CBSE Sample Paper Class 12 Maths Set 8

SUBJECT: MATHEMATICS CLASS : XII | **MAX. MARKS : 100**
DURATION : 3 HRS

General Instruction:

(i) **All** questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Question **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.
(iv) Question **5-12** in **Section B** are short-answer type questions carrying **2** marks each.
(v) Question **13-23** in **Section C** are long-answer-I type questions carrying **4** marks each.
(vi) Question **24-29** in **Section D** are long-answer-II type questions carrying **6** marks each.

SECTION – A

Questions 1 to 4 carry 1 mark each.

1. If f : R $\rightarrow$ R and g : R $\rightarrow$ R are given by f (x) = sin x and g(x) = $5x^2$, find gof (x).

2. Find $\frac{dy}{dx}$, if $x^2 + y^2 = 5$.

3. Write the degree of the differential equation $\left(\frac{dy}{dx}\right)^4 + 3x\frac{d^2y}{dx^2} = 0$.

4. Find a unit vector in the direction of $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$.

OR

If $\overrightarrow{AB} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{BC} = 6\hat{i} + 3\hat{j} - 6\hat{k}$, can we say that the points A, B, C are collinear?

SECTION – B

Questions 5 to 12 carry 2 marks each.

5. Let f(x) = $\frac{\alpha x}{x+1}$, x $\neq$ –1, then find the value(s) of for which fof is identity function $\alpha \in \{\sqrt{2}, -\sqrt{2}, 1, -1\}$.

6. If $\begin{bmatrix} 3 & 4 \\ 2 & x \end{bmatrix}\begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} 19 \\ 15 \end{bmatrix}$, find the value of x.

7. Find $\int \tan^3 x\, dx$

8. Find $\int (e^x + 3x)^2 (e^x + 3) dx$

OR

Find $\int \frac{2}{1+\cos 2x} dx$

9. Form the differential equation representing the family of curves y = e^{2x} (A + Bx), where A and B are constants.

10. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

OR

If two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 2, |\vec{b}| = 1$ and $\vec{a}.\vec{b}$ = 1, then find the value of $(3\vec{a} - 5\vec{b}).(2\vec{a} + 7\vec{b})$.

11. For 6 trials of an experiment, let X be a binomial variate which satisfies the relation 9P(X = 4) = P(X = 2). Find the probability of success.

12. If P(A) = 0.4, P(B) = p and P(A ∪ B) = 0.7. Find the value of p, if A and B are independent events.

OR

The probability that at least one of the events A and B occurs is $\frac{3}{5}$. If A and B occur simultaneously with probability $\frac{1}{5}$. Find $P(\overline{A})+P(\overline{B})$

SECTION – C

Questions 13 to 23 carry 4 marks each.

13. Show that the binary operation * on A = R – {–1} defined as a * b = a + b + ab for all a, b ∈ A is commutative and associative on A. Also find the identity element of * in A and prove that every element of A is invertible.

OR

If f (x) = $\frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that fof(x) = x for all. What is the inverse of f?

14. Solve for x: $\sin^{-1}\frac{2\alpha}{1+\alpha^2}+\sin^{-1}\frac{2\beta}{1+\beta^2}=2\tan^{-1}x$

15. Using properties of determinants, show that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

16. Differentiate $x^{x\cos x} + \frac{x^2+1}{x^2-1}$ w.r.t. x

OR

If $x=\sqrt{a^{\sin^{-1}t}}, x=\sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx}=-\frac{y}{x}$.

17. If f(x) = $\begin{cases} \frac{1-\cos 4x}{x^2}, when \quad x<0 \\ a \quad , when \quad x=0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, when \quad x>0 \end{cases}$ and function is continuous at x = 0, find the value of a.

18. Find the interval in which the function f (x) = $2x^3 - 15x^2 + 36x + 17$ is strictly increasing or strictly decreasing.

19. Evaluate $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\cot x}}$

20. Find the following integral $\int\frac{x^2+1}{x^2-5x+6}dx$.

21. Find the particular solution of the differential equations $(3xy + y^2)dx + (x^2 + xy)dy = 0$: for x = 1, y = 1.

OR

Solve the following differential equation $x\frac{dy}{dx}+2y=x^2\log x$.

22. Find a unit vector perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ where $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$.

23. Find the image of the point (1, 6, 3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.

SECTION – D

Questions 24 to 29 carry 6 marks each.

24. Prove without expanding that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$.

OR

Solve the system of the following equations: $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4;\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1;\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$

25. Show that the altitude of a right circular cone of maximum volume that can be inscribed in a sphere of radius R is $\frac{4R}{3}$.

26. Using integration, find the area of the region bounded by the triangle whose vertices are (–1, 2), (1, 5) and (3, 4).

OR

Find the area of the region bounded by the curves $\frac{x^2}{4}+\frac{y^2}{9}=1$ and $\frac{x^2}{9}+\frac{y^2}{4}=1$.

27. Find the equation of the plane passing through the points (–1, 2, 0), (2, 2, –1) and parallel to the line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$.

OR

Find the value of k for which the following lines are perpendicular to each other: $\frac{x+3}{k-5}=\frac{y-1}{1}=\frac{5-z}{-2k-1}$ and $\frac{x+2}{-1}=\frac{2-y}{-k}=\frac{z}{5}$.

28. A company manufactures three kinds of calculators: A, B and C in its two factories I and II. The company has got an order for manufacturing at least 6400 calculators of kind A, 4000 of kind B and 4800 of kind C. The daily output of factory I is of 50 calculators of kind A, 50 calculators of kind B, and 30 calculators of kind C. The daily output of factory II is of 40 calculators of kind A, 20 of kind B and 40 of kind C. The cost per day to run factory I is Rs. 12,000 and of factory II is Rs. 15,000. How many days do the two factories have to be in operation to produce the order with the minimum cost? Formulate this problem as an LPP and solve it graphically.

29. In an examination, an examinee either guesses or copies or knows the answer of multiple choice questions with four choices. The probability that he makes a guess is $\frac{1}{3}$ and probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct, given that he copied it, is $\frac{1}{8}$. Find the probability that he knew the answer to the question, given that he correctly answered it.