and the rate of change of flux through the coil *B* at that instant.



Solution

N_A = 200 turns; N_B = 1000 turns;
$r_A = 20 \times 10^{-2}$ m; $r_B = 2 \times 10^{-2}$ m;
dt = 0.04 s; di_A = 6−2 = 4A

Let i_A be the current flowing in coil *A*, then the magnetic field B_A at the centre of the circular coil *A* is

$$B_A = \frac{\mu_\circ N_A i_A}{2r_A} = \frac{4\pi\times10^{-7}N_A i_A}{2r_A}$$

$$= \frac{10^{-7}\times2\times3.14\times200}{20\times10^{-2}}\times i_A$$

$$= 6.28\times10^{-4} i_A \ Wbm^{-2}$$

The magnetic flux linkage of coil *B* is

$$N_B\Phi_B = N_B B_A A_B$$

$$= 1000\times6.28\times10^{-4}\times i_A\times3.14\times\left(2\times10^{-2}\right)^2$$

$$= 7.89\times10^{-4} i_A \quad Wb\ turns$$

The mutual inductance of the coil *B* with respect to coil *A* is

$$M_{BA} = \frac{N_B\Phi_B}{i_A} = 7.89\times10^{-4}H$$

Induced emf in coil *B* is

$$\varepsilon_B = -M_{BA}\frac{di_A}{dt}$$

Considering only the magnitude,

$$\varepsilon_B = \frac{7.89\times10^{-4}\times(6-2)}{0.04}$$

$$\varepsilon_B = \frac{7.89\times10^{-4}\times(4)}{4\times10^{-2}}$$

$$\varepsilon_B = 7.89\times10^{-2}V$$

$$\varepsilon_B = 78.9mV$$

The rate of change of magnetic flux of coil *B* is

$$\frac{d(N_B\Phi_B)}{dt} = \varepsilon_B = 78.9\ mWbs^{-1}$$

4.4 METHODS OF PRODUCING INDUCED EMF

4.4.1 Introduction

Electromotive force is the characteristic of any energy source capable of driving electric charge around a circuit. We have already learnt that it is not actually a force. It is the work done in moving unit electric charge around the circuit. It is measured in $J\,C^{-1}$ or volt.

Some examples of energy source which provide emf are electrochemical cells, thermoelectric devices, solar cells and electrical generators. Of these, electrical generators are most powerful machines. They are used for large scale power generation.

According to Faraday's law of electromagnetic induction, an emf is induced in a circuit when magnetic flux linked with it changes. This emf is called induced emf. The magnitude of the induced emf is given by

$$\varepsilon = \frac{d\Phi_B}{dt} \quad or$$

$$\varepsilon = \frac{d}{dt}(BA\cos\theta) \qquad (4.26)$$

From the above equation, it is clear that induced emf can be produced by changing magnetic flux in any of the following ways.

(i) By changing the magnetic field B

(ii) By changing the area A of the coil and

(iii) By changing the relative orientation θ of the coil with magnetic field

4.4.2 Induction of emf by changing the magnetic field

From Faraday's experiments on electromagnetic induction, it was discovered that an emf is induced in a circuit on changing the magnetic flux of the field through it. The change in flux is brought about by (i) relative motion between the circuit and the magnet (First experiment) (ii) variation in current flowing through the nearby coil (Second experiment).

4.4.3 Induction of emf by changing the area of the coil

Consider a conducting rod of length l moving with a velocity v towards left on a rectangular metallic framework as shown in Figure 4.24. The whole arrangement is placed in a uniform magnetic field $\vec{B}$ whose magnetic lines are perpendicularly directed into the plane of the paper.

As the rod moves from AB to DC in a time dt, the area enclosed by the loop and hence the magnetic flux through the loop decreases.

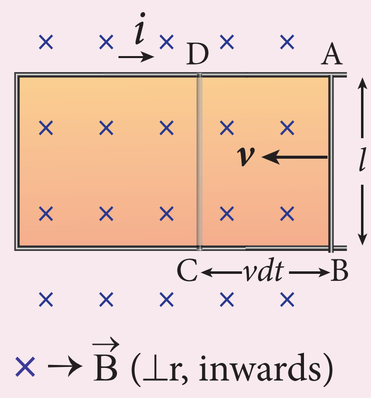


Figure 4.24 Induction of emf by changing the area enclosed by the loop

The change in magnetic flux in time dt is

$$d\Phi_B = B \times \text{ change in area}$$

$$= B \times \text{ Area ABCD}$$

$$= Blvdt \qquad \text{since Area ABCD} = l(vdt)$$

$$\text{(or) } \frac{d\Phi_B}{dt} = Blv$$

As a result of change in flux, an emf is generated in the loop. The magnitude of the induced emf is

$$\varepsilon = \frac{d\Phi_B}{dt}$$

$$\varepsilon = Blv \qquad (4.27)$$

This emf is called **motional emf**. The direction of induced current is found to be clockwise from Fleming's right hand rule.

EXAMPLE 4.14

A circular metal of area 0.03 m^2 rotates in a uniform magnetic field of 0.4 T. The axis of rotation passes through the centre and perpendicular to its plane and is also parallel to the field. If the disc completes 20 revolutions in one second and the resistance of the disc is 4 Ω, calculate the induced emf between the axis and the rim and induced current flowing in the disc.

Solution

A = 0.03 m^2; B = 0.4 T; f = 20 rps;
R = 4 Ω

Area covered in 1 sec = Area of the disc × frequency

= 0.03 × 20
= 0.6 m^2

Induced emf, ε= Rate of change of flux

$$= \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt}$$

$$\varepsilon = \frac{0.4 \times 0.6}{1}$$

$$\varepsilon = 0.24 \text{ V}$$

Induced current, $i = \frac{\varepsilon}{R} = \frac{0.24}{4} = 0.06\,A$

Note Emf can be induced by changing relative orientation between the coil and the magnetic field. This can be achieved either by rotating a coil in a magnetic field or by rotating a magnetic field within a stationary coil. Here rotating coil type is considered.

4.4.4 Induction of emf by changing relative orientation of the coil with the magnetic field

Consider a rectangular coil of N turns kept in a uniform magnetic field $\vec{B}$ as shown in Figure 4.25(a). The coil rotates in anti-clockwise direction with an angular velocity ω about an axis, perpendicular to the field.

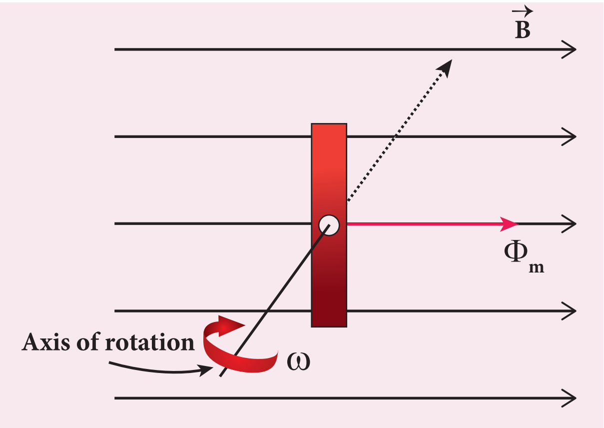


Figure 4.25(a) Top view of the coil with its plane perpendicular to the magnetic field

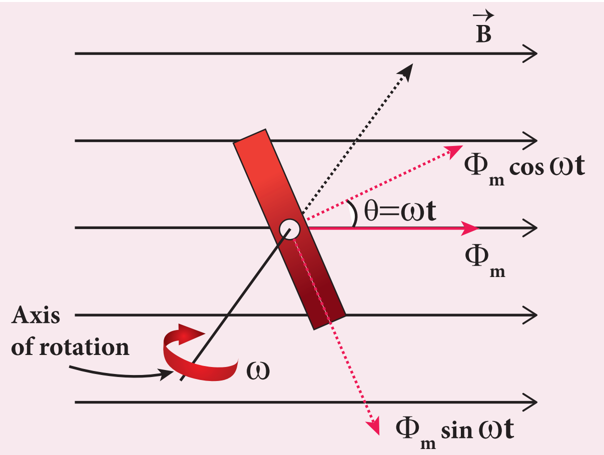


Figure 4.25(b) The coil has rotated through an angle $\theta = \omega t$

At time = 0, the plane of the coil is perpendicular to the field and the flux linked with the coil has its maximum value $\Phi_m = BA$ (where A is the area of the coil).

In a time t seconds, the coil is rotated through an angle θ (= ωt) in anti–clockwise direction. In this position, the flux linked is Φ_m cos ωt, a component of Φ_m normal to the plane of the coil (Figure 4.25(b)). The component parallel to the plane (Φ_m sin ωt) has no role in electromagnetic induction. Therefore, the flux linkage at this deflected position is

$$N\Phi_B = N\Phi_m \cos\omega t$$

According to Faraday's law, the emf induced at that instant is

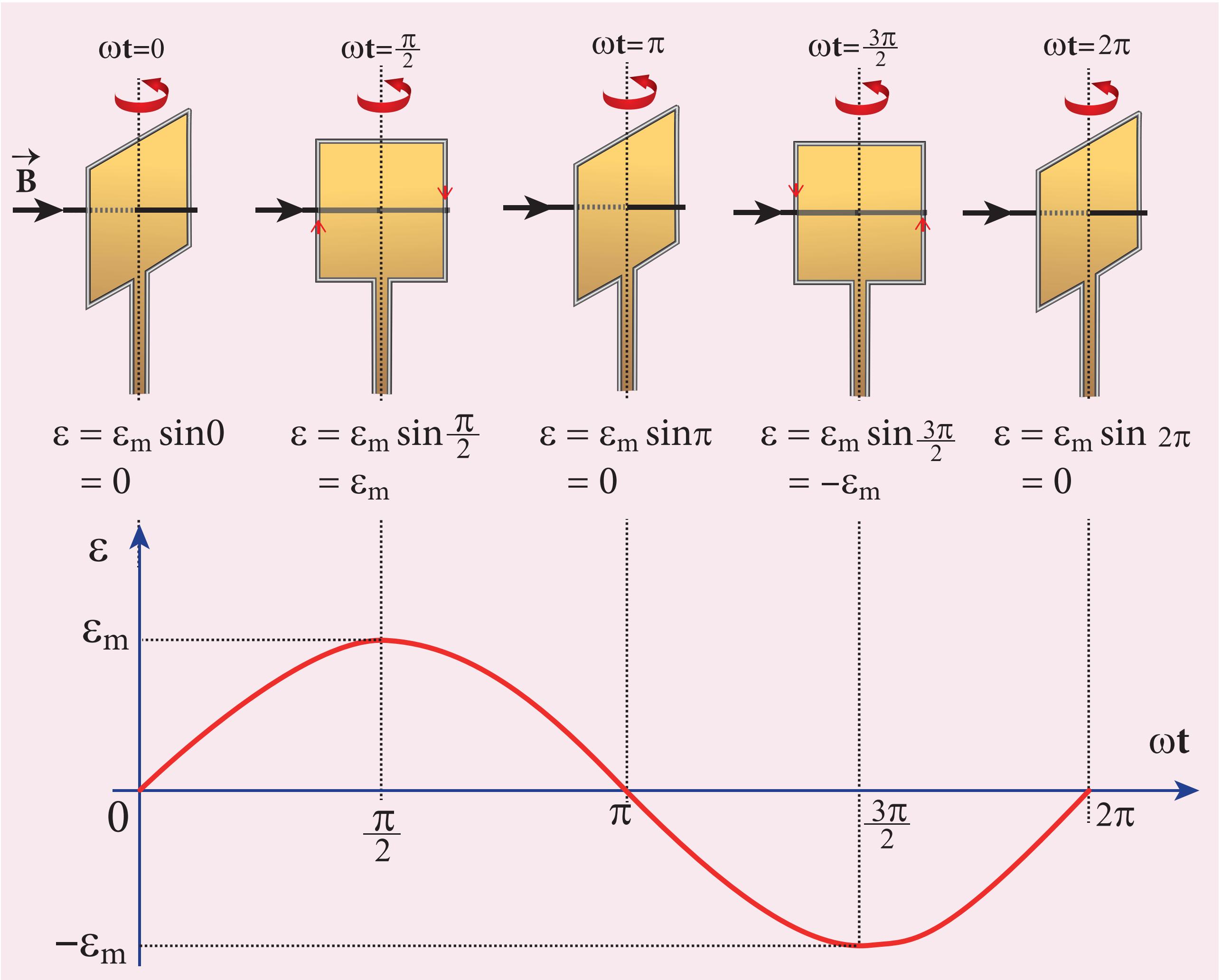


Figure 4.26 Variation of induced emf as a function of ωt

$$\varepsilon = -\frac{d}{dt}(N\Phi_B) = -\frac{d}{dt}(N\Phi_m \cos\omega t)$$

$$= -N\Phi_m(-\sin\omega t)\omega$$

$$= N\Phi_m \omega \sin\omega t$$

When the coil is rotated through 90° from initial position, sin ωt = 1. Then the maximum value of induced emf is

$$\varepsilon_m = N\Phi_m \omega$$

$$\varepsilon_m = NBA\omega \qquad \text{since } \Phi_m = BA$$

Therefore, the value of induced emf at that instant is then given by

$$\varepsilon = \varepsilon_m \sin\omega t \qquad (4.28)$$

It is seen that the induced emf varies as sine function of the time angle *ωt*. The graph between induced emf and time angle for one rotation of coil will be a sine curve (Figure 4.26) and the emf varying in this manner is called sinusoidal emf or alternating emf.

If this alternating voltage is given to a closed circuit, a sinusoidally varying current flows in it. This current is called alternating current and is given by

$$i = I_m \sin\omega t \qquad (4.29)$$

where I_m is the maximum value of induced current.

EXAMPLE 4.15

A rectangular coil of area 70 cm^2 having 600 turns rotates about an axis perpendicular to a magnetic field of 0.4 Wb m^{-2}. If the coil completes 500 revolutions in a minute, calculate the instantaneous emf when the plane of the coil is (i) perpendicular to the field (ii) parallel to the field and (iii) inclined at 60° with the field.

Solution

$A = 70 \times 10^{-4}\text{m}^2$; $N = 600$ turns

$B = 0.4$ Wbm^{-2}; $f = 500$ rpm

The instantaneous emf is

$$\varepsilon = \varepsilon_m \sin\omega t$$

$$\text{since } \varepsilon_m = N\Phi_m\omega = N(BA)(2\pi f)$$

$$\varepsilon = NBA \times 2\pi f \times \sin\omega t$$

(i) When $\omega t = 0^\circ$,

$$\varepsilon = \varepsilon_m \sin 0 = 0$$

(ii) When $\omega t = 90^\circ$,

$$\varepsilon = \varepsilon_m \sin 90^\circ = NBA \times 2\pi f \times 1$$

$$= 600 \times 0.4 \times 70 \times 10^{-4} \times 2 \times \frac{22}{7} \times \left(\frac{500}{60}\right)$$

$$= 88\,V$$

(iii) When ωt = 90° – 60° = 30°,

$$\varepsilon = \varepsilon_m \sin 30^\circ = 88 \times \frac{1}{2} = 44\ V$$

4.5 AC GENERATOR

4.5.1 Introduction

AC generator or alternator is an energy conversion device. It converts mechanical energy used to rotate the coil or field magnet into electrical energy. Alternator produces a large scale electrical power for use in homes and industries. AC generator and its components are shown in Figure 4.27.

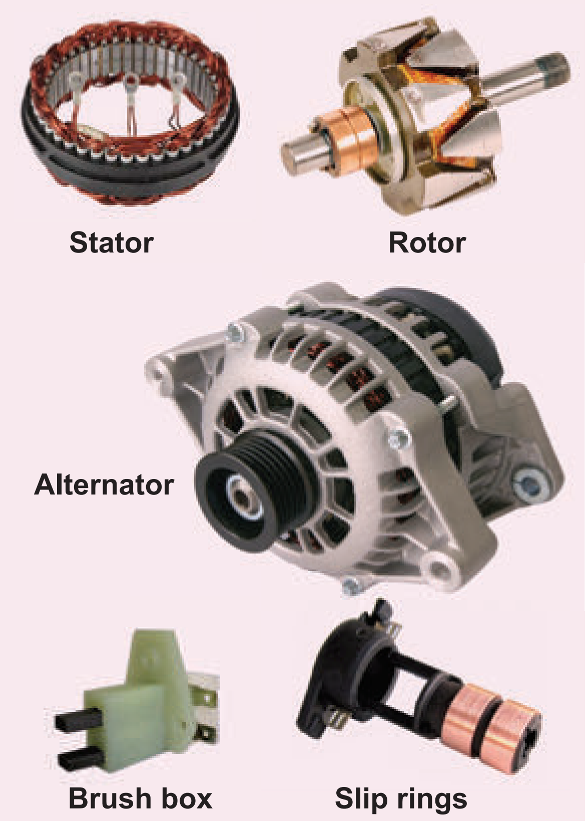


Figure 4.27 AC generator and its components

4.5.2 Principle

Alternators work on the principle of electromagnetic induction. The relative motion between a conductor and a magnetic field changes the magnetic flux linked with the conductor which in turn, induces an emf. The magnitude of the induced emf is given by Faraday's law of electromagnetic induction and its direction by Fleming's right hand rule.

Note Alternating emf is generated by rotating a coil in a magnetic field or by rotating a magnetic field within a stationary coil.
The first method is used for small AC generators while the second method is employed for large AC generators. The rotating-field method is the one which is mostly used in power stations.

4.5.3 Construction

Alternator consists of two major parts, namely stator and rotor. As their names suggest, stator is stationary while rotor rotates inside the stator. In any standard construction of commercial alternators, the armature winding is mounted on stator and the field magnet on rotor.

The construction details of stator, rotor and various other components involved in them are given below.

i) Stator

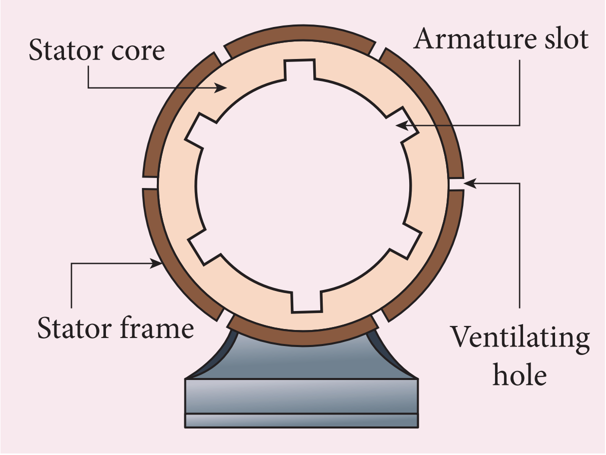


Figure 4.28 Stator and its components

The stationary part which has armature windings mounted in it is called stator. It has three components, namely stator frame, stator core and armature winding.

Stator frame

This is the outer frame used for holding stator core and armature windings in proper position. Stator frame provides best ventilation with the help of holes provided in the frame itself (Figure 4.28).

Stator core

Stator core or armature core is made up of iron or steel alloy. It is a hollow cylinder and is laminated to minimize eddy current loss. The slots are cut on inner surface of the core to accommodate armature windings.

Armature winding

Armature winding is the coil, wound on slots provided in the armature core. One or more than one coil may be employed, depending on the type of alternator.

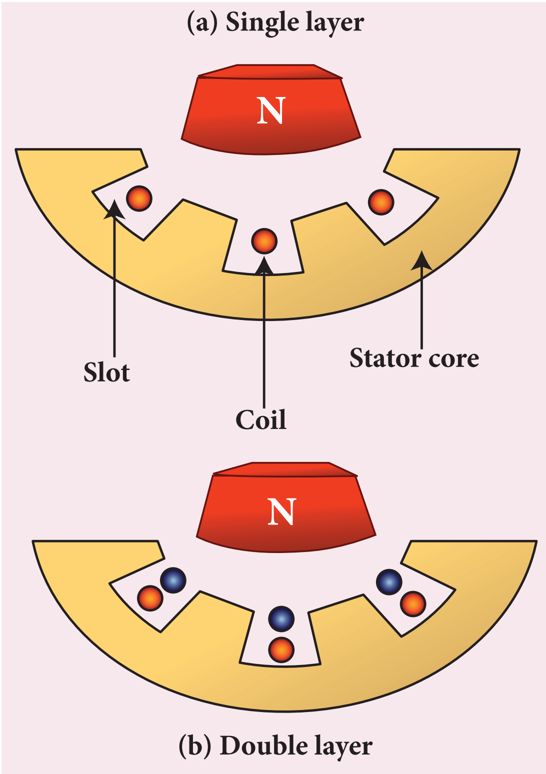


Figure 4.29 Armature windings

Two types of windings are commonly used. They are i) single-layer winding and ii) double-layer winding. In single-layer winding, a slot is occupied by a coil as a single layer. But in double-layer winding, the coils are split into two layers such as top and bottom layers (Figure 4.29).

ii) Rotor

Rotor contains magnetic field windings. The magnetic poles are magnetized by DC source. The ends of field windings are connected to a pair of slip rings, attached to a common shaft about which rotor rotates. Slip rings rotate along with rotor. To maintain connection between the DC source and field windings, two brushes are used which continuously slide over the slip rings.

There are 2 types of rotors used in alternators i) salient pole rotor and ii) cylindrical pole rotor.

MN5C7X

Salient pole rotor

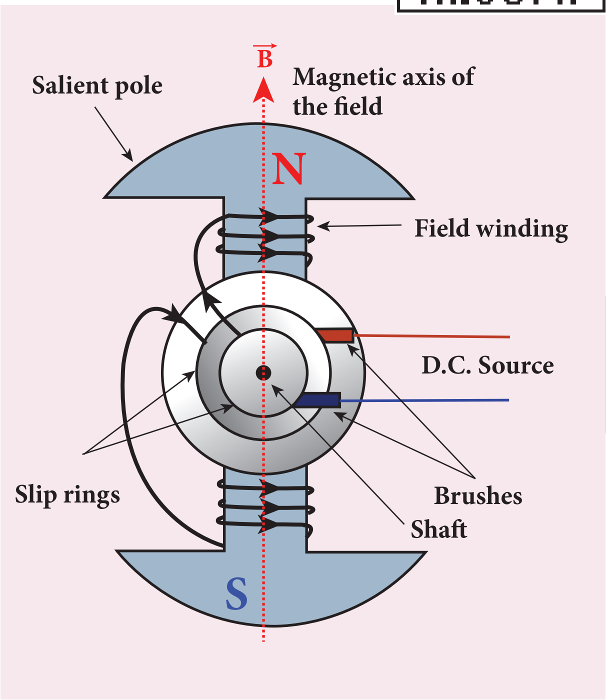


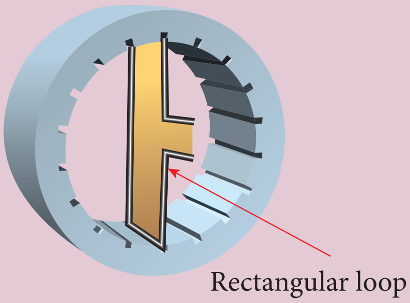
Figure 4.30 Salient 2-pole rotor

Construction of AC generator (Not for examination)

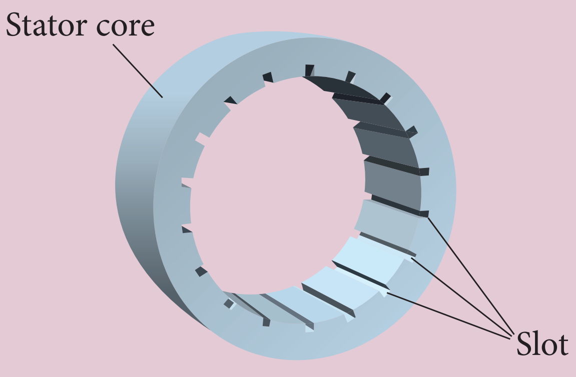
Alternator consists of two major parts, namely stator and rotor. (This box is given for better understanding of constructional details)

i) Stator

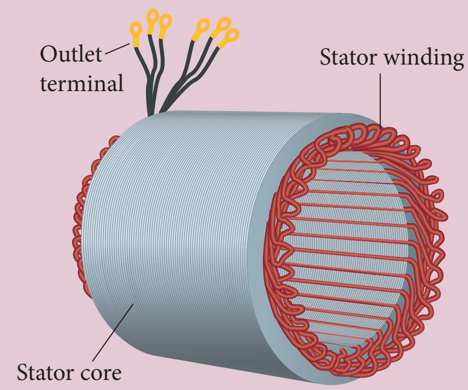
Stator has three components, namely stator frame, stator core and armature winding.



Figure(b): Stator core with rectangular loop



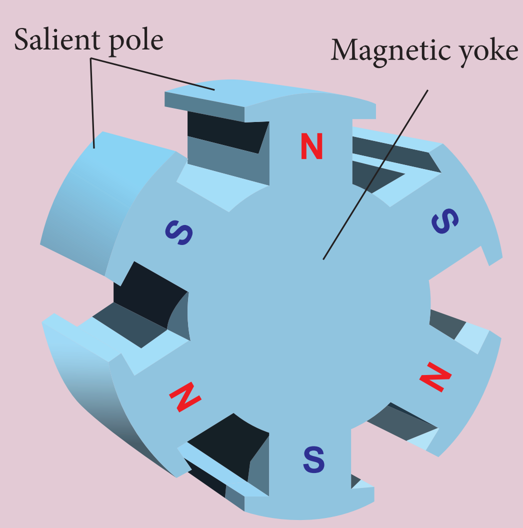
Figure(a): Stator core with empty slots



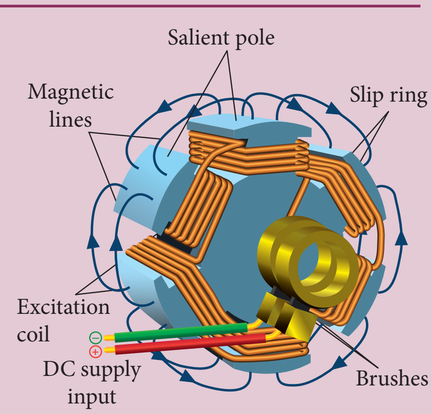
Figure(c): Stator core with armature windings

ii) Rotor

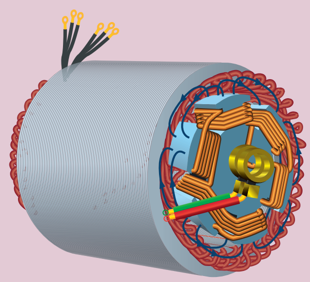
Rotor contains magnetic field windings, slip rings and brushes mounted on the same shaft.



Figure(d): Salient 6-pole rotor



Figure(e): Salient 6-pole rotor with field windings, slip rings and brushes



Figure(f): Stator core and rotor

The word salient means projecting. This rotor has a number of projecting poles having their bases riveted to the rotor. It is mainly used in low-speed alternators. The salient 2-pole rotor is shown in the Figure 4.30.

Cylindrical pole rotor

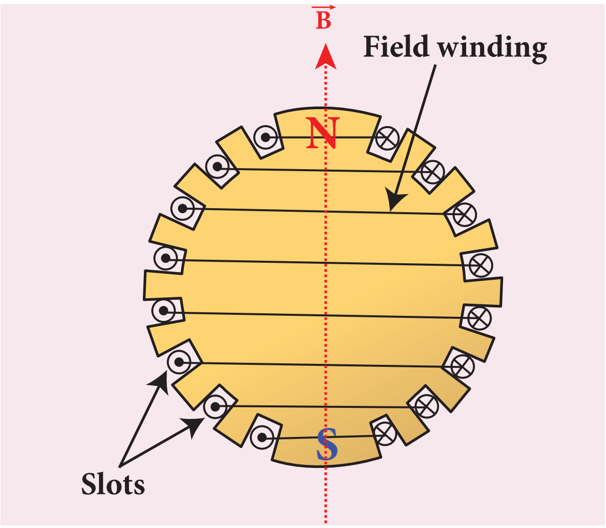


Figure 4.31 Cross-sectional view of cylindrical 2-pole rotor

This rotor consists of a smooth solid cylinder. The slots are cut on the outer surface of the cylinder along its length. It is suitable for very high speed alternators (Figure 4.31).

The frequency of alternating emf induced is directly proportional to the rotor speed. In order to maintain the frequency constant, the rotor must run at a constant speed.

These are standard construction details of alternators. Based on the type of alternator being constructed, the details like number of poles, pole type, number of coils and type of windings would change from one another.

We will discuss the construction and working of two examples, namely single phase and three phase AC generators in the following sections.

4.5.4 Advantages of stationary armature-rotating field alternator

Alternators are generally high current and high voltage machines. The stationary armature-rotating field construction has many advantages. A few of them include:

1) The current is drawn directly from fixed terminals on the stator without the use of brush contacts.
2) The insulation of stationary armature winding is easier.
3) The number of sliding contacts (slip rings) is reduced. Moreover, the sliding contacts are used for low-voltage DC Source.
4) Armature windings can be constructed more rigidly to prevent deformation due to any mechanical stress.

4.5.5 Single phase AC generator

In a single phase AC generator, the armature conductors are connected in series so as to form a single circuit which generates a single-phase alternating emf and hence it is called single-phase alternator.

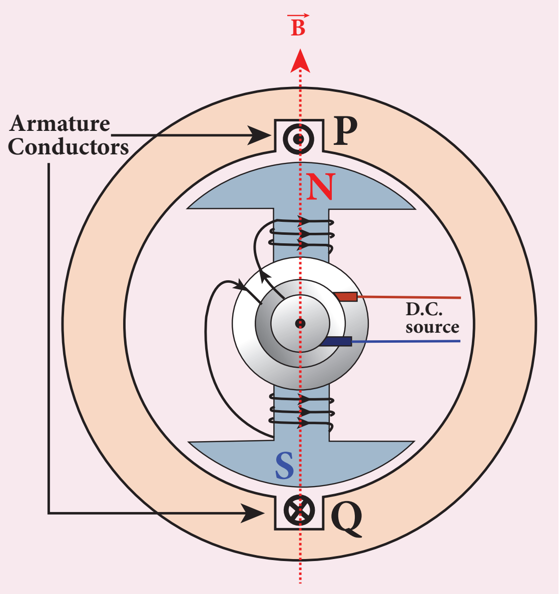


Figure 4.32 Stator core with a rectangular loop and 2-pole rotor

The simplified version of a AC generator is discussed here. Consider a stator core consisting of 2 slots in which 2 armature conductors PQ and RS are mounted to form single-turn rectangular loop PQRS as shown in Figure 4.33. Rotor has 2 salient poles with field windings which can be magnetized by means of DC source.

Working

The loop PQRS is stationary and is perpendicular to the plane of the paper. When field windings are excited, magnetic field is produced around it. The direction of magnetic field passing through the armature core is shown in Figure 4.33. Let the field magnet be rotated in clockwise direction by the prime mover. The axis of rotation is perpendicular to the plane of the paper.

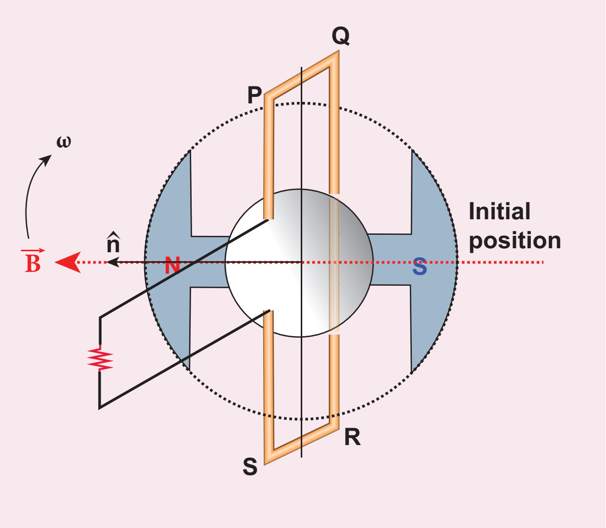


Figure 4.33 The loop PQRS and field magnet in its initial position

Assume that initial position of the field magnet is horizontal. At that instant, the direction of magnetic field is perpendicular to the plane of the loop PQRS. The induced emf is zero (Refer case (iii) of section 4.4). This is represented by origin O in the graph between induced emf and time angle (Figure 4.34).

When field magnet rotates through 90°, magnetic field becomes parallel to PQRS. The induced emfs across PQ and RS would become maximum. Since they are connected in series, emfs are added up and the direction of total induced emf is given by Fleming's right hand rule.

Care has to be taken while applying this rule; the thumb indicates the direction of the motion of the conductor with respect to field. For clockwise rotating poles, the conductor appears to be rotating anti-clockwise. Hence, thumb should point to the left. The direction of the induced emf is at right angles to the plane of the paper. For PQ, it is downwards and for RS upwards. Therefore, the current flows along PQRS. The point A in the graph represents this maximum emf.

For the rotation of 180° from the initial position, the field is again perpendicular to PQRS and the induced emf becomes zero. This is represented by point B.

The field magnet becomes again parallel to PQRS for 270° rotation of field magnet. The induced emf is maximum but the direction is reversed. Thus the current flows along SRQP. This is represented by point C.

On completion of 360°, the induced emf becomes zero and is represented by the point D. From the graph, it is clear that emf induced in PQRS is alternating in nature.

Therefore, when field magnet completes one rotation, induced emf in PQRS finishes one cycle. For this construction, the frequency of the induced emf depends on the speed at which the field magnet rotates.

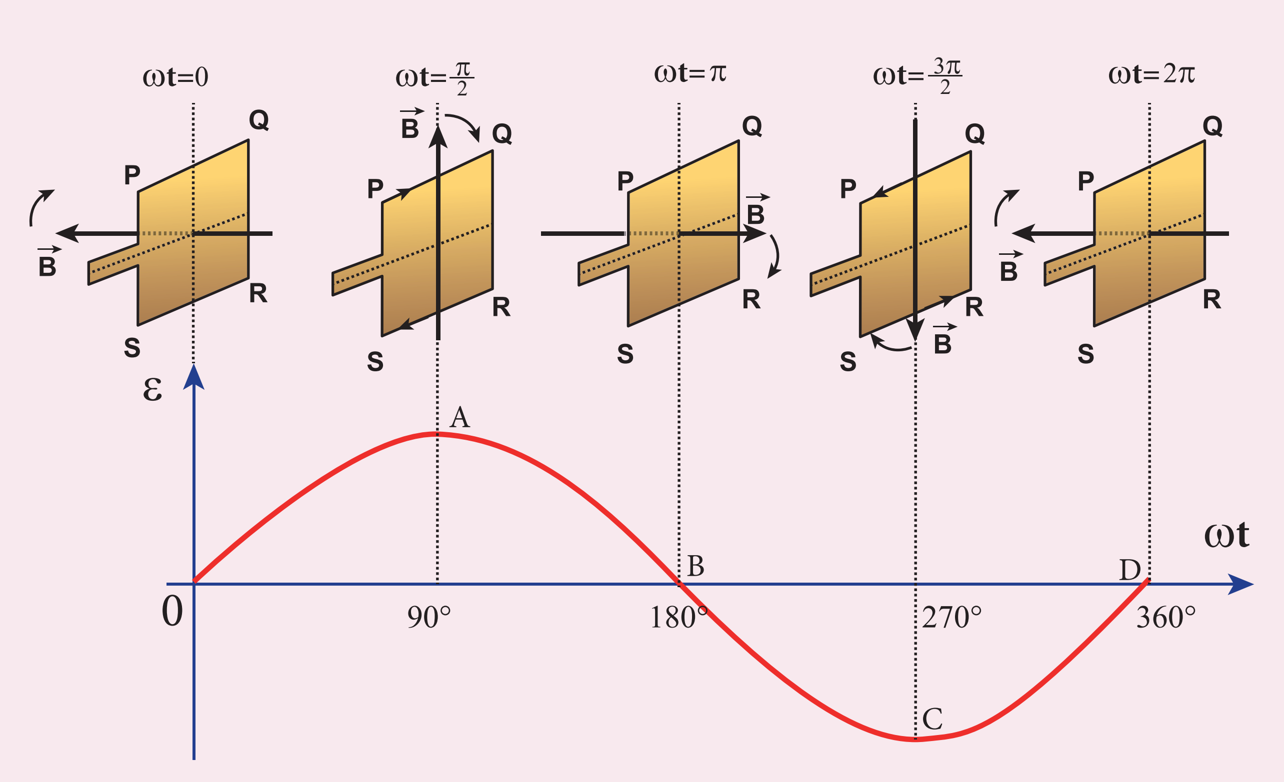
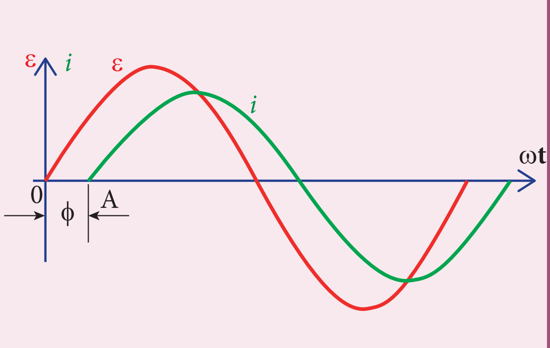


Figure 4.34 Variation of induced emf with respect to time angle

> **Note**
>
> **Phase difference**
>
> If two alternating quantities of same frequency do not pass through a particular point, say zero point, in the same direction at the same instant, they are said to have a phase difference. The angle between zero points is the angle of phase difference.
>
> 
>
>
> For the graph shown above, the phase difference between ε and i is given by OA = ϕ.

4.5.6 Three-phase AC generator

Some AC generators may have more than one coil in the armature core and each coil produces an alternating emf. In these generators, more than one emf is produced. Thus they are called poly-phase generators.

If there are two alternating emfs produced in a generator, it is called two-phase generator. In some AC generators, there are three separate coils, which would give three separate emfs. Hence they are called three-phase AC generators.

In the simplified construction of three-phase AC generator, the armature core has 6 slots, cut on its inner rim. Each slot is 60° away from one another. Six armature conductors are mounted in these slots. The conductors 1 and 4 are joined in series to form coil 1. The conductors 3 and 6 form coil 2 while the conductors 5 and 2 form coil 3. So, these coils are retangular in shape and are 120° apart from one another (Figure 4.35).

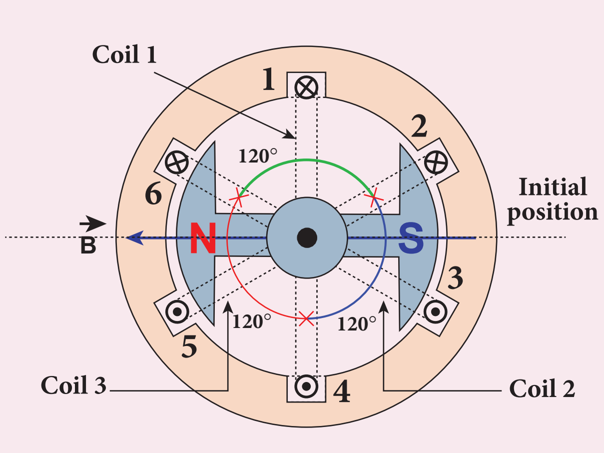


Figure 4.35 Construction of three-phase AC generator

The initial position of the field magnet is horizontal and field direction is perpendicular to the plane of the coil 1. As it is seen in single phase AC generator, when field magnet is rotated from that position in clockwise direction, alternating emf ε_1 in coil 1 begins a cycle from origin O. This is shown in Figure 4.36.

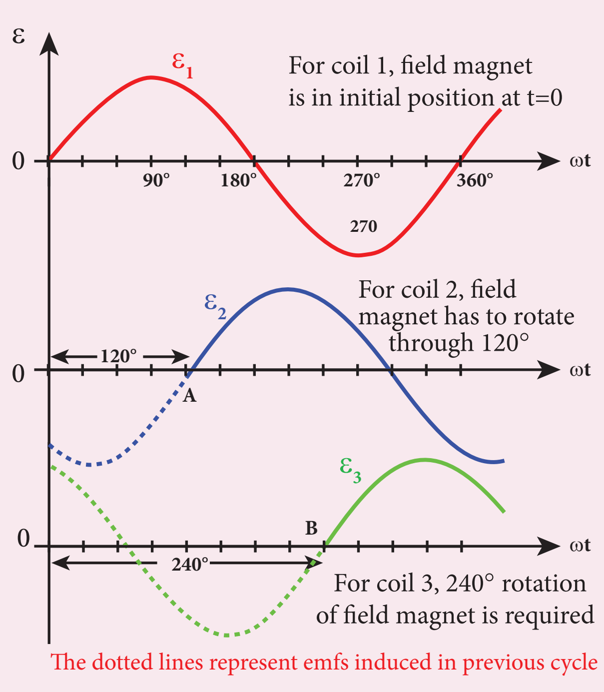


Figure 4.36 Variation of emfs ε_1, ε_2 and ε_3 with time angle.

The corresponding cycle for alternating emf ε_2 in coil 2 starts at point A after field magnet has rotated through 120°. Therefore, the phase difference between ε_1 and ε_2 is 120°. Similarly, emf ε_3 in coil 3 would begin its cycle at point B after 240° rotation of field magnet from initial position. Thus these emfs produced in the three phase AC generator have 120° phase difference between one another.

4.5.7 Advantages of three-phase alternator

Three-phase system has many advantages over single-phase system. Let us see a few of them.

1) For a given dimension of the generator, three-phase machine produces higher power output than a single-phase machine.
2) For the same capacity, three-phase alternator is smaller in size when compared to single phase alternator.
3) Three-phase transmission system is cheaper. A relatively thinner wire is sufficient for transmission of three-phase power.

4.6 TRANSFORMER

Transformer is a stationary device used to transform electrical power from one circuit to another without changing its frequency. The applied alternating voltage is either increased or decreased with corresponding decrease or increase of current in the circuit.

If the transformer converts an alternating current with low voltage into an alternating current with high voltage, it is called step-up transformer. On the contrary, if the transformer converts alternating current with high voltage into an alternating current with low voltage, then it is called step-down transformer.

4.6.1 Construction and working of transformer

Principle

The principle of transformer is the mutual induction between two coils. That is, when an electric current passing through a coil changes with time, an emf is induced in the neighbouring coil.

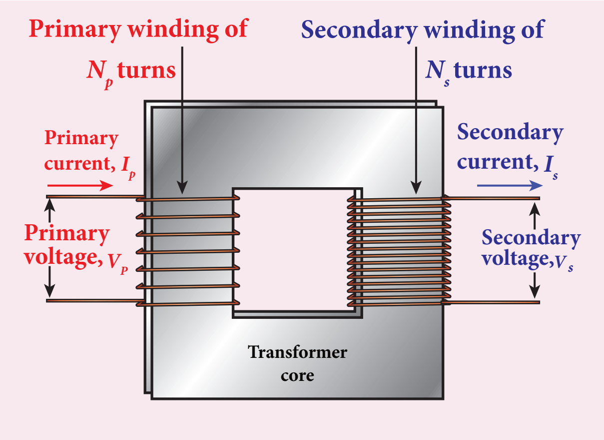


Figure 4.37(a) Construction of transformer

Figure 4.37(b) Roadside transformer

Construction

In the simple construction of transformers, there are two coils of high mutual inductance wound over the same transformer core. The core is generally laminated and is made up of a good magnetic material like silicon steel. Coils are electrically insulated but magnetically linked via transformer core (Figure 4.37).

The coil across which alternating voltage is applied is called primary coil *P* and the coil from which output power is drawn out is called secondary coil *S*. The assembled core and coils are kept in a container which is filled with suitable medium for better insulation and cooling purpose.

Working

If the primary coil is connected to a source of alternating voltage, an alternating magnetic flux is set up in the laminated core. If there is no magnetic flux leakage, then whole of magnetic flux linked with primary coil is also linked with secondary coil. This means that rate at which magnetic flux changes through each turn is same for both primary and secondary coils.

As a result of flux change, emf is induced in both primary and secondary coils. The emf induced in the primary coil ε_p is almost equal and opposite to the applied voltage υ_p and is given by

$$\upsilon_p = \varepsilon_p = -N_p \frac{d\Phi_B}{dt} \qquad (4.30)$$

The frequency of alternating magnetic flux in the core is same as the frequency of the applied voltage. Therefore, induced emf in secondary will also have same frequency as that of applied voltage. The emf induced in the secondary coil ε_s is given by

$$\varepsilon_s = -N_s \frac{d\Phi_B}{dt}$$

where N_p and N_s are the number of turns in the primary and secondary coil respectively. If the secondary circuit is open, then $\varepsilon_s = \upsilon_s$ where υ_s is the voltage across secondary coil.

$$\upsilon_s = \varepsilon_s = -N_s \frac{d\Phi_B}{dt} \quad (4.31)$$

From equations (4.30) and (4.31),

$$\frac{\upsilon_s}{\upsilon_p} = \frac{N_s}{N_p} = K \quad (4.32)$$

This constant K is known as voltage transformation ratio. For an ideal transformer,

Input power $\upsilon_p i_p$ = Output power $\upsilon_s i_s$

where i_p and i_s are the currents in the primary and secondary coil respectively. Therefore,

$$\frac{\upsilon_s}{\upsilon_p} = \frac{N_s}{N_p} = \frac{i_p}{i_s} \quad (4.33)$$

Equation 4.33 is written in terms of amplitude of corresponding quantities,

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s} = K$$

i) If $N_s > N_p \left(\text{or } K > 1\right), \therefore\ V_s > V_p$ and $I_s < I_p$. This is the case of step-up transformer in which voltage is increased and the corresponding current is decreased.

ii) If $N_s < N_p \left(\text{or } K < 1\right), \therefore\ V_s < V_p$ and $I_s > I_p$. This is step-down transformer where voltage is decreased and the current is increased.

Efficiency of a transformer:

The efficiency η of a transformer is defined as the ratio of the useful output power to the input power. Thus

$$\eta = \frac{\text{Output power}}{\text{Input power}} \times 100\% \quad (4.34)$$

Transformers are highly efficient devices having their efficiency in the range of 96 – 99%. Various energy losses in a transformer will not allow them to be 100% efficient.

4.6.2 Energy losses in a transformer

Transformers do not have any moving parts so that its efficiency is much higher than that of rotating machines like generators and motors. But there are many factors which lead to energy loss in a transformer.

i) Core loss or Iron loss

This loss takes place in transformer core. Hysteresis loss (Refer section 3.6) and eddy current loss are known as core loss or Iron loss. When transformer core is magnetized and demagnetized repeatedly by the alternating voltage applied across primary coil, hysteresis takes place due to which some energy is lost in the form of heat. Hysteresis loss is minimized by using steel of high silicon content in making transformer core.

Alternating magnetic flux in the core induces eddy currents in it. Therefore there is energy loss due to the flow of eddy current, called eddy current loss which is minimized by using very thin laminations of transformer core.

ii) Copper loss

Transformer windings have electrical resistance. When an electric current flows through them, some amount of energy is dissipated due to Joule heating. This energy loss is called copper loss which is minimized by using wires of larger diameter.

iii) Flux leakage

Flux leakage happens when the magnetic lines of primary coil are not completely linked with secondary coil. Energy loss due to this flux leakage is minimized by winding coils one over the other.

4.6.3 Advantages of AC in long distance power transmission

Electric power is produced in a large scale at electric power stations with the help of AC generators. These power stations are classified based on the type of fuel used as thermal, hydro electric and nuclear power stations. Most of these stations are located at remote places. Hence the electric power generated is transmitted over long distances through transmission lines to reach towns or cities where it is actually consumed. This process is called power transmission.

But there is a difficulty during power transmission. A sizable fraction of electric power is lost due to Joule heating (i^2R) in the transmission lines which are hundreds of kilometer long. This power loss can be tackled either by reducing current i or by reducing resistance R of the transmission lines. The resistance R can be reduced with thick wires of copper or aluminium. But this increases the cost of production of transmission lines and other related expenses. So this way of reducing power loss is not economically viable.

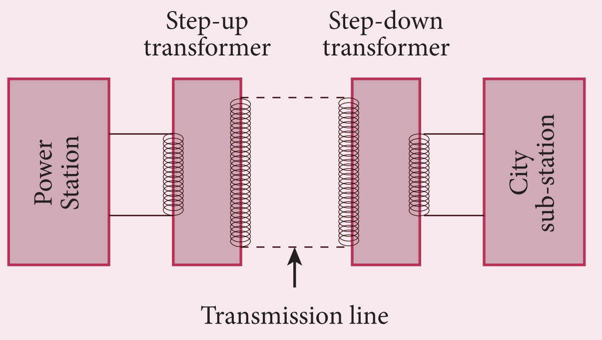


Figure 4.38 Long distance power transmissions

Since power produced is alternating in nature, there is a way out. The most important property of alternating voltage that it can be stepped up and stepped down by using transformers could be exploited in reducing current and thereby reducing power losses to a greater extent.

At the transmitting point, the voltage is increased and the corresponding current is decreased by using step-up transformer (Figure 4.38). Then it is transmitted through transmission lines. This reduced current at high voltage reaches the destination without any appreciable loss. At the receiving point, the voltage is decreased and the current is increased to appropriate values by using step-down transformer and then it is given to consumers. Thus power transmission is done efficiently and economically.

Illustration:

An electric power of 2 MW is transmitted to a place through transmission lines of total resistance, say R = 40 Ω, at two different voltages. One is lower voltage (10 kV) and the other is higher (100 kV). Let us now calculate and compare power losses in these two cases.

Case (i):

P = 2 MW; R = 40 Ω; V = 10 kV

Power, $P = VI$

$$\therefore \text{Current, } I = \frac{P}{V} = \frac{2\times10^6}{10\times10^3} = 200A$$

Power loss = Heat produced

$$= I^2R = (200)^2 \times 40 = 1.6\times10^6 W$$

$$\%\text{ of power loss} = \frac{1.6\times10^6}{2\times10^6}\times100\%$$

$$= 0.8\times100\% = 80\%$$

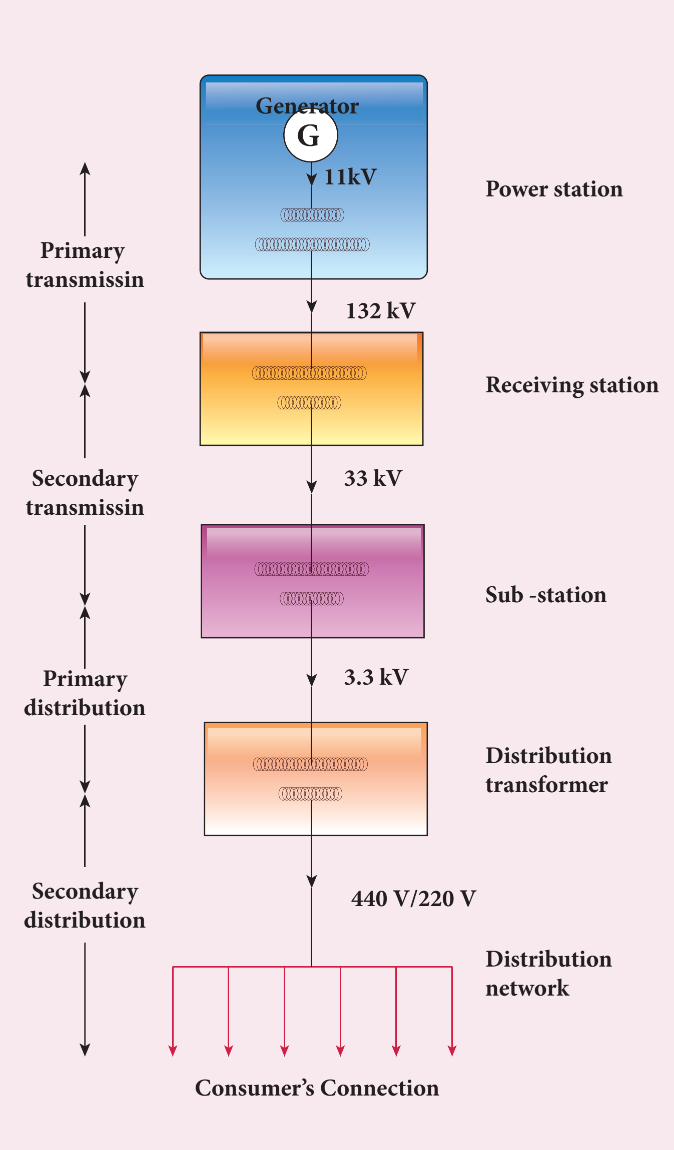
Power system at a glance (Not for Examination)

The generating stations present in a region are interconnected to form a common electrical network and are operated in parallel. This is to ensure uninterrupted power supply to a large number of consumers in the case of failure of any power station or a sudden increase of load beyond the capacity of the generating station.

The various elements such as generating stations, transmission lines, the substations and distributors etc are all tied together for continuous generation and consumption of electric energy.This is called power system. A part of power system consisting of the sub-stations and transmission lines is known as a grid.

In a power system, the transfer of electric power produced to the consumer is carried out in two stages which are further sub-divided into two as given below.

1) Transmission stage
   a) Primary transmission stage
   b) Secondary transmission stage
2) Distribution stage
   a) Primary distribution stage
   b) Secondary transmission stage



and then it is supplied to individual consumers. These two stages of power transmission is presented in a single-line diagram shown in figure. The central system usually generates power at 11 kV which is stepped up to 132 kV and is transmitted through transmission lines. This is known as primary or high-voltage transmission.

This high-voltage power reaches receiving station at the outskirts of the city where it is stepped down to 33 kV and is transmitted as secondary or low-voltage transmission to sub-stations situated within the city limits.

In the primary distribution system, the voltage is reduced from 33 kV to 3.3kV at sub-stations and is given to distribution sub-stations. The voltage is finally brought down to 440V or 230V at distribution sub-station from where secondary distribution is done to factories (440V) and homes (230V) via distribution networks.

Case (ii):

$P = 2$ MW; $R = 40\ \Omega$; $V = 100$ kV

$$\therefore \text{Current, } I = \frac{P}{V}$$

$$= \frac{2\times10^6}{100\times10^3} = 20A$$

Power loss = I^2R

$$= (20)^2 \times 40 = 0.016\times10^6 W$$

$$\% \text{ of power loss} = \frac{0.016\times10^6}{2\times10^6}\times100\%$$

$$= 0.008\times100\% = 0.8\%$$

Thus it is clear that when an electric power is transmitted at higher voltage, the power loss is reduced to a large extent.

EXAMPLE 4.16

An ideal transformer has 460 and 40,000 turns in the primary and secondary coils respectively. Find the voltage developed per turn of the secondary if the transformer is connected to a 230 V AC mains. The secondary is given to a load of resistance $10^4\,\Omega$. Calculate the power delivered to the load.

Solution

N_P = 460 turns; N_S = 40,000 turns

V_P = 230 V; $R_S = 10^4\ \Omega$

(i) Secondary voltage,

$$V_S = \frac{V_P N_S}{N_P} = \frac{230\times40{,}000}{460}$$

$$= 20{,}000V$$

$$\text{Secondary voltage per turn, } \frac{V_S}{N_S} = \frac{20{,}000}{40{,}000}$$

$$= 0.5\ V$$

(ii) Power delivered

$$= V_S I_S = \frac{V_S^2}{R_S} = \frac{20{,}000\times20{,}000}{10^4} = 40\ kW$$

EXAMPLE 4.17

An inverter is common electrical device which we use in our homes. When there is no power in our house, inverter gives AC power to run a few electronic appliances like fan or light. An inverter has inbuilt step-up transformer which converts 12 V AC to 240 V AC. The primary coil has 100 turns and the inverter delivers 50 mA to the external circuit. Find the number of turns in the secondary and the primary current.

Solution

V_p = 12 V; V_s = 240 V

I_s = 50mA; N_p = 100 turns

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S} = K$$

$$\text{Transformation ratio, } K = \frac{240}{12} = 20$$

The number of turns in the secondary

$$N_S = N_P \times K = 100\times20 = 2000$$

Primary current,

$I_p = K \times I_s = 20 \times 50$ mA = 1 A

4.7 ALTERNATING CURRENT

4.7.1 Introduction

In section 4.5, we have seen that when the orientation of the coil with the magnetic field is changed, an alternating emf is induced and hence an alternating current flows in the closed circuit. **An alternating voltage is the voltage which changes polarity at regular intervals of time and the direction of the resulting alternating current also changes accordingly.**

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In the Figure 4.39(a), an alternating voltage source is connected to a resistor R in which the upper terminal of the source is positive and lower terminal negative at an instant. Therefore, the current flows in clockwise direction. After a short time, the polarities of the source are reversed so that current now flows in anti-clockwise direction (Figure 4.39(b)). This current which flows in alternate directions in the circuit is called alternating current.

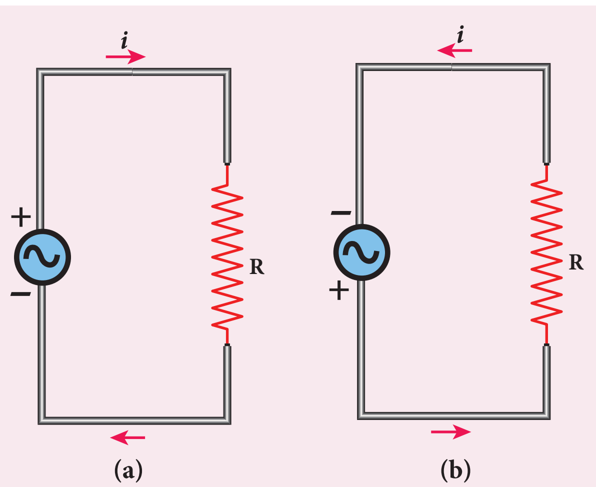


Figure 4.39 Alternating voltage and the corresponding alternating current

Sinusoidal alternating voltage

If the waveform of alternating voltage is a sine wave, then it is known as sinusoidal alternating voltage which is given by the relation.

$$\upsilon = V_m \sin \omega t \qquad (4.35)$$

where υ is the instantaneous value of alternating voltage; V_m is the maximum value (amplitude) and ω is the angular frequency of the alternating voltage. When sinusoidal alternating voltage is applied to a closed circuit, the resulting alternating current is also sinusoidal in nature and its relation is

$$i = I_m \sin \omega t \qquad (4.36)$$

where I_m is the maximum value (amplitude) of the alternating current. The direction of sinusoidal voltage or current is reversed after every half-cycle and its magnitude is also changing continuously as shown in Figure 4.40.

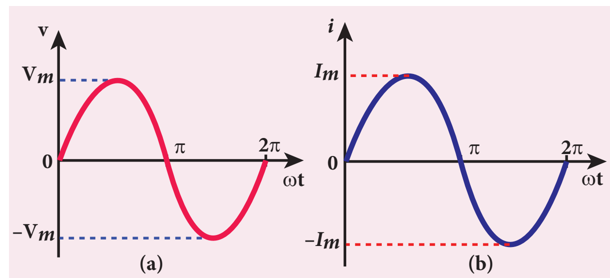


Figure 4.40 (a) Sinusoidal alternating voltage (b) Sinusoidal alternating current

> **Note** Interestingly, sine waves are very common in nature. The periodic motions like waves in water, swinging of pendulum are associated with sine waves. Thus sine wave seems to be nature's standard. Also refer unit 11 of XI physics text book.

4.7.1 Mean or Average value of AC

The current and voltage in a DC system remain constant over a period of time so that there is no problem in specifying their magnitudes. However, an alternating current or voltage varies from time to time. Then a question arises how to express the magnitude of an alternating current or voltage. Though there are many ways of expressing it, we limit our discussion with two ways, namely mean value and RMS (Root Mean Square) value of AC.

Mean or Average value of AC

We have learnt that the magnitude of an alternating current in a circuit changes from one instant to other instant and its direction also reverses for every half cycle. During positive half cycle, current is taken as positive and during negative cycle it is negative. Therefore mean or average value of symmetrical alternating current over one complete cycle is zero.

Therefore the average or mean value is measured over one half of a cycle. These electrical terms, average current and average voltage can be used in both AC and DC circuit analysis and calculations.

The average value of alternating current is defined as the average of all values of current over a positive half-cycle or negative half-cycle.

The instantaneous value of sinusoidal alternating current is given by the equation $i = I_m \sin\omega t$ or $i = I_m \sin\theta$ (where $\theta = \omega t$) whose graphical representation is given in Figure 4.41.

The sum of all currents over a half-cycle is given by area of positive half-cycle (or negative half-cycle). Therefore,

$$I_{av} = \frac{\text{Area of positive half-cycle (or negative half-cycle)}}{\text{Base length of half-cycle}} \quad (4.37)$$

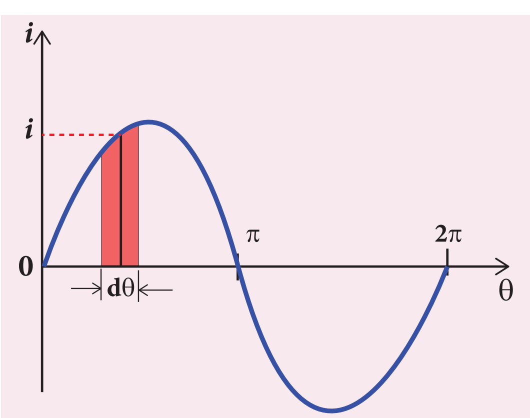


Figure 4.41 Sine wave of an alternating current

Consider an elementary strip of thickness $d\theta$ in the positive half-cycle of the current wave (Figure 4.41). Let i be the mid-ordinate of that strip.

Area of the elementary strip = $i\, d\theta$

Area of positive half-cycle

$$= \int_0^{\pi} i d\theta = \int_0^{\pi} I_m \sin\theta d\theta$$

$$= I_m \left[-\cos\theta\right]_0^{\pi} = -I_m\left[\cos\pi - \cos 0\right] = 2I_m$$

Substituting this in equation (4.37), we get (The base length of half-cycle is π)

Average value of AC, $I_{av} = \frac{2I_m}{\pi}$

$$I_{av} = 0.637\, I_m \quad (4.38)$$

Hence the average value of AC is 0.637 times the maximum value I_m of the alternating current. For negative half-cycle, $I_{av} = -0.637\, I_m$.

Note

For example, if we consider n currents in a half-cycle of AC, namely $i_1, i_2, \ldots i_n$, then average value is given by

$$I_{av} = \frac{\text{Sum of all currents over half-cycle}}{\text{Number of currents}}$$

$$I_{av} = \frac{i_1 + i_2 + \ldots. + i_n}{n}$$

4.7.2 RMS value of AC

The term RMS refers to time-varying sinusoidal currents and voltages and not used in DC systems.

The root mean square value of an alternating current is defined as the square root of the mean of the squares of all currents over one cycle. It is denoted by

I_{RMS}. For alternating voltages, the RMS value is given by V_{RMS}.

The alternating current $i = I_m \sin\omega t$ or $i = I_m \sin\theta$, is represented graphically in Figure 4.42. The corresponding squared current wave is also shown by the dotted lines.

The sum of the squares of all currents over one cycle is given by the area of one cycle of squared wave. Therefore,

$$I_{RMS} = \sqrt{\frac{\text{Area of one cycle of squared wave}}{\text{Base length of one cycle}}} \qquad (4.39)$$

An elementary area of thickness $d\theta$ is considered in the first half-cycle of the squared current wave as shown in Figure 4.42. Let i^2 be the mid-ordinate of the element.

Area of the element = $i^2 d\theta$

Area of one cycle of squared wave $= \int_0^{2\pi} i^2 d\theta$

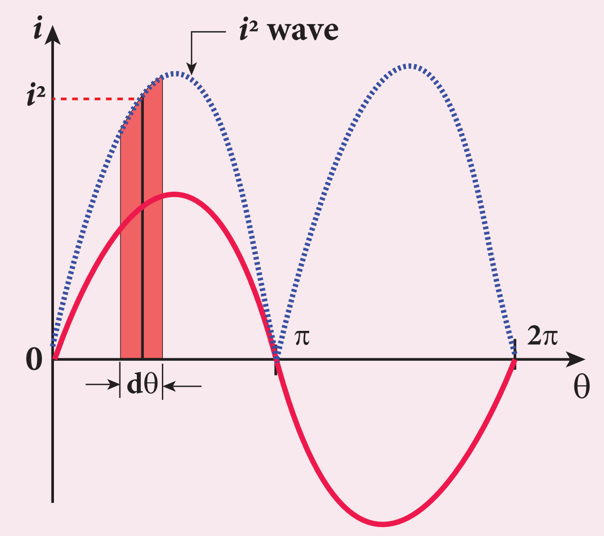


Figure 4.42 Squared wave of AC

$$= \int_0^{2\pi} I_m^2 \sin^2\theta d\theta = I_m^2 \int_0^{2\pi} \sin^2\theta d\theta \qquad (4.40)$$

$$= I_m^2 \int_0^{2\pi} \left[\frac{1-\cos 2\theta}{2}\right] d\theta$$

$$\text{since } \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \frac{I_m^2}{2}\left[\int_0^{2\pi} d\theta - \int_0^{2\pi} \cos 2\theta d\theta\right]$$

$$= \frac{I_m^2}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{I_m^2}{2}\left[\left(2\pi - \frac{\sin 2\times 2\pi}{2}\right) - \left(0 - \frac{\sin 0}{2}\right)\right]$$

$$= \frac{I_m^2}{2} \times 2\pi = I_m^2 \pi \qquad [\because \sin 0 = \sin 4\pi = 0]$$

Substituting this in equation (4.39), we get

$$I_{RMS} = \sqrt{\frac{I_m^2 \pi}{2\pi}} = \frac{I_m}{\sqrt{2}}$$ [Base length of one cycle is 2π]

$$I_{rms} = 0.707\, I_m \qquad (4.41)$$

Thus we find that for a symmetrical sinusoidal current rms value of current is 70.7 % of its peak value.

Similarly for alternating voltage, it can be shown that

$$V_{rms} = 0.707\, V_m \qquad (4.42)$$

Note

RMS value of alternating current is also called effective value and is represented as I_{eff}. It is used to compare RMS current of AC to an equivalent steady current.

RMS value is also defined as that value of the steady current which when flowing through a given circuit for a given time produces the same amount of heat as produced by the alternating current when flowing through the same circuit for the same time. The effective value of an alternating voltage is represented by V_{eff}.

Note

For example, if we consider n currents in one cycle of AC, namely i_1, i_2, ... i_n, then RMS value is given by

$$I_{RMS} = \sqrt{\frac{\text{Sum of squares of all currents over one cycle}}{\text{Number of currents}}}$$

$$I_{RMS} = \sqrt{\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n}}$$

DO YOU KNOW?

For common household appliances, the voltage rating and current rating are generally specified in terms of their RMS value. The domestic AC supply is 230V, 50 Hz. It is the RMS or effective value. Its peak value will be $V_m = \sqrt{2}\ V_{rms} = \sqrt{2} \times 230 = 325V$.

EXAMPLE 4.18

Write down the equation for a sinusoidal voltage of 50 Hz and its peak value is 20 V. Draw the corresponding voltage versus time graph.

Solution

$f = 50\,Hz;\qquad V_m = 20\ V$

Instantaneous voltage, $\upsilon = V_m \sin\omega t$

$= V_m \sin 2\pi \nu t$

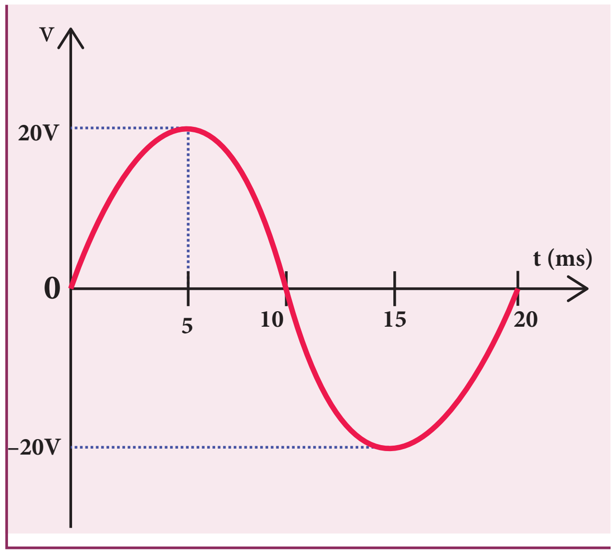
$= 20\sin(2\pi \times 50)\,t = 20\sin(100 \times 3.14)\,t$

$\upsilon = 20\sin 314\,t$

Time for one cycle, $T = \dfrac{1}{f} = \dfrac{1}{50} = 0.02\,s$

$= 20 \times 10^{-3}\,s \quad = 20\,ms$

The wave form is given below.



EXAMPLE 4.19

The equation for an alternating current is given by $i = 77\ sin\ 314t$. Find the peak value, frequency, time period and instantaneous value at t = 2 ms.

Solution

$i = 77 \sin 314t\,; \quad t = 2\text{ ms} = 2 \times 10^{-3}\,\text{s}$

The general equation of an alternating current is $i = I_m \sin\omega t$. On comparsion,

(i) Peak value, $I_m = 77\,A$

(ii) Frequency, $f = \dfrac{\omega}{2\pi} = \dfrac{314}{2 \times 3.14} = 50\ Hz$

(iii) Time period, $T = \dfrac{1}{f} = \dfrac{1}{50} = 0.02\ s$

(iv) At t = 2 m s,
Instantaneous value,
$i = 77\sin(314 \times 2 \times 10^{-3})$
$i = 45.24\,A$

Phasor and phasor diagram

Phasor

A sinusoidal alternating voltage (or current) can be represented by a vector which rotates about the origin in anti-clockwise direction at a constant angular velocity ω. Such a rotating vector is called a phasor. A phasor is drawn in such a way that

- the length of the line segment equals the peak value V_m (or I_m) of the alternating voltage (or current)
- its angular velocity ω is equal to the angular frequency of the alternating voltage (or current)
- the projection of phasor on any vertical axis gives the instantaneous value of the alternating voltage (or current)
- the angle between the phasor and the axis of reference (positive x-axis) indicates the phase of the alternating voltage (or current).

The notion of phasors is introduced to analyse phase relationship between voltage and current in different AC circuits.

Phasor diagram

The diagram which shows various phasors and their phase relations is called phasor diagram. Consider a sinusoidal alternating voltage $v = V_m \sin \omega t$ applied to a circuit. This voltage can be represented by a phasor, namely $\overrightarrow{OA}$ as shown in Figure 4.43.

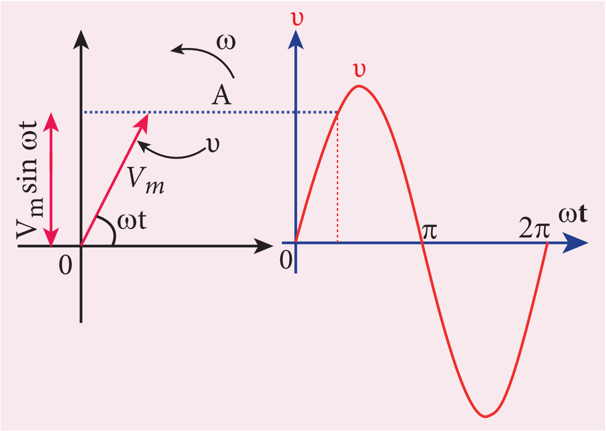


Figure 4.43 Phasor diagram for an alternating voltage $v = V_m \sin \omega t$

Here the length of $\overrightarrow{OA}$ equals the peak value (V_m), the angle it makes with x-axis gives the phase (ωt) of the applied voltage. Its projection on y-axis provides the instantaneous value ($V_m \sin \omega t$) at that instant.

When $\overrightarrow{OA}$ rotates about O with angular velocity ω in anti-clockwise direction, the

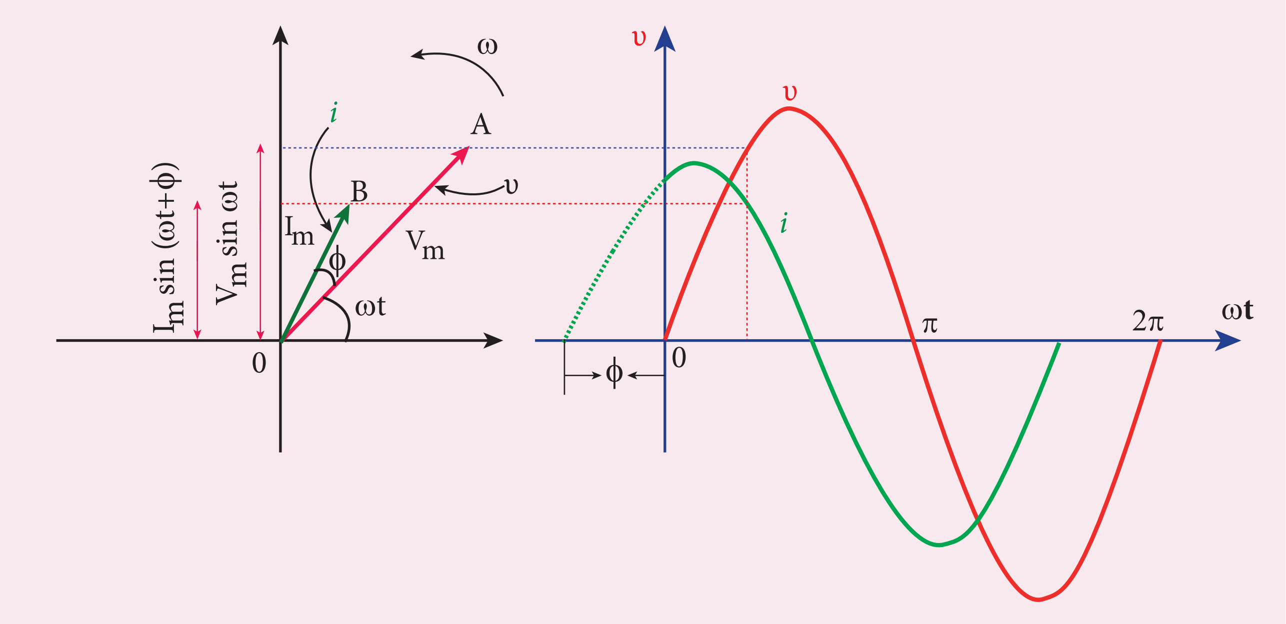


Figure 4.44 Phasor diagram and wave diagram say that i leads v by ϕ

waveform of the voltage is generated. For one full rotation of $\overrightarrow{OA}$, one cycle of voltage is produced.

The alternating current in the same circuit may be given by the relation $i = I_m \sin(\omega t + \phi)$ which is represented by another phasor $\overrightarrow{OB}$. Here ϕ is the phase angle between voltage and current. In this case, the current leads the voltage by phase angle ϕ which is shown in Figure 4.44. If the current lags behind the voltage, then we write $i = I_m \sin(\omega t - \phi)$.

4.7.3 AC circuit containing pure resistor

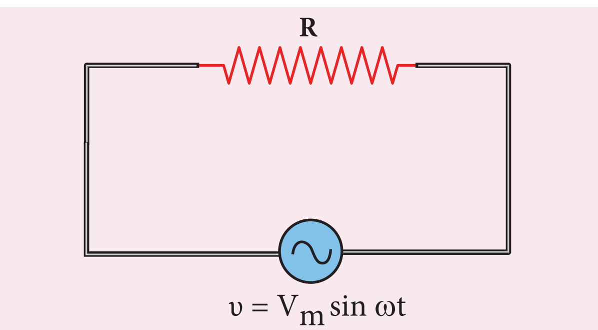


Figure 4.45 AC circuit with resistance

Consider a circuit containing a pure resistor of resistance R connected across an alternating voltage source (Figure 4.45). The instantaneous value of the alternating voltage is given by

$$\upsilon = V_m \sin \omega t \qquad (4.43)$$

An alternating current i flowing in the circuit due to this voltage, develops a potential drop across R and is given by

$$V_R = iR \qquad (4.44)$$

Kirchoff's loop rule (Refer section 2.4) states that the algebraic sum of potential differences in a closed circuit is zero. For this resistive circuit,

$$\upsilon - V_R = 0$$

From equation (4.43) and (4.44),

$$V_m \sin \omega t = iR$$

$$i = \frac{V_m}{R} \sin \omega t$$

$$i = I_m \sin \omega t \qquad (4.45)$$

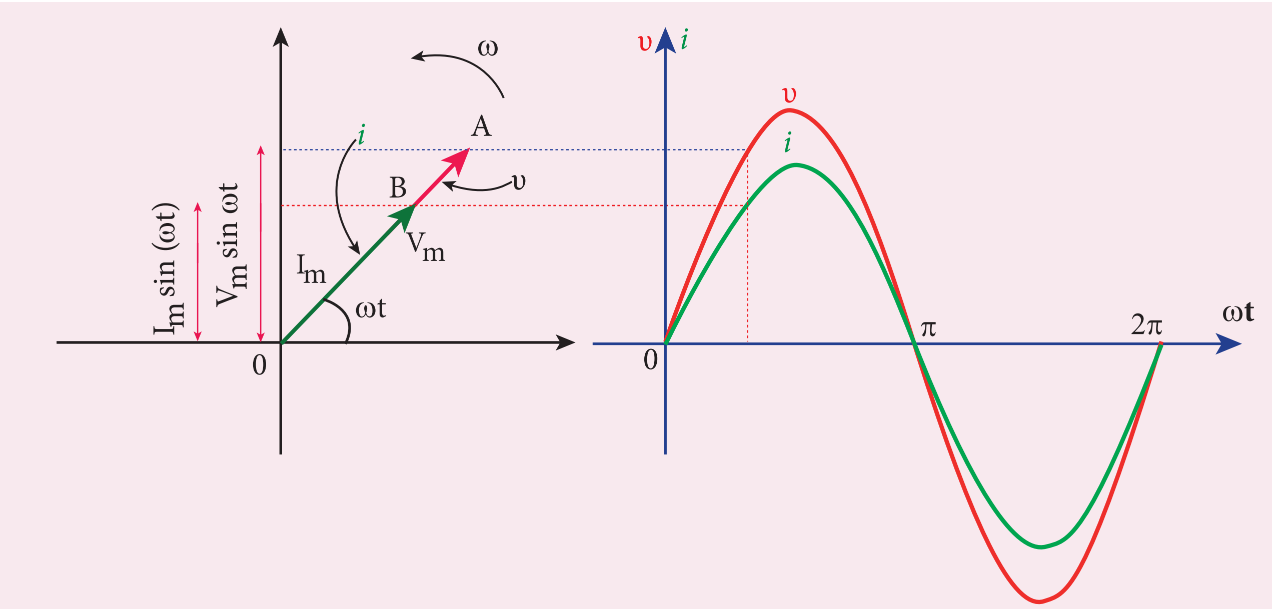


Figure 4.46 Phasor diagram and wave diagram for AC circuit with R

where $\frac{V_m}{R} = I_m$, the peak value of alternating current in the circuit. From equations (4.43) and (4.45), it is clear that the applied voltage and the current are in phase with each other in a resistive circuit. It means that they reach their maxima and minima simultaneously. This is indicated in the phasor diagram (Figure 4.46). The wave diagram also depicts that current is in phase with the applied voltage (Figure 4.46).

4.7.4 AC circuit containing only an inductor

Consider a circuit containing a pure inductor of inductance L connected across an alternating voltage source (Figure 4.47). The alternating voltage is given by the equation.

$$\upsilon = V_m \sin\omega t \qquad (4.46)$$

The alternating current flowing through the inductor induces a self-induced emf or back emf in the circuit. The back emf is given by

$$\text{Back emf}, \varepsilon = -L\frac{di}{dt}$$

By applying Kirchoff's loop rule to the purely inductive circuit, we get

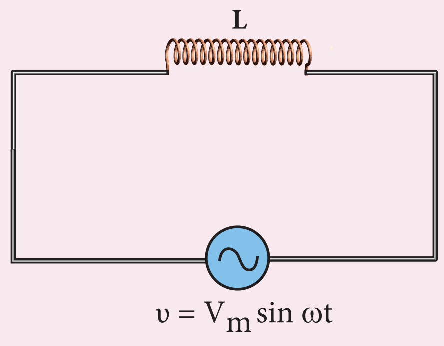


Figure 4.47 AC circuit with inductance

$$\upsilon + \varepsilon = 0$$

$$V_m \sin\omega t = L\frac{di}{dt}$$

$$di = \frac{V_m}{L}\sin\omega t\ dt$$

Integrating both sides, we get

$$i = \frac{V_m}{L}\int \sin\omega t\, dt$$

$$i = \frac{V_m}{L\omega}\left(-\cos\omega t\right) + \text{constant}$$

The integration constant in the above equation is independent of time. Since the voltage in the circuit has only time dependent part, we can set the time independent part in the current (integration constant) into zero.

$$\left(\begin{array}{l}\cos\omega t = \sin\left(\pi/_2 - \omega t\right)\\ -\sin\left(\pi/_2 - \omega t\right) = \sin\left(\omega t - \pi/_2\right)\end{array}\right)$$

$$i = \frac{V_m}{\omega L}\sin\left(\omega t - \pi/_2\right)$$

$$\text{or} \quad i = I_m \sin\left(\omega t - \pi/_2\right) \qquad (4.47)$$

where $\frac{V_m}{\omega L} = I_m$, the peak value of the alternating current in the circuit. From equation (4.46) and (4.47), it is evident that current lags behind the applied voltage by $\pi/_2$ in an inductive circuit. This fact is depicted in the phasor diagram. In the wave diagram also, it is seen that current lags the voltage by 90° (Figure 4.48).

Inductive reactance X_L

The peak value of current I_m is given by $I_m = \frac{V_m}{\omega L}$. Let us compare this equation with $I_m = \frac{V_m}{R}$ from resistive circuit. The quantity ωL plays the same role as the resistance in resistive circuit. This is the resistance offered by the inductor, called inductive reactance (X_L). It is measured in ohm.

$$X_L = \omega L$$

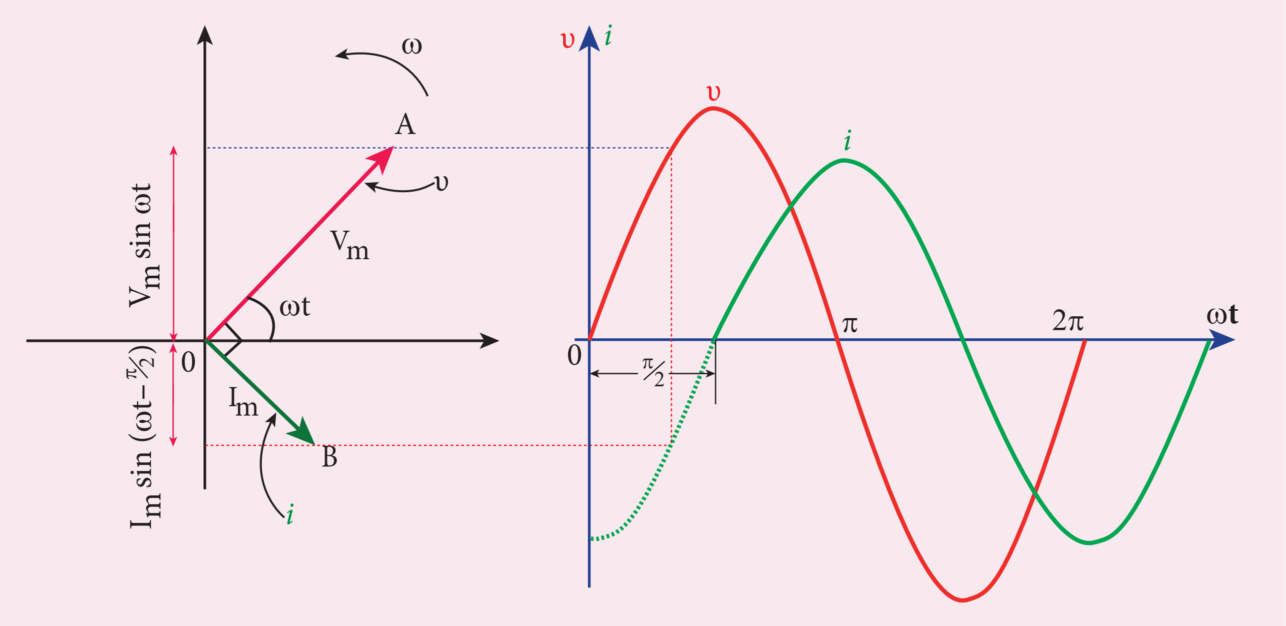


Figure 4.48 Phasor diagram and wave diagram for AC circuit with L

An inductor blocks AC but it allows DC. Why? and How?

An inductor L is a closely wound helical coil. The steady DC current flowing through L produces uniform magnetic field around it and the magnetic flux linked remains constant. Therefore there is no self-induction and self-induced emf (back emf). Since inductor behaves like a resistor, DC flows through an inductor.

The AC flowing through L produces time-varying magnetic field which in turn induces self-induced emf (back emf). This back emf, according to Lenz's law, opposes any change in the current. Since AC varies both in magnitude and direction, its flow is opposed in L. For an ideal inductor of zero ohmic resistance, the back emf is equal and opposite to the applied emf. Therefore L blocks AC.

The inductive reactance (X_L) varies directly as the frequency.

$$X_L = 2\pi f L \qquad (4.48)$$

where f is the frequency of the alternating current. For a steady current, $f = 0$. Therefore, $X_L = 0$. Thus an ideal inductor offers no resistance to steady DC current.

4.7.5 AC circuit containing only a capacitor

Consider a circuit containing a capacitor of capacitance C connected across an alternating voltage source (Figure 4.49). The alternating voltage is given by

$$\upsilon = V_m \sin \omega t \qquad (4.49)$$

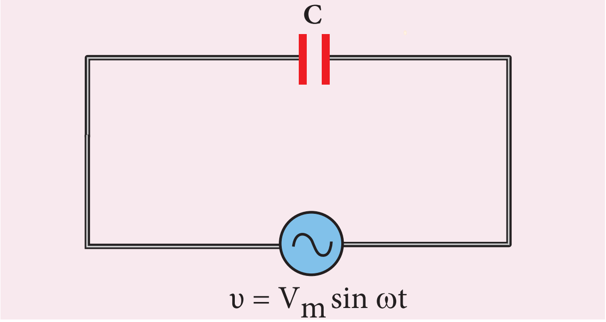


Figure 4.49 AC circuit with capacitance

Let q be the instantaneous charge on the capacitor. The emf across the capacitor at that instant is $\frac{q}{C}$. According to Kirchoff's loop rule,

$$\upsilon - \frac{q}{C} = 0$$

$$q = CV_m \sin \omega t$$

By the definition of current,

$$i = \frac{dq}{dt} = \frac{d}{dt}\left(CV_m \sin \omega t\right)$$

$$= CV_m \frac{d}{dt}\left(\sin \omega t\right)$$

$$= CV_m \,\omega \cos \omega t$$

$$\text{or} \quad i = \frac{V_m}{1/C\omega} \sin\left(\omega t + \pi/2\right)$$

Instantaneous value of current,

$$i = I_m \sin\left(\omega t + \pi/2\right) \qquad (4.50)$$

where $\frac{V_m}{1/C\omega} = I_m$, the peak value of the alternating current. From equations (4.49) and (4.50), it is clear that current leads the applied voltage by $\pi/2$ in a capacitive circuit. This is shown pictorially in Figure 4.50. The wave diagram for a capacitive circuit also shows that the current leads the applied voltage by 90°.

Capacitive reactance X_C

The peak value of current I_m is given by $I_m = \frac{V_m}{1/C\omega}$. Let us compare this equation with $I_m = \frac{V_m}{R}$ from resistive circuit. The quantity $1/C\omega$ plays the same role as the resistance R in resistive circuit. This is the resistance offered by the capacitor, called capacitive reactance (X_C). It measured in ohm.

$$X_c = \frac{1}{\omega C} \qquad (4.51)$$

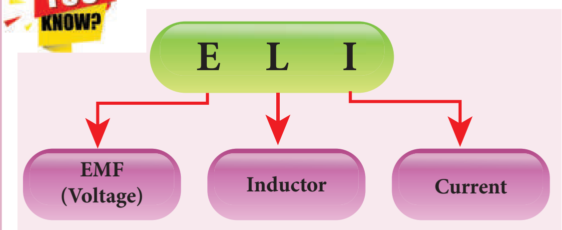
The capacitive reactance (X_C) varies inversely as the frequency. For a steady current, $f = 0$.

$$\therefore X_C = \frac{1}{\omega C} = \frac{1}{2\pi f\, C} = \frac{1}{0} = \infty$$

Thus a capacitive circuit offers infinite resistance to the steady current. So that steady current cannot flow through the capacitor.

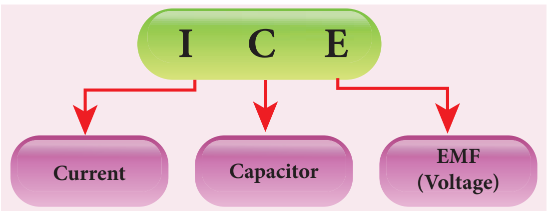
DO YOU KNOW?

What is ELI?



ELI is an acronym which means that EMF (voltage) leads the current in an inductive circuit.

What is ICE?



ICE is an acronym which means that the current leads the EMF (voltage) current in a capacitive circuit.

It is easier to may remember the results of AC circuits with the mnemonic 'ELI the ICE man'.

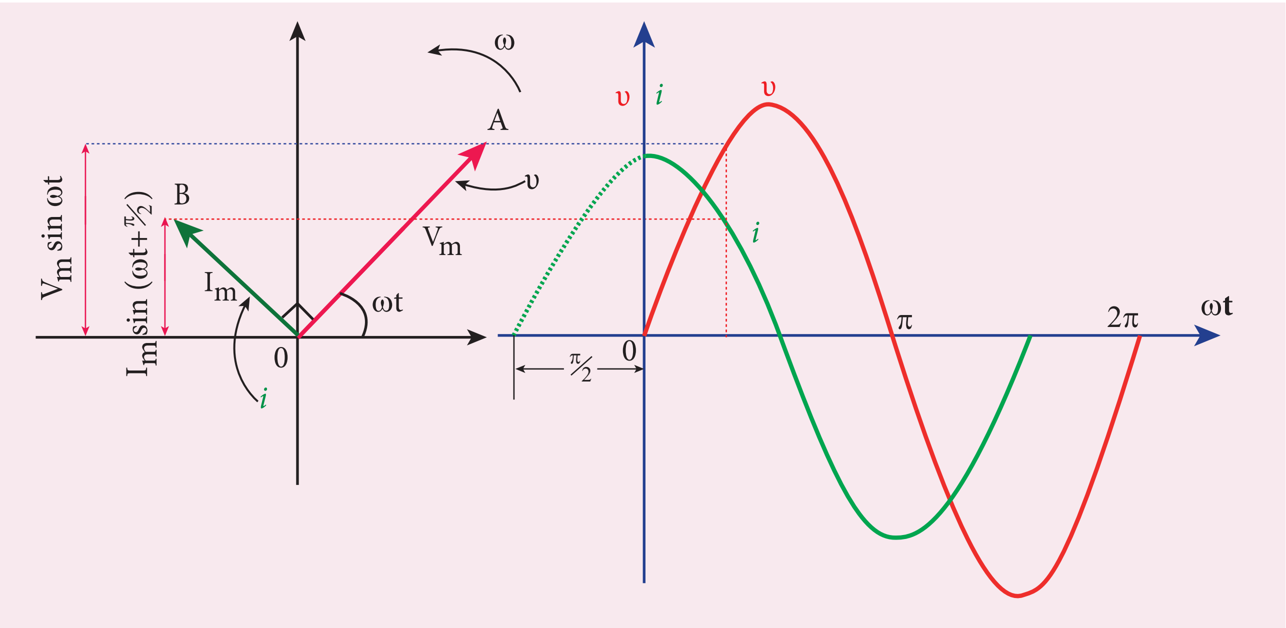


Figure 4.50 Phasor diagram and wave diagram for AC circuit with C

EXAMPLE 4.20

A 400 mH coil of negligible resistance is connected to an AC circuit in which an effective current of 6 mA is flowing. Find out the voltage across the coil if the frequency is 1000 Hz.

Solution

$L = 400 \times 10^{-3}$ H; $I_{eff} = 6 \times 10^{-3}$A

$f = 1000$ Hz

Inductive reactance, $X_L = L\omega = L \times 2\pi f$

$= 2 \times 3.14 \times 1000 \times 0.4$

$= 2512\ \Omega$

Voltage across L,

$V = I X_L = 6 \times 10^{-3} \times 2512$

$V = 15.072\, V (RMS)$

EXAMPLE 4.21

A capacitor of capacitance $\frac{10^2}{\pi}$ μF is connected across a 220 V, 50 Hz A.C. mains. Calculate the capacitive reactance, RMS value of current and write down the equations of voltage and current.

Solution

$C = \frac{10^2}{\pi} \times 10^{-6} F,\ V_{RMS} = 220\ V;\ f = 50 Hz$

(i) Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$= \frac{1}{2 \times \pi \times 50 \times \frac{10^{-4}}{\pi}} = 100\,\Omega$$

(ii) RMS value of current,

$$I_{RMS} = \frac{V_{RMS}}{X_C} = \frac{220}{100} = 2.2\,A$$

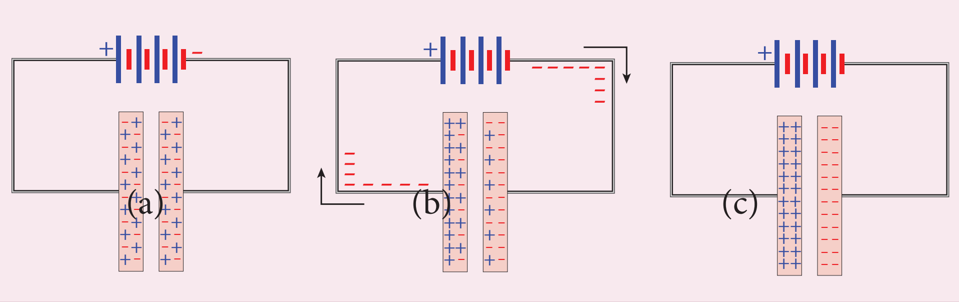
(iii) $V_m = 220 \times \sqrt{2} = 311\ V$

A capacitor blocks DC but it allows AC. Why? and How? (Not for examination)

Capacitors have two parallel metallic plates placed close to each other and there is a gap between plates. Whenever a source of voltage (either DC voltage or AC voltage) is connected across a capacitor C, the electrons from the source will reach the plate and stop. They cannot jump across the gap between plates to continue its flow in the circuit. Therefore the electrons flowing in one direction (i.e. DC) cannot pass through the capacitor. But the electrons from AC source seem to flow through C. Let us see what really happens!

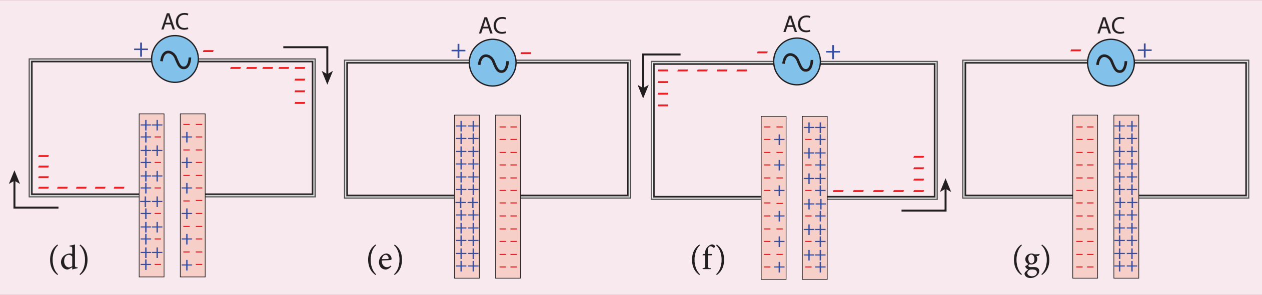
DC cannot flow through a capacitor:

Consider a parallel plate capacitor whose plates are uncharged (same amount of positive and negative charges). A DC source (battery) is connected across C as shown in Figure (a).



As soon as battery is connected, electrons start to flow from the negative terminal and are accumulated at the right plate, making it negative. Due to this negative potential, the electrons present in the nearby left plate are repelled and are moved towards positive terminal of the battery. When electrons leave the left plate, it becomes positively charged. This process is known as charging. The direction of flow of electrons is shown by arrows.

The charging of the plates continues till the level of the battery. Once C is fully charged and current will stop. At this time, we say that capacitor is blocking DC Figure (c).



AC flows (?!) through a capacitor:

Now an AC source is connected across C. At an instant, the right side of the source is at negative potential, then the electrons flow from negative terminal to the right plate and from left plate to the positive terminal as shown in Figure (d) but no electron crosses the gap between the plates. These electron-flows are represented by arrows. thus, the charging of the plates takes place and the plates become fully charged (Figure (e)).

After a short time, the polarities of AC source are reversed and the right side of the source is now positive. The electrons which were accumulated in the right plate start to flow to the positive terminal and the electrons from negative terminal flow to the left plate to neutralize the positive charges stored in it. As a result, the net charges present in the plates begin to decrease and this is called discharging. These electron-flows are represented by arrows as shown in Figure (f). Once the charges are exhausted, C will be charged again but with reversed polarities as shown in Figure (g).

Thus the electrons flow in one direction while charging the capacitor and its direction is reversed while discharging (the conventional current is also opposite in both cases). Though electrons flow in the circuit, no electron crosses the gap between the plates. In this way, AC flows through a capacitor.

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$$I_m = 2.2 \times \sqrt{2} = 3.1A$$

Therefore,

$$\upsilon = 311 \sin 314t$$

$$i = 3.1 \sin\left(314t + \pi/2\right)$$

4.7.6 AC circuit containing a resistor, an inductor and a capacitor in series – Series RLC circuit

Consider a circuit containing a resistor of resistance R, a inductor of inductance L and a capacitor of capacitance C connected across an alternating voltage source (Figure 4.51). The applied alternating voltage is given by the equation.

$$\upsilon = V_m \sin \omega t \qquad (4.52)$$

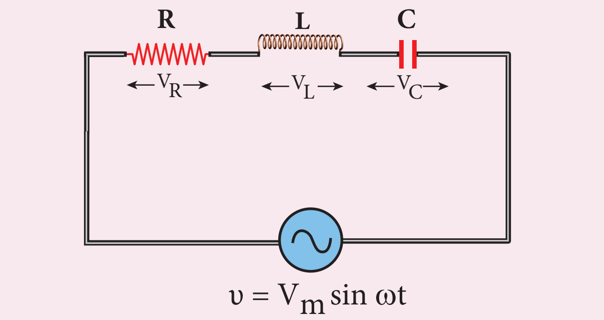

Figure 4.51 AC circuit containing R, L and C

Let i be the resulting circuit current in the circuit at that instant. As a result, the voltage is developed across R, L and C.

We know that voltage across R (V_R) is in phase with i, voltage across L (V_L) leads i by $\pi/2$ and voltage across C (V_C) lags i by $\pi/2$.

The phasor diagram is drawn with current as the reference phasor. The current is represented by the phasor $\overrightarrow{OI}$, V_R by $\overrightarrow{OA}$; V_L by $\overrightarrow{OB}$ and V_C by $\overrightarrow{OC}$ as shown in Figure 4.52.

The length of these phasors are

$OI = I_m$, $OA = I_m R$, $OB = I_m X_L$; $OC = I_m X_C$

The circuit is either effectively inductive or capacitive or resistive that depends on the value of V_L or V_C. Let us assume that $V_L > V_C$ so that net voltage drop across L-C combination is $V_L - V_C$ which is represented by a phasor $\overrightarrow{AD}$.

By parallelogram law, the diagonal $\overrightarrow{OE}$ gives the resultant voltage υ of V_R and $(V_L - V_C)$ and its length OE is equal to V_m. Therefore,

$$V_m^2 = V_R^2 + \left(V_L - V_C\right)^2$$

$$= \sqrt{\left(I_m R\right)^2 + \left(I_m X_L - I_m X_C\right)^2}$$

$$= I_m \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

$$\text{or} \quad I_m = \frac{V_m}{\sqrt{R^2 + \left(X_L - X_C\right)^2}} \qquad (4.53)$$

$$\text{or} \quad I_m = \frac{V_m}{Z}$$

$$\text{where} \quad Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} \qquad (4.54)$$

Z is called impedance of the circuit which refers to the effective opposition to the circuit current by the series RLC circuit. The voltage triangle and impedance triangle are given in the Figure 4.53.

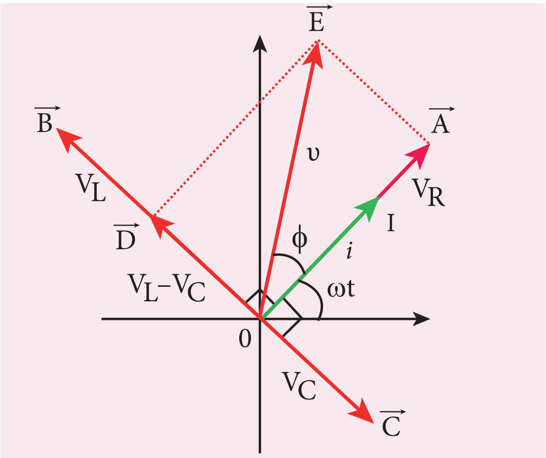

Figure 4.52 Phasor diagram for a series RLC – circuit when $V_L > V_C$

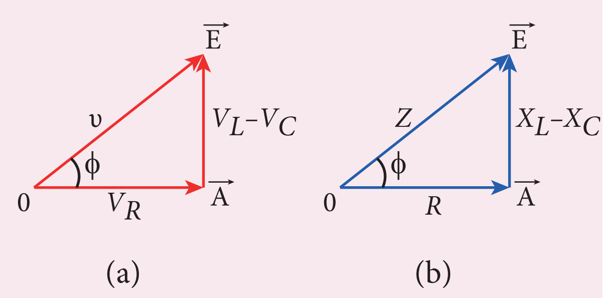

Figure 4.53 Voltage and impedance triangle when $X_L > X_C$

From phasor diagram, the phase angle between υ and i is found out from the following relation

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} \quad (4.55)$$

Special cases

(i) If $X_L > X_C$, $(X_L - X_C)$ is positive and phase angle ϕ is also positive. It means that the applied voltage leads the current by ϕ (or current lags behind voltage by ϕ). The circuit is inductive.

$$\therefore \upsilon = V_m \sin\omega t; \; i = I_m \sin(\omega t - \phi)$$

(ii) If $X_L < X_C$, $(X_L - X_C)$ is negative and ϕ is also negative. Therefore current leads voltage by ϕ and the circuit is capacitive.

$$\therefore \upsilon = V_m \sin\omega t; \; i = I_m \sin(\omega t + \phi)$$

(iii) If $X_L = X_C$, ϕ is zero. Therefore current and voltage are in the same phase and the circuit is resistive.

$$\therefore \upsilon = V_m \sin\omega t; \quad i = I_m \sin\omega t$$

4.7.7 Resonance in series RLC Circuit

When the frequency of the applied alternating source (ω_r) is equal to the natural frequency $\left[{1}/{\sqrt{LC}} \right]$ of the RLC circuit, the current in the circuit reaches its maximum value. Then the circuit is said to be in electrical resonance. The frequency at which resonance takes place is called resonant frequency.

Resonant angular frequency, $\omega_r = \frac{1}{\sqrt{LC}}$

$$\text{or} \quad f_r = \frac{1}{2\pi\sqrt{LC}} \quad (4.56)$$

At series resonance,

$$\omega_r = \frac{1}{\sqrt{LC}} \quad \text{or} \quad \omega_r^2 = \frac{1}{LC}$$

$$\omega_r L = \frac{1}{\omega_r C} \quad \text{or}$$

$$X_L = X_C \quad (4.57)$$

Table 4.1 Summary of results of AC circuits

Type of Impedance	Value of Impedance	Phase angle of current with voltage	Power factor
Resistance	R	0°	1
Inductance	$X_L = \omega L$	90° lag	0
Capacitance	$X_C = {1}/{\omega C}$	90° lead	0
R- L - C	$\sqrt{R^2 + \left(\omega L - {1}/{\omega C}\right)^2}$	Between 0° and 90° lag or lead	Between 0 and 1

Since X_L and X_C are frequency dependent, the resonance condition $(X_L = X_C)$ can be achieved by the varying the frequency of the applied voltage.

Effects of series resonance

When series resonance occurs, the impedance of the circuit is minimum and is equal to the resistance of the circuit. As a result of this, the current in the circuit becomes maximum. This is shown in the resonance curve drawn between current and frequency (Figure 4.54).

At resonance, the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R \quad \text{since} X_L = X_C$$

Therefore, the current in the circuit is

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$I_m = \frac{V_m}{R} \qquad (4.58)$$

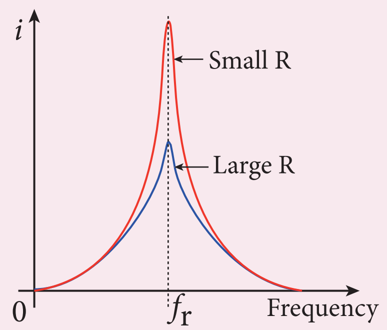

Figure 4.54 Resonance curve

The maximum current at series resonance is limited by the resistance of the circuit. For smaller resistance, larger current with sharper curve is obtained and vice versa.

Applications of series RLC resonant circuit

RLC circuits have many applications like filter circuits, oscillators, voltage multipliers etc. An important use of series RLC resonant circuits is in the tuning circuits of radio and TV systems. The signals from many broadcasting stations at different frequencies are available in the air. To receive the signal of a particular station, tuning is done.

The tuning is commonly achieved by varying capacitance of a parallel plate variable capacitor, thereby changing the resonant frequency of the circuit. When resonant frequency is nearly equal to the frequency of the signal of the particular station, the amplitude of the current in the circuit is maximum. Thus the signal of that station alone is received.

Note The phenomenon of electrical resonance is possible when the circuit contains both *L* and *C*. Only then the voltage across *L* and *C* cancel one another when V_L and V_C are 180° out of phase and the circuit becomes purely resistive. This implies that resonance will not occur in a *RL* and *RC* circuits.

4.7.8 Quality factor or Q–factor

The current in the series *RLC* circuit becomes maximum at resonance. Due to the increase in current, the voltage across *L* and *C* are also increased. This magnification of voltages at series resonance is termed as Q–factor.

It is defined as the ratio of voltage across L or C to the applied voltage.

$$\text{Q-factor} = \frac{\text{Voltage across L or C}}{\text{Applied voltage}}$$

At resonance, the circuit is purely resistive. Therefore, the applied voltage is equal to the voltage across R.

$$\text{Q-factor} = \frac{I_m X_L}{I_m R} = \frac{X_L}{R}$$

$$\text{Q-factor} = \frac{\omega_r L}{R} \quad (4.59)$$

$$\text{Q-factor} = \frac{L}{R\sqrt{LC}} \quad \text{since } \omega_r = \frac{1}{\sqrt{LC}}$$

$$\text{Q-factor} = \frac{1}{R}\sqrt{\frac{L}{C}} \quad (4.60)$$

The physical meaning is that Q–factor indicates the number of times the voltage across L or C is greater than the applied voltage at resonance.

EXAMPLE 4.22

Find the impedance of a series RLC circuit if the inductive reactance, capacitive reactance and resistance are 184 Ω, 144 Ω and 30 Ω respectively. Also calculate the phase angle between voltage and current.

Solution

$X_L = 184\ \Omega$; $X_C = 144\ \Omega$

$R = 30\ \Omega$

(i) The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{30^2 + (184-144)^2}$$

$$= \sqrt{900+1600}$$

Impedance, $Z = 50\ \Omega$

(ii) Phase angle is

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$= \frac{184-144}{30} = 1.33$$

$$\phi = 53.1^\circ$$

Since the phase angle is positive, voltage leads current by 53.1° for this inductive circuit.

EXAMPLE 4.23

A 500 μH inductor, $\frac{80}{\pi^2}\,pF$ capacitor and a 628 Ω resistor are connected to form a series RLC circuit. Calculate the resonant frequency and Q-factor of this circuit at resonance.

Solution

$$L = 500\times10^{-6}\text{H};\ C = \frac{80}{\pi^2}\times10^{-12}F;\ \ R = 628\Omega$$

(i) Resonant frequency is

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{500\times10^{-6}\times\frac{80}{\pi^2}\times10^{-12}}}$$

$$= \frac{1}{2\sqrt{40{,}000\times10^{-18}}}$$

$$= \frac{10{,}000\times10^{3}}{4}$$

$$f_r = 2500\ KHz$$

(ii) Q–factor

$$= \frac{\omega_r L}{R} = \frac{2\times3.14\times2500\times10^3\times500\times10^{-6}}{628}$$

$$Q = 12.5$$

EXAMPLE 4.24

Find the instantaneous value of alternating voltage $\upsilon = 10\sin(3\pi\times10^4 t)$ volt at i) 0 s ii) 50 μs iii) 75 μs.

Solution

The given equation is $\upsilon = 10\sin\left(3\pi\times10^4 t\right)$

(i) At t = 0 s,

$$\upsilon = 10\sin 0 = 0\ V$$

(ii) At t = 50 μs,

$$\upsilon = 10\sin\left(3\pi\times10^4\times50\times10^{-6}\right)$$
$$= 10\sin\left(150\pi\times10^{-2}\right)$$
$$= 10\sin(4.71\ rad)$$
$$= 10\times -0.99$$
$$= -9.9\ V$$

(iii) At t = 75 μs,

$$\upsilon = 10\sin\left(3\pi\times10^4\times75\times10^{-6}\right)$$
$$= 10\sin\left(225\pi\times10^{-2}\right)$$
$$= 10\sin(7.071\ rad)$$
$$= 10\times0.709$$
$$= 7.09\ V$$

EXAMPLE 4.25

The current in an inductive circuit is given by 0.3 sin ($200t$ – 40°) A. Write the equation for the voltage across it if the inductance is 40 mH.

Solution

$L = 40\times10^{-3}$ H; $i = 0.1\sin(200t - 40^\circ)$

$X_L = \omega L = 200\times40\times10^{-3} = 8\ \Omega$

$V_m = I_m X_L = 0.3\times8 = 2.4$ V

In an inductive circuit, the voltage leads the current by 90° Therefore,

$$\upsilon = V_m\sin\left(\omega t + 90^\circ\right)$$

$$\upsilon = 2.4\sin\left(200t - 40^\circ + 90^\circ\right)$$

$$\upsilon = 2.4\sin\left(200t + 50^\circ\right) volt$$

4.8 POWER IN AC CIRCUITS

4.8.1 Introduction of power in AC circuits

Power of a circuit is defined as the rate of consumption of electric energy in that circuit. It is given by the product of the voltage and current. In an AC circuit, the voltage and current vary continuously with time. Let us first calculate the power at an instant and then it is averaged over a complete cycle.

The alternating voltage and alternating current in the series *RLC* circuit at an instant are given by

$$\upsilon = V_m\sin\omega t \quad \text{and} \quad i = I_m\sin\left(\omega t + \phi\right)$$

where ϕ is the phase angle between υ and i. The instantaneous power is then written as

$$P = \upsilon\, i$$
$$= V_m I_m \sin\omega t\sin\left(\omega t + \phi\right)$$
$$= V_m I_m \sin\omega t\left[\sin\omega t\cos\phi - \cos\omega t\sin\phi\right]$$
$$P = V_m I_m\left[\cos\phi\sin^2\omega t - \sin\omega t\cos\omega t\sin\phi\right] \qquad (4.61)$$

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Here the average of $\sin^2 \omega t$ over a cycle is $\frac{1}{2}$ and that of $\sin\omega t \cos\omega t$ is zero. Substituting these values, we obtain average power over a cycle.

$$P_{av} = V_m I_m \cos\phi \times \frac{1}{2}$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi$$

$$P_{av} = V_{RMS}\, I_{RMS} \cos\phi \qquad (4.62)$$

where $V_{RMS} I_{RMS}$ is called apparent power and $\cos\phi$ is power factor. The average power of an AC circuit is also known as the true power of the circuit.

Special Cases

(i) For a purely resistive circuit, the phase angle between voltage and current is zero and $\cos\phi = 1$.

$\therefore P_{av} = V_{RMS}\, I_{RMS}$

(ii) For a purely inductive or capacitive circuit, the phase angle is $\pm \frac{\pi}{2}$ and $\cos\left(\pm\frac{\pi}{2}\right) = 0$.

$\therefore P_{av} = 0$

(iii) For series RLC circuit, the phase angle $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

$\therefore P_{av} = V_{RMS}\, I_{RMS}\, \cos\phi$

(iv) For series RLC circuit at resonance, the phase angle is zero and $\cos\phi = 1$.

$\therefore P_{av} = V_{RMS}\, I_{RMS}$

4.8.2 Wattless current

Consider an AC circuit in which there is a phase angle of ϕ between V_{RMS} and I_{RMS} and voltage is assumed to be leading the current by ϕ as shown in the phasor diagram (Figure 4.55).

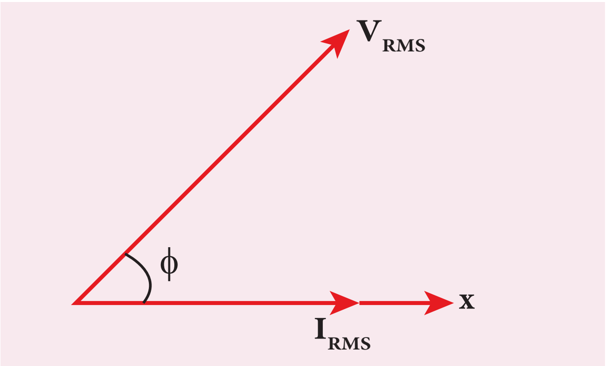

Figure 4.55 V_{RMS} leads I_{RMS} by ϕ

Now, I_{RMS} is resolved into two perpendicular components namely, $I_{RMS}\cos\phi$ along V_{RMS} and $I_{RMS}\sin\phi$ perpendicular to V_{RMS} as shown in Figure 4.56.

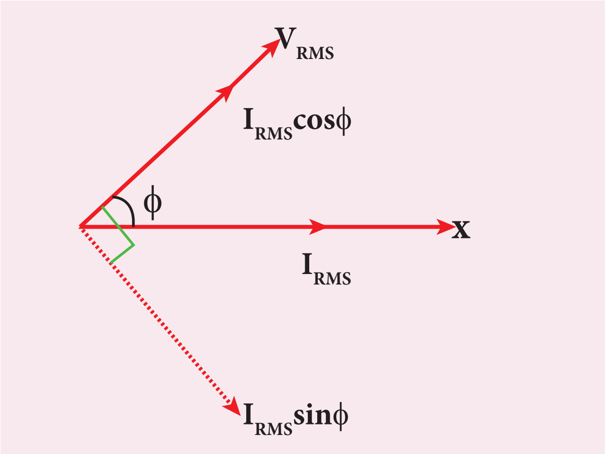

Figure 4.56 The components of I_{RMS}

(i) The component of current $(I_{RMS}\cos\phi)$ which is in phase with the voltage is called active component. The power consumed by this current $= V_{RMS} I_{RMS}\cos\phi$. So that it is also known as 'Wattful' current.

(ii) The other component $(I_{RMS}\sin\phi)$ which has a phase angle of $\frac{\pi}{2}$ with the voltage is called reactive component. The power consumed is zero. So that it is also known as 'Wattless' current.

The current in an AC circuit is said to be wattless current if the power consumed by it is zero. This wattless current happens in a purely inductive or capacitive circuit.

4.8.3 Power factor

The power factor of a circuit is defined in one of the following ways:

(i) **Power factor = cos ϕ = cosine of the angle of lead or lag**

(ii) **Power factor =** $\frac{R}{Z} = \frac{\text{Resistance}}{\text{Impedance}}$

(iii) **Power factor =** $\frac{VI\cos\phi}{VI}$

$$= \frac{\text{True power}}{\text{Apparent power}}$$

Some examples for power factors:

(i) Power factor = cos 0° = 1 for a pure resistive circuit because the phase angle ϕ between voltage and current is zero.

(ii) Power factor $= \cos\left(\pm \pi/2\right) = 0$ for a purely inductive or capacitive circuit because the phase angle ϕ between voltage and current is $\pm \pi/2$.

(iii) Power factor lies between 0 and 1 for a circuit having R, L and C in varying proportions.

4.8.4 Advantages and disadvantages of AC over DC

There are many advantages and disadvantages of AC system over DC system.

Advantages:

(i) The generation of AC is cheaper than that of DC.

(ii) When AC is supplied at higher voltages, the transmission losses are small compared to DC transmission.

(iii) AC can easily be converted into DC with the help of rectifiers.

Disadvantages:

(i) Alternating voltages cannot be used for certain applications e.g. charging of batteries, electroplating, electric traction etc.

(ii) At high voltages, it is more dangerous to work with AC than DC.

EXAMPLE 4.26

A series RLC circuit which resonates at 400 kHz has 80 μH inductor, 2000 pF capacitor and 50 Ω resistor. Calculate (i) Q-factor of the circuit (ii) the new value of capacitance when the value of inductance is doubled and (iii) the new Q-factor.

Solution

$L = 80 \times 10^{-6}$H; $C = 2000 \times 10^{-12}$ F

$R = 50\ \Omega$; $f_r = 400 \times 10^3$Hz

(i) Q-factor, $Q_1 = \frac{1}{R}\sqrt{\frac{L}{C}}$

$$= \frac{1}{50}\sqrt{\frac{80\times10^{-6}}{200\times10^{-12}}} = 4$$

(ii) When $L_2 = 2$ L

$= 2 \times 80 \times 10^{-6}$ H $= 160 \times 10^{-6}$ H,

$$C_2 = \frac{1}{4\pi^2 f_r^2 L_2}$$

$$= \frac{1}{4\times3.14^2\times\left(400\times10^3\right)^2\times160\times10^{-6}}$$

$$\simeq 1000\times10^{-12}\,F$$

$$C_2 \simeq 1000\,pF$$

(iii) $Q_2 = \frac{1}{R}\sqrt{\frac{L_2}{C_2}} = \frac{1}{50}\sqrt{\frac{160\times10^{-6}}{1000\times10^{-12}}}$

$$= \frac{1}{50}\sqrt{\frac{16\times10^{-5}}{10^{-9}}} = \frac{4\times10^2}{50} = 8$$

EXAMPLE 4.27

A capacitor of capacitance $\frac{10^{-4}}{\pi}F$, an inductor of inductance $\frac{2}{\pi}H$ and a resistor of resistance 100 Ω are connected to form a series *RLC* circuit. When an AC supply of 220 V, 50 Hz is applied to the circuit, determine (i) the impedance of the circuit (ii) the peak value of current flowing in the circuit (iii) the power factor of the circuit and (iv) the power factor of the circuit at resonance.

Solution

$$L=\frac{2}{\pi}H; C=\frac{10^{-4}}{\pi}F; R=100\Omega$$
$$V_{RMS}=220\ V; f=50\,Hz$$

$$X_L=2\pi fL=2\pi\times 50\times {}^{2}\!/_{\pi}=200\Omega$$

$$X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi\times 50\times\frac{10^{-4}}{\pi}}=100\Omega$$

(i) Impedance, $Z=\sqrt{R^2+\left(X_L-X_C\right)^2}$

$$=\sqrt{100^2+\left(200-100\right)^2}=141.4\ \Omega$$

(ii) Peak value of current,

$$I_m=\frac{V_m}{Z}=\frac{\sqrt{2}V_{RMS}}{Z}$$
$$=\frac{\sqrt{2}\times 220}{141.4}=2.2\ \text{A}$$

(iii) Power factor of the circuit,

$$\cos\phi=\frac{R}{Z}=\frac{100}{141.4}=0.707$$

(iv) Power factor at resonance

$$\cos\phi=\frac{R}{Z}=\frac{R}{R}=1$$

4.9 OSCILLATION IN LC CIRCUITS

4.9.1 Energy conversion during LC oscillations

We have learnt that energy can be stored in both inductors and capacitors (Refer sections 1.8.2 and 4.3.2). In inductors, the energy is stored in the form of magnetic field while in capacitors; it is stored as the electric field.

Whenever energy is given to a circuit containing a pure inductor of inductance *L* and a capacitor of capacitance *C*, the energy oscillates back and forth between the magnetic field of the inductor and the electric field of the capacitor. Thus the electrical oscillations of definite frequency are generated. These oscillations are called *LC* oscillations.

Generation of LC oscillations

Let us assume that the capacitor is fully charged with maximum charge Q_m at the initial stage. So that the energy stored in the capacitor is maximum and is given by $U_E={Q_m^2}/{2C}$. As there is no current in the inductor, the energy stored in it is zero i.e., $U_B=0$. Therefore, the total energy is wholly electrical. This is shown in Figure 4.57(a).

The capacitor now begins to discharge through the inductor that establishes current *i* in clockwise direction. This current produces a magnetic field around the inductor and the energy stored in the inductor is given by $U_B={Li^2}/{2}$. As the charge in the capacitor decreases, the energy stored in it also decreases and is given by

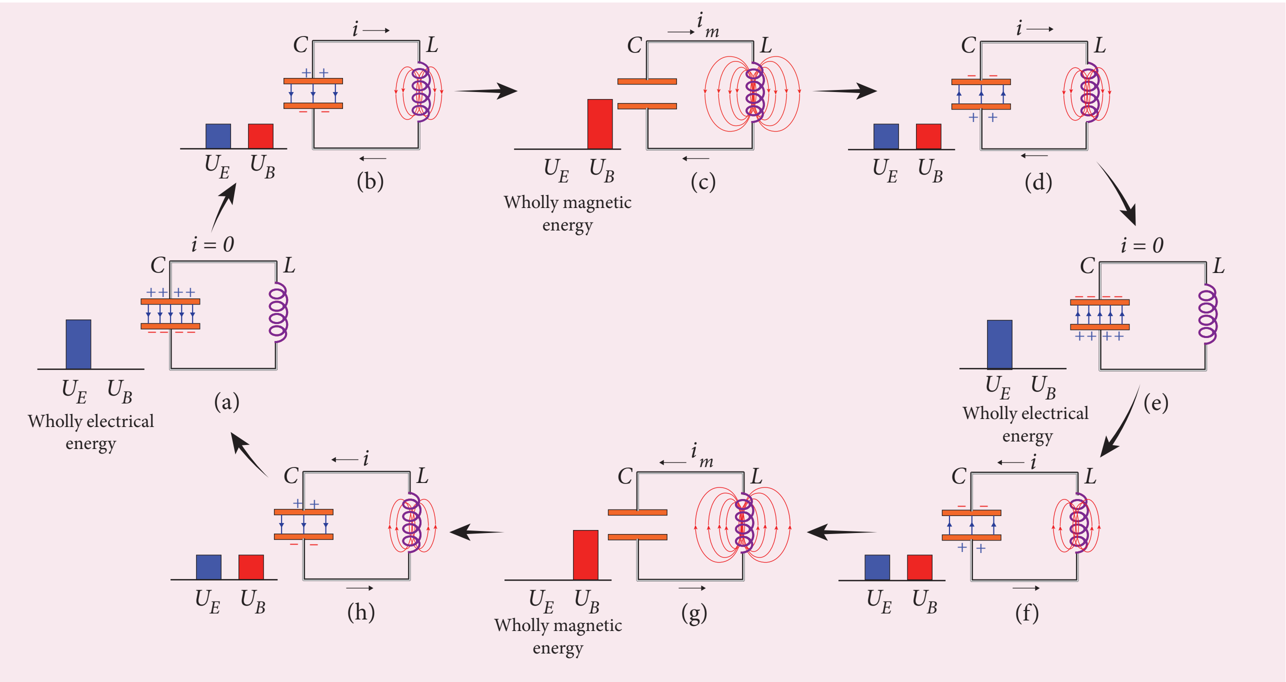

Figure 4.57 LC oscillations

$U_E = q^2/2C$. Thus there is a transfer of some part of energy from the capacitor to the inductor. At that instant, the total energy is the sum of electrical and magnetic energies (Figure 4.57(b)).

When the charges in the capacitor are exhausted, its energy becomes zero i.e., $U_E = 0$. The energy is fully transferred to the magnetic field of the inductor and its energy is maximum. This maximum energy is given by $U_B = LI_m^2/2$ where I_m is the maximum current flowing in the circuit. The total energy is wholly magnetic (Figure 4.57(c)).

Even though the charge in the capacitor is zero, the current will continue to flow in the same direction because the inductor will not allow it to stop immediately. The current is made to flow with decreasing magnitude by the collapsing magnetic field of the inductor. As a result of this, the capacitor begins to charge in the opposite direction. A part of the energy is transferred from the inductor back to the capacitor. The total energy is the sum of the electrical and magnetic energies (Figure 4.57(d)).

When the current in the circuit reduces to zero, the capacitor becomes fully charged in the opposite direction. The energy stored in the capacitor becomes maximum. Since the current is zero, the energy stored in the inductor is zero. The total energy is wholly electrical (Figure 4.57(e)).

The state of the circuit is similar to the initial state but the difference is that the capacitor is charged in opposite direction. The capacitor then starts to discharge through the inductor with anti-clockwise current. The total energy is the sum of the electrical and magnetic energies (Figure 4.57(f)).

As already explained, the processes are repeated in opposite direction (Figure 4.57(g) and (h)). Finally, the circuit returns to the initial state (Figure 4.57(a)). Thus, when the circuit goes through these stages, an alternating current flows in the circuit. As this process is repeated again and again, the electrical oscillations of definite

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frequency are generated. These are known as LC oscillations.

In the ideal LC circuit, there is no loss of energy. Therefore, the oscillations will continue indefinitely. Such oscillations are called undamped oscillations.

Note But in practice, the Joule heating and radiation of electromagnetic waves from the circuit decrease the energy of the system. Therefore, the oscillations become damped oscillations.

4.9.2 Conservation of energy in LC oscillations

During LC oscillations in LC circuits, the energy of the system oscillates between the electric field of the capacitor and the magnetic field of the inductor. Although, these two forms of energy vary with time, the total energy remains constant. It means that LC oscillations take place in accordance with the law of conservation of energy.

$$\text{Total energy}, U = U_E + U_B = \frac{q^2}{2C} + \frac{1}{2}Li^2$$

Let us consider 3 different stages of LC oscillations and calculate the total energy of the system.

Case (i) When the charge in the capacitor, $q = Q_m$ and the current through the inductor, $i = 0$, the total energy is given by

$$U = \frac{Q_m^2}{2C} + 0 = \frac{Q_m^2}{2C} \quad (4.63)$$

The total energy is wholly electrical.

Case (ii) When charge = 0 ; current = I_m, the total energy is

$$U = 0 + \frac{1}{2}LI_m^2 = \frac{1}{2}LI_m^2$$

$$= \frac{L}{2} \times \left(\frac{Q_m^2}{LC}\right) \quad \text{since } I_m = Q_m\omega = \frac{Q_m}{\sqrt{LC}}$$

$$= \frac{Q_m^2}{2C} \quad (4.64)$$

The total energy is wholly magnetic.

Case (iii) When charge = q; current = i, the total energy is

$$U = \frac{q^2}{2C} + \frac{1}{2}Li^2$$

Since $q = Q_m \cos\omega t$, $i = -\frac{dq}{dt} = Q_m \omega \sin\omega t$.

The negative sign in current indicates that the charge in the capacitor decreases with time.

$$U = \frac{Q_m^2 \cos^2 \omega t}{2C} + \frac{L\omega^2 Q_m^2 \sin^2 \omega t}{2}$$

$$= \frac{Q_m^2 \cos^2 \omega t}{2C} + \frac{LQ_m^2 \sin^2 \omega t}{2LC} \quad \text{since } \omega^2 = \frac{1}{LC}$$

$$= \frac{Q_m^2}{2C}\left(\cos^2 \omega t + \sin^2 \omega t\right)$$

$$U = \frac{Q_m^2}{2C} \quad (4.65)$$

From above three cases, it is clear that the total energy of the system remains constant.

4.9.3 Analogies between LC oscillations and simple harmonic oscillations

(i) Qualitative treatment

The electromagnetic oscillations of LC system can be compared with the mechanical oscillations of a spring-mass system.

There are two forms of energy involved in LC oscillations. One is electrical energy

Table 4.3 Energy in two oscillatory systems

LC oscillator		Spring-mass system	
Element	**Energy**	**Element**	**Energy**
Capacitor	Electrical Energy $= \frac{1}{2}\left(\frac{1}{C}\right)q^2$	Spring	Potential energy $= \frac{1}{2}k\,x^2$
Inductor	Magnetic energy $= \frac{1}{2}Li^2 \quad i = \frac{dq}{dt}$	Mass	Kinetic energy $= \frac{1}{2}mv^2 \quad v = \frac{dx}{dt}$

of the charged capacitor; the other magnetic energy of the inductor carrying current.

Likewise, the mechanical energy of the spring-mass system exists in two forms; the potential energy of the compressed or extended spring and the kinetic energy of the mass. The Table 4.3 lists these two pairs of energy.

By examining the Table 4.3, the analogies between the various quantities can be understood and these correspondences are given in the Table 4.4.

The angular frequency of oscillations of a spring-mass is given by (Refer equation 10.22 of section 10.4.1 of XI physics text book).

$$\omega = \sqrt{\frac{k}{m}}$$

From Table 4.4, $k \to {}^{1}\!/_{C}$ and $m \to L$. Therefore, the angular frequency of LC oscillations is given by

$$\omega = \frac{1}{\sqrt{LC}} \qquad (4.66)$$

(ii) Quantitative treatment

The mechanical energy of the spring-mass system is given by

$$E = \frac{1}{2}mv^2 + \frac{1}{2}k\,x^2 \qquad (4.67)$$

The energy E remains constant for varying values of x and v. Differentiating E with respect to time, we get

Table 4.4 Analogies between electrical and mechanical quantities

Electrical system	Mechanical system
Charge q	Displacement x
Current $i = \frac{dq}{dt}$	Velocity $v = \frac{dx}{dt}$
Inductance L	Mass m
Reciprocal of capacitance $\frac{1}{C}$	Force constant k
Electrical energy $= \frac{1}{2}\left(\frac{1}{C}\right)q^2$	Potential energy $= \frac{1}{2}k\,x^2$
Magnetic energy $= \frac{1}{2}Li^2$	Kinetic energy $= \frac{1}{2}mv^2$
Electromagnetic energy $U = \frac{1}{2}\left(\frac{1}{C}\right)q^2 + \frac{1}{2}Li^2$	Mechanical energy $E = \frac{1}{2}k\,x^2 + \frac{1}{2}mv^2$

$$\frac{dE}{dt} = \frac{1}{2}m\left(2v\frac{dv}{dt}\right) + \frac{1}{2}k\left(2x\frac{dx}{dt}\right) = 0$$

$$\text{or} \quad m\frac{d^2x}{dt^2} + kx = 0 \qquad (4.68)$$

$$\text{since } \frac{dx}{dt} = v \text{ and } \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

This is the differential equation of the oscillations of the spring-mass system. The general solution of equation (4.68) is of the form

$$x(t) = X_m \cos(\omega t + \phi) \quad (4.69)$$

where X_m is the maximum value of $x(t)$, ω the angular frequency and ϕ the phase constant.

Similarly, the electromagnetic energy of the LC system is given by

$$U = \frac{1}{2}Li^2 + \frac{1}{2}\left(\frac{1}{C}\right)q^2 = \text{constant} \quad (4.70)$$

Differentiating U with respect to time, we get

$$\frac{dU}{dt} = \frac{1}{2}L\left(2i\frac{di}{dt}\right) + \frac{1}{2C}\left(2q\frac{dq}{dt}\right) = 0$$

$$\text{or} \quad L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0 \quad (4.71)$$

$$\text{since } i = \frac{dq}{dt} \text{ and } \frac{di}{dt} = \frac{d^2q}{dt^2}$$

The general solution of equation (4.71) is of the form

$$q(t) = Q_m \cos(\omega t + \phi) \quad (4.72)$$

where Q_m is the maximum value of $q(t)$, ω the angular frequency and ϕ the phase constant.

Current in the LC circuit

The current flowing in the LC circuit is obtained by differentiating $q(t)$ with respect to time.

$$i(t) = \frac{dq}{dt} = \frac{d}{dt}\left[Q_m \cos(\omega t + \phi)\right]$$

$$= -Q_m \omega \sin(\omega t + \phi) \qquad \text{since } I_m = Q_m \omega$$

$$\text{or} \quad i(t) = -I_m \sin(\omega t + \phi) \quad (4.73)$$

The equation (4.73) clearly shows that current varies as a function of time t. In fact, it is a sinusoidally varying alternating current with angular frequency ω.

Angular frequency of LC oscillations

By differentiating equation (4.72) twice, we get

$$\frac{d^2q}{dt} = -Q_m \omega^2 \cos(\omega t + \phi) \quad (4.74)$$

Substituting equations (4.72) and (4.74) in equation (4.71), we obtain

$$L\left[-Q_m \omega^2 \cos(\omega t + \phi)\right] + \frac{1}{C}Q_m \cos(\omega t + \phi) = 0$$

Rearranging the terms, the angular frequency of LC oscillations is given by

$$\omega = \frac{1}{\sqrt{LC}} \quad (4.75)$$

This equation is the same as that obtained from qualitative analogy.

Oscillations of electrical and magnetic energy

The electrical energy of the LC oscillator is

$$U_E = \frac{q^2}{2C} = \frac{Q_m^2}{2C}\cos^2(\omega t + \phi) \quad (4.76)$$

The magnetic energy is

$$U_B = \frac{1}{2}Li^2 = \frac{Q_m^2}{2C}\sin^2(\omega t + \phi) \quad (4.77)$$

If the two energies are plotted with an assumption of $\phi = 0$, we obtain Figure 4.58.

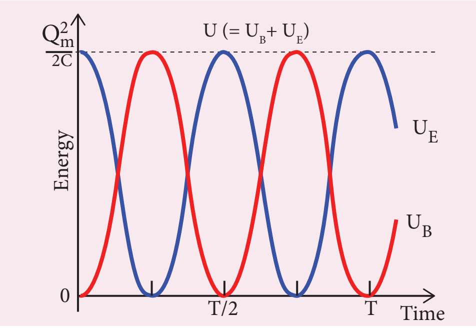


Figure 4.58 The variation of U_E and U_B as a function of time

From the graph, it can be noted that

(i) At any instant $U_E + U_B = \frac{Q_m^2}{2C}$ =constant

(ii) The maximum values of U_E and U_B are both $\frac{Q_m^2}{2C}$.

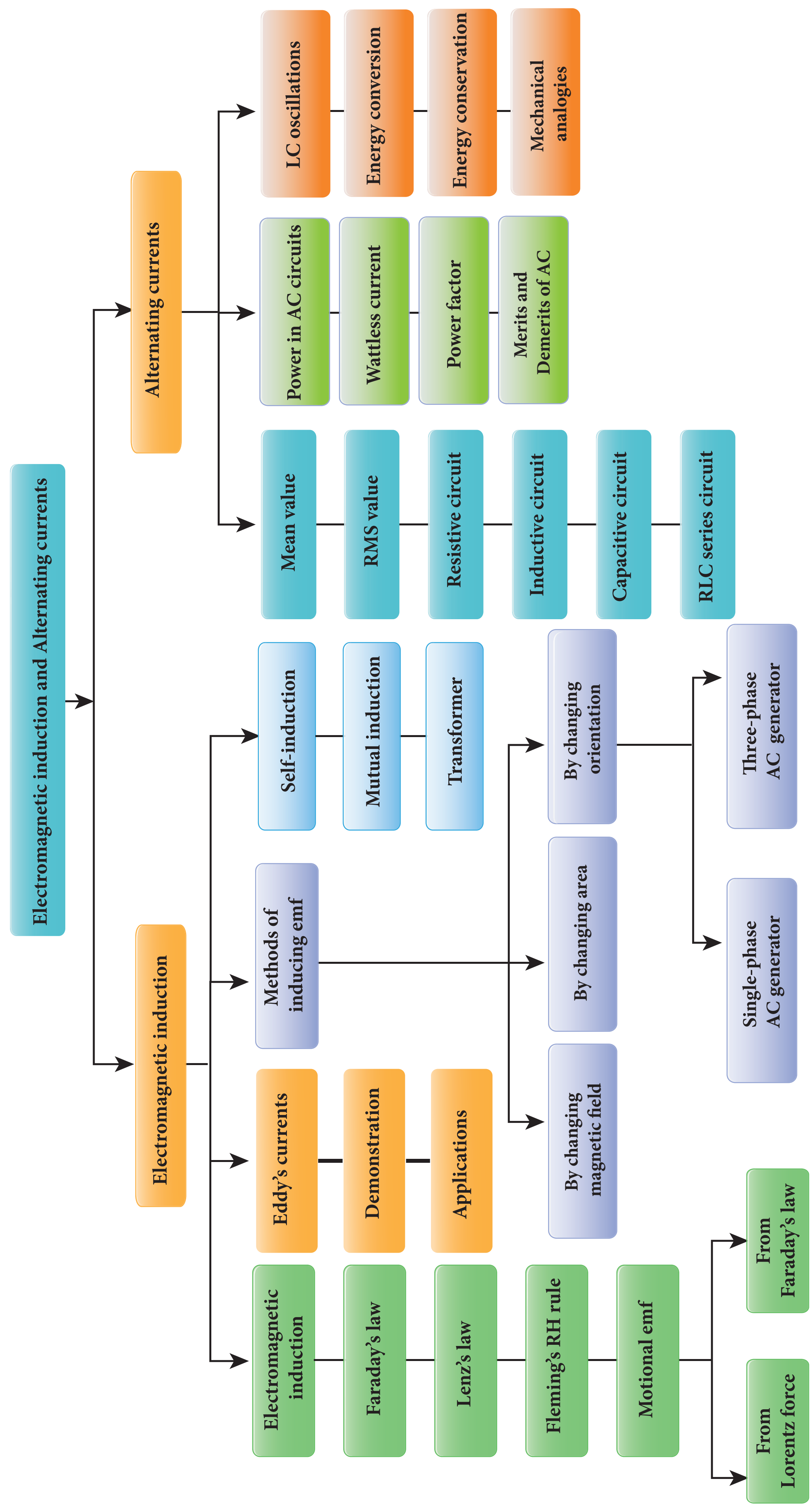
(iii) When U_E is Maximum, U_B is zero and vice versa.

SUMMARY

- Whenever the magnetic flux linked with a closed coil changes, an emf is induced and hence an electric current flows in the circuit. This phenomenon is known as electromagnetic induction.
- Faraday's first law states that whenever magnetic flux linked with a closed circuit changes, an emf is induced in the circuit.
- Faraday's second law states that the magnitude of induced emf in a closed circuit is equal to the time rate of change of magnetic flux linked with the circuit.
- Lenz's law states that the direction of the induced current is such that it always opposes the cause responsible for its production.
- Lenz's law is established on the basis of the law of conservation of energy.
- Fleming's right hand rule states that if the index finger points the direction of the magnetic field and the thumb indicates the direction of motion of the conductor, then the middle finger will indicate the direction of the induced current.
- Even for a conductor in the form of a sheet or a plate, an emf is induced when magnetic flux linked with it changes. The induced currents flow in concentric circular paths called Eddy currents or Foucault currents.
- Inductor is a device used to store energy in a magnetic field when an electric current flows through it.
- If the flux linked with the coil is changed, an emf is induced in that same coil. This phenomenon is known as self-induction. The emf induced is called self-induced emf.
- When an electric current passing through a coil changes with time, an emf is induced in the neighbouring coil. This phenomenon is known as mutual induction and the emf is called mutually induced emf.
- AC generator or alternator is an energy conversion device. It converts mechanical energy used to rotate the coil or field magnet into electrical energy.
- In some AC generators, there are three separate coils, which would give three separate emfs. Hence they are called three-phase AC generators.
- Transformer is a stationary device used to transform AC electric power from one circuit to another without changing its frequency.
- The efficiency of a transformer is defined as the ratio of the useful output power to the input power.
- An alternating voltage is a voltage which changes polarity at regular intervals of time and the resulting alternating current changes direction accordingly.
- The average value of alternating current is defined as the average of all values of current over a positive half-circle or negative half-circle.
- The root mean square value or effective value of an alternating current is defined as the square root of the mean of the squares of all currents over one cycle.

- A sinusoidal alternating voltage (or current) can be represented by a vector which rotates about the origin in anti-clockwise direction at a constant angular velocity. Such a rotating vector is called a phasor.
- When the frequency of the applied alternating source is equal to the natural frequency of the RLC circuit, the current in the circuit reaches its maximum value. Then the circuit is said to be in electrical resonance.
- The magnification of voltages at series resonance is termed as Q–factor.
- Power of a circuit is defined as the rate of consumption of electric energy in that circuit. It depends on the components of the circuit.
- Whenever energy is given to a LC circuit, the electrical oscillations of definite frequency are generated. These oscillations are called LC oscillations.
- During LC oscillations, the total energy remains constant. It means that LC oscillations take place in accordance with the law of conservation of energy.

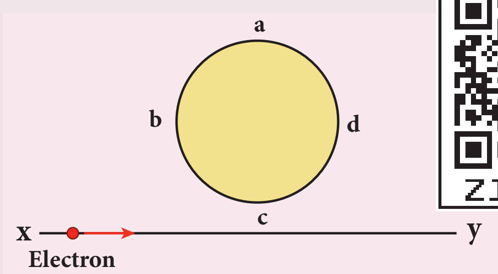
CONCEPT MAP


Electromagnetic induction and Alternating currents
Electromagnetic induction
Alternating currents
Electromagnetic induction
Faraday's law
Lenz's law
Fleming's RH rule
Motional emf
From Lorentz force
From Faraday's law
Eddy's currents
Demonstration
Applications
Methods of inducing emf
By changing magnetic field
By changing area
By changing orientation
Single-phase AC generator
Three-phase AC generator
Self-induction
Mutual induction
Transformer
Mean value
RMS value
Resistive circuit
Inductive circuit
Capacitive circuit
RLC series circuit
Power in AC circuits
Wattless current
Power factor
Merits and Demerits of AC
LC oscillations
Energy conversion
Energy conservation
Mechanical analogies

I Multiple Choice Questions

1. An electron moves on a straight line path XY as shown in the figure. The coil *abcd* is adjacent to the path of the electron. What will be the direction of current, if any, induced in the coil?

(NEET – 2015)



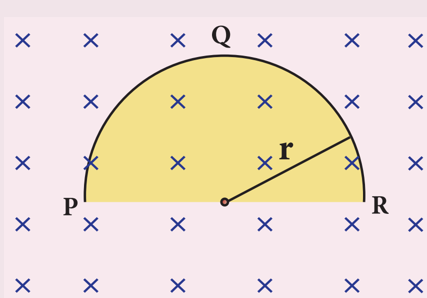
(a) The current will reverse its direction as the electron goes past the coil

(b) No current will be induced

(c) abcd

(d) adcb

2. A thin semi-circular conducting ring (PQR) of radius r is falling with its plane vertical in a horizontal magnetic field B, as shown in the figure.



The potential difference developed across the ring when its speed v, is

(NEET 2014)

(a) Zero

(b) $\frac{Bv\pi r^2}{2}$ and P is at higher potential

(c) πrBv and R is at higher potential

(d) $2rBv$ and R is at higher potential

3. The flux linked with a coil at any instant t is given by $\Phi_B = 10t^2 - 50t + 250$. The induced emf at t = 3s is

(a) $-190\ V$ (b) $-10\ V$

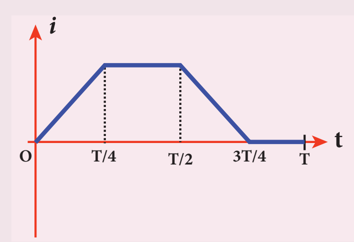
(c) $10\ V$ (d) $190\ V$

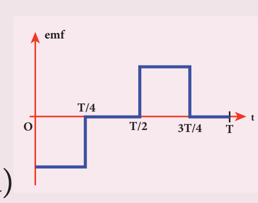
4. When the current changes from +2A to −2A in 0.05 s, an emf of 8 V is induced in a coil. The co-efficient of self-induction of the coil is

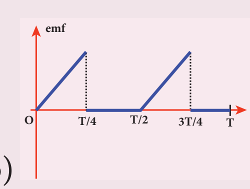
(a) 0.2 H (b) 0.4 H

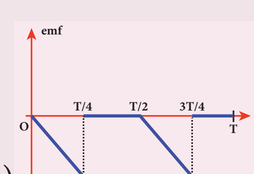
(c) 0.8 H (d) 0.1 H

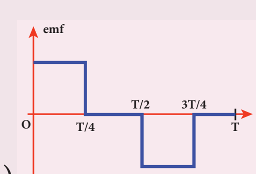
5. The current *i* flowing in a coil varies with time as shown in the figure. The variation of induced emf with time would be (NEET – 2011)



(a)


(b)


(c)


(d)


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6. A circular coil with a cross-sectional area of 4 cm^2 has 10 turns. It is placed at the centre of a long solenoid that has 15 turns/cm and a cross-sectional area of 10 cm^2. The axis of the coil coincides with the axis of the solenoid. What is their mutual inductance?

(a) 7.54 µH (b) 8.54 µH

(c) 9.54 µH (d) 10.54 µH

7. In a transformer, the number of turns in the primary and the secondary are 410 and 1230 respectively. If the current in primary is 6A, then that in the secondary coil is

(a) 2 A (b) 18 A

(c) 12 A (d) 1 A

8. A step-down transformer reduces the supply voltage from 220 V to 11 V and increase the current from 6 A to 100 A. Then its efficiency is

(a) 1.2 (b) 0.83

(c) 0.12 (d) 0.9

9. In an electrical circuit, *R*, *L*, *C* and AC voltage source are all connected in series. When *L* is removed from the circuit, the phase difference between the voltage and current in the circuit is $\pi/3$. Instead, if C is removed from the circuit, the phase difference is again $\pi/3$. The power factor of the circuit is

(NEET 2012)

(a) $1/2$ (b) $1/\sqrt{2}$

(c) 1 (d) $\sqrt{3}/2$

10. In a series RL circuit, the resistance and inductive reactance are the same. Then the phase difference between the voltage and current in the circuit is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{6}$ (d) zero

11. In a series resonant *RLC* circuit, the voltage across 100 Ω resistor is 40 V. The resonant frequency ω is 250 rad/s. If the value of C is 4 µF, then the voltage across L is

(a) 600 V (b) 4000 V

(c) 400V (d) 1 V

12. An inductor 20 *mH*, a capacitor 50 *µF* and a resistor 40 Ω are connected in series across a source of emf v = 10 sin 340 t. The power loss in AC circuit is

(a) 0.76 W (b) 0.89 W

(c) 0.46 W (d) 0.67 W

13. The instantaneous values of alternating current and voltage in a circuit are

$i = \frac{1}{\sqrt{2}}\sin(100\pi t)$ A and

$v = \frac{1}{\sqrt{2}}\sin\left(100\pi t + \frac{\pi}{3}\right)$ V.

The average power in watts consumed in the circuit is (IIT Main 2012)

(a) $\frac{1}{4}$ (b) $\frac{\sqrt{3}}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{8}$

14. In an oscillating LC circuit, the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic fields is

(a) $\frac{Q}{2}$ (b) $\frac{Q}{\sqrt{3}}$

(c) $\frac{Q}{\sqrt{2}}$ (d) Q

15. $\frac{20}{\pi^2} H$ inductor is connected to a capacitor of capacitance C. The value of C in order to impart maximum power at 50 Hz is

(a) 50 μF (b) 0.5 μF

(c) 500 μF (d) 5 μF

Answers

1) a	2) d	3) b	4) d	5) a
6) a	7) a	8) b	9) c	10) a
11) c	12) c	13) d	14) c	15) d

II Short Answer Questions

1. What is meant by electromagnetic induction?
4. State Faraday's laws of electromagnetic induction.
5. State Lenz's law.
6. State Fleming's right hand rule.
7. How is Eddy current produced? How do they flow in a conductor?
8. Mention the ways of producing induced emf.
9. What for an inductor is used? Give some examples.
10. What do you mean by self-induction?
11. What is meant by mutual induction?
12. Give the principle of AC generator.
13. List out the advantages of stationary armature-rotating field system of AC generator.
14. What are step-up and step-down transformers?
15. Define average value of an alternating current.
16. How will you define RMS value of an alternating current?
17. What are phasors?
18. Define electric resonance.
19. What do you mean by resonant frequency?
20. How will you define Q-factor?
21. What is meant by wattles current?
22. Give any one definition of power factor.
23. What are LC oscillations?

III Long Answer Questions

1. Establish the fact that the relative motion between the coil and the magnet induces an emf in the coil of a closed circuit.
2. Give an illustration of determining direction of induced current by using Lenz's law.
3. Show that Lenz's law is in accordance with the law of conservation of energy.
4. Obtain an expression for motional emf from Lorentz force.
5. Using Faraday's law of electromagnetic induction, derive an equation for motional emf.
6. Give the uses of Foucault current.
7. Define self-inductance of a coil interms of (i) magnetic flux and (ii) induced emf.
8. How will you define the unit of inductance?
9. What do you understand by self-inductance of a coil? Give its physical significance.
10. Assuming that the length of the solenoid is large when compared to

its diameter, find the equation for its inductance.

11. An inductor of inductance L carries an electric current i. How much energy is stored while establishing the current in it?

12. Show that the mutual inductance between a pair of coils is same (M_{12} = M_{21}).

13. How will you induce an emf by changing the area enclosed by the coil?

14. Show mathematically that the rotation of a coil in a magnetic field over one rotation induces an alternating emf of one cycle.

15. Elaborate the standard construction details of AC generator.

16. Explain the working of a single-phase AC generator with necessary diagram.

17. How are the three different emfs generated in a three-phase AC generator? Show the graphical representation of these three emfs.

18. Explain the construction and working of transformer.

19. Mention the various energy losses in a transformer.

20. Give the advantage of AC in long distance power transmission with an example.

21. Find out the phase relationship between voltage and current in a pure inductive circuit.

22. Derive an expression for phase angle between the applied voltage and current in a series RLC circuit.

23. Define inductive and capacitive reactance. Give their units.

24. Obtain an expression for average power of AC over a cycle. Discuss its special cases.

25. Show that the total energy is conserved during LC oscillations.

26. Prove that energy is conserved during electromagnetic induction.

27. Compare the electromagnetic oscillations of LC circuit with the mechanical oscillations of block-spring system to find the expression for angular frequency of LC oscillators mathematically.

IV. Numerical problems

1. A square coil of side 30 cm with 500 turns is kept in a uniform magnetic field of 0.4 T. The plane of the coil is inclined at an angle of 30° to the field. Calculate the magnetic flux through the coil. (Ans: 9 Wb)

2. A straight metal wire crosses a magnetic field of flux 4 mWb in a time 0.4 s. Find the magnitude of the emf induced in the wire. (Ans: 10 mV)

3. The magnetic flux passing through a coil perpendicular to its plane is a function of time and is given by $\Phi_B = (2t^3 + 4t^2 + 8t + 8)\ Wb$. If the resistance of the coil is 5 Ω, determine the induced current through the coil at a time t = 3 second. (Ans: 17.2 A)

4. A closely wound coil of radius 0.02 m is placed perpendicular to the magnetic field. When the magnetic field is changed from 8000 T to 2000 T in 6 s, an emf of 44 V is induced. Calculate the number of turns in the coil.

(Ans: 35 turns)

5. A rectangular coil of area 6 cm^2 having 3500 turns is kept in a uniform magnetic field of 0.4 T. Initially, the plane of the coil is perpendicular to the field and is then rotated through an angle of 180°. If the resistance of the coil is 35 Ω, find the amount of charge flowing through the coil.

(Ans: 48×10^{-3} C)

6. An induced current of 2.5 mA flows through a single conductor of resistance 100 Ω. Find out the rate at which the magnetic flux is cut by the conductor.

(Ans: 250 $mWbs^{-1}$)

7. A fan of metal blades of length 0.4 m rotates normal to a magnetic field of $4 \times 10^{-3} T$. If the induced emf between the centre and edge of the blade is 0.02 V, determine the rate of rotation of the blade.

(Ans: 9.95 revolutions/second)

8. A bicycle wheel with metal spokes of 1 m long rotates in Earth's magnetic field. The plane of the wheel is perpendicular to the horizontal component of Earth's field of $4 \times 10^{-5} T$. If the emf induced across the spokes is 31.4 mV, calculate the rate of revolution of the wheel.

(Ans: 250 revolutions/second)

9. Determine the self-inductance of 4000 turn air-core solenoid of length 2m and diameter 0.04 m. (Ans: 12.62 mH)

10. A coil of 200 turns carries a current of 4 A. If the magnetic flux through the coil is 6×10^{-5} Wb, find the magnetic energy stored in the medium surrounding the coil. (Ans: 0.024 J)

11. A 50 cm long solenoid has 400 turns per cm. The diameter of the solenoid is 0.04 m. Find the magnetic flux of a turn when it carries a current of 1 A.

(Ans: 1.26 Wb)

12. A coil of 200 turns carries a current of 0.4 A. If the magnetic flux of 4 mWb is linked with the coil, find the inductance of the coil. (Ans: 2H)

13. Two air core solenoids have the same length of 80 cm and same cross–sectional area 5 cm^2. Find the mutual inductance between them if the number of turns in the first coil is 1200 turns and that in the second coil is 400 turns. (Ans: 0.38 mH)

14. A long solenoid having 400 turns per cm carries a current 2A. A 100 turn coil of cross-sectional area 4 cm^2 is placed co-axially inside the solenoid so that the coil is in the field produced by the solenoid. Find the emf induced in the coil if the current through the solenoid reverses its direction in 0.04 sec.

(Ans: 0.20 V)

15. A 200 turn coil of radius 2 cm is placed co-axially within a long solenoid of 3 cm radius. If the turn density of the solenoid is 90 turns per cm, then calculate mutual inductance of the coil.

(Ans: 2.84 mH)

16. The solenoids S_1 and S_2 are wound on an iron-core of relative permeability 900. The area of their cross-section and their length are the same and are 4 cm^2 and 0.04 m respectively. If the number of turns in S_1 is 200 and that in S_2 is 800, calculate the mutual inductance between the coils. The current in solenoid 1 is increased form 2A to 8A

in 0.04 second. Calculate the induced emf in solenoid 2.

(Ans: 1.81H; 271.5 V)

17. A step-down transformer connected to main supply of 220 V is made to operate 11V,88W lamp. Calculate (i) Transformation ratio and (ii) Current in the primary.

(Ans: 1/20 and 0.4A)

18. A 200V/120V step-down transformer of 90% efficiency is connected to an induction stove of resistance 40 Ω. Find the current drawn by the primary of the transformer.

(Ans: 2A)

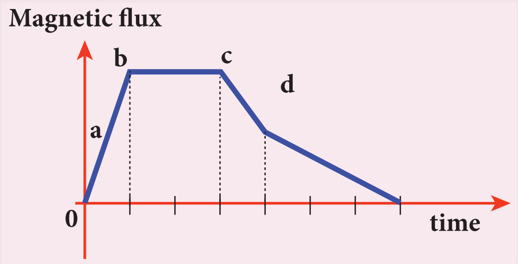
19. The 300 turn primary of a transformer has resistance 0.82 Ω and the resistance of its secondary of 1200 turns is 6.2 Ω. Find the voltage across the primary if the power output from the secondary at 1600V is 32 kW. Calculate the power losses in both coils when the transformer efficiency is 80%.

(Ans: 8.2 kW and 2.48 kW)

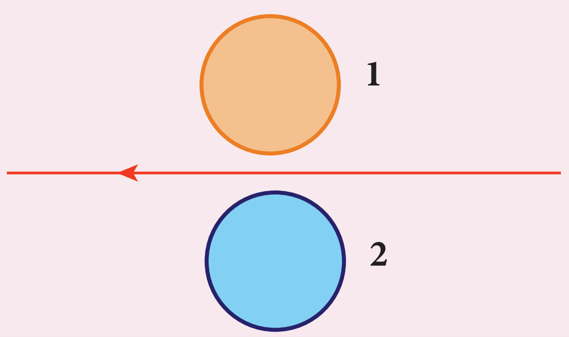
20. Calculate the instantaneous value at 60^{o}, average value and RMS value of an alternating current whose peak value is 20 A. (Ans: 17.32A, 12.74A, 14.14 A)

IV Conceptual Questions

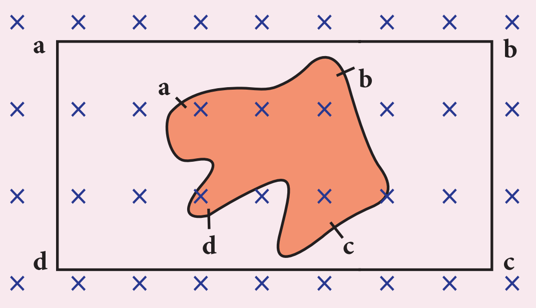
1. A graph between the magnitude of the magnetic flux linked with a closed loop and time is given in the figure. Arrange the regions of the graph in ascending order of the magnitude of induced emf in the loop.



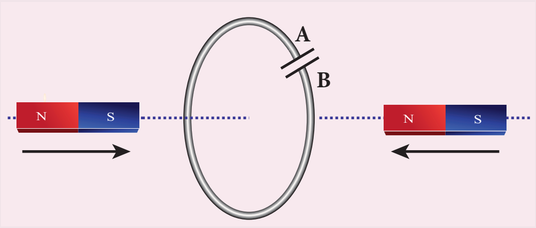
2. Using Lenz's law, predict the direction of induced current in conducting rings 1 and 2 when current in the wire is steadily decreasing.



3. A flexible metallic loop abcd in the shape of a square is kept in a magnetic field with its plane perpendicular to the field. The magnetic field is directed into the paper normally. Find the direction of the induced current when the square loop is crushed into an irregular shape as shown in the figure.



4. Predict the polarity of the capacitor in a closed circular loop when two bar magnets are moved as shown in the figure.



5. In series LC circuit, the voltages across L and C are 180° out of phase. Is it correct? Explain.
6. When does power factor of a series RLC circuit become maximum?
7. Draw graphs showing the distribution of charge in a capacitor and current through an inductor during LC oscillations with respect to time. Assume that the charge in the capacitor is maximum initially.

BOOK FOR REFERENCES

1. H.C.Verma, Concepts of Physics, Volume 1 and 2, Bharathi Bhawan publishers.
2. Halliday, Resnick and Walker, Principles of Physics, Wiley publishers.
3. D.C.Tayal, Electricity and Magnetism, Himalaya Publishing House.
4. K.K.Tewari, Electricity and Magnetism with Electronics, S.Chand Publishers.
5. B.L.Theraja and A.K.Theraja, A text book of Electrical Technology, Volume 1 and 2, S.Chand publishers.

ICT CORNER

Electromagnetic induction and alternating current

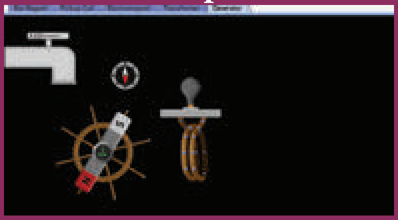
In this activity you will be able to
(1) understand electromagnetic induction.
(2) verify Faraday's laws in virtual lab.

Topic: Faraday's electromagnetic lab

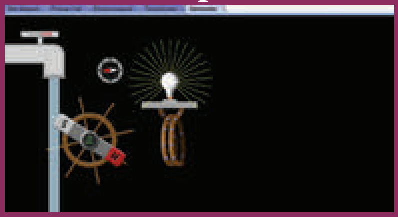
STEPS:

- Open the browser and type "phet.colorado.edu" in the address bar. Click play with simulation tab. Search Faraday's electromagnetic lab in the search box.
- Select 'pick coil' tab. Move the magnet through the coil. Note what happens when the magnetic field linked with the coil changes. Change the loop area, flux change and observe the intensity of current with the help of glowing bulb.
- Select 'Electromagnet' tab, Change the current flowing through the coil and observe the change in magnetic flux generated.
- Select 'Generator' tab. Observe induced emf in the coil if you change the angular velocity of the coil.

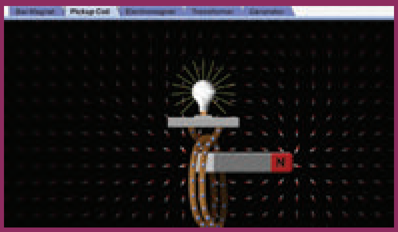
Step1



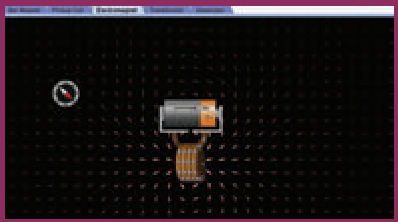
Step2



Step3



Step4



Note:

Install Java application if it is not in your system. You can download all the phet simulation and works in off line from https://phet.colorado.edu/en/offline-access.

URL:

https://phet.colorado.edu/en/simulation/legacy/faraday

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

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