EXERCISE 6.1

PAGE NO: 55

Write the correct answer in each of the following:
1. In Fig. 6.1, if AB || CD || EF, PQ || RS, ∠RQD = 25° and ∠CQP = 60°, then ∠QRS is equal to
(A) 85°
(B) 135°
(C) 145°
(D) 110°

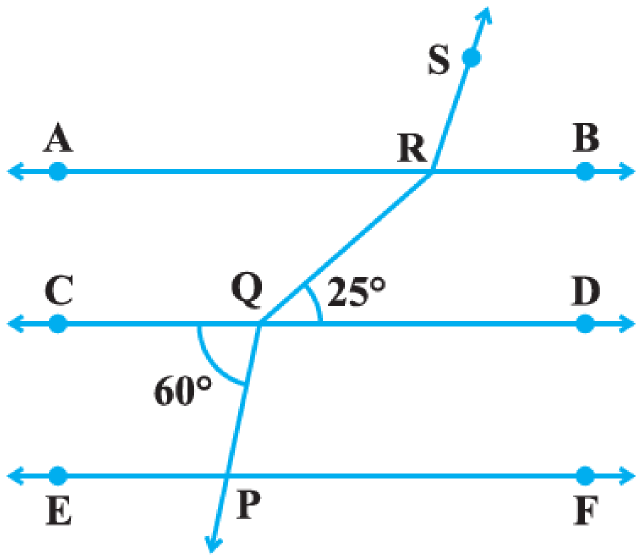


Fig. 6.1

Solution:

(C) 145°

Explanation:

According to the given figure, we have

AB || CD || EF
PQ || RS
∠RQD = 25°
∠CQP = 60°
PQ || RS.

We know that,
If a transversal intersects two parallel lines, then each pair of alternate exterior angles is equal.
Now, since, PQ || RS
⇒ ∠PQC = ∠BRS
We have ∠PQC = 60°
⇒ ∠BRS = 60° … eq.(i)
We also know that,
If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.
Now again, since, AB || CD
⇒ ∠DQR = ∠QRA
We have ∠DQR = 25°

⇒ ∠QRA = 25° … eq.(ii)
Using linear pair axiom,
We get,
∠ARS + ∠BRS = 180°
⇒ ∠ARS = 180° - ∠BRS
⇒ ∠ARS = 180° - 60° (From (i), ∠BRS = 60°)
⇒ ∠ARS = 120° … eq.(iii)
Now, ∠QRS = ∠QRA + ∠ARS
From equations (ii) and (iii), we have,
∠QRA = 25° and ∠ARS = 120°
Hence, the above equation can be written as:
∠QRS = 25° + 120°
⇒ ∠QRS = 145°
Therefore, option (C) is the correct answer.

2. If one angle of a triangle is equal to the sum of the other two angles, then the triangle is
(A) An isosceles triangle
(B) An obtuse triangle
(C) An equilateral triangle
(D) A right triangle
Solution:
(D) A right triangle
Explanation:
Let the angles of △ABC be ∠A, ∠B and ∠C
Given that ∠A= ∠B+∠C …(eq1)
But, in any △ABC,
Using angle sum property, we have,
∠A+∠B+∠C=180° …(eq2)
From equations (eq1) and (eq2), we get
∠A+∠A=180°
⇒2∠A=180°
⇒∠A=180°/2 = 90°
⇒∠A = 90°
Hence, we get that the triangle is a right triangle
Therefore, option (D) is the correct answer.

3. An exterior angle of a triangle is 105° and its two interior opposite angles are equal. Each of these equal angles is
(A) 37 ½°
(B) 52 ½°
(C) 72 ½°
(D) 75°
Solution:
(B) 52 ½°
Explanation:
According to the question,

Exterior angle of triangle= 105°
Let the two interior opposite angles of the triangle = x
We know that,
Exterior angle of a triangle = sum of interior opposite angles
Then, we have the equation,
$105° = x + x$
$2x = 105°$
$x = 52.5°$
$x = 52\frac{1}{2}$
Therefore, option (B) is the correct answer.

4. The angles of a triangle are in the ratio 5 : 3 : 7. The triangle is
(A) An acute angled triangle
(B) An obtuse angled triangle
(C) A right triangle
(D) An isosceles triangle
Solution:

(A) An acute angled triangle
Explanation:
According to the question,
The angles of a triangle are of the ratio 5 : 3 : 7
Let 5:3:7 be 5x, 3x and 7x
Using the angle sum property of a triangle,
$5x + 3x + 7x = 180$
$15x = 180$
$x = 12$
Substituting the value of x, x = 12, in 5x, 3x and 7x we get,
$5x = 5\times12 = 60^o$
$3x = 3\times12 = 36^o$
$7x = 7\times12 = 84^o$
Since all the angles are less than 90^o, the triangle is an acute angled triangle.
Therefore, option (A) is the correct answer.

EXERCISE 6.2 PAGE NO: 56

1. For what value of x + y in Fig. 6.4 will ABC be a line? Justify your answer.

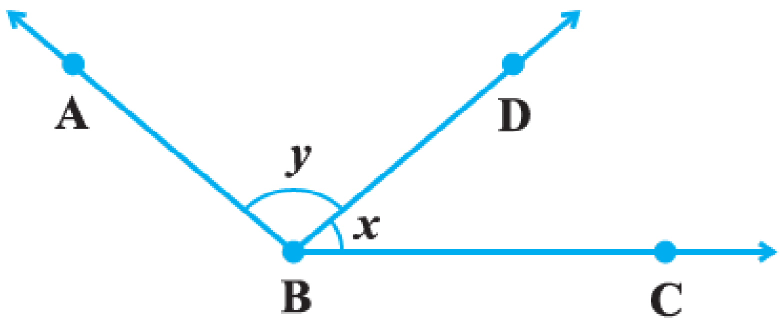


Fig. 6.4

Solution:

Value of x + y should be 180º for ABC to be a line.

Justification:

From the figure we can say that,

BD is a ray that intersects AB and BC at the point B which results in

∠ABD = y

and, ∠DBC = x

We know,

If a ray stands on a line, then the sum of two adjacent angles so formed is 180°.

⇒ If the sum of two adjacent angles is 180°, then a ray stands on a line.

Thus, for ABC to be a line,

The sum of ∠ABD and ∠DBC should be equal to 180°.

⇒ ∠ABD + ∠DBC = 180°

⇒ x + y = 180°

Therefore, the value of x + y should be equal to 180° for ABC to be a line.

2. Can a triangle have all angles less than 60°? Give reason for your answer.

Solution:

No. A triangle cannot have all angles less than 60°

Justification:

According to angle sum property,

We know that the sum of all the interior angles of a triangle should be = 180º.

Suppose, all the angles are 60º,

Then we get, 60º + 60º + 60º = 180º.

Now, considering angles less than 60º,

Let us take 59º, which is the highest natural number less than 60º.

Then we have,

59º +59º + 59º = 177º ≠ 180º

Hence, we can say that if all the angles are less that 60º, the measure of the angles won't satisfy the angle sum property.

Therefore, a triangle cannot have all angles less than 60°.

3. Can a triangle have two obtuse angles? Give reason for your answer.

Solution:

No. A triangle cannot have two obtuse angles

Justification:

According to angle sum property,

We know that the sum of all the interior angles of a triangle should be = 180°.

An obtuse angle is one whose value is greater than 90° but less than 180°.

Considering two angles to be equal to the lowest natural number greater than 90°, i.e., 91°.

According to the question,

If the triangle has two obtuse angles, then there are two angles which are at least 91° each.

On adding these two angles,

Sum of the two angles = 91° + 91°

⇒ Sum of the two angles = 182°

The sum of these two angles already exceeds the sum of three angles of the triangle, even without considering the third angle.

Therefore, a triangle cannot have two obtuse angles.

4. How many triangles can be drawn having its angles as 45°, 64° and 72°? Give reason for your answer.

Solution:

No triangle can be drawn having its angles 45°, 64° and 72°.

Justification:

According to angle sum property,

We know that the sum of all the interior angles of a triangle should be = 180°.

But, according to the question,

We have the angles 45°, 64° and 72°.

Sum of these angles = 45° + 64° + 72°

= 181°, which is greater than 180°.

Hence, the angles do not satisfy the angle sum property of a triangle.

Therefore, no triangle can be drawn having its angles 45°, 64° and 72°.

5. How many triangles can be drawn having its angles as 53°, 64° and 63°? Give reason for your answer.

Solution:

Infinitely many triangles can be drawn having its angles as 53°, 64° and 63°.

Justification:

According to angle sum property,

We know that the sum of all the interior angles of a triangle should be = 180°.

According to the question,

We have the angles 53°, 64°, and 63°.

Sum of these angles = 53° + 64° + 63°

= 180°

Hence, the angles satisfy the angle sum property of a triangle.

Therefore, infinitely many triangles can be drawn having its angles as 53°, 64° and 63°.

EXERCISE 6.3 PAGE NO: 58

1. In Fig. 6.9, OD is the bisector of ∠AOC, OE is the bisector of ∠BOC and OD ⊥ OE. Show that the points A, O and B are collinear.

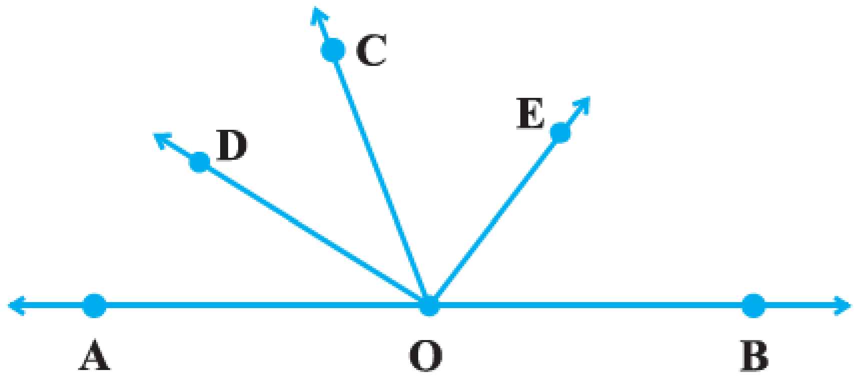


Fig. 6.9

Solution:

According to the question,

In figure,

OD ⊥ OE,

OD and OE are the bisector of ∠AOC and ∠BOC.

To prove: Points A, O and B are collinear

i.e., AOB is a straight line.

Proof:

Since, OD and OE bisect angles ∠AOC and ∠BOC respectively.

∠AOC = 2∠DOC …(eq.1)

And ∠COB = 2∠COE …(eq.2)

Adding (eq.1) and (eq.2), we get

∠AOC = ∠COB = 2∠DOC + 2∠COE

∠AOC +∠COB = 2(∠DOC +∠COE)

∠AOC + ∠COB = 2∠DOE

Since, OD⊥OE

We get,

∠AOC +∠COB = 2×90°

∠AOC +∠COB =180°

∠AOB =180°

So, ∠AOC + ∠COB are forming linear pair.

Therefore, AOB is a straight line.

Hence, points A, O and B are collinear.

2. In Fig. 6.10, ∠1 = 60° and ∠6 = 120°. Show that the lines m and n are parallel.

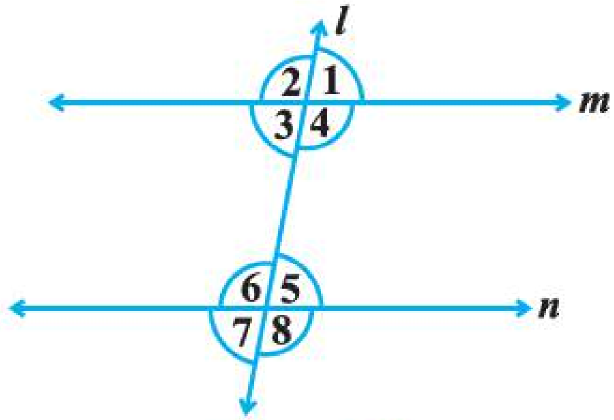


Fig. 6.10

Solution:

According to the question,
We have from the figure ∠1 = 60° and ∠6 = 120°
Since, ∠1 = 60° and ∠6 = 120°
Here, ∠1 = ∠3 [since they are vertically opposite angles]
∠3 = ∠1 = 60°
Now, ∠3 + ∠6 = 60° + 120°
⇒ ∠3 + ∠6 = 180°
We know that,
If the sum of two interior angles on same side of *l* is 180°, then the lines are parallel.
Therefore, m || n

3. AP and BQ are the bisectors of the two alternate interior angles formed by the intersection of a transversal t with parallel lines l and m (Fig. 6.11). Show that AP || BQ.

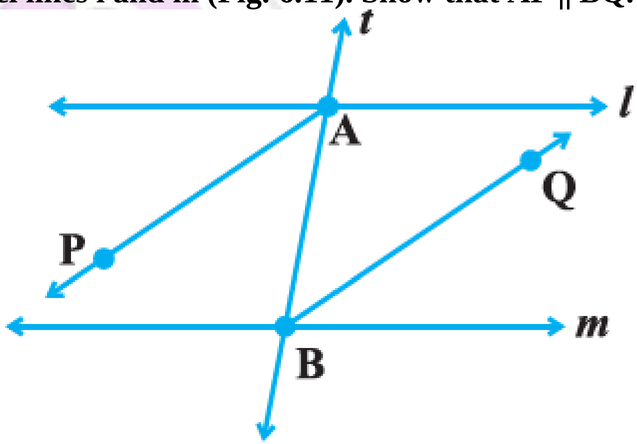
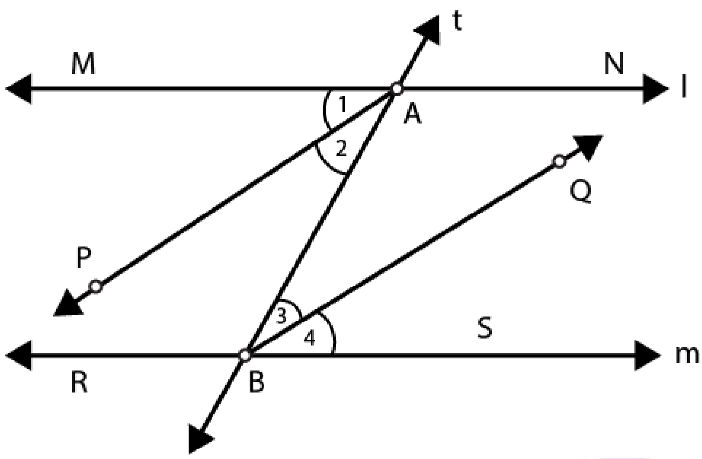


Fig. 6.11

Solution:



l || m and t is the transversal
∠MAB = ∠SBA [alternate angles]
⇒ ½ ∠MAB = ½ ∠SBA
⇒ ∠PAB = ∠QBA
⇒ ∠2 = ∠3
But, ∠2 and ∠3 are alternate angles.
Hence, AP||BQ.

4. If in Fig. 6.11, bisectors AP and BQ of the alternate interior angles are parallel, then show that *l* || m.

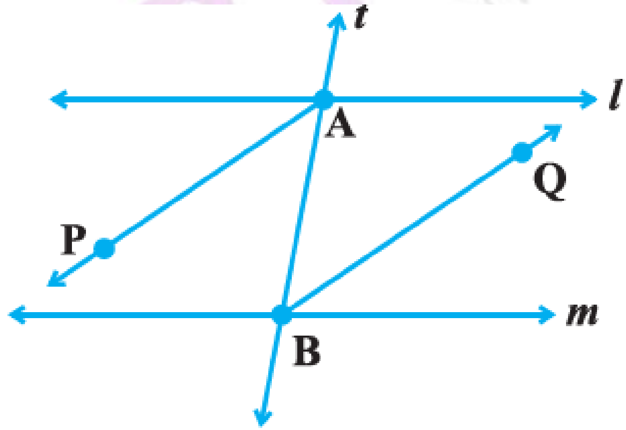
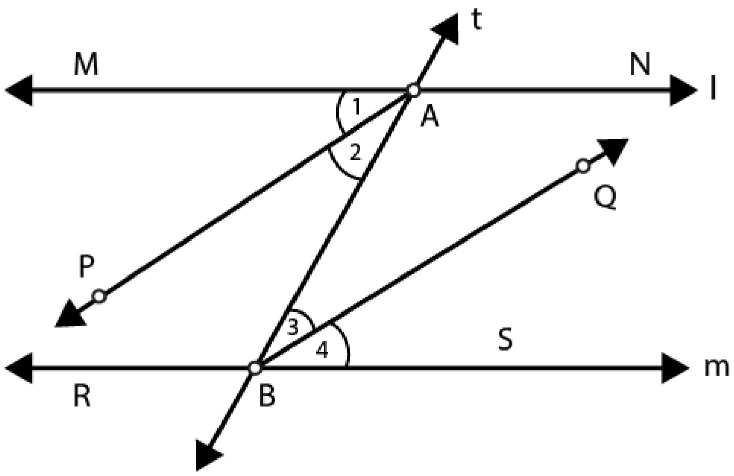


Fig. 6.11

Solution:

AP is the bisector of ∠MAB
BQ is the bisector of ∠SBA.
Given: AP||BQ.
As AP||BQ,
We have,



So ∠2 = ∠3 [Alternate angles]
2∠2 = 2∠3
⇒ ∠2 + ∠2 = ∠3 +∠3
From figure, we have ∠1= ∠2and ∠3 = ∠4
⇒ ∠1+ ∠2 = ∠3 +∠4
⇒ ∠MAB = ∠SBA
But, we know that these are alternate angles.
Hence, the lines *l* and m are parallel, i.e., *l* ||m.

5. In Fig. 6.12, BA || ED and BC || EF. Show that ∠ABC = ∠DEF [Hint: Produce DE to intersect BC at P (say)].

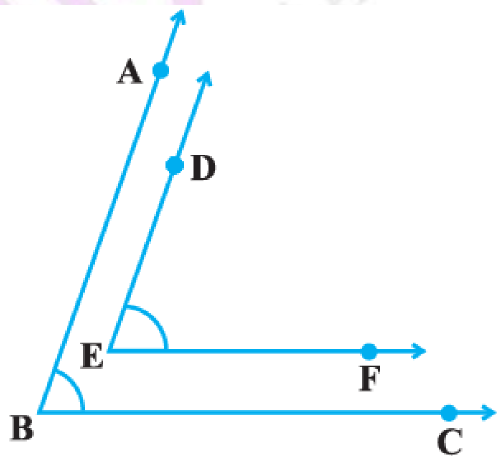


Fig. 6.12

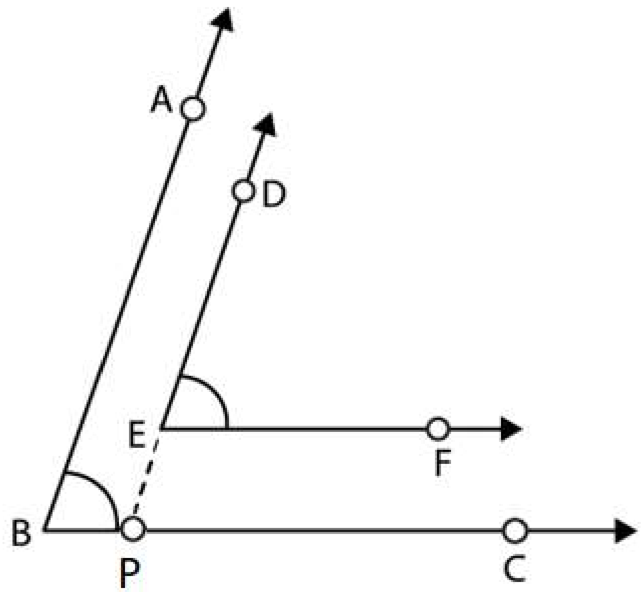
Solution:

Construction:
Extend DE to intersect BC at point, P.
Given, EF||BC and DP is the transversal,

$\angle DEF = \angle DPC$ …(eq.1) [Corresponding angles]

Also given, AB||DP and BC is the transversal,

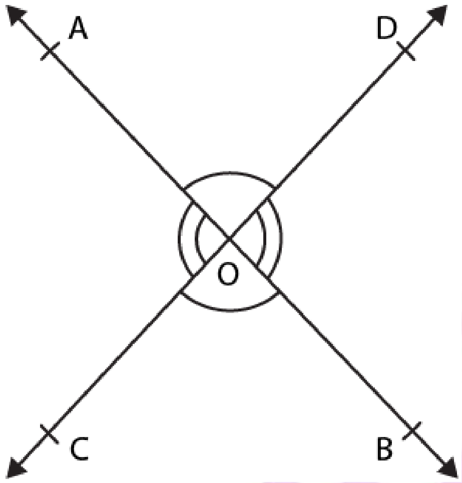
$\angle DPC = \angle ABC$ …(eq.2) [Corresponding angles]



From (eq.1) and (eq.2), we get
∠ABC = ∠DEF
Hence, Proved.

EXERCISE 6.4 PAGE NO: 61

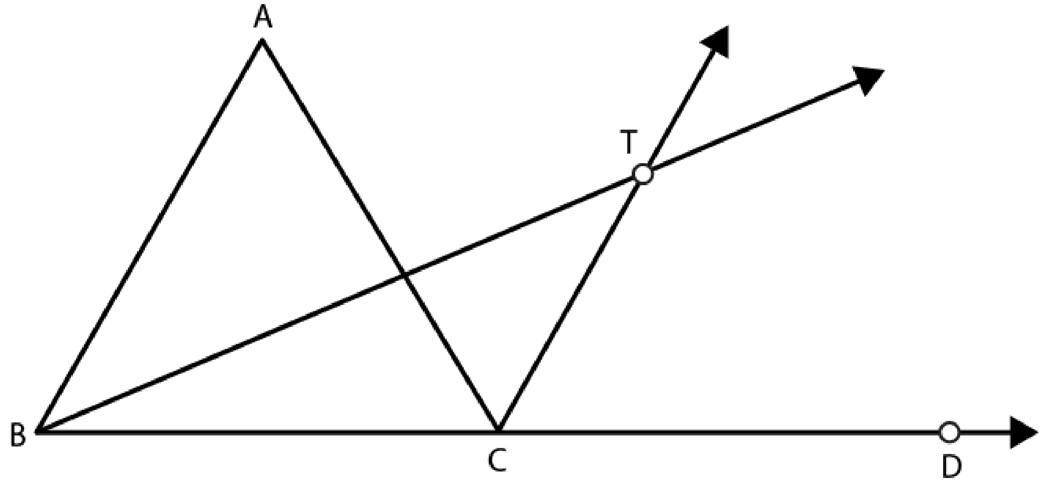
1. If two lines intersect, prove that the vertically opposite angles are equal.
Solution:



From the figure, we know that,
AB and CD intersect each other at point O.
Let the two pairs of vertically opposite angles be,
1st pair - ∠AOC and ∠BOD
2nd pair - ∠AOD and ∠BOC
To prove:
Vertically opposite angles are equal,
i.e., ∠AOC = ∠BOD, and ∠AOD = ∠BOC
From the figure,
The ray AO stands on the line CD.
We know that,
If a ray lies on a line then the sum of the adjacent angles is equal to 180°.
⇒ ∠AOC + ∠AOD = 180° (By linear pair axiom) … (i)
Similarly, the ray DO lies on the line AOB.
⇒ ∠AOD + ∠BOD = 180° (By linear pair axiom) … (ii)
From equations (i) and (ii),
We have,
∠AOC + ∠AOD = ∠AOD + ∠BOD
⇒ ∠AOC = ∠BOD - - - - (iii)
Similarly, the ray BO lies on the line COD.
⇒ ∠DOB + ∠COB = 180° (By linear pair axiom) - - - - (iv)
Also, the ray CO lies on the line AOB.
⇒ ∠COB + ∠AOC = 180° (By linear pair axiom) - - - - (v)
From equations (iv) and (v),
We have,
∠DOB + ∠COB = ∠COB + ∠AOC
⇒ ∠DOB = ∠AOC - - - - (vi)
Thus, from equation (iii) and equation (vi),

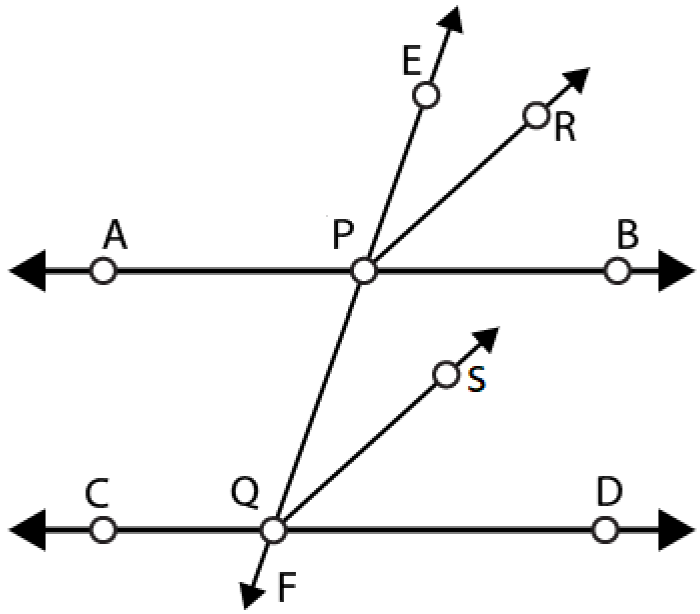
We have,
∠AOC = ∠BOD, and ∠DOB = ∠AOC
Therefore, we get, vertically opposite angles are equal.
Hence Proved.

2. Bisectors of interior ∠B and exterior ∠ACD of a Δ ABC intersect at the point T. Prove that ∠ BTC = ½ ∠ BAC.
Solution:
Given: △ ABC, produce BC to D and the bisectors of ∠ABC and ∠ACD meet at point T.



To prove:
∠BTC = ½ ∠BAC
Proof:
In △ABC,∠ACD is an exterior angle.
We know that,
Exterior angle of a triangle is equal to the sum of two opposite angles,
Then,
∠ACD = ∠ABC + ∠CAB
Dividing L.H.S and R.H.S by 2,
⇒ ½ ∠ACD = ½ ∠CAB + ½ ∠ABC
⇒ ∠TCD = ½ ∠CAB + ½ ∠ABC …(1)
[∵CT is a bisector of ∠ACD⇒ ½ ∠ACD = ∠TCD]
We know that,
Exterior angle of a triangle is equal to the sum of two opposite angles,
Then in △ BTC,
∠TCD = ∠BTC +∠CBT
⇒ ∠TCD = ∠BTC + ½ ∠ABC …(2)
[∵BT is bisector of △ ABC ⇒∠CBT = ½ ∠ABC]
From equation (1) and (2),
We get,
½ ∠CAB + ½ ∠ABC = ∠BTC + ½ ∠ABC
⇒ ½ ∠CAB = ∠BTC or ½ ∠BAC = ∠BTC
Hence, proved.

3. A transversal intersects two parallel lines. Prove that the bisectors of any pair of corresponding angles so formed are parallel.
Solution:



Let,
AB ‖ CD
EF be the transversal passing through the two parallel lines at P and Q respectively.
PR and QS are the bisectors of ∠EPB and ∠PQD.
We know that the corresponding angles of parallel lines are equal,
So, ∠EPB = ∠PQD
½ ∠EPB = ½ ∠PQD
∠EPR = ∠PQS
But, we also know that they are corresponding angles of PR and QS
Since the corresponding angles are equal,
We have,
PR ‖ QS
Hence Proved.