CBSE Sample Paper Class 12 Maths Set 10

SUBJECT: MATHEMATICS CLASS : XII | **MAX. MARKS : 100**
DURATION : 3 HRS

General Instruction:

(i) **All** questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Question **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.
(iv) Question **5-12** in **Section B** are short-answer type questions carrying **2** marks each.
(v) Question **13-23** in **Section C** are long-answer-I type questions carrying **4** marks each.
(vi) Question **24-29** in **Section D** are long-answer-II type questions carrying **6** marks each.

SECTION – A

Questions 1 to 4 carry 1 mark each.

1. The binary operation * : R × R → R is defined as a * b = 2a + b. Find (2 * 3) * 4.

2. Differentiate $\log_e (\sin x)$ with respect to x.

3. Find the value of m and n, where m and n are order and degree of differential equation $\dfrac{4\left(\dfrac{d^2y}{dx^2}\right)^3}{\dfrac{d^3y}{dx^3}} + \dfrac{d^3y}{dx^3} = x^2 - 1$.

4. Find λ when the projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units.

OR

Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively and when $|\vec{a} \times \vec{b}| = \sqrt{3}$

SECTION – B

Questions 5 to 12 carry 2 marks each.

5. Let f and g be two real functions defined as f(x) = 2x – 3; g(x) = $\dfrac{3+x}{2}$. Find fog and gof. Can you say one is inverse of the other?

6. Find the value of x + y from the following equation: $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

7. Find $\int \cot x \log(\sin x) dx$

OR

Evaluate: $\int \sqrt{1 + \sin \dfrac{x}{4}}\ dx$

8. If $\int_0^a 3x^2 dx = 8$, write the value of a.

9. Form the differential equation of the family of parabolas having vertex at the origin and axis along positive y-axis.

10. If P(A) = $\dfrac{7}{13}$, P(B) = $\dfrac{9}{13}$ and P(A ∩ B) = $\dfrac{4}{13}$, find P(A′/B).

11. Find λ, if $(2\hat{i}+6\hat{j}+14\hat{k})\times(\hat{i}-\lambda\hat{j}+7\hat{k})=\vec{0}$.

OR

Find the volume of a parallelepiped whose continuous edges are represented by vectors $\vec{a}=2\hat{i}-3\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{c}=2\hat{i}+\hat{j}-\hat{k}$

12. Find the binomial distribution for which mean is 4 and variance 3.

OR

A random variable has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	2p	2p	3p	p^2	$2p^2$	$7p^2$	2p

Find the value of p.

SECTION – C

Questions 13 to 23 carry 4 marks each.

13. For any relation R in a set A, we can define the inverse relation R^{-1} by a R^{-1} b if and only if bRa. Prove that R is symmetric if and only if $R = R^{-1}$.

OR

Let A = R – {2} and B = R – {1}. If f : A   B is a function defined by f(x) =   , show that f is one-one and onto. Hence, find f^{-1}.

14. Solve for x: $\cos^{-1}\left(\dfrac{x^2-1}{x^2+1}\right)+\dfrac{1}{2}\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)=\dfrac{2\pi}{3}$.

15. Using properties of determinants, solve for x: $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}=0$.

16. Find $\dfrac{dy}{dx}$, if y = $(\log x)^x + x^{\log x}$.

OR

Find $\dfrac{dy}{dx}$ of the functions expressed in parametric form sin x = $\dfrac{2t}{1+t^2}$, tan y = $\dfrac{2t}{1-t^2}$.

17. If y = $\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)\dfrac{d^2y}{dx^2}-3x\dfrac{dy}{dx}-y=0$

18. Find the equation of the tangent to the curve y = $\sqrt{3x-2}$ which is parallel to the line 4x – 2y + 5 = 0.

19. Evaluate the following integral $\int_0^{\pi}\log(1+\cos x)dx$

20. Evaluate $\int\left\{\log(\log x)+\dfrac{1}{(\log x)^2}\right\}dx$

21. Show that each of the given three vectors is a unit vector: $\dfrac{1}{7}(2\hat{i}+3\hat{j}+6\hat{k})$, $\dfrac{1}{7}(3\hat{i}-6\hat{j}+2\hat{k})$ and $\dfrac{1}{7}(6\hat{i}+2\hat{j}-3\hat{k})$. Also, show that they are mutually perpendicular to each other.

22. Find the coordinates of the point, where the line $\dfrac{x-2}{3}=\dfrac{y+1}{4}=\dfrac{z-2}{2}$ intersects the plane x – y + z – 5 = 0. Also find the angle between the line and the plane.

23. Find the general solution of the following differential equation: $(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$

OR

For the differential equations, find the general solution of
$(1 + x)(1 + y^2)dx + (1 + y)(1 + x^2)dy = 0$.

SECTION – D

Questions 24 to 29 carry 6 marks each.

24. If $x \neq y \neq z$ and $\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then show that $1 + xyz = 0$.

OR

Given $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, verify that BA = 6I, use the result to solve the system $x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$.

25. Prove that, the radius of the right circular cylinder of greatest curved surface which can be inscribed in a given cone, is half of that of the cone.

26. Using the method of integration find the area of the ΔABC, coordinates of whose vertices are A(2, 0), B(4, 5) and C(6, 3).

OR

Find the area of the region $\{(x, y): y^2 \leq 6ax \text{ and } x^2 + y^2 \leq 16a\}$ using method of integration.

27. Show that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ intersect. Also, find their point of intersection.

OR

Find the coordinates of the point where the line through (3, –4, –5) and (2, –3, 1) crosses the plane, passing through the points (2, 2, 1), (3, 0, 1) and (4, –1, 0).

28. A company produces soft drinks that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B go into each bottle of the drink. The chemicals are available in prepared mix packets from two different suppliers. Supplier S had a packet of mix of 4 units of A and 2 units of B that costs 10. The supplier T has a packet of mix of 1 unit of A and 1 unit of B that costs 4. How many packets of mixes from S and T should the company purchase to honour the contract requirement and yet minimise cost? Make an LPP and solve graphically.

29. Three bags contain balls as shown in the table below:

Bag	Number of white balls	Number of black balls	Number of red balls
I	1	2	3
II	2	1	1
III	4	3	2

A bag is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they came from the III bag?