CBSE Sample Paper Class 12 Maths Set 6

SUBJECT: MATHEMATICS CLASS : XII **MAX. MARKS : 100**
DURATION : 3 HRS

General Instruction:

(i) **All** questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Questions **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.
(iv) Questions **5-12** in **Section B** are short-answer type questions carrying **2** marks each.
(v) Questions **13-23** in **Section C** are long-answer-I type questions carrying **4** marks each.
(vi) Questions **24-29** in **Section D** are long-answer-II type questions carrying **6** marks each.

SECTION – A

Questions 1 to 4 carry 1 mark each.

1. If $|\vec{a}|=2, |\vec{b}|=3$ and $\vec{a}.\vec{b} = 4$, find $|\vec{a} - \vec{b}|$.

OR

Find the projection of the vector $\vec{a}=\hat{i}+3\hat{j}+7\hat{k}$ on the vector $\vec{b}=2\hat{i}-\hat{j}+8\hat{k}$.

2. Write the value of the determinant $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

3. Find the principal values of $\cos ec^{-1}\left(-\sqrt{2}\right)$

4. Let * *be* a binary operation, on the set of all non-zero real numbers, given by $a*b=\frac{ab}{5}$ for all $a,b \in R-\{0\}$. Find the value of *x*, given that 5 * (*x* * 2) = 20.

SECTION – B

Questions 5 to 12 carry 2 marks each.

5. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{\pi}{2}<x<\frac{\pi}{2}$ in the simplest form.

6. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix}+y\begin{bmatrix} -1 \\ 1 \end{bmatrix}=\begin{bmatrix} 10 \\ 5 \end{bmatrix}$, write the value of *x*.

7. If $y=\tan^{-1}\frac{\sin x}{1+\cos x}$, find $\frac{dy}{dx}$.

8. Evaluate: $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$

OR

Evaluate: $\int \sin^2(2x+5)dx$

9. If the radius of a sphere is measured as 7 m with an error of 0.02 m, then find the approximate error in calculating its volume.

10. Form the differential equation of the family of circles touching the *y*-axis at origin.

11. Find $|\vec{a}|$ and $|\vec{b}|$, if $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=8$ and $|\vec{a}|=8|\vec{b}|$.

OR

Find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).

12. If $P(A)=\frac{6}{11}, P(B)=\frac{5}{11}$ and $P(A\cup B)=\frac{7}{11}$, then find P(A/B).

SECTION – C

Questions 13 to 23 carry 4 marks each.

13. Evaluate: $\int \frac{x^2}{(x^2+1)(x^2+4)}dx$

14. Find the value of a and b so that the function

$f(x)=\begin{cases}(x^2+3x+a), when & x\le 1\\ (bx+2), & when\ \ x=1\end{cases}$ is differentiable at each $x\in R$.

OR

For what value of λ is the function defined by $f(x)=\begin{cases}\lambda(x^2-2x), & if\ \ x\le 0;\\ 4x+1, & if\ \ x>0\end{cases}$ is continuous at x = 0? What about continuity at x = 1?

15. Using matrices, solve the following system of equations:
$x-y+z=4$; $2x+y-3z=0$; $x+y+z=2$

16. Find the equations of the tangent and normal to the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=2$ at (1, 1).

OR

Find the absolute maximum and minimum values of the function f given by
$f(x)=\cos^2 x+\sin x, x\in[0,\pi]$

17. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

18. Find the general solution of the differential equation $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x$

OR

Solve the following differential equation: $(xdy-ydx)y\sin\left(\frac{y}{x}\right)=(ydx+xdy)x\cos\left(\frac{y}{x}\right)$

19. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean and variance of the number of kings.

20. If $x=\frac{\sin^3 t}{\sqrt{\cos 2t}}, y=\frac{\cos^3 t}{\sqrt{\cos 2t}}$, find $\frac{dy}{dx}$.

21. Prove that the area of a right-angled triangle of a given hypotenuse is maximum when the triangle is isosceles.

22. Find the shortest distance between the lines whose vector equations are
$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k})$ and $\vec{r}=(4\hat{i}+5\hat{j}+6\hat{k})+\mu(2\hat{i}+3\hat{j}+\hat{k})$.

23. If $\vec{a}+\vec{b}+\vec{c}=0$, prove that, $(\vec{a}\times\vec{b})=(\vec{b}\times\vec{c})=(\vec{c}\times\vec{a})$ for any three vectors $\vec{a},\ \vec{b},\ \vec{c}$

SECTION – D

Questions 24 to 29 carry 6 marks each.

24. Let Z be the set of all integers and R be the relation on Z defined as $R = \{(a, b)\ ;\ a, b \in Z$, and $(a - b)$ is divisible by 5.$\}$ Prove that R is an equivalence relation.

OR

Let $A = R - \{2\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$ defined by $f(x)=\left(\frac{x-3}{x-2}\right)$.

Show that f is one-one and onto and hence find f^{-1}.

25. Using properties of determinants, prove that $\begin{vmatrix} \frac{(a+b)^2}{c} & c & c \\ a & \frac{(b+c)^2}{a} & a \\ b & b & \frac{(c+a)^2}{b} \end{vmatrix} = 2(a+b+c)^3$

OR

If $p \neq 0, q \neq 0$ and $\begin{vmatrix} p & q & p\alpha+q \\ q & r & q\alpha+r \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$, then, using properties of determinants, prove that at least one of the following statements is true: (a) p, q, r are in G. P. (b) α is a root of the equation $px^2 + 2qx + r = 0$

26. Evaluate: $\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2 \sin^2 x}\,dx$

OR

Find $\int_0^4 (x+e^{2x})\,dx$ as the limit of a sum.

27. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$.

28. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})-1=0$ and $\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})+4=0$ and parallel to x –axis.

29. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on the grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs. 25 and that from a shade is Rs. 15. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit. Formulate an LPP and solve it graphically.