EXERCISE 7.1

In each of the following, write the correct answer:

1. Which of the following is not a criterion for congruence of triangles?

(A) SAS

(B) ASA

(C) SSA

(D) SSS

Solution:

(C) SSA

Explanation:

We know that,

Two triangles are congruent, if the side(S) and angles (A) of one triangle is equal to another.

And criterion for congruence of triangle are SAS, ASA, SSS, and RHS.

SSA is not the criterion for congruency of a triangle.

Hence, option C is the correct answer.

2. If AB = QR, BC = PR and CA = PQ, then

(A) Δ ABC ≅ ΔPQR

(B) ΔCBA ≅ ΔPRQ

(C) ΔBAC ≅ Δ RPQ

(D) Δ PQR ≅ Δ BCA

Solution:

(B) ΔCBA ≅ ΔPRQ

Explanation:

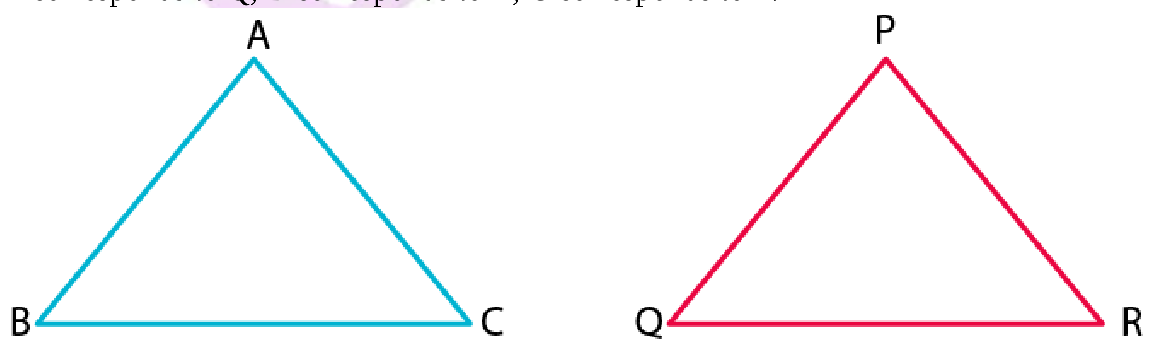
According to the question,

AB = QR, BC = PR and CA = PQ

Since, AB = QR, BC = PR and CA = PQ

We can say that,

A corresponds to Q, B corresponds to R, C corresponds to P.



Hence, (B) ΔCBA ≅ ΔPRQ

Hence, option B is the correct answer.

3. In Δ ABC, AB = AC and ∠B = 50°. Then ∠C is equal to

(A) 40° (B) 50° (C) 80° (D) 130°

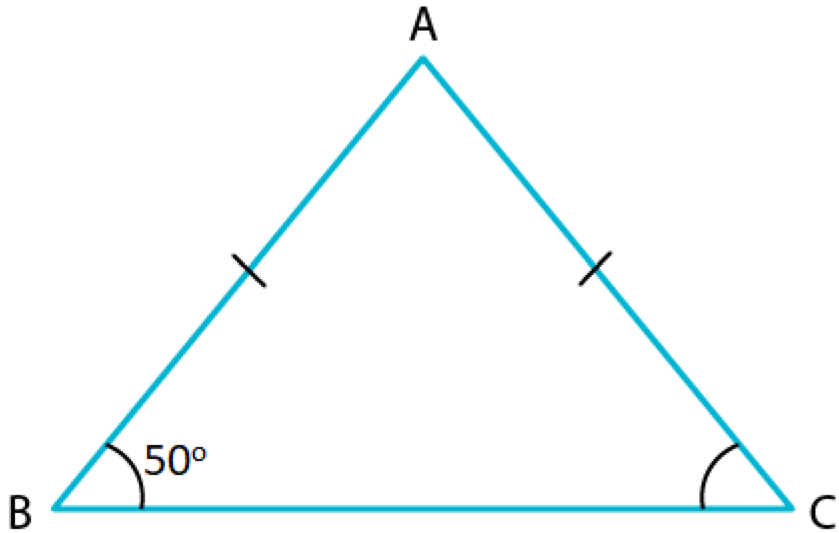
Solution:

(B) 50°

Explanation:

According to the question,

Δ ABC, AB = AC and ∠B = 50°.



Since, AB = AC

Δ ABC is an isosceles triangle.

Hence, ∠B = ∠C

∠B = 50° (given)

⇒ ∠C = 50°

Hence, option B is the correct option.

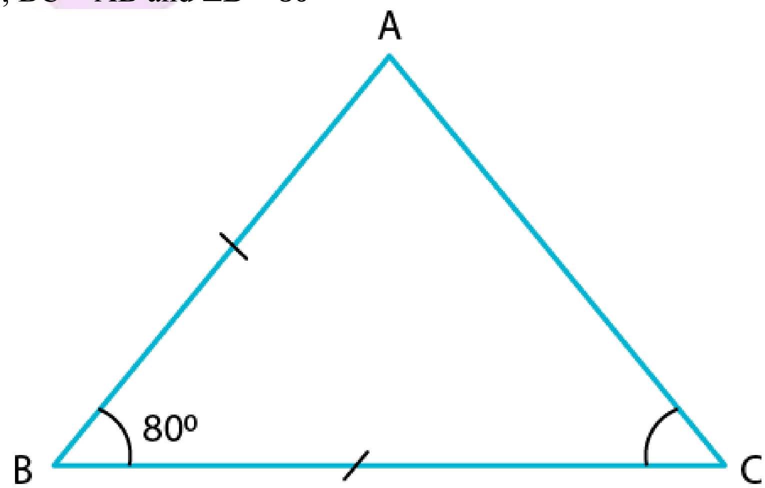
4. In Δ ABC, BC = AB and ∠B = 80°. Then ∠A is equal to
(A) 80° (B) 40° (C) 50° (D) 100°

Solution:

(C) 50°

Explanation:

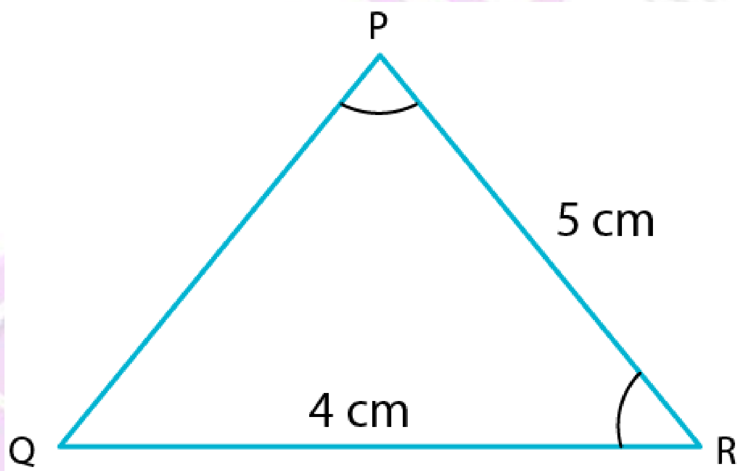
Given: Δ ABC, BC = AB and ∠B = 80°



Since, BC = AB

Δ ABC is an isosceles triangle.
Let, ∠C = ∠A = x
∠B = 80° (given)
We know that,
Using angle sum property,
Sum of interior angles of a triangle should be = 180°
∠A + ∠B + ∠C = 180°
⇒ x + 80° + x = 180°
⇒ 2x = 180° - 80°
⇒ 2x = 100°
⇒ x = 50°
Therefore, ∠C = ∠A = 50°
Hence, option C is the correct answer.

5. In Δ PQR, ∠R = ∠P and QR = 4 cm and PR = 5 cm. Then the length of PQ is
(A) 4 cm (B) 5 cm (C) 2 cm (D) 2.5 cm
Solution:
Given: ΔPQR, ∠R = ∠P and QR = 4 cm and PR = 5 cm

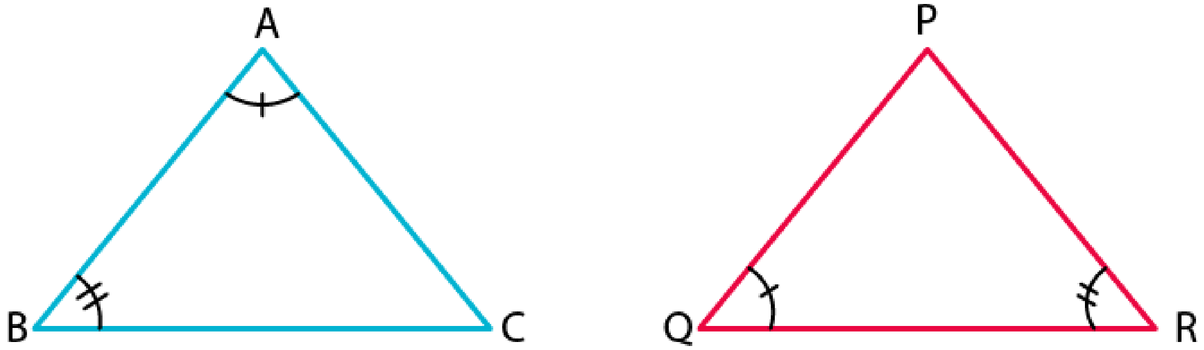


Since, ∠R = ∠P
Δ PQR is an isosceles triangle.
Hence, PQ = QR
⇒ PQ = 4cm
Hence, option A is the correct answer.

EXERCISE 7.2 PAGE NO: 65

1. In triangles ABC and PQR, ∠A = ∠Q and ∠B = ∠R. Which side of Δ PQR should be equal to side AB of Δ ABC so that the two triangles are congruent? Give reason for your answer.

Solution:



In triangle ABC and PQR, we have

∠A = ∠Q [Given]

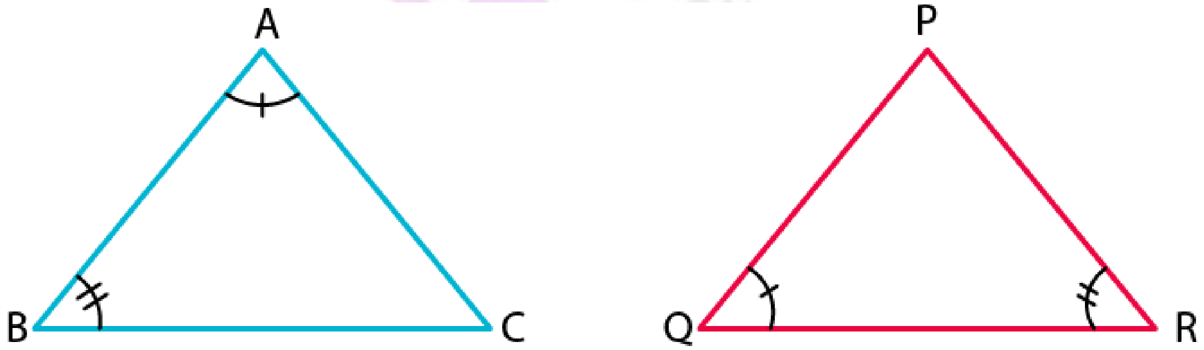
∠B = ∠R [Given]

For the triangle to be congruent, AB should be equal to QR.

Hence, triangle ABC and PQR can be congruent by ASA congruence rule.

2. In triangles ABC and PQR, ∠A = ∠Q and ∠B = ∠R. Which side of Δ PQR should be equal to side BC of Δ ABC so that the two triangles are congruent? Give reason for your answer.

Solution:



In triangle ABC and PQR, we have

$\angle A = \angle Q$ and $\angle B = \angle R$ [Given]

For the triangles to be congruent, we must have

BC = RP

Hence, triangle ABC and PQR will be congruent by AAS congruence rule.

3. "If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent." Is the statement true? Why?

Solution:

No, the statement, "if two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent" is false.

<u>Justification:</u>

Because by the congruent rule,

The two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle, i.e., SAS rule.

4. "If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent." Is the statement true? Why?

Solution:

The statement, "If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent." is true.

Justification:

The statement is true because, the triangles will be congruent either by ASA rule or AAS rule. This is because two angles and one side are enough to construct two congruent triangles.

5. Is it possible to construct a triangle with lengths of its sides as 4 cm, 3 cm and 7 cm? Give reason for your answer.

Solution:

No, it is not possible to construct a triangle with lengths of sides 4 cm, 3 cm and 7 cm.

Justification:

We know that,

Sum of any two sides of a triangle is always greater than the third side.

Here, the sum of two sides whose lengths are 4 cm and 3 cm = 4 cm + 3 cm = 7 cm,

Which is equal to the length of third side, i.e., 7 cm.

Hence, it is not possible to construct a triangle with lengths of sides 4 cm, 3 cm and 7 cm.

6. It is given that $\Delta ABC \cong \Delta RPQ$. Is it true to say that BC = QR? Why?

Solution:

It is False that BC = QR because BC = PQ as $\Delta ABC \cong \Delta RPQ$.

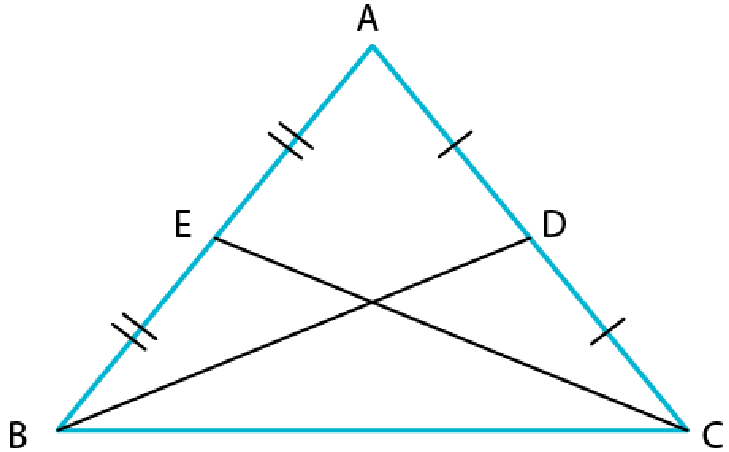
EXERCISE 7.3 PAGE NO: 67

1. ABC is an isosceles triangle with AB = AC and BD and CE are its two medians. Show that BD = CE.

Solution:

According to the question,

ΔABC is an isosceles triangle and AB = AC, BD and CE are two medians



From ΔABD and ΔACE,

AB = AC (given)

2 AE = 2 AD (as D and E are mid points)

So, AE = AD

∠A = ∠A (common)

Hence, ΔABD ≅ ΔACE (using SAS)

⇒ BD = CE (by CPCT)

Hence proved.

2. In Fig.7.4, D and E are points on side BC of a Δ ABC such that BD = CE and AD = AE. Show that Δ ABD ≅ Δ ACE.

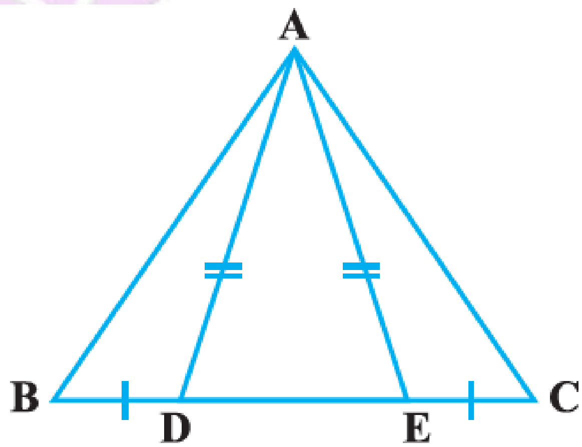


Fig. 7.4

Solution:

According to the question,

In ΔABC,
BD = CE and AD = AE.
In ΔADE,
AD = AE
Since opposite angles to equal sides are equal,
We have,
$\angle ADE = \angle AED$ … (1)
Now, $\angle ADE + \angle ADB = 180°$ (linear pair)
$\angle ADB = 180° - \angle ADE$ ... (2)
Also, $\angle AED + \angle AEC = 180°$ (linear pair)
$\angle AEC = 180° - \angle AED$
Since, $\angle ADE = \angle AED$
$\angle AEC = 180° - \angle ADE$ … (3)
From equation (2) and (3)
$\angle ADB = \angle AEC$ ... (4)
Now, In ΔADB and ΔAEC,
AD = AE (given)
BD = EC (given)
$\angle ADB = \angle AEC$ (from (4)
Hence, $\Delta ABD \cong \Delta ACE$ (by SAS)

3. CDE is an equilateral triangle formed on a side CD of a square ABCD (Fig.7.5). Show that Δ ADE $\cong$ Δ BCE.

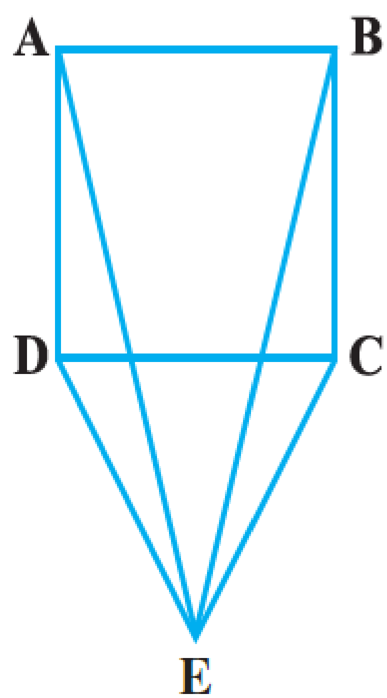


Fig. 7.5

Solution:

According to the question,
CDE is an equilateral triangle formed on a side CD of a square ABCD.
In ΔADE and ΔBCE,
DE = CE (sides of equilateral triangle)
Now,
∠ADC = ∠BCD = 90°
And, ∠EDC = ∠ECD = 60°
Hence, ∠ADE = ∠ADC + ∠CDE = 90° + 60° = 150°
And ∠BCE = ∠BCD + ∠ECD = 90° + 60° = 150°
⇒ ∠ADE = ∠BCE
AD = BC (sides of square)
Hence, ΔADE ≅ΔBCE (by SAS)

4. In Fig.7.6, BA ⊥ AC, DE ⊥ DF such that BA = DE and BF = EC. Show that Δ ABC ≅ Δ DEF.

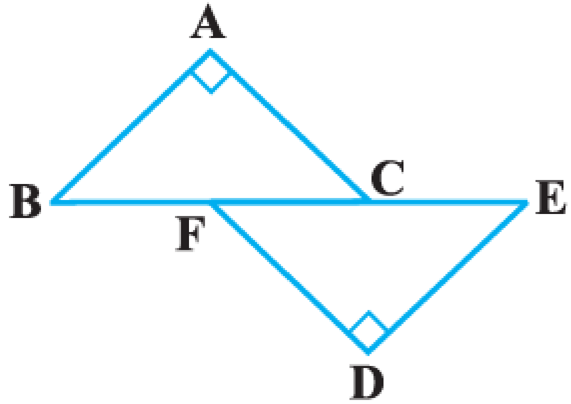


Fig. 7.6

Solution:

According to the question,
BA ⊥ AC, DE ⊥ DF such that BA = DE and BF = EC.
In ΔABC and ΔDEF
BA = DE (given)
BF = EC (given)
∠A = ∠D (both 90°)
BC = BF + FC
EF = EC + FC = BF + FC (∵ EC = BF)
⇒ EF = BC
Hence, ΔABC ≅ ΔDEF (by RHS)

5. Q is a point on the side SR of a Δ PSR such that PQ = PR. Prove that PS > PQ.

Solution:

Given: in ΔPSR, Q is a point on the side SR such that PQ = PR.
In ΔPRQ,
PR = PQ (given)
⇒ ∠PRQ = ∠PQR (opposite angles to equal sides are equal)
But ∠PQR > ∠PSR (exterior angle of a triangle is greater than each of opposite interior angle)

⇒ ∠PRQ > ∠PSR
⇒ PS > PR (opposite sides to greater angle is greater)
⇒ PS > PQ (as PR = PQ)

EXERCISE 7.4 PAGE NO: 69

1. Find all the angles of an equilateral triangle.
Solution:

In equilateral triangle,
All the sides are equal.
Therefore, all angles are also equal
Let the angles of an equilateral triangle = x
According to angle sum property,
We know that the sum of the interior angles is equal to 180^o.
$x+x+x=180^o$
$3x=180$
$x=60°$
Therefore, all the angles of an equilateral triangle are 60°

2. The image of an object placed at a point A before a plane mirror LM is seen at the point B by an observer at D as shown in Fig. 7.12. Prove that the image is as far behind the mirror as the object is in front of the mirror.
[Hint: CN is normal to the mirror. Also, angle of incidence = angle of reflection].

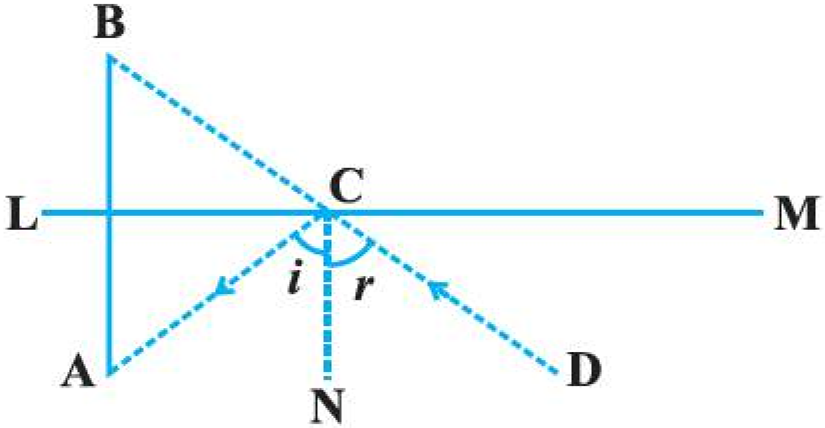
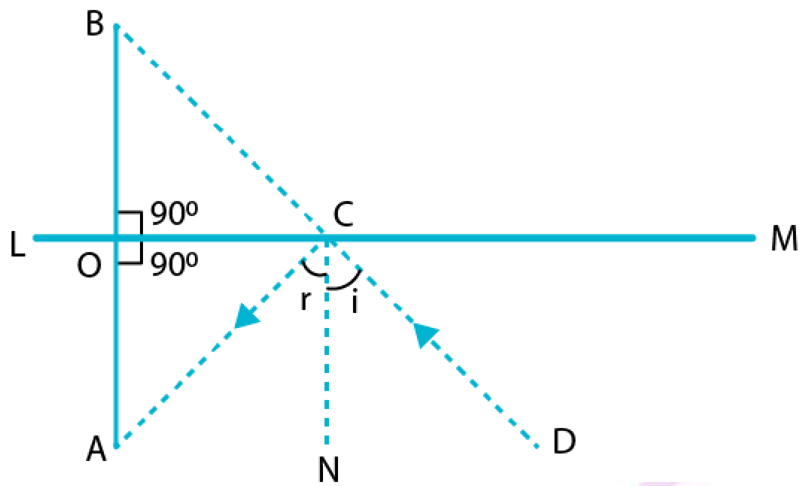


Fig. 7.12

Solution:

Let AB intersect LM at O. We have to prove that AO = BO.
Now, ∠i = ∠r …(1)
[∵Angle of incidence = Angle of reflection]
∠B = ∠i [Corresponding angles] …(2)
And ∠A = ∠r [Alternate interior angles] …(3)



From (1), (2) and (3), we get
∠B = ∠A
⇒ ∠BCO = ∠ACO
In ΔBOC and ΔAOC we have
∠1 = ∠2 [Each = 90º]
OC = OC [Common side]
And ∠BCO = ∠ACO [Proved above]
ΔBOC ≅ ΔAOC [ASA congruence rule]
Hence, AO = BO [CPCT]

3. ABC is an isosceles triangle with AB = AC and D is a point on BC such that AD ⊥ BC (Fig. 7.13). To prove that ∠BAD = ∠CAD, a student proceeded as follows:
In Δ ABD and Δ ACD,
AB = AC (Given)
∠B = ∠C (because AB = AC)
And ∠ADB = ∠ADC
Therefore, Δ ABD Δ Δ ACD (AAS)
So, ∠BAD = ∠CAD (CPCT)

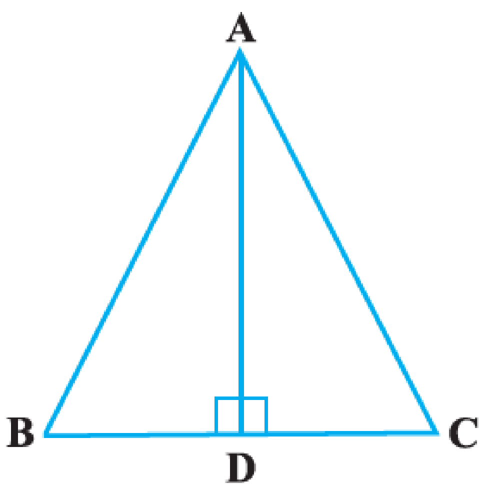
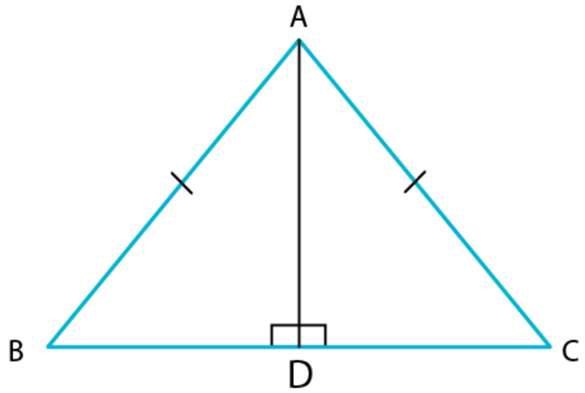


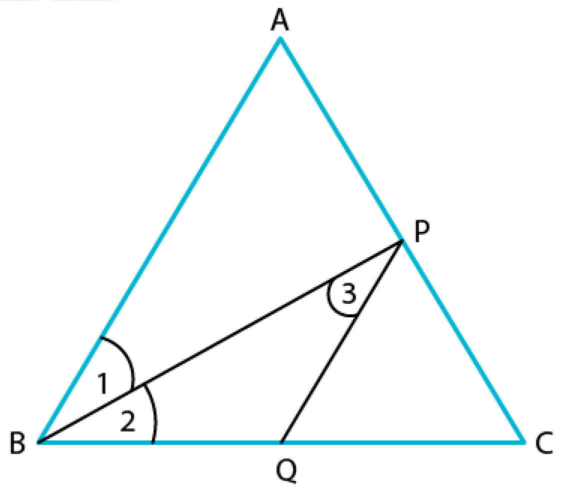
Fig. 7.13

What is the defect in the above arguments?
[Hint: Recall how ∠B = ∠C is proved when AB = AC].
Solution:



In Δ ABD and Δ ADC, we have
∠ADB = ∠ADC
According to the question,
AB = BC
AD = AD [Common side]
By RHS criterion of congruence,
We have,
Δ ABD ≅ Δ ACD
∠BAD = ∠CAD [CPCT]
Hence Proved.

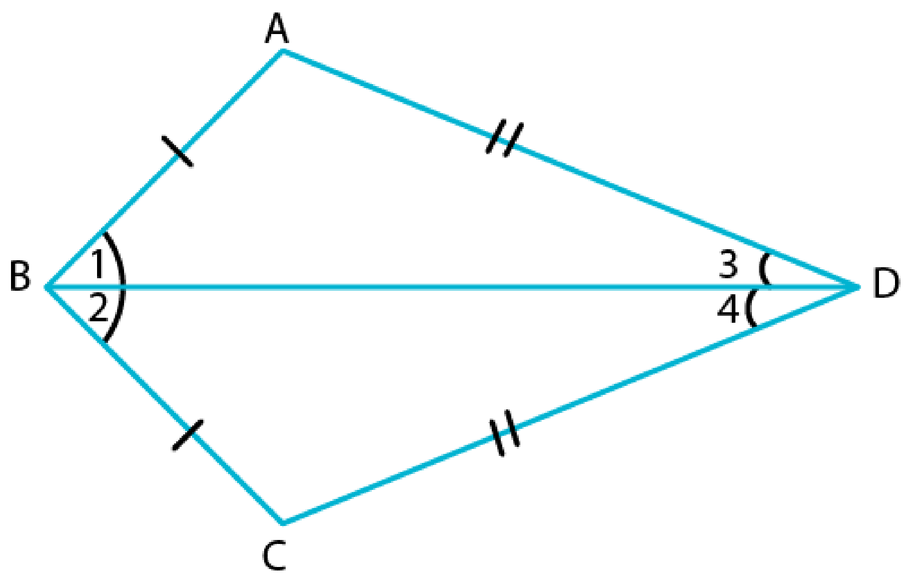
4. P is a point on the bisector of ∠ABC. If the line through P, parallel to BA meet BC at Q, prove that BPQ is an isosceles triangle.
Solution:



To prove: BPQ is an isosceles triangle.
According to the question,
Since, BP is the bisector of ∠ABC,
∠1 = ∠2 … (1)
Now, PQ is parallel to BA and BP cuts them

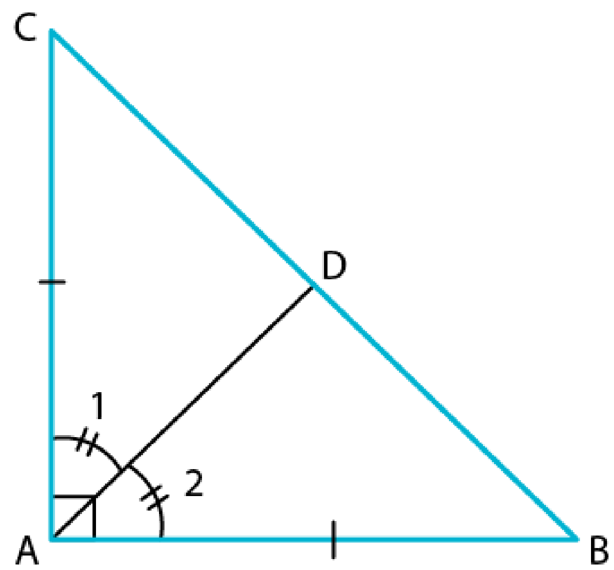
∠1 = ∠3 [Alternate angles] … (2)
From equations, (1) and (2),
We get
∠2 = ∠3
In Δ BPQ,
We have
∠2 = ∠3
PQ = BQ
Hence, BPQ is an isosceles triangle.

5. ABCD is a quadrilateral in which AB = BC and AD = CD. Show that BD bisects both the angles ABC and ADC.
Solution:



According to the question,
In ΔABC and ΔCBD,
We have
AB = BC
AD = CD
BD = BD [Common side]
ΔABC ≅ ΔCBD [By SSS congruence rule]
⇒ ∠1 = ∠2 [CPCT]
And ∠3 = ∠4
Hence, BD bisects both ∠ABC and ∠ADC.

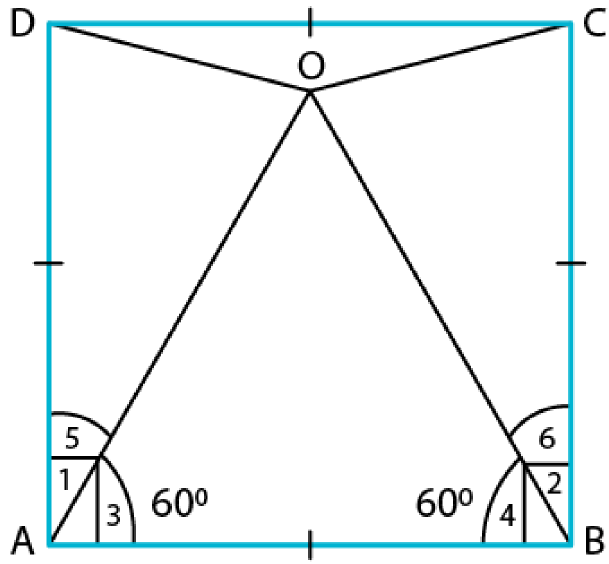
6. ABC is a right triangle with AB = AC. Bisector of ∠A meets BC at D. Prove that BC = 2 AD.
Solution:
Given: A right angles triangle with AB = AC bisector of ∠A meets BC at D.
To prove: BC = 2AD



Proof:
According to the question,
In right Δ ABC,
AB = AC
Since, hypotenuse is the longest side,
BC is hypotenuse
∠BAC = 90°
Now,
In Δ CAD and Δ BAD,
We have,
AC = AB
Since, AD is the bisector of ∠A,
∠1 = ∠2
AD = AD [Common side]
Now,
By SAS criterion of congruence,
We get,
Δ CAD ≅ Δ BAD
CD = BD [CPCT]
Since, Mid-point of hypotenuse of a right triangle is equidistant from the 3 vertices of a triangle.
AB = BD = CD …(1)
Now, BC = BD + CD
⇒ BC = AD + AD [Using eq.(1)]
⇒ BC = 2AD
Hence, proved.

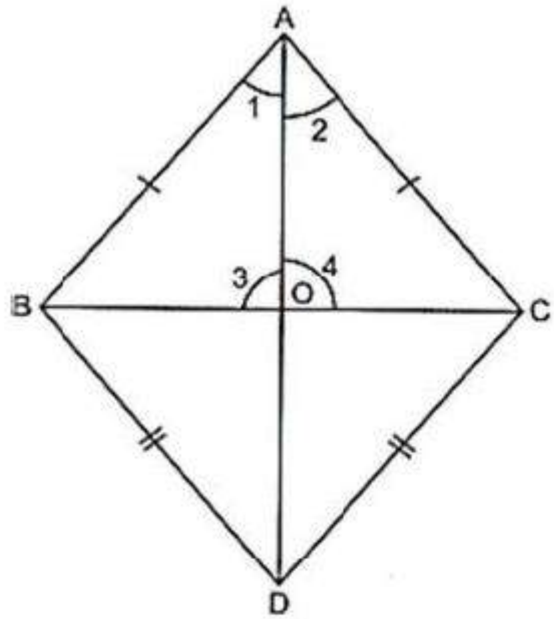
7. O is a point in the interior of a square ABCD such that OAB is an equilateral triangle. Show that Δ OCD is an isosceles triangle.
Solution:

According to the question,
It is given that: A square ABCD and OA = OB = AB.



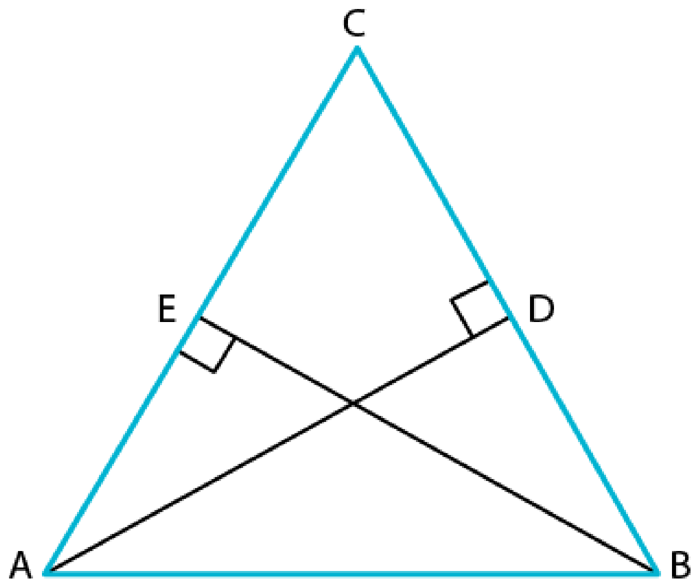
To prove: Δ OCD is an isosceles triangle.
Proof:
In square ABCD,
Since ∠1 and ∠2 is equal to 90^o
∠1 = ∠2 …(1)
Now, in Δ OAB , we have
Since ∠3 and ∠4 is equal to 60^o
∠3 = ∠4 …(2)
Subtracting equations (2) from (1),
We get
∠1−∠3 = ∠2 −∠4
⇒ ∠5 = ∠6
Now,
In Δ DAO and Δ CBO,
AD = BC [Given]
∠5 = ∠6 [Proved above]
OA = OB [Given]
By SAS criterion of congruence,
We have
Δ DAO ≅ Δ CBO
OD = OC
⇒ Δ OCD is an isosceles triangle.
Hence, proved.

8. ABC and DBC are two triangles on the same base BC such that A and D lie on the opposite sides of BC, AB = AC and DB = DC. Show that AD is the perpendicular bisector of BC.
Solution:



Given: Δ ABC and Δ DBC on the same base BC. Also, AB = AC and BD = DC.
To prove: AD is the perpendicular bisector of BC i.e., OB = OC
Proof: In Δ BAD and Δ CADwe have
AB = AC [Given]
BD = CD [Given]
AD = AD [common side]
So, by SSS criterion of congruence, we have
Δ BAD ≅ Δ CAD
∠1 = ∠2 [CPCT]
Now, in Δ BAO and Δ CAO, we have
AB = AC [Given]
∠1 = ∠2 [Proved above]
AO = AO [Common side]
So, by SAS criterion of congruence, we have
Δ BAO ≅ Δ CAO
BO = CO [CPCT]
And, ∠3 = ∠4 [CPCT]
But, $\angle 3+\angle 4 = 180^o$ [Linear pair axiom]
⇒ $\angle 3+\angle 3 = 180$
⇒ $2\angle 3 = 180$
⇒ $\angle 3 = 180/2$
⇒ $\angle 3 = 90^o$
Since BO = CO and $\angle 3 = 90^o$,
AD is perpendicular bisector of BC.
Hence, proved.

9. ABC is an isosceles triangle in which AC = BC. AD and BE are respectively two altitudes to sides BC and AC. Prove that AE = BD.
Solution:

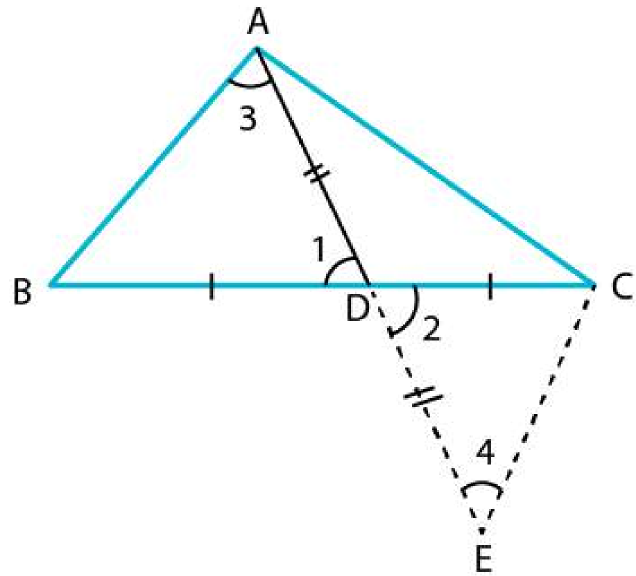


According to the question,
In Δ ADC and Δ BEC,
We have
AC = BC [Given] …(1)
Since ∠ADC and ∠BEC = 90°
∠ADC = ∠BEC
∠ACD = ∠BCE [Common angle]
Δ ADC ≅ Δ BEC [By SSS congruence rule]
CE = CD … (2) [CPCT]
Subtracting equation (2) from (1),
We get
AC – CE = BC – CD
⇒ AE = BD
Hence, proved.

10. Prove that sum of any two sides of a triangle is greater than twice the median with respect to the third side.

Solution:

According to the question,
We have, Δ ABC with median AD.
To prove:
AB + AC > 2AD
AB + BC > 2AD
BC + AC > 2AD
Construction:
Extend AD to E such that DE = AD
Join EC.
Proof:



In Δ ADB and Δ EDC ,
AD = ED [By construction]
∠1 = ∠2 [Vertically opposite angles are equal]
DB = DC [Given]
So, by SAS criterion of congruence, we have
Δ ADB ≅ Δ EDC
AB = EC [CPCT]
And ∠3 = ∠4 [CPCT]
Now, in Δ AEC,
Since sum of the lengths of any two sides of a triangle must be greater than the third side,
We have
AC + CE > AE
⇒ AC + CE > AD + DE
⇒ AC +CE > AD+ AD [\AD = DE]
⇒ AC + CE > 2AD
⇒ AC + AB > 2AD [∵AB = CE]
Similarly,
We get,
AB + BC > 2AD and BC + AC >2AD.
Hence, proved.