SECTION – A

1. Given $g(x) = 4e^{2x} + 3$

Let $y = 4e^{2x} + 3$

$\frac{y-3}{4} = e^{2x}$

Taking logarithms on both sides we get

$\ln\left(\frac{y-3}{4}\right) = 2x$

$x = \frac{1}{2}\ln\left(\frac{y-3}{4}\right) = ln\sqrt{\frac{y-3}{4}}$

$g^{-1}(x) = ln\sqrt{\frac{x-3}{4}}$

The domain of g^{-1} is same as the range of $g(x) = 4e^{2x} + 3$

The range of $g(x) = 4e^{2x} + 3$ is $[3, \infty]$ because the +3 shifts up the graph of the function

So, the domain of g^{-1} is $[3, \infty]$

2. As the three points (2, -3), (3, 1) and (5, n) are said to be collinear, we can form the 3×3 determinant where the first column is the x's for all the points, the second column is the y's for all the points, and the last column is all ones.

Then equate that determinant to zero to get the value of n

$\begin{vmatrix} 2 & -3 & 1 \\ 3 & 1 & 1 \\ 5 & n & 1 \end{vmatrix} = 0$

$2\begin{vmatrix} 1 & 1 \\ n & 1 \end{vmatrix} - 3\begin{vmatrix} -3 & 1 \\ n & 1 \end{vmatrix} + 5\begin{vmatrix} -3 & 1 \\ 1 & 1 \end{vmatrix} = 0$

$2(1-n) - 3(-3-n) + 5(-3-1) = 0$

$n = 9$

3. X={1,2,3,5,6,10,15,30}

f(2,5,15) =(2+5)*(5+15) =10*(30/5 +15)=210

4. $\vec{c} = \left(3\vec{a} + \vec{b}\right) + \left(\vec{a} - 4\vec{b}\right) = 4\vec{a} - 3\vec{b}$

Now $|\vec{c}|^2 = \left|4\vec{a} - 3\vec{b}\right|^2 = 16|\vec{a}|^2 - 24\vec{a}.\vec{b} + 9\left|\vec{b}\right|^2 = 16(36) + 9(64) - 24(6)cos180 = 16(144)$

$|\vec{c}| = \sqrt{16(144)} = 48$

(OR)

Given $a = \hat{\imath} + \hat{\jmath} + \hat{k}, b = \hat{\imath} - \hat{\jmath} + 2\hat{k}$

$c = x\hat{\imath} + (x-2)\hat{\jmath} - \hat{k}$

Since c lies on the plane of vectors a and b

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix}$=0

$1(1 - 2x + 4) - 1(-1 - 2x) + x - 2 + x = 0$

$x = 2$

SECTION – B

5. Let A=$\begin{bmatrix} 3 & 7 \\ 1 & 4 \end{bmatrix}$ then A'=$\begin{bmatrix} 3 & 1 \\ 7 & 4 \end{bmatrix}$

Now A+A' = $\begin{bmatrix} 3 & 7 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 8 & 8 \end{bmatrix}$

Let $P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 6 & 8 \\ 8 & 8 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 4 & 4 \end{bmatrix}$

$P' = \begin{bmatrix} 3 & 4 \\ 4 & 4 \end{bmatrix}$

Since P=P', so P is a symmetric matrix

Now A-A' = $\begin{bmatrix} 3 & 7 \\ 1 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$

Let $Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

$Q' = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = -Q$

So, Q is a skew-symmetric matrix

Representing A as the sum of P and Q

$P + Q = \begin{bmatrix} 3 & 4 \\ 4 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 4 \end{bmatrix} = A$

(OR)

$X = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$

$g(x) = x^2 - 3x + 7$

$g(X) = X^2 - 3X + 7$

$X^2 = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} -7 & -12 \\ 24 & 17 \end{bmatrix}$

$3X = \begin{bmatrix} 3 & -6 \\ 12 & 15 \end{bmatrix}$

$X^2 - 3X + 7 = \begin{bmatrix} -7 & -12 \\ 24 & 17 \end{bmatrix} - \begin{bmatrix} 3 & -6 \\ 12 & 15 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$g(X) = \begin{bmatrix} -3 & -6 \\ 12 & 9 \end{bmatrix}$

$g(X) + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix} = \begin{bmatrix} -3 & -6 \\ 12 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

6. $f(\alpha) = \begin{cases} \frac{1-sin^3\alpha}{3cos^2\alpha}, if\ \alpha < \pi/2 \\ i, if\ \ \alpha = \pi/2 \\ j\left(\frac{1-\sin\alpha}{(\pi-2\alpha)^2}\right), if\ \alpha > \pi/2 \end{cases}$

$\lim_{\alpha\to\frac{\pi}{2}^-} \frac{1-sin^3\alpha}{3cos^2\alpha} = \lim_{\alpha\to\frac{\pi}{2}^+} j\left(\frac{1-\sin\alpha}{(\pi-2\alpha)^2}\right) = i$

$\lim_{\alpha\to\frac{\pi}{2}^-} \frac{3sin^2\alpha cos\alpha}{6cos\alpha sin\alpha} = \lim_{\alpha\to\frac{\pi}{2}^+} \left(\frac{-jcos\alpha}{2(\pi-2\alpha)^1(-2)}\right) = i$

$\lim_{\alpha\to\frac{\pi}{2}^-} \frac{sin\alpha}{2} = \lim_{\alpha\to\frac{\pi}{2}^+} \left(\frac{jsin\alpha}{8}\right) = i$

$\frac{1}{2} = \frac{j}{2} = i$

$i = \frac{1}{2}$

$j = 1$

7. $u = e^{v+e^{v+e^{v+\cdots}}} = e^{v+u}$

$u = e^{v+u}$

Differentiating both sides, we get

$1 = e^{v+u}[v+1]$

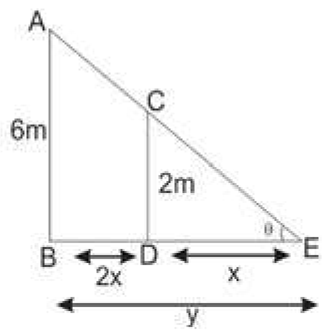
$e^{-u-v} - 1 = v'$ => $\ln u = v + u$

$e^{-\ln u} - 1 = v'$=> $v = \ln u - u$

$u^{-1} - 1 = v'$

$\frac{1}{u} - 1 = v'$

8.



In triangle CDE

$\tan\theta = \frac{2}{x} - - - (i)$

In triangle ABE

$\tan\theta = \frac{6}{y} - - - (ii)$

$\frac{2}{x} = \frac{6}{y}$

$y = 3x$

$BD = y - x = 3x - x = 2x$

Differentiate $BD = d = 2x$

$6 = 2 \times \frac{dx}{dt}$

Therefore $\frac{dx}{dt} = 3\ km/hr = u$ km/hr

$u = 3$

9. Given that

$R = \{(m, n): m, n \in Z\ and\ 5\ divides\ (m - n)\}$

(i) As $(m - m) = 0$ and we know that every non-integer divides zero that is 0 is divisible by every integer other than 0. So, 5 divides 0.

$$(m - m) \in R\ \forall\ m\ \in R$$

Hence R is reflexive

(ii) Let $(m, n) \in R$

So 5 $divides\ (m - n)$

This implies that 5 divides $(-(n - m))$

Thus, 5 $divides\ (n - m)$

So $(n, m) \in R$

Hence R is symmetric.

(iii) Let $(m, n) \in R$ and $(n, l) \in Z$

Then 5 divides $(m-n)$ and 5 divides $(n-l)$
5 divides $[(m-n)+(n-l)]$
This implies 5 divides $(m-l)$

$$(m,l) \in R$$

Therefore, R is transitive

Hence R is an equivalence relation.

10. The probability of randomly selecting a student from the auditorium who is NOT born in a leap year is $\frac{3}{4}$

Among 2 students, the probability that none of them was born in a leap is $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

The probability that at least one student of the selected two students, is born in leap year is $1 - \frac{9}{16} = \frac{7}{16} < \frac{1}{2}$

So, we need such value of 'n' so that

$$1 - \left(\frac{3}{4}\right)^n > \frac{1}{2}$$

Substituting each option, we get n=3

(OR)

Given that the signal is red on evening commute is 20%. Then the probability p = 0.2

Let X be the binomial random variable. It denotes the number of days that the signal shows red with n=5 (because 5 evenings are considered here) and p=0.2

The probability that the signal shows red on exactly one day over five evenings is

$$P(X=1) = \binom{5}{1} 0.2^1 (1-0.2)^{5-1} = 0.4096$$

11. The points denoted by the given position vectors are

(1,1,1), (2,3,0), (3,5,-2) and (0,-1,1)

Find the length of each sides

$\sqrt{1^2+2^2+(-1)^2}, \sqrt{1^2+2^2+(-2)^2}, \sqrt{(-3)^2+(-6)^2+(3)^2}, \sqrt{1^2+2^2}$=$\sqrt{6}, \sqrt{9}, \sqrt{54}, \sqrt{5}$

Since all sides are of different lengths, thus the type of quadrilateral is a trapezium.

(OR)

Let x be the radius and y be the height of the cone.

Volume V = $\pi r^2 h = \pi x^2 y$

Differentiating, we get

dV = $\pi 2xydx + \pi x^2 dy$

Multiply by 100 and divide by volume

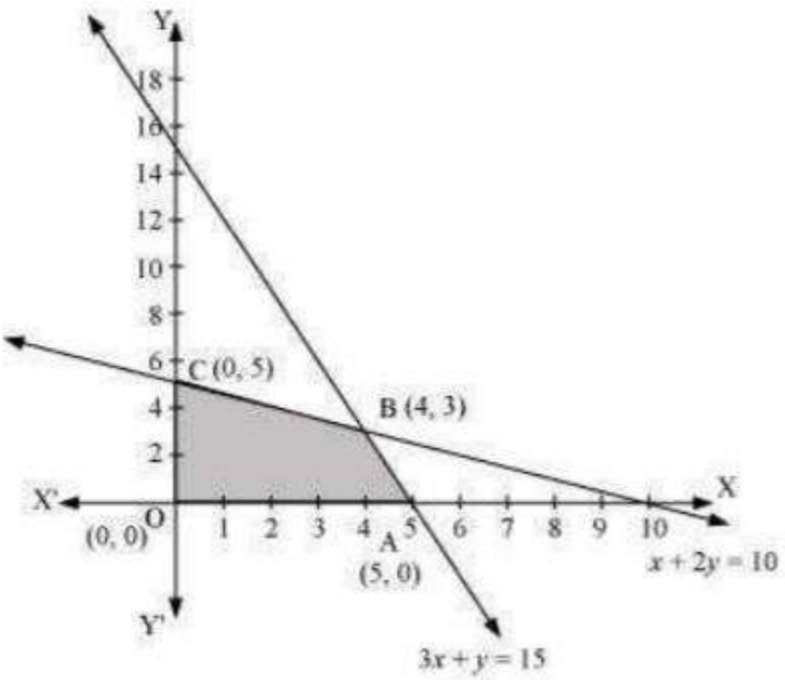
$100 \times \frac{dV}{V} = 100 \times \frac{\pi 2xydx+\pi x^2 dy}{\pi x^2 y} = 100 \times \frac{2dx}{x} + 100 \times \frac{dy}{y}$

Given that $dx = dy = \pm 0.2\ cm$

Substituting x=5, y=16, dx = dy $= \pm 0.2\ cm$

$100 \times \frac{2dx}{x} + 100 \times \frac{dy}{y} = \pm 0.0925$

12. The feasible region can be obtained from the given constraints as follows:



The points A (5,0) , B(4,3) and (0,5) are observed to be the corner points of the feasible region.

Thus the value of Z for each of the corner point is as follows:

$A(5,0): Z = 3(5) + 2(0) = 15$

$B(4,3): Z = 3(4) + 2(3) = 12 + 6 = 18$

$C(0,5): Z = 3(0) + 2(5) = 10$

Maximum of values for Z is 18 at the corner point B (4, 3).

SECTION-C

13. Let the equation of the line be

$\frac{x-a_1}{b_1} = \frac{y-a_2}{b_2} = \frac{z-a_3}{b_3} = k$

(a_1, a_2, a_3) is a point on the line and $(b_1\ b_2, b_3)$ is the direction ratio of the line.

But given that the line passes through the origin$(a_1, a_2,\ a_3)$ = (0,0,0)

Then the equation of the line

$\frac{x-0}{b_1} = \frac{y-0}{b_2} = \frac{z-0}{b_3} = k$ -(i)

The equation of the other line is

x - 2y – z + 3 = 0 which can be rewritten as

$\frac{x-3}{2} = \frac{y-3}{1} = \frac{z-0}{1} = j$ --(ii)

Let the point of intersection of the above two lines (i) and (ii) be P

P (2j+3, j+3, j)

Direction ratio of the line segment from origin to point P is (2j+3, j+3, j)

Since line (i) also passes through the origin

Direction ratio $d_1 = (b_1, b_2, b_3) = (2j+3, j+3, j)$

Direction ratio $d_2 = (2,1,1)$

$$d_1 d_2 = 2(2j+3) + 1(j+3) + 1.j = 6j + 9$$

$$|d_1| = \sqrt{(2j+3)^2 + (j+3)^2 + j^2} = \sqrt{6j^2 + 18j + 18}$$

$$|d_2| = \sqrt{4+1+1} = \sqrt{6}$$

$$\cos 60 = \left(\frac{d_1 d_2}{|d_1||d_2|}\right)$$

$$\frac{1}{2} = \frac{6j+9}{\sqrt{6j^2+18j+18}\sqrt{6}}$$

Solving for j, we get j=-1, -2

$$d_1 = (1,2,-1) \text{ or } (-1,1,-2)$$

14. If k=0

$$|X| = l^2m^2n^2 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Therefore k^2 is a factor of det X since 3 rows are identical

Let k=l, Therefore

$$|X| = \begin{vmatrix} l^2 & 0 & 0 \\ (l-m)^2 & m^2 & (l-m)^2 \\ (l-n)^2 & (l-n)^2 & n^2 \end{vmatrix} = \begin{vmatrix} l^2 & 0 & 0 \\ (n)^2 & m^2 & (n)^2 \\ (m)^2 & (m)^2 & n^2 \end{vmatrix} = 0$$

Therefore k-l is a factor of det X

Similarly, k-m, k-n are also factors. But determinant X is a sixth-degree polynomial.

Therefore, the sixth factor is of the form d (a + b + c)

So, determinant X =d (l + m + n)k^2 (k - l)(k - m)(k - n)

Let l = m = 0, n = 2, k = 1

Substituting these values becomes

$$|X| = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 4 \end{vmatrix} = -2d$$

$$d = 1$$

$$As\ k = a$$

$$det\ X = 2k^3\ (k-l)(k-m)(k-n)$$

15. Expanding the power of 3 in $P(m) = 300 + 0.25m - 0.5\left(\frac{m}{1000}\right)^3$, we get

$$P(m) = 300 + 0.25m - 5 \times 10^{-10}m^3$$

Differentiating this, we get

$$\frac{dP}{dm} = 0.25 - 1.5\ \times 10^{-9}m^2$$

Substitute m =2000, we get

$$P(2000) = 796$$

The estimated cost of 2001st camera is obtained by using the derivative estimate.

$$\mathrm{P(2001) - P(2000)} \approx \mathrm{P'(2000)(1)} = 0.25 - 1.5 \times 10^{-9} \times 2000^2 = 0.244$$

(OR)

First, we will need to find the equation of the line passing through (-6, 2 ,3) that is at right angles to the plane 3x-4y+5z-9=0

From the equation of the plane we can say that the normal vector to the plane is (3,-4, 5)

Thus, the line is defined as

$x = 3k - 6$

$y = -4k + 2$

$z = 5k + 3$

So, to determine the value of k, we need to substitute these values in the equation of the plane

$3(3k-6)-4(-4k+2)+5(5k+3)-9=0$

$k=\frac{2}{5}$

Substituting this value of k in x, y, z

We get $\left(\frac{-24}{5},\frac{2}{5},5\right)$

The vector from this point to point X is $(3k,-4k,5k)$ =$(\frac{-6}{5},\frac{8}{5},-2)$

So, we now find the negation of the vector

=$(\frac{6}{5},\frac{-8}{5},2)$

Now add this to the point of intersection $(\frac{-24}{5},\frac{2}{5},5)$ to get the image of point X

=$\left(\frac{-6}{5},\frac{8}{5},-2\right)+\left(\frac{-24}{5},\frac{2}{5},5\right)=\left(\frac{-18}{5},\frac{-6}{5},7\right)$

Hence i + j + k= $\frac{-18}{5}+\frac{-6}{5}+7=\frac{11}{5}$

16. $\tan\left(\frac{\pi}{4}+x\right)=\frac{1+tan\frac{\pi}{4}tanx}{1-tan\frac{\pi}{4}tanx}=\frac{1+tanx}{1-tanx}$

$$\lim_{x\to0}f(x)=\lim_{x\to0}\left(\frac{1+tanx}{1-tanx}\right)^{\frac{1}{x}}$$
$$\lim_{x\to0}f(x)=\lim_{x\to0}\ [(1+tanx)^{\frac{1}{tanx}}]^{\frac{tanx}{x}}\times[(1-tanx)^{\frac{-1}{tanx}}]^{\frac{tanx}{x}}$$

$$=e\times e=e^2$$

(OR)

The slope of the curve at the point (x, y) is given as

$$\frac{dy}{dx}=\frac{y}{x}+\sec\left(\frac{y}{x}\right)$$

Let y = vx

$$\frac{d(vx)}{dx}=\frac{vx}{x}+\sec\left(\frac{vx}{x}\right)$$

$$v+\frac{xdv}{dx}=v+secv$$

$$\frac{xdv}{dx}=sec\ v$$

$$\frac{dv}{sec\ v}=\frac{dx}{x}$$

Integrating on both sides,

$$\int \frac{dv}{sec\ v} = \int \frac{dx}{x}$$

$$\int \cos\ v\, dv = \int \frac{dx}{x}$$

$$sin\ v = \log v + \log C$$

$$\sin\left(\frac{y}{x}\right) = logx + \log C = \log(Cx)$$

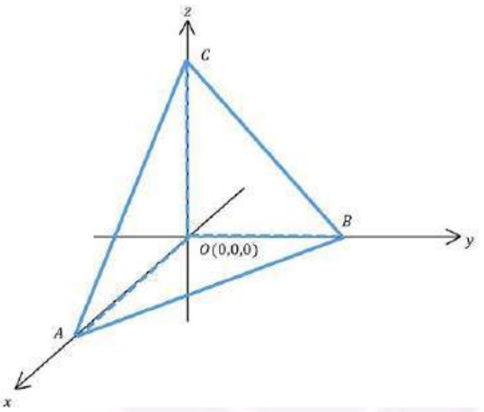
The curve passes through $\left(1\ ,\ \frac{\pi}{3}\right)$

$$\sin\left(\frac{\pi}{3}\right) = logc$$

$$\frac{\sqrt{3}}{2} = logc$$

The equation of the curve is $\sin\left(\frac{y}{x}\right) = logx + \frac{\sqrt{3}}{2}$

17.



The four vertices of the tetrahedron would be obtained by plotting the points where the given plane 2x + 4y + 6z = 8 intersects the axes.

When it intersects x-axis where y = 0 and z = 0,

So, 2x = 8 which implies x = 4. This is the point (4, 0, 0).

Similarly, it goes through $(0, 2, 0)$ and $\left(0, 0, \frac{4}{3}\right)$.

Hence four vertices of the tetrahedron are (0,0,0), (4, 0, 0), (0, 2, 0) and $\left(0, 0, \frac{4}{3}\right)$.

The given plane can be rewritten as

$$\frac{\mathrm{x}}{4} + \frac{\mathrm{y}}{2} + \frac{\mathrm{z}}{\frac{4}{3}} = 1$$

The distance of the given plane from origin is

$$D = \frac{\left|\frac{0}{4}+\frac{0}{2}+\frac{0}{\frac{4}{3}}-1\right|}{\sqrt{\left(\frac{1}{4}\right)^2+\left(\frac{1}{2}\right)^2+\left(\frac{1}{\frac{4}{3}}\right)^2}}$$

$$= \frac{1}{\sqrt{\left(\frac{1}{4}\right)^2+\left(\frac{1}{2}\right)^2+\left(\frac{1}{\frac{4}{3}}\right)^2}}$$

$$\text{The distance of the given plane from origin is } = \frac{4}{\sqrt{14}}$$

18. The graph cuts at y-axis (x=0) at (0,10) and given that

$$\frac{dy}{dc}\text{at }(0,10) = 24$$

at x=0

$$8c = 24$$

C=3

$\frac{dy}{dx} = 0$ at $S(-2,0)$ as it touches x-axis

$$48a - 24b + 8c = 0$$

$$24a - 12b + 24 = 0$$

$$2a - b = -1 \ldots\ldots..(i)$$

Also, S lies on the curve

$4ax^3 + 6bx^2 + 8cx + 10 = -32a + 24b - 16c + 10 = 0$
$-16a + 12b = 19 \ldots\ldots\ldots (ii)$

Solving (i) and (ii)

Multiplying (i) by -8

$-16a + 8b = 8$

Now substitute (ii) in (i)

$19 - 12b + 8b = 8$

$-4b = -11$

$$b = \frac{11}{4}$$

$$a = \frac{7}{8}$$

19. When a pair of dice is thrown for the first time, the number of possible events that would occur is $6 \times 6 = 36$

Doublet means the same number appearing on both the dice.

The number of possible doublets that could occur are {(1,1),(2,2,),(3,3,),(4,4,),(5,5),(6,6)}

The probability of getting a doublet when a pair of dice is thrown for one time = P = $\frac{6}{36} = \frac{1}{6}$

The probability of not getting a doublet = p'=1 $-\frac{1}{6} = \frac{5}{6}$

If Y is a binomial distribution for the probability of not more than 4 success events in 6 Bernoulli trials.

$$P(Y \le 4) = 1 - [P(Y=5) + P(Y=6)] = 1 - \left(6C_5\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right)^{6-5} + 6C_6\left(\frac{1}{6}\right)^6\left(\frac{5}{6}\right)^{6-6}\right)$$

$$1 - \left(6 \times \frac{1}{6^5} \times \frac{5}{6} + 1 \times \frac{1}{6^6}\right) = \frac{46625}{46656} = 0.99$$

20. Let x be the sum of the events of picking two tiles

X	2	3	4	5	6	7	8
P(X)	4/36	4/36	9/36	8/36	6/36	4/36	1/36

Mean $\sum_{x=2}^{8} xP(x) = 2 \times \frac{4}{36} + 3 \times \frac{4}{36} + 4 \times \frac{9}{36} + 5 \times \frac{8}{36} + 6 \times \frac{6}{36} + 7 \times \frac{4}{36} + 8 \times \frac{1}{36} = 4.67$

X^2	4	9	16	25	36	49	64
P(X)	4/36	4/36	9/36	8/36	6/36	4/36	1/36

$$E(x^2) = 4 \times \frac{4}{36} + 9 \times \frac{4}{36} + 16 \times \frac{9}{36} + 25 \times \frac{8}{36} + 36 \times \frac{6}{36} + 49 \times \frac{4}{36} + 64 \times \frac{1}{36} = 24.22$$

Variance = $E(x^2) - \left(E(x)\right)^2 = 24.22 - (4.67)^2 = 2.4111$

21. Let X be the random variable which would denote the number of questions for which Jill had ticked the right options.

X follows binomial distribution.

Conditions:

1) Probability of guessing correct answer is fixed for any of the 25 questions because every question has only 4 options

$$\frac{1}{4} = 0.25$$

2) A finite number '25' is defined as the number of questions in the test. This is independent of whether Jill answers correctly or not.

Probability of mass function for the binomial distribution is as follows.

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

If more than 20 questions are answered correctly

$$P(X > 20) = P(X = 21) + P(X = 22) + P(X = 23) + P(X = 24) + P(X = 25)$$

$P(X > 20) = \binom{25}{21} 0.25^{21}(1-0.25)^{25-21} + \binom{25}{22} 0.25^{22}(1-0.25)^{25-22} + \binom{25}{23} 0.25^{23}(1-0.25)^{25-23} + \binom{25}{24} 0.25^{24}(1-0.25)^{25-24} + \binom{25}{25} 0.25^{25}(1-0.25)^{25-25}$ = 9.68×10^{-10}

22. $e^{2ulnu} - 2e^{ulnu} cotv - 1 = 0$

$$\frac{e^{2ulnu} - 1}{e^{ulnu}} = 2\cot v$$

$$e^{ulnu}\left[e^{2ulnu}\left(2u \times \frac{1}{u} + 2lnu\right)\right] = -2cosec^2 v \, . v'$$

$$= \frac{-(e^{2ulnu} - 1)e^{ulnu}\left(u \times \frac{1}{u} + 1 \times \ln u\right)}{(e^{ulnu})^2}$$

$$\frac{e^1 \ln 1 \, [e^{2ln1}(2 + 2\ln 1)] - (e^{2.1ln1} - 1)e^{2.1\ln 1}(1 + \ln 1)}{(e^{1ln1})^2} = 2cosec^2 v . v'$$

$$\frac{2}{2} = -1 = v'.cosec^2 v$$

$$u^{2u} - 2u^u \cot v - 1 = 0$$

At u=1

$$1^{2(1)} - 2(1)^1 \cot v - 1 = 0$$

$$\cot v = 0$$

$$v = \frac{\pi}{2}$$

$$-1 = v'.cosec^2 \frac{\pi}{2}$$

$$v' = -1$$

23. The Corner Point Method is a method used for solving a linear programming problem. The steps involved for this method are as follows:

(i) Determine the feasible region for the given linear programming problem.

(ii) Determine the corner points of the obtained feasible region by solving the equations of the two lines intersecting at the point. The corner points also can be obtained by inspection.

(iii) Determine the value of Z for each corner point using the objective function Z=ax + by

(iv) (a) If the feasible region is bounded, the largest and smallest values of Z would be the maximum and minimum values of Z.

(b) If the feasible region is unbounded, graph the inequality ax + by > M where M is the maximum value of Z. If the resulting open half plane has no point which is in common with the feasible region, M is the maximum value of Z. Otherwise, there is no maximum value for Z.

(c) If the feasible region is unbounded, graph the inequality ax + by < m where m is the minimum value of Z. If the resulting open half plane has no point which is in common with the feasible region, m is the minimum value of Z. Otherwise, there is no minimum value for Z.

(OR)

We notice that:

$$\left(a+\frac{1}{a}\right)' = 1-\frac{1}{a^2}$$

And so:

$$a\left(a+\frac{1}{a}\right)' = a\left(1-\frac{1}{a^2}\right) = a-\frac{1}{a}$$

So, for

$$u(a) = a+\frac{1}{a}$$

We have to integrate a form that can be represented like this:

$$\left(1+a-\frac{1}{a}\right)e^{a+\frac{1}{a}} = 1\times e^{u(a)} + au'(a)\times e^{u(a)}$$

Can be written like this

$$a'\times e^{u(a)} + au'(a)\times e^{u(a)}$$

And it is the derivative of $ae^{u(a)}$

And therefore, we see that

$$\int\left(1+a-\frac{1}{a}\right)e^{a+\frac{1}{a}}da = \int\left(a'\times e^{u(a)} + au'(a)\times e^{u(a)}\right)da$$

$$= \int\left(a'\times e^{u(a)} + a\left[e^{u(a)}\right]'\right)da$$

$$\int\left(a\left[e^{u(a)}\right]\right)'da$$

= $a\left[e^{u(a)}\right] + C$

Substituting u (a) in above equation, we get

$$a\left[e^{a+\frac{1}{a}}\right]+C$$

SECTION-D

24. $\frac{x-3}{4}=\frac{y-4}{1}=\frac{z-1}{1}=\ t$

$\frac{x+1}{12}=\frac{y-7}{6}=\frac{z-5}{3}$

Which means x = 4t + 3, y = t + 4, z = t+1

And x = 12t -1, y = 6t+7, z = 3t + 5

Putting t = 0, we get

From equation (1), x = 3, y = 4 , z = 1

From equation (2) x =-1, y = 7 , z= 5

n=uxv = $\begin{vmatrix} i & j & k \\ -4 & 3 & 4 \\ 1 & -4 & 4 \end{vmatrix}$

n= i (12+16) – j (-16-4)+k(16-4) = 28i +20j +12 k = (28,20,12)

Equation of plane is given by:

3(x-28) + 4(y-20) + 1(z-12) = 0

3x +4y +z = 176

Hence, we get two points that lies in the plane:

p1 = (3,4,1) and p2 = (-1,7,5)

As given, the plane is parallel to x - 4y + 4z = -9

The vector between the points: u= ((-1-3), (7-4), (5-1))

u = (-4, 3, 4)

Direction vector for parallel plane: n = (1,-4, 4)

Normal vector for plane is given by :

$$n=uxv=\begin{vmatrix} i & j & k \\ -4 & 3 & 4 \\ 1 & -4 & 4 \end{vmatrix}$$

n= i (12+16) - j(-16-4)+k(16-4) = 28i +20j +12 k = (28,20,12)

Equation of plane is given by:

3(x-28) + 4(y-20)+ 1(z-12)=0

3x +4y +z = 176

(OR)

To find the projection A' of point A(3, 17, 3) onto the plane:

$$\frac{x_1-15}{13}=\frac{y_1+12}{-9}=\frac{z_1-17}{16}=t$$

x=13t+15, y=-9t-12, z=16t+17

Now dot product =0

(13t+15)(13) + (-9t-12)(-9)+(16t+17)(16)=0

$$t=-\frac{25}{22}$$

$$A'(\frac{5}{22},-\frac{39}{22},-\frac{26}{22})$$

Similarly, for B'

$$\frac{x_1-0}{13}=\frac{y_1-3}{-9}=\frac{z_1-6}{16}=t$$

$$x=13t, y=-9t+3, z=16t+6$$

$$(13t)(13)\ +\ (-9t+3)(-9)+(16t+6)(16)=0$$

$$t=-\frac{3}{22}$$

$$B'\left(-\frac{39}{22},\frac{93}{22},\frac{84}{22}\right)$$

$$Length\ of\ Projection=\left[\left(\frac{-39-5}{22}\right)^2+\left(\frac{93+39}{22}\right)^2+^2\left(\frac{84+26}{22}\right)^2\right]^{1/2}$$

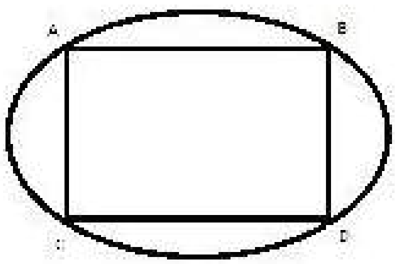
$=\sqrt{65}$

25. As we know the parallelogram having equal diagonals and do not bisect each other at right angles is a rectangle.

So, we need to find the area of the largest rectangle

Suppose that the upper right-hand corner of the rectangle is at the point ⟨x, y⟩.

By symmetry, the rectangle with the largest area will be one with its sides parallel to the ellipse's axes. Consider any point B(x, y) on the ellipse located in the first quadrant.



You can easily see that A(−x, y), D(x, −y), and C(−x,−y).

So, Area = 4xy, and you know that

Ellipse $\frac{x^2}{25} + \frac{y^2}{49} = 1$

Thinking of the area as a function of x, we have

$$\frac{dA}{dx} = \frac{4xdy}{dx} + 4y.$$

Differentiating (1) with respect to x, we have

$$\frac{2x}{25} + (\frac{2y}{49})(\frac{dy}{dx}) = 0$$

$$\frac{dy}{dx} = -\frac{49x}{25y} = \frac{dA}{dx} = 4y - 4(\frac{49x^2}{25y})$$

Setting this to 0 and simplifying, we have $y^2 = \frac{49x^2}{25}$ From (1) we know that

$$y^2 = 49 - \frac{49x^2}{25}.$$

Thus, $y^2 = 49 - y^2$, $2y^2 = 49$ and $\frac{y^2}{49} = \frac{1}{2}$. Clearly, then, $\frac{x^2}{25} = \frac{1}{2}$ as well, and the area is maximized when

$$x = \frac{5\sqrt{2}}{2} = 2.5\sqrt{2}$$

$$y = \frac{7\sqrt{2}}{2} = 3.5\sqrt{2}$$

Area of rectangle = $xy = 4 \times 2.5\sqrt{2} \times 3.5\sqrt{2} = 70$

26. Let the invested amount in the stock mutual fund be x, the invested amount in the bond fund be y and the invested amount in the money market fund be z

$$x + y + z = 10{,}00{,}000$$

The expected returns are as follows

$$0.10x + 0.07y + 0.05z = 0.08(of\ 10{,}00{,}000)$$

The financial planner advice

$$x = y + z$$

From the above three equations, we need to solve for x, y, z using matrices

Given system of linear equations

$$x + y + z = 10{,}00{,}000$$
$$0.10x + 0.07y + 0.05z = 80000$$
$$x - y - z = 0$$

We can write these equations as

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.1 & 0.07 & 0.05 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1000000 \\ 80000 \\ 0 \end{bmatrix}$$

As above is of the form AX=B

$$X = A^{-1}B$$

First, we need to find the $\begin{bmatrix} 1 & 1 & 1 \\ 0.1 & 0.07 & 0.05 \\ 1 & -1 & -1 \end{bmatrix}^{-1}$ which means the inverse of $\begin{bmatrix} 1 & 1 & 1 \\ 0.1 & 0.07 & 0.05 \\ 1 & -1 & -1 \end{bmatrix}$

First, we need to find the determinant of A = $\begin{bmatrix} 1 & 1 & 1 \\ 0.1 & 0.07 & 0.05 \\ 1 & -1 & -1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 0.1 & 0.07 & 0.05 \\ 1 & -1 & -1 \end{vmatrix}$$

$$= 1[(0.07)(-1) - (-1)(0.05)] - 1[(0.1)(-1) - (0.05)(1)] + 1[(0.1)(-1) - (0.07)(1)]$$

$$=1[-0.07 + 0.05] - 1[-0.1 - 0.05] + 1[-0.1 - 0.07]$$

$$= 1[-0.02 + 0.15 - 0.17] = -0.04$$

To find the inverse, we need to do the following steps

Step 1: Calculate the Matrix of Minors

For each element of the matrix, ignore the values on the current row and column and calculate the determinant of the remaining values. Put those determinants into a matrix (the "Matrix of Minors")

Step 2: Convert the Matrix of Minors into the Matrix of Co-factors

Change the sign of the values in (first row, second column), (second row, first column), (second row, third column) and (third row, second column)

Step 3: Transpose the obtained matrix.

Change the values in rows to columns and columns to rows

Step 4: Multiply the resulting matrix by 1/Determinant of the original matrix.

By doing these steps, the inverse of = $\begin{bmatrix} 1 & 1 & 1 \\ 0.1 & 0.07 & 0.05 \\ 1 & -1 & -1 \end{bmatrix}$ is = $\begin{bmatrix} 0.5 & 0 & 0.5 \\ -3.75 & 50 & -1.25 \\ 4.25 & -50 & -0.75 \end{bmatrix}$

Thus

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0.5 \\ -3.75 & 50 & -1.25 \\ 4.25 & -50 & -0.75 \end{bmatrix} \begin{bmatrix} 1000000 \\ 80000 \\ 0 \end{bmatrix} = \begin{bmatrix} 500000 \\ 250000 \\ 250000 \end{bmatrix}$$

Hence Pooja should invest Rs 5 lakhs in stock fund.

27. $\int_0^{\frac{\pi}{2}} \frac{cos3\alpha+1}{2cos\alpha-1} d\alpha = \int_0^{\frac{\pi}{2}} \frac{cos3\alpha-\cos(\frac{3\pi}{3})}{2(cos\alpha-\frac{1}{2})} d\alpha = \int_0^{\frac{\pi}{2}} \frac{cos3\alpha-\cos(\frac{3\pi}{3})}{2(cos\alpha-\cos(\frac{\pi}{3}))} d\alpha$

$$= \int_0^{\frac{\pi}{2}} \frac{(4\cos^3\alpha - 3\cos\alpha) - (4\cos^3\frac{\pi}{3} - 3\cos\left(\frac{\pi}{3}\right))}{2(\cos\alpha - \cos\left(\frac{\pi}{3}\right))} d\alpha$$

$$= \int_0^{\frac{\pi}{2}} \frac{4(\cos^3\alpha - \cos^3\frac{\pi}{3}) - 3(\cos\alpha - \cos\left(\frac{\pi}{3}\right))}{2(\cos\alpha - \cos\left(\frac{\pi}{3}\right))} d\alpha$$

$$= \int_0^{\frac{\pi}{2}} \frac{4(\cos^3\alpha - \cos^3\frac{\pi}{3})}{2(\cos\alpha - \cos\left(\frac{\pi}{3}\right))} d\alpha - \int_0^{\frac{\pi}{2}} \frac{3(\cos\alpha - \cos\left(\frac{\pi}{3}\right))}{2(\cos\alpha - \cos\left(\frac{\pi}{3}\right))} d\alpha$$

$$= 2\int_0^{\frac{\pi}{2}} \frac{(\cos^3\alpha - \cos^3\frac{\pi}{3})}{(\cos\alpha - \cos\left(\frac{\pi}{3}\right))} d\alpha - \int_0^{\frac{\pi}{2}} \frac{3}{2} d\alpha$$

$$= 2\int_0^{\frac{\pi}{2}} \frac{(\cos\alpha - \cos\left(\frac{\pi}{3}\right))(\cos^2\alpha + \cos^2\frac{\pi}{3} + \cos\alpha\cos\left(\frac{\pi}{3}\right))}{(\cos\alpha - \cos\left(\frac{\pi}{3}\right))} d\alpha - \frac{3}{2}\int_0^{\frac{\pi}{2}} d\alpha$$

$$= 2\int_0^{\frac{\pi}{2}} (\cos^2\alpha + \frac{1}{4} + \frac{\cos\alpha}{2})\, d\alpha - \frac{3}{2}\int_0^{\frac{\pi}{2}} d\alpha$$

$$= \int_0^{\frac{\pi}{2}} (2\cos^2\alpha + \frac{1}{2} + \cos\alpha) d\alpha - \frac{3}{2}[\alpha]_0^{\frac{\pi}{2}}$$

$$\int_0^{\frac{\pi}{2}} (2\cos^2\alpha + \frac{1}{2} + \cos\alpha) d\alpha - \frac{3}{2}[\alpha]_0^{\frac{\pi}{2}} = \int_0^{\frac{\pi}{2}} (1 + \cos 2\alpha + \frac{1}{2} + \cos\alpha) d\alpha - \frac{3}{2}\left[\frac{\pi}{2} - 0\right]$$

$$= \int_0^{\frac{\pi}{2}} (\cos 2\alpha + \frac{3}{2} + \cos\alpha) d\alpha - \frac{3\pi}{4}$$

$$[\frac{\sin 2\alpha}{2} + \frac{3}{2}\alpha + \sin\alpha]_0^{\frac{\pi}{2}} - \frac{3\pi}{4}$$

$$\frac{\sin 2(\frac{\pi}{2})}{2} - \frac{\sin 0}{2} + \frac{3}{2}\left[\frac{\pi}{2} - 0\right] + \sin\frac{\pi}{2} - \sin 0 - \frac{3\pi}{4} = \frac{3\pi}{4} + 1 - \frac{3\pi}{4} = 1$$

(OR)

Let x be the cost ABC has charged for LED TV, y be the cost ABC has charged for Smart TV and z be the cost ABC has charged for Flat Screen TV.

4x + 6y + 10z = 114000

8x + 3y + 5z = 72000

2x + 9y + 5z = 81000

Writing in matrix form, we get

$$\begin{bmatrix} 4 & 6 & 10 \\ 8 & 3 & 5 \\ 2 & 9 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 114000 \\ 72000 \\ 81000 \end{bmatrix}$$

As above is of the form AX=B

$$X = A^{-1}B$$

First, we need to find the $\begin{bmatrix} 4 & 6 & 10 \\ 8 & 3 & 5 \\ 2 & 9 & 5 \end{bmatrix}^{-1}$ which means the inverse of $\begin{bmatrix} 4 & 6 & 10 \\ 8 & 3 & 5 \\ 2 & 9 & 5 \end{bmatrix}$

First, we need to find the determinant of A = $\begin{bmatrix} 4 & 6 & 10 \\ 8 & 3 & 5 \\ 2 & 9 & 5 \end{bmatrix}$

$$|A| = \begin{vmatrix} 4 & 6 & 10 \\ 8 & 3 & 5 \\ 2 & 9 & 5 \end{vmatrix}$$

$$= 4[(3)(5) - (5)(9)] - 6[(8)(5) - (2)(5)] + 10[(8)(9) - (3)(2)]$$

$$=4[15 - 45] - 6[40 - 10] + 10[72 - 6] = 4(-30) - 6(30) + 10(66)$$

$$= [-120 - 180 + 660] = 360$$

To find the inverse, we need to do the following steps

Step 1: Calculate the Matrix of Minors

For each element of the matrix, ignore the values on the current row and column and calculate the determinant of the remaining values. Put those determinants into a matrix (the "Matrix of Minors")

Step 2: Convert the Matrix of Minors into the Matrix of Cofactors

Change the sign of the values in (first row, second column), (second row, first column), (second row, third column) and (third row, second column)

Step 3: Transpose the obtained matrix.

Change the values in rows to columns and columns to rows

Step 4: Multiply the resulting matrix by 1/Determinant of the original matrix.

By doing these steps, the inverse of = $\begin{bmatrix} 4 & 6 & 10 \\ 8 & 3 & 5 \\ 2 & 9 & 5 \end{bmatrix}$ is $\begin{bmatrix} -\frac{30}{360} & \frac{60}{360} & 0 \\ -\frac{30}{360} & 0 & \frac{60}{360} \\ \frac{66}{360} & -\frac{24}{360} & -\frac{36}{360} \end{bmatrix}$

Thus

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{30}{360} & \frac{60}{360} & 0 \\ -\frac{30}{360} & 0 & \frac{60}{360} \\ \frac{66}{360} & -\frac{24}{360} & -\frac{36}{360} \end{bmatrix} \begin{bmatrix} 114000 \\ 72000 \\ 81000 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{30}{360} \times 114000 + \frac{60}{360} \times 72000 + 0 \\ -\frac{30}{360} \times 114000 + 0 \times 72000 + \frac{60}{360} \times 81000 \\ \frac{66}{360} \times 114000 - \frac{24}{360} \times 72000 - \frac{36}{360} \times 81000 \end{bmatrix} = \begin{bmatrix} 2500 \\ 4000 \\ 8000 \end{bmatrix}$$

The cost ABC has charged for Smart TV is 4000 Euros.

28. $I=\int_0^{\pi}\frac{u\tan u}{\sec u+\tan u}$ $----(1)$

Using the property $\int_0^a f(x)dx=\int_0^a f(a-x)dx$, we have

$$I=\int_0^{\pi}\frac{(\pi-u)\tan(\pi-u)}{\sec(\pi-u)+\tan(\pi-u)}du$$

$$I=\int_0^{\pi}\frac{(\pi-u)(-\tan u)}{-\sec u-\tan u}du$$

$$I=\int_0^{\pi}\frac{\pi(\tan u)}{\sec u+\tan u}du-\int_0^{\pi}\frac{u(\tan u)}{\sec u+\tan u}du\quad ----(2)$$

Adding (1) and (2), we have

$$2I=\pi\int_0^{\pi}\frac{\tan u}{\sec u+\tan u}du$$

$$2I=\int_0^{\pi}\frac{\sin u}{1+\sin u}du$$

$$[f(x)=f(2a-x)]then\ \int_0^{2a}f(x)dx=2.\int_0^{a}f(x)dx,$$

$$2I=\pi\times 2\times\int_0^{\frac{\pi}{2}}\frac{\sin u}{1+\sin u}du,$$

$$I=\pi\int_0^{\frac{\pi}{2}}\frac{\sin u+1-1}{1+\sin u}du,$$

$$I=\pi\int_0^{\frac{\pi}{2}}du\ -\pi\int_0^{\frac{\pi}{2}}\frac{1}{1+\sin u}du$$

$I=\frac{\pi^2}{2}-\pi\int_0^{\frac{\pi}{2}}\frac{1}{1+\cos u}du$, Using the property $\int_0^a f(x)dx=\int_0^a f(a-x)dx$,

$$I=\frac{\pi^2}{2}-\pi\int_0^{\frac{\pi}{2}}\frac{1}{2\cos^2\frac{u}{2}}du$$

$$I=\frac{\pi^2}{2}-\frac{\pi}{2}\int_0^{\frac{\pi}{2}}\sec^2\frac{u}{2}du$$

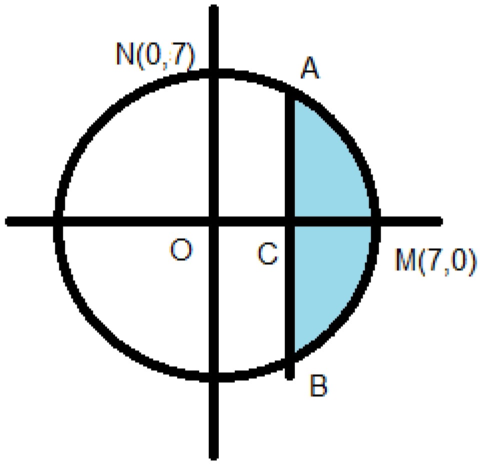
$$I=\frac{\pi^2}{2}-\frac{\pi}{2}\left[\frac{\tan\frac{u}{2}}{\frac{1}{2}}\right]_0^{\frac{\pi}{2}}$$

$$I=\frac{\pi^2}{2}-\pi$$

(OR)

Equation of the given circle is $x^2+y^2=49$

Radius r=7



So as 7 is positive

$x = \frac{7}{\sqrt{2}}$ will lie on positive side of x-axis

Let AB represent the line $x = 72$

We have to find the area of MAB

The area of MAB will be twice of area of MAC

$$Area_{MAB} = 2 \times \int_{\frac{7}{\sqrt{2}}}^{7} ydx$$

We know that

$$x^2 + y^2 = 7^2$$

$$y^2 = 7^2 - x^2$$

$$y = \pm\sqrt{7^2 - x^2}$$

Since MAC is in first quadrant

$$y = \sqrt{7^2 - x^2}$$

$$Area_{MAB} = 2 \times \int_{\frac{7}{\sqrt{2}}}^{7} \sqrt{7^2 - x^2}dx$$

It is of the form

$$\int \sqrt{a^2 - x^2}dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

$$Area_{MAB} = 2 \times \int_{\frac{7}{\sqrt{2}}}^{7} \sqrt{7^2 - x^2}dx$$

$$= 2[\frac{1}{2}x\sqrt{7^2 - x^2} + \frac{7^2}{2}\sin^{-1}(\frac{x}{7})]_{7/\sqrt{2}}^{7}$$

When simplified we get

Area of smaller portion $\frac{49}{2}(\frac{\pi}{2}-1)$

Subtracting this from area of circle, we get area of larger portion

$$(7)^2\pi - \frac{49}{2}\left(\frac{\pi}{2}-1\right)$$

$$49\pi - \frac{49\pi}{4} + \frac{49}{2} = 49\left(\frac{3\pi}{4}+\frac{1}{2}\right)$$

29. Using the relations

(i) $\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2}$,

(ii) $\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$

(iii) $\tan(A-B) = \frac{(\tan A - \tan B)}{1+\tan A\tan B}$

First we take the LHS

LHS = $\tan^{-1}\frac{1}{sec\alpha(1+\sin\alpha)} = \tan^{-1}\frac{\cos\alpha}{(1+\sin\alpha)}$

$$= \tan^{-1}\left(\frac{\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2},}{(\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}, + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2})}\right)$$

$$= \tan^{-1}\left(\frac{(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2})\,(\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2})}{(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2})^2}\right)$$

$$= \tan^{-1}\left(\frac{(\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2})}{(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2})}\right)$$

On dividing the numerator and denominator by $\cos\frac{\alpha}{2}, we\ get$

$$\tan^{-1}\left(\frac{(\frac{\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2})}{\cos\frac{\alpha}{2}}}{(\frac{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2})}{\cos\frac{\alpha}{2}}}\right) = \tan^{-1}\left(\frac{(1-\tan\frac{\alpha}{2})}{(1+\tan\frac{\alpha}{2})}\right) = \tan^{-1}\left(\frac{(\tan\frac{\pi}{4}-\tan\frac{\alpha}{2})}{(\tan\frac{\pi}{4}+\tan\frac{\alpha}{2})}\right)$$

$$\tan^{-1}(\tan(\frac{\pi}{4}-\frac{\alpha}{2})) = \frac{\pi}{4}-\frac{\alpha}{2} = RHS$$

Hence proved.

OR

What is the maximum value of $(\cos^{-1}\alpha)^2 + (\sin^{-1}\alpha)^2$?

Answer:

$$\left(\frac{\pi}{2} - \sin^{-1}\alpha\right)^2 + (\sin^{-1}\alpha)^2 = 2(\sin^{-1}\alpha)^2 - \pi\sin^{-1}\alpha + \frac{\pi^2}{4}$$

$$= 2\left((\sin^{-1}\alpha)^2 - 2\sin^{-1}\alpha \times \frac{\pi}{4} + \frac{\pi^2}{16} - \frac{\pi^2}{16} + \frac{\pi^2}{8}\right)$$

$$= 2\left((\sin^{-1}\alpha - \frac{\pi}{4})^2 + \frac{\pi^2}{16}\right)$$

Maximum value will be obtained when $\sin^{-1}\alpha = \frac{\pi}{2}$

$$2\left(\left(\frac{\pi}{2} - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right) = \frac{5\pi^2}{4}$$