1. An isosceles right triangle has area 8 cm$^2$. The length of its hypotenuse is
(A) $\sqrt{32}$ cm  
(B) $\sqrt{16}$ cm  
(C) $\sqrt{48}$ cm  
(D) $\sqrt{24}$ cm  

Solution:
(A) $\sqrt{32}$ cm  
Explanation:
Let height of triangle = $h$
As the triangle is isosceles,
Let base = height = $h$
According to the question,
Area of triangle = 8 cm$^2$
$\Rightarrow \frac{1}{2} \times \text{Base} \times \text{Height} = 8$
$\Rightarrow \frac{1}{2} \times h \times h = 8$
$\Rightarrow h^2 = 16$
$\Rightarrow h = 4$ cm
Base = Height = 4 cm
Since the triangle is right angled,
Hypotenuse$^2 = \text{Base}^2 + \text{Height}^2$
$\Rightarrow $Hypotenuse$^2 = 4^2 + 4^2$
$\Rightarrow $Hypotenuse$^2 = 32$
$\Rightarrow $Hypotenuse = $\sqrt{32}$
Hence, Options A is the correct answer.

2. The perimeter of an equilateral triangle is 60 m. The area is
(A) 10$\sqrt{3}$ m$^2$
(B) 15$\sqrt{3}$ m$^2$
(C) 20$\sqrt{3}$ m$^2$
(D) 100$\sqrt{3}$ m$^2$

Solution:
(D) 100$\sqrt{3}$ m$^2$
Explanation:
Area of an equilateral triangle of side $a = \frac{\sqrt{3}}{4} a^2$
According to the question,
Perimeter of triangle = 60m
$\Rightarrow a + a + a = 60$
$\Rightarrow 3a = 60$
$\Rightarrow a = 20$ m
Area of the triangle $= \left(\frac{\sqrt{3}}{4}\right) a^2$
$= \left(\frac{\sqrt{3}}{4}\right) (20)^2$
$= \left(\frac{\sqrt{3}}{4}\right) (400)$
$= 100\sqrt{3}$
3. The sides of a triangle are 56 cm, 60 cm and 52 cm long. Then the area of the triangle is
(A) 1322 cm²
(B) 1311 cm²
(C) 1344 cm²
(D) 1392 cm²
Solution:
(C) 1344 cm²
Explanation:
According to the question, Sides of a triangle, a = 56, b = 60, c = 52
s = (a + b + c)/2
⇒ s = (56 + 60 + 52)/2
= 168/2 = 84.
Area of triangle = √s(s-a)(s-b)(s-c)
= √84(84-56)(84-60)(84-52)
= √84×28×24×32
= 1344 cm²
Hence, Options C is the correct answer.

4. The area of an equilateral triangle with side $2\sqrt{3}$ cm is
(A) 5.196 cm²
(B) 0.866 cm²
(C) 3.496 cm²
(D) 1.732 cm²
Solution:
(A) 5.196 cm²
Explanation:
Area of an equilateral triangle of side a = $\sqrt{3}/4$ a²
According to the question, a = $2\sqrt{3}$
Area of triangle = $(\sqrt{3}/4)$ a²
= $(\sqrt{3}/4)$ $(2\sqrt{3})^2$
= $(\sqrt{3}/4)(12)$
= $3\sqrt{3}$
= 5.196
Hence, Options A is the correct answer.
Write True or False and justify your answer:

1. The area of a triangle with base 4 cm and height 6 cm is 24 cm$^2$.
   Solution:
   False
   Justification:
   \[
   \text{Area of triangle} = \frac{1}{2} \times \text{Base} \times \text{Altitude}
   \]
   \[
   = \frac{1}{2} \times 4 \times 6
   \]
   \[
   = 12 \text{cm}^2
   \]
   Hence, the statement “the area of a triangle with base 4 cm and height 6 cm is 24 cm$^2$” is False.

2. The area of \( \triangle ABC \) is 8 cm$^2$ in which \( AB = AC = 4 \text{ cm} \) and \( \angle A = 90^\circ \).
   Solution:
   True
   Justification:
   \[
   \text{Area of triangle} = \frac{1}{2} \times \text{Base} \times \text{Altitude}
   \]
   \[
   = \frac{1}{2} \times 4 \times 4
   \]
   \[
   = 8 \text{cm}^2
   \]
   Hence, the statement is “area of \( \triangle ABC \) is 8 cm$^2$ in which \( AB = AC = 4 \text{ cm} \) and \( \angle A = 90^\circ \)” is True.

3. The area of the isosceles triangle is \( \frac{5}{4} \sqrt{11} \text{cm}^2 \), if the perimeter is 11 cm and the base is 5 cm.
   Solution:
   True
   Justification:
   According to the question,
   Perimeter = 11 cm
   And base, \( a = 5 \)
   As the triangle is isosceles, \( b = c \)
   Perimeter = 11
   \[
   \Rightarrow a + b + c = 11
   \]
   \[
   \Rightarrow 5 + b + b = 11
   \]
   \[
   \Rightarrow 5 + 2b = 11
   \]
   \[
   \Rightarrow 2b = 6
   \]
   \[
   \Rightarrow b = 3
   \]
   So, we have,
   \( a = 5, b = 3, c = 3 \)
   \[
   s = \frac{(a + b + c)}{2}
   \]
   \[
   \Rightarrow s = \frac{(5 + 3 + 3)}{2} = 11/2
   \]
   Area of triangle = \( \sqrt{s(s-a)(s-b)(s-c)} \)
\[
\Rightarrow \text{Area of triangle} = \frac{5\sqrt{11}}{4}\text{cm}^2
\]

Hence, the statement “The area of the isosceles triangle is \(\frac{5}{4} \sqrt{11}\text{cm}^2\), if the perimeter is 11 cm and the base is 5 cm” is True.

4. The area of the equilateral triangle is \(20\sqrt{3}\text{ cm}^2\) whose each side is 8 cm.

Solution:
False

Justification:
Area of an equilateral triangle of side \(a = \frac{\sqrt{3}}{4} a^2\)
According to the question,
Area of a triangle = \(20\sqrt{3}\text{ cm}^2\)
\[\Rightarrow \frac{\sqrt{3}}{4} a^2 = 20\sqrt{3}\]
\[\Rightarrow a^2 = 20 \times 4\]
\[\Rightarrow a^2 = 80\]
\[\Rightarrow a = 4\sqrt{5}\text{ cm}\]

Hence, the statement “the area of the equilateral triangle is \(20\sqrt{3}\text{ cm}^2\) whose each side is 8 cm” is False.
1 Find the cost of laying grass in a triangular field of sides 50 m, 65 m and 65 m at the rate of Rs 7 per m².

Solution:

According to the question,
Sides of the triangular field are 50 m, 65 m and 65 m.
Cost of laying grass in a triangular field = Rs 7 per m²

Let \(a = 50, \ b = 65, \ c = 65\)

\[s = \frac{(a + b + c)}{2}\]

\[\Rightarrow s = \frac{(50 + 65 + 65)}{2}\]

\[= 180/2\]

\[= 90.\]

Area of triangle = \(\sqrt{s(s-a)(s-b)(s-c)}\)

\[= \sqrt{90(90-50)(90-65)(90-65)}\]

\[= \sqrt{90\times40\times25\times25}\]

\[= 1500m^2\]

Cost of laying grass = \(\text{Area of triangle} \times \text{Cost per m}^2\)

\[= 1500 \times 7\]

\[= Rs.10500\]

2 The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 13 m, 14 m and 15 m. The advertisements yield an earning of Rs 2000 per m² a year. A company hired one of its walls for 6 months. How much rent did it pay?

Solution:

According to the question,
The sides of the triangle are 13 m, 14 m and 15 m

Let \(a = 13, \ b = 14, \ c = 15\)

\[s = \frac{(a + b + c)}{2}\]

\[\Rightarrow s = \frac{(13 + 14 + 15)}{2}\]

\[= 42/2\]

\[= 21.\]

Area of triangle = \(\sqrt{s(s-a)(s-b)(s-c)}\)

\[= \sqrt{21(21-13)(21-14)(21-15)}\]

\[= \sqrt{21\times8\times7\times6}\]

\[= 84m^2\]

Cost of advertisements for a year = \(\text{Area of triangle} \times \text{Cost per m}^2\)

\[= 84\times2000\]

\[= Rs. 168000\]

Since the board is rented for only 6 months:
Cost of advertisements for 6 months = \((6/12) \times 168000\)

\[= Rs. 84000\]

3 From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm. Find the area of the triangle.
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Solution:
According to the question,
The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm.
We know that, 
Area of an equilateral triangle of side $a = \frac{\sqrt{3}}{4} a^2$
We divide the triangle into three triangles,
Area of triangle = $(\frac{1}{2} \times a \times 14) + (\frac{1}{2} \times a \times 10) + (\frac{1}{2} \times a \times 6)$
\[
\frac{\sqrt{3}}{4} a^2 = \frac{1}{2} \times a \times (14 + 10 + 6)
\]
\[
\frac{\sqrt{3}}{4} a^2 = \frac{1}{2} \times a \times 30
\]
\[
a = \frac{60}{\sqrt{3}}
\]
\[
= 20\sqrt{3}
\]
Area of the triangle = \[
\frac{\sqrt{3}}{4} a^2
\]
=\[
\frac{\sqrt{3}}{4} \times (20\sqrt{3})^2
\]
= $300\sqrt{3}$ cm$^2$

4 The perimeter of an isosceles triangle is 32 cm. The ratio of the equal side to its base is 3 : 2. Find the area of the triangle.
Solution:
According to the question,
Perimeter of the isosceles triangle = 32 cm
It is also given that,
Ratio of equal side to base = 3 : 2
Let the equal side = 3x
So, base = 2x
Perimeter of the triangle = 32
\[
\Rightarrow 3x + 3x + 2x = 32
\]
\[
\Rightarrow 8x = 32
\]
\[
\Rightarrow x = 4.
\]
Equal side = 3x = 3×4 = 12
Base = 2x = 2×4 = 8
The sides of the triangle = 12cm, 12cm and 8cm.
Let $a = 12$, $b = 12$, $c = 8$
$s = (a + b + c)/2$
\[
\Rightarrow s = (12 + 12 + 8)/2
\]
\[
= 32/2
\]
\[
= 16.
\]
Area of the triangle = \[
\sqrt{s(s-a)(s-b)(s-c)}
\]
\[
= \sqrt{16(16-12)(16-12)(16-8)}
\]
\[
= \sqrt{16\times4\times4\times8}
\]
\[
= 32\sqrt{2}$ cm$^2$

5 Find the area of a parallelogram given in Fig. 12.2. Also find the length of the altitude from vertex A on the side DC.
Solution:
We know that,
Area of parallelogram(ABCD) = Area(\(\triangle BCD\)) + Area(\(\triangle ABD\))
For Area (\(\triangle BCD\)),
We have,
\[a = 12, \quad b = 17, \quad c = 25\]
\[s = \frac{a + b + c}{2} = \frac{54}{2} = 27.\]
Area of (\(\triangle BCD\)) = \[\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{27 \times 15 \times 10 \times 2} = 90 \text{ cm}^2\]
Since, ABCD is a parallelogram,
Area(\(\triangle BCD\)) = Area(\(\triangle ABD\))
Area of parallelogram(ABCD) = Area(\(\triangle BCD\)) + Area(\(\triangle ABD\))
\[= 90 + 90 = 180 \text{ cm}^2\]
Let altitude from A be \(x\)
Also, Area of parallelogram(ABCD) = CD \times (\text{Altitude from A})
\[\Rightarrow 180 = 12 \times x\]
\[\Rightarrow x = 15 \text{ cm}\]
1. How much paper of each shade is needed to make a kite given in Fig. 12.4, in which ABCD is a square with diagonal 44 cm.

Solution:

According to the figure,
\[ AC = BD = 44\text{cm} \]
\[ AO = 44/2 = 22\text{cm} \]
\[ BO = 44/2 = 22\text{cm} \]
From \( \triangle AOB \),
\[ AB^2 = AO^2 + BO^2 \]
\[ \Rightarrow AB^2 = 22^2 + 22^2 \]
\[ \Rightarrow AB^2 = 2 \times 22^2 \]
\[ \Rightarrow AB = 22\sqrt{2} \text{ cm} \]
Area of square = (Side)\(^2\)
Area of each triangle (I, II, III, IV) = Area of square /4
= 968 /4
= 242 cm²

To find area of lower triangle,
Let a = 28, b = 28, c = 14

\[
s = \frac{(a + b + c)}{2} = \frac{(28 + 28 + 14)}{2} = \frac{70}{2} = 35.
\]

Area of the triangle
\[
= \sqrt{s(s-a)(s-b)(s-c)}
= \sqrt{35(35-28)(35-28)(35-14)}
= \sqrt{35 \times 7 \times 7 \times 21}
= 49\sqrt{15} = 189.77 cm²
\]

Therefore,
We get,
Area of Red = Area of IV = 242 cm²
Area of Yellow = Area of I + Area of II = 242 + 242 = 484 cm²
Area of Green = Area of III + Area of lower triangle = 242 + 189.77 = 431.77 cm²

2. The perimeter of a triangle is 50 cm. One side of a triangle is 4 cm longer than the smaller side and the third side is 6 cm less than twice the smaller side. Find the area of the triangle.

Solution:
Let the smaller side be \(x\) cm
Then, larger side = \((x + 4)\) cm
And, third side = \((2x - 6)\) cm

Given that,
Perimeter = 50 cm
\[x + (x + 4) + (2x - 6) = 50 \Rightarrow 4x - 2 = 50 \Rightarrow 4x = 52 \Rightarrow x = 13\]

Therefore, smaller side = 13 cm
Larger side = \(x + 4 = 13 + 4 = 17\) cm
Third side = \(2x - 6 = 2 \times 13 - 6 = 26 - 6 = 20\) cm

To find area of triangle,
Let \(a = 13, \ b = 17, \ c = 20\)

\[
s = \frac{(a + b + c)}{2} = \frac{(13 + 17 + 20)}{2} = \frac{50}{2} = 25.
\]

Area of triangle
\[
= \sqrt{s(s-a)(s-b)(s-c)}
= \sqrt{25(25-13)(25-17)(25-20)}
= \sqrt{25 \times 12 \times 8 \times 5}
\]
3. The area of a trapezium is 475 cm\(^2\) and the height is 19 cm. Find the lengths of its two parallel sides if one side is 4 cm greater than the other.

Solution:

Let the given trapezium be \(\text{PQRS}\), given in the figure.

According to the question,

\(\text{PQ} = 19\text{ cm}\)

Let \(\text{RQ} = x \text{ cm}\)

Then,

\(\text{PS} = (x + 4)\text{ cm}\)

Construction:

Draw a perpendicular from \(\text{R}\) on \(\text{PS}\) which will also be parallel to \(\text{PQ}\).

Now,

We get,

\(\text{PQRT}\) is a rectangle,

Area of rectangle \(\text{PQRT} = \text{PQ} \times \text{QR}\)

\(\Rightarrow\) Area\((\text{PQRT}) = 19 \times x = 19x\)

Now,

\(\text{PS} = \text{PT} + \text{TS}\)

Since \(\text{PT} = \text{QR} = x \text{ cm}\)

\((x + 4) = x + \text{TS}\)

\(\Rightarrow\) \(\text{TS} = 4\text{ cm}\)

Area of triangle \(\text{RST} = \frac{1}{2} \times \text{RT} \times \text{ST}\)

Since \(\text{RT} = \text{PQ} = 19\text{ cm}\)

\(\Rightarrow\) Area\((\Delta \text{RST}) = \frac{1}{2} \times 19 \times 4 = 38\text{ cm}^2\)

Area\((\text{PQRS}) = \text{Area}(\text{PQRT}) + \text{Area}(\Delta \text{RST})\)

\(\Rightarrow 475 = 19x + 38\)

\(\Rightarrow 19x = 475 - 38\)

\(\Rightarrow 19x = 437\)

\(\Rightarrow x = 23 \text{ cm}\)

\((x + 4) = 23 + 4 = 27\text{ cm}\)
Therefore, lengths of parallel sides is 23cm and 27cm.

4. A rectangular plot is given for constructing a house, having a measurement of 40 m long and 15 m in the front. According to the laws, a minimum of 3 m, wide space should be left in the front and back each and 2 m wide space on each of other sides. Find the largest area where house can be constructed.

Solution:

Let the given rectangle be rectangle PQRS.
According to the question, 
PQ = 40m and QR = 15m
As 3m is left in both front and back,  
AB = PQ - 3 -3  
⇒ AB = 40 -6  
⇒ AB = 34m
Also,  
Given that 2m has to be left at both the sides,
BC = QR -2 – 2  
⇒ BC = 15 -4  
⇒ BC = 11m
Now, Area left for house construction is area of ABCD.
Hence,  
Area(ABCD) = AB × CD  
= 34 × 11  
= 374 m²