SAMPLE PAPER: MATHEMATICS

CLASS–XII: 2014–15

TYPOLOGY

	VSA (1 M)	LA–I (4 M)	LA–II (6 M)	100
Remembering	2, 5	11, 15, 19	24	20
Understanding	1, 4	8, 12	23	16
Applications	6	14, 18, 13	21, 26	25
HOTS	3	10, 17	20, 22	21
Evaluation & MD	–	7, 9, 16	25	18

SECTION–A

Question number 1 to 6 carry 1 mark each.

1. The position vectors of points A and B are $\vec{a}$ and $\vec{b}$ respectively.

   P divides AB in the ratio 3 : 1 and Q is mid-point of AP. Find the position vector of Q. 1

2. Find the area of the parallelogram, whose diagonals are $\vec{d}_1=5\hat{\imath}$ and $\vec{d}_2=2\hat{\jmath}$ 1

3. If P(2, 3, 4) is the foot of perpendicular from origin to a plane, then write the vector equation of this plane. 1

4. If $\Delta = \begin{vmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{vmatrix}$, Write the cofactor of a_{32} (the element of third row and 2nd column). 1

5. If m and n are the order and degree, respectively of the differential equation $y\left(\frac{dy}{dx}\right)^3 + x^3\left(\frac{d^2y}{dx^2}\right)^2 - xy = \sin x$, then write the value of m+n. 1

6. Write the differential equation representing the curve $y^2 = 4ax$, where a is an arbitrary constant. 1

SECTION-B

Question numbers 7 to 19 carry 4 marks each.

7. To raise money for an orphanage, students of three schools A, B and C organized an exhibition in their locality, where they sold paper bags, scrap-books and pastel sheets made by them using recycled paper, at the rate of Rs. 20, Rs.15 and Rs. 5 per unit respectively. School A sold 25 paper-bags 12 scrap-books and 34 pastel sheets. School B sold 22 paper-bags, 15 scrapbooks and 28 pastel-sheets while school C sold 26 paper-bags, 18 scrap-books and 36 pastel sheets. Using matrices, find the total amount raised by each school.

   By such exhibition, which values are inculcated in the students? 4

8. Let $A = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$, then show that $A^2 - 4A + 7I = O$.

Using this result calculate A^3 also.

OR

If $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{pmatrix}$, find A^{-1} , using elementary row operations. 4

9. If x, y, z are in GP, then using properties of determinants, show that

$\begin{vmatrix} px+y & x & y \\ py+z & y & z \\ 0 & px+y & py+z \end{vmatrix}$ = o, where $x \neq$ y $\neq$ z and p is any real number. 4

10. Evaluate : $\int_{-1}^{1} |x \cos\pi\, x| \mathrm{d}x$. 4

11. Evaluate : $\int \frac{1+\sin 2x}{1+\cos 2x}$. e^{2x} dx. 4

OR

Evaluate : $\int \frac{x^4}{(x-1)(x^2+1)}$ dx

12. Consider the experiment of tossing a coin. If the coin shows tail, toss it again but if it shows head, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 3' given that 'there is at least one head'. 4

OR

How many times must a man toss a fair coin so that the probability of having at least one head is more than 90%?

13. For three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ if $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \times \vec{c} = \vec{b}$, then prove that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors, $|\vec{b}| = |\vec{a}|$ and $|\vec{a}| = 1$ 4

14. Find the equation of the line through the point (1,-1,1) and perpendicular to the lines joining the points (4,3,2), (1,-1,0) and (1,2,-1), (2,1,1) 4

OR

Find the position vector of the foot of perpendicular drawn from the point P(1,8,4) to the line joining A(O,-1,3) and B(5,4,4). Also find the length of this perpendicular.

15. Solve for x: $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}x = -\frac{\pi}{2}$

OR

Prove that: $2 \sin^{-1} \frac{3}{5} - \tan^{-1} \frac{17}{31} = \frac{\pi}{4}$ 4

16. If $x = \sin t$, $y = \sin kt$, show that

$(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + k^2y = 0$ 4

17. If $y^x + x^y + x^x = a^b$, find $\frac{dy}{dx}$ 4

18. It is given that for the function $f(x) = x^3 + bx^2 + ax + 5$ on [1, 3], Rolle's theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$.

Find values of a and b. 4

19. Evaluate : $\int \frac{3x+1}{\sqrt{5-2x-x^2}} dx$ 4

SECTION-C

Question numbers 20 to 26 carry 6 marks each.

20. Let A = {1, 2, 3, ... , 9} and R be the relation in A x A defined by (a, b) R (c, d) if a+d = b+c for a, b, c, d ∈ A.

Prove that R is an equivalence relation. Also obtain the equivalence class [(2, 5)]. 6

OR

Let $f : N \longrightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$.

Show that $f : N \longrightarrow S$ is invertible, where S is the range of f. Hence find inverse of f.

21. Compute, using integration, the area bounded by the lines

$x+2y = 2$, $y-x=1$ and $2x+y= 7$ 6

22. Find the particular solution of the differential equation 6

$xe^{\frac{y}{x}} - y\sin\left(\frac{y}{x}\right) + x\frac{dy}{dx}\, sin\left(\frac{y}{x}\right) = o$, given that

y = 0, when x = 1

OR

Obtain the differential equation of all circles of radius r.

23. Show that the lines $\vec{r} = (-3\hat{\imath} + \hat{\jmath} + 5\hat{k}) + \lambda\ (-3\hat{\imath} + \hat{\jmath} + 5\hat{k})$ and $\vec{r} = \left(-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}\right) + \mu\left(-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}\right)$ are coplanar. Also, find the equation of the plane containing these lines. 6

24. 40% students of a college reside in hostel and the remaining reside outside. At the end of year, 50% of the hosteliers got A grade while from outside students, only 30% got A grade in the examination. At the end of year, a student of the college was chosen at random and was found to get A grade. What is the probability that the selected student was a hostelier? 6

25. A man rides his motorcycle at the speed of 50km/h. He has to spend Rs. 2 per km on petrol. If he rides it at a faster speed of 80km/h, the petrol cost increases to Rs. 3 per km. He has atmost Rs. 120 to spend on petrol and one hour's time. Using LPP find the maximum distance he can travel. 6

26. A jet of enemy is flying along the curve y = x^2+2 and a soldier is placed at the point (3, 2). Find the minimum distance between the soldier and the jet. 6