EXERCISE 12.1 PAGE NO: 113

1. An isosceles right triangle has area 8 cm^2. The length of its hypotenuse is
(A) √32 cm
(B) √16 cm
(C) √48 cm
(D) √24 cm
Solution:

(A) √32 cm
Explanation:
Let height of triangle = h
As the triangle is isosceles,
Let base = height = h
According to the question,
Area of triangle = $8cm^2$
⇒ ½ × Base × Height = 8
⇒ ½ × h × h = 8
⇒ $h^2 = 16$
⇒ h = 4cm
Base = Height = 4cm
Since the triangle is right angled,
$Hypotenuse^2 = Base^2 + Height^2$
⇒ $Hypotenuse^2 = 4^2 + 4^2$
⇒ $Hypotenuse^2 = 32$
⇒ Hypotenuse = √32
Hence, Options A is the correct answer.

2. The perimeter of an equilateral triangle is 60 m. The area is
(A) 10√3 m^2
(B) 15√3 m^2
(C) 20√3 m^2
(D) 100√3 m^2
Solution:

(D) 100√3 m^2
Explanation:
Area of an equilateral triangle of side a = √3/4 a^2
According to the question,
Perimeter of triangle = 60m
⇒ a + a + a = 60
⇒ 3a = 60
⇒ a = 20m
Area of the triangle = (√3/4) a^2
= (√3/4) $(20)^2$
= (√3/4) (400)
= 100√3

Hence, Options D is the correct answer.

3. The sides of a triangle are 56 cm, 60 cm and 52 cm long. Then the area of the triangle is
(A) 1322 cm^2
(B) 1311 cm^2
(C) 1344 cm^2
(D) 1392 cm^2
Solution:

(C) 1344 cm^2

Explanation:

According to the question,
Sides of a triangle,
a = 56, b = 60, c = 52
s = (a + b + c)/2
⇒ s = (56 + 60 + 52)/2
= 168/2 = 84.
Area of triangle = √s(s-a)(s-b)(s-c)
= √84(84-56)(84-60)(84-52)
= √84×28×24×32
= 1344cm^2

Hence, Options C is the correct answer.

4. The area of an equilateral triangle with side 2√3 cm is
(A) 5.196 cm^2
(B) 0.866 cm^2
(C) 3.496 cm^2
(D) 1.732 cm^2
Solution:

(A) 5.196 cm^2

Explanation:

Area of an equilateral triangle of side a = √3/4 a^2
According to the question,
a = 2√3
Area of triangle = (√3/4) a^2
= (√3/4) $(2\sqrt{3})^2$
= (√3/4)(12)
= 3√3
= 5.196

Hence, Options A is the correct answer.

EXERCISE 12.2 PAGE NO: 115

Write True or False and justify your answer:

1. The area of a triangle with base 4 cm and height 6 cm is 24 cm^2.

Solution:

False

Justification:

Area of triangle = ½ × Base × Altitude

= ½ × 4 × 6

= 12cm^2

Hence, the statement "the area of a triangle with base 4 cm and height 6 cm is 24 cm^2" is False.

2. The area of △ ABC is 8 cm^2 in which AB = AC = 4 cm and ∠A = 90º.

Solution:

True

Justification:

Area of triangle = ½ × Base × Altitude

= ½ × 4 × 4

= 8cm^2

Hence, the statement is "area of △ABC is 8 cm^2 in which AB = AC = 4 cm and ∠A = 90º" is True.

3. The area of the isosceles triangle is 5/4 √11cm^2, if the perimeter is 11 cm and the base is 5 cm.

Solution:

True

Justification:

According to the question,

Perimeter = 11cm

And base, a = 5

As the triangle is isosceles, b = c

Perimeter = 11

⇒ a + b + c = 11

⇒ 5 + b + b = 11

⇒ 5 + 2b = 11

⇒ 2b = 6

⇒ b = 3

So, we have,

a = 5, b = 3, c = 3

s = (a + b + c)/2

⇒ s = (5 + 3 + 3)/2 = 11/2

Area of triangle = √s(s-a)(s-b)(s-c)

$$= \sqrt{\frac{11}{2}\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-3\right)\left(\frac{11}{2}-3\right)}$$

$$= \sqrt{\frac{11}{2}\left(\frac{1}{2}\right)\left(\frac{5}{2}\right)\left(\frac{5}{2}\right)}$$

⇒ Area of triangle = $(5\sqrt{11})/4 cm^2$

Hence, the statement "The area of the isosceles triangle is $5/4 \sqrt{11} cm^2$, if the perimeter is 11 cm and the base is 5 cm" is True.

4. The area of the equilateral triangle is $20\sqrt{3}$ cm^2 whose each side is 8 cm.

Solution:

False

<u>Justification:</u>

Area of an equilateral triangle of side a = $\sqrt{3}/4\ a^2$

According to the question,

Area of a triangle = $20\sqrt{3}$ cm^2

⇒ $\sqrt{3}/4\ a^2 = 20\sqrt{3}$

⇒ $a^2 = 20 \times 4$

⇒ $a^2 = 80$

⇒ $a = 4\sqrt{5}$ cm

Hence, the statement "the area of the equilateral triangle is $20\sqrt{3}$ cm^2 whose each side is 8 cm" is False.

EXERCISE 12.3 PAGE NO: 117

1 Find the cost of laying grass in a triangular field of sides 50 m, 65 m and 65 m at the rate of Rs 7 per m^2.

Solution:

According to the question,
Sides of the triangular field are 50 m, 65 m and 65 m.
Cost of laying grass in a triangular field = Rs 7 per m^2
Let a = 50, b = 65, c = 65
s = (a + b + c)/2
⇒ s = (50 + 65 + 65)/2
= 180/2
= 90.
Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
= $\sqrt{90(90-50)(90-65)(90-65)}$
= $\sqrt{90\times40\times25\times25}$
= $1500m^2$
Cost of laying grass = Area of triangle ×Cost per m^2
= 1500×7
= Rs.10500

2 The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 13 m, 14 m and 15 m. The advertisements yield an earning of Rs 2000 per m^2 a year. A company hired one of its walls for 6 months. How much rent did it pay?

Solution:

According to the question,
The sides of the triangle are 13 m, 14 m and 15 m
Let a = 13, b = 14, c = 15
s = (a + b + c)/2
⇒ s = (13 + 14 + 15)/2
= 42/2
= 21.
Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
= $\sqrt{21(21-13)(21-14)(21-15)}$
= $\sqrt{21\times8\times7\times6}$
= $84m^2$
Cost of advertisements for a year = Area of triangle × Cost per m^2
= 84×2000
= Rs. 168000
Since the board is rented for only 6 months:
Cost of advertisements for 6 months = (6/12) × 168000
= Rs. 84000

3 From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm. Find the area of the triangle.

Solution:

According to the question,

The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm.

We know that,

Area of an equilateral triangle of side a = $\sqrt{3}/4\ a^2$

We divide the triangle into three triangles,

Area of triangle = (1/2 × a × 14) + (1/2 × a × 10) + (1/2 × a × 6)

$\sqrt{3}/4\ a^2 = ½ × a × (14 + 10 + 6)$

$\sqrt{3}/4\ a^2 = ½ × a × 30$

$a = 60/\sqrt{3}$

$= 20\sqrt{3}$

Area of the triangle $= \sqrt{3}/4\ a^2$

$=\sqrt{3}/4\ (20\sqrt{3})^2$

$= 300\sqrt{3}\ cm^2$

4 The perimeter of an isosceles triangle is 32 cm. The ratio of the equal side to its base is 3 : 2. Find the area of the triangle.

Solution:

According to the question,

Perimeter of the isosceles triangle = 32 cm

It is also given that,

Ratio of equal side to base = 3 : 2

Let the equal side = 3x

So, base = 2x

Perimeter of the triangle = 32

⇒ 3x + 3x + 2x = 32

⇒ 8x = 32

⇒ x = 4.

Equal side = 3x = 3×4 = 12

Base = 2x = 2×4 = 8

The sides of the triangle = 12cm, 12cm and 8cm.

Let a = 12, b = 12, c = 8

s = (a + b + c)/2

⇒ s = (12 + 12 + 8)/2

= 32/2

= 16.

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16(16-12)(16-12)(16-8)}$

$= \sqrt{16×4×4×8}$

$= 32\sqrt{2}\ cm^2$

5 Find the area of a parallelogram given in Fig. 12.2. Also find the length of the altitude from vertex A on the side DC.

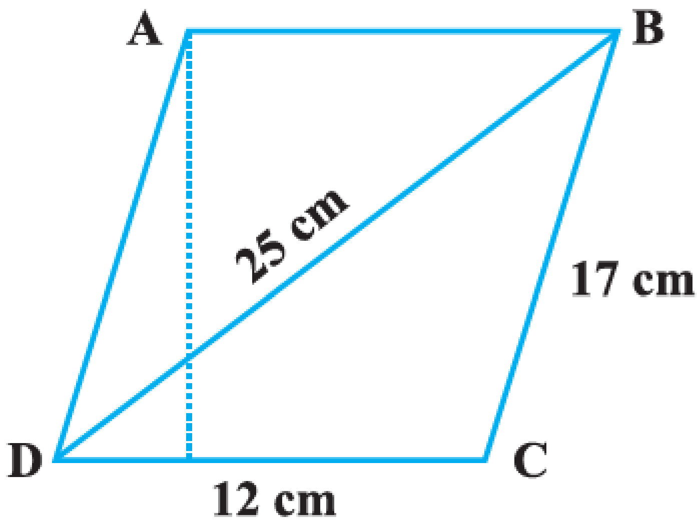


Fig. 12.2

Solution:

We know that,

Area of parallelogram(ABCD) = Area(ΔBCD) + Area(ΔABD)

For Area (ΔBCD),

We have,

$a = 12$, $b = 17$, $c = 25$

$s = (a + b + c)/2$

$\Rightarrow s = (12 + 17 + 25)/2 = 54/2 = 27.$

Area of (ΔBCD) $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{27(27-12)(27-17)(27-25)}$

$= \sqrt{27\times15\times10\times2}$

$= 90 \text{ cm}^2$

Since, ABCD is a parallelogram,

Area(ΔBCD) = Area(ΔABD)

Area of parallelogram(ABCD) = Area(ΔBCD) + Area(ΔABD)

$= 90 + 90$

$= 180 \text{ cm}^2$

Let altitude from A be = x

Also, Area of parallelogram(ABCD) = CD × (Altitude from A)

$\Rightarrow 180 = 12 \times x$

$\Rightarrow x = 15 \text{ cm}$

EXERCISE 12.4 PAGE NO: 118

1. How much paper of each shade is needed to make a kite given in Fig. 12.4, in which ABCD is a square with diagonal 44 cm.

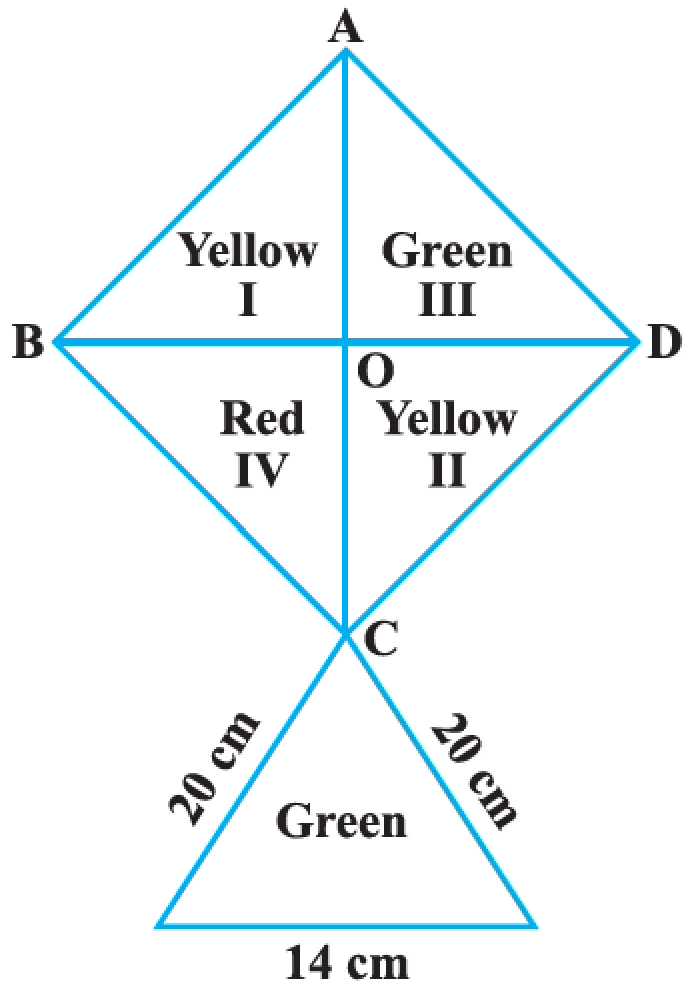


Fig. 12.4

Solution:

According to the figure,

AC = BD = 44cm

AO = 44/2 = 22cm

BO = 44/2 = 22cm

From ΔAOB,

$AB^2 = AO^2 + BO^2$

$\Rightarrow AB^2 = 22^2 + 22^2$

$\Rightarrow AB^2 = 2 \times 22^2$

$\Rightarrow AB = 22\sqrt{2}$ cm

Area of square = $(\text{Side})^2$

$= (22\sqrt{2})^2$
$= 968 \text{ cm}^2$

Area of each triangle (I, II, III, IV) = Area of square /4
= 968 /4
$= 242 \text{ cm}^2$

To find area of lower triangle,
Let a = 28, b = 28, c = 14
s = (a + b + c)/2
⇒ s = (28 + 28 + 14)/2 = 70/2 = 35.
Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{35(35-28)(35-28)(35-14)}$
$= \sqrt{35\times7\times7\times21}$
$= 49\sqrt{15} = 189.77 \text{cm}^2$

Therefore,
We get,
Area of Red = Area of IV
$= 242 \text{ cm}^2$
Area of Yellow = Area of I + Area of II
= 242 + 242
$= 484 \text{ cm}^2$
Area of Green = Area of III + Area of lower triangle
= 242 + 189.77
$= 431.77 \text{ cm}^2$

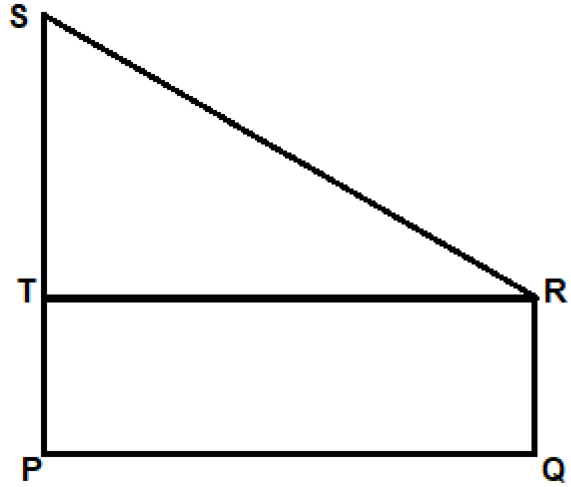
2. The perimeter of a triangle is 50 cm. One side of a triangle is 4 cm longer than the smaller side and the third side is 6 cm less than twice the smaller side. Find the area of the triangle.

Solution:

Let the smaller side be = x cm
Then, larger side = (x + 4) cm
And, third side = (2x-6) cm
Given that,
Perimeter = 50 cm
⇒ x + (x + 4) + (2x-6) = 50
⇒ 4x-2 = 50
⇒ 4x = 52
⇒ x = 13
Therefore, smaller side = 13cm
Larger side = x + 4 = 13 + 4 = 17cm
Third side = 2x-6 = 2×13 – 6 = 26-6 = 20cm
To find area of triangle,
Let a = 13, b = 17, c = 20
s = (a + b + c)/2
⇒ s = (13 + 17 + 20)/2 = 50/2 = 25.
Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{25(25-13)(25-17)(25-20)}$
$= \sqrt{25\times12\times8\times5}$

$= 20\sqrt{30}\ cm^2$

3. The area of a trapezium is 475 cm^2 and the height is 19 cm. Find the lengths of its two parallel sides if one side is 4 cm greater than the other.
Solution:

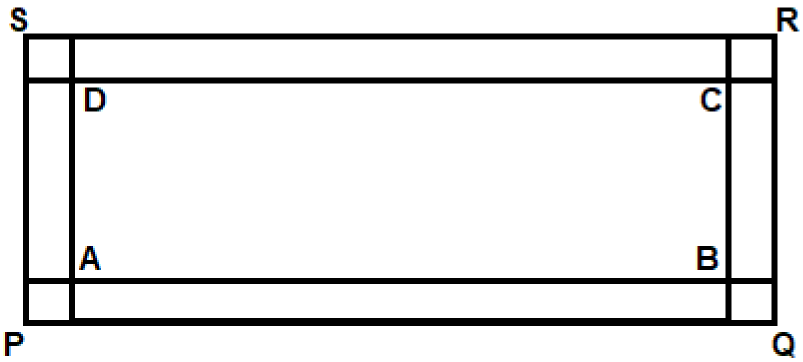


Let the given trapezium be PQRS, given in the figure.
According to the question,
PQ = 19cm
Let RQ = x cm
Then,
PS = (x + 4)cm
Construction:
Draw a perpendicular from R on PS which will also be parallel to PQ.
Now,
We get,
PQRT is a rectangle,
Area of rectangle PQRT = PQ × QR
⇒ Area(PQRT) = 19×x = 19x
Now,
PS = PT + TS
Since PT = QR = x cm
(x + 4) = x + TS
⇒ TS = 4cm
Area of triangle RST = ½ × RT × ST
Since RT = PQ = 19cm
⇒ Area(ΔRST) = ½ × 19 × 4
$= 38cm^2$
Area(PQRS) = Area(PQRT) + Area(ΔRST)
⇒ 475 = 19x + 38
⇒ 19x = 475 -38
⇒ 19x = 437
⇒ x = 23 cm
(x + 4) = 23 + 4 = 27cm

Therefore, lengths of parallel sides is 23cm and 27cm.

4. A rectangular plot is given for constructing a house, having a measurement of 40 m long and 15 m in the front. According to the laws, a minimum of 3 m, wide space should be left in the front and back each and 2 m wide space on each of other sides. Find the largest area where house can be constructed.

Solution:



Let the given rectangle be rectangle PQRS,
According to the question,
PQ = 40m and QR = 15m
As 3m is left in both front and back,
AB = PQ -3 -3
⇒ AB = 40 -6
⇒ AB = 34m
Also,
Given that 2m has to be left at both the sides,
BC = QR -2 – 2
⇒ BC = 15 -4
⇒ BC = 11m
Now, Area left for house construction is area of ABCD.
Hence,
Area(ABCD) = AB × CD
= 34 × 11
= 374 m^2