CBSE Sample Paper Class 12 Maths Set 4

SUBJECT: MATHEMATICS CLASS : XII | **MAX. MARKS : 100 DURATION : 3 HRS**

General Instruction:

(i) **All** questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Question **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.
(iv) Question **5-12** in **Section B** are short-answer type questions carrying **2** marks each.
(v) Question **13-23** in **Section C** are long-answer-I type questions carrying **4** marks each.
(vi) Question **24-29** in **Section D** are long-answer-II type questions carrying **6** marks each.

SECTION – A

Questions 1 to 4 carry 1 mark each.

1. Evaluate : $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$

2. Let * *be* a binary operation, on the set of all non-zero real numbers, given by $a*b=\frac{ab}{5}$ for all $a,b\in R-\{0\}$. Find the value of *x*, given that 2 * (*x* * 5) = 10.

3. If *A* is a square matrix and |*A*| = 2, then write the value of $|AA'|$, where *A*' is the transpose of matrix *A*.

4. If a line has direction ratios 2, – 1, – 2, determine its direction cosines.

OR

Find the distance of the plane 3x – 4y + 12z = 3 from the origin.

SECTION – B

Questions 5 to 12 carry 2 marks each.

5. Simplify : $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right), 0<x<\pi$

6. Find the value of *a* if $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix}=\begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

7. Evaluate: $\int e^{x}\left(\sin^{-1}x+\frac{1}{\sqrt{1-x^{2}}}\right)dx$

OR

Evaluate: $\int$ cot x log sin x *dx*

8. Use differential to approximate $\sqrt{36.6}$.

9. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+\vec{b}$ is also a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

Find $|\vec{a}$ x $\vec{b}|$, if $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+5\hat{j}-2\hat{k}$.

10. If $y=\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{x}}{1+\sqrt{ax}}\right)$, find $\frac{dy}{dx}$.

11. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that atleast one is a girl?

OR

A fair die is rolled. Consider events E = {1,3,5}, F = {2,3} and G = {2,3,4,5}. Find P(E|F) and P(F|E)

12. Form the differential equation representing the family of parabolas having vertex at origin and axis along positive direction of x-axis.

SECTION – C

Questions 13 to 23 carry 4 marks each.

13. Evaluate: $\int \frac{x^2}{(x^2+1)(x^2+4)}dx$

14. Find the value of k such that the following function is continuous at x = 2:

$$f(x)=\begin{cases}2x+1, & x<2\\ k, & x=2\\ 3x-1, & x>2\end{cases}$$

OR

Find the value of k such that the following function is continuous at $x=\frac{\pi}{2}$:

$$f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & if\quad x\neq\frac{\pi}{2}\\ 3, & if\quad x=\frac{\pi}{2}\end{cases}$$

15. Use product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the system of equations

$x - y + 2z = 1$, $2y - 3z = 1$ and $3x - 2y + 4z = 2$

16. In a group of 50 scouts in a camp, 30 are well trained in first aid techniques while the remaining are well trained in hospitality but not in first aid. Two scouts are selected at random from the group. Find the probability distribution of number of selected scouts who are well trained in first aid. Find the mean of the distribution also.

17. Find the general solution of the differential equation $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$

OR

Solve the differential equation $ye^{\frac{x}{y}}dx=(xe^{\frac{y}{x}}+y^2)dy, y\neq 0$

18. Find the coordinates of the point where the line through (3, – 4, – 5) and (2, – 3, 1) crosses the plane $2x + y + z = 7$.

19. Show that the vectors $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar if $\vec{a},\vec{b},\vec{c}$ are coplanar.

20. If $x=a(\theta+\sin\theta)$ and $y=a(1-\cos\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{2}$

21. Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is actually a six. Find the probability that it is actually a six.

22. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

23. Find points on the curve $\frac{x^2}{4}+\frac{y^2}{25}=1$ at which the tangents are (i) parallel to x-axis (ii) parallel to y-axis.

OR

Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is (a) strictly increasing (b) strictly decreasing

SECTION – D

Questions 24 to 29 carry 6 marks each.

24. Show that the function $f: R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}, x \in R$ is one-one and onto function.

OR

Consider $f: R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y)=\left(\frac{\sqrt{y+6}-1}{3}\right)$.

25. If a, b and c are real numbers, and $\Delta=\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}=0$. Show that either $a + b + c = 0$ or $a = b = c$.

OR

If $A^{-1}=\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

26. Evaluate: $\int_0^{\pi} \frac{x}{a^2\cos^2 x+b^2\sin^2 x}dx$

OR

Find $\int_1^3 (x^2+5x)dx$ as the limit of a sum.

27. Find the area of the region $\{(x, y) : 0 \le y \le x^2+1, 0 \le y \le x+1, 0 \le x \le 2\}$

28. Find the equation of the plane passing through the point (– 1, 3, 2) and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

29. A merchant plans to sell two types of personal computers — a desktop model and a portable model that will cost Rs. 25,000 and Rs. 40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs. 70 lakhs and his profit on the desktop model is Rs. 4,500 and on the portable model is Rs. 5,000. Make an L.P.P. and solve it graphically.