Marking Scheme (Sample Paper)

Section A

Q1. $-\frac{24}{25}$ (1)

Q2. We have 1, 2 $\in$ Z such that 1 * 2 = 8 and 2 * 1 = 2. This implies that 1 * 2 $\neq$ 2 * 1. Hence, * is not commutative. (1)

Q3. $\begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix}$ (1)

Q4. 4% (1)

Q5. 5 (1)

Q6. -14 (1)

Section B

Q7. Here, $fof : W \to W$ is such that, if n is odd, $fof(n) = f(f(n)) = f(n-1) = n-1+1 = n$

(1+1/2)

and if n is even, $fof(n) = f(f(n)) = f(n+1) = n+1-1 = n$ (1+1/2)

Hence, $fof = I$ This implies that f is invertible and $f^{-1} = f$ (1)

OR

Let $(a,b) \in N \times N$. Then $\because a^2 + b^2 = b^2 + a^2 \therefore (a,b)R(a,b)$ Hence, R is reflexive. (1)

Let $(a,b),(c,d) \in N \times N$ be such that $(a,b)R(c,d)$

$\Rightarrow a^2 + d^2 = b^2 + c^2$

$\Rightarrow c^2 + b^2 = d^2 + a^2$

$\Rightarrow (c,d)R(a,b)$

Hence, R is symmetric. (1)

Let $(a,b),(c,d),(e,f) \in N \times N$ be such that $(a,b)R(c,d),(c,d)R(e,f)$.

$\Rightarrow a^2 + d^2 = b^2 + c^2$ and $c^2 + f^2 = d^2 + e^2$

$\Rightarrow a^2 + d^2 + c^2 + f^2 = b^2 + c^2 + d^2 + e^2$

$\Rightarrow a^2 + f^2 = b^2 + e^2$

$\Rightarrow (a,b)R(e,f)$

Hence, R is transitive. (1+1/2)

Since R is reflexive, symmetric and transitive. Therefore R is an equivalence relation. (1/2)

Q8.

$$\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right) = \tan^{-1}\left(\frac{\sqrt{2\cos^2\frac{x}{2}}+\sqrt{2\sin^2\frac{x}{2}}}{\sqrt{2\cos^2\frac{x}{2}}-\sqrt{2\sin^2\frac{x}{2}}}\right) \quad (1)$$

$$= \tan^{-1}\left(\frac{-\sqrt{2}\cos\frac{x}{2}+\sqrt{2}\sin\frac{x}{2}}{-\sqrt{2}\cos\frac{x}{2}-\sqrt{2}\sin\frac{x}{2}}\right) \quad \left(\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4} \Rightarrow \cos\frac{x}{2} < 0, \sin\frac{x}{2} > 0\right) \quad (1+1/2)$$

$$= \tan^{-1}\left(\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right) = \tan^{-1}(\tan(\frac{\pi}{4}-\frac{x}{2})) \quad (1)$$

$$= \frac{\pi}{4}-\frac{x}{2} \quad \left(-\frac{\pi}{4} > \frac{\pi}{4}-\frac{x}{2} > -\frac{\pi}{2}\right) \quad (1/2)$$

Q9. $\text{adj}A = \begin{bmatrix} 4 & -3 \\ -1 & -2 \end{bmatrix}' = \begin{bmatrix} 4 & -1 \\ -3 & -2 \end{bmatrix}$ (2)

$$(\text{adj}A)A = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix} = -11\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ (1/2)

$$A(\text{adj}A) = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix} = -11\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ (1/2)

$$|A| = \begin{vmatrix} -2 & 1 \\ 3 & 4 \end{vmatrix} = -11$$ (1/2)

Hence, $A(\text{adj}A) = (\text{adj}A)A = |A|I$ verified. (1/2)

Q(10) $$\text{LHS} = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 3 & 4+3p & 2+4p+3q \\ 4 & 7+4p & 2+7p+4q \end{vmatrix} = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & -1+p \\ 0 & 3 & -2+3p \end{vmatrix} (R_2 \to R_2 - 3R_1, R_3 \to R_3 - 4R_1)$$

(2)

$$= \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & -1+p \\ 0 & 0 & 1 \end{vmatrix} (R_3 \to R_3 - 3R_2)$$ (1)

=1 =RHS. Hence, proved. (1)

OR

Let $$\Delta = \begin{vmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{vmatrix} = \begin{vmatrix} 0 & -2 & 3 \\ 2 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix}$$ (interchanging rows and columns) (1 + 1/2)

$$= (-1)(-1)(-1)\begin{vmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{vmatrix}$$ (1 +1/2))

$$= -\Delta$$ (1/2)

$$\Rightarrow 2\Delta = 0 \Rightarrow \Delta = 0$$ (1/2)

Q11. $AB=\begin{bmatrix}2&0\\0&2\end{bmatrix}=2I$ (1/2)

$$\Rightarrow A(\frac{1}{2}B)=I\Rightarrow A^{-1}=\frac{1}{2}B=\begin{bmatrix}2&-3\\-1&2\end{bmatrix}$$ (1)

The given system of equations is equivalent to $A'X=C$, where $X=\begin{bmatrix}x\\y\end{bmatrix}, C=\begin{bmatrix}4\\1\end{bmatrix}$ (1/2)

$$X=(A')^{-1}C=(A^{-1})'C$$ (1)

$$=\begin{bmatrix}2&-1\\-3&2\end{bmatrix}\begin{bmatrix}4\\1\end{bmatrix}=\begin{bmatrix}7\\-10\end{bmatrix}\Rightarrow x=7, y=-10$$ (1)

Q12. Since, f is differentiable at x = 2, therefore, f is continuous at x = 2. (1/2)

$$\Rightarrow \lim_{x\to 2^-} f(x)=\lim_{x\to 2^+} f(x)=f(2)\Rightarrow \lim_{x\to 2^-} x^2=\lim_{x\to 2^+}(ax+b)=4\Rightarrow 4=2a+b$$ (1+1/2)

Since, f is differentiable at x = 2,

$$\therefore Lf'(2)=Rf'(2)\Rightarrow \lim_{h\to 0}\frac{f(2-h)-f(2)}{-h}=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}\quad (h>0)$$
$$\Rightarrow \lim_{h\to 0}\frac{(2-h)^2-4}{-h}=\lim_{h\to 0}\frac{a(2+h)+b-4}{h}$$
$$\Rightarrow \lim_{h\to 0}(-h+4)=\lim_{h\to 0}\frac{4+ah-4}{h}\Rightarrow 4=a$$

(1+1/2)

b = -4 (1/2)

Q13.

$$\text{Let } u=(\log x)^x.\text{Then } \log u=x\log(\log x)\Rightarrow \frac{1}{u}\frac{du}{dx}=\frac{1}{\log x}+\log(\log x)$$
$$\Rightarrow \frac{du}{dx}=(\log x)^x\left[\frac{1}{\log x}+\log(\log x)\right]$$

(1+1/2)

Let $v = x^{x\cos x}$. Then $\log v = x\cos x\log x \Rightarrow \frac{1}{v}\frac{dv}{dx} = \frac{x\cos x}{x} + \cos x(\log x) - x\sin x\log x$

$\Rightarrow \frac{dv}{dx} = x^{x\cos x}\left[\cos x + \cos x(\log x) - x\sin x\log x\right]$

(1+1/2)

$y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right] +$

$x^{x\cos x}\left[\cos x + \cos x(\log x) - x\sin x\log x\right]$

(1)

OR

$\frac{dx}{dt} = ap\cos pt, \frac{dy}{dt} = -bp\sin pt, \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{b}{a}\tan pt$ (1+1/2)

$\frac{d^2y}{dx^2} = -\frac{b}{a}p\sec^2 pt \times \frac{dt}{dx}$ (1+1/2)

$\frac{d^2y}{dx^2} = -\frac{b}{a^2\cos^3 pt}$ (1/2)

$\Rightarrow (\frac{d^2y}{dx^2})_{t=0} = -\frac{b}{a^2}$ (1/2)

Q14. Given integral =

$$\int \frac{1}{\sin x - \sin 2x}dx = \int \frac{1}{\sin x(1-2\cos x)}dx = \int \frac{\sin x}{(1+\cos x)(1-\cos x)(1-2\cos x)}dx$$

$= -\int \frac{dt}{(1+t)(1-t)(1-2t)}$ $(\cos x = t \Rightarrow -\sin x dx = dt)$ (1)

$$\frac{1}{(1+t)(1-t)(1-2t)}=\frac{A}{1+t}+\frac{B}{1-t}+\frac{C}{1-2t}$$
$$\therefore 1=A(1-t)(1-2t)+B(1+t)(1-2t)+C(1-t^2)\ \text{(An identity)}$$

Putting, t= 1, ½, -1, we get A = 1/6, B= -1/2, C = 4/3 (1+1/2)

Therefore, the given integral

$$=-\frac{1}{6}\log|1+t|-\frac{1}{2}\log|1-t|+\frac{4}{6}\log|1-2t|+c$$
$$=-\frac{1}{6}\log|1+\cos x|-\frac{1}{2}\log|1-\cos x|+\frac{2}{3}\log|1-2\cos x|+c$$

(1+1/2)

OR

$$\int\frac{\sin\phi}{\sqrt{\sin^2\phi+2\cos\phi+3}}d\phi=\int\frac{\sin\phi}{\sqrt{1-\cos^2\phi+2\cos\phi+3}}d\phi$$ (1/2)

$$=\int\frac{\sin\phi}{\sqrt{-\cos^2\phi+2\cos\phi+4}}d\phi=\int\frac{-1}{\sqrt{-t^2+2t+4}}dt\ (\cos\phi=t\Rightarrow-\sin\phi d\phi=dt)$$ (1)

$$=-\int\frac{1}{\sqrt{(\sqrt{5})^2-(t-1)^2}}dt$$ (1+1/2)

$$=-\sin^{-1}\frac{t-1}{\sqrt{5}}+c=-\sin^{-1}\frac{\cos\phi-1}{\sqrt{5}}+c$$ (1)

Q15. Let

$$I=\int_{-\pi}^{\pi}\frac{2x(1+\sin x)}{1+\cos^2 x}dx=\int_{-\pi}^{\pi}\frac{2x}{1+\cos^2 x}dx+\int_{-\pi}^{\pi}\frac{2x\sin x}{1+\cos^2 x}dx$$
$$=0+2\int_{0}^{\pi}\frac{2x\sin x}{1+\cos^2 x}dx$$

(as $\frac{2x}{1+\cos^2 x}$ is odd and

$\frac{2x\sin x}{1+\cos^2 x}$ is even) (1)

$$= 4\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx \, .$$

Let

$$I_1 = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}dx$$

$$I_1 = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}dx \quad (1)$$

$$\text{Adding, } 2I_1 = \pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$$

$$= -\pi\int_1^{-1} \frac{dt}{1+t^2} \ (\cos x = t \Rightarrow -\sin xdx = dt) \quad (1)$$

$$= \pi\left[\tan^{-1} t\right]_{-1}^{1} \quad (1/2)$$

$$I_1 = \frac{\pi^2}{4}. \text{ Hence, } I = \pi^2 \quad (1/2)$$

Q16. Given differential equation is

$$\frac{dy}{dx} = \frac{y}{x} + \frac{\sqrt{x^2+y^2}}{x}, x > 0 \text{ or, } \frac{dy}{dx} = \frac{y}{x} + \sqrt{1+(\frac{y}{x})^2} = f(\frac{y}{x}), \text{ hence, homogeneous.} \quad (1/2)$$

Put y = v x $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$. The differential equation becomes $v + x\frac{dv}{dx} = v + \sqrt{1+v^2}$ (1)

or, $\frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$ (1/2).

Integrating, we get $\log\left|v+\sqrt{1+v^2}\right| = \log|x| + \log k$ (1)

$$\Rightarrow \log\left|v+\sqrt{1+v^2}\right| = \log|x|k \Rightarrow \left|v+\sqrt{1+v^2}\right| = |x|k$$
$$\Rightarrow v+\sqrt{1+v^2} = \pm kx \Rightarrow \frac{y}{x} + \sqrt{1+(\frac{y}{x})^2} = cx$$
$$\Rightarrow y+\sqrt{x^2+y^2} = cx^2,$$

which gives the general solution. (1)

Q17. We have the following differential equation: $\frac{dx}{dy} = \frac{(\tan^{-1}y - x)}{1+y^2}$ 0r, $\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$,

which is linear in x (1/2)

$$\text{I.F.} = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$$ (1)

Multiplying both sides by I. F. and integrating we get $xe^{\tan^{-1}y} = \int e^{\tan^{-1}y}\frac{\tan^{-1}y}{1+y^2}dy$ (1/2)

$$\Rightarrow xe^{\tan^{-1}y} = \int e^t t\,dt \quad (\tan^{-1}y = t \Rightarrow \frac{1}{1+y^2}dy = dt)$$
$$\Rightarrow xe^{\tan^{-1}y} = te^t - e^t + c \Rightarrow xe^{\tan^{-1}y} = \tan^{-1}y\,e^{\tan^{-1}y} - e^{\tan^{-1}y} + c$$

which gives the general solution of the differential equation. (2)

Q18. The vector equations of the given lines are

$$\vec{r} = \hat{i}+2\hat{j}-\hat{k}+\lambda(2\hat{i}-3\hat{j}+4\hat{k}), \vec{r} = -2\hat{i}+3\hat{j}+\mu(-\hat{i}+2\hat{j}+3\hat{k})$$
$$\vec{a}_1 = \hat{i}+2\hat{j}-\hat{k}, \vec{b}_1 = 2\hat{i}-3\hat{j}+4\hat{k}, \vec{a}_2 = -2\hat{i}+3\hat{j}, \vec{b}_2 = -\hat{i}+2\hat{j}+3\hat{k}$$

(1)

$$\vec{a}_2-\vec{a}_1=-3\hat{i}+\hat{j}+\hat{k}, \vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -3 & 4\\ -1 & 2 & 3\end{vmatrix}=-17\hat{i}-10\hat{j}+\hat{k}$$ (2)

The required shortest distance $=\dfrac{\left|(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$ (1/2)

$=\dfrac{42}{\sqrt{390}}$ units (1/2)

Q19. Let us define the following events: E = A solves the problem, F = B solves the problem, G = C solves the problem, H = D solves the problem (1/2)

(i) The required probability = $P(E\cup F\cup G\cup H)$ (1/2)

$= 1 - P(\bar{E}\cap\bar{F}\cap\bar{G}\cap\bar{H})$

$= 1 - P(\bar{E})\times P(\bar{F})\times P(\bar{G})\times P(\bar{H})$ (1)

$= 1 - \dfrac{2}{3}\times\dfrac{3}{4}\times\dfrac{4}{5}\times\dfrac{1}{3}=\dfrac{13}{15}$ (1/2)

(ii)The required probability =

$$P(\bar{E})\times P(\bar{F})\times P(\bar{G})\times P(\bar{H})+P(E)\times P(\bar{F})\times P(\bar{G})\times P(\bar{H})+P(\bar{E})\times P(F)\times P(\bar{G})\times P(\bar{H})$$
$$+P(\bar{E})\times P(\bar{F})\times P(G)\times P(\bar{H})+P(\bar{E})\times P(\bar{F})\times P(\bar{G})\times P(H)$$ (1)

$$=\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{1}{3}+\frac{1}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{1}{3}+\frac{2}{3}\times\frac{1}{4}\times\frac{4}{5}\times\frac{1}{3}$$
$$+\frac{2}{3}\times\frac{3}{4}\times\frac{1}{5}\times\frac{1}{3}+\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{2}{3}=\frac{5}{18}$$ (1/2)

Section C

Q20. $f'(x)=(x-1)^2(x+2)(5x+4)$ (1/2)

$f'(x)=0 \Rightarrow x=1,-2,\frac{-4}{5}$ (1/2)

In the interval	Sign of f′(x)	Nature of the function
$(-\infty,-2)$	(+ve)(-ve)(-ve)= +ve	f is strictly increasing in $(-\infty,-2]$
$(-2,-\frac{4}{5})$	(+ve)(+ve)(-ve)= -ve	f is strictly decreasing in $\left[-2,-\frac{4}{5}\right]$
$(-\frac{4}{5},1)$	(+ve)(+ve)(+ve)= +ve	f is strictly increasing in $\left[-\frac{4}{5},1\right]$
$(1,\infty)$	(+ve)(+ve)(+ve)= +ve	f is strictly increasing in $[1,\infty)$

(2+1/2)

Hence, f is strictly increasing in $(-\infty,-2]$ and $\left[-\frac{4}{5},\infty\right)$. f is strictly decreasing in $\left[-2,-\frac{4}{5}\right]$

(1/2)

In the left nhd of -2, f′(x)>0, in the right nhd of -2, f′(x)<0 and f′(-2) = 0, therefore, by the first derivative test, -2 is a point of local maximum. (1)

In the left nhd of -4/5, f′(x)<0, in the right nhd of -4/5, f′(x)>0 and f′(-4/5) = 0, therefore, by the first derivative test, -4/5 is a point of local minimum. (1)

Q21. We have

$\vec{a}.\vec{b}=0, \vec{a}.\vec{c}=0 \Rightarrow \vec{a} \perp$ both $\vec{b}$ and $\vec{c}$ (as $\vec{a},\vec{b},\vec{c}$ are nonzero vectors)
$\Rightarrow \vec{a} \parallel \vec{b}\times\vec{c}$ (1)

Let $\vec{a} = \lambda(\vec{b} \times \vec{c})$ (1)

Then

$$|\vec{a}| = |\lambda||(\vec{b} \times \vec{c})| \Rightarrow \frac{|\vec{a}|}{|(\vec{b} \times \vec{c})|} = |\lambda| \Rightarrow |\lambda| = \frac{1}{\sin\frac{\pi}{6}} = 2 \Rightarrow \lambda = \pm 2$$

$\therefore \vec{a} = \pm 2(\vec{b} \times \vec{c})$

(2)

Now $\left[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}\right] = \left\{\left(\vec{a}+\vec{b}\right) \times \left(\vec{b}+\vec{c}\right)\right\}.\left(\vec{c}+\vec{a}\right) = \left(\vec{a} \times \vec{b}\right).\vec{c} + \left(\vec{b} \times \vec{c}\right).\vec{a}$ (As the scalar triple product = 0 if any two vectors are equal.)

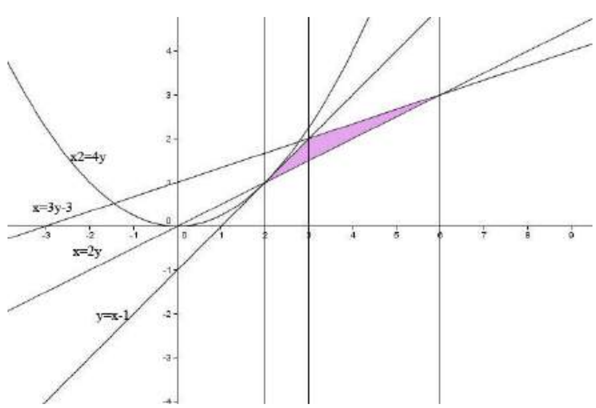
$\vec{a}.(\vec{b} \times \vec{c}) + (\vec{a} \times \vec{b}).\vec{c} = \vec{a}.(\vec{b} \times \vec{c}) + \vec{a}.(\vec{b} \times \vec{c}) = 2\vec{a}.(\vec{b} \times \vec{c})$ (1+1/2)

$= 2\vec{a}.(\pm\frac{1}{2}\vec{a}) = \pm 1$ (1/2)

Q22. We have the curve

$4y = x^2 \Rightarrow 4\frac{dy}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{x}{2} \Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = 1$ (1)

The equation of the tangent is $y = x - 1$ (1)



Graph sketch (1)

The required area = the shaded area =

$$\int_2^3\left[(x-1)-\frac{x}{2}\right]dx+\int_3^6\left[\frac{(x+3)}{3}-\frac{x}{2}\right]dx=\int_2^3(x-1)dx+\frac{1}{3}\int_3^6(x+3)dx-\frac{1}{2}\int_2^6 xdx \qquad (1)$$

$$=\left[\frac{x^2}{2}-x\right]_2^3+\frac{1}{3}\left[\frac{x^2}{2}+3x\right]_3^6-\frac{1}{4}\left[x^2\right]_2^6 \qquad (1+1/2)$$

= 1 square units (1/2)

Q23. The equation of the line passing through the point(3, -2, 1) and parallel to the given line is

$$\frac{x-3}{2}=\frac{y+2}{-3}=\frac{z-1}{1} \qquad (1)$$

Any point on this line is $(2\lambda+3,-3\lambda-2,\lambda+1)$ (1/2)

If it lies on the plane, we have $3(2\lambda+3)-3\lambda-2-\lambda-1+2=0\Rightarrow\lambda=-4$ (1)

Hence, the point common to the plane and the line is (-5, 10, -3). (1/2)

Hence, the required distance $=\sqrt{(3+5)^2+(-2-10)^2+(1+3)^2}$ units $=4\sqrt{14}$ units (1)

The equation of the line passing through (3, -2, 1) and perpendicular to the plane is

$$\frac{x-3}{3}=\frac{y+2}{1}=\frac{z-1}{-1} \qquad (1/2)$$

Any point on it is $(3\mu+3,\mu-2,-\mu+1)$ (1/2)

If it lies on the plane, we get $3(3\mu+3)+\mu-2+\mu-1+2=0\Rightarrow\mu=\frac{-8}{7}$ (1/2)

The required foot of the perpendicular = $(\frac{-3}{7},\frac{-22}{7},\frac{15}{7})$ (1/2)

OR

Any plane through the line of intersection of the given planes is

$$\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})+1+\lambda(\ \vec{r}.(\hat{i}+\hat{j}-2\hat{k}))=0$$
$$\text{or}, \vec{r}.((2+\lambda)\hat{i}+(3+\lambda)\hat{j}+(-1-2\lambda)\hat{k})=-1$$

(2)

If it contains the point (3, -2, -1), we have

$$(3)(2+\lambda)+(-2)(3+\lambda)+(-1)(-1-2\lambda)=-1 \Rightarrow \lambda=\frac{-2}{3}$$

(1)

The required equation of the plane is

$$\vec{r}.((2-\frac{2}{3})\hat{i}+(3-\frac{2}{3})\hat{j}+(-1+\frac{4}{3})\hat{k})=-1 \text{or}, \vec{r}.(4\hat{i}+7\hat{j}+\hat{k})=-3 \quad (1)$$

If θ be the angle between the normals to the two given planes, then θ is the angle between the planes and $\cos\theta=\frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}=\frac{2+3+2}{\sqrt{14}\sqrt{6}}=\frac{7}{2\sqrt{21}}$ (2)

Q24. Let us define the following events: E_1 = Two white balls are transferred, E_2 = Two red balls are transferred, E_3 = One red and one white balls are transferred, A = The ball drawn from the Bag II is red (1/2)

$$P(E_1)=\frac{{}^4C_2}{{}^9C_2}=\frac{4\times 3}{9\times 8} \quad (1)$$

$$P(E_2)=\frac{{}^5C_2}{{}^9C_2}=\frac{4\times 5}{9\times 8} \quad (1)$$

$$P(E_3)=\frac{{}^5C_1\times{}^4C_1}{{}^9C_2}=\frac{4\times 5\times 2}{9\times 8} \quad (1)$$

$P(A/E_1)=\frac{3}{8}$, $P(A/E_2)=\frac{5}{8}$, $P(A/E_3)=\frac{4}{8}$ $(\frac{1}{2}+\frac{1}{2}+\frac{1}{2})$

The required probability, $P(E_3/A)$, by Baye's Theorem,

$$=\frac{P(E_3)\times P(A/E_3)}{P(E_1)\times P(A/E_1)+P(E_2)\times P(A/E_2)+P(E_3)\times P(A/E_3)}$$ (1/2)

= 20/37 (1/2)

OR

Let X represent the random variable. Then X = 0, 1, 2, 3 (1/2)

$P(X=0)=P(r=0)={}^3C_0(\frac{1}{6})^0(\frac{5}{6})^3=\frac{125}{216}$ (1/2)

$P(X=1)=P(r=1)={}^3C_1(\frac{1}{6})^1(\frac{5}{6})^2=\frac{75}{216}$ (1/2)

$P(X=2)=P(r=2)={}^3C_2(\frac{1}{6})^2(\frac{5}{6})^1=\frac{15}{216}$ (1/2)

$P(x=3)=p(r=3)={}^3C_3(\frac{1}{6})^3(\frac{5}{6})^0=\frac{1}{216}$ (1/2)

x_i	p_i	x_ip_i	$(x_i)^2p_i$
0	125/216	0	0
1	75/216	75/216	75/216
2	15/216	30/216	60/216
3	1/216	3/216	9/216
Total		1/2	2/3

(2)

Mean = $\sum x_i p_i = \frac{1}{2}$, $\text{var}(X) = \sum x_i{}^2 p_i - (\sum x_i p_i)^2 = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$ (1)

Standard deviation $= \sqrt{\text{var}(X)} = \frac{\sqrt{15}}{6}$ (1/2)

Q25. Let the radius of the circular garden be r m and the side of the square garden be x m. Then

$600 = 2\pi r + 4x \Rightarrow x = \frac{600 - 2\pi r}{4}$ (1)

The sum of the areas = A = $\pi r^2 + x^2 \Rightarrow A = \pi r^2 + \left(\frac{600 - 2\pi r}{4}\right)^2$ (1)

$\frac{dA}{dr} = 2\pi r + \frac{2}{16}(600 - 2\pi r)(-2\pi) = \frac{\pi}{2}(4r - 300 + \pi r), \frac{dA}{dr} = 0 \Rightarrow r = \frac{300}{\pi + 4}$ (1)

$\frac{d^2A}{dr^2} = \frac{\pi}{2}(4 + \pi), (\frac{d^2A}{dr^2})_{r=\frac{300}{\pi+4}} > 0$ (1)

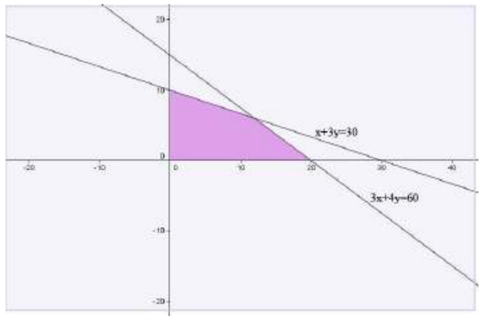
Therefore, A is minimum when $r = \frac{300}{\pi + 4}$ For this value of r, $x = 2r$ (1)

To achieve any goal, there is every possibility that energy, time and money are required to be invested. One must plan in such a manner that least energy, time and money are spent. A good planning and execution, therefore, is essentially required. (1)

Q26. Let the number of pieces of model A to be manufactured be = x and the number of pieces of model B to be manufactured be = y. (1/2)

Then to maximize the profit, P = Rs (8000x+12000y) (1/2)

subject to the constraints $9x + 12y \leq 180$, or, $3x + 4y \leq 60, x + 3y \leq 30, x \geq 0, y \geq 0$ (2)



Graph work (on the actual graph paper) (1+1/2)

At	Profit
(0,0)	Rs 0
(20,0)	Rs 160000
(12,6)	Rs 168000 (maximum)
(0.10)	Rs 120000

(1)

The number of pieces of model A =12, the number of pieces of model B =6 and the maximum profit = Rs 168000. (1/2)