SAMPLE QUESTION PAPER
CLASS-XII (2016-17)
MATHEMATICS (041)

Time allowed: 3 hours
Maximum Marks: 100

General Instructions:

(i) All questions are compulsory.
(ii) This question paper contains 29 questions.
(iii) Question 1-4 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
(v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
(vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

SECTION A
Questions from 1 to 4 are of 1 mark each.

1. What is the principal value of \( \tan^{-1} \left( \tan \frac{2\pi}{3} \right) \)?

2. A and B are square matrices of order 3 each, \(|A| = 2\) and \(|B| = 3\). Find \(|3AB|\)

3. What is the distance of the point \((p, q, r)\) from the x-axis?

4. Let \( f : \mathbb{R} \to \mathbb{R} \) be defined by \( f(x) = 3x^2 - 5 \) and \( g : \mathbb{R} \to \mathbb{R} \) be defined by \( g(x) = \frac{-x}{x^2 + 1} \). Find \( g \circ f \)

SECTION B
Questions from 5 to 12 are of 2 marks each.

5. How many equivalence relations on the set \( \{1,2,3\} \) containing \((1,2)\) and \((2,1)\) are there in all? Justify your answer.

6. Let \( l_i, m_i, n_i \); \( i = 1, 2, 3 \) be the direction cosines of three mutually perpendicular vectors in space. Show that \( AA' = I_3 \), where \( A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \).

7. If \( e^x (x + 1) = 1 \), show that \( \frac{dy}{dx} = -e^y \)

8. Find the sum of the order and the degree of the following differential equations:
   \[ \frac{d^2y}{dx^2} + 3 \sqrt{\frac{dy}{dx}} + (1 + x) = 0 \]
9. Find the Cartesian and Vector equations of the line which passes through the point \((-2, 4, 5)\) and parallel to the line given by \(\frac{x+3}{3} = \frac{y-4}{5} = \frac{8-z}{-6}\).

10. Solve the following Linear Programming Problem graphically:
    Maximize \(Z = 3x + 4y\)
    subject to
    \(x + y \leq 4, x \geq 0\) and \(y \geq 0\).

11. A couple has 2 children. Find the probability that both are boys, if it is known that (i) one of them is a boy (ii) the older child is a boy.

12. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which its area increases, when side is 10 cm long.

**SECTION-C**

Questions from 13 to 23 are of 4 marks each.

13. If \(A + B + C = \pi\), then find the value of
    \[
    \begin{vmatrix}
    \sin(A + B + C) & \sin B & \cos C \\
    -\sin B & 0 & \tan A \\
    \cos(A + B) & -\tan A & 0
    \end{vmatrix}
    
    OR
    Using properties of determinant, prove that
    \[
    \begin{vmatrix}
    \frac{b + c}{b} & \frac{a - b}{a} & \frac{a}{a} \\
    \frac{c + a}{c} & \frac{b - c}{b} & \frac{b}{b} \\
    \frac{a + b}{a} & \frac{c - a}{c} & \frac{c}{c}
    \end{vmatrix} = 3abc - a^3 - b^3 - c^3
    
14. It is given that for the function \(f(x) = x^3 - 6x^2 + ax + b\) Rolle’s theorem holds in \([1, 3]\) with \(c = 2 + \frac{1}{\sqrt{3}}\). Find the values of ‘a’ and ‘b’

15. Determine for what values of \(x\), the function \(f(x) = x^3 + \frac{1}{x}\) \((x \neq 0)\) is strictly increasing or strictly decreasing

    OR
    Find the point on the curve \(y = x^3 - 11x + 5\) at which the tangent is \(y = x - 11\)

16. Evaluate \(\int_0^2 (x^2 + 3) \, dx\) as limit of sums.

17. Find the area of the region bounded by the y-axis, \(y = \cos x\) and \(y = \sin x\), \(0 \leq x \leq \frac{\pi}{2}\).

18. Can \(y = ax + \frac{b}{a}\) be a solution of the following differential equation?
    \[
    y = x \frac{dy}{dx} + \frac{b}{a}
    \]
    If no, find the solution of the D.E.\(*\).

    OR
    Check whether the following differential equation is homogeneous or not
    \[
    x^2 \frac{dy}{dx} - xy = 1 + \cos \left(\frac{y}{x}\right), \, x \neq 0
    \]
    Find the general solution of the differential equation using substitution \(y = vx\).
19. If the vectors $\vec{p} = a\hat{i} + b\hat{j} + c\hat{k}$, $\vec{q} = d\hat{i} + e\hat{j} + f\hat{k}$ and $\vec{r} = g\hat{i} + h\hat{j} + i\hat{k}$ are coplanar, then for $a, b, c \neq 1$ show that

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

20. A plane meets the coordinate axes in $A, B$ and $C$ such that the centroid of $\Delta ABC$ is the point $(\alpha, \beta, \gamma)$. Show that the equation of the plane is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$

21. If a 20 year old girl drives her car at 25 km/h, she has to spend Rs 4/km on petrol. If she drives her car at 40 km/h, the petrol cost increases to Rs 5/km. She has Rs 200 to spend on petrol and wishes to find the maximum distance she can travel within one hour. Express the above problem as a Linear Programming Problem. Write any one value reflected in the problem.

22. The random variable $X$ has a probability distribution $P(X)$ of the following form, where $k$ is some number:

$$P(X) = \begin{cases}
  k, & \text{if } x = 0 \\
  2k, & \text{if } x = 1 \\
  3k, & \text{if } x = 2 \\
  0, & \text{otherwise}
\end{cases}$$

(i) Find the value of $k$ (ii) Find $P(X < 2)$ (iii) Find $P(X \leq 2)$ (iv) Find $P(X \geq 2)$

23. A bag contains $(2n + 1)$ coins. It is known that ‘$n$’ of these coins have a head on both its sides whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, find the value of ‘$n$’.

SECTION-D

Questions from 24 to 29 are of 6 marks each

24. Using properties of integral, evaluate $\int_{0}^{\pi} \frac{x}{1+\sin x} \, dx$

OR

Find: $\int \frac{\sin x}{\sin^3 x + \cos^3 x} \, dx$

25. Does the following trigonometric equation have any solutions? If Yes, obtain the solution(s):

$$\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = -\tan^{-1}7$$

OR

Determine whether the operation $*$ define below on $\mathbb{Q}$ is binary operation or not.

$$a * b = ab + 1$$

If yes, check the commutative and the associative properties. Also check the existence of identity element and the inverse of all elements in $\mathbb{Q}$. 

26. Find the value of \( x, y \) and \( z \), if \( A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \) satisfies \( A' = A^{-1} \)

OR

Verify: \( A(adj \ A) = (adj \ A)A = |A||I \) for matrix \( A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} \)

27. Find \( \frac{dy}{dx} \), if \( y = e^{sin^2 x} \left( 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right) \)

28. Find the shortest distance between the line \( x - y + 1 = 0 \) and the curve \( y^2 = x \)

29. Define skew lines. Using only vector approach, find the shortest distance between the following two skew lines:

\( \vec{r} = (8 + 3 \lambda) \hat{i} - (9 + 16 \lambda) \hat{j} + (10 + 7 \lambda) \hat{k} \)

\( \vec{r} = 15 \hat{i} + 29 \hat{j} + 5 \hat{k} + \mu (3 \hat{i} + 8 \hat{j} - 5 \hat{k}) \)