EXERCISE 8.1

Choose the correct answer from the given four options:

1. If cos A = 4/5 , then the value of tan A is

(A) 3/5 (B) ¾ (C) 4/3 (D) 5/3

Solution:

According to the question,

cos A = 4/5 …(1)

We know,

tan A = sinA/cosA

To find the value of sin A,

We have the equation,

$\sin^2 \theta + \cos^2 \theta = 1$

So, $\sin \theta = \sqrt{(1-\cos^2 \theta)}$

Then,

$\sin A = \sqrt{(1-\cos^2 A)}$ …(2)

$\sin^2 A = 1-\cos^2 A$

$\sin A = \sqrt{(1-\cos^2 A)}$

Substituting equation (1) in (2),

We get,

$\text{Sin } A = \sqrt{(1-(4/5)^2)}$

$= \sqrt{(1-(16/25))}$

$= \sqrt{(9/25)}$

= ¾

Therefore,

$$\tan A = \frac{3}{5} \times \frac{5}{4} = \frac{3}{4}$$

2. If sin A = ½ , then the value of cot A is

(A) √3 (B) 1/√3 (C) √3/2 (D) 1

Solution:

According to the question,

Sin A = ½ … (1)

We know that,

$$\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A} \text{ … (2)}$$

To find the value of cos A.

We have the equation,

$\sin^2 \theta + \cos^2 \theta = 1$

So, $\cos \theta = \sqrt{(1-\sin^2 \theta)}$

Then,

$\cos A = \sqrt{(1-\sin^2 A)}$ … (3)

$\cos^2 A = 1-\sin^2 A$

$\cos A = \sqrt{(1-\sin^2 A)}$

Substituting equation 1 in 3, we get,

cos A = √(1-1/4) = √(3/4) = √3/2

Substituting values of sin A and cos A in equation 2, we get

cot A = (√3/2) × 2 = √3

3. The value of the expression [cosec (75° + θ) – sec (15° – θ) – tan (55° + θ) + cot (35° – θ)] is

(A) – 1 (B) 0 (C) 1 (D) 3 2

Solution:

According to the question,

We have to find the value of the equation,

cosec(75°+θ) – sec(15°-θ) - tan(55°+θ) + cot(35°-θ)

= cosec[90°-(15°-θ)] – sec(15°-θ) – tan(55°+θ) + cot[90°-(55°+θ)]

Since, cosec (90°- θ) = sec θ

And, cot(90°-θ) = tan θ

We get,

= sec(15°-θ) – sec(15°-θ) - tan(55°+θ) + tan(55°+θ)

= 0

4. Given that sinθ = a b , then cosθ is equal to

(A) $b/\sqrt{(b^2- a^2)}$ (B) b/a (C) $\sqrt{(b^2-a^2)}/b$ (D) $a/\sqrt{(b^2-a^2)}$

Solution:

According to the question,

sin θ =a/b

We know, $\sin^2 \theta + \cos^2 \theta = 1$

$\sin^2 A = 1-\cos^2 A$

sin A = $\sqrt{(1-\cos^2 A}$

So, cos θ = $\sqrt{(1-a^2/b^2)} = \sqrt{((b^2-a^2)/b^2)} = \sqrt{(b^2-a^2)}/b$

Hence, cos θ = $\sqrt{(b^2 - a^2)}/b$

5. If cos (α + β) = 0, then sin (α – β) can be reduced to

(A) cos β (B) cos 2β (C) sin α (D) sin 2α

Solution:

According to the question,

cos(α+β) = 0

Since, cos 90° = 0

We can write,

cos(α+β)= cos 90°

By comparing cosine equation on L.H.S and R.H.S,

We get,

(α+β)= 90°

α = 90°-β

Now we need to reduce sin (α -β),

So, we take,

sin(α-β) = sin(90°-β-β) = sin(90°-2β)

sin(90°-θ) = cos θ

So, sin(90°-2β) = cos 2β

Therefore, sin(α-β) = cos 2β

6. The value of (tan1° tan2° tan3° ... tan89°) is

(A) 0 (B) 1 (C) 2 (D) ½

Solution:

tan 1°. tan 2°.tan 3° …… tan 89°

= tan1°.tan 2°.tan 3°…tan 43°.tan 44°.tan 45°.tan 46°.tan 47°…tan 87°.tan 88°.tan 89°

Since, tan 45° = 1,

= tan1°.tan 2°.tan 3°…tan 43°.tan 44°.1.tan 46°.tan 47°…tan 87°.tan 88°.tan 89°

= tan1°.tan 2°.tan 3°…tan 43°.tan 44°.1.tan(90°-44°).tan(90°-43°)…tan(90°-3°). tan(90°-2°).tan(90°-1°)

Since, tan(90°-θ) = cot θ,

= tan1°.tan 2°.tan 3°…tan 43°.tan 44°.1.cot 44°.cot 43°…cot 3°.cot 2°.cot 1°

Since, tan θ = (1/cot θ)

= tan1°.tan 2°.tan 3°…tan 43°.tan 44°.1. (1/tan 44°). (1/tan 43°)… (1/tan 3°). (1/tan 2°). (1/tan 1°)

$= \left(\tan 1° \times \frac{1}{\tan 1°}\right).\left(\tan 2° \times \frac{1}{\tan 2°}\right) \ldots \left(\tan 44° \times \frac{1}{\tan 44°}\right)$

= 1

Hence, tan 1°.tan 2°.tan 3° …… tan 89° = 1

7. If cos 9α = sinα and 9α < 90° , then the value of tan5α is

(A) 1/√3 (B) √3 (C) 1 (D) 0

Solution:

According to the question,

cos 9∝ = sin ∝ and 9∝<90°

i.e. 9α is an acute angle

We know that,

sin(90°-θ) = cos θ

So,

cos 9∝ = sin (90°-∝)

Since, cos 9∝ = sin(90°-9∝) and sin(90°-∝) = sin∝

Thus, sin (90°-9∝) = sin∝

90°-9∝ =∝

10∝ = 90°

∝ = 9°

Substituting ∝ = 9° in tan 5∝, we get,

tan 5∝ = tan (5×9) = tan 45° = 1

∴, tan 5∝ = 1

EXERCISE 8.2 PAGE NO: 93

Write 'True' or 'False' and justify your answer in each of the following:

1. tan 47°/cot 43 ° = 1

Solution:

True

Justification:

Since, tan (90° -θ) = cot θ

$$\frac{\tan 47^\circ}{\cot 43^\circ} = \frac{\tan(90^\circ - 43^\circ)}{\cot 43^\circ}$$

$$\frac{\tan 47^\circ}{\cot 43^\circ} = \frac{\cot 43^\circ}{\cot 43^\circ}$$

$$\frac{\tan 47^\circ}{\cot 43^\circ} = 1$$

2. The value of the expression ($\cos^2 23° – \sin^2 67°$) is positive.

Solution:

False

Justification:

Since, $(a^2-b^2) = (a+b)(a-b)$

$\cos^2 23° - \sin^2 67°$ =(cos 23°+sin 67°)(cos 23°-sin 67°)

= [cos 23°+sin(90°-23°)] [cos 23°-sin(90°-23°)]

= (cos 23°+cos 23°)(cos 23°-cos 23°) (∵sin(90°-θ) = cos θ)

= (cos 23°+cos 23°).0

= 0, which is neither positive nor negative

3. The value of the expression (sin 80° – cos 80°) is negative.

Solution:

False

Justification:

We know that,

sin θ increases when 0° ≤ θ ≤ 90°

cos θ decreases when 0° ≤ θ ≤ 90°

And (sin 80°-cos 80°) = (increasing value-decreasing value)

= a positive value.

Therefore, (sin 80°-cos 80°) > 0.

4. $\sqrt{((1-\cos^2\theta) \sec^2 \theta)} = \tan \theta$

Solution:

True

Justification:

LHS: $\sqrt{((1-\cos^2\theta)\sec^2\theta)}$

$= \sqrt{\sin^2\theta \sec^2\theta}$

$(\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1-\cos^2\theta)$

$= \sqrt{\frac{\sin^2\theta}{\cos^2\theta}}$ (Since, $\sec^2\theta = \frac{1}{\cos^2\theta}$)

$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$

= RHS

5. If cosA + cos²A = 1, then sin²A + sin⁴A = 1.

Solution:

True

Justification:

According to the question,

$\cos A + \cos^2 A = 1$

i.e., $\cos A = 1 - \cos^2 A$

Since,

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta)$

We get,

$\cos A = \sin^2 A$ …(1)

Squaring L.H.S and R.H.S,

$\cos^2 A = \sin^4 A$ …(2)

To find $\sin^2 A + \sin^4 A = 1$

Adding equations (1) and (2),

We get

$\sin^2 A + \sin^4 A = \cos A + \cos^2 A$

Therefore, $\sin^2 A + \sin^4 A = 1$

6. (tan θ + 2) (2 tan θ + 1) = 5 tan θ + sec² θ.

Solution:

False

Justification:

L.H.S $= (\tan\theta+2)(2\tan\theta+1)$

$= 2\tan^2\theta + \tan\theta + 4\tan\theta + 2$

$= 2\tan^2\theta + 5\tan\theta + 2$

Since, $\sec^2\theta - \tan^2\theta = 1$, we get, $\tan^2\theta = \sec^2\theta - 1$

$= 2(\sec^2\theta - 1) + 5\tan\theta + 2$

$= 2\sec^2\theta - 2 + 5\tan\theta + 2$

$= 5\tan\theta + 2\sec^2\theta \neq$ R.H.S

$\therefore$, L.H.S $\neq$ R.H.S

EXERCISE 8.3

Prove the following (from Q.1 to Q.7):
1. sin θ/(1+cos θ) + (1+ cos θ)/sin θ = 2cosec θ
Solution:

L.H.S=

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$$

Taking the L.C.M of the denominators,
We get,

$$= \frac{\sin^2\theta+(1+\cos\theta)^2}{(1+\cos\theta).\sin\theta}$$

$$= \frac{\sin^2\theta+1+\cos^2\theta+2\cos\theta}{(1+\cos\theta).\sin\theta}$$

Since, $\sin^2\theta + \cos^2\theta = 1$

$$= \frac{1+1+2\cos\theta}{(1+\cos\theta).\sin\theta}$$

$$= \frac{2+2\cos\theta}{(1+\cos\theta).\sin\theta}$$

$$= \frac{2(1+\cos\theta)}{(1+\cos\theta).\sin\theta}$$

Since, 1/sin θ = cosec θ

$$= \frac{2}{\sin\theta} = 2\text{ cosec }\theta$$

R.H.S
Hence proved.

2. tan A/(1+secA) - tan A/(1-secA) = 2cosec A
Solution:

L.H.S:

$$\frac{\tan A}{1+\sec A} - \frac{\tan A}{1-\sec A}$$

Taking LCM of the denominators,

$$= \frac{\tan A(1-\sec A)-\tan A(1+\sec A)}{(1+\sec A)(1-\sec A)}$$

Since, $(1 + \sec A)(1 - \sec A) = 1 - \sec^2 A$)

$$= \frac{\tan A(1-\sec A-1-\sec A)}{1-\sec^2 A}$$

$$= \frac{\tan A(-2\sec A)}{1-\sec^2 A}$$

$$= \frac{2\tan A.\sec A}{\sec^2 A-1}$$

Since,
$\sec^2 A - \tan^2 A = 1$
$\sec^2 A - 1 = \tan^2 A$

$= \frac{2 \tan A . \sec A}{\tan^2 A}$

Since, sec A = (1/cos A) and tan A = (sin A/ cos A)

$= \frac{2 \sec A}{\tan A} = \frac{2 \cos A}{\cos A \sin A}$

$= \frac{2}{\sin A}$

$= 2 \text{ cosec } A \ (\because \frac{1}{\sin A} = \text{cosec } A)$

= R.H.S

Hence proved.

3. If tan A = ¾, then sinA cosA = 12/25

Solution:

According to the question,

tan A = ¾

We know,

tan A = perpendicular/ base

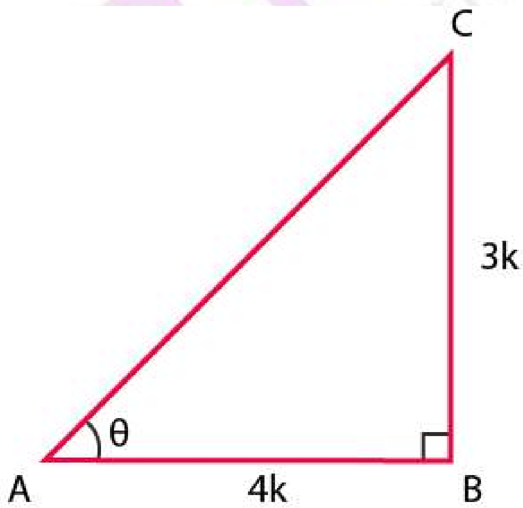
So,

tan A = 3k/4k

Where,

Perpendicular = 3k

Base = 4k



Using Pythagoras Theorem,

$(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$(\text{hypotenuse})^2 = (3k)^2 + (4k)^2 = 9k^2+16k^2 = 25k^2$

hypotenuse = 5k

To find sin A and cos A,

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{3k}{5k} = \frac{3}{5}$$
$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{4k}{5k} = \frac{4}{5}$$
Multiplying sin A and cos A,
$$\sin A \cos A = \frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$$
Hence, proved.

4. (sin α + cos α) (tan α + cot α) = sec α + cosec α
Solution:

L.H.S:
(sin α + cos α) (tan α + cot α)
As we know,
$$\tan A = \frac{\sin A}{\cos A}$$
$$\cot A = \frac{\cos A}{\sin A}$$
$$= (\sin\alpha + \cos\alpha)\left(\frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}\right)$$
$$= (\sin\alpha + \cos\alpha)\left(\frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha\cos\alpha}\right)$$
$$[\because \sin^2\alpha + \cos^2\alpha = 1]$$
$$= (\sin\alpha + \cos\alpha)\left(\frac{1}{\sin\alpha\cos\alpha}\right)$$
$$= \frac{\sin\alpha}{\sin\alpha\cos\alpha} + \frac{\cos\alpha}{\sin\alpha\cos\alpha}$$
$$= \frac{1}{\cos\alpha} + \frac{1}{\sin\alpha}$$
$$= \sec\alpha + \operatorname{cosec}\alpha \; [\because \frac{1}{\cos\alpha} = \sec\alpha \text{ and } \frac{1}{\sin\alpha} = \operatorname{cosec}\alpha]$$
= R.H.S
Hence, proved.

5. (√3+1) (3 – cot 30°) = $\tan^3$ 60° – 2 sin 60°
Solution:

L.H.S: (√3 + 1) (3 – cot 30°)
= (√3 + 1) (3 – √3) [∵cos 30° = √3]
= (√3 + 1) √3 (√3 - 1) [∵(3 – √3) = √3 (√3 - 1)]
= $((\sqrt{3})^2 - 1)$ √3 [∵ (√3+1)(√3-1) = $((\sqrt{3})^2 - 1)$]
= (3-1) √3
= 2√3
Similarly solving R.H.S: $\tan^3$ 60° - 2 sin 60°
Since, tan 60^o = √3 and sin 60^o = √3/2,
We get,

$(\sqrt{3})^3 - 2.(\sqrt{3}/2) = 3\sqrt{3} - \sqrt{3}$

$= 2\sqrt{3}$

Therefore, L.H.S = R.H.S

Hence, proved.

6. 1 + (cot^2 α/1+cosec α = cosec α

Solution:

L.H.S:

Since,

$\cot^2\alpha = \frac{\cos^2\alpha}{\sin^2\alpha}$ and $\operatorname{cosec}\alpha = \frac{1}{\sin\alpha}$]

We get,

$1 + \frac{\cot^2\alpha}{1+\operatorname{cosec}\alpha} = 1 + \frac{\cos^2\alpha/\sin^2\alpha}{1+1/\sin\alpha}$

$= 1 + \frac{\cos^2\alpha/\sin^2\alpha}{\frac{\sin\alpha+1}{\sin\alpha}}$

$= 1 + \frac{\cos^2\alpha}{\sin\alpha(1+\sin\alpha)}$

$= \frac{\sin\alpha+\sin^2\alpha+\cos^2\alpha}{\sin\alpha+\sin^2\alpha}$

And, we know that,

$\sin^2\alpha + \cos^2\alpha = 1$

$= \frac{1+\sin\alpha}{\sin\alpha(1+\sin\alpha)}$

Since,

$\frac{1}{\sin\alpha} = \operatorname{cosec}\alpha$]

$= \frac{1}{\sin\alpha} = \operatorname{cosec}\alpha$

= R.H.S

7. tan θ + tan (90° – θ) = sec θ sec (90° – θ)

Solution:

L.H.S=

Since, tan (90° - θ) = cot θ

tan θ + tan (90° - θ)= tan θ + cot θ

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

Since,

$\sin^2\theta + \cos^2\theta = 1$

$= \frac{1}{\sin\theta\cos\theta}$

$= \frac{1}{\cos\theta} \times \frac{1}{\sin\theta}$

$= \sec\theta \operatorname{cosec}\theta$

Since,

$\operatorname{cosec}\theta = \sec(90° - \theta)$

$= \sec\theta \sec(90° - \theta)$

= R.H.S

Hence, proved.

EXERCISE 8.4 PAGE NO: 99

1. If cosecθ + cotθ = p, then prove that cosθ = $(p^2 – 1)/ (p^2 + 1)$.

Solution:

According to the question,

cosec θ + cot θ = p

Since,

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} \ \& \ \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = p$$

$$\frac{1+\cos\theta}{\sin\theta} = p$$

Squaring on L.H.S and R.H.S,

$$\left(\frac{1+\cos\theta}{\sin\theta}\right)^2 = p^2$$

$$\frac{1+\cos^2\theta+2\cos\theta}{\sin^2\theta} = p^2$$

Applying component and dividend rule,

$$\frac{(1+\cos^2\theta+2\cos\theta)-\sin^2\theta}{(1+\cos^2\theta+2\cos\theta)+\sin^2\theta} = \frac{p^2-1}{p^2+1}$$

$$\frac{(1-\sin^2\theta)+\cos^2\theta+2\cos\theta}{\sin^2\theta+\cos^2\theta+1+2\cos\theta} = \frac{p^2-1}{p^2+1}$$

Since,

$$1-\sin^2\theta = \cos^2\theta \ \& \ \sin^2\theta+\cos^2\theta = 1$$

$$\frac{\cos^2\theta+\cos^2\theta+2\cos\theta}{1+1+2\cos\theta} = \frac{p^2-1}{p^2+1}$$

$$\frac{2\cos^2\theta+2\cos\theta}{2+2\cos\theta} = \frac{p^2-1}{p^2+1}$$

$$\frac{2\cos\theta\,(\cos\theta+1)}{2\,(\cos\theta+1)} = \frac{p^2-1}{p^2+1}$$

$$\cos\theta = \frac{p^2-1}{p^2+1}$$

Hence, proved.

2. Prove that $\sqrt{(\sec^2\theta + \operatorname{cosec}^2\theta)} = \tan\theta + \cot\theta$

Solution:

L.H.S=

$\sqrt{(\sec^2\theta + \operatorname{cosec}^2\theta)}$

Since,

$\sec^2\theta = \frac{1}{\cos^2\theta}$ & $\text{cosec}^2\theta = \frac{1}{\sin^2\theta}$

$= \sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}$

$= \sqrt{\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}}$

Since,

$\sin^2\theta + \cos^2\theta = 1$

$= \sqrt{\frac{1}{\cos^2\theta\sin^2\theta}}$

$= \frac{1}{\cos\theta\sin\theta}$

Since,

$1 = \sin^2\theta + \cos^2\theta$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$

$= \frac{\sin^2\theta}{\cos\theta\sin\theta} + \frac{\cos^2\theta}{\cos\theta\sin\theta}$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

Since,

$\frac{\sin\theta}{\cos\theta} = \tan\theta$ & $\frac{\cos\theta}{\sin\theta} = \cot\theta$

$= \tan\theta + \cot\theta$

= R.H.S

Hence, proved.

3. The angle of elevation of the top of a tower from certain point is 30°. If the observer moves 20 metres towards the tower, the angle of elevation of the top increases by 15°. Find the height of the tower.

Solution:

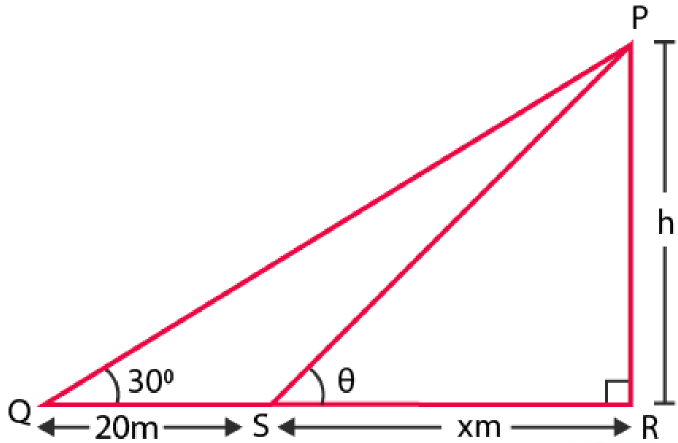
Let PR = h meter, be the height of the tower.

The observer is standing at point Q such that, the distance between the observer and tower is QR = (20+x) m, where

QR = QS + SR = 20 + x

∠PQR = 30°

∠ PSR = θ



In ΔPQR,

$\tan 30° = \frac{h}{20+x}$ [∵, $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$]

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20+x}$ [∵, $\tan 30° = \frac{1}{\sqrt{3}}$]

Rearranging the terms,
We get $20 + x = \sqrt{3}h$
$\Rightarrow x = \sqrt{3}h - 20$ …eq.1
In ΔPSR,
$\tan\theta = h/x$
Since, angle of elevation increases by 15° when the observer moves 20 m towards the tower.
We have,
$\theta = 30° + 15° = 45°$
So,
$\tan 45° = h/x$
$\Rightarrow 1 = h/x$
$\Rightarrow h = x$
Substituting x=h in eq. 1, we get
$h = \sqrt{3}\,h - 20$
$\Rightarrow \sqrt{3}\,h - h = 20$
$\Rightarrow h(\sqrt{3} - 1) = 20$

$\Rightarrow h = \frac{20}{\sqrt{3}-1}$

Rationalizing the denominator, we have

$\Rightarrow h = \frac{20}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow h = \frac{20(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{20(\sqrt{3}+1)}{3-1}$

$= \frac{20(\sqrt{3}+1)}{2}$

$= 10(\sqrt{3} + 1)$
Hence, the required height of the tower is $10(\sqrt{3} + 1)$ meter.

4. If 1 + $\sin^2\theta$ = 3sinθ cosθ , then prove that tanθ = 1 or ½.
Solution:

Given: $1+\sin^2\theta = 3\sin\theta\cos\theta$
Dividing L.H.S and R.H.S equations with $\sin^2\theta$,
We get,

$$\frac{1+\sin^2\theta}{\sin^2\theta} = \frac{3\sin\theta\cos\theta}{\sin^2\theta}$$

$$\Rightarrow \frac{1}{\sin^2\theta} + 1 = \frac{3\cos\theta}{\sin\theta}$$

$\text{cosec}^2\theta + 1 = 3\cot\theta$
Since,
$\text{cosec}^2\theta - \cot^2\theta = 1 \Rightarrow \text{cosec}^2\theta = \cot^2\theta + 1$
$\Rightarrow \cot^2\theta + 1 + 1 = 3\cot\theta$
$\Rightarrow \cot^2\theta + 2 = 3\cot\theta$
$\Rightarrow \cot^2\theta - 3\cot\theta + 2 = 0$
Splitting the middle term and then solving the equation,
$\Rightarrow \cot^2\theta - \cot\theta - 2\cot\theta + 2 = 0$
$\Rightarrow \cot\theta(\cot\theta - 1) - 2(\cot\theta + 1) = 0$
$\Rightarrow (\cot\theta - 1)(\cot\theta - 2) = 0$
$\Rightarrow \cot\theta = 1, 2$
Since,
$\tan\theta = 1/\cot\theta$
$\tan\theta = 1, ½$
Hence, proved.

5. Given that sinθ + 2cosθ = 1, then prove that 2sinθ – cosθ = 2.
Solution:

Given: $\sin\theta + 2\cos\theta = 1$
Squaring on both sides,
$(\sin\theta + 2\cos\theta)^2 = 1$
$\Rightarrow \sin^2\theta + 4\cos^2\theta + 4\sin\theta\cos\theta = 1$
Since, $\sin^2\theta = 1 - \cos^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$
$\Rightarrow (1 - \cos^2\theta) + 4(1 - \sin^2\theta) + 4\sin\theta\cos\theta = 1$
$\Rightarrow 1 - \cos^2\theta + 4 - 4\sin^2\theta + 4\sin\theta\cos\theta = 1$
$\Rightarrow - 4\sin^2\theta - \cos^2\theta + 4\sin\theta\cos\theta = - 4$
$\Rightarrow 4\sin^2\theta + \cos^2\theta - 4\sin\theta\cos\theta = 4$
We know that,
$a^2 + b^2 - 2ab = (a - b)^2$
So, we get,
$(2\sin\theta - \cos\theta)^2 = 4$
$\Rightarrow 2\sin\theta - \cos\theta = 2$
Hence proved.

6. The angle of elevation of the top of a tower from two points distant s and t from its foot are complementary. Prove that the height of the tower is √st .

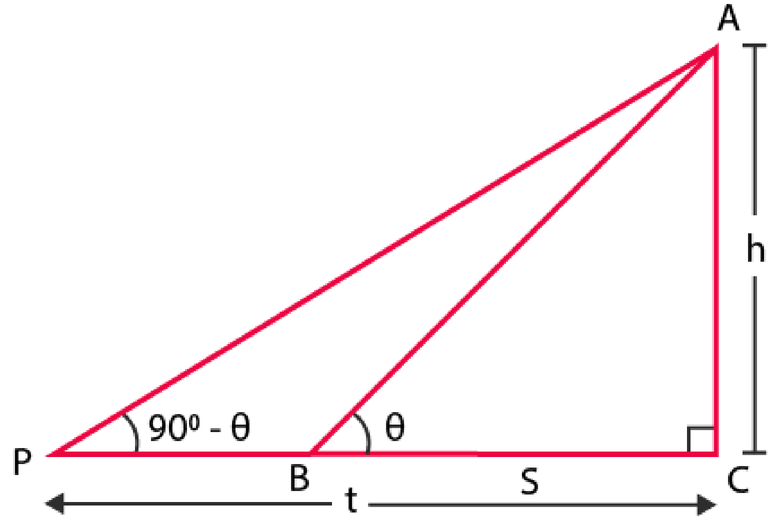
Solution:

Let BC = s; PC = t

Let height of the tower be AB = h.

∠ABC = θ and ∠APC = 90° - θ

(∵ the angle of elevation of the top of the tower from two points P and B are complementary)



In $\Delta ABC, \tan\theta = \frac{AC}{BC} = \frac{h}{s}$ ... eq. 1 $[\because, \tan\theta = \frac{\text{perpendicular}}{\text{base}}]$

In $\Delta APC, \tan(90° - \theta) = \frac{AC}{PC} = \frac{h}{t}$

$\Rightarrow \cot\theta = \frac{h}{t}$ ... eq. 2

Multiplying eq. 1 and eq. 2, we get

$\tan\theta \times \cot\theta = \frac{h}{s} \times \frac{h}{t}$

$\Rightarrow 1 = \frac{h^2}{st}$ $[\because \tan\theta \times \cot\theta = 1 \text{ as } \cot\theta = \frac{1}{\tan\theta}]$

$\Rightarrow h^2 = st$

$\Rightarrow h = \sqrt{st}$

Hence the height of the tower is $\sqrt{st}$.

7. The shadow of a tower standing on a level plane is found to be 50 m longer when Sun's elevation is 30° than when it is 60°. Find the height of the tower.

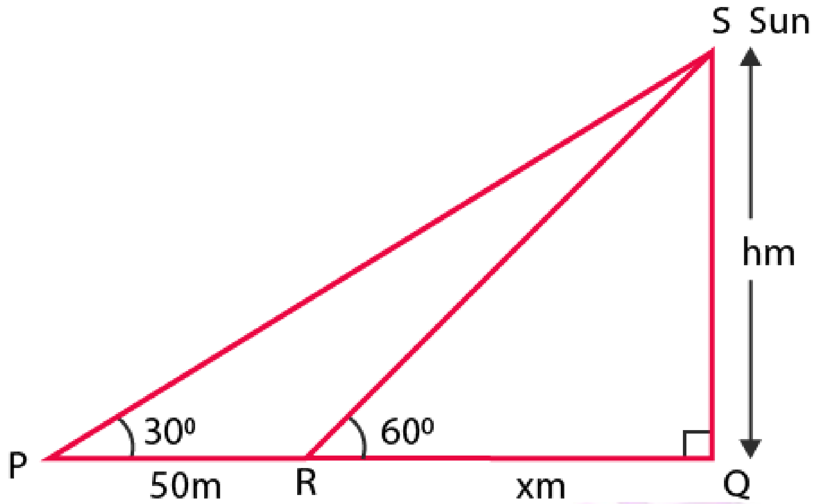
Solution:

Let SQ = h be the tower.

∠SPQ = 30° and ∠SRQ = 60°

According to the question, the length of shadow is 50 m long hen angle of elevation of the sun is 30° than when it was 60°. So,

PR = 50 m and RQ = x m



So in ΔSRQ, we have

$\tan 60° = \frac{h}{x}$

$[\because \tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$\Rightarrow \tan 60° = \frac{SQ}{RQ}]$

$\Rightarrow \sqrt{3} = \frac{h}{x}$ $[\because \tan 60° = \sqrt{3}]$

$\Rightarrow x = \frac{h}{\sqrt{3}}$

In ΔSPQ,

$\tan 30° = \frac{h}{50+x}$

$[\because \tan 30° = \frac{SQ}{PQ} = \frac{SQ}{PR+PQ}]$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{50+x}$ $[\because, \tan 30° = \frac{1}{\sqrt{3}}]$

$\Rightarrow 50 + x = \sqrt{3}h$

Substituting the value of x in the above equation, we get

$\Rightarrow 50 + \frac{h}{\sqrt{3}} = \sqrt{3}h$

$\Rightarrow \frac{50\sqrt{3}+h}{\sqrt{3}} = \sqrt{3}h$

$\Rightarrow 50\sqrt{3}+h = 3h$

$\Rightarrow 50\sqrt{3} = 3h - h$

$\Rightarrow 3h - h = 50\sqrt{3}$

$\Rightarrow 2h = 50\sqrt{3}$

$\Rightarrow h = (50\sqrt{3})/2$

$\Rightarrow h = 25\sqrt{3}$

Hence, the required height is $25\sqrt{3}$ m.

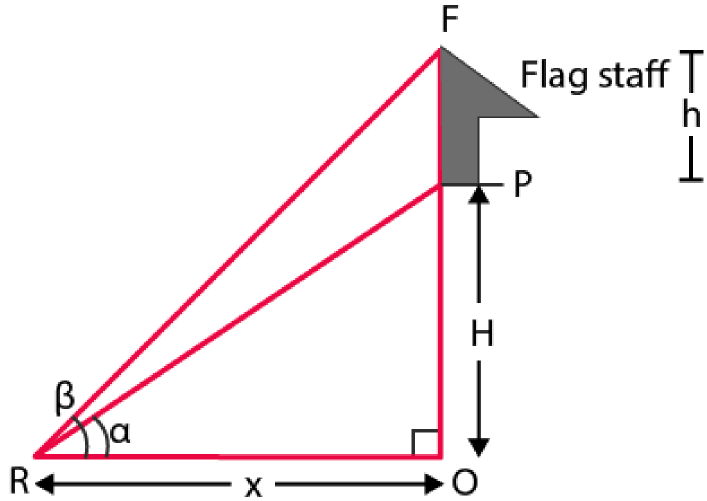
8. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height

h. At a point on the plane, the angles of elevation of the bottom and the top of the flag staff are α and β, respectively. Prove that the height of the tower is [h tan α/(tan β – tan α)].

Solution:

Given that a vertical flag staff of height h is surmounted on a vertical tower of height H(say), such that FP = h and FO = H.

The angle of elevation of the bottom and top of the flag staff on the plane is ∠PRO = α and ∠FRO = β respectively



In ΔPRO, we have

$$\tan\alpha = \frac{PO}{RO} = \frac{H}{x}$$

$$[\because \tan\theta = \frac{\text{perpendicular}}{\text{base}}]$$

$$\Rightarrow x = \frac{H}{\tan\alpha} \ \ldots \text{eq. 1}$$

And in ΔFRO, we have

$$\tan\beta = \frac{FO}{RO} = \frac{FP+PO}{RO} = \frac{h+H}{x}$$

$$\Rightarrow x = \frac{h+H}{\tan\beta} \ \ldots \text{eq. 2}$$

Comparing eq. 1 and eq. 2,

$$\Rightarrow \frac{H}{\tan\alpha} = \frac{h+H}{\tan\beta}$$

Solving for H,

$\Rightarrow H \tan\beta = (h+H) \tan\alpha$

$\Rightarrow H \tan\beta - H \tan\alpha = h \tan\alpha$

$\Rightarrow H (\tan\beta - \tan\alpha) = h \tan\alpha$

$$\Rightarrow H = \frac{h\tan\alpha}{\tan\beta - \tan\alpha}$$

Hence, proved.

9. If tanθ + secθ = l, then prove that secθ = (l^2 + 1)/2*l*.

Solution:

Given: tan θ+ sec θ = l …eq. 1

Multiplying and dividing by (sec θ – tan θ) on numerator and denominator of L.H.S,

$$\frac{(\sec\theta+\tan\theta)\,(\sec\theta-\tan\theta)}{\sec\theta-\tan\theta} = l$$

$$\Rightarrow \frac{\sec^2\theta-\tan^2\theta}{\sec\theta-\tan\theta} = l$$

Since, $\sec^2\theta - \tan^2\theta = 1$

$$\Rightarrow \frac{1}{\sec\theta-\tan\theta} = l$$

So, sec θ – tan θ = 1 …eq.2

Adding eq. 1and eq. 2, we get

(tan θ + sec θ) + (sec θ – tan θ) = 1

$$\Rightarrow 2\sec\theta = \frac{l^2+1}{l}$$

$$\Rightarrow \sec\theta = \frac{l^2+1}{2l}$$

Hence, proved.