Time: 3 hrs Total marks: 100

General Instructions:

- 1. All questions are compulsory.
- 2. The questions paper consists of 29 questions.
- 3. Questions 1-4 in Section A are very short answer type questions carrying 1 mark each.
- 4. Questions 5 12 in Section B are short-answer type questions carrying 2 marks each.
- 5. Questions 13 23 in Section C are long-answer type I questions carrying 4 marks each.
- 6. Questions 24 29 in Section D are long-answer type II questions carrying 6 marks each.

SECTION - A

- **1.** Find the value of x, when $\sin(\sin^{-1}\frac{3}{5} + \cos^{-1}x) = 1$.
- **2.** Express $(3 + 7i)^2$ in the form of a + ib.
- 3. Evaluate: $\begin{vmatrix} \cos 18^{\circ} & \sin 18^{\circ} \\ \sin 72^{\circ} & \cos 72^{\circ} \end{vmatrix}$

OR

If
$$\begin{vmatrix} x & 10 \\ 5 & 2x \end{vmatrix} = 0$$
, then find the value of x.

4. Evaluate: $\lim_{x\to 1} \frac{x^{45}-1}{x^{40}-1}$.

SECTION - B

5. What is the value of x, when $\tan^{-1}\frac{1}{\sqrt{3}} + \cot^{-1}x$ is $\frac{\pi}{2}$?

6. Simplify:
$$\sec \theta \begin{bmatrix} \sec \theta & \tan \theta \\ \tan \theta & \sec \theta \end{bmatrix} - \tan \theta \begin{bmatrix} \tan \theta & \sec \theta \\ \sec \theta & \tan \theta \end{bmatrix}$$
.

OF

If $A = \begin{bmatrix} \alpha & 1 \\ 1 & \alpha \end{bmatrix}$ and $|A^3| = 512$, then, find the value of α .

7. How many terms are there in the AP: 32,36,40,44,,320?

OR

Find *n* , if $(n + 2)! = 90 \times n!$.

8. Express $\frac{6+\sqrt{5}i}{1-\sqrt{5}i}$ in the form of a+ib.

9. If $f(x) = 256x^4$ and $g(x) = x^{\frac{1}{4}}$, then find gof(x).

OR

If g is the inverse function of f and $f'(x) = \frac{1}{1+x^{19}}$, then, find the value of g'(x).

10. If the Cartesian equation of the line is $\frac{5-x}{7} = \frac{y}{4} = \frac{3-z}{4}$. Find

- 1. Direction ratio of the line and
- 2. Direction cosine of the line is

11. Expand $(x^3 + 4y)^4$ by the binomial theorem.

12. Find the equation of a line for which $\tan \theta = \frac{1}{5}$ and x - intercept is equal to 6 units.

SECTION - C

- **13.** If $f: R \to R$ is given by f(x) = 11x 13, then find $f^{-1}(x)$.
- **14.** Find the intervals in which the function $f(x) = \frac{6}{4}x^4 2x^3 6x^2 + 32$ is
 - 1. Strictly increasing
 - 2. Strictly decreasing
- **15.** How many different words can be formed with the letter of the word 'PUNAM', if begin with P and does not end with M?
- **16.** What is the co-efficient of x^4 in the given expansion of the product $(1+3x)^6(1-x)^7$?
- **17.** Find the distance of the point (1, -3, 6) from the plane x y + z = 6 measured along the line x = y = z.
- **18.** Find the equation of the ellipse for which $e = \frac{3}{5}$ and whose vertices are $(0, \pm 5)$.
- 19. Using properties of determinants, prove that

$$\begin{vmatrix} x+\lambda & 6x & 6x \\ 6x & x+\lambda & 6x \\ 6x & 6x & x+\lambda \end{vmatrix} = (13x+\lambda)(\lambda-5x)^2.$$

If $A = \begin{bmatrix} 2 & 6 \\ 6 & 8 \end{bmatrix}$ and $A^2 - kA - 20I_2 = 0$, then find the value of k.

20. Prove that $\cos^{-1}\frac{12}{13} + \tan^{-1}\frac{4}{3} = \tan^{-1}\frac{63}{16}$.

OR

Solve for x, $\tan^{-1}(x+3) + \tan^{-1}(x-3) = \tan^{-1}\frac{2}{3}$, x > 0.

21. Find the value of x, when $1 + 21 + 41 + 61 + \cdots + x = 622500$

OR

Find the complex root of x, when the given quadratic equation is $x^2 + 3x + 3 = 0$.

- **22.** The Boolean expression $\sim (m \lor n) \lor (\sim m \land n)$ is equivalent to:
- **23.** Raju and Akash are given to solve a mathematical problem. The probability that they will solve this problem is $\frac{1}{3}$ and $\frac{3}{4}$ respectively. Then, find the probability that both, Raju and Akash will solve any random mathematical problem given to them after sufficient time?

SECTION - D

24. Find the value of g'(0), when $f(x) = |\log 11 - \sin x|$ and g(x) = f(f(x)) for all $x \in R$.

OR

If the function

$$f(x) = \begin{cases} (1 + |sin\theta|^{\frac{a}{|sin\theta|}}, -\frac{\pi}{6} < \theta < 0 \\ b, \theta < 0 & \text{is continuous at } x = 0. \text{ Then, find the value of } a \\ e^{tan\,7\theta/tan\,8\theta}, 0 < \theta < \frac{\pi}{6} \end{cases}$$
 and b .

- **25.** The sum of an infinite geometric series is 8 and the sum of the squares of infinite term is 4, then, find the first term and common ratio.
- 26. Using elementary row transformation, find the inverse of matrix

$$\begin{bmatrix} -3 & 3 & 6 \\ 3 & 6 & 9 \\ 9 & 3 & 3 \end{bmatrix}.$$

27. If
$$(\tan x)^y = (\tan y)^x$$
, prove that $\frac{dy}{dx} = \frac{(\tan y)((\tan x) \cdot \log(\tan y) - y \cdot \sec^2 x)}{(\tan x)((\tan y) \cdot \log(\tan x) - x \cdot \sec^2 x)}$.

OR

Using mathematical induction, show that $51^n - 14^n$ is multiple of 37, $\forall n \in \mathbb{N}$.

- **28.** Let f be the function defined by $f(x) = x^3 2x + 9$ is neither increasing nor decreasing in (-1,1), then, prove that f(x) is increasing or decreasing (-1,1). Also, find the interval in which f(x) is:
 - 1. Strictly increasing
 - 2. Strictly decreasing
- **29.** Find the area (in sq units) bounded by the curves $y = \sqrt{x}$, y x + 2 = 0, X axis and lying in the first quadrant.

OR

Find the value of λ , so that following lines are perpendicular to each other $\frac{x-1}{2} = \frac{y-3}{\lambda} = \frac{z+1}{-1}$ and $\frac{x+1}{\lambda} = \frac{y-1}{2} = \frac{z-2}{2}$.