CBSE Sample Paper Class 12 Maths Set 9

SUBJECT: MATHEMATICS CLASS : XII **MAX. MARKS : 100**
DURATION : 3 HRS

General Instruction:

(i) **All** questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Question **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.
(iv) Question **5-12** in **Section B** are short-answer type questions carrying **2** marks each.
(v) Question **13-23** in **Section C** are long-answer-I type questions carrying **4** marks each.
(vi) Question **24-29** in **Section D** are long-answer-II type questions carrying **6** marks each.

SECTION – A

Questions 1 to 4 carry 1 mark each.

1. If the binary operation * on the set Z of integers is defined by a * b = a + b – 5, then write the identity element for the operation * in Z.

2. If xy = 9, find $\frac{dy}{dx}$.

3. Write the degree of the differential equation $x\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + x^3 = 0$.

4. If $|\vec{a}| = \sqrt{3}, |\vec{b}| = 2$ and angle between $\vec{a}$ and $\vec{b}$ is 60°, find $\vec{a}.\vec{b}$

OR

Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a}.\vec{b} = 8$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

SECTION – B

Questions 5 to 12 carry 2 marks each.

5. Let f : {1, 3, 4} $\rightarrow$ {1, 2, 5} and g : {1, 2, 5} $\rightarrow$ {1, 3} be given by f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Write down gof.

6. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find (A + 2B)′.

7. Evaluate: $\int_0^1 \frac{dx}{1+x^2}$

8. Find $\int \frac{2}{1+\cos 2x} dx$

OR

Find $\int (e^{2x} + 5x)^2 (2e^{2x} + 5)\, dx$

9. Show that $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$, if $\vec{a}$ and $\vec{b}$ are along adjacent sides of a rectangle.

OR

If vectors $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$ then find the value of λ.

10. Show that the differential equation of which $y = 2(x^2 - 1) + ce^{-x2}$ is a solution, is $\frac{dy}{dx} + 2xy = 4x^3$.

11. Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$.

12. If the sum of the mean and variance of a binomial distribution for 5 trials be 1.8, find the distribution.

OR

A die is thrown three times, if the first throw results in 4, then find the probability of getting 15 as a sum.

SECTION – C

Questions 13 to 23 carry 4 marks each.

13. Show that f : [–1, 1] $\rightarrow$ R, given by f(x) = is one-one.
Find the inverse of the function f : [–1, 1] $\rightarrow$ Range f.

OR

Determine whether the relation R defined on the set R of all real numbers as R = {(a, b) : a, b $\in$ R and $a - b + \sqrt{3} \in$ S, where S is the set of all irrational numbers}, is reflexive, symmetric and transitive.

14. Prove that $\tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right); |x| < \frac{1}{\sqrt{3}}$

15. Using the properties of determinants, prove that $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

16. If $y = \sin(m \sin^{-1} x)$, prove that $(1 - x^2)\, y_2 - xy_1 + m^2 y = 0$.

OR

If $y = \log\left(\frac{x}{a+bx}\right)^x$, prove that $x^3 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

17. For what value of k, the function $f(x) = \begin{cases} \frac{\sin 3x}{\tan 2x} & ,if \quad x < 0 \\ k & ,if \quad x = 0 \\ \frac{\log(1+3x)}{e^{2x} - 1} & ,if \quad x > 0 \end{cases}$ is continuous at x = 0?

18. Find the equation of the normal at a point on the curve $x^2 = 4y$ which passes through the point (1, 2). Also find the equation of the corresponding tangent.

19. Evaluate : $\int_0^{\pi/4} \frac{\sin x + \cos x}{3 + \sin 2x} dx$

20. Evaluate: $\int \frac{1}{\cos^4 x + \sin^4 x} dx$

21. Solve the differential equation $(x^2 - yx^2)\, dy + (y^2 + x^2y^2)\, dx = 0$.

OR

Find the general solution of the differential equation $(1 + x^2) \frac{dy}{dx} + y = \tan^{-1} x$.

22. Show that the four points A, B, C and D with position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $4(-\hat{i} + \hat{j} + \hat{k})$ respectively are coplanar.

23. Find the Cartesian equation of the plane passing through the points A(0, 0, 0) and B(3, –1, 2) and parallel to the line $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$

SECTION – D

Questions 24 to 29 carry 6 marks each.

24. Using properties of determinants, prove that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

OR

If $A=\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1} and hence prove that $A^2 - 4A - 5I = O$.

25. Show that the semi-vertical angle of a right circular cone of given total surface area and maximum volume is $\sin^{-1}\frac{1}{3}$.

26. Using the method of integration find the area of the region bounded by the lines $3x - 2y + 1 = 0$, $2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$.

OR

Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.

27. Find the shortest distance between the following pair of lines and hence write whether the lines are intersecting or not: $\frac{x-1}{2}=\frac{y+1}{3}=z$ and $\frac{x+1}{5}=\frac{y-2}{1}; z=2$

OR

Find the equation of the plane passing through the point P(1, 1, 1) and containing the line $\vec{r}=(-3\hat{i}+\hat{j}+5\hat{k})+\lambda(3\hat{i}-\hat{j}-5\hat{k})$. Also, show that the plane contains the line $\vec{r}=(-\hat{i}+2\hat{j}+5\hat{k})+\mu(\hat{i}-2\hat{j}-5\hat{k})$.

28. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs. 50 per kg to produce food I and Rs. 70 per kg to produce food II. Find the minimum cost of such a mixture. Formulate the above as an LPP mathematically and then solve it.

29. In a bulb factory, machines A, B and C manufacture 60%, 30% and 10% bulbs respectively. 1%, 2% and 3% of the bulbs produced respectively by A, B and C are found to be defective. A bulb is picked up at random from the product and is found to be defective. Find the probability that this bulb was produced by the machine A.