CBSE Sample Paper Class 12 Maths Set 7

SUBJECT: MATHEMATICS CLASS : XII
MAX. MARKS : 100
DURATION : 3 HRS

General Instruction:

(i) **All** questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Question **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.
(iv) Question **5-12** in **Section B** are short-answer type questions carrying **2** marks each.
(v) Question **13-23** in **Section C** are long-answer-I type questions carrying **4** marks each.
(vi) Question **24-29** in **Section D** are long-answer-II type questions carrying **6** marks each.

SECTION – A

Questions 1 to 4 carry 1 mark each.

1. If f : R $\rightarrow$ R be defined by $f(x) = (3 - x^3)^{1/3}$, then find fof (x).

2. If $y = e^{-3\log x}$, then find $\frac{dy}{dx}$.

3. What is the degree of the following differential equation? $5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$

4. If $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - 5\hat{k}$, find a unit vector in the direction of $\vec{a} - \vec{b}$

OR

Find the angle between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$

SECTION – B

Questions 5 to 12 carry 2 marks each.

5. Let * be a binary operation defined on Q. Find which of the following binary operations are associative: $a * b = ab^2$ for a, b $\in$ Q.

6. Find x and y, if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$.

7. Find $\int \frac{x^4+1}{x^2+1}dx$

8. Find $\int (e^{x\log a} + e^{a\log x} + e^{a\log a})dx$

OR

Find $\int \frac{\sec^2 x}{3+\tan x}dx$

9. Form the differential equation of the family of parabolas having vertex at the origin and axis along positive y-axis.

10. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$

OR

If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}, \vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ show that $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular to each other.

11. Find the binomial distribution for which mean is 4 and variance 3.

12. If P(A) = 0.4, P(B) = p and P(A ⋃ B) = 0.7. Find the value of p, if A and B are independent events.

OR

A die is thrown three times, if the first throw results in 4, then find the probability of getting 15 as a sum.

SECTION – C

Questions 13 to 23 carry 4 marks each.

13. Show that the function f : R → R defined by f(x) = $\frac{x}{x^2+1}$, ∀ x ∈ R is neither one-one nor onto.

OR

Let N denote the set of all natural numbers and R be the relation on N × N defined by (a, b) R (c, d) if ad (b + c) = bc (a + d). Show that R is an equivalence relation.

14. Solve for x, $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$,x>0.

15. By using properties of determinants prove that: $\begin{vmatrix}\sin\alpha & \cos\alpha & \cos(\alpha+\delta)\\ \sin\beta & \cos\beta & \cos(\beta+\delta)\\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta)\end{vmatrix}=0$.

16. Verify the Rolle's Theorem for the function f (x) = sin x – sin 2x in [0, 2□].

17. Find the value of a for which the function f defined by $f(x)=\begin{cases} a\sin\frac{\pi}{2}(x+1), x\le 0\\ \frac{\tan x-\sin x}{x^3}, x>0\end{cases}$ is continuous at x = 0.

OR

Differentiate the given function with respect to x : $y=\tan^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

18. Find the equation of the tangent to the curve x^2 + 3y – 3 = 0, which is parallel to the line y = 4x – 5.

19. Find $\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx$.

20. Evaluate $\int\frac{dx}{\sqrt{5-4x-2x^2}}$

21. Solve the following differential equation $(1+e^{x/y})dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$.

OR

Find the general solution of the differential equation $\cos^2 x\frac{dy}{dx}$ + y = tan x.

22. Show that the area of the parallelogram having diagonals $3\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}-3\hat{j}+4\hat{k}$ is $5\sqrt{3}$ sq units.

23. Find the foot of the perpendicular drawn from the point A(1, 0, 3) to the join of the points B(4, 7, 1) and C(3, 5, 3).

SECTION – D

Questions 24 to 29 carry 6 marks each.

24. Using integration find the area of the region bounded by the lines y = 2 + x, y = 2 – x and x = 2.

OR

Draw the rough sketch of the region $\{(x, y) : y^2 \le 3x, 3x^2 + 3y^2 \le 16\}$ and find the area of the region enclosed by using the method of integration.

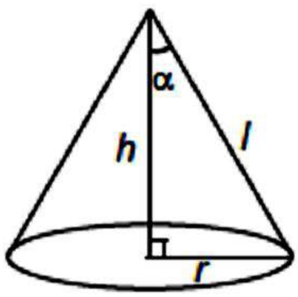
25. Using properties of determinants, prove that

$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = (b^2-ac)(ax^2+2bxy+cy^2)$$

OR

If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

26. Prove that the semi-vertical angle of the right circular cone of given volume and least curved surface area is $\cot^{-1}\sqrt{2}$.



27. Find the equation of the plane passing through the intersection of planes 4x – y + z = 10 and x + y – z = 4 and parallel to the line with direction ratios, 2, 1, 1. Find the perpendicular distance of the point (1, 1, 1) from this plane.

OR

Find the equations of the lines of shortest distance between the lines, $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$. Also find the shortest distance between the lines.

28. A diet for a sick person must contain at least 4,000 units of vitamins, 50 units of minerals and 1,400 calories. Two foods X and Y are available at a cost of 4 and 3 per unit respectively. 1 unit of the food X contains 200 units of vitamins, 1 unit of minerals and 40 calories, whereas 1 unit of food Y contains 100 units of vitamins, 2 units of minerals and 40 calories. Find what combination of X and Y should be used to have least cost, satisfying the requirements?

29. In a factory which manufactures bolts, machines A, B and C manufacture respectively 30%, 50% and 20% of the bolts. Of their outputs 3, 4 and 1 per cent respectively are defective bolts. A bolt is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine B.