EXERCISE 9.1 PAGE NO: 85

Write the correct answer in each of the following:
1. The median of a triangle divides it into two
(A) triangles of equal area
(B) congruent triangles
(C) right triangles
(D) isosceles triangles
Solution:

(A) triangles of equal area

Explanation:

The median of a triangle divides it into triangle of equal area.
Hence, option (A) is the correct answer.

2. In which of the following figures (Fig. 9.3), you find two polygons on the same base and between the same parallels?

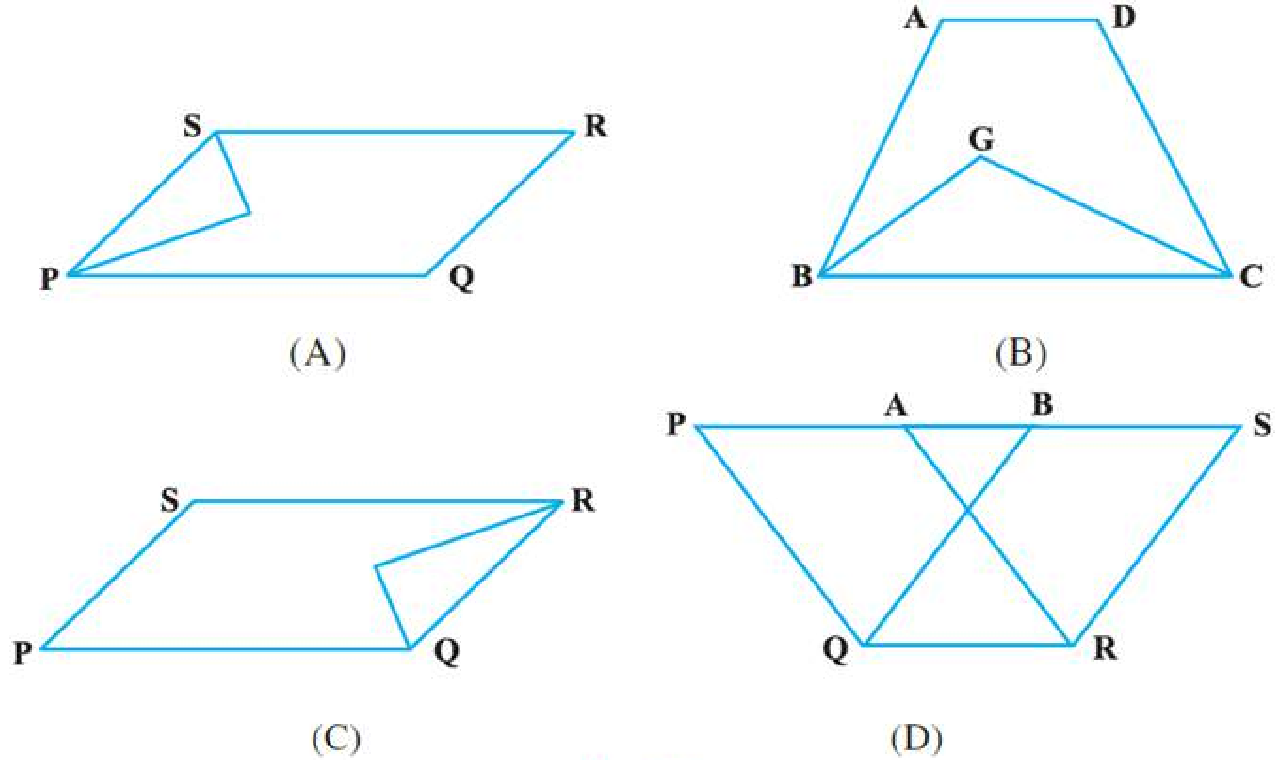


Fig. 9.3

Solution:

(D)

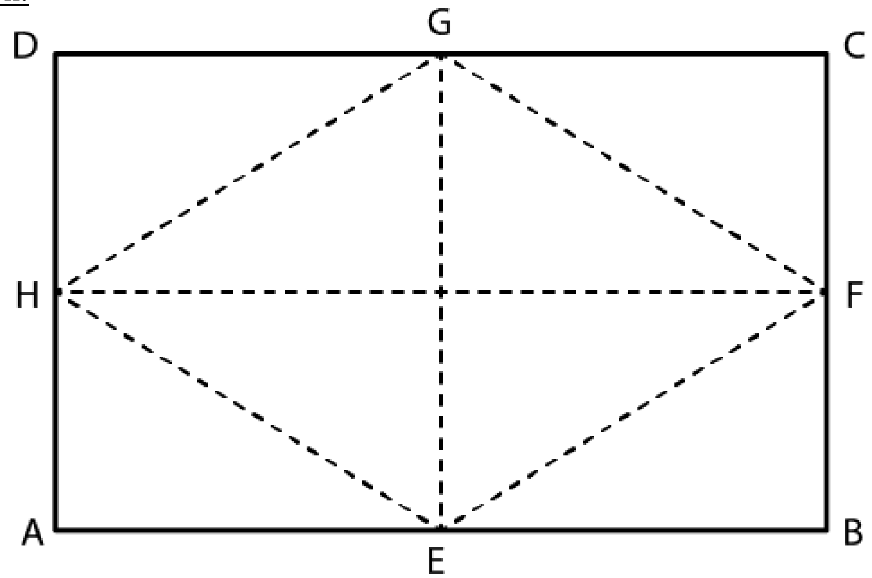
Explanation:

In figure (D), the parallelograms, PQRA and BQRS are on the same base QR and between the same parallels QR and PS.
Hence, option (D) is the correct answer.

3. The figure obtained by joining the mid-points of the adjacent sides of a rectangle of sides 8 cm and 6 cm is :
(A) a rectangle of area 24 cm^2
(B) a square of area 25 cm^2
(C) a trapezium of area 24 cm^2
(D) a rhombus of area 24 cm^2
Solution:

(D) a rhombus of area 24 cm^2

Explanation:



According to the question,
Let ABCD be the rectangle.
E is the mid-point of the side AB
F is the mid-point of the side BC
G is the mid-point of the side CD
H is the mid-point of the side DA
The figure obtained by joining the midpoints E, F, G and H is rhombus.
Sides of the rectangle is given,
We know that,
Side of the rectangle AB = diagonal of the rhombus FH = 8cm
Similarly,
Side of the rectangle AD = diagonal of the rhombus EG = 6cm
Then,
Area of the rhombus $= \frac{1}{2} \times EG \times FH$
$= \frac{1}{2} \times 6 \times 8$
$= 24 \text{ cm}^2$
Hence, option (D) is the correct answer.

4. In Fig. 9.4, the area of parallelogram ABCD is:
(A) AB × BM

(B) BC × BN
(C) DC × DL
(D) AD × DL

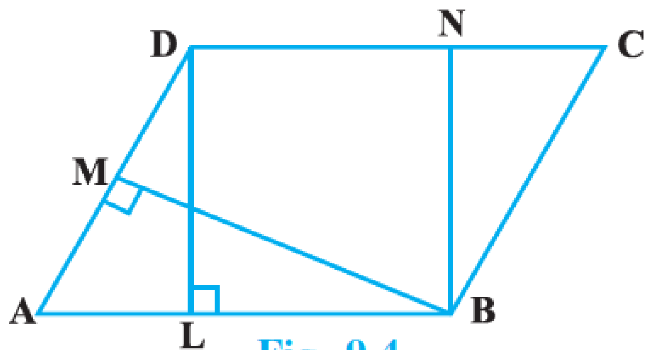


Fig. 9.4

Solution:

(C) DC × DL

Explanation:

Area of parallelogram = Base × Corresponding altitude

= AB × DL … (eq 1)

Since, opposite sides of a parallelogram are equal,

We get,

AB = DC

Substituting this in eq(1), we get,

Area of parallelogram = AB × DL

= DC × DL

Hence, option (C) is the correct answer.

5. In Fig. 9.5, if parallelogram ABCD and rectangle ABEF are of equal area, then :
(A) Perimeter of ABCD = Perimeter of ABEM
(B) Perimeter of ABCD < Perimeter of ABEM
(C) Perimeter of ABCD > Perimeter of ABEM
(D) Perimeter of ABCD = ½ (Perimeter of ABEM)

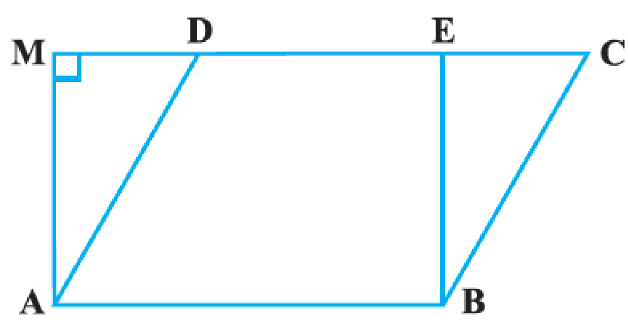


Fig. 9.5

Solution:

(C) Perimeter of ABCD > Perimeter of ABEM

Explanation:

In rectangle ABEM,
AB = EM …(eq.1) [sides of rectangle]
In parallelogram ABCD,
CD = AB …(eq.2)
Adding, equations (1) and (2),
We get
AB + CD = EM + AB …(i)
We know that,
Perpendicular distance between two parallel sides of a parallelogram is always less than the length of the other parallel sides.
BE < BC and AM < AD
[because, in a right angled triangle, the hypotenuse is greater than the other side]
On adding both above inequalities, we get
SE + AM <BC + AD or BC + AD> BE + AM
On adding AB + CD both sides, we get
AB + CD + BC + AD> AB + CD + BE + AM
⇒ AB+BC + CD + AD> AB+BE + EM+ AM [∴ CD = AB = EM]
Hence,
We get,
Perimeter of parallelogram ABCD > perimeter of rectangle ABEM
Hence, option (C) is the correct answer.

EXERCISE 9.2 PAGE NO: 88

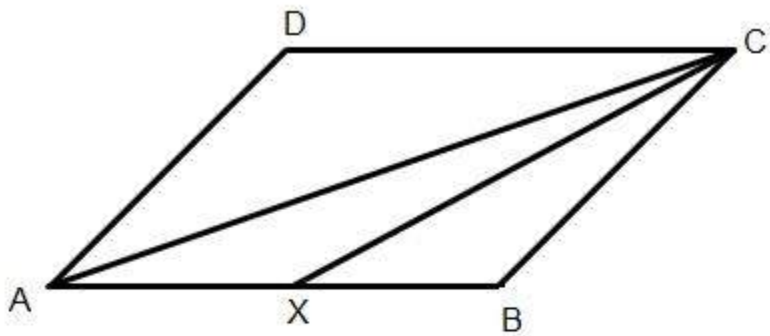
Write True or False and justify your answer:

1. ABCD is a parallelogram and X is the mid-point of AB. If ar (AXCD) = 24 cm^2, then ar (ABC) = 24 cm^2.

Solution:

False

Explanation:



In ΔABC, X is midpoint of AB

Considering the given question,

Hence ar(ΔAXC) = ar(ΔBXC) = ½ ar(ΔABC)

Therefore,

ar(ΔAXC) = ar(ΔBXC) = 12 cm^2

Area of ||gm ABCD = 2 x ar(ΔABC)

= 2 x 24 = 48 cm^2

But, according to the question,

area of ||gm ABCD = ar(AXCD) + ar(ΔBXC)

= 24 + 12 = 36 cm^2

Hence, we find a contradiction here.

So, If ar (AXCD) = 24 cm^2, then ar (ABC) ≠ 24 cm^2

2. PQRS is a rectangle inscribed in a quadrant of a circle of radius 13 cm. A is any point on PQ. If PS = 5 cm, then ar (PAS) = 30 cm^2.

Solution:

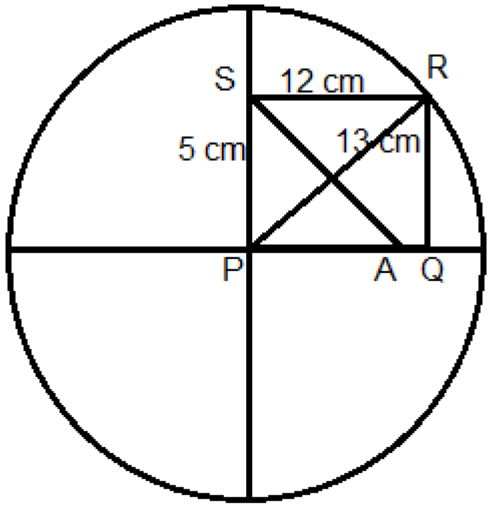
False. Since A is any point on PQ, then ar (PAS) ≠ 30 cm^2

But, the statement can be true if PA is equal to PS.

Justification:

According to the question,

PQRS is a rectangle inscribed in a quadrant of a circle of radius 13 cm.



Given,
A is any point on PQ
PA<PQ
ar (△PQR) = ½ ×PQ×QR
= ½ ×12×5
= $30cm^2$
PS=5 cm
Suppose PA<PQ,
ar(△PAS) < ar(△PQR)
ar(△PAS) < 30 cm^2
Suppose PA=PQ,
ar (△PAS) = ½ ×PQ×PS
= ½ ×12×5
= $30cm^2$

EXERCISE 9.3

1. In Fig.9.11, PSDA is a parallelogram. Points Q and R are taken on PS such that PQ = QR = RS and PA || QB || RC. Prove that ar (PQE) = ar (CFD).

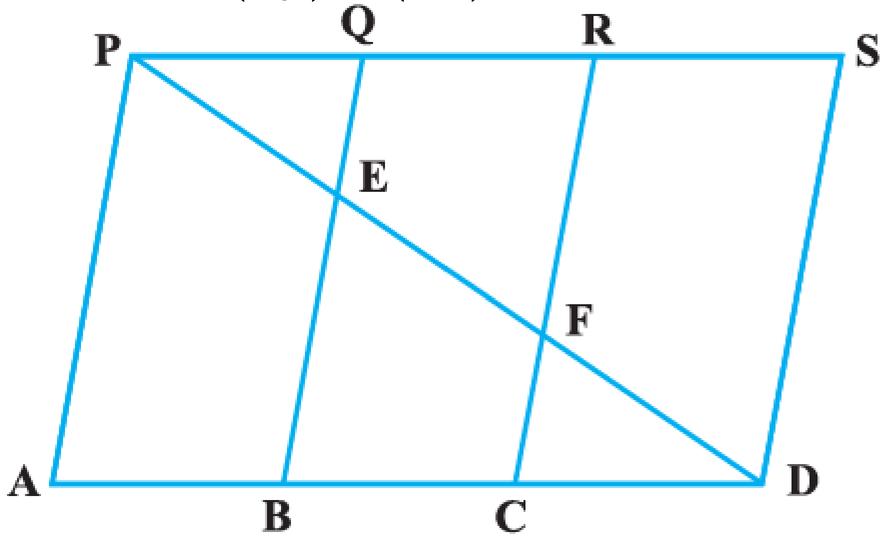


Fig. 9.11

Solution:

According to the question,

PA ∥ QB ∥ RC ∥ SD & PQ=QR=RS

According to the Equal intercept theorem,

We know that if three or more parallel lines make equal intercept on traversal, then they make equal intercept on any other form of traversal.

Hence, we get,

PE=EF=FD& AB=BC=CD

From ΔPQE & ΔDCF,

We get,

∠PEQ=∠DFC

PE=DF

∠QPE=∠CDF

So,

ΔPQE ≅ ΔDCF

Since Congruent figures have equal areas,

We get,

ar ΔPQE= ar ΔDCF

Hence proved.

2. X and Y are points on the side LN of the triangle LMN such that LX = XY = YN. Through X, a line is drawn parallel to LM to meet MN at Z (See Fig. 9.12). Prove that ar (LZY) = ar (MZYX)

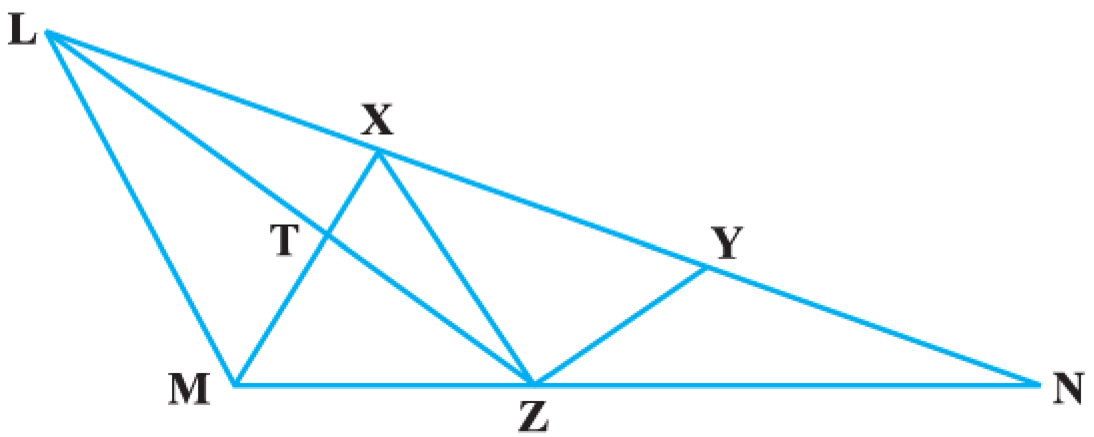


Fig. 9.12

Solution:

According to the question,

LX=XY=YN

XZ II LM

We have,

ar(LZX)+(XZY)=ar(LZY) --- (1)

ar(MXZ)+ar(XZY)=ar(MZYX) --- (2)

Both triangles LZX and MXZ are on the same base XZ and between same parallels LM and XZ

ar(LZX)=ar(MXZ)

Adding equation (1) and (2),

We get,

ar(LZY)=ar(MZYX)

Hence proved

3. The area of the parallelogram ABCD is 90 cm^2 (see Fig.9.13). Find

(i) ar (ABEF)

(ii) ar (ABD)

(iii) ar (BEF)

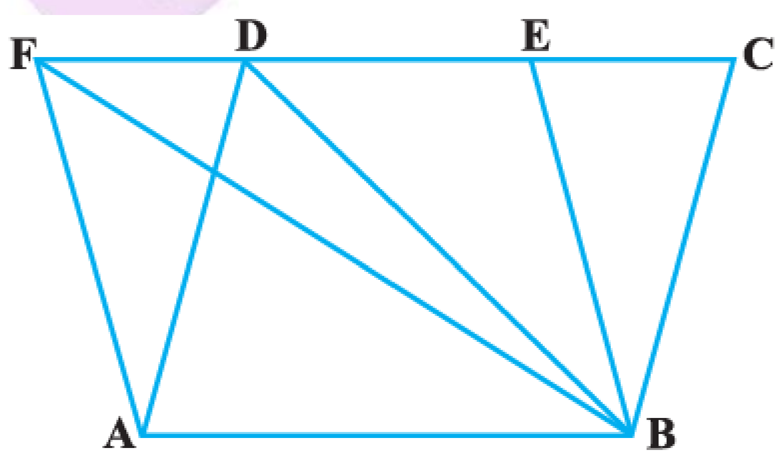


Fig. 9.13

Solution:

According to the question,
Area of parallelogram, ABCD = 90 cm^2
(i)
We know that,
Parallelograms on the same base and between the same parallel are equal in areas.
Here,
The parallelograms ABCD and ABEF are on same base AB and between the same parallels AB and CF.
Therefore, ar (ABEF) = ar (ABCD) = 90 cm^2
(ii)
We know that,
If a triangle and a parallelogram are on the same base and between the same parallels, then area of triangle is equal to half of the area of the parallelogram.
Here,
ΔABD and parallelogram ABCD are on the same base AB and between the same parallels AB and CD.
Therefore, ar (ΔABD) = ½ ar (ABCD)
= ½ x 90 = 45 cm^2
(iii)
We know that,
If a triangle and a parallelogram are on the same base and between the same parallels, then area of triangle is equal to half of the area of the parallelogram.
Here,
ΔBEF and parallelogram ABEF are on the same base EF and between the same parallels AB and EF.
Therefore, ar (ΔBEF) = ½ ar (ABEF)
= ½ x 90 = 45 cm^2

4. In △ ABC, D is the mid-point of AB and P is any point on BC. If CQ || PD meets AB in Q (Fig. 9.14), then prove that ar (BPQ) = ½ ar (ABC).

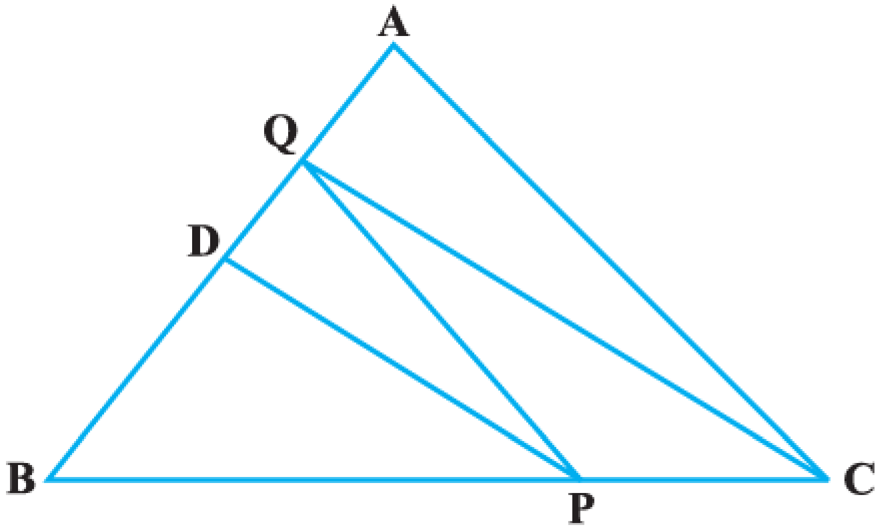
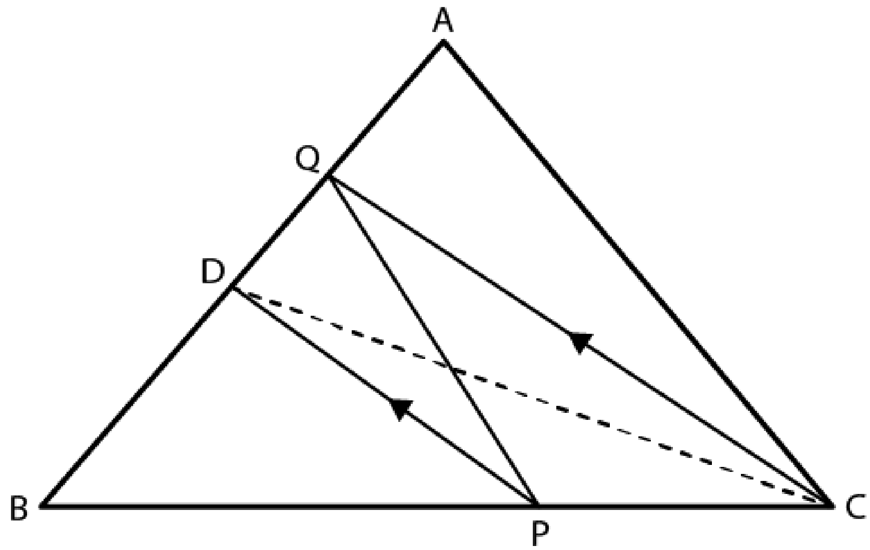


Fig. 9.14

Solution:

According to the question,
We have the figure,

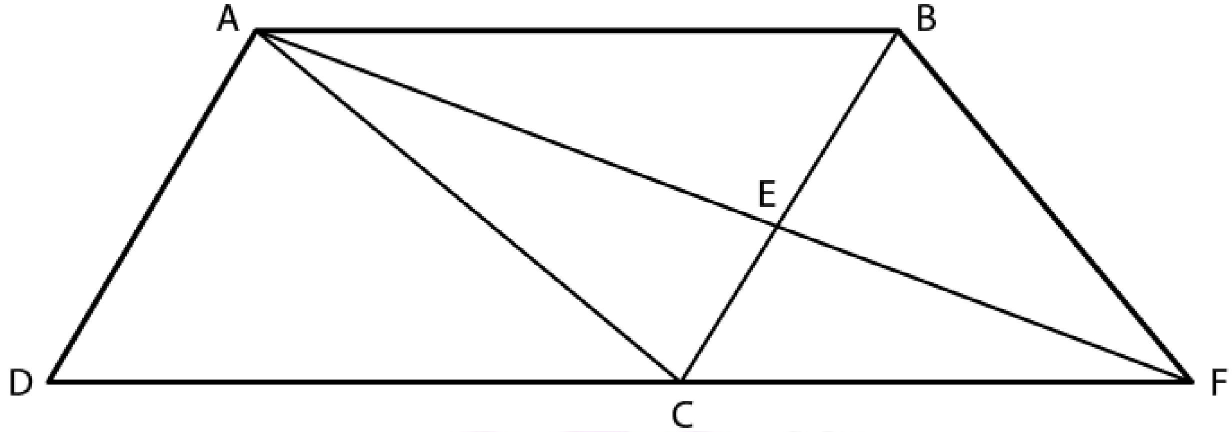


In △ ABC,
D is the mid-point of AB
P is any point on BC.
If CQ || PD meets AB at Q
To prove: ar (BPQ) = ½ ar (ABC)
Construction: Join DC
Proof:- Since D is the mid-point of AB. so, in △ ABC, CD is the median.
ar(△ BCD) = ½ ar (△ ABC) ..... (1)
We know that,
△ PDQ and △ PDC are on the same base PD and between the same parallels lines PD QC.
Therefore, ar(△ PDQ) = ar(△ PDC) .... (2)
From (1) and (2)
ar(△ BCD) = ½ ar(△ ABC)
ar(△ BCD = ar(△ BPD) + ar(△ PDC)
= ½ ar(△ ABC)
Area of △ PDC = PDQ
So,
ar(△ BPQ) = ar(△ BPD) + ar(△ PDQ)
= ½ ar(△ ABC)
Therefore, ar(△BPQ) = ½ ar(△ ABC)
Hence Proved

EXERCISE 9.4 PAGE NO: 94

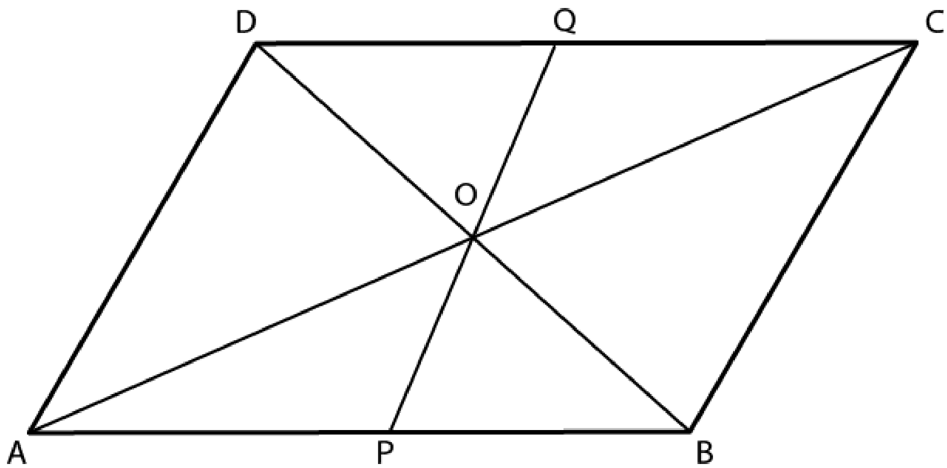
1. A point E is taken on the side BC of a parallelogram ABCD. AE and DC are produced to meet at F. Prove that ar (ADF) = ar (ABFC)
Solution:

According to the question,
ABCD is a parallelogram.
E is a point on BC.
AE and DC are produced to meet at F.



To prove: area (ΔADF) = area (ABFC).
Proof:
We know that,
Triangles on the same base and between the same parallels are equal in area.
Here,
We have,
ΔABC and ΔABF are on same base AB and between same parallels, AB || CF.
Area (ΔABC) = area (ΔABF) ... (1)
We also know that,
Diagonal of a Parallelogram divides it into two triangles of equal area
So,
Area (ΔABC) = area (ΔACD) ... (2)
Now,
Area (ΔADF) = area (ΔACD) + area (ΔACF)
∴ Area (ΔADF) = area (ΔABC) + area (ΔACF) … (From equation (2))
⇒ Area (ΔADF) = area (ΔABF) + area (ΔACF) … (From equation (1))
⇒ Area (ΔADF) = area (ABFC)

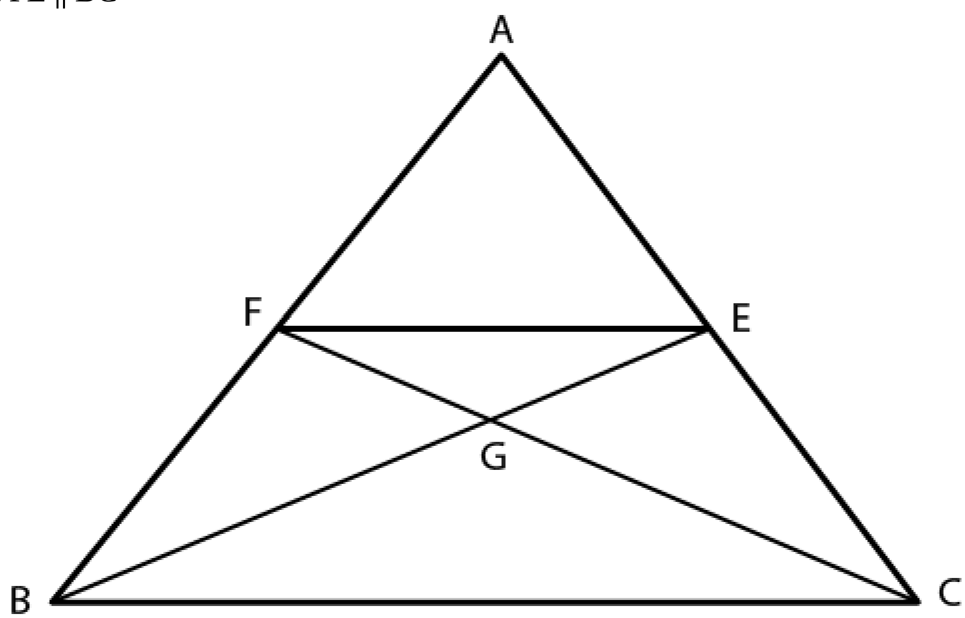
2. The diagonals of a parallelogram ABCD intersect at a point O. Through O, a line is drawn to intersect AD at P and BC at Q. Show that PQ divides the parallelogram into two parts of equal area.
Solution:



According to the question,
The diagonals of a parallelogram ABCD intersect at a point O.
Through O, a line is drawn to intersect AD at P and BC at Q.
To Prove:
Ar (parallelogram PDCQ) = ar (parallelogram PQBA).
Proof:
AC is a diagonal of || gm ABCD
∴ ar(ΔABC) = ar(ΔACD)
= ½ ar (||gm ABCD) …(1)
In ΔAOP and ΔCOQ,
AO = CO
Since, diagonals of a parallelogram bisect each other,
We get,
∠AOP = ∠COQ
∠OAP = ∠OCQ (Vertically opposite angles)
∴ ΔAOP = ΔCOQ (Alternate interior angles)
∴ ar(ΔAOP) = ar(ΔCOQ) (By ASA Congruence Rule)
We know that,
Congruent figures have equal areas
So,
ar(ΔAOP) + ar(parallelogram OPDC) = ar(ΔCOQ) + ar(parallelogram OPDC)
⇒ ar(ΔACD) = ar(parallelogram PDCQ)
⇒ ½ ar(|| gm ABCD) = ar(parallelogram PDCQ)
From equation (1),
We get,
ar(parallelogram PQBA) = ar(parallelogram PDCQ)
⇒ ar(parallelogram PDCQ) = ar(parallelogram PQBA).
Hence Proved.

3. The medians BE and CF of a triangle ABC intersect at G. Prove that the area of △ GBC = area of the quadrilateral AFGE.
Solution:
According to the question,

We have,
BE & CF are medians
E is the midpoint of AC
F is the midpoint of AB
∴ ΔBCE = ΔBEA … (i)
ΔBCF = ΔCAF
Construct:
Join EF,
By midpoint theorem,
We get FE || BC



We know that,
Δ on the same base and between same parallels are equal in area
∴ ΔFBC = ΔBCE
ΔFBC - ΔGBC = ΔBCE - ΔGBC
⇒ ΔFBG = ΔCGE (ΔGBC is common)
⇒ ΔCGE = ΔFBG …(ii)
Subtracting equation (ii) from (i)
We get,
ΔBCE - ΔCGE = ΔBEA - ΔFBG
∴ ΔBGC =quadrilateral AFGE.

4. In Fig. 9.24, CD || AE and CY || BA. Prove that ar (CBX) = ar (AXY).

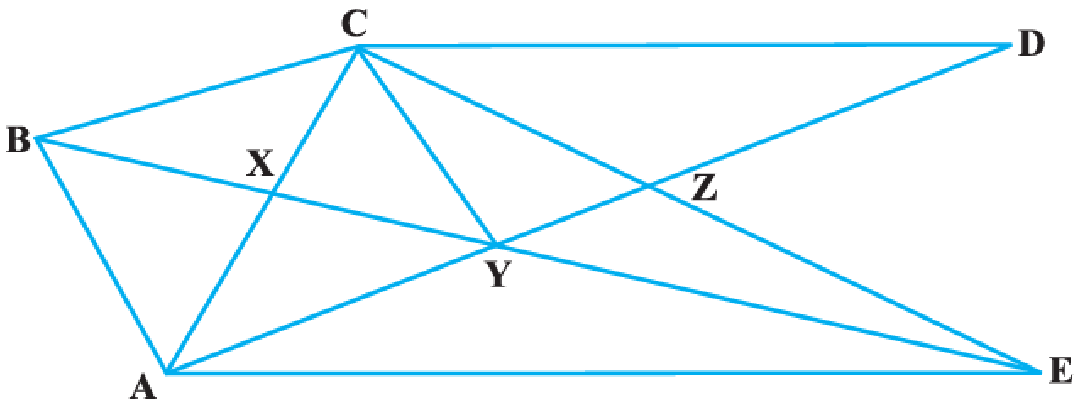
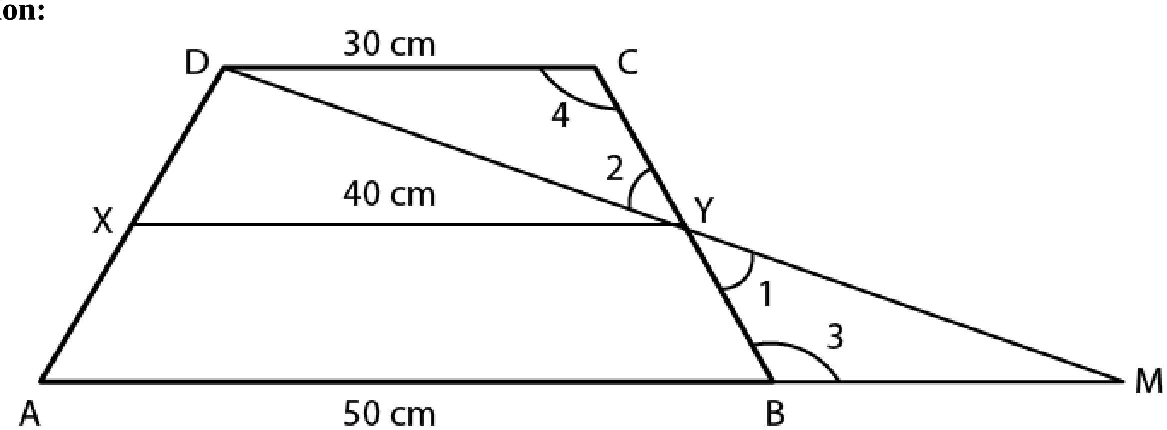


Fig. 9.24

Solution:

According to the question,
From figure,
We get,
CD||AE
and CY || BA
To prove:
ar (ΔCBX) = ar (ΔAXY) .
Proof:
We know that,
Triangles on the same base and between the same parallels are equal in areas.
Here,
ΔABY and ΔABC both lie on the same base AB and between the same parallels CY and BA.
ar (ΔABY) = ar (ΔABC)
⇒ ar (ABX) + ar (AXY) = ar (ABX) + ar (CBX)
Solving and cancelling ar (ABX),
We get,
⇒ ar (AXY) = ar (CBX)

5. ABCD is a trapezium in which AB || DC, DC = 30 cm and AB = 50 cm. If X and Y are, respectively the mid-points of AD and BC, prove that ar (DCYX) = 7/9 ar (XYBA)
Solution:



According to the question,
We have,
ABCD is a trapezium with AB || DC
Construction: Join DY and produce it to meet AB produced at P.
In ΔBYP and ΔCYD
∠BYP = ∠CYD (vertically opposite angles)
Since, alternate opposite angles of DC || AP and BC is the transversal
∠DCY = ∠PBY
Since, Y is the midpoint of BC,
BY = CY
Thus ΔBYP ≅ ΔCYD (by ASA cogence criterion)

So, DY = YP and DC = BP
⇒ Y is the midpoint of AD
∴ XY || AP and XY = ½ AP (by midpoint theorem)
⇒ XY = ½ AP
= ½ (AB + BP)
= ½ (AB + DC)
= ½ (50 + 30)
= ½ × 80 cm
= 40 cm

Since X is the midpoint of AD
And Y is the midpoint of BC
Hence, trapezium DCYX and ABYX are of same height, *h*, cm
Now

$$\frac{\text{area (DCYX)}}{\text{area (ABYX)}} = \frac{\frac{1}{2}(DC+XY)\times h}{\frac{1}{2}(AB \times XY)\ h} = \frac{30+40}{50+40} = \frac{70}{90} = \frac{7}{9}$$

⇒ 9 ar(DCXY) = 7 ar(XYBA)
⇒ ar(DCXY) = 7/9 ar(XYBA)
Hence Proved.