

CBSE Sample Paper Class 12 Maths Set 5

SUBJECT: MATHEMATICS
CLASS : XII

MAX. MARKS : 100
DURATION : 3 HRS

General Instruction:

- (i) All questions are compulsory.
- (ii) This question paper contains 29 questions.
- (iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
- (iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
- (v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
- (vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

SECTION – A

Questions 1 to 4 carry 1 mark each.

1. If A is a square matrix of order 3 and $|2A| = k|A|$, then find the value of k.
2. If \vec{a} and \vec{b} are two nonzero vectors such that $|\vec{a} + \vec{b}| = \vec{a} \cdot \vec{b}$, then find the angle between a and b.

OR

Find the projection of the vector $\vec{a} = \hat{i} - 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$.

3. Write $f \circ g$, if $f: R \rightarrow R$ and $g: R \rightarrow R$ are given by $f(x) = 8x^3$ and $g(x) = x^{1/3}$.
4. If the binary operation $*$ on the set Z of integers is defined by $a * b = a + b - 5$, then write the identity element for the operation $*$ in Z.

SECTION – B

Questions 5 to 12 carry 2 marks each.

5. Find the value of $\tan^{-1} \frac{x}{y} - \tan^{-1} \frac{x-y}{x+y}$
6. Prove that the diagonal elements of a skew symmetric matrix are all zeros.
7. If $y = \tan^{-1} \frac{5x}{1-6x^2}$, $-\frac{1}{\sqrt{6}} < x < \frac{1}{\sqrt{6}}$, find $\frac{dy}{dx}$.
8. Evaluate: $\int \frac{\tan^{-1} x}{(1+x)^2} dx$

OR

Evaluate: $\int \cos^3 x dx$

9. If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 3$, find the angle between \vec{a} and \vec{b} .

OR

Find the area of a triangle having the points A(1, 1, 1), B(1, 2, 3) and C(2, 3, 1) as its vertices.

10. A black and a red dice are rolled. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

OR

If the sum of the mean and variance of a binomial distribution for 5 trials be 1.8, find the distribution.

11. If x changes from 4 to 4.01, then find the approximate change in $\log x$

12. Form the differential equation of the family of ellipses having foci on y -axis and centre at origin.

SECTION – C

Questions 13 to 23 carry 4 marks each.

13. Evaluate: $\int \frac{\sin x}{(\cos^2 x + 1)(\cos^2 x + 4)} dx$

14. Determine the value of k so that the function

$$f(x) = \begin{cases} \frac{\sqrt{3} \sin x + \cos x}{x + \frac{\pi}{6}}, & \text{if } x \neq -\frac{\pi}{6}; \\ k, & \text{if } x = -\frac{\pi}{6} \end{cases} \text{ is continuous at } x = -\frac{\pi}{6}.$$

OR

Discuss the differentiability of the function $f(x) = \begin{cases} 2x - 1, & \text{if } x < \frac{1}{2}; \\ 3 - 6x, & \text{if } x \geq \frac{1}{2} \end{cases}$ at $x = \frac{1}{2}$.

15. If $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$ then using A^{-1} , solve the following system of equations: $x - 2y = -1$, $2x + y = 2$

16. Find the equation of tangents to the curve $y = \cos(x + y)$, $-2\pi \leq x \leq 2\pi$ that are parallel to the line $x + 2y = 0$.

OR

Find the intervals in which the function f given by $f(x) = x^3 + \frac{1}{x^3}$, $x \neq 0$ is (i) increasing (ii) decreasing.

17. A speaks truth in 60% of the cases, while B in 90% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact?

18. Find the general solution of the differential equation $x \frac{dy}{dx} + y - x + xy \cot x = 0$ ($x \neq 0$)

OR

Solve the following differential equation:

$$\left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y dx = \left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x dy$$

19. Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Find the mean and variance of the number of red cards.

20. If $x = a \left[\cos \theta + \log \tan \frac{\theta}{2} \right]$, $y = a \sin \theta$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$.

21. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.
22. Find the coordinates of the point where the line through the points A (3, 4, 1) and B(5, 1, 6) crosses the XY-plane.
23. Prove that, $\vec{a} \cdot \{(\vec{b} + \vec{c}) \times (\vec{a} + 2\vec{b} + 3\vec{c})\} = [\vec{a} \ \vec{b} \ \vec{c}]$ for any three vectors $\vec{a}, \vec{b}, \vec{c}$

SECTION – D

Questions 24 to 29 carry 6 marks each.

24. Let $f : [0, \infty) \rightarrow R$ be a function defined by $f(x) = 9x^2 + 6x - 5$. Prove that f is not invertible. Modify, only the codomain of f to make f invertible and then find its inverse.

OR

Let $*$ be a binary operation defined on $Q \times Q$ by $(a, b) * (c, d) = (ac, b + ad)$, where Q is the set of rational numbers. Determine, whether, is commutative and associative. Find the identity element for, and the invertible elements of $Q \times Q$.

25. Using properties of determinants, prove that

$$\begin{vmatrix} a^2 & a^2 - (b-c)^2 & bc \\ b^2 & b^2 - (c-a)^2 & ca \\ c^2 & c^2 - (a-b)^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a^2 + b^2 + c^2)$$

OR

Show that $\Delta = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$

26. Evaluate: $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$

OR

Find $\int_1^3 (2x^2 + 3x) dx$ as the limit of a sum.

27. Using the method of integration find the area of the region bounded by lines: $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$
28. Find the vector equation of the line passing through (1, 2, 3) and parallel to the planes $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) - 5 = 0$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) - 6 = 0$
29. A company produces soft drinks that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B go into each bottle of the drink. The chemicals are available in prepared mix packets from two different suppliers. Supplier S had a packet of mix of 4 units of A and 2 units of B that costs Rs. 10. The supplier T has a packet of mix of 1 unit of A and 1 unit of B costs Rs.4. How many packets of mixed from S and T should the company purchase to honour the contract requirement and yet minimize cost? Make a LPP and solve graphically.