

CBSE Sample Paper Class 11 Maths Set 1 Solution

1. Solution:

We have,

$$\sin(\sin^{-1}\frac{3}{5} + \cos^{-1}x) = 1$$

$$\Rightarrow \sin^{-1}\frac{3}{5} + \cos^{-1}x = \sin^{-1}(1) \quad [\because \sin \theta = x \Rightarrow \theta = \sin^{-1}x]$$

$$\Rightarrow \sin^{-1}\frac{3}{5} + \cos^{-1}x = \sin^{-1}\left(\sin\frac{\pi}{2}\right) \quad \left[\because \sin\frac{\pi}{2} = 1\right]$$

$$\Rightarrow \sin^{-1}\frac{3}{5} + \cos^{-1}x = \frac{\pi}{2} \quad \dots (1)$$

We know that,

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}, \quad x \in [-1,1] \quad \dots (2)$$

Equating equations (1) and (2), we get

LHS of both equations are equal.

$$\sin^{-1}\frac{3}{5} + \cos^{-1}x = \sin^{-1}x + \cos^{-1}x$$

$$\Rightarrow \sin^{-1}\frac{3}{5} = \sin^{-1}x$$

$$\text{Hence, } x = \frac{3}{5}$$

2. Solution:

We have, $(3 + 7i)^2$

$$(3 + 7i)^2 = 3^2 + (7i)^2 + 2(3)(7i)$$

$$= 9 + 49i^2 + 42i$$

$$= 9 - 49 + 42i \quad [\because i^2 = -1]$$

$$= -40 + 42i$$

3. Solution:

$$\text{Let } A = \begin{vmatrix} \cos 18^\circ & \sin 18^\circ \\ \sin 72^\circ & \cos 72^\circ \end{vmatrix}$$

On expanding, we get

$$\begin{aligned} A &= (\cos 18^\circ \cdot \cos 72^\circ - \sin 18^\circ \cdot \sin 72^\circ) \\ &= \cos(18^\circ + 72^\circ) [\because \cos A \cdot \cos B - \sin A \cdot \sin B = \cos(A + B)] \\ &= \cos 90^\circ = 0 [\because \cos 90^\circ = 0] \end{aligned}$$

OR

Solution:

$$\text{We have, } \begin{vmatrix} x & 10 \\ 5 & 2x \end{vmatrix} = 0$$

On expanding, we get

$$\Rightarrow x \cdot 2x - 5 \cdot 10 = 0$$

$$\Rightarrow 2x^2 - 50 = 0$$

$$\Rightarrow 2x^2 = 50$$

$$\Rightarrow x^2 = 25$$

$$\Rightarrow x = \pm 5$$

Hence, $x = \pm 5$.

4. Solution:

$$\text{We have, } \lim_{x \rightarrow 1} \frac{x^{45} - 1}{x^{40} - 1}$$

$$= \lim_{x \rightarrow 1} \left\{ \left(\frac{x^{45} - 1}{x - 1} \right) \div \left(\frac{x^{40} - 1}{x - 1} \right) \right\}$$

$$= \lim_{x \rightarrow 1} \left(\frac{x^{45} - 1}{x - 1} \right) \div \lim_{x \rightarrow 1} \left(\frac{x^{40} - 1}{x - 1} \right)$$

$$= (45 \times 1^{44}) \div (40 \times 1^{39}) = \frac{45}{40} = \frac{9}{8}$$

Hence, $\lim_{x \rightarrow 1} \frac{x^{45} - 1}{x^{40} - 1} = \frac{9}{8}$.

SECTION B

5. Solution:

We have,

$$\tan^{-1} \frac{1}{\sqrt{3}} + \cot^{-1} x = \frac{\pi}{2} \quad \dots (1)$$

We know that,

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, \quad x \in R \quad \dots (2)$$

Equating equations (1) and (2), we get

$$\tan^{-1} \frac{1}{\sqrt{3}} + \cot^{-1} x = \tan^{-1} x + \cot^{-1} x$$

$$\Rightarrow \tan^{-1} \frac{1}{\sqrt{3}} = \tan^{-1} x$$

Hence, $x = \frac{1}{\sqrt{3}}$.

6. Solution:

We have,

$$\begin{aligned} & \sec \theta \begin{bmatrix} \sec \theta & \tan \theta \\ \tan \theta & \sec \theta \end{bmatrix} - \tan \theta \begin{bmatrix} \tan \theta & \sec \theta \\ \sec \theta & \tan \theta \end{bmatrix} \\ &= \begin{bmatrix} \sec^2 \theta & \sec \theta \cdot \tan \theta \\ \sec \theta \cdot \tan \theta & \sec^2 \theta \end{bmatrix} - \begin{bmatrix} \tan^2 \theta & \sec \theta \cdot \tan \theta \\ \sec \theta \cdot \tan \theta & \tan^2 \theta \end{bmatrix} \\ &= \begin{bmatrix} \sec^2 \theta - \tan^2 \theta & \sec \theta \cdot \tan \theta - \sec \theta \cdot \tan \theta \\ \sec \theta \cdot \tan \theta - \sec \theta \cdot \tan \theta & \sec^2 \theta - \tan^2 \theta \end{bmatrix} \\ &= \begin{bmatrix} \sec^2 \theta - \tan^2 \theta & 0 \\ 0 & \sec^2 \theta - \tan^2 \theta \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I = \text{Unit matrix } [\because \sec^2 \theta - \tan^2 \theta = 1] \end{aligned}$$

OR

Solution:

Since, $|A^3| = 512$

$$\Rightarrow |A^3| = 8^3$$

$$\therefore A = \begin{bmatrix} \alpha & 1 \\ 1 & \alpha \end{bmatrix}$$

$$\Rightarrow \alpha^2 - 1 = 8$$

$$\Rightarrow \alpha^2 = 8 + 1$$

$$\Rightarrow \alpha^2 = 9$$

$$\Rightarrow \alpha = \pm 3$$

Hence, $\alpha = \pm 3$.

7. Solution:

Here, $a = 32, d = 36 - 32 = 4$ and $a_n = 320$

Let n be the number of terms.

$$\text{Now, } a_n = 320 \Leftrightarrow a + (n - 1)d = 320$$

$$\Leftrightarrow 32 + (n - 1)(4) = 320 \quad [\because a = 32, d = 4]$$

$$\Leftrightarrow 32 + 4n - 4 = 320$$

$$\Leftrightarrow 4n + 28 = 320 \Leftrightarrow 4n = 320 - 28 = 292$$

$$\Leftrightarrow n = \frac{292}{4} = 73$$

Hence, the given AP contains 73 terms.

OR

Solution:

We have,

$$(n + 2)(n + 1) \times n! = 90 \times n!$$

$$\Rightarrow (n + 2)(n + 1) = 90$$

$$\Rightarrow n^2 + 2n + n + 2 = 90$$

$$\Rightarrow n^2 + 3n - 88 = 0$$

$$\Rightarrow n^2 + 11n - 8n - 88 = 0$$

$$\Rightarrow n(n + 11) - 8(n + 11) = 0$$

$$\Rightarrow (n + 11)(n - 8) = 0$$

$$\Rightarrow n = 8$$

$$[\because n \geq 0]$$

Hence, $n = 8$.

8. Solution:

We have, $\frac{6+\sqrt{5}i}{1-\sqrt{5}i}$

$$\begin{aligned}\frac{6+\sqrt{5}i}{1-\sqrt{5}i} &= \frac{6+\sqrt{5}i}{1-\sqrt{5}i} \times \frac{1+\sqrt{5}i}{1+\sqrt{5}i} \\ &= \frac{6+6\sqrt{5}i+\sqrt{5}i+(\sqrt{5}i)^2}{1^2-(\sqrt{5}i)^2} \quad [\because a^2 - b^2 = (a + b)(a - b)] \\ &= \frac{6+7\sqrt{5}i+5i^2}{1-5i^2} = \frac{6+7\sqrt{5}i-5}{1+5} \quad [\because i^2 = -1] \\ &= \frac{1+7\sqrt{5}i}{6} = \frac{1}{6} + \frac{7\sqrt{5}}{6}i\end{aligned}$$

$$\text{Hence, } \frac{6+\sqrt{5}i}{1-\sqrt{5}i} = \frac{1}{6} + \frac{7\sqrt{5}}{6}i.$$

9. Solution:

Given, $f(x) = 256x^4$ and $g(x) = x^{\frac{1}{4}}$

$$\therefore g \circ f(x) = g[f(x)] = g(256x^4)$$

$$= (256x^4)^{\frac{1}{4}} = (256)^{\frac{1}{4}} \cdot (x^4)^{\frac{1}{4}}$$

$$= (4^4)^{\frac{1}{4}} \cdot (x^4)^{\frac{1}{4}} = 4^{4 \times \frac{1}{4}} \cdot x^{4 \times \frac{1}{4}} = 4x$$

Hence, $gof(x) = 4x$.

OR

Solution:

Here, g is the inverse of $f(x)$.

$$\Rightarrow fog(x) = x$$

On differentiating with respect to x , we get

$$f'\{g(x)\} \times g'(x) = 1$$

$$\Rightarrow g'(x) = \frac{1}{f'\{g(x)\}}$$

$$= \frac{1}{\frac{1}{1+\{g(x)\}^{19}}} [\because f'(x) = \frac{1}{1+x^{19}}]$$

$$\Rightarrow g'(x) = 1 + \{g(x)\}^{19}$$

Hence, the value of $g'(x)$ is $1 + \{g(x)\}^{19}$.

10. Solution:

The given equation of the line is $\frac{5-x}{7} = \frac{y}{4} = \frac{3-z}{4}$.

It can be rewritten in standard form

$$\frac{x-5}{-7} = \frac{y}{4} = \frac{z-3}{-4}$$

1. Direction ratio of the line is $(-7, 4, -4)$.

$$\text{Now, } \sqrt{(-7)^2 + (4)^2 + (-4)^2}$$

$$= \sqrt{49 + 16 + 16} = \sqrt{81} = 9 \text{ Units}$$

2. Direction cosine of the line is $\left(-\frac{7}{9}, \frac{4}{9}, -\frac{4}{9}\right)$.

11. Solution:

We have, $(x^3 + 4y)^4$

Using the binomial theorem, we get

$$\begin{aligned}(x^3 + 4y)^4 &= C_0^4(x^3)^4 + C_1^4(x^3)^3(4y) + C_2^4(x^3)^2(4y)^2 + C_3^4(x^3)^1(4y)^3 + C_4^4(4y)^4 \\ &= 1 \cdot x^{12} + 4 \cdot x^9 \cdot (4y) + 6 \cdot x^6 \cdot 16 \cdot y^2 + 4 \cdot x^3 \cdot 64 \cdot y^3 + 1 \cdot 256 \cdot y^4 \\ &= x^{12} + 36 \cdot x^9 \cdot y + 96 \cdot x^6 \cdot y^2 + 256 \cdot x^3 \cdot y^3 + 256 \cdot y^4\end{aligned}$$

$$\text{Hence, } (x^3 + 4y)^4 = x^{12} + 36x^9y + 96x^6y^2 + 256x^3y^3 + 256y^4$$

12. Solution:

We know that the equation of a line with slope m and x - *intercept* d is given by:

$$y = m(x - d).$$

$$\text{Here, } \tan \theta = \frac{1}{5} \text{ and } d = 6.$$

Hence, the required equation of the line is:

$$y = \frac{1}{5}(x - 6)$$

$$\Rightarrow 5y = x - 6$$

$$\Rightarrow x - 5y - 6 = 0$$

13. Solution:

Clearly, $f : R \rightarrow R$ is a one - one function.

So, it is invertible.

$$\text{Let } (x) = y.$$

$$\text{Then, } 11x - 13 = y \Rightarrow 11x = y + 13$$

$$\therefore x = \frac{y + 13}{11}.$$

$$\Rightarrow f^{-1}(y) = \frac{y + 13}{11}$$

$$\text{Hence, } f^{-1}(x) = \frac{x+13}{11}.$$

14. Solution:

The given function is:

$$f(x) = \frac{6}{4}x^4 - 2x^3 - 6x^2 + 32$$

On differentiating both sides w. r. t. x , we get

$$f'(x) = \frac{6}{4} \cdot 4 \cdot x^3 - 2 \cdot 3 \cdot x^2 - 6 \cdot 2 \cdot x + 0$$

$$\Rightarrow f'(x) = 6x^3 - 6x^2 - 12x$$

For strictly increasing or strictly decreasing,

Put $f'(x) = 0$, we get

$$6x^3 - 6x^2 - 12x = 0$$

$$\Rightarrow 6x(x^2 - x - 2) = 0$$

$$\Rightarrow 6x(x^2 - 2x + x - 2) = 0$$

$$\Rightarrow 6x(x + 1)(x - 2) = 0$$

$$\Rightarrow x = 0, -1 \text{ or } 2$$

Now, we find intervals in which $f(x)$ is strictly increasing or strictly decreasing.

Interval	$f'(x) = 6x(x + 1)(x - 2)$	Sign of $f'(x)$
$x < -1$	$(-)(-)(-)$	$-ve$
$-1 < x < 0$	$(-)(+)(-)$	$+ve$
$0 < x < 2$	$(+)(+)(-)$	$-ve$
$x > 2$	$(+)(+)(+)$	$+ve$

We know that, a function $f(x)$ is said to be strictly increasing, if $f'(x) > 0$ and it is said to be strictly decreasing, if $f'(x) < 0$. So, the given function $f(x)$ is

1. Strictly increasing on the interval $(-1, 0)$ and $(2, \infty)$.

2. Strictly decreasing on the interval $(-\infty, -1)$ and $(0, 2)$.

15. Solution:

There are 5 letters in the word 'PUNAM', of which all are each of its own kind.

After fixing P at first place and M at last place, we have 3 letters out of which all are its own kind.

So, total number of words

$$= 3! = 3 \times 2 \times 1 = 6$$

After fixing P at first place, we have 4 letters out of which are all the each of its own kind

So, total number of words

$$= 4! = 4 \times 3 \times 2 \times 1 = 24$$

\therefore Number of words begin with P and does not end with M = Number of words begin with P -
Number of words begin with P and end with M

$$= 24 - 6 = 18$$

16. Solution:

We have,

$$\begin{aligned} (1 + 3x)^6(1 - x)^7 &= [1 + C_1^6(3x) + C_2^6(3x)^2 + C_3^6(3x)^3 + C_4^6(3x)^4 + C_5^6(3x)^5 + C_6^6(3x)^6] \\ &\times [1 - C_1^7(x) + C_2^7(x)^2 - C_3^7(x)^3 + C_4^7(x)^4 - C_5^7(x)^5 + C_6^7(x)^6 - C_7^7(x)^7] \\ &= [1 + 7 \times 3x + 15 \times 9x^2 + 20 \times 27x^3 + 15 \times 81x^4 + 6 \times 243x^5 + 1 \times 729x^6] \\ &\times [1 - 7 \times x + 21 \times x^2 - 35 \times x^3 + 35 \times x^4 - 21 \times x^5 + 7 \times x^6 - 1 \times x^7] \\ &= [1 + 21x + 135x^2 + 540x^3 + 1215x^4 + 1458x^5 + 729x^6] \\ &\times [1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7] \end{aligned}$$

\therefore Co-efficient of the x^4 in the product

$$= 1 \times 35 + 21 \times (-35) + 135 \times 21 + 540 \times (-7) + 1215 \times 1$$

$$= 35 - 735 + 2835 - 3780 + 1215 = -430$$

Hence, the co-efficient of the product of x^4 in the given expansion is -430 .

17. Solution:

The equation of a line passing through the points $(1, -3, 6)$ and parallel to $x = y = z$ is

$$\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-6}{1} = \lambda \text{ (Let)}$$

Thus, any point on this line is of the form $(\lambda + 1, \lambda - 3, \lambda + 6)$.

Now, if $P(\lambda + 1, \lambda - 3, \lambda + 6)$ is the point of intersection of line and plane, then

$$\lambda + 1 - (\lambda - 3) + \lambda + 6 = 6$$

$$\therefore \lambda + 1 = -4 + 1 = -3, \lambda - 3 = -4 - 3 = -7, \lambda + 6 = -4 + 6 = 2$$

\therefore Coordinates of point P are $(-3, -7, 2)$.

Hence, required distance

$$= \sqrt{(-3 - 1)^2 + (-7 + 3)^2 + (2 - 6)^2}$$

$$= \sqrt{4^2 + 4^2 + 4^2}$$

$$= \sqrt{16 + 16 + 16} = \sqrt{48} = 4\sqrt{3} \text{ units}$$

18. Solution:

\therefore The vertices of the ellipse lie on the y - axis, it is a vertical ellipse.

Let the required equation be $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, where $a^2 > b^2$.

Its vertices are $(0, \pm a)$ and therefore, $a = 5$.

$$\text{Let } c^2 = a^2 - b^2$$

$$\text{Then, } e = \frac{c}{a} \Rightarrow c = ae = 5 \times \frac{3}{5} = 3$$

$$\text{Now, } c^2 = a^2 - b^2 \Leftrightarrow b^2 = a^2 - c^2 = 5^2 - 3^2 = 25 - 9 = 16$$

$$\therefore a^2 = 5^2 = 25 \text{ and } b^2 = 16$$

Hence, the required equation is $\frac{x^2}{16} + \frac{y^2}{25} = 1$.

19. Solution:

$$L.H.S. = \begin{vmatrix} x + \lambda & 6x & 6x \\ 6x & x + \lambda & 6x \\ 6x & 6x & x + \lambda \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$L.H.S. = \begin{vmatrix} 13x + \lambda & 6x & 6x \\ 13x + \lambda & x + \lambda & 6x \\ 13x + \lambda & 6x & x + \lambda \end{vmatrix}$$

$$L.H.S. = (13x + \lambda) \begin{vmatrix} 1 & 6x & 6x \\ 1 & x + \lambda & 6x \\ 1 & 6x & x + \lambda \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - R_1$

$$L.H.S. = (13x + \lambda) \begin{vmatrix} 1 & 6x & 6x \\ 0 & \lambda - 5x & 0 \\ 0 & 0 & \lambda - 5x \end{vmatrix}$$

$$L.H.S. = (13x + \lambda)(\lambda - 5x)^2 = R.H.S.$$

Hence, it is proved.

OR

Solution:

$$\text{We have, } A = \begin{bmatrix} 2 & 6 \\ 6 & 8 \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} 2 & 6 \\ 6 & 8 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ 6 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 36 & 12 + 48 \\ 12 + 48 & 36 + 64 \end{bmatrix} = \begin{bmatrix} 40 & 60 \\ 60 & 100 \end{bmatrix}$$

$$\therefore A^2 - 20I_2 = \begin{bmatrix} 40 & 60 \\ 60 & 100 \end{bmatrix} - 20 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 40 & 60 \\ 60 & 100 \end{bmatrix} - \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}$$

$$\therefore A^2 - 20I_2 = \begin{bmatrix} 20 & 60 \\ 60 & 80 \end{bmatrix} = 10 \begin{bmatrix} 2 & 6 \\ 6 & 8 \end{bmatrix} = 10A$$

$$\Rightarrow A^2 - 20I_2 = 10A$$

$$\therefore kA = 10A$$

$$\Rightarrow k = 10$$

Hence, $k = 10$.

20. Solution:

$$\text{Let } \cos^{-1} \frac{12}{13} = \theta. \text{ Then, } \cos \theta = \frac{12}{13}.$$

$$\therefore \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{1 - \cos^2 \theta}}{\cos \theta} \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= \frac{\sqrt{1 - \frac{144}{169}}}{\frac{12}{13}} = \frac{5}{13} \times \frac{13}{12} = \frac{5}{12}$$

$$\therefore \theta = \tan^{-1} \frac{5}{12}$$

$$L.H.S. = \tan^{-1} \frac{5}{12} + \tan^{-1} \frac{4}{3}$$

$$= \tan^{-1} \left(\frac{\frac{5}{12} + \frac{4}{3}}{1 - \frac{5}{12} \cdot \frac{4}{3}} \right) \quad \left(\because \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right) \right)$$

$$= \tan^{-1} \left(\frac{\frac{5+16}{12}}{\frac{36-20}{36}} \right) = \tan^{-1} \left(\frac{21}{12} \times \frac{36}{16} \right) = \tan^{-1} \left(\frac{7}{4} \times \frac{9}{4} \right)$$

$$= \tan^{-1} \left(\frac{63}{16} \right) = R.H.S.$$

OR

Solution:

$$\text{Given, } \tan^{-1}(x+3) + \tan^{-1}(x-3) = \tan^{-1} \frac{2}{3}, x > 0$$

$$\Rightarrow \tan^{-1} \left(\frac{(x+3)+(x-3)}{1-(x+3)(x-3)} \right) = \tan^{-1} \frac{2}{3}$$

$$\left(\because \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right) \right)$$

$$\Rightarrow \tan^{-1} \left(\frac{x+3+x-3}{1-(x^2-9)} \right) = \tan^{-1} \frac{2}{3}$$

$$\Rightarrow \tan^{-1} \left(\frac{2x}{1-x^2+9} \right) = \tan^{-1} \frac{2}{3}$$

$$\Rightarrow \frac{2x}{10-x^2} = \frac{2}{3}$$

$$\Rightarrow 6x = 20 - 2x^2$$

$$\Rightarrow 2x^2 + 6x - 20 = 0$$

$$\Rightarrow x^2 + 3x - 10 = 0$$

$$\Rightarrow x^2 + 5x - 2x - 10 = 0$$

$$\Rightarrow x(x+5) - 2(x+5) = 0$$

$$\Rightarrow (x+5)(x-2) = 0$$

$$\Rightarrow x = 2 \text{ or } -5$$

But it is given that, $x > 0$.

$$\therefore x = 2.$$

21. Solution:

$$1 + 21 + 41 + 61 + \dots + x = 622500$$

The given series is an AP

Here, first term (a) = 1, Common difference (d) = $21 - 1 = 20$ and

x be the number of term

$$S_x = 622500$$

Using formula,

$$S_n = \frac{n}{2} \{2a + (n-1)d\}$$

$$S_x = \frac{x}{2} \{2 \cdot 1 + (x-1)20\} = 622500$$

$$\Rightarrow \frac{x}{2}\{20x - 18\} = 622500$$

$$\Rightarrow 10x^2 - 9x - 622500 = 0$$

$$\therefore x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \cdot 10 \cdot (-622500)}}{2 \cdot 10} = \frac{9 \pm \sqrt{81 + 40 \times 622500}}{20}$$

$$= \frac{9 \pm \sqrt{24900081}}{20} = \frac{9 \pm 4990}{20}$$

$$= \frac{9+4990}{20}, \frac{9-4990}{20}$$

$$= 249, -249.5 \text{ (Negative sign neglected)}$$

Hence, $x = 249$.

OR

Solution:

We have, $x^2 + 3x + 3 = 0$

Here, $a = 1, b = 3$ and $c = 3$

$$\therefore D = b^2 - 4ac$$

$$= 3^2 - 4 \cdot 1 \cdot 3$$

$$= 9 - 12 = -3 < 0$$

So, the given equation has complex roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{-3}}{2 \cdot 1}$$

$$= \frac{-3 \pm i\sqrt{3}}{2} \quad [\because i = \sqrt{-1}]$$

$$\therefore \text{Solution set} = \left\{ \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2} \right\}$$

22. Solution:

We have, $\sim (m \vee n) \vee (\sim m \wedge n)$

$\sim (m \vee n) \vee (\sim m \wedge n) \equiv (\sim m \wedge \sim n) \vee (\sim m \wedge n)$

[\because De-Morgan's law $\sim (x \vee y) = (\sim x \wedge \sim y)$]

$\equiv \sim m \wedge (\sim m \vee n)$ [\because Distributive law $\sim x \vee y = t$]

$\equiv \sim m \wedge t$

$\equiv \sim m$

23. Solution:

Let the probability that Raju can solve a problem be denoted by $P(R)$.

So, $P(R) = \frac{1}{3}$

Let the probability that Akash can solve a problem be denoted by $P(A)$.

So, $P(A) = \frac{3}{4}$

Also, $P(X \cap Y) = P(X) \cdot P(Y)$

Hence,

$$P(R \cap A) = P(R) \cdot P(A) = \frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$$

Also, $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$P(R \cup A)$ represents that both of them will solve the problem.

$$\therefore P(R \cup A) = P(R) + P(A) - P(R \cap A)$$

$$P(R \cup A) = \frac{1}{3} + \frac{3}{4} - \frac{1}{4}$$

$$= \frac{4+9-3}{12} = \frac{10}{12} = \frac{5}{6}$$

Hence, required answer is $\frac{5}{6}$

24. Solution:

We have, $f(x) = |\log 11 - \sin x|$ and $(x) = f(f(x))$, $x \in R$

Note that $x \rightarrow 0, \log 11 > \sin x$

$$\therefore f(x) = \log 11 - \sin x$$

$$\Rightarrow g(x) = \log 11 - \sin f(x) = \log 11 - \sin(\log 11 - \sin x)$$

Clearly, $g(x)$ is differentiable at $x = 0$ as $\sin x$ is differentiable.

Now,

$$g'(x) = 0 - \cos(\log 11 - \sin x)(-\cos x)$$

$$= \cos x \cdot \cos(\log 11 - \sin x)$$

$$\Rightarrow g'(0) = \cos 0 \cdot \cos(\log 11 - \sin 0) = 1 \cdot \cos(\log 11)$$

Hence, $g'(0) = \cos(\log 11)$.

OR

Solution:

Given,

$$f(x) = \begin{cases} (1 + |\sin\theta|^{\frac{a}{|\sin\theta|}}), & -\frac{\pi}{6} < \theta < 0 \\ b, & \theta = 0 \\ e^{\tan 7\theta / \tan 8\theta}, & 0 < \theta < \frac{\pi}{6} \end{cases}$$

Since, $f(x)$ is continuous at $x = 0$, therefore

$$\lim_{x \rightarrow 0^-} f(x) = f(0) = \lim_{x \rightarrow 0^+} f(x)$$

$$\Rightarrow \lim_{x \rightarrow 0^-} 1 + |\sin\theta|^{\frac{a}{|\sin\theta|}} = b = \lim_{x \rightarrow 0^+} e^{\tan 7\theta / \tan 8\theta}$$

$$\Rightarrow e^a = b = e^{\frac{7}{8}}$$

$$\Rightarrow a = \frac{7}{8} \text{ and } a = \log_e b$$

Hence, $a = \frac{7}{8}$ and $a = \log_e b$.

25. Solution:

Let a be the first term and r be the common ratio.

$$a + ar + ar^2 + \dots + \infty = 8$$

$$\Rightarrow \frac{a}{1-r} = 8$$

Squaring both sides, we get

$$\frac{a^2}{(1-r)^2} = 64$$

$$\Rightarrow a^2 = 64(1-r)^2 \quad \dots (1)$$

Also, $a^2 + a^2r^2 + a^2r^4 + \dots + \infty = 4$

$$\frac{a^2}{1-r^2} = 4 \quad \dots (2)$$

Putting the value of (1) in (2), we get

$$\frac{64(1-r)^2}{1-r^2} = 4$$

$$\Rightarrow \frac{16(1-r)^2}{(1+r)(1-r)} = 1$$

$$\Rightarrow \frac{16(1-r)}{(1+r)} = 1$$

$$\Rightarrow 16 - 16r = 1 + r \Rightarrow 17r = 15$$

$$\therefore r = \frac{15}{17}$$

Putting $r = \frac{15}{17}$ in $\frac{a}{1-r} = 8$, we get

$$\frac{a}{1-\frac{15}{17}} = 8 \Rightarrow a = \frac{16}{17}$$

Hence, first term (a) = $\frac{16}{17}$ and common difference (r) = $\frac{15}{17}$.

26. Solution:

$$\text{Given matrix is } A = \begin{bmatrix} -3 & 3 & 6 \\ 3 & 6 & 9 \\ 9 & 3 & 3 \end{bmatrix}$$

$$\text{Let } A = IA$$

$$\Rightarrow \begin{bmatrix} -3 & 3 & 6 \\ 3 & 6 & 9 \\ 9 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 + R_1, R_3 \rightarrow R_3 + 3R_1$, we get

$$\Rightarrow \begin{bmatrix} -3 & 3 & 6 \\ 0 & 9 & 15 \\ 0 & 12 & 21 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow (-1)R_1$, we get

$$\Rightarrow \begin{bmatrix} 3 & -3 & -6 \\ 0 & 9 & 15 \\ 0 & 12 & 21 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - R_3$, we get

$$\Rightarrow \begin{bmatrix} 3 & -3 & -6 \\ 0 & -3 & -6 \\ 0 & 12 & 21 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & 1 & -1 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - R_2, R_3 \rightarrow R_3 + 4R_2$, we get

$$\Rightarrow \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & -6 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 1 & -1 \\ -5 & 4 & -3 \end{bmatrix} A$$

Applying $R_2 \rightarrow (-1)R_2$, we get

$$\Rightarrow \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 6 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 1 \\ -5 & 4 & -3 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 + 2R_3$, we get

$$\Rightarrow \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ -5 & 4 & -3 \end{bmatrix} A$$

Applying $R_3 \rightarrow (-1)R_3$, we get

$$\Rightarrow \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} A$$

$$\Rightarrow (27) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} A$$

$$\Rightarrow A^{-1} = \frac{1}{27} \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$$

27. Solution:

We have, $(\tan x)^y = (\tan y)^x$,

On taking log both sides, we get

$$y \log(\tan x) = x \log(\tan y) \quad [\because \log m^n = n \log m] \dots (1)$$

On differentiating both sides of (1) w.r.t. x , we get

$$y \cdot \frac{d}{dx}(\log(\tan x)) + \log(\tan x) \cdot \frac{d}{dx}(y) = x \cdot \frac{d}{dx}(\log(\tan y)) + \log(\tan y) \cdot \frac{d}{dx}(x)$$

$$\Rightarrow y \cdot \frac{1}{(\tan x)} \cdot \frac{d}{dx}(\tan x) + \log(\tan x) \cdot \frac{dy}{dx} = x \cdot \frac{1}{(\tan y)} \cdot \frac{d}{dx}(\tan y) + \log(\tan y) \cdot 1$$

$$\Rightarrow y \cdot \frac{1}{(\tan x)} \cdot \sec^2 x + \log(\tan x) \cdot \frac{dy}{dx} = x \cdot \frac{1}{(\tan y)} \cdot \sec^2 x \cdot \frac{dy}{dx} + \log(\tan y)$$

$$\Rightarrow \log(\tan x) \cdot \frac{dy}{dx} - x \cdot \frac{1}{(\tan y)} \cdot \sec^2 x \cdot \frac{dy}{dx} = \log(\tan y) - y \cdot \frac{1}{(\tan x)} \cdot \sec^2 x$$

$$\Rightarrow (\log(\tan x) - x \cdot \frac{1}{(\tan y)} \cdot \sec^2 x) \frac{dy}{dx} = \frac{(\tan x) \cdot \log(\tan y) - y \cdot \sec^2 x}{(\tan x)}$$

$$\Rightarrow \left(\frac{(\tan y) \cdot \log(\tan x) - x \cdot \sec^2 x}{(\tan y)} \right) \frac{dy}{dx} = \frac{(\tan x) \cdot \log(\tan y) - y \cdot \sec^2 x}{(\tan x)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(\tan y) \cdot ((\tan x) \cdot \log(\tan y) - y \cdot \sec^2 x)}{(\tan x) \cdot ((\tan y) \cdot \log(\tan x) - x \cdot \sec^2 x)}$$

Hence, it is proved.

OR

Solution:

Let $P(n): 51^n - 14^n, \forall n \in N$

For $n = 1$, the given expression becomes

$$51^1 - 14^1 = 51 - 14 = 37, \text{ which is multiple of } 37.$$

So, the given statement is true for 1, i. e., $P(1)$ is true.

Let $P(k)$ be true.

Then, $P(k): 51^k - 14^k$ is multiple of 37.

$$\Rightarrow 51^k - 14^k = 37m \text{ for some natural number } m. \dots (1)$$

$$\text{Now, } 51^{k+1} - 14^{k+1} = (51^{k+1} - 51 \cdot 14^k) + (51 \cdot 14^k - 14^{k+1})$$

[On subtracting and adding $51 \cdot 14^k$]

$$= 51(51^k - 14^k) + 14^k(51 - 14)$$

$$= 51 \cdot 37m + 37 \cdot 14^k \text{ [Using(1)]}$$

$$= 37(51m + 14^k), \text{ Which is multiple of } 37.$$

$\therefore P(k + 1): 51^{k+1} - 14^{k+1}$ is multiple of 37.

$\Rightarrow P(k + 1)$ is true, whenever $P(k)$ be true.

Thus, $P(1)$ is true and $P(k + 1)$ is true, whenever $P(k)$ be true.

Hence, by principle of mathematical induction, $P(n)$ is true for all $n \in N$.

28. Solution:

The given function is

$$f(x) = x^3 - 2x + 9$$

On differentiating both sides w. r. t. x , we get

$$f'(x) = 3x - 2$$

On putting $f'(x) = 0$, we get

$$\Rightarrow 3x - 2 = 0$$

$$\Rightarrow 3x = 2$$

$$\Rightarrow x = \frac{2}{3}$$

Now, we find intervals in which $f(x)$ is strictly increasing or strictly decreasing.

Interval	$f'(x) = 3x - 2$	Sign of $f'(x)$
$x < \frac{2}{3}$	(-)	-ve
$x > \frac{2}{3}$	(+)	+ve

We know that, a function $f(x)$ is said to be strictly increasing, if $f'(x) > 0$ and it is said to be strictly decreasing, if $f'(x) < 0$. So, the given function $f(x)$ is:

1. Strictly increasing on the interval $(\frac{2}{3}, \infty)$.

2. Strictly decreasing on the interval $(-\infty, \frac{2}{3})$.

Hence, $f(x)$ is increasing nor decreasing in $(-\infty, \frac{2}{3})$.

29. Solution:

Given curves are $y = \sqrt{x}$ (1)

and $y - x + 2 = 0$ (2)

On solving (1) and (2), we get

$$\sqrt{x} - \sqrt{x}^2 + 2 = 0$$

$$\Rightarrow \sqrt{x}^2 - \sqrt{x} - 2 = 0$$

$$\Rightarrow \sqrt{x}^2 - 2\sqrt{x} + \sqrt{x} - 2 = 0$$

$$\Rightarrow \sqrt{x}(\sqrt{x} - 2) + 1(\sqrt{x} - 2) = 0$$

$$\Rightarrow (\sqrt{x} - 2)(\sqrt{x} + 1) = 0$$

$$\Rightarrow \sqrt{x} = 2 \quad [\because \sqrt{x} = -1 \text{ is not possible}]$$

Hence, required area = $\int_0^2 (x_2 - x_1) dy = \int_0^2 (y + 2) dy - \int_0^2 y^2 dy$

$$\begin{aligned} &= \left[\frac{y^2}{2} + 2y - \frac{y^3}{3} \right]_0 \\ &= 2 + 4 - \frac{8}{3} = \frac{10}{3} \text{ sq. units} \end{aligned}$$

OR

Solution:

Given equations of lines are:

$$\frac{x-1}{2} = \frac{y-3}{\lambda} = \frac{z+1}{-1} \text{ and } \frac{x+1}{\lambda} = \frac{y-1}{2} = \frac{z-2}{2}.$$

The given lines are parallel to the vectors $\vec{b}_1 = 2\hat{i} + \lambda\hat{j} - \hat{k}$ and $\vec{b}_2 = \lambda\hat{i} + 2\hat{j} + 2\hat{k}$ respectively. The lines are perpendicular if $\vec{b}_1 \cdot \vec{b}_2 = 0$

$$\Rightarrow (2\hat{i} + \lambda\hat{j} - \hat{k})(\lambda\hat{i} + 2\hat{j} + 2\hat{k}) = 0$$

$$\Rightarrow 2 \times \lambda + \lambda \times 2 + (-1) \times 2 = 0$$

$$\Rightarrow 4\lambda - 2 = 0$$

$$\Rightarrow 4\lambda = 2$$

$$\text{Hence, } \lambda = \frac{1}{2}.$$