

CBSE Sample Paper Class 12 Maths Set 3

SUBJECT: MATHEMATICS CLASS :
12

MAX. MARKS : 100
DURATION : 3 HRS

General Instruction:

- All questions are compulsory.
- This question paper contains 29 questions.
- Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
- Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
- Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
- Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

SECTION – A

Questions 1 to 4 carry 1 mark each.

- Let * be a binary operation on N given by $a * b = \text{HCF}(a, b)$, $a, b \in N$. Write the value of $22 * 4$.
- If $f(x)$ is an invertible function, find the inverse of $f(x) = \frac{3x-2}{5}$
- If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x .
- Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $2\hat{i} - 3\hat{j} + 6\hat{k}$.

OR

If \vec{a} is a unit vector and $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$, then find $|\vec{x}|$.

SECTION – B

Questions 5 to 12 carry 2 marks each.

- Simplify : $\tan^{-1} \frac{\sqrt{1+x^2}-1}{x}$, $x \neq 0$
- For what value of x , is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix?
- If $y = \sec^{-1} \left(\frac{x^2+1}{x^2-1} \right)$, find $\frac{dy}{dx}$.
- Evaluate: $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

OR

Evaluate: $\int \sin^3 x dx$

- Find $\vec{a} \cdot (\vec{b} \times \vec{c})$, if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$.

OR

Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

10. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its volume.
11. A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once?
- OR**
- A black and a red dice are rolled. Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.
12. Form the differential equation of the family of circles touching the x -axis at origin.

SECTION – C

Questions 13 to 23 carry 4 marks each.

13. Evaluate: $\int \frac{x^2 + 1}{x^2 - 5x + 6} dx$

14. Find the value of a and b such that the following function is continuous:

$$f(x) = \begin{cases} 5, & \text{when } x \leq 2 \\ ax + b, & \text{when } 2 < x < 10 \\ 21, & \text{when } x \geq 10 \end{cases}$$

OR

Find the value of a such that the following function is continuous at $x = 0$:

$$f(x) = \begin{cases} a \sin \frac{\pi}{2}(x+1), & \text{if } x \leq 0; \\ \frac{\tan x - \sin x}{x^3}, & \text{if } x > 0 \end{cases}$$

15. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence find A^{-1} .

16. In a hockey match, both teams A and B scored same number of goals up to the end of the game, so to decide the winner, the referee asked both the captains to throw a die alternately and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team A was asked to start, find their respective probabilities of winning the match.

17. Find the general solution of the differential equation $y dx - (x + 2y^2) dy = 0$.

OR

Find the general solution of the differential equation $(1 + x^2) dy + 2xy dx = \cot x dx$, $x \neq 0$

18. Bag I contains 3 red and 4 black balls while another Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag II.

19. If $x = a (\cos t + t \sin t)$ and $y = a (\sin t - t \cos t)$, find $\frac{d^2y}{dx^2}$

20. Find the coordinates of the foot of the perpendicular drawn from the origin to the plane $2x - 3y + 4z - 6 = 0$.

21. Find a unit vector perpendicular to each of the vectors $\vec{a} + 2\vec{b}$ and $2\vec{a} + \vec{b}$, where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$.
22. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.
23. Find the equation of the tangent to the curve $y = \frac{(x-7)}{(x-2)(x-3)}$ at the point where it cuts the x -axis.

OR

Find local maximum and local minimum values of the function f given by $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$

SECTION – D

Questions 24 to 29 carry 6 marks each.

24. Let $f: \mathbb{N} \rightarrow \mathbb{R}$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: \mathbb{N} \rightarrow S$, where, S is the range of f , is invertible. Find the inverse of f .

OR

Prove that the relation R in the set $A = \{5, 6, 7, 8, 9\}$ given by $R = \{(a, b) : |a - b|, \text{ is divisible by } 2\}$, is an equivalence relation. Find all elements related to the element 6.

25. Using properties of determinants, prove that
$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$

OR

Using properties of determinants, prove that
$$\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = (\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma)$$

26. Evaluate: $\int_0^{\pi/2} \log \sin x dx$

OR

Find $\int_0^4 (x + e^{2x}) dx$ as the limit of a sum.

27. Using integration find the area of region bounded by the triangle whose vertices are (1, 0), (2, 2) and (3, 1).
28. Find the vector equation of the line passing through the point (1, 2, -4) and perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+19}{-3} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$
29. A factory makes two types of items A and B , made of plywood. One piece of item A requires 5 minutes for cutting and 10 minutes for assembling. One piece of item B requires 8 minutes for cutting and 8 minutes for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours for assembling. The profit on one piece of item A is Rs 5 and that on item B is Rs 6. How many pieces of each type should the factory make so as to maximise profit? Make it as an L.P.P. and solve it graphically.