MATHEMATICS

CLASS-6

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FOREWORD

Mathematics is not only limited to its scope as a subject alone, but it also has an important role in understanding other subjects. The main objective of studying mathematics in class VI is to assimilate its universalisation in terms of different characteristics of geometrical figures, the concept of negative numbers (integers) and utilise the already attained understanding of Mathematics in various fields of life.

Mathematics is not a subject to be spoken of or understood. It is a subject where things have to be conceptualised in our minds and when one her/himself solves several problems related to an area, these concepts are strengthened.

Attempts have been made in this book to allow a student to form concepts in Mathematics and establish them for her/himself and also use these concepts in related environments in different fields of life. To obtain this objective the student has to read the book attentively follow the activities described and do them so as to draw conclusions from the experiences. One is also suggested to keep a written record of the activities and observations.

No book is complete in itself. Hence, if readers of this text have suggestions with respect to difficulties in the book and suggestions for improvement are brought forth, they can be very well taken up in favour for the students of this state in future.

We express our heartiest thankfulness and gratitude towards the teachers of several government & private schools, DIETs, professors from colleges & NGOs as well as senior citizens of the state who have steered and guided through the making of the book.

The National Council of Educational Research and Training (NCERT) sets some clear and measurable goals for class 1 to 8th. They are known as 'Learning outcomes'.

We have made some necessary changes in this textbook in reference with 'Learning out comes'. Some new contents have been added and some chapters have been transferred from one class to another. Do not let the teachers and the students get confused.

Director
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The glorious contribution of India to Mathematics

Till now you have studied mathematics and language as two main subjects. While studying mathematics, you must have thought how this subject came into existance? What could have been the contribution of Indians in its evolution?

India has a glorious history in the development of Mathematics. The decimal system that the whole world uses today was evolved in India. We could not decode the graphics of the Indus civilization, but the remains of the Harappan culture tell us that their knowledge of Mathematics, espacially mensuration was quite advanced. The Aryans used the knowledge of mensuration that was developed during the Indus valley civilization in the making of their worship-altars (yagya-vedi).

When research in mathematics in Europe witnessed a dark age, India had scholars in mathematics like Aryabhatt, Brahmagupta, Mahaviracharya & Bhaskaracharya. Who could build the tradition of mathematics & mathematicians for the world. **Aryabhatt** founded the rules of estimating squares, square roots, cubes, cube-roots, areas of rectangle, triangle and sphere. The ratio of circumference and diameter denoted by π (pie) and got the decimal values accurated to the fourth place.

Brahmagupta evolved the tradition of Aryabhatt further and was the first mathematician who divided mathematics into algebra and arithmatic and was the first to use zero in algebra.

Mahaviracharya was also one who had the pride of developing the tradition of mathematics in India. He gave many rules about addition and subtraction of fraction and gave many entertaining examples for fraction.

Bhaskaracharya was the first mathematician who considered zero as a very small number and said that a number divided by zero, gives us infinitive.

Thus, many mathematicians had their great contributions to the field of mathematics that made India and Indian tradition of mathematics great.

FOR THE TEACHERS

What is Mathematics ?

Mathematics is that branch of study which basically includes numbers. The relationship between their characterstics features and spatial understanding. This would also assimilate congruence, angles & their drawings, quality and their quantifications etc.

Mathematics is used essentially not only in all areas of teaching & learning but also has its important role in life. Therefore, Mathematics is one of the core subjects for education at the primary level also. Generally, the basic concept of Mathematics is attained through experience related to concrete objects which is then propogated into abstract thoughts gradually. Understanding of Mathematics, therefore, advances step by step. At every step concepts are made more generalised and the scope of generalisation increases.

The relationship between Language and the learning of Mathematics : The meaning of logic and its proof.

Language plays a vital role in the learning of mathematics. Language is required to understand arithmatical logic and concept as well as to express them. Language also contributes a great extent to concretize. The mathematical concepts which are otherwise abstract or basically experiential.

Language is the basis of developing and using sequential logics along with its great utility in helping one use mathematics in everyday life. Hence, the use of language in its proper form and propogation cannot be denied. Through language we attempt to give a structure to some provable statements of mathematics which are derived from assumptions & concepts. Therefore, the basis of structuring mathematics is logic or justification. It is essential to bring children to experience this process which is a major part of the nature of mathematics and the way it is learnt.

During the study of mathematics in classes VI-VIII, we will begin to learn more wide and relatively more abstract theorems of mathematics.

Number of objects in a group, understanding of numbers and their general arithmatical processing will now move on to generalising number concepts, the concept of variables, proving theorems, extended rules and applied mathematics. Along with these, we shall understand types of shapes and then construct and shall try to find out such objects around ourselves. We shall also start classification of numbers into a presentation and try to draw conclusions from a given data.

The main point about teaching mathematics is that we get ample scope about lending children to solve problems & draw their own conclusions because mathematics cannot be taught by spoken deliverance or discussions by a figurative concept that gets built up in ones own brain and this skill needs to be developed in the students.

The teaching of Mathematics

The study of mathematics is utilised in all activities of life but it is not limited to its use. Though its origination needs concrete experience & concrete objects and the propogation of mathematics is sequentially understood through abstract thoughts. This makes it clear that teaching of mathematics centrally takes care of the fact that learners are able to conceptualise ideas and theories in mathematics. We do not really need to explain questions in mathematics but is more related to solving of problems by students.

Therefore, we should provide help to the students' about widened mathematical concepts, concrete objects and experience taken into consideration while moving ahead. Therefore, for practising every lesson, it should be underlined which mathematical rule you are following up & universalising during the teaching of the lesson. At the end of the lesson, the child should be able to derive that rule and use it in the follow up of the exercises.

General Objectives of Teaching Mathematics at the Upper Primary Level

1. Strengthening of knowledge at the primary level.
2. Developing concepts of commerce, mathematics & elementary statistics.
3. The ability to understand the basics of mathematics and make it useful for everyday activities.
4. To develop the ability to solve easy problems related to commerce, mathematics, trigonometry, mensuration and elementary statistics.
5. To develop the competence of solving geometrical questions, to understand the

relationship between different stages of questions & assumption of logical analysis & mathematical concepts.

6. To understand the primary foundations of elementary algebra.
7. To attain competence, in understanding statistical graphs, pictures, charts, models & their use.
8. Logical ability, derivations & proofs and recognition of patterns.
9. Understanding problems & solving them.
10. Developing awareness about rational integration, similarity, environmental security, ideas of a small family, social evils, social equality.
11. Negative numbers, concept of variable, equation, number groups & their characters and fractions needs to be emphasised because they form the basis of mathematics ahead. Similarly, geometry also begins in this class and hence much practice is to be given at this level for such concepts.

Director
State Council of Educational Research
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Content

Chapter 1

NATURAL NUMBERS

Let us take some examples of counting:

1. Sudha's parents pluck tendu leaves and Sudha helps them to make bundles of 50-50 leaves.
2. Radha helps her mother to sell the vegetables on holidays and keeps account of the items sold.
3. Suresh's father owns a dairy farm. He counts the animals in the morning and evening everyday and also keeps the account of milk.

In the same way, you also do several kinds of counting everyday. Some figures are given below. What names would you give to these groups of figures? Name the figures in the boxes given below. One of the pictures has been named as example:

ACTIVITY 1

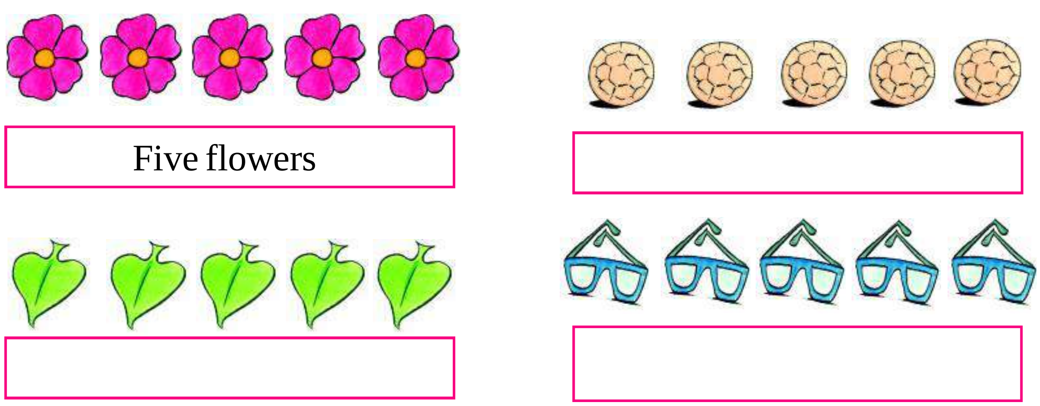


Fig 1

The above figures can be named as five flowers, five balls, five leaves and five goggles or spectacles. This means the numbers obtained by counting are not connected with any particular thing. It is only a thought. This idea is advocated in different languages by different symbols in written form such as five shown in Hindi as ५ , 5 in English and V in Roman. In every number system, each number has a particular symbol.

In ancient times, though men did not have any symbol for counting, yet counting was done by several means. For examples: stones, seeds or putting knots on a rope etc. were used as different means of counting. When things were counted, then in place of each object, one stone, or one seed was kept or a knot was put on a rope. Which is known as one to one correspondence.

If in a class there are 10 desks, for all the 10 desks. 10 chairs are required. Then one desk corresponds to one chair. For each desk one chair is required. So, there is a one to one correspondence between a desk and a chair.

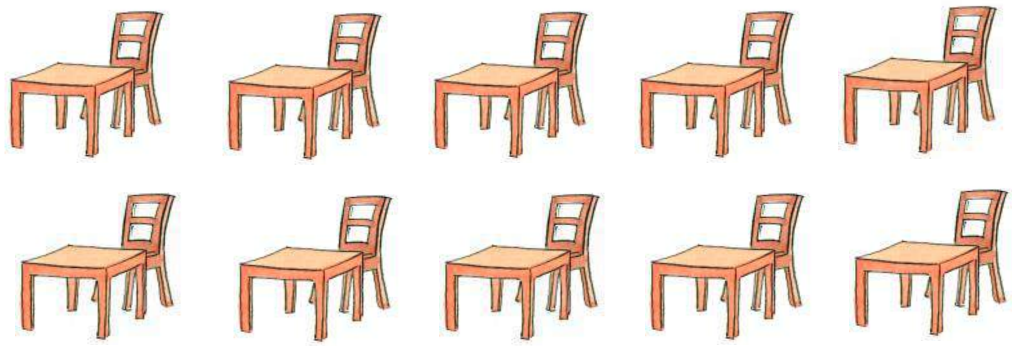


Fig 2

Can you tell the number of students present by counting the school bags in your class?

Since each student is related to one school bag, so, there is one to one correspondence between a student and a school bag. Therefore, 32 school bags in the class would mean 32 students are present in that class.

What are the numbers that you need while counting? From where do you get the numbers that you need for counting? Let us find out the answers for these questions.

For counting, we use 10 symbols: 1,2,3,4,5,6,7,8,9,0. counting begins with 1. These digits are combined to write numbers. These numbers that are used for counting are called NATURAL NUMBERS. The group of natural numbers denoted by "N".

i.e. Natural Numbers (N) = {1, 2, 3, ...}

The smallest Natural Number is 1.

One of the properties of the natural numbers is that each number is greater than the next number by one! Which means that if one is added to each natural number, we shall get the next natural number.

Secondly, the list of names of these natural numbers goes on increasing i,e, the list seems to be endless. Let us verify these two properties by some examples.

ACTIVITY 2

Write the numbers given below in increasing and decreasing orders:

Numbers	**Ascending (Increasing) order**	**Descending (Decreasing) order**
15,12,27,9,13,31,49,18	9,12,13,15,18,27,31,49	49,31,27,18,15,13,12,9
98,33,62,49,107,		
67,78,75,57,25,		
103,113,131,301,331		

On this basis, we can say that 9<12<13<15<18<27<36<49

or 49>31>27>18>15>13>12>9.

Think and find out which will be the largest natural number?

Is there any number greater than 10 lakhs?

Any number that is greater than 10 crore?

Then which will be the largest number?

EXERCISE 1

1. Which is the smallest natural number?
2. Which number is greater – 41600 or 41006?
3. Fill in the blanks by using the correct symbol >,< or =
   (1) 45……. 21
   (2) 543 …….. 345
   (3) 15 …… 15
   (4) 5304 ……5340
   (5) 10991 ……. 10091
   (6) 99876 ……. 99786
4. How many times do we use 9 to write the numbers between 1 to 100?
5. Find the difference between the greatest four digit natural number and smallest three digit natural numbers.

What Have We Learnt ?

1. The numbers that we use for counting are called natural numbers.
2. 1,2,3,4,5,6 ……………… etc. are all natural numbers.
3. We denote natural numbers by 'N', Thus, N= {1,2,3,4,5 etc.}
4. The smallest natural numbers is 1.
5. By adding 1 to a natural number, The next natural number can be obtained.
6. The greatest natural number can never be obtained, which means, every time 1 is added to a natural number, the next number will be obtained and the chain goes on.

Fig 3

Chapter 2

WHOLE NUMBERS AND OPERATIONS WITH WHOLE NUMBERS

Whole Number

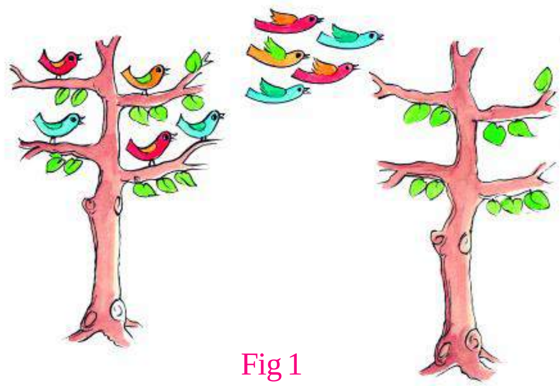


Fig 1

In the previous lesson, we have learnt about counting of numbers and natural numbers.

1, 2, 3, 4, 5.....etc. are natural numbers. Can you say what will be the remainder when any natural number is subtracted from the same natural number? Yes, the remainder will always be zero.

For example 2-2=0, 5-5=0. Is the zero (0) that is obtained here, a natural number?

No, zero is not a natural number. But we need this number. Suppose, 5 birds are sitting on a tree and all the five birds have flown away, then how many birds will remain sitting on the tree?

To answer this question, along with counting, we will need the number zero also. The group of numbers in which zero is included in the process of counting are known as WHOLE NUMBERS.

Whole numbers are denoted by "W".

i.e. whole numbers (W) = {0, 1, 2, 3, 4, 5, ...etc.}

Let us try to understand the concept of zero.

1. Sangeeta had Rs.10/- she brought a copy for Rs.7/- and a pen for Rs.3/-. How much money does she have now?

i.e. 10-7=3 (less the cost of a copy)

3-3=0 (less the cost of a pen)

Sangeeta has zero rupees left with her now, which means she has no money remaining. This state is denote by 0.

2. Ramu's mother gave 5 Laddus to Ramu. Ramu gave 2 Laddus to Mohan to eat and Ramu ate 3 Laddus himself. How many Laddus remain with Ramu?
3. Rahim has a note book of 100 pages. He has written Maths in 80 pages and Science in 20 pages. How many pages remain blank in his note book?

Representing Whole Numbers on A Number Line

To show whole numbers on a number line, we draw a straight line as in the example given below, on which many marks are put at equal distances.

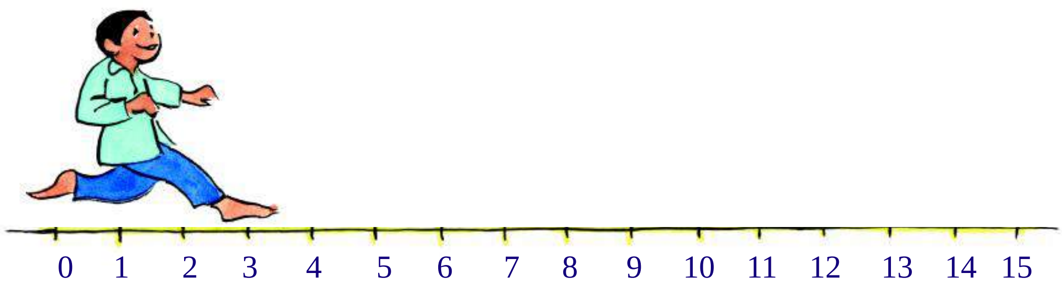


Fig 2

In this figure, the initial point is indicated by a “0”. To the right of the zero (0), we write the numbers 1, 2, 3, 4 in order on the points marked at equal distances. Now, looking at this number line, can you tell, which number is greater? Think, whether the number on the left of any number would be greater than that number or smaller?

The Properties of Whole Number

You now know that 0, 1, 2, 3, 4, 5….etc. are whole numbers. Now, let us study its properties.

1. All the properties of natural numbers are applicable to Whole Numbers.
2. The smallest Whole Number is zero (0).
3. On a number line, the numbers to the right of zero (0) are in increasing (ascending) order.

i.e. 0+1=1, 1+1=2, …………….. 101+1=102, 102+1=103, 103+1=104, etc.

4. On the number line if we move from the right towards left, we find that the numbers are in decreasing (descending) order, like 4, 3, 2, 1, 0.
5. No last or greatest whole number can be shown because if you think of a very-very big number, even then on adding 1 to that number, you can get the next number, that is the succeeding number.
6. The predecessor of 50 is 49, the predecessor of 17 is 16. Is there any whole number that precedes zero?

Operations on the Number Line

1. Addition of Whole Numbers

ACTIVITY 1

Show 3+2=5 on the number line.

Steps:

1. Draw a number line.
2. Move 3 steps to the right of zero (reach 3).
3. Now, move 2 steps to the right from 3.
4. Now, the distance from 0 position is 5 steps. Therefore 3+2=5.

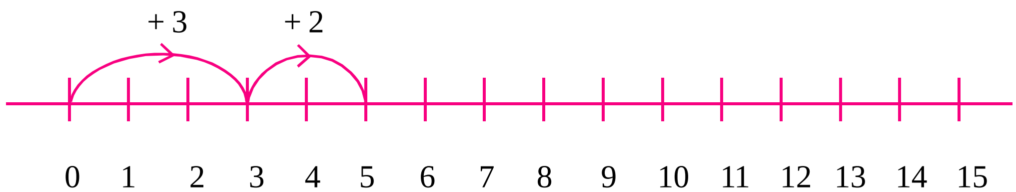


Fig 3

Practice 1

a. Now, draw number lines yourself and check the following questions.

1) 4 + 5 2) 6 + 4 3) 5 + 7

b. Is 3 + 4 = 4 + 3. Verify this on the number line.

2. Subtraction of Whole Numbers

From a greater whole number, the smaller whole number can be subtracted. If a whole number is subtracted from the same number, we get 0. Let us do another activity.

ACTIVITY 2

How to show 8 - 5 = 3 on the number line?

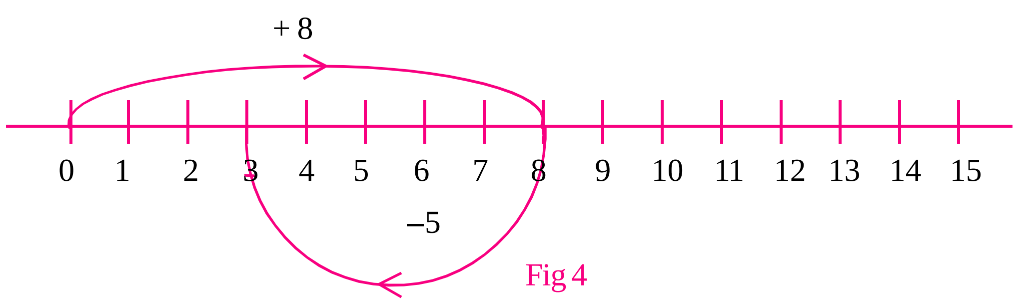


Fig 4

Steps :

1. Draw a number line.
2. Move 8 steps from 0 towards right.
3. Now, move back towards left from 8 by 5 steps (because on subtraction we move back towards left).
4. The position obtained is 3 steps to the right of 0. So, the answer we get is 8 - 5 = 3.

   Suppose, a greater number is subtracted from the smaller number, shall we get a whole number?

Practice 2

Draw number line and verify the following :

(i) 6 - 2 (ii) 7 - 4 (iii) 8 - 3

3. Mutiplication of Whole Numbers

The multiplication of whole numbers can be represented on the number line.

Example : 3 × 4 = 12 or 3 + 3 + 3 + 3 = 12

Multiplication is repeated addition of a number. Let us do this on the number line.

ACTIVITY 3

First of all, draw a number line.

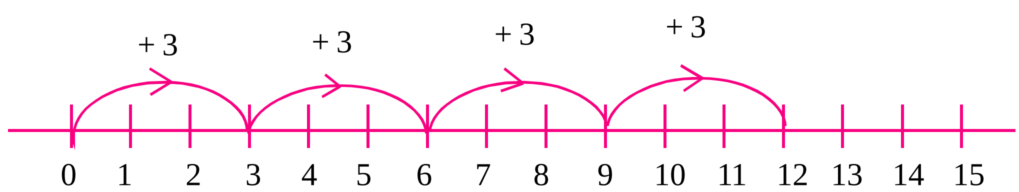


Fig 5

When we move 3 ,3 steps four times from 0, we represent it by moving from 0 to 3, 3 to 6, 6 to 9 and 9 to 12.

Therefore, 3 × 4 = 12.

Practice 3

1. Represent the following on the number line.

   (i) 4 × 3 (ii) 3 × 2 (iii) 0 × 2

   (iv) 2 × 3 (v) 3 × 3

4. Division of Whole Numbers

Can you tell how many times shall we move towards left in 3 , 3 steps from 12 so that we reach to zero? To find out this, let us take up an activity.

ACTIVITY 4

You know that division is repeated subtraction.

Therefore, In 12 ÷ 3 we shall have

12 - 3 = 9 (once)

9 - 3 = 6 (twice)

6 - 3 = 3 (three times)

3 - 3 = 0 (four times)

So, if we move 3,3 steps from 12 four times, we shall reach zero.

Therefore, 12 ÷ 3 = 4.

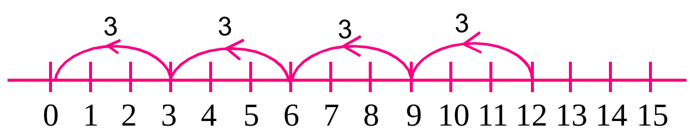


Fig 6

Can you represent on the number line and verify whether 8 is completely divisible by 3?

Practice 4

1. Show the following divisions of whole number on the number line.

   (i) 8 ÷ 2 (ii) 8 ÷ 4 (iii) 8 ÷ 1 (iv) 8 ÷ 8

Face and Place Value

For counting we use the ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. This system is called the decimal system. In decimal system, the value of a digit in the ten's place is 10 times the unit's place. The place value of a digit in the hundred's place is 10 times, the place value of ten's place. The place value of a digit in the thousand's place is 10 times the place value of the hundred's place.

We can thus count places for bigger numbers in the number system ahead.

Example. $769 = 7 \times 100 + 6 \times 10 + 9 \times 1$

In the number 769, the place value for 7, 6 and 9 are 700, 60 and 9 respectively.

Plate	**Hundreds**	**Tens**	**Units**
Place valvue	$7 \times 100 = 700$	$6 \times 10 = 60$	$9 \times 1 = 9$

Hence in expanded notation 769 is written as 700 + 60 + 9.

Example 1.

Find the place value of 5 in the number 4579.

Solution :

In the number 4579, 5 is in hundred's place.

$\therefore$ The place value of $5 = 5 \times 100 = 500$.

Example 2.

Verify the number 3214 with the help of place value of each digit in the number.

Solution :

In the number 3214, 4 is in the unit's place. Similarly, 1 is in ten's, 2 is in hundred's and 3 is in the thousand's place.

Thus, the place value of $4 = 4 \times 1 = 4$

the place value of $1 = 1 \times 10 = 10$

the place value of $2 = 2 \times 100 = 200$

the place value of $3 = 3 \times 1000 = 3000$

Verification:

$$\begin{aligned} 3214 &= 3000 + 200 + 10 + 4 \\ &= 3214. \end{aligned}$$

Example 3.

Find out the number next to 393237310.

Solution :

The number that comes after 393237310 is one more than this number

$= 393237310 + 1$

$= 393237311$

Example 4.

Find out the number that comes before 393237310.

Solution :

The predecessor or number before 393237310 will be 1 less than this number

$= 393237310 - 1$

$= 393237309$

EXERCISE 2.1

1. What is the smallest natural number?
2. Find the whole number which is not a natural number?
3. Which whole number comes before 5?
4. Write three consecutive numbers after 45.
5. Which is greater, 41608 or 41806?
6. Which of the following statements are true or false.
   (i) The smallest natural number is zero. (..................)
   (ii) The smallest whole number is zero. (..................)
   (iii) If any natural number is subtracted from the same natural number, we get zero. (..................)
   (iv) In 4215, the place value of 2 is 200. (..................)
   (v) In 4215, the face value of 2 is 2. (..................)
   (vi) The greatest whole number cannot be obtained. (..................)
   (vii) In 3857, 8 is in the thousands place. (..................)
7. Write the predecessor of the given number -
   (i) 25 (ii) 79 (iii) 520 (iv) 1100 (v) 52332
8. Write the successor (the number that comes after) of the numbers -
   (i) 25 (ii) 79 (iii) 520 (iv) 1100 (v) 52332
9. Write the smallest six digit whole number.
10. Write the greatest five digit whole number.
11. Find the difference between greatest five digit number and smallest six digit number.
12. Write the number in increasing (ascending) order -
    252, 557, 18, 421, 497, 731
13. Write the number in decreasing (descending) order -
    252, 458, 69, 59, 617
14. Which of the following numbers is 7 lac 5 thousand and six-
    (i) 750006 (ii) 705006 (iii) 7005006
    (iv) 75006

15. Write the natural number for the expanded form -

$6 \times 1000 + 3 \times 100 + 8 \times 10 + 7 \times 1$

16. Represent on a number line to check whether the solution is correct -

(i) a. $4 + 3 = 7$ b. $3 + 4 = 7$

c. $0 + 2 = 2$ d. $2 + 0 = 2$

e. $4 + 3 = 3 + 4$

(ii) a. $4 - 3 = 1$ b. $7 - 4 = 3$

c. $6 - 2 = 4$ d. $10 - 5 = 5$

e. Verify $5 - 2$ and $2 - 5$.

(iii) a. $2 \times 3 = 6$ b. $3 \times 2 = 6$

c. $2 \times 5 = 10$ d. $5 \times 2 = 10$

(iv) a. $6 \div 2 = 3$ b. $8 \div 4 = 2$

You know that the sum of two whole numbers is always a whole number. This is the **closure property** of whole numbers.

If the multiplication of two whole numbers is always a whole number, then the whole numbers follow the closure property of whole numbers. Similarly, the rule will exist for divisions of two whole numbers giving always a whole number as the quotient and for the difference of two whole number is always a whole number, then the rule will follow for subtraction too.

Let us see, which of operations follow the closure property of whole numbers.

ACTIVITY 5

You are given a list of the whole numbers, look at the numbers filled in the table as example and fill in the rest of the blank spaces yourself after serial number 8. (See table in next page)

Observe the above table and verify in which of the operations the result is always a whole number and in which it is not a whole number. Also think about the conclusion we draw from this.

It is clear that whole numbers are added, the sum is always whole number and the product of two whole numbers is also always a whole number. But the subtraction and quotient of two whole numbers divided, may not always be a whole number. So the subsequent rule follows the property of whole numbers of addition and multiplication but the rule is not follow the process of subtraction and division.

Activity 5

S. No.	First whole number	Second whole no.	First W.N.+ Second W.N.	Sum is a whole no. Yes/No	First W.N.- Second W.N.	Diffi. is a whole no. Yes/No	First W.N. × Second W.N.	Product is a whole no. Yes/No	First W.N.÷ Second W.N.	Quotient is a whole no. Yes/No
1.	3	12	$3 + 12 = 15$	Yes	$3 - 12 = ?$	No	$3 \times 12 = 36$	Yes	$3 \div 12 = ?$	No
2.	39	13								
3.	5	0								
4.	0	8								
5.	11	15								
6.	99	1								
7.	27	9								
8.	111	11								
9.										
10.										
11.										
12.										

The Other Properties of Operation on Addition

Let us consider three friends A, B and C. In another situation, in one situation, A and B meet first then they both meet C. B and C meet first, then they meet A. What is difference between these two situations? Are these two conditions the same? In these two conditions, A, B and C meet together. If any two conditions have similar nature, then in mathematics, they are said to follow the Associative rule. Is this property true for addition of whole numbers?

Let us look at one example :-

Let us take the numbers 3, 4 and 5.

First add (3 + 4) and to their sum, add 5, the result will be -

$(3 + 4) + 5 = 7 + 5 = 12.$

Now, add 3 to the sum of (4 + 5), the result will be -

$3 + (4 + 5) = 3 + 9 = 12.$

So, what do you see? Are the sums equal in both the conditions?

ACTIVITY 6

Check the associative law for addition, for the following numbers -

1. 2, 3, 4
2. 6, 7, 8
3. 0, 1, 2
4. 4, 13, 17, 20

Is this relation true for subtraction also?

$(13 - 6) - 5 = 13 - (6 - 5)$

Are they equal? Verify.

You will observe that Associative Law is not true for subtraction.

Take 3 examples of each addition and subtration and examine the Associative Law.

Practice 5

Fill in the blanks with the correct whole numbers-

1. $(4 + 6) + 5 = \boxed{}$
2. $4 + (6 + 5) = \boxed{}$
3. $12 + (6 + \boxed{}) = 20$
4. $(\boxed{} + 6) + 2 = 20$
5. $(8 + 9) + \boxed{} = 25$
6. $(12 + 8) + \boxed{} = \boxed{} + (8 + 10)$
7. $(6 + 2) + \boxed{} = \boxed{} + (2 + 3)$

The Study of Multiplication

1. Fill in the blanks for following multiplications :-
   To check whether
   whole number × whole number = the product, whole number or not.

Example -

$7 \times 9 =$ $\boxed{\text{63, whole number}}$ $8 \times 12 =$ $\boxed{}$

$23 \times 15 =$ $\boxed{}$ $12 \times 0 =$ $\boxed{}$

$0 \times 12 =$ $\boxed{}$

Can you think two whole numbers whose product is not a whole number.

ACTIVITY 8

On the basis of multiplication (×), put the correct whole number in the boxes, In some boxes, the solution is given.

×	0	1	2	3
0	0			
1			2	
2		2		
3				9

The Commutative Law

$12 \times 5 = \boxed{60}$

Now, if we change the order of these numbers, we have

$5 \times 12 = \boxed{60}$

In both the conditions, are the products equal?

If $357 \times 486 = 173502$, then find without multiplying that the value of

$486 \times 357 =$ $\boxed{}$?

Practice 6

Fill in the blanks :-

(i) $87 \times 887 = 887 \times \boxed{}$

(ii) $279 \times \boxed{} = 481 \times 279$

(iii) $303 \times 117 = \boxed{} \times 303$

(iv) $\boxed{} \times 583 = 583 \times 179$

Test of Associative Law for multiplication

You have learnt the multiplication of two numbers, let us now verify the multipliaction of three numbers.

Take three numbers 2, 5 and 6, multiply them in different ways and write their products in the boxes.

$2 \times (5 \times 6)$	=	$\square$	$(2 \times 5) \times 6$	=	$\square$
$5 \times (6 \times 2)$	=	$\square$	$(5 \times 6) \times 2$	=	$\square$
$6 \times (5 \times 2)$	=	$\square$	$(6 \times 5) \times 2$	=	$\square$
$2 \times (6 \times 5)$	=	$\square$	$(2 \times 6) \times 5$	=	$\square$
$5 \times (2 \times 6)$	=	$\square$	$(5 \times 2) \times 6$	=	$\square$
$(6 \times 2) \times 5$	=	$\square$	$6 \times (2 \times 5)$	=	$\square$

Are the product of all the boxes different? If not, then we can multiply 3 numbers in different ways and see that the result is the same. This is called the Associative Law, you can take any three other numbers and test the law. (Note that, In the process of multplication, first the digits in brackets are multiplied and then the product is multiplied to the digit outside the brackets.)

Practice 7

Fill in the blanks -

(i) $4 \times (5 \times 6) = (4 \times \square) \times 6$ (ii) $8 \times (4 \times 2) = \square \times 2$

(iii) $3 \times (7 \times 5) = (3 \times \square) \times 5$ (iv) $2 \times (8 \times \square) = 8 \times (\square \times 4)$

(v) $7 \times (3 \times 5) = 7 \times (\square \times 5)$

Divisor, Dividend, Quotient and Remainder

You have already learnt these in previous classes, let us revise -

Example 1.

$20 \div 5$

$$\begin{array}{r} 5)\ 20\ (4 \\ 20 \\ \hline 00 \end{array}$$

Here, 5 is divisor, and 4 is quotient.

Is there any relationship among divisor, dividend and quotient?

$$20 = 5 \times 4$$

Dividend = divisor × quotient.

Example 2.

21 ÷ 5

Dividend

Divisor → 5) 21 (4 ← Quotient

J 20

×1 ← Remainder

1. The number which is divided is called dividend, 21 is dividend.
2. The number by which any number is divided is called the divisor, 5 is divisor.
3. The number of times it gets divided is called the quotient, 4 is quotient.
4. After division a number that is smaller than the divisor remains. This is called the remainder. Here 1 is remainder.

Therefore, $21 = 5 \times 4 + 1$

Now, Divide 22 by 5

5) 22 (4

J 20

×2

Here, 5 is divisor, 22 is the dividend, 4 is the quotient and 2 is the remainder.

Write the relationship amongst, divisor, dividend, quotient and remainder and test in your notebooks the relationship with the help of an exercise.

Practice 8

Divide

(i) $48 \div 7$ (ii) $36 \div 5$ (iii) $78 \div 9$

In this way, the law you have made is known as the Law of Divisibility, which is -

Divident = Divisor× Quotient + Remainder

Can the remainder be greater than divisor?

Practice 9

Fill in the blanks -

(i) In $8 \div 4$, the quotient is = - - - - - - - - - , remainder = - - - - - - - - -

(ii) In $5 \div 2$, the quotient is = - - - - - - - - -, remainder = - - - - - - - - -

(iii) In $6 \div 4$, the quotient is = - - - - - - - - -, remainder = - - - - - - - - -

(iv) In $7 \div 2$, the quotient is = - - - - - - - - -, remainder = - - - - - - - - -

The Properties of Zero

Let us now learn about zero (0) -

1. $5 + 0 = 5$
2. $0 + 5 = 5$
3. $5 - 0 = 5$
4. $5 \times 0 = 0$
5. $0 \times 5 = 0$
6. $0 \div 5 = 0$
7. $5 \div 0$ = No solution.

Practice 10

1. Fill in the blanks with the correct numbers -

(i)	$0 + 0$ = - - - - - - - - -	(ii)	$0 - 0$ = - - - - - - - - -
(iii)	$7 + 0$ = - - - - - - - - -	(iv)	$0 + 7$ = - - - - - - - - -
(v)	$7 - 0$ = - - - - - - - - -	(vi)	7×0 = - - - - - - - - -
(vii)	0×7 = - - - - - - - - -	(viii)	$0 \div 7$ = - - - - - - - - -

From the above exercise you have understood the following properties of zero.

1. If '0' is added to any whole number, the value of that number remains unchanged.So **"0"** is known as **additive identity**. Think and write four examples.
2. If '0' is subtracted to any whole number, the value of that number remains unchanged. Write four examples.
3. If '0' is multiplied by whole number, then the product is zero.
4. If '0' is divided by whole number, then the quotient is zero.

   $5 \div 0 = ?$ We may subtract 0 from 5, as many times, we wish, but we always get the same number. We repeat this process many times, but the number remains unchanged.

   This means, if we divide any whole number by zero, we will not get any particular number as the quotient. In the same way $0 \div 0$ is also not defined.

   Discuss this with your teacher.

EXERCISE 2.2

Oral Questions

Q.1 On the basis of the following information, solve the following without multiplying or adding -

(i)	$17 \times 23 = 391$	then,	23×17 = ...
(ii)	$15 + 25 = 40$	then,	$25 + 15$ = ...
(iii)	$40 + 0 = 40$	then,	$0 + 40$ = ...
(iv)	$39 \times 1 = 39$	then,	1×39 = ...
(v)	$a \times b = c$	then,	$b \times a$ = ...

Q.2 Add the given number by putting them in such an order that the addition becomes easy.

(i)	23589 + 411 + 1248	(ii)	32 + 2546 + 68 + 544
(iii)	247 + 376 + 153	(iv)	143 + 456 + 857
(v)	32958 + 5000 + 12042		

Q.3 Which whole number will be obtained if "0" is multiplied to any number?

Q.4 What is the closure property for addition?

Q.5 Fill in the blank on the basis of characteristics/properties of operations.

(i) 2376+4559= .... + 2376

(ii) 1×0= 0×1= ....

(iii)
$$\begin{array}{r} 8\;\;7\;\;6 \\ -\;\square\;\;3\;\;\square \\ \hline 6\;\;\square\;\;7 \\ \hline \end{array}$$

Q.6 Which is the number, that when divided by the same number, gives the same number again?

Q.7 What is the product of the largest 4-digit number and the smallest 1-digit?

Q.8 If 76 × 16 = 1216, then 1216 $\square$ 76 = 16 (Put right symbol in box.)

Written Questions :

Q.9 Rama planted a total 544 plants in 17 rows. How many plants did she put in each row?

Q.10 In a city, out of every 15 people, 1 person is a government servant. If 1354 persons in that city are in government jobs, what is the total population of that city?

Q.11 Find out the quotient and remainder and verify the Law of Divisibility.

(i) 7772 ÷ 36 (ii) 12425 ÷ 835 (iii) 92845 ÷ 300

Q.12 Write down a suitable number in every blank.

(i)
$$\begin{array}{ccc} 7 & 3 & 5 \\ -4 & 2 & \square \\ \hline \boxed{\ldots} & \boxed{..} & 6 \end{array}$$

(ii)
$$\begin{array}{cccc} 4 & 9 & 3 & 1 \\ -\boxed{\ldots} & \square & 7 & 8 \\ \hline 1 & 8 & \boxed{..} & \boxed{..} \end{array}$$

Q.13 Manjulata went to the market with Rs.1800/-. She bought a purse for Rs.135/- , a handkerchief for Rs.75/- and a gold chain for Rs.1200/-. Find out how much money remained with her after the purchasing.

Q.14 Ashok deposited Rs.4539/- in the bank on Tuesday. On Saturday, he withdrew Rs.2556/- and the next week, he again deposited Rs.1431/-. How much money does he have in his bank account?

Q.15 In a Model school, the fees for a student of class VI is Rs.95/-. If the number of students is 335, find out total amount to be paid as fees?

Q.16 The product of two numbers is 117. If one of the numbers is 13, find the other one.

Q.17 Nisha purchased 24 radio sets for Rs.18720/-. If all radio sets are of the same cost, find out the cost of one radio set.

Q.18 For the given magic square, add the numbers in the square, vertically, horizontally and diagonally.
Is the sum the same every time?

14	1	9
3	8	13
7	15	2

Q.19 Fill in the blanks in the given magic square. Remember it the addition of numbers in the square, vertically, horizontally, diagonally should be same.

22		6	13	20
	10	12	19	
9	11	18	25	
15	17	24	26	
16			7	14

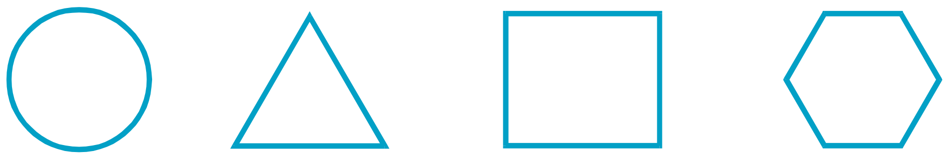
What Have We Learnt ?

1. Zero is a whole number.
2. Two Whole Numbers when added or multiplied give another Whole Number.
3. The Commutative Law is applicable to operations with addition multiplication for Whole Numbers but not applicable to operations with subtraction and division.
4. The Associative Law is applicable to operations with addition and multiplication in Whole Numbers but is not applicable to operations with division and subtraction.
5. Zero is known as the additive elements.
6. 1 is known as the identical multiplication elements.
7. 0 added to or subtracted from any Whole Number does not change its value.
8. If 1 is multiplied to any Whole Number, the value of the number remains unchanged.
9. If 0 is multiplied to any Whole Number, its value becomes 0.
10. Any whole number divided by zero, is not defined.
11. Divident = divisor × quotient + remainder.

Chapter 3

LINE SEGMENT

In learning Mathematics, you have already come across various kinds of figures like the circle, triangle, quadrilateral, hexagon etc.



Can you dawn a circle in your notebook?

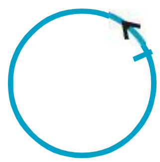


Fig 1

If you move a pencil along the circle that you have drawn, you will observe that:

1. You start at a point and reach the same point again.
2. While starting at a point on the circle and coming back to the same point again, you don't have to pass twice through any point.

Such figures with the two above mentioned properties are known as closed figures. Now look at another figure.

P

Fig 2

Is "P" a closed figure?

No, because starting at any point on the figure "P", to reach or come back at the same point again without lifting your pencil, you will have to pass through some points on the figure two times.

Now, look at the figures given below and say whether they are open or close figures. Also identify whether they are made up of straight lines, curved lines or both.

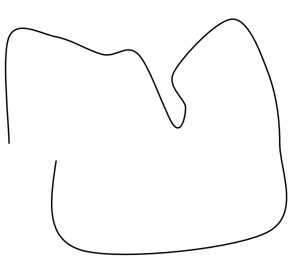


Fig 3

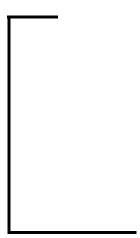


Fig 4

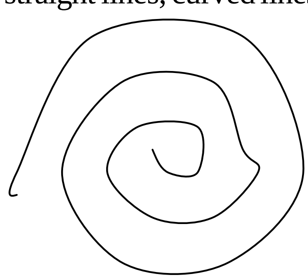


Fig 5

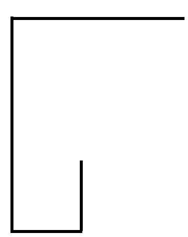


Fig 6

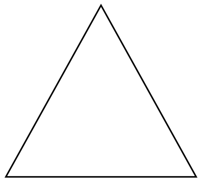


Fig 7


Fig 8

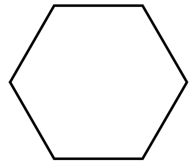


Fig 9

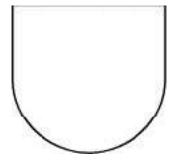


Fig 10

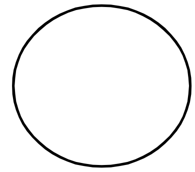


Fig 11

Write about figures in the following table -

S. No.	Fig No.	Close curve or Open curve	Made up of straight lines or curve lines or both type of lines
1.	3	Open curve	Curve lines
2.	4		
3.	5		
4.	6		
5.	7		
6.	8		
7.	9		
8.	10		
9.	11		

Can you make a list of figures that can be seen around you which are made up of straight lines and curved lines?

Till now you have used two types of lines- one is a curved line that is uneven and the other is straight line, about which you have already studied in your previous classes. Let us now know something more about straight lines.

You all know how to draw a straight line. Can you draw a horizontal straight line on the blackboard? This straight line can be as long as the width of the blackboard on which you have drawn it. Now, suppose we increase the width of the blackboard two times of what it is now, then the straight line can also be extended two times its length. If the blackboard is extended more and more and more. This means a straight line can be increased in length from both sides endlessly.

Therefore, a straight line is a non curve line which extended at both ends will never come to stop. Can you draw a straight line on your note book?

If you can draw it, describe how you drew it and if you can't, say why you can't draw it.

In trying to draw a straight line you have found that you can draw straight line only as long as your note book, but this line doesn't ever come to an end, so how can we draw it in our note books?

We really can't. We can only draw it symbolically. Can you suggest some way in which a straight line can be represented in your note book ?

Your suggestion could be:

……………………………………

Drawing A Straight Line Using A Symbol

Fig 12

In figure 12, a straight line is shown with two arrow heads at both its ends. The arrow heads on both the sides symbolize the fact that the straight line has no end point, it doesn't come to a stop and moves on, and on.

Similarly, if we draw a straight line starting from a stated point that moves on in one direction without coming to a stop, we then call it a **ray**. Draw a **ray** in your note book and write the description of how you drew it. Since a ray begins at a particular point and moves on and on, therefore a ray is shown with an arrow head on one side of the straight line.

A ray starting at point "O"

Fig 13

In other words we can say that a ray is a straight line that has only one end point.

Thus, a straight line that begins at a point and moves on in one direction is known as a ray. It doesn't have fixed length.

Think of other examples of rays in your daily life and write them down.

ACTIVITY 1

Can you identify the straight lines and ray in the following figures :

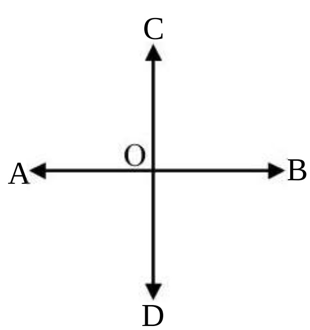


Fig 14

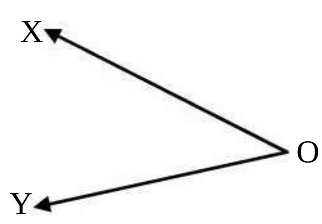


Fig 15

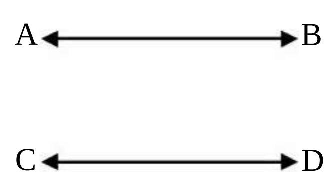


Fig 16

Write about the figures as you identify them.

ACTIVITY 2

You are given two points below. How many straight lines can you draw passing through point P? Draw them. How many straight lines can you draw from point Q? Draw such lines.

Fig 17

Answer the following :

1. Can you count the number of straight lines passing through point P?

   ----

2. Can you count the number of straight lines passing through point Q?

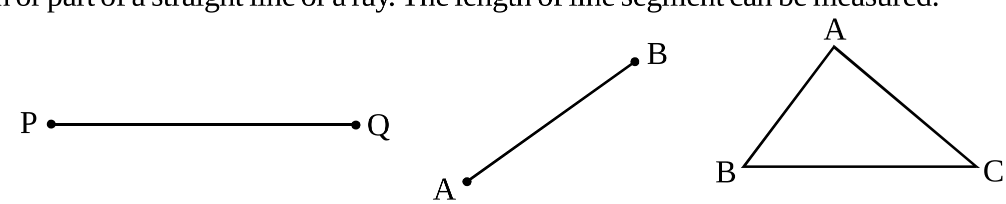
   ----

3. How many straight lines can you draw passing through both points P and Q?

From the above activities, you can say that from a given point we can draw infinite number of straight lines. But, through two given points, we can draw only one straight lines.

A Line Segment

Now look at these straight lines that are used in making this figure begin from a definite point and end at definite points. Such lines are called line segments. so, a line segment is a portion or part of a straight line or a ray. The length of line segment can be measured.



ACTIVITY 3

Study the figures 18 and 19 and answer the questions given below:

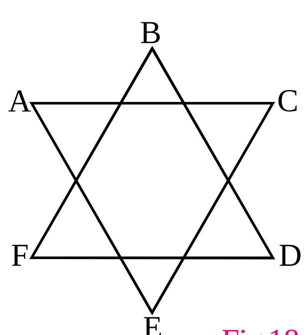


Fig 18

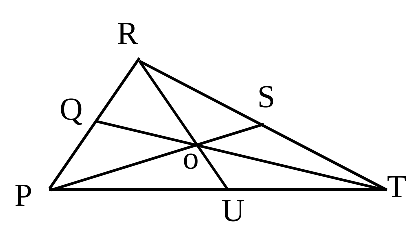


Fig 19

Identify the lines segments in figure18. Write the name of all the line segments. How many line segments are there?

How many points of intersection can be seen in figure 18 and 19? Which line segment intersect the other and at which points?

What is a line segment? Write in your words.

ACTIVITY 4

Take two points P and Q. The points are joined in three ways in the figure, you can try more ways.

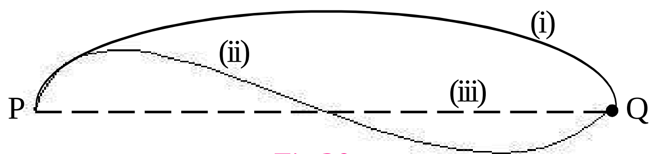


Fig 20

Points P and Q can be joined in many ways. But they can be joined only in one way by a line segment. So, a line segment is the smallest line to join two points.

Fig 21

Meena says that the smallest distance between P and Q is a line segment. Do you agree? Why?

Collinear Points

Observe the points located on a surface-

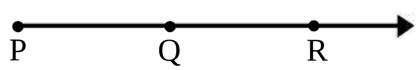


Fig 22

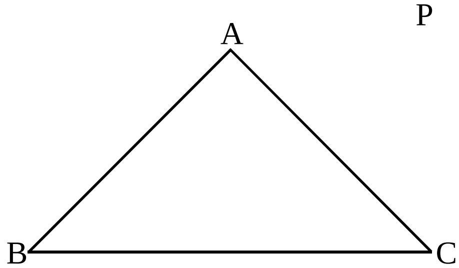


Fig 23

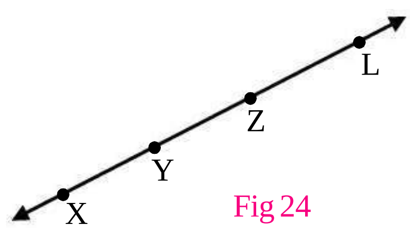


Fig 24

In fig.22, Points P, Q and R are on the same straight line. They are called collinear points.

In fig.23, Points A, B and C are not on the same straight line. They are not called collinear points. Why?

In fig.24, Points X, Y, Z and L are on the same straight line. Are they called collinear points?

Comparison of Line Segments

The comparison of two line segments means to find which line segment is longer?

(1) To compare by observation-

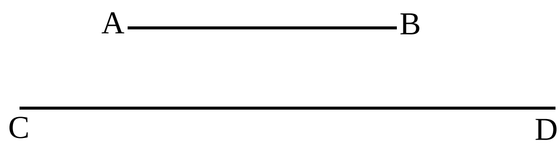


Fig 25

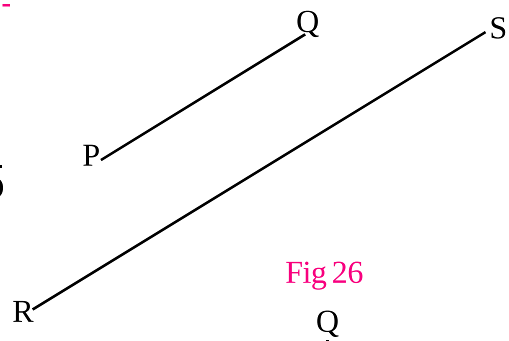
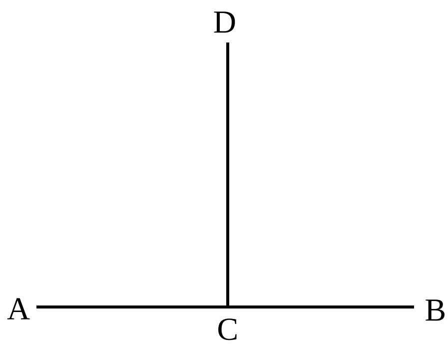


Fig 26



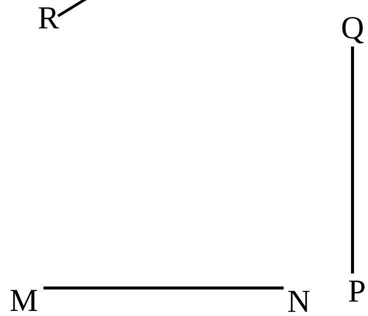


Fig 27

Fig 28

Just by looking at the figures 25 and 26, it can be said that line segment CD is longer than AB and RS is longer than PQ or in fig 25, CD > AB and in fig 26, PQ < RS. In fig 27 and 28, can you find out, which line segment is bigger? Let us measure them.

(2) Correct measurement with the help of a scale-

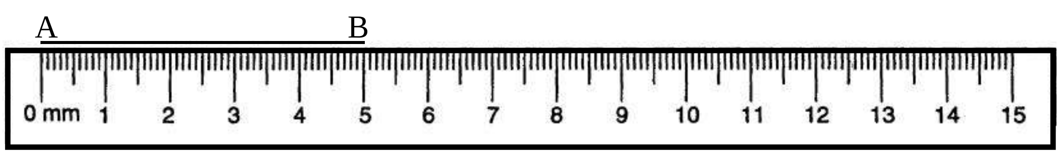


Fig 29

In fig 29, the length of line segment AB is shown with the help of a scale.

Method of Measurement

Let us measure the line segment AB. We place a ruler with its edge along the line segment AB in such a ways that the zero mark of the ruler coincides with the point A. Then we read the mark on the ruler which is against the point B. This gives us the length of the line segment AB (fig 27). Thus the lengths of line segment is 5cm or we write $\overline{AB} = 5cm$.

(3) Measurement by Divider

You have seen that the length of a line segment can be measured with the help of a scale where the scale can be put on a straight surface. Can we measure the distance between inner walls of a box or a glass with the help of scale?

Take a divider, put it on the mouth of the tumbler stretching it in such a way that the point of divider touches the inner walls of the glass.

Now take out the divider without disturbing it on the scale as shown in fig 31 and measure the length.

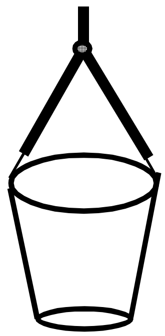


Fig 30

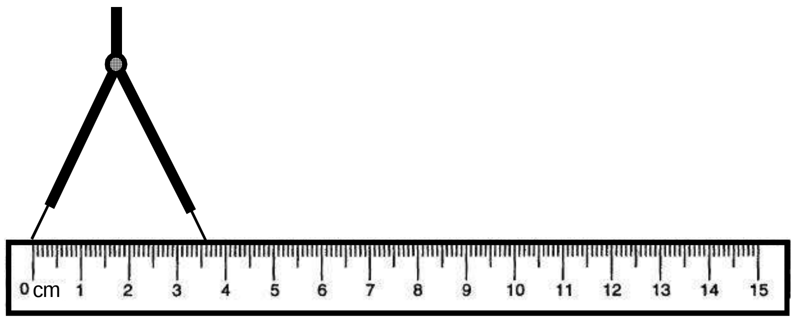


Fig 31

For the teacher - It should be noted that beginning the measurement from any point of the scale, does not affect the measure of the length using the old scale, not starting from zero, how can you measure any length? Discuss this with students. Let them practice measuring any length starting at any point on the scale.

ACTIVITY 5

Compare the measures of line segments in the following figures with the help of divider or compasses. Write the measures of each figure in their increasing order.

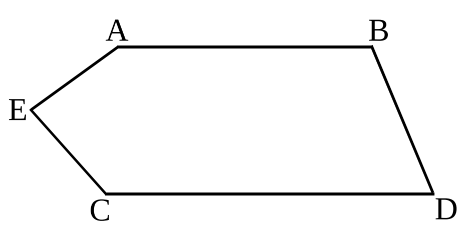


Fig 32

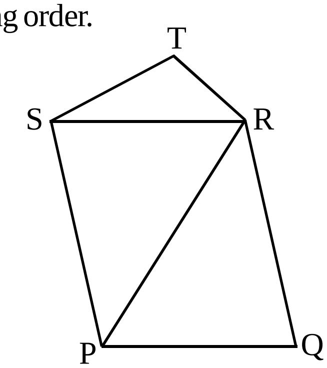


Fig 33

Measure the following line segments with the help of scale and compass and write their measurements.

Fig 34

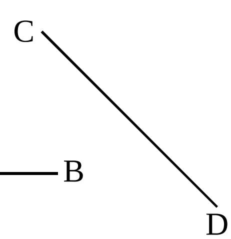


Fig 35

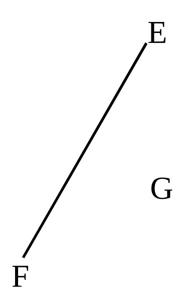


Fig 36

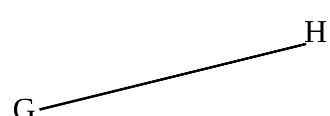


Fig 37

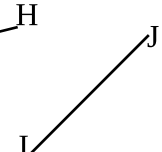


Fig 38

Drawing line segments of lengths equivalent to the sum of two or more line segments

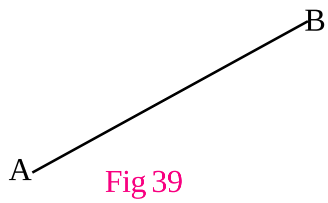


Fig 39

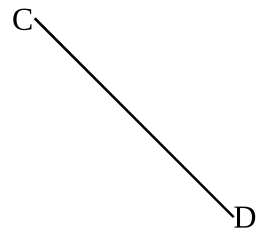


Fig 40

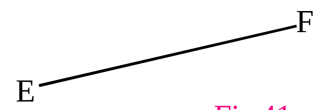


Fig 41

Fig 42

Steps :-

(1) Draw a line XY.

(2) Stretch the compass of length equal to that of line segment AB.

(3) Taking point P as a centre on line XY, draw an arc equal to the length of line segment AB and cut line XY at point Q.

(4) Take measure on the compass equal to line segment CD and draw an arc taking Q as the centre, cut line XY in the opposite direction of the arc at point P. Let the point be R.

(5) Similarly, take an arc equal to line segment EF and with R as the centre, cut on line segment XY. Let this point be S.

Is AB = PQ, CD = QR, EF = RS? Why?

Write in your notebook.

(6) PS = PQ + QR + RS

PS = AB + CD + EF

Now measure the length of PS. PS would be the sum of the given line segments.

Drawing line segment of length equivalent to the difference of line segments.

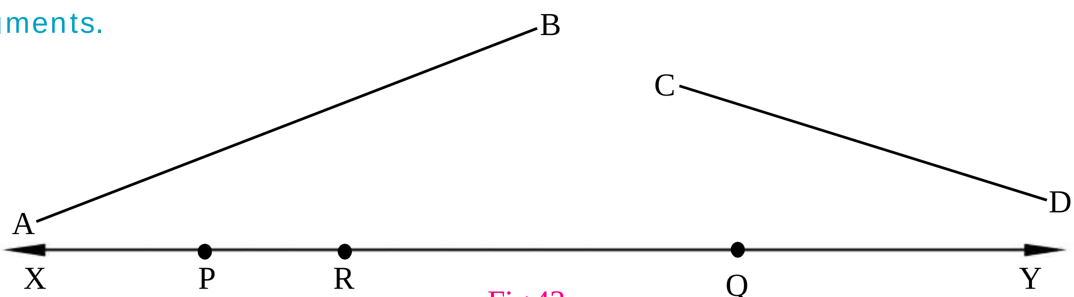


Fig 43

(1) Draw a straight line XY. Mark a point P on it.

(2) Stretch the compass to a length equal to $\overline{AB}$.

(3) Draw an arc at length of equal to AB and cut an arc on XY. Taking P as the centre. It cuts at point Q.

(4) Now take a measure on the compass equal to CD and with Q as centre, cut an arc on XY in the direction P. It cuts at R. Is AB = PQ and CD = QR? If yes, then why?

Write in your notebook. Here, why are the arcs are put in a direction opposite to PQ? While adding the arcs were put in the same direction. Write the reasons in your notebook.

(5) PQ - QR = PR

or AB - CD = PR

Measure the length of PR, the difference of line segments will be PR.

Parallel Lines

Suppose the teacher asks students to draw two lines on the note book and the students draw lines that looked like this:

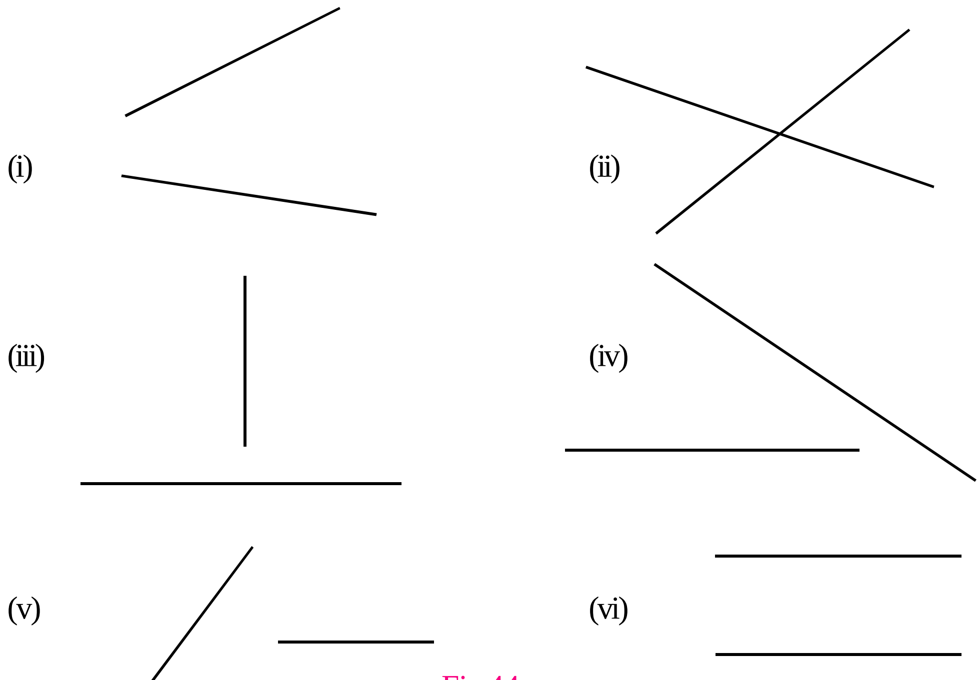


Fig 44

Now, you also draw a few pairs of lines in your note book.

Now look at the pairs of lines given below:

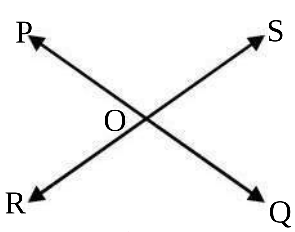


Fig 45(1)

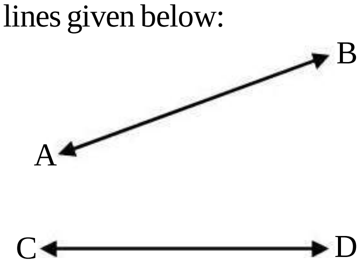


Fig 45(2)

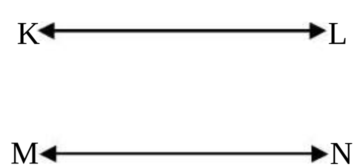


Fig 45(3)

In fig 45(1) the lines PQ and RS cut each other at point O. Therefore the lines PQ and RS are intersecting lines and point O is common to them.

In fig 45(2) the lines AB and CD are not cutting each other in the present state, if they are extended forwards or backward, they would intersect each other. Therefore, fig 45(2) would also come in the category of intersecting lines like fig 45(1).

In fig 45(3) the lines KL and MN are neither intersecting each other in the present state, nor will they intersect if they are extended in any direction.

How can we verify that they will not intersect each other?

Observe the lines in figure 45(3)

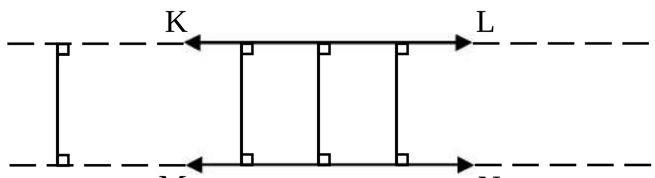


Fig 46

The perpendicular distance between KL and MN is equal at any point. If the lines are extended in any direction to any extent, the perpendicular distance between them would remain the same. You can also verify the perpendicular distance between the two lines by measuring it with the help of a divider. Are they equal? These lines are known as **parallel lines**. This means parallel lines always exist at equal distances. They neither come nearer to each other nor go farther.

Observe the things around you in your classroom, the blackboard in your class, the windows, doors, walls, your geometry box, desk, book, scale, note book etc. In all these things the edges will give you the notion of being parallel.

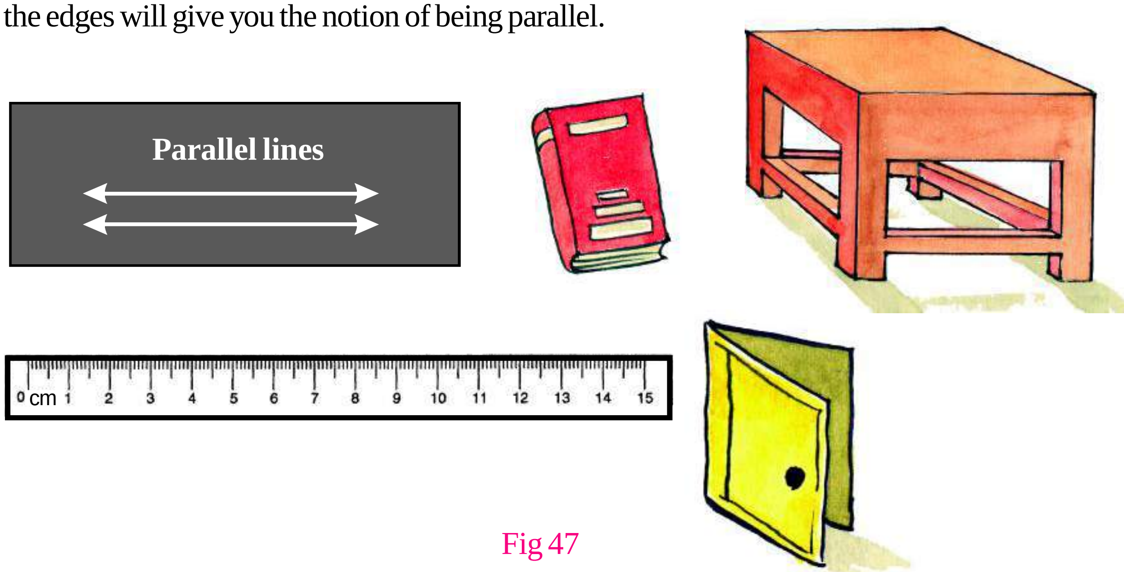


Fig 47

Look at these pictures. In these pictures, where, all do you see the examples of parallel line segments?

Make a list of examples of five intersecting and five parallel lines segments in your notebook.

EXERCISE 3

Q.1. Say whether the following statements are true or false -

(i) Passing through one point, infinite number of line segments can be drawn.

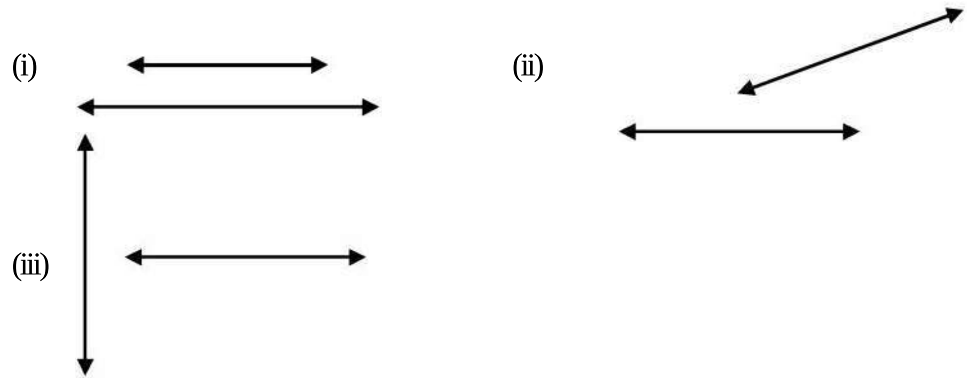
(ii) Passing through two points, infinite number of lines can be drawn.

(iii) A line segment has only length and no breadth.

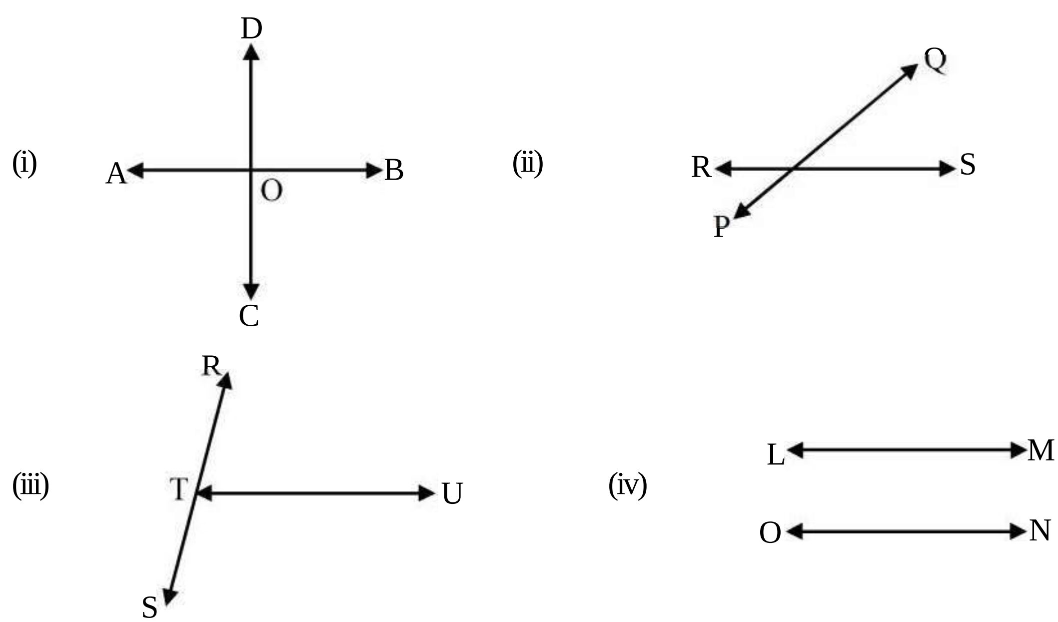
(iv) If four points are taken on a line segment, they are all collinear points.

(v) At the most 2 line segments can be drawn passing through 3 collinear points.

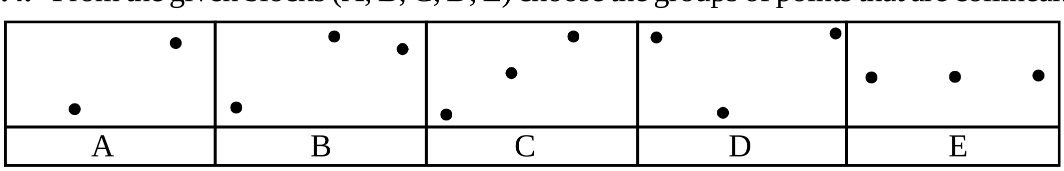
Q.2. Which of the pairs along the following figures are not intersecting lines?



Q.3. From the given pairs of lines select the pairs of intersecting lines.



Q.4. From the given blocks (A, B, C, D, E) choose the groups of points that are collinear.



Q.5. Draw line segments of the following measures:

6 cm   5 cm   4.5 cm   2.3 cm

Q.6. Draw 3 line segments 3 cm, 5 cm, 6.5 cm on one straight line.

Q.7. Some line segments are given below. Draw a line segment equal to the sum of their lengths.

(i) A ———— B C ———— D

(ii) A ———— B C ———— D

Q.8. Draw line segments equal to the difference of the given pairs of line segments.

(i) A ———— B C ———— D

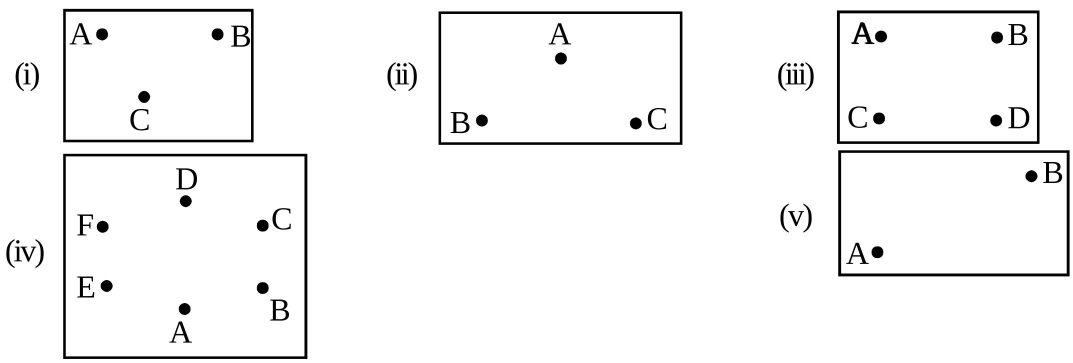
(ii) A ———— B C ———— D

Q.9. Draw a line segment AB = 12 cm and divide it into 3 line segments of length 3 cm, 4 cm and 5 cm AC, CD and DB respectively. Now verify the following situations.

(i) AD – AC =CB - DB

(ii) AB – CD =AC + DB

Q.10. How many line segments can be drawn with the help of the points given in each condition. Draw the line segments and write their numbers?



What Have We Learnt ?

1. A ray has an initial point and it moves in one direction.
2. The straight line moves on and on in both sides.
3. A line segment is a portion or part of a straight line or a ray. Which has an initial and end point also. The length of a line segment can be measured.
4. Length of a straight line and a ray cannot be measured.
5. Two lines intersect each other at only one point.
6. Passing through one point infinite number of line can be drawn and passing through one point infinite number of ray can be drawn.

Chapter 4

INTEGERS

One day, the students of the class requested their teacher to let them play some games. The teacher happily agreed to this and said, “Why not? let us play a game of numbers today”.

She said, “All of you write any single digit number in your notebooks, multiply it with 2, subtract 12 from the obtained product and tell me the answer”.

Directions	Fatima	Kamli	Monu
Number considered	7	6	5
After multiplying by 2	$7 \times 2 = 14$	$6 \times 2 = 12$	$5 \times 2 = 10$
Subtracting 12 from the product	$14 - 12 = 2$	$12 - 12 = 0$	$10 - 12 = ?$

The number of students could have three types of possibilities:

(1) Some students might have a result like that of Fatima.

(2) Some students might have a result like that of Kamli.

(3) Some other students might have a problem like that of Monu, they might not be able to subtract the number.

All the children who were in a situation like Monu, asked the teacher, “What should we do?” the teacher said “Let us understand the problem first.”

Suppose, there is a bamboo fixed vertically in your village pond. An insect of the pond climbs 10 feet straight up the bamboo from the surface of water and then slips back 12 feet down wards.

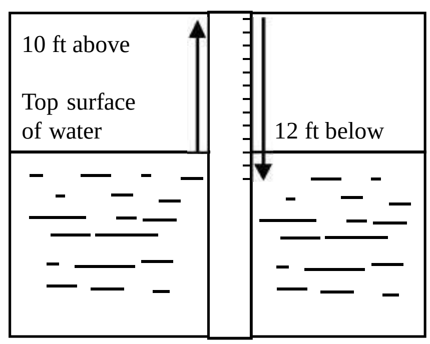


Fig 1

This means, the insect could not climb up and is now 2 feet under the surface of water. If the level of water be considered 0, then how will you show the measurement below the point zero?

But Monu couldn’t yet understand what number could be used to show 10-12. he said “I still can’t understand what to do with my problem. Then the teacher played another game of dice and scale. She said “I have a big scale on which 0 is written in the center. On both left and right sides of the 0, ten divisions made on the right side of 0, number 1-10 are written and we have two dices- one red and the other green.”

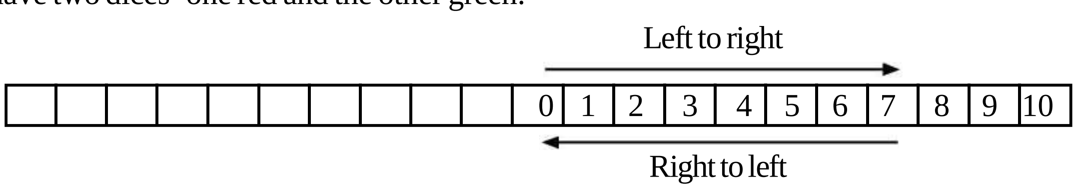


Fig 2

There are two conditions for the game.

First: We shall move as many numbers towards right on the scale which will be equal to the number of points that be seen on the face of the red dice.

Second: We shall move as many numbers towards left on the scale which equal to the number of points that will be seen on the face of the green dice. This move will begin from that point where we have stopped after making the move for the red dice.

The Game Starts Now

First of all, Fatima throws the dices. Fatima's red dice showed 5 places and green dice showed 3 points. According to the conditions, Fatima would move 5 places towards right from zero and then come back 3 places. This means her position will be at 2 on the scale as shown in figure 3.

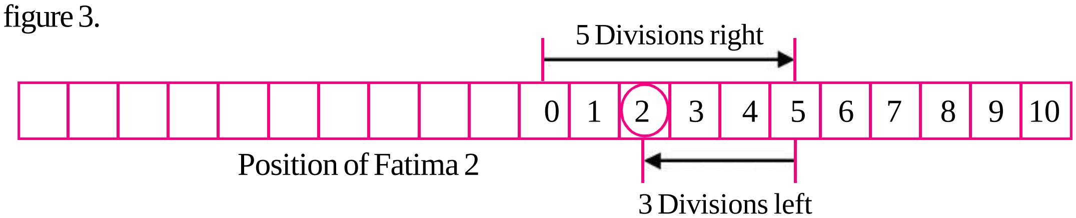


Fig 3

Now Kamli throws the two dices. Both the dices showed 4 points. As per the conditions, Kamli would move 4 places towards right and come back four places. So, her position would be at 0 as in figure 4.

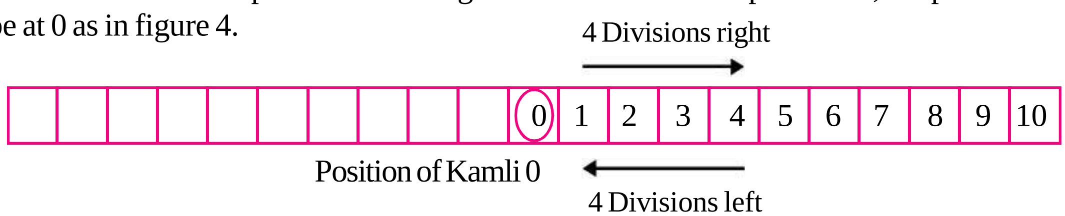


Fig 4

At last Rakesh throws both dices. On his red dice, he got 2 points and the green dice showed 5. As per the conditions, Rakesh has to move 2 places towards, the right and from there comes back 5 places.

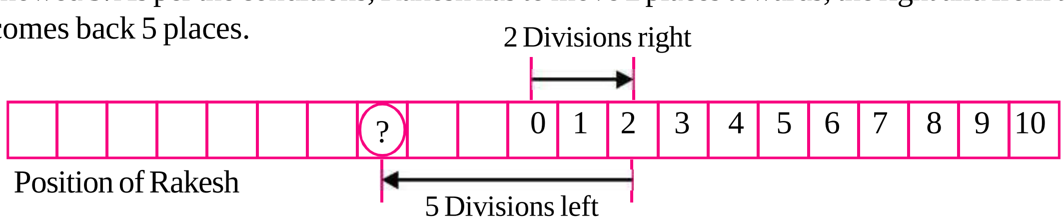


Fig 5

Towards left which means he crosses 0 and stops at the third place to the left of 0. Once again the children are not able to understand how this position can be represented by a numbers.

But Kamli and other students are slightly able to understand the fact that numbers are increasing by one, to the right of zero which means to move towards right 1 added to the previous number to get the next number. Similarly, to go towards left 1 is subtracted from the original number to get the preceding number.

"So, the numbers towards the right of 0 are obtained by adding one and there towards the left of 0 are obtained by subtracting one." "Did you understand what I said," asked the teacher.Monu said "Does that means that the numbers to the right of 0 are positive and those to the left of 0 are negative? The teacher said, "Exactly, that's rightly."

Example :

0 + 1	=	1
1 + 1	=	2
2 + 1	=	3
3 + 1	=	4
• + •	=	•

3 – 1	=	2
2 – 1	=	1
1 – 1	=	0
0 – 1	=	–1
–1–1	=	–2
• – •	=	•

If we subtract one-one number from zero .

The 0-1= - 1, -1-1=-2, -2-1=-3 etc. will be obtained. Therefore, Rakesh's position is at -3, because this will be the value of the third division to the left of 0. similarly, all the numbers towards the left of zero. -1,-2,-3,-4,-5 etc would be negative numbers. As we move towards left, the number value would become less and greater negative numbers would be obtained.

					–5	–4	–3	–2	–1	0	1	2	3	4	5	6	7	8	9	10

Numbers left to zero ⟵ | ⟶ Numbers right to zero

Fig 6

Fatima says, "this means numbers gradually increase from zero towards right. 1>0,2>1, 3>2, 4>3, 5>4 ….. etc and the numbers towards left will get reduced. i.e. -1<0, -2<-1, -3<-2 etc. (To understand negative numbers, you must practice playing this game at home as well as in your classroom. If you don't get dices, take papers of two different colors are write the numbers 1 to 6, separately and fold it in a way that the numbers are not seen. Now pick out one paper of each color and open the chits to find the numbers. You can then play the game.)

On the basis of your experience, put the > or < symbol in the appropriate boxes. If required.

0		-1	-1		-2
50		70	100		101
-5		5	-53		-5

Think of some more such pairs and ask your friends to solve them.

Negative Numbers

ACTIVITY 1

$3 + \boxed{4} = 7$

$3 + \boxed{} = 6$

$3 + \boxed{} = 5$

$3 + \boxed{} = 4$

$3 + \boxed{} = 3$

$3 + \boxed{} = 2$

$3 + \boxed{} = 1$

$3 + \boxed{} = 0$

From the examples, we find that just as we have positive numbers, we have negative numbers also. If we include these numbers, we can work on some more new operations like subtract 14 from 12 and show an answer for it. On adding 3 and 4, we get 7. If 3 is kept constant, then which number should be added, so that we get the numbers 6,5,4,3,2,1 and 0 consecutively, write these value in the empty boxes. Think of more such questions and solve them. Can you tell the value of the smaller and the greatest negative numbers.

Integers

You know about Natural numbers and whole numbers. What will happen if you add the negative numbers also to these numbers? Numbers to the right of zero are Natural numbers and those towards the left are negative numbers. Positive numbers, negative numbers and zero, all the three together make Integers. Integers are denoted by I or Z, which means:

Integer I = {.........., -3, -2, -1, 0, 1, 2, 3, 4, 5, ...}

Just as we do not have any whole number that is the greatest number, similarly, we do not have the largest integer also. Can you think of a number that will be the smallest integer ?

Representing Integers on the numberline

Draw a straight line. Put some points/marks on the line at equal distance. On the line, write zero in the middle and write positive numbers on the right of '0' and negative numbers to the left of '0' (according to fig. 7).

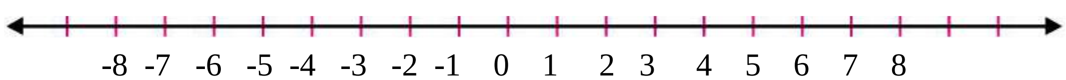


Fig 7

Any line drawn like this is known as a number line.

Representation of Integers through pictures

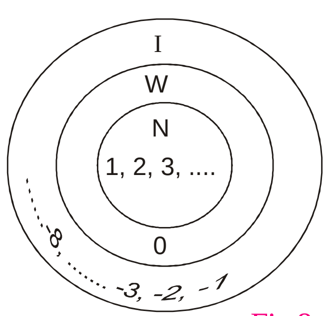


Fig 8

Where

N = Natural number

W = Whole number

I = Integer number

Look at the symbols used above and identify the numbers that are included in integers as represented in figure 8. What numbers do you think are included in whole numbers?

Representing Operations With Integers on the Number Line

Addition of Integers:

When both the numbers are positive. $3 + 5 = ?$

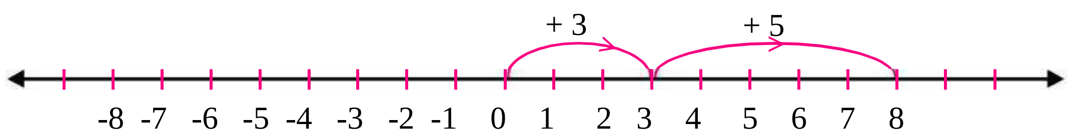


Fig. 9

First we move from zero to 3 in the positive directions then from 3, we move 5 places in the positive direction, we reach the number 8.

Hence, $3 + 5 = 8$.

When both the numbers are negative,

e.g. $(-2) + (-5)$ then,

First we move 2 places in the negative direction. Then from that point, we move 5 places towards left and reach -7.

i.e. $(-2) + (-5) = -7$

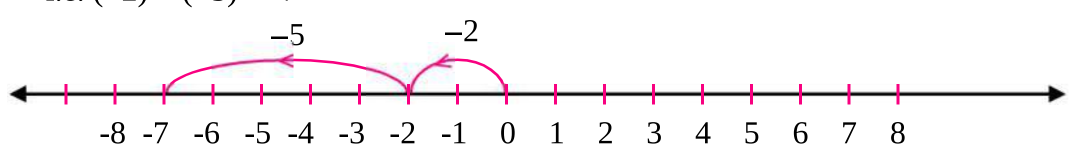


Fig 10

When one is positive and the other a negative number

(a) $8 + (-5) = ?$

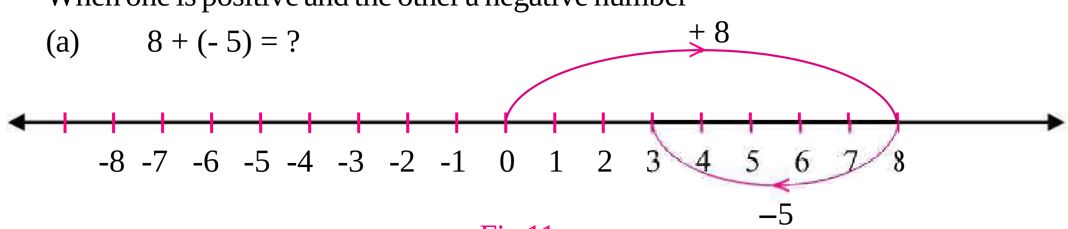


Fig 11

First we move from zero to 8 in the positive direction and then move 5 places back in the negative direction from the 8^{th} place. Considering the direction with respect to zero, we reach at the 3rd place in the positive direction, therefore,

$8 + (-5) = 3$.

(b) $-8 + 5 = ?$

In this situation we shall move 8 places from zero in the negative direction and then move 5 places towards zero (in the positive direction) so that finally we reach the place 3 in the negative direction, which means,

- 8 + 5 = - 3

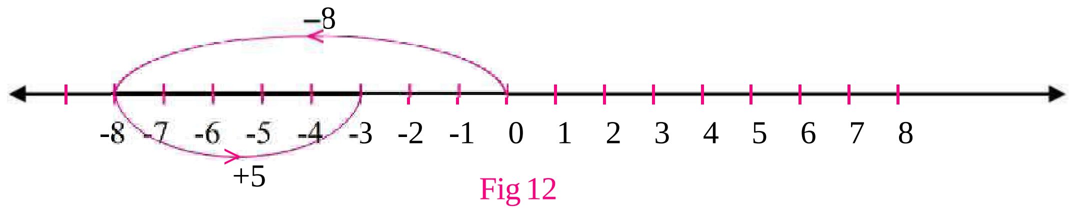


Fig 12

Thus from the examples, we observe that :

(1) On addition of two same signs, the result retain the same sign as that of the added numbers.

(2) When the signs used in two integers are different, the result of the addition takes the signs of the greater number. This means the result depends on the numbers added.

In addition integers abide by all the characterstics that are followed by whole numbers:

1. The addition or sum of two integers is an integer.
2. Commutative law is applicable in the addition of all integers.
3. The sum of two integers is always an integer. This is the closure rule for Integers.
4. There's no change in the value of an integer when zero is added to it.

Subtraction of Integers

Just as in whole numbers subtraction is an operation that is opposite to addition, similarly, in integers also the operation of subtraction is opposite to that of addition.

Observe the following examples:

(a) 14 - 18 = - 4

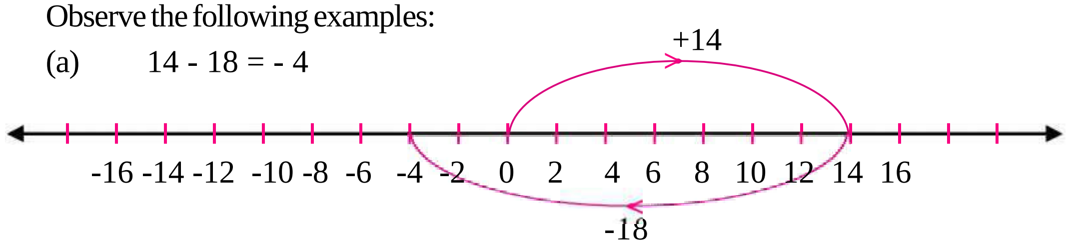


Fig 13

(b) 5 - 3 = 2

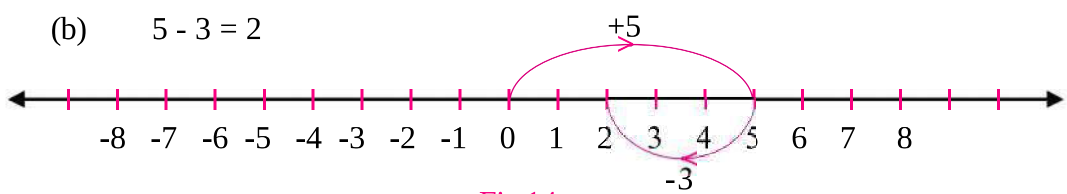


Fig 14

(c) 8 - 4 = 4

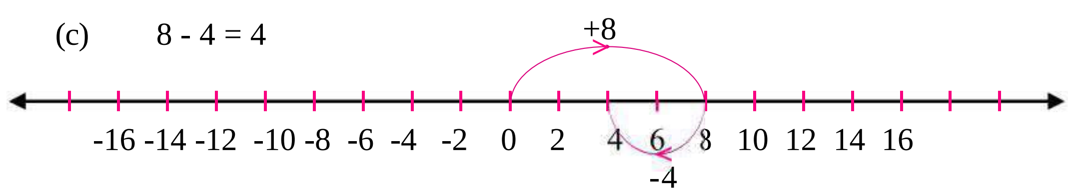


Fig 15

You are already familiar with the process of subtraction. But if you are asked to subtract - 6 from 10. Then you might face problem because we often don't understand how negative numbers can be brought into the process of subtration. Let us clarify this :

You know that 5 + 0 = 5, 8 + 0 = 8, 111 + 0 = 111.

This means if zero is added to any number, the sum will remain the same number. There by **zero is known as the additive identity**.

An Additive Inverse is a number which when added to any would give the additive identity or zero as the sum.

Example : 5 + (additive inverse of 5) = Additive Identity

What should then be added to 5 so as to get zero as the sum? The answer is -5.

Similarly, the additive inverse of 8 will be -8 and 13 would be -13.

Thence, -7 + (additive inverse of -7) = Additive Identity

Here, what should be added to - 7, to get zero ? Your answer would be +7 which means -7 + (+ 7) = 0. Similarly, the additive inverse of -3 would be 3 and that of -9 would be +9.

Now, find out the additive inverse of the numbers given below:

Number		Additive Inverse of the number	Additive Identity
35	+	(-----------------------)	0
- 40	+	(-----------------------)	0
- 17	+	(-----------------------)	0
- 35	+	(-----------------------)	0
- 13	+	(-----------------------)	0

Subtracting the first number from the second number means adding the additive the inverse of the second number to the first number. Think, Is this true?

e.g. 12 – (5) = 12 + (additive inverse of 5)

= 12 + (- 5)

Similarly, 12 - (- 5) = 12 + (additive inverse of -5)

= 12 + (+ 5)

= 12 + 5 = 17

Now, can you subtract -6 from 10?

Practice 1

Find the additive inverse of the number to be subtracted and solve the following questions:

(i) - 3 - (- 7) (ii) 12 - (- 10) (iii) 15 - (+ 7)

(iv) 7 - (+ 18) (v) 19 - (- 7)

In the above examples, you can see that the negative of a negative number is a positive number and the negative of a positive number is a negative number.

Thus, - (- 3) = + 3

$(-1) \times (-3) \quad = +3$

$(-5) \times (-3) \quad = +15$

This means, the product of two negative Integers is always a positive Integers.

Similarly,

$-(+7) \quad = -7$

$(-1) \times (+7) \quad = -7$

$(-5) \times (+3) \quad = -15$

This means, the product of a negative Integer and a positive Integer is always negative Integer.

Short Cut Method of Subtraction

When we have to subtract any number from 10, 100, 1000 etc. then we will convert highest place value of 1 to its smaller unit. let us take some examples.

Example 1

$$\begin{array}{r} 1\,0\,0 \\ -\quad 7 \\ \hline \end{array}$$

Solution – From unit place of 100 we cannot subtract 7. There is zero in ten's place also so we cannot borrow from ten's place. You can convert one hundred into ten, ten's value and from this tens we will change one ten in to 10 unit values.

H	T	O		H	T	O	
1	0	0	→	0	9	10	(100= 90+10)

Means one hundred will change in to 9 tens and 10 unit values. In this manner it is easy to subtract any value which is less then hundred.

Subtract –

$$\begin{array}{ccc} H & T & O \\ 0 & 9 & 10 \\ & & 7 \\ \hline & 9 & 3 \end{array}$$

(100 – 7 = 90 + 10 – 7)

(= 90 + 3 = 93)

One more Example –

Example 2

$$\begin{array}{r} 1\,0\,0\,0\,0 \\ -\quad 2\,8\,7\,4 \end{array}$$

\

Given

$$\begin{array}{cccccc} & Tth & Th & H & T & O \\ & 1 & 0 & 0 & 0 & 0 \\ - & & 2 & 8 & 7 & 4 \\ \hline \end{array}$$

Change ten thousand into smaller units

Have given

	Tth	Th	H	T	O
		9	9	9	10
–		2	8	7	4
		7	1	2	6

(10000 = 9990 + 10)

Example 3 1000 – 8 7 6

	9	9	(10)
–	8	7	6

From these examples we can see that we are subtracting any unit place value from ten and other place values from 9.

Now you can write the answer directly.

100 – 23 = (9 – 2) (10 – 3) = 77

100 – 69 = (9 – 6) (10 – 9) = 31

1000 – 512 = (9 – 5) (9 – 1) (10 – 2) = 488

1000 – 32 = (9 – 0) (9 – 3) (10 – 2) = 968

1000 – 8 = 992

10,000 – 982 = 9018

10,000 – 8374 = 1626

Properties Related to Subtraction of Integers

1. The difference of two integers is always an integer. (Closure property)
2. Zero subtracted from an integer does not change the value of the integer.
3. Every number that is an integer, has a predecessor.

   e.g. The predecessor of 0 is - 1, that of -1 is - 2 and that -5 is - 6.

Verification of Addition And Subtraction by Digit Sum (Bijank) Method

You know that for obtaining digit sum we add each digit of number until we not get a single digit. The single digit obtained in last, is the digit sum of given number.

Example –

Digit sum of 45 = 4 + 5 = 9

and digit sum of 457 = 4 + 5 + 7 = 16

two digits are there in 16 = 1 + 6 = 7

So, digit sum of 457 is 7

We can check our solved problems with the help of digit sum.

Verification of Addition

For verification of addition we have to find digit sum of number to be added and their sum.

If the sum of digit sum of numbers to be added and their sum is equal then our result is correct.

Digit sum of 453 is 3 and digit sum of 158 is 5 and the digit sum of result obtained (611) is 8 which is equal to sum of digit sum of numbers to be added.

In this manner we can check our solved questions of addition.

Verification of Subtraction

If the sum of digit sum of subtrahend and digit sum of difference is equal to digit sum of minuend then our result is correct.

let us see an example – 587 – 235 = 352

The digit sum of difference 352 is 1

and digit sum of subtrahend 235 is 1

The sum of their digit sum is 2

and the digit sum of minuend 587 is also 2. Means our calculation is right.

Now, by using this method verify your solved problems.

Multiplication of Integers.

ACTIVITY 1

In the given table the product of integers is shown. Some blank spaces are given in the table, fill it :-

S. No.	First Number	Second number	First no. × Second no.	Product	Conclusion
01	3	4	3 × 4	+12	The product of two positive integer is a positive integer.
02	–6	–2	(–6) × (–2)	+12	The product of two negative integer is a positive integer.
03	–5	2	(–5) × (+2)	–10	The product of a positive integer and a negative integer is a negative integer.
04	3	–6	(3) × (–6)	–18	....
05	–5	–4	....	....	....
06	–7	2	....	....	....
07	–8	–12	....	....	....
08	15	–13	....	....	....
09	–17	–19	....	....	....

Multiplication of 9, 99, 999 .... etc.

Multiplication of one, two or three digit number to 9, 99, 999 respectively gives an interesting pattern. Let us see some examples –

$8 \times 9 = 72$ $\qquad$ $47 \times 99 = \underline{46}\ \underline{53}$

$7 \times 9 = 63$ $\qquad$ $78 \times 99 = \underline{77}\ \underline{22}$

$5 \times 9 = 45$

You can see that the multiplicand and multiplier are one digit number. Ten's place digit of product is always one less than multiplicand and the difference between digit obtained in one's place and 9 is digit obtained tens place.

Does this pattern will also obtained in other numbers?

	Ten's	Unit value	T.	O	
6 × 9	= (6 – 1)	(9 – 5)	= 5	4	= 54 is true
4 × 9	= (4 – 1)	(9 – 3)	= 3	6	= 36 is true

What will happen if multiplicand and multiplier are two digit number? in the product we will get a 4 digit number. (Except 10 x 99)

	Th H	T O	Th	H	T	O	
10 x 99	= (10 -1)	(99 - 9)	=	9	9	0	= 990
75 x 99	= (75 - 1)	(99 - 74)	= 7	4	2	5	= 7425
84 x 99	= (84 - 1)	(99 - 83)	= 8	3	1	6	= 8316

take it slightly more -

solve 100 × 999

	T Th Th	H T O	
100 × 999	= (100 – 1)	(999 – 99)	
	99	900	= 99900

If we take any three digit number greater than, 100, then its products will be of 6 digits.

217 × 999	= (217 – 1)	(999 – 216)	
	216	783	= 2 1 6 7 8 3
999 × 999	= (999 – 1)	(999 – 998)	
	998	001	= 9 9 8 0 0 1

Let us think, How does all this happen -

$$8 \times 9 = 8 \times (10 - 1) = 80 - 8 = 70 + 10 - 8$$
$$= 70 + 9 - 7$$
$$= 70 + 2$$
$$= 72$$

$$(7 + 1)\,(10 - 1) = 70 + 10 - 7 - 1$$
$$= 70 + 10 - 1 - 7$$
$$= 70 + 9 - 7$$
$$= 70 + 2$$
$$= 72$$

Now look at these examples carefully-

$3 \times 2 = 6$,	$2 \times 1 = 2$,	$4 \times 2 = 8$,	$1 \times 4 = 4$
$5 \times 3 = 15$,	$2 \times 8 = 16$,	$7 \times 3 = 21$,	$9 \times 9 = 81$

In the first four examples given above, you are seeing that on multiplying unit place digit of multiplicand to unit place digit of multiplier we are getting unit place digit of product and in the last four examples given above on multiplying unit place digit of multiplicand to unit digit of multiplier we are getting ten's place digit along with unit place digit of product.

In this manner we can see that on multiply ten's place digit of multiplicand unit place digit of multiplier we will get hundred's place digit and ten's place digit or ten's place digit only.

i.e.	$20 \times 3 = 60$,	$30 \times 1 = 30$,	$10 \times 4 = 40$
	$40 \times 3 = 120$,	$50 \times 5 = 250$,	$30 \times 7 = 210$

On multiplying 20 means 2 tens to 3 we will get 60 units means 6 tense and on multiplying 4 tense (40) to 3 we will get 12 tens or 1 hundred and 2 tens.

If you keep these things in mind and do multiplication then we can summarise multiplication process. Let us see more example to understand it more.

Example 1 13×12

Solution

$$\begin{array}{r} 13 \\ \times\ 12 \end{array}$$

Step 1 -

$$\begin{array}{r} 13 \\ \times\ 12 \\ \hline 6 \end{array}$$

On multiplying 3 and 2 of unit place we will get 6 units write it in unit place.

Step 2 -

$$\begin{array}{r} 1\ 3 \\ \times\ 1\ 2 \\ \hline 5\ 6 \end{array}$$

$$\begin{array}{c} 1\ \ 3 \\ \times \\ 1\ \ 2 \end{array}$$

On multiplying unit place digit 3 of first number to ten's place digit 1 of second number we will get 3 tens and on multiplying unit place digit 2 of second number to ten's place digit 1 of second number we will get 2 tens. We get total $(3 + 2) = 5$ tens write it in ten’s place.

$(3 \times 1) + (2 \times 1) = 3 + 2 = 5$ tens

Step 3 -

$$\begin{array}{r} 1\ 3 \\ \times\ 1\ 2 \\ \hline 1\ 5\ 6 \end{array}$$

on multiply ten's place digit 1 of first number to ten's place digit 1 of second number we will get 1 hundred. Write it in hundred's place. We get 156.

We can see all three steps like this

H	T	O		
1	1	3	3	
$\updownarrow$	$\times$		$\updownarrow$	
1	1	2	2	
1	(3 + 2)		6	
1	5	6	=	156

Example 2- Solve 12 × 31

Solution -

Step 1

$$\begin{array}{r} 1\ \ 2 \\ \times\ 3\ \ 1 \\ \hline 2 \end{array}$$

Unit 2 x Unit + 1 = 2 Units

(Write in unit's place)

Step 2 -

$$\begin{array}{r} 1\ \ 2 \\ 3\ \ 1 \\ \hline 7\ \ 2 \end{array}$$

Unit 2 x Ten's 3 = 6 tens

Unit 1 x Ten's 1 = 1 tens

Total = 7 tens

(Write in ten's place)

Set 3 -

$$\begin{array}{r} 1\ \ 2 \\ \times\ 3\ \ 1 \\ \hline 3\,7\ \ 2 \end{array}$$

tens 1 × tens 3 = 1 × 3 = 3 hundred

(Write in hundred's place)

12 × 31 = 372 (result)

We did not get the carry on multiplication in these two example but it we take some greater number then this condition will arrive.

Let's see

Example 3 - Solve 43 x 12

Step 1

$$\begin{array}{r} 4\,3 \\ \updownarrow \\ \times\ 1\,2 \\ \hline 6 \end{array}$$

Unit 3 x Unit 2 = 6 Units

(Write in unit's place)

Step 2

$$\begin{array}{r} 4\ 3 \\ \times \\ 1\ 2 \\ \hline 1\ 6 \end{array}$$

(unite 3 x tens 1) + (unit 2 x Tens 4)

= 3 + 8 = 11 Tens

11 tens = 1 Hundred + 1 Ten

Write 1 ten in ten's place. We keep 1 hundred for further carry

Step 3

$$\begin{array}{r} 4\ 3 \\ \updownarrow \\ \times\ \ 1\ 2 \\ \hline 5\ 1\ 6 \\ \hline \end{array}$$

Tens 4 x Tens 1 = 4 hundreds

= + 1 hundred (carry)

= 5 hundreds

Write in the hundred's place

Product = 43 x 12 = 516

Example 4 - Solve 76 x 58

Step 1

$$\begin{array}{r} 7\ 6 \\ \updownarrow \\ \times\ \ 5\ 8 \\ \hline 8 \\ (4) \end{array}$$

6 × 8 = 48 8 Ones

4 Tens (of carry)

Step 2

$$\begin{array}{r} 7\ 6 \\ \times \\ 5\ 8 \\ \hline 0\ 8 \\ (9)\ (4) \end{array}$$

(65) + (87)

30 + 56 =	8 6	tens
	+ 4	tens (carry)
	9 0	tens

= 9 hundreds + 0 tens

Write 0 in Tens and 9 in hundred (for carry)

Step 3

$$\begin{array}{r} 7\ 6 \\ \updownarrow \\ \times\ \ 5\ 8 \\ \hline 4\ 4\ 0\ 8 \end{array}$$

7 × 5	= 35	hundreds
	+ 9	hundreds (of Carry)
	44	hundreds

4 thousands and 4 Hundreds

Write them in their respective places

We get the solution 76 × 58 = 4408

Properties of Multiplication of Integers

1. The product of two integers is always an integer. It is called closure property.

   Thus, $3 \times (-6) = -18$ (Here, the product of 3 and - 6 is - 18 and it is an integer.)

2. The product of integers follow the commutative rule,

   As, $(-7) \times 2 = 2 \times (-7) = -14$.

3. **1** is multiplied to any integer, we get the same integer.

   As, $(-4) \times 1 = 1 \times (-4) = -4$.

   Here, the number **1** called multiplicative identity.

4. Any integer is multiplied by its multiplicative inverse 1 is always obtianed.

   As, $5 \times \frac{1}{5} = 1$.

   Here, multiplicative inverse of 5 is $\frac{1}{5}$.

5. Multiplication of zero - If any integer is multipied by zero, zero is obtained.

   As, $(-3) \times 0 = 0 \times (-3) = 0$.

6. The product of integers follows the Associative law.

   For example : $-3 \times (4 \times 5) = (-3 \times 4) \times 5$

7. Distributive property - The operation of product in integers is distributive over the operation of addition.

   e.g., $3 \times (-4 + 5) = 3 \times (-4) + 3 \times 5$

   or $3(-4 + 5) = 3(-4) + 3 \times 5$

   $= 3$.

Division of Integers

In the previous lesson, you have learnt the division of whole numbers. We have already seen examples of multiplication of whole numbers. On this basis, we can understand the division of integers.

$3 \times 4 = 12$	$-5 \times 6 = -30$	$(-7) \times (-2) = 14$
$12 \div 3 = ?$	$-30 \div -5 = ?$	$14 \div (-2) = ?$
$12 \div 4 = ?$	$30 \div 6 = ?$	$14 \div (-7) = ?$

Just as we have seen in whole numbers, similarly, apart from zero, for all numbers multiplication and division can be considered as operations opposite to each other. Therefore, for the above problems put numbers in place of the question marks.

Properties of Division in Integers

1. Closure properties are not always applicable to the operation of division. e.g. In $3 \div 4$, the quotient is not an integer.
2. Every integer (excluding zero) divided by the same integer would always give 1 as the quotient e.g. $7 \div 7 = 1$.
3. Excluding zero, all integers when divided by their additive inverse give -1 as the quotient e.g. $15 \div (-15) = -1$.
4. Zero divided by any integer would always give zero. e. g. $0 \div 16 = 0$
5. No integer can be divided by zero. This means the quotient for the division of an integer by zero is not defined, e.g. $4 \div 0$ = not defined.

EXERCISE 4

1. Represent the following numbers on the number line and write the results:-

(i) $2 + (-4)$ (ii) $-3 + 5$ (iii) $(-6) + (-3)$
(iv) $6 + 4 + (-2)$ (v) $4 + (-3) + (-5)$ (vi) $0 + 3$
(vii) $0 + (-5)$ (viii) $9 + 0 + (-1)$

2. Find the sums of :-

(i) 1531, (-503) (ii) -55, -211 (iii) 117, -81
(iv) -18, 172

3. Fill in the blanks with >, =, or <, so that the statements become true:

(i) $8 + (-3)$ .... $-3 + 8$
(ii) $-28 + 25$ .... $-25 + 28$
(iii) $-4 + 0$ .... $4 + 0$
(iv) $0 + 9$ .... $9 + 0$
(v) $25 + (+25)$ .... $+ 25 - (-25)$
(vi) $208 + 53$ .... $208 - 53$

4. Find the product of the following:

(i) $(+2) \times (3) \times (5)$ (ii) $3 \times (-5) \times (-6)$
(iii) $(-4) \times 3 \times (-2)$ (iv) $(-6) \times (-4) \times 1$
(v) $3 \times 0 \times (-2)$ (vi) $2 \times (-7) \times (-3)$

5. Fill in the blanks with symbols >, =, or <

(i) $(2) \times (5)$ .... $(-3) \times (5)$
(ii) $2 \times -4 \times -3$ .... 8×3
(iii) $4 \times -3 \times -1$ 28
(iv) $(-8) \times (-5)$ .... 2×20
(v) $2 \times -3 \times 0$ .... 0×-8
(vi) $4 \times 5 \times (-3)$ .... $-4 \times (+5) \times (-3)$
(vii) $3 \times 8 \times (-5)$ .... $3 \times 8 \times (-5)$

6. **The sum of two integers is 69. If one of them is 56. Find the other integer.**

7. **The sum of two integers is 85. If one of them is -15. Find the other integer.**

8. **Find the quotients for each of the following divisions:**

(i) $30 \div 2$ (ii) $40 \div (-4)$ (iii) $-48 \div 12$

(iv) $24 \div 0$ (v) $-14 \div 1$ (vi) $95 \div (-5)$

9. **Fill in the blanks:**

(i) -80 ÷ ....= - 20 (ii) 46 ÷ .... = -23

(iii) -24 ÷ .... = 24 (iv) 12 ÷ .... = -1

10. **Find the additive inverse of the following integers:**

(i) 17 (ii) -23 (iii) 68

(iv) -75

11. **Fill in the blanks:**

(i) -18 + ... = 0 (ii) 26 + ... = 0

(iii) 161 + ... = 0 (iv) -79 + ... = 0

What Have We Learnt ?

1. When two integers with the same sign are added, the sum is also represented with the same sign as that of the added integers.
2. The sum of two integers is always an integer. This is the closure property for Integers.
3. Zero added to any integer doesn't change its value.
4. When a positive number is multiplied by a negative number the product is always negative number, e.g. $(+1) \times (-1) = -1$ or $(-1) \times (+1) = -1$.
5. On multiplying 1 to any integer there is no change in it's value. Integer 1 is called as multiplicative indenity.
6. A negative number multiplied by another negative number always gives a positive number as the product $(-1) \times (-1) = +1$.
7. The difference of two Integers is an integer.
8. When zero (0) is subtracted from an integer, its value does not change.
9. Every integer has a predecessor.
10. The additive inverse of a negative number is a positive number and the additive inverse of a positive number is a negative number.
11. The closure property is not always applicable on the division of integers. e.g. In $3 \div 4$, the quotient is not an integer.
12. Except zero dividing an integer with the same integer, always gives **1** as the quotient.
13. Except zero, all integers when divided by their additive inverse would give - 1 as the quotient.
14. There is no existence of multiplicative inverse of zero.
15. Properties of Integers :-

Properties	**Addition**	**Subtraction**	**Multiplication**	**Division**
Closure	✓	✓	✓	✗
Commutative	✓	✗	✓	✗
Associative	✓	✗	✓	✗

Chapter 5

THE CIRCLE

In your previous class, you have read about the circle. You also know the shape of the circle. For example: bangles. The wheels of the bullock cart etc.

Below are given a few figures, observe whether they are circular or not and fill in the given blanks

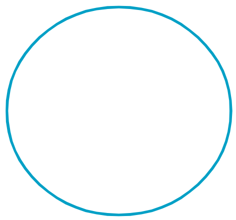


Fig 1

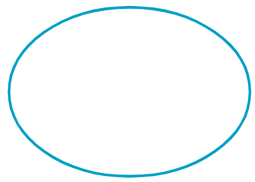


Fig 2

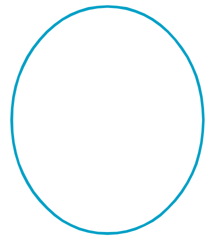


Fig 3

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There are many such objects around you with the help of which you can make circles in your notebooks.

Can you identify some such objects? Make a list of such objects and make circles using three of them.

The objects that you have used to make circles might not always be a complete circle; it might have uneven edges also. Therefore, to make a perfect (accurate) circle, we use a compass.

ACTIVITY 1

Drawing a Circle with the Help of a Compass

Take the compass from your geometry box. Fix a small pencil in the space meant for holding the pencil in the compass (fig. 4). Now spread the arms of the compass a little, so that you can rotate the arm with the pencil around the pointed arm of the compass which you have kept fixed in the centre of your notebook.

You must be careful not to displace the pointed end from the centre of your notebook. The figure like this is known as a circle.

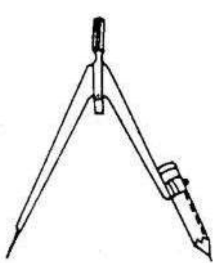


Fig 4

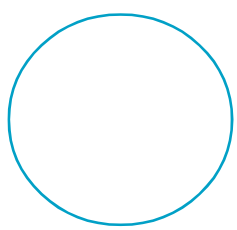


Fig 5

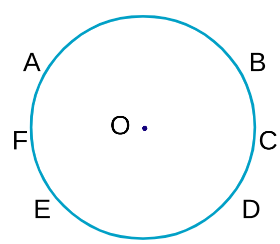


Fig 6

Put a dot at the point at which you had fixed the pointed end of the compass and name it as O. Now on the circle mark many points like A, B, C, D, E, F and measure their distances from point O.

1.	OA =	2.	OD =	3.	OB =
4.	OE =	5.	OC =	6.	OF =

Are all the measures equal?

Will any point on the circle give the same measure?

Similarly, make two more circles with the help of compass and verify the above results.

So, now you have learnt how to make a circle. We shall now do some activity to know the parts of a circle.

ACTIVITY 2

Make circle on piece of paper and cut out the shape of the circle from the paper with a pair of scissors. Now fold the circle in such a way that one half overlaps the other. Now, fold this semicircle into two equal parts again (again, one part will completely overlap the other.) Next open the paper and draw lines over the folds with the pencil.

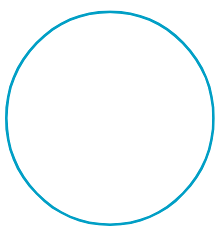


Fig 7

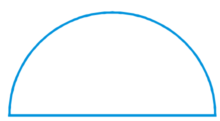


Fig 8

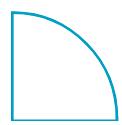


Fig 9

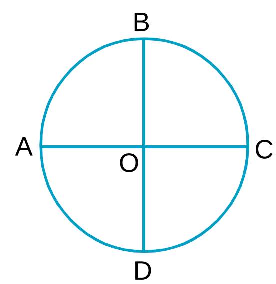


Fig 10

- Measure the lengths of AC and BD. Are they equal?
- Measure the distances from the point of intersection O for O to A, O to C, O to B and O to D and see whether their lengths are equals.

We observe that the distances of A, B, C and D from O are equal. This is known as the radius of the circle.

ACTIVITY 3

Make a circle in your copy with the help of the compass. Identify the centre of the circle. Now, take any point A on the circle. Now, join the different point of the circle to point A, in such a way that at least one line segment passes through the centre. Now measure the different line segments that you have drawn and note the measures in your notebook.

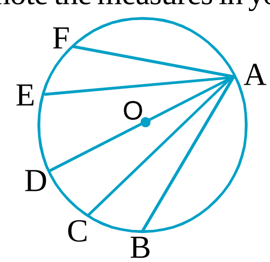


Fig 11

Now, find out the solutions to the questions given below:

1. Which is the longest line segment?
2. Does the longest line segment pass through the centre?
3. In one circle, how many such line can be drawn?

While doing this activity, you must have drawn several line segments connecting two points on a circle. These line segments are called chords. The longest chord passes through the centre of the circle and is called the diameter e.g. line segment (chord AD).

Since any line segment that passes through the centre is the longest chord and you also know that infinite number of lines can pass through a point; therefore in a circle, you can draw infinite number of diameters.

ACTIVITY 4

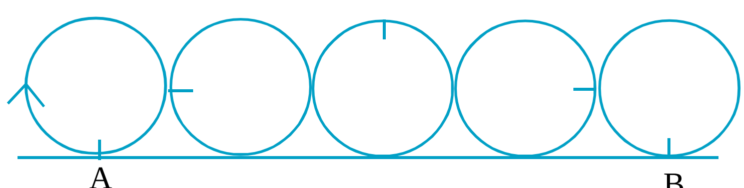
Draw three circles of different measures in your notebook and complete the following table.

S. no.	Length of the diameter Length of the radius	Diameter ÷ Radius
1.		
2.		
3.		

In the above activity, you find that the diameter of any circle is double the length of the radius which means a radius is half of the length of the diameter.

Now, measure the complete area covered by the circle. This is the circumference of the circle and we shall try to understand the relationship between the circumference and the radius.

You know that in any close figure, the perimeter of the figure is the length of the circumference. To find out the perimeter of a circle cut out a circle from a piece of cardboard, mark a point on its edge. Now draw a straight line on your notebook and mark a point A on this line at one of its ends. Make the cardboard circle stand on the straight line in a way that point A on the cardboard coincides with point A on the straight line.



One complete Rotation

Fig 12

Now roll the cardboard circle over and till the mark A on the circle comes in the contact of the straight line again. Mark this point on the straight line as B.

Next, measure the distance between the point A and B.

S.No.	Measure of the radius	Perimeter/ circumferenc of the circle	Diameter of the circle	Circumference ÷ diameter
1.	3·5 cm			
2.	7 cm			
3.	10·5 cm			

In the above table, the circumference ÷ diameter is nearly same for all the places, which means the ratio of circumference and diameter for any circle is always the same. This constant is indicated by the Greek letter π (pie) and its value is $\frac{22}{7}$ or 3.14 approximately.

EXERICSE 5

1. **Draw circles of the given measures -**

   (i) radius = 2 cm (ii) radius = 3.5 cm

   (iii) radius = 4.2 cm (iv) radius = 5 cm

2. **Draw circles of the given measurements -**

   (i) diameter = 3 cm (ii) diameter = 6 cm

   (iii) diameter = 6.8 cm (iv) diameter = 7.4 cm

3. **In a circle of radius 3.2 cm, draw a chord of 6.4 cms.**

4. **Draw a circle whose longest chord is 8 cm in length.**

5. **If the radius of a circle is 7 cm, what would be its circumference or perimeter?**

6. **Fill in the blanks -**

   (i) diameter = 2 × __________

   (ii) The largest chord in a circle is known as a ________________.

   (iii) The diameter of a circle passes through its ________________.

   (iv) The radii of a circle are ________________.

   (v) Two diameters in a circle intersect each other at the ___________.

   (vi) The line segment joining any point on the circumference of a circle from the centers known as its ________________.

   (vii) The line segment joining in any two points on the circumference of its__________.

What Have We Learnt ?

1. The line segment joining the centre of the circle to a point on the circumference of the circle is the radius of the circle.
2. All the radii of a circle are equal.
3. The line segment joining two points on the circumference of a circle is known as its chord.
4. The longest chord of the circle is its diameter which passes through the centre of the circle.
5. The diameter of the circle is twice its radius.
6. The ratio between the (perimeter) circumference of a circle and its diameter is always constant. This is known as π (pie) and its value is $\frac{22}{7}$ or 3.14 approxima-tely.

Chapter 6

FACTORS AND MULTIPLES

In the chapter on Integer, you have learnt that the operation on division doesn't follow the closure property. This means if an integer is divided by another integer, we don't always get an integer. Think about this and find out by which numbers 8 gets divided and doesn't give any remainder. And by which numbers does 7 get divided?

Factors

In $2 \times 5 = 10$, you have observed that 10 can be divided by 2 and 5 completely. The factors of a number are those numbers that divide the number completely.

$10 \div 2 = 5$

$10 \div 5 = 2$ thus, 2 and 5 are the factors of 10.

The factors of a number are all those numbers that can divide that number.

Every number at least gets compulsorily divided by 1 and itself.

For example: 12 get completely divided by 1.

12 get completely divided by 12.

Do you know any number that doesn't get completely divided by 1 or by the same number?

Divisible Numbers

Those numbers that get completely divided by numbers other than 1 and itself are known as *divisible numbers* or *composite numbers*.

Those numbers which get completely divided by any number are known as *factors* of that number.

Lets find out how many numbers can divide 12

$12 = 1 \times 12$

$12 = 2 \times 6$

$12 = 3 \times 4$

$12 = 3 \times 2 \times 2$

So, 12 can get divided by 1,2,3,4,6,12. Hence the number 12 is divisible by 1,2,3,4,6,12.

Indivisible or Prime Number

The numbers whose only factors are 1 and the number itself are called Prime numbers. For example: 13 can only get divided completely by 1 and 13. Other such numbers are 2, 3, 5, 7, 11 etc.

Numbers having more than 2 factors are called *Composite numbers*.

ACTIVITY 1

In the table below are given the factors of some numbers. Write down all the possible factors for the remaining numbers in the blank spaces and list out the numbers with one, two and more than two factors separately.

Number	All factors	Number	All factors
1	1	6	1, 6, 2, 3
2	1, 2	7	....
3	....	8	....
4	1, 4, 2	9	....
5	....	10	....

The table shows three types of numbers –

1. The numbers with only one factor. Such a number is 1. It is neither a (indivisible) prime nor a composite (divisible) number and therefore makes a separate category.
2. The numbers with two factors: 2, 3, 5, 7 etc. are numbers which have only two factors. These get divisible by that number and one only. Therefore these are prime numbers. Write such five more examples in your notebooks.
3. Numbers that have more than two factors e.g. 4, 6, 8, 9, 10 etc. which are known as composite numbers.

ACTIVITY 2

The Sieve of Eratosthenes

We can find prime numbers from 1 to 100 with the help of an easier method without actually checking the factors of a number. This method was given by a Greek mathematician Eratosthenes in 3rd century B.C. His method is known by the name the Sieve of Eratosthenes that helps us to separate Prime numbers and composite numbers.

The Sieve of Eratosthenes

Below, numbers from 1 to 100 are given in table. Now follow the instructions given:

Instructions:

1. Strike out 1, because 1 is not a prime number.
2. Encircle 2 and strike out all the numbers that get divided by 2, e.g. 4,6,8,10,12 etc.
3. Now take the next number that has not been struck out that is 3. Encircle 3 first and then strike out the numbers that get divided by 3.
4. Similarly, take the next number that has not been struk out, encircle it and strike off the remaining numbers in the list that get divided by the encircled number.

5. Repeat the process till all the numbers in the list upto 100 get crossed out.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	9	~~10~~
11	~~12~~	13	~~14~~	15	~~16~~	17	~~18~~	19	~~20~~
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

All the encircled numbers in the table are prime numbers. All the struck out numbers, other than 1 are composite numbers.

This method is known as the Sieve of Eratosthenes.

ACTIVITY 3

You know about the prime and composite numbers, now let us play the game of finding factors.

Draw some circles in you notebook. Beneath every circle write the numbers 1, 2, 3, 4, 5… etc. serially and then follow the instructions given below.

Instructions:

Write 1 in all those circles for the numbers that get divided by 1.

Write 2 in the circles for numbers that get divided by 2.

Write 3 in the circles for numbers that get divided by 3.

Now carry on this process for as many numbers you can and then answers the questions that follow:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

Q 1. How many circles contain only one numbers? Write down the figures outside those circles.

____

Q 2. How many circles have the two numbers? Write down the figures outside those circles.

____

Q 3. How many circles contain more than two numbers? Write down the figures outside those circles.

____

The numbers inside the circles are the factors for the numbers outside the circles. **All those numbers which have only two factors are the prime numbers.**

Co Prime Number

Let us discuss on factors of 8 and 15

Factors of 8 = 1,2,4,8

Factors of 15 = 1,3,5,15

It is clear that there is no common factor for 8 and 15 except 1. In this condition 8 and 15 will know as Co prime number.

Like this, Let us think on factors 9, 10 and 49

Factors of 9 = 1,3,9

Factors of 10 = 1,2,5,10

Factors of 49 = 1,7,49

In the example given above only 1 is common factor for 9, 10 and 49. No other common factor is there for 9, 10 and 49. So 9, 10 and 49 are Co prime numbers.

Numbers, which do not have any common factor between them other than 1 are called Co prime numbers.

Practice 1

1. Write down all the prime numbers between 1 to 100.
2. Write the composite numbers between 75 to 100.
3. Which number between 70 to 80 has the maximum number of factors?
4. Whether the 12 and 25 are Co prime numbers?
5. Can two consecutive number be Co prime?

Some other Types of Numbers

1. **EVEN NUMBERS :** These numbers that are completely divided by 2 are called even numbers. Example: 2, 4, 6, 8, 10, 12 etc.

2. **ODD NUMBERS :** These numbers that are not completely divisible by 2 are called odd number. Example: 1, 3, 5, 7, 9, 11 etc.

Below are given some odd numbers, separate out the prime and composite numbers from the list.

41, 45, 47, 53, 55, 57, 63, 67, 69.

71, 73, 77, 81, 87, 89, 91, 93, 95, 97, 99.

COMPOSITE NUMBERS	PRIME NUMBERS

Are all odd numbers prime?

Prime Factors

Let us now find out what are the prime factors of 42.

$42 = 14 \times 3$, here 3 is a prime number.

Is 14 also a prime number?

Number 14 will be written as 2×7.

That is, $42 = 2 \times 7 \times 3$

Now, here 2, 7, and 3 are all prime numbers. These are known as prime factors of 42.

What are the prime factors of 6? Take some more numbers and find out the prime factors.

Determining Prime Factors

How should we find out the prime factor of a number? Shall we divide each number several times to do so? Generally, we use the method shown below to find out the prime factors of any number. First, divide the given number by 2. If the number is divisible by 2. Then the number and its quotients are continuously divided by 2 till they get divided by 2. Then, if the number is divisible by 3. Similarly if for numbers like 5, 7, 11……..etc this process is repeated until the quotient obtained is 1.

Example 1: Let us observe the prime factors of 24.

2	30
3	15
5	5
	1

$\therefore$ 24 = 2 × 2 × 2 × 3.

Now take another number.

Example 2: The prime factor of 30.

2	24
2	12
2	6
3	3
	1

$\therefore$ 30 = 2 × 3 × 5.

Practice 2

1. Find out the prime factors for the following numbers:
   (i) 16 (ii) 48 (iii) 60 (iv) 84

You have learnt how to find out prime factors. Now let us think about all the multiple factors of any number.

Example 3: Is 18, a factor of 108?

1st method: If 18, a factor of 108, then 108 must be completely divisible by 18.

18)108(6
108
0

Remainder is zero. Therefore, 18 is a factor of 108.

2nd method: (i) Find out the prime factors of 18.
(ii) Find out the prime factors of 108.

Solution:

18 = 2 × 3 × 3
108 = 2 × 2 × 3 × 3 × 3.

Since all the numbers included in the prime factors of 18 are also the prime factors of 108 and they are repeated more times in 108, therefore 18 is a factor for 108.

By all factors of any number, we mean that 1, the number itself and all those numbers that completely divide the number for which we are finding factors.

Example 4: Write down all the factors of 18.

1st Method: As you already know that 18 gets divided by these numbers 1, 18, 2, 3, 6 and 9, remainder is zero.

Therefore, 1, 18, 2, 3, 6, 9 are the factors or dividends if 18.

2nd Method: Here all the dividends of 18 can be found out like this too.
18 = 1 × 18
18 = 2 × 9
18 = 3 × 6.

Therefore the divisors or factors of 18 would be 1, 2, 3, 6, 9, 18.

Example 5:

Write down all the factors of 60.

Solution:

60 = 1 × 60
60 = 2 × 30
60 = 3 × 20
60 = 4 × 15
60 = 5 × 12
60 = 6 ×10
Therefore the factors of 60 should be 1, 2, 3, 4, 5, 6, 10, 12, 20, 30, 60.

Practice 3

1. Write down all the dividends (factors) of the following numbers.

(i) 28 (ii) 36 (iii) 45 (iv) 72

You might have found that determining factors takes time. But now you don't know the rules of verifying divisibility, because of which without equally dividing the number, you are not able to say whether a number can be divided by 3, 5, 7 etc. or not.

Let us now learn some techniques for verifying divisibility.

Verification Rule of Divisibility

1. Verification of Divisibility by 2

If the unit's place of any number has 0, 2, 4, 6 and 8, then this number is completely divisible by 2.

20, 62, 34, 26, 18, ... are divisible by 2.

21, 63, 33, 35, 17, ... are not divisible by 2.

here, 18 is divisible by 2. Let us verify this by division.

2)18(9
–18
0

Therefore, it is completely divisible by 2.

2) 21(10
– 2
01
00
1

Remainder is 1
Therefore, it is not completely divisible by 2.

2. Verification of divisibility by 3

If the sum of all the digits of a number is divisible by 3, then the number is divisible by 3. For example: In 111111,the sum of all the digits would be 1 + 1 + 1 + 1 + 1 + 1 = 6, therefore the number is divisible by 3.

Similarly in 5112 the sum of all the numbers 5 + 1 + 1 + 2 = 9 and this number is divisible by 3.

The digits in 412 will give the sum 7, therefore this number will not be divisible by 3.

3. Verification of divisibility by 6

If any number is divisible by 2 and 3 separatly, then the number will be divisible by 6.

216, it is divisible by 2 (The digit in unit's place is 2)

It is divisible by 3 .(The sum of digits is 9)

So, it will be divisible by 6.

643212, is divisible by 2 (because the digit in unit's a place is 2)

is divisible by 3 (because the sum of digits is 18).

4. Verification of divisibility by 9

If the sum of its digits is divisible by 9, then the number will be divisible by 9.

The number 3663, is divisible by 9 (because the sum is 3 + 6 + 6 + 3 = 18, divisible by 9).

1827, is divisible by 9 (the of digits is 18, which is divisible by 9).

1227, is divisible by 9 (the of digits is 12, which is not divisible by 9).

5. Verification of divisibility by 5

If the digit in the unit's place is 0 or 5, the number will be divisble by 5.

e.g. 1045, is divisible by 5 (because the digit in the unit's place is 5).

940, is divisible by 5 (because the digit in the unit's place is 0).

6. Verification of divisibility by 10

If any number has 0 in its unit's place, then the number is divisible by 10.

Example:

1000, is divisible by 10 (digit in the unit's place is 0).

2130, is divisible by 10 (digit in the unit's place is 0).

5003, is not divisible by 10 (digit in the unit's place is 3).

7. Verification of divisibility by 4

If the number made by the ten's and unit's place digits of any number is divisible by 4 or the ten's and unit's place has zero, then the number is divisible by 4.

For example:

In 79412, the digits in ten's and unit's place are 1 and 2, so the number made by these two digits is 12. Since 12 is divisible by 4, therefore, 79412 will be divisible by 4.

1300 will be divisible by 4 (because the digits in ten's and unit's place are 0).

413 will not be divisible by 4 (because the digits 13 is not completely divisible by 4).

8. Verification of divisibility by 8

If the number made by unit's, ten's and hundredth places is divisible by 8, or the number contains 0 in all these three places, then the number would be divisible by 8.

31000	(divisible by 8)
1816	(divisible by 8, because 816 is divisible by 8)
12317	(not divisible by 8, because 317 is not divisible by 8)

9. Verification of divisibility by 7

Take a number and double its last digit. Now subtract this doubled number from the rest of the digits of the original number.

Repeat the process till the result is a digit like 1 or 2. If the obtained number is divisible by 7, then the original number also be divisible by 7.

In 1729, the last digit is 9, twice of 9 is 18

172 - 18 = 154

In 154, the last digit is 4, twice of 4 is 8

15 - 8 = 7, 7 is the last digit.

Therefore the number will be divisible by 7.

Do you know that 1729 is also known as the Ramanujan Number?

When the great Indian mathematician, Ramanujan was in England, he once became very ill. Prof. Hardy met Ramanujan in the hospital and this was the dialogue that followed-

Ramanujan : How did you come here, sir?

Prof. Hardy : By a taxi.

Ramanujan : What was the number of the taxi?

Prof. Hardy : 1729, not a very special number.

Ramanujan : No, professor, you're mistaken.

Its very interesting number that can be written as the sum of cubes of two numbers. In two different ways.

i.e. $1729 = 1^3 + 12^3$

$= 9^3 + 10^3$

10. Verification of divisibility by 11

For any number find out the sum of the digits in the odd places and the sum of the digits in the even places. If the difference between the sum of digits at odd places and the sum of digits in even places are 0, 11 or multiple of 11, then the number would be divisible by 11.

Example: In 856592,

the sum of digits in odd places

$= 8 + 6 + 9 = 23$

the sum of digits in even places

$= 5 + 5 + 2 = 12$

The difference between the two sums :

$23 - 12 = 11$

Therefore, the number is divisible by 11.

Example 6 :

Verify whether the number 805130425 is divisible by 11.

Solution :

In 805130425,

(1) The sum of digits in odd places = 8 + 5 + 3 + 4 +5 = 25

(2) The sum of digits in even places = 0 + 1 + 0 +2 = 3

The difference in the sums = 25 - 3

= 22

22 is divisible by 11, therefore, the number 805130425 is divisible by 11.

Practice 4

1. Put the ✓ mark on divisible number:

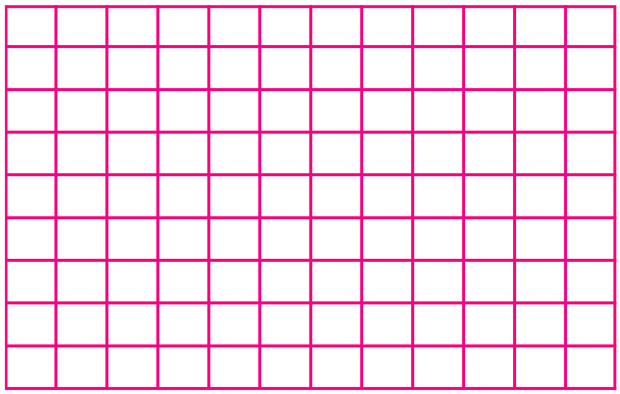
S.No.	Number	Number by which the number gets competely divided									
		1	2	3	4	5	6	7	8	9	10
1.	2550	✓	✓	✓		✓	✓				✓
2.	4914										
3.	9432										
4.	7332										
5.	13310										
6.	872										
7.	1210										

2. By which number is 27720 divisible?
3. Determine which of the statements are true or false.
   (1) 78 can be divided by 2.
   (2) 375 is completely divisible by 3, 5 and 10.
   (3) The number in which unit's place is 0, will be divisible by 5.
   (4) If the difference between sum of the digits in the even places and the sum of the digits in the odd places is zero, then the number would be divisible by 11.
   (5) The number 10080 is completely divisible by 2, 3, 4,5, 6, 7, 8, 9 respectively.

Highest Common Factor

ACTIVITY 4

Suppose you need to prepare a wooden scale to measure a 12 feet long and 9 feet wide room. What would be the maximum length of the long scale which can be used to measure both the length.



What lengths of scales can be used to measure this length of 12 feet. You will find that the length of 12 feet can be measured by scales that are 1, 2, 3, 4, 6 or 12 feet long. Note that all these numbers are multiple factors of 12.

Similarly, the measure of 9 feet can be taken by scales that are 1, 3 or 9 feet long. These are all the factors of 9. But we actually need a big scale that would measure both the lengths 12 feet and 9 feet. Note that the largest common factor for both 12 and 9 would be 3.

Since three is the highest common factor for 12 and 9; this is also known as the **Highest Common Factor** or **H.C.F.**

Let us find out the H.C.F. of some more numbers with the help of divisions or factors.

Example 7:

All multiple factors of 48: 1, 2, 3, 4, 6, 8, 12, 24, 48

Multiple factors of 64: 1, 2, 4, 8, 16, 32, 64

Multiple factors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

Encircle all the common factors and you'll see that the common (even) divisors are 1, 2, 4 and 8.

Therefore, the highest common factor for 48, 64 and 72 is 8.

This means 8 is the largest number that completely divides the numbers 48, 64 and 72. Hence the H.C.F. of 48, 64 and 72 is 8.

The highest or greatest common factor (divisor) out of the common factors for two or more than two number which completely divides the given number is known as the highest common factor for those numbers. This means,

H.C.F. = The largest equivalent or like factor

Method of Determining the H.C.F.

Example 8 : Find out the H.C.F. of 24, 36 and 60.

(1) The prime factor method :

24

$$\begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

36

$$\begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

60

$$\begin{array}{r|l} 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$24 = 2 \times 2 \times 2 \times 3$ $36 = 2 \times 2 \times 3 \times 3$ $60 = 2 \times 2 \times 3 \times 5$

Therefore the common multiple factor for 24, 36, 60 would be :

$$24 = 2 \times 2 \times 2 \times 3$$

$$36 = 2 \times 2 \times 2 \times 3$$

$$60 = 2 \times 2 \times 3 \times 5$$

$$= 2 \times 2 \times 3 = 12$$

(2) The factorisation method :

The factors of 24 1, 2, 3, 4, 6, 8, 12, 24

The factors of 36 1, 2, 3, 4, 6, 9, 12, 18, 36

The factors of 60 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Therefore, the common factors of 24, 36, 60 are 1, 2, 3, 4, 6, 12.

Highest common divisor = 12

Therefore, H.C.F. = 12

(3) The Division Method :

There are two methods in this process.

First Method :

Example 9 : Find out the H.C.F. for 16 and 36.

2	16, 36
2	8, 18
	4, 9

Steps:

(1) Divide 16 and 36 by the smallest prime number 2.

(2) Divide 8 and 18 by 2.

(3) It is not possible to divide 4 and 9 by any prime number.

(4) The highest common factor will be the product of these prime numbers which can act as divisor for all numbers.

Hence H.C.F. = $2 \times 2 = 4$

Example 10: Find out the highest common factor for 60,90 and 210.

2	60, 90, 210
3	30, 45, 105
5	10, 15, 35
	2, 3, 7

Steps:

(1) Divide 60, 90, 210 by 2.

(2) Divide 30, 45, 105 by 3.

(3) Divide 10, 15, 35 by 5.

Since 2, 3, 7 cannot be divided by any other prime number. Therefore,

H.C.F. = $2 \times 3 \times 5 = 30$

Second Method :

To find out the H.C.F. by this method, divide the bigger number by the smaller number till a remainder obtained is smaller than the divisor. Now consider the divisor as the dividend (number to be divided) and the remainder as the divisor and find the solution. Repeat this process till you get zero as the remainder. The divisor that gives a zero remainder will be the H.C.F. for the number. Let us work at these examples -

Example 11: Find out the H.C.F. of 15 and 63.

Solution : 15) 63 (4
– 60
3

Then, 3) 15 (5
– 15
0

Therefore, 3 is the highest commom factor for 15 and 63.

Example 12: Find out the largest common number which when used to divide 18 and 55, would give 2 and 3 as the remainder respectively.

Solution: Since we get remainder 2 when 18 is divided by the number to be found out. Therefore, the number is 18 - 2 = 16.

Similarly, when the remainder is 3, the other number would be 55 - 3 = 52.

This can be verified by finding out the H.C.F. of 16 and 52.

16) 52 (3
– 48
4) 16 (4
– 16
0

Thus, the H.C.F. of 16 and 52 is 4.

The Properties / Characteristics of Highest Common Factors

Let us understand the characteristics of H.C.F. through a few examples.

Example 13: Find out the H.C.F. of 15 and 60.

Since 60 gets completely divided by 15, therefore 15 will be the H.C.F. of 15 and 60.

Example 14: Determine the H.C.F. of 12, 36.

Since 36 is completely divisible by 12, therefore 12 will be the H.C.F. for 12, 36.

Example 15: Find out H.C.F. for 20, 40.

Since 40 is completely divisible by 20, therefore, the H.C.F of 20 and 40 is 20.

Example 16: Find the H.C.F.for the number 15, 16 and 13, 17.

Except 1 there is no common multiple factor for 15 and 16 except 1. Therefore, the H.C.F. for 15, 16 will be 1.

Similarly except 1 there is no common multiple factors for 13 and 17. Therefore the H.C.F. for 13 and 17 will be 1.

Properties :

(1) For two given numbers, if the bigger number is completely divisible by the smaller number, then the smaller number is the H.C.F. for the two numbers.

(2) The H.C.F. for those numbers which do not have any other common factor except 1, is 1.

EXERCISE 6.1

(1) Find out H.C.F. for the following with the help of prime factorisation method.

(i) 120, 104 (ii) 144, 198

(iii) 150, 140, 210 (iv) 108, 135, 162

(2) Find out H.C.F. by the division method.

(i) 252, 576 (ii) 300, 450

(iii) 72, 96, 144 (iv) 120, 300, 105

(3) What would be the H.C.F. for two consecutive numbers?

(4) Divide the numerator and denominators of the given fractions by there highest common factors and write the fractions in simplified forms.

(i) 1444/256 (ii) 2211/3025

(5) Two small tankers contain 85 and 68 liters of petrol respectively. Find out the maximum capacity of the measuring vessel with which the total quantity of petrol can be measured.

(6) Find out the largest divisor for the numbers 389, 436 and 542 that would give 4, 7 and 3 respectively as remainders.

(7) In classes (VI, VII and VIII) 6, 7, and 8 of a school, there are 220, 176 and 132 students respectively. Find the maximum number of students that can be included in a group so that the groups have equal number of students and can be made in each class.

(8) Hamida has 527 apples, 646 chickoos and 748 oranges. These fruits have to be distributed in equal heaps. What would be the number of fruits in the largest heap? How many such heaps will be formed?

(9) A rectangular room is 122 m long and 92 m wide from outside. If its walls are 1 m thick then find out the maximum length of a scale that can be used to measure the inner length and width of that room.

Hints : Inner length = 122 - 2 = 120 m.
Inner width = 92 -2 = 90 m.
(Determine the H.C.F. of 120 and 90)

Multiples

You must have learnt multiplication tables in your previous classes. you have also used tables while doing multiplications and divisions.

You know that $7 \times 3 = 21$, which means that the product 7 and 3 is 21, i.e. both 7 and 3 have a multiple 21.

All the number that occur in the multiplication table of 2 are the multiples of 2. Similarly 13, 26, 39, 52, 65, 78 etc. are all multiples of 13.

ACTIVITY 5

Below are given some numbers, write down their first five multiples in the boxes provided.

Multiple of	4	4	8	12	16	20
Multiple of	7					
Multiple of	10					
Multiple of	12					
Multiple of	15					
Multiple of	16					
Multiple of	20					

Lowest Common Multiple

We find several examples of lowest common multiples in our everyday life. Let us discuss some such situations.

Example 17 :

Ram went to the market to buy fruits. The shopkeeper showed him two types of bananas. The first type of bananas were Rs.10 for 6 bananas and the second kind were Rs. 10 for 8 bananas. Ram wanted to buy equal number of both the kinds of bananas using some 10 rupee note without getting any money as change. How many bananas of both kinds can he buy?

Solution : To find this out, we shall write down the multiples of 6 and 8 first.

Multiples of 6 : 6, 12, 18,24 30, 36, 42, 48, 54, 60, 66, 72, 78.

Multiples of 8 : 8, 16, 24, 32, 40, 48, 56, 64, 72

This means that Ram can have 6, 12, 18, 24,................ numbers of bananas of the first kind without getting any change for one, two, three and four 10-10 rupee notes and so on.

Similarly he can buy 8, 16, 24,.........etc. numbers of bananas of the second kind with 10-10 rupee notes without getting any change. To buy the same number of both type of bananas, he will have to buy the number of bananas that are common to both groups.

This means, he will have to take the number of bananas that is common for multiples of both 6 and 8.

These number are 24, 48, 72 etc.(These number have been encircled)

To buy the same number of bananas of both kinds 24, 48 or 72 bananas can be purchased with the help of 10 rupee notes.

Let us took at another example.

Example 18 :

A shopkeeper wants to purchase pens for his shop from the wholesale market. He chooses two kinds of pen. The first type of pen is available in 12 pieces per packet and the second type of pen as 15 pieces per packet. The wholesale dealer doesn't sell the loose pens, that is, he doesn't open the packets. Can you find out the minimum number of packets the shopkeeper will need to buy so that the number of each type of pen bought is equal?

Let us solve this problem with the help of a table.

No. of pens in the packets	Total number of pens				
	In 1 packet	In 2 packets	In 3 packets	In 4 packets	In 5 packets
12	12	24	36	48	60
15	15	30	45	60	75

Now you can see that if 5 packets containing 12 pens each are bought, the shopkeeper will have 60 pens and if 4 packets containing 15 pens each are taken, then again they'll be 60 pens.

To solve the abo ve problem, you have found the multiple of 12 and 15 and the common multiple which is the smallest among these, is the expected answer.

Smallest Common Multiple is called the Lowest Common Multiple or L.C.M.

Can you find the lowest common multiple of 10, 12 and 15.

Write down the multiples of 10 = ...

Write down the multiples of 12 = ...

Write down the multiples of 15 = ...

The smallest common multiple or the lowest common multiple = ...

Find out the L.C.M. of the numbers in table given below.

S. No.	Numbers	The multiples of the numbers	L.C.M.
1.	3, 5, 6	Multiples of 3 = 3, 6, 9, 12, 15, 18, 21 24, 27, 30 Multiples of 5 = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50 Multiples of 6 = 6, 12, 18, 24, 30, 36, 42, 48, 54, 60	30
2.	4, 6, 9		
3.	4, 9, 12		
4.	6, 15, 18		

So, we can say that

(1) The Lowest Common Multiple (LCM) of two or more given numbers is the lowest (or smallest or least) of their common multiples.

(2) L.C.M. of two or more than two numbers is that smallest number which can be divided by each of the given numbers.

Method of Determining L.C.M.

(1) Prime Factorisation Method

Example 19 : (i) Find out the prime factors of the given numbers : 16, 24.

$$\begin{array}{c|l} \multicolumn{2}{c}{\textbf{16}} \\ 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array} \qquad \begin{array}{c|l} \multicolumn{2}{c}{\textbf{24}} \\ 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$16 = 2 \times 2 \times 2 \times 2$ $\qquad$ $24 = 2 \times 2 \times 2 \times 3$

(ii) Take the smallest prime factors out of all these numbers. Write down the factors occuring maximum number of times in any number.

(iii) Choose the next greater prime factor and write down the factor occurs maximum number of times in any number.

(iv) Similarly, write all prime factors. Their multiplication will give L.C.M.

Thus $16 = 2 \times 2 \times 2 \times 2$

$24 = 2 \times 2 \times 2 \times 3$

The lowest prime factor 2 occures 4 times in 16. The lowest prime factor 3 occures only once in 24.

$$\text{L.C.M.} = 2 \times 2 \times 2 \times 2 \times 3 = 48$$

(2) The Division Method

Example 20 : Find out the L.C.M. of 12, 16, 24.

$$\begin{array}{r|l}
2 & 12, 16, 24 \\ \hline
2 & 6, 8, 12 \\ \hline
2 & 3, 4, 6 \\ \hline
2 & 3, 2, 3 \\ \hline
3 & 3, 1, 3 \\ \hline
 & 1, 1, 1
\end{array}$$

All numbers are divisible by 2.
All numbers are divisible by 2.
Two numbers are divisible by 2.
One numbers are divisible by 2.
Two numbers is divisible by 3.

The product of all the divisors is the L.C.M.

$2 \times 2 \times 2 \times 2 \times 3 = 48$

Relationship Between L.C.M. and H.C.F. and the Product of two Numbers

Example 21: Consider two numbers 12 and 16.

Let us multiply the two numbers, where the first number is 12 and the second number is 16.

Multiplication of the two numbers $= 1^{st}$ number $\times$ 2^{nd} number

$= 12 \times 16$

$= 192$

Now we shall find the H.C.F. and L.C.M. of the two numbers also.

H.C.F.

$$\begin{array}{r|l}
2 & 12, 16 \\ \hline
2 & 6, 8 \\ \hline
 & 3, 4
\end{array}$$

H.C.F. $= 2 \times 2 = 4$

L.C.M.

$$\begin{array}{r|l}
2 & 12, 16 \\ \hline
2 & 6, 8 \\ \hline
2 & 3, 4 \\ \hline
2 & 3, 2 \\ \hline
3 & 3, 1 \\ \hline
 & 1, 1
\end{array}$$

L.C.M. $= 2 \times 2 \times 2 \times 2 \times 3 = 48$

Therefore, H.C.F. × L.C.M.

$= 4 \times 48 = 192$

The product obtained in both the situations for the numbers 12 and 16 is the same (192).

So, we can now say that

First number × second number = H.C.F. × L.C.M.

or **Product of two numbers = H.C.F. × L.C.M.**

ACTIVITY 6

Verify the above stated relationship by finding H.C.F. and L.C.M. of the numbers given in the table.

First Number	Second Number	H.C.F.	L.C.M.	H.C.F. × L.C.M.	1^{st} no. × 2^{nd} no.
6	8	2	24	2 × 24 = 48	6 × 8 = 48
4	9				
30	36				
42	48				
108	18				
21	105				

EXERCISE 6.2

Oral Questions :

(1) The H.C.F. of two numbers is 2 and their L.C.M. is 12. If one number is 6. What will be the other number?

(2) If the product of two numbers are 338, what is the product of their L.C.M. and H.C.F.?

(3) What would be the L.C.M. of 2, 6, 8?

(4) Is the L.C.M. of 7 and 14 greater than 7 or smaller than 7?

(5) Can the L.C.M. of 15 and 30 be lesser than 30?

Written Questions :

(1) Find the L.C.M. of the following by factorisation method?

(i) 14, 28 (ii) 108, 162

(iii) 12, 15, 45 (iv) 40, 36, 126

(2) Find the L.C.M. of the following by division method?

(i) 28, 56 (ii) 112, 168

(iii) 36, 45, 72 (iv) 180, 184, 144

(3) On a field 55m long and 22m wide square shaped matresses have to be spread out. Find out the minimum number of same size matresses, that would be needed to cover field.

(4) If six bells starts ringing together and if they ring at intervals of 1, 2, 4, 6, 8, 10 and 12 seconds in a sequence, then how many times would they ring together in 30 minutes?

(5) A trader goes to Raipur every fourth day while another trader goes to Raipur every tenth day. If both the traders are goning to Raipur together on 3 January, what will be the next date on which they would go to Raipur together?

(6) If two numbers 24 and 36 have the H.C.F. 12, find out their L.C.M.

(7) The H.C.F. of two numbers is 13 and their L.C.M. is 1989. If one of the number is 117, find out the other number

(8) Shashank saves 4.65 rupees everyday. In atleast how many days, will he be able to save the complete amount of rupee value?

[**Hint** - In Rs. 4.65, Rs. 4 is in whole number. Now find the L.C.M. of 65 paise and 100 paise. Then number of whole days = $\frac{\text{L.C.M}}{65}$, which represent total amount in rupees]

(9) Can two numbers have a H.C.F. of 14 and L.C.M. of 204? Give reasons for your answer.

(10) On a particular day buses from Ratanpur to Raipur run at intevals of every 40 minutes and buses from Raipur to Ratanpur run every 45 minutes. If buses from two opposite directions cross each other on a bridge at 10.15 a.m., then at what time at the earliest next buses from opposite directions will cross each other at the bridge.

[**Hints :** L.C.M. of 40 and 45 = 360 minutes

$$\frac{360}{60} = 6 \text{ hours}$$

$$10.15 + 6.00 = 16.15$$

$$16.15 - 12.00 = 4.15 \text{ p.m.}]$$

What Have We Learnt ?

(1) A number is totally divisible by its factor.

(2) The multiples of a number is completely divisible by that number.

(3) Every number is a multiple and factor of itself.

(4) 1 is the factor for all numbers and it is neither prime nor composite.

(5) Only 2 is even prime number.

(6) The H.C.F. of two or more numbers is its greatest common factor.

(7) The L.C.M. of two or more given numbers is their lowest or smallest of their common multiples.

(8) The product of two numbers is equal to the product of their H.C.F. and L.C.M.

(9) All the multiple of 2 are even numbers.

(10) Those numbers which are not multiples of 2, are odd numbers.

(11) The H.C.F. of two numbers is one factor of their L.C.M.

(12) The H.C.F. of numbers, cannot be greater than the numbers themselves.

(13) The L.C.M. of numbers cannot be smaller than the numbers themselves.

(14) The numbers which have only one common factor (1) are called Co prime numbers

Chapter 7

FRACTIONS

Raju has studied fractions in his previous classes, but he is worried why are fractions actually necessary? He never required to divide any number while counting things, then why should a number need to be divided? Will he need to divide a rupee coin into four equal parts to get Rs.1.25.

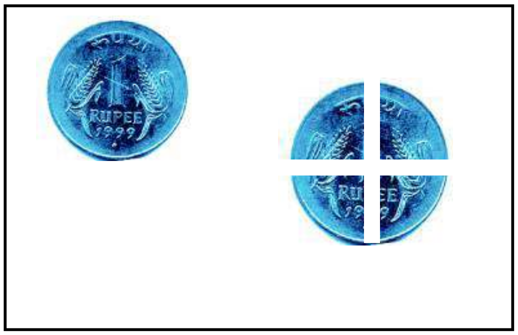


Fig. 1

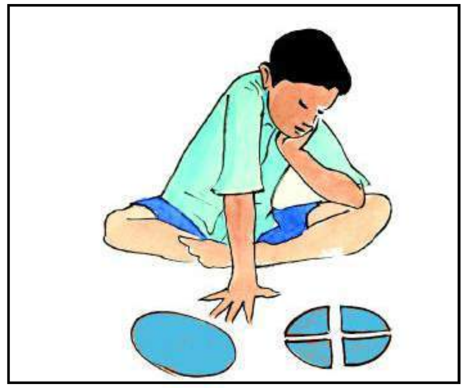


Fig. 2

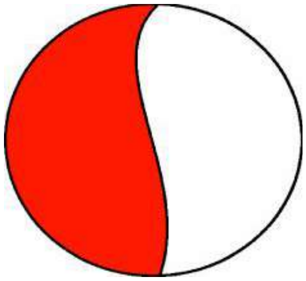
Just then Dolly called out, “Raju, Rashmi, Farida, come, it’s lunch time. Let us take our tiffins.” On opening the tiffin boxes, 10 puries were in front of them. Now the problem was, she needed to divide the 10 puris equally among four of them.

To distribute equally, initially Dolly gave two puris to each. Two ‘puris’ still remained, which she needed to distribute equally among all four of them. So, she halved each of the two puris and gave one half to each of the four members. Thus every one got two and half puris to eat. Raju felt that this half puri should be divided into two equal parts, he tore the half puri into two equal pieces again, and showing one part of it and asked, “how much do we call this part of a puri?”

Farida also divided her share of half puri into two equal parts and then putting the two parts of her half puri and the other two parts of Raju’s half puri together, said, “Look, this becomes one whole puri now. Since this puri has been divided into four equal parts, so each part is one fourth part of the whole puri, which means one divided by four”. Raju immediately asked, “ Well, will 2 pieces then become equal to $^2/_4$?” Rashmi said, “yes and three piece then will equal $^3/_4$ and all the four pieces will make $^4/_4$ which is equal to 1, that means a whole puri. If we have five such pieces of puri then it will mean 1 and $^1/_4$ that is, $1\,^1/_4$.

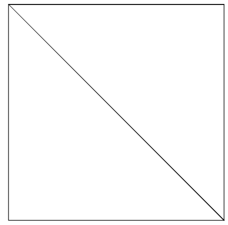
Now Raju began to wonder that when 3 out of 4 equal pieces of a puri are taken, it shows $^3/_4$, then when $^3/_5$ of any object will be required, we shall need to make five equal parts of the thing and take 3 out of them.

Raju has began to understand something about fractions now. Would you like to verify if you have understand it? Below are given some figures. Some numbers are written beneath the figures. Look at them and shade the figures according to the numbers that are provided with them.



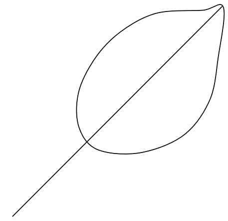
$\frac{1}{2}$ Part

Fig 3



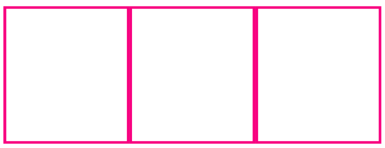
$\frac{1}{2}$ Part

Fig 4



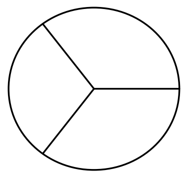
$\frac{1}{2}$ Part

Fig 5



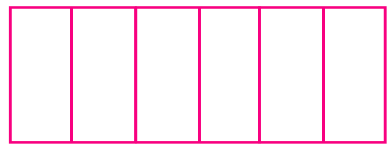
$\frac{1}{3}$ Part

Fig 6



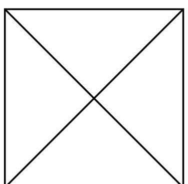
$\frac{1}{3}$ Part

Fig 7



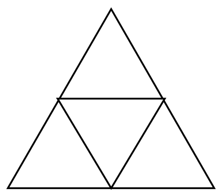
$\frac{2}{6}$ Part

Fig 8



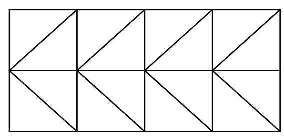
$\frac{1}{4}$ Part

Fig 9



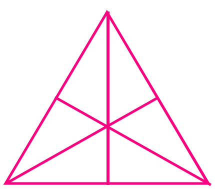
$\frac{3}{4}$ Part

Fig 10



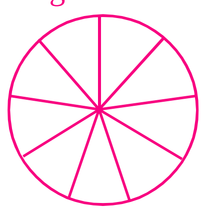
$\frac{12}{16}$ Part

Fig 11



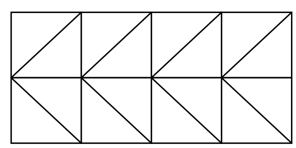
$\frac{1}{2}$ Part

Fig 12



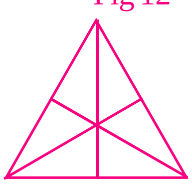
$\frac{7}{9}$ Part

Fig 13



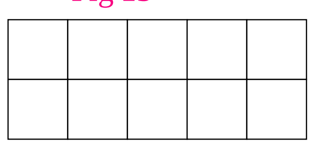
$\frac{3}{4}$ Part

Fig 14



$\frac{3}{6}$ Part

Fig 15



$\frac{1}{5}$ Part

Fig 16

$\frac{2}{10}$ Part

Fig 17

From the above shaded fractions which fraction did occupy the same area, identify them and complete the table

Now fill in the table these fractions which have made you shade equal parts of the given figures. Identify them and complete the table given.

Figure No.	**Fractional value of 1st fraction**	**Fractional value of 2nd fraction**	**Conclusion**
Fig. 6 and 8	$\frac{1}{3}$	$\frac{2}{6}$	$\frac{1}{3} = \frac{2}{6}$

In all the above examples, you can see that, if the numerator and denominator of a fraction are multiplied by the same number, or any digit is used to divide the numerator as well as denominator of a fraction, the value of the fraction does not change. This maens any fraction can be represented in more than one way. Some examples are as follows :

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$$

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}$$

When fractions are represented in different ways like this, we call them *equivalent fractions*.

ACTIVITY 1

Complete the table given below. One example is given for you.

Fractions	**Equivalent fractions obtained**				
	Multiplying by $\frac{2}{2}$	Multiplying by $\frac{3}{3}$	Multiplying by $\frac{4}{4}$	Multiplying by $\frac{5}{5}$	Multiplying by $\frac{6}{6}$
$\frac{2}{7}$	$\frac{2}{7} \times \frac{2}{2} = \frac{4}{14}$	$\frac{2}{7} \times \frac{3}{3} = \frac{6}{21}$	$\frac{2}{7} \times \frac{4}{4} = \frac{8}{28}$	$\frac{2}{7} \times \frac{5}{5} = \frac{10}{35}$	$\frac{2}{7} \times \frac{6}{6} = \frac{12}{42}$
$\frac{3}{8}$					
$\frac{4}{5}$					
$\frac{5}{9}$					
$\frac{4}{6}$					

ACTIVITY 2

Below are given some fractions. Write the appropriate numerator or denominator in the boxes that would make them *equivalent fractions*.

(i) $\frac{3}{5} = \frac{\square}{30}$ (ii) $\frac{4}{7} = \frac{12}{\square}$ (iii) $\frac{7}{9} = \frac{35}{\square}$

(iv) $\frac{34}{51} = \frac{2}{\square}$ (v) $\frac{26}{65} = \frac{\square}{5}$ (vi) $\frac{37}{74} = \frac{\square}{2}$

(vii) $\frac{10}{36} = \frac{5}{\square}$ (viii) $\frac{27}{81} = \frac{\square}{3}$ (ix) $\frac{30}{36} = \frac{\square}{6}$

(x) $\frac{3}{4} = \frac{21}{\square}$ (xi) $\frac{4}{9} = \frac{\square}{54}$ (xii) $\frac{11}{13} = \frac{55}{\square}$

What method did you use in finding out equivalent fractions in the above questions?

In Activity 2 (i), the denominator is 5. The fraction has to be changed in such a way that the denominator becomes 30. 5 multiplied by 6 makes 30. Therefore to make an equivalent fraction we can multiply the numerator also by 6.

$$\frac{3}{5} \times \frac{6}{6} = \frac{18}{30}$$

ACTIVITY 3

Given below are pairs of fractions. Change the pairs into equivalent pairs with common denominator and write down the equivalent fractions in the given table.

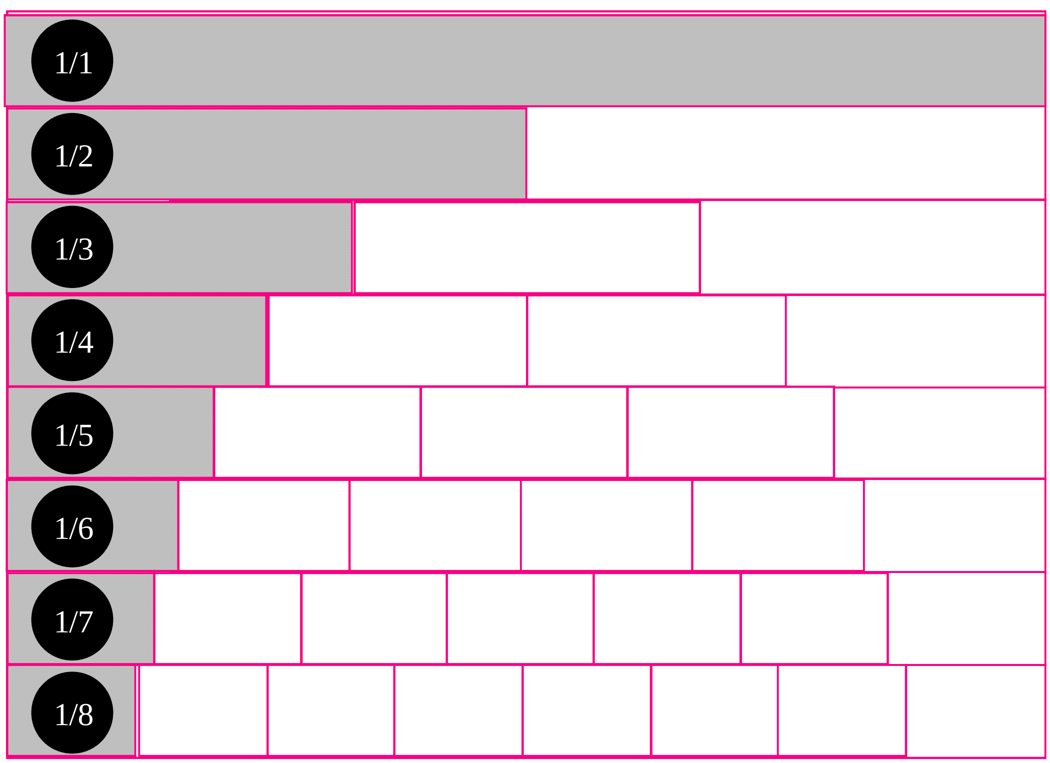
S. No.	Fraction	Denominator	Common denominator	Fractions with equivalent denominator
1.	$\frac{1}{2}$ and $\frac{1}{3}$	2, 3	6	$\frac{3}{6}$ and $\frac{2}{6}$
2.	$\frac{3}{5}$ and $\frac{4}{7}$			
3.	$\frac{1}{3}$ and $\frac{3}{4}$			
4.	$\frac{4}{4}$ and $\frac{1}{6}$			
5.	$\frac{3}{5}$ and $\frac{5}{7}$			

6.	$\frac{2}{6}$ and $\frac{1}{9}$			
7.	$\frac{7}{7}$ and $\frac{3}{5}$			
8.	$\frac{5}{3}$ and $\frac{7}{9}$			
9.	$\frac{5}{8}$ and $\frac{1}{6}$			
10.	$\frac{5}{6}$ and $\frac{4}{9}$			
11	$\frac{4}{15}$ and $\frac{3}{20}$			
12	$\frac{5}{12}$ and $\frac{7}{18}$			

In activity 3, you have converted all the fractions into fractions with same denominators. These are known as fractions with equal denominator or *equi-denominatoral fractions*. But in example 8, 9, 10 and 11, you must have observed that if the common denominator is obtained by taking the L.C.M. of the denominators of the two given fractions, then we get the simplest form of the numerator and denominator of the fractions.

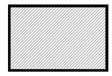
Example 1:

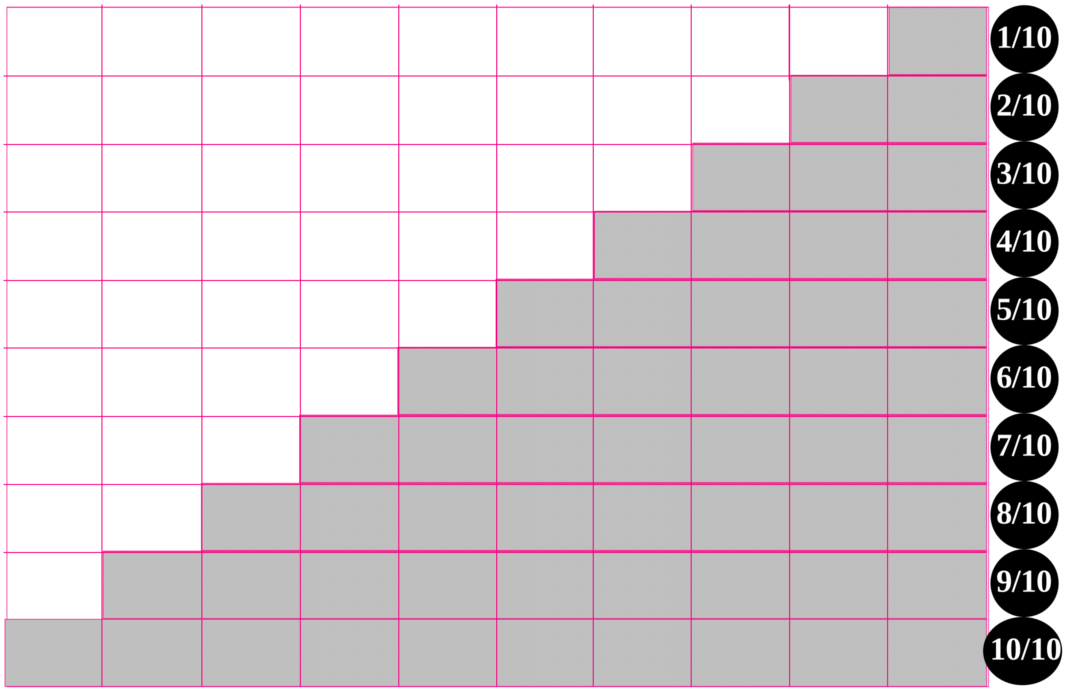
Order or sequence in fractions



Therefore we see that $\frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \frac{1}{5} > \frac{1}{6} > \frac{1}{7} > \frac{1}{8}$. This means when the numerators are equal, then the greater denominator gives a smaller value to the fraction.

Example 2:

$\frac{1}{10}$ is represented by 



The above picture clearly shows that

$\frac{1}{10} < \frac{2}{10} < \frac{3}{10} < \frac{4}{10} < \frac{5}{10} < \frac{6}{10} < \frac{7}{10} < \frac{8}{10} < \frac{9}{10} < \frac{10}{10}$.

This means when the denominators are equal, larger numerator will give greater value to the fraction.

Therefore, when the denominators of two given fractions are not equal, they are changed into equivalent fractions with the same denominators to find out which of them is greater.

Example : Which is a greater fraction between $\frac{5}{12}$ and $\frac{7}{18}$.

Solution : The L.C.M. of 12 and 18 is 36.

So, equivalent fractions would be $\frac{15}{36}$ and $\frac{14}{36}$.

Therefore, $\frac{15}{36} > \frac{14}{36}$ or $\frac{5}{12} > \frac{7}{18}$.

Practice 1

Write the given fractions in their increasing order.

(1) $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ (2) $\frac{7}{6}, \frac{6}{7}, \frac{5}{9}$

(3) $\frac{7}{9}, \frac{11}{15}, \frac{13}{18}$ (4) $\frac{3}{7}, \frac{8}{9}, \frac{5}{12}$

(5) $\frac{11}{12}, \frac{11}{13}, \frac{11}{14}$

So, now you have learnt how to write fractions in their ascending or descending orders by changing them into fractions with the same denominators.

Similarly, we can add or subtract fractions by changing them into fractions with equal denominators as in following example.

Example 3: Solve : $\frac{3}{5}+\frac{7}{9}+\frac{2}{3}$

To find a solution, we will first have to make fractions with common denominators so that we can add equal fractions. To get equivalent fractions with equal denominators we find out the L.C.M. of the three denominators.

$$\begin{array}{c|c} 3 & 5, 9, 3 \\ \hline & 5, 3, 1 \\ \hline \end{array} \quad 3 \times 5 \times 3 \times 1$$

The L.C.M. would be 45.

Now, we shall get equivalent fractions with the same denominators like

$$\frac{27}{45}, \frac{35}{45}, \frac{30}{45}$$

Therefore, $\frac{3}{5}+\frac{7}{9}+\frac{2}{3} = \frac{27}{45}+\frac{35}{45}+\frac{30}{45}$

Since the denominator are same in all the three fractions, only the numerator can be added, that is :

$$= \frac{27+35+30}{45} = \frac{92}{45}.$$

Example 4: Solve: $\frac{1}{3}+\frac{3}{5}-\frac{8}{12}$

(1) The L.C.M. of 3, 5 and 12 is

$$\begin{array}{c|c} 3 & 3, 5, 12 \\ \hline & 1, 5, 4 \\ \hline \end{array} \quad \text{L.C.M.} = 3 \times 1 \times 5 \times 4 = 60$$

Hence $\frac{1}{3}+\frac{3}{5}-\frac{8}{12} = \frac{20}{60}+\frac{36}{60}-\frac{40}{60} = \frac{20+36-40}{60} = \frac{16}{60} = \frac{4}{15}$.

[A fraction is said to be in the simplest (or lowest) form if its numerator and denominator have no common factor except 1.] You also know that the numerator and denominator of a fraction are divided by the same number, there is no change in the value of the fraction. Therefore, dividing the numerator and denominator of $\frac{16}{60}$ by 4. We have got, $\frac{16}{60} = \frac{4}{15}$.

Practice 2

Solve :

S. No.	Question	L.C.M. of the denominator	Conversion of fractions with equal denominator using L.C.M.	Addition and subraction of fraction with equal denominator	Solution	Simple fraction
1.	$\frac{3}{5}+\frac{7}{9}+\frac{1}{15}$	45	$\frac{27}{45}+\frac{35}{45}+\frac{3}{45}$	27+35+3=65	$\frac{65}{45}$	$\frac{13}{9}$
2.	$\frac{2}{3}+\frac{3}{5}-\frac{1}{6}$	30	$\frac{20}{30}+\frac{18}{30}-\frac{5}{30}$	20+18+5=33	$\frac{33}{30}$	$\frac{11}{10}$
3.	$\frac{1}{6}-\frac{4}{7}+\frac{8}{4}$					
4.	$\frac{2}{5}-\frac{11}{13}+\frac{15}{4}$					
5.	$\frac{6}{7}+\frac{11}{14}-\frac{9}{21}$					
6.	$\frac{3}{26}-\frac{5}{39}+\frac{1}{13}$					

While solving the questions given above you have found that when the value of the numerator is greater than the denominator, this fraction is known as an **Improper Fractions**.

Example : In $\frac{13}{9}$ the numerator 13 is greater than the denominator 9 i.e. 13>9.

Therefore, $\frac{13}{9}$ is as improper fraction.

Similarly, $\frac{11}{10}$ is an improper fraction. $\frac{13}{9}$ can also be written as $1+\frac{4}{9}$ or $\frac{13}{9}$.

This representation is known as a Mixed fraction. When the numerator of the fraction is smaller than its denominator, it is known as a **proper fraction**.

Like $\frac{3}{9}, \frac{5}{7}, \frac{101}{106}$ etc.

ACTIVITY 4

Of the given fractions, identity the proper and improper fractions and write the improper fractions as mixed fractions.

S. No.	Fractions	Proper or improper	If improper then write in the form of mixed fraction
1.	$\frac{127}{29}$	Improper	$4\frac{11}{29}$
2.	$\frac{29}{127}$	Proper	
3.	$\frac{29}{133}$		
4.	$\frac{81}{10}$		
5.	$\frac{126}{127}$		
6.	$\frac{36}{39}$		
7.	$\frac{103}{13}$		
8.	$\frac{335}{33}$		

Multiplication and Division of Fractions

When two fractions are multiplied, then the numerator of one is multiplied with the numerator of the other, and the denominator of one is multiplied by the denominator of the other. For example, If we have to find out $\frac{1}{2}$ of $\frac{1}{2}$, we shall get it by $\frac{1}{2} \times \frac{1}{2}$.

Similarly, half of $\frac{3}{4}$ would be $\frac{3}{4} \times \frac{1}{2}$.

We also know that the half of $\frac{1}{2}$ is $\frac{1}{4}$ and the double of $\frac{1}{2}$ is 1. This means if denominator is multiplied by denominator and the numerator by numerator, then we get the answer :

$$\frac{3}{8} \times \frac{2}{5} = \frac{3}{8} \times \frac{2}{5} = \frac{6}{40} = \frac{3}{20}.$$

Let us understand the operation of division by the following examples :

$6 \div 3$ means; how many times 3 comes in 6. Now think how many times does $\frac{1}{4}$ occur in $\frac{1}{2}$. Obviously, both the problems have the same answer 2. Similarly, $\frac{1}{2}$ occurs three times in $\frac{3}{2}$.

$$3 \div 5 = \frac{3}{1} \div \frac{5}{1} = \frac{3}{1} \times \frac{1}{5} = \frac{3}{5}$$

or $$8 \div 9 = \frac{8}{1} \div \frac{9}{1} = \frac{8}{1} \times \frac{1}{9} = \frac{8}{9}$$

or $$\frac{3}{2} \div \frac{5}{7} = \frac{3}{2} \times \frac{7}{5} = \frac{21}{10}$$

or $$\frac{8}{7} \div \frac{11}{13} = \frac{8}{7} \times \frac{13}{11} = \frac{104}{77}.$$

Thus, when one fraction is divided by another fraction. Then the fraction which is the divisor is written as an inverse, that is the denominator becomes the numerator and the numerator becomes the denominator and the sign of division is put on sign of multiplication.

Practice 3

Write the given fractions in their simplest forms :

(1) $\frac{1}{3} \div \frac{5}{7}$ (2) $\frac{121}{70} \div \frac{11}{35}$

(3) $\frac{27}{8} \div \frac{81}{16}$ (4) $\frac{33}{28} \div \frac{11}{4}$

Make more such problems and solve with your friends.

Place Value of Numbers in Fractional Forms

Till now you have played with numbers in many ways. You have learnt addition, subtraction, multiplication and division of numbers. You have also learnt to put numbers in places-units, tens, hundreds and thousands. Let us now discuss something more about place values.

How many 3 digits numbers can you make by changing the sequence of 3, 6 and 8.

(i) 368 (ii) 386 (iii) ______

(iv) _____ (v) ____ (vi) ______

Note that each time you are using the same digits 3, 6 and 8 but why are the values of the numbers different each time? Discuss with your friends and write down the reasons for these differences?

____

____

____.

Mary said to Hamida -

The place value of 8 in 368 is 8

The place value of 8 in 386 is 80

The place value of 8 in 836 is 800.

Thus, the value of the same number is different for different places. If we write eight thousand eight hundred and eighty eight (8888), then the value of 8 in one place is 8000, at the next place it is 800, in another place it is 80 and at the other place it is 8.

Let us add two numbers :-

Thousands	Hundreds	Tens	Units
	3	6	8
	8	9	5
	11	15	13

Can we add up the numbers like this?

On addition of the unit places we get 13, the tens places on addition gives 15 and the addition of hundredth place give 11. If we place the sums in the place value chart, we find 11 hundreds, 15 tens and 13 units. Therefore, this can be displayed in the following manner also:

11 hundreds + 15 tens + 13 units, but the largest digit that can be at any place is 9. because when it is 10; the number retained at that place would be 0 and 1 will be shifted to the next place to be added at that place. In the above example, addition of 8 and 5 gives 13 units. In the number 13, 3 is in units place and 1 is in the tens place, so 3 is kept in the unit's place and 1 being in ten's place is added up with 6 and 9 in the ten's place.

Thus, adding all tens place numbers would give 6 + 9 + 1 = 16 tens. In 16 tens, 10 tens is equal to one hundred, therefore, only 6 will be written in the ten's place, while the 1 or 10 hundreds will be added up in the hundredth place. This will give 3 + 8 + 1 = 12 hundreds. Thinking ahead in the same sequence in 12 hundreds, 10 hundreds equal 1 thousand. So, we separate the digits, to write only 2 in the hundred's place. Thereby, the value of remaining 10 hundreds being 1 thousand, 1 will be placed in the thousandth place.

Thus, the sum of the addition would be :

Thousands	Hundreds	Tens	Units	
1	2	6	3	= 1263

Find out the sum of the given numbers as in the above example:

(1)

Thousands	Hundreds	Tens	Units
	7	8	5
	6	1	8

(2)

Thousands	Hundreds	Tens	Units
	5	6	8
	4	3	9

(3)

Thousands	Hundreds	Tens	Units
	8	6	4
	3	9	5
	9	2	7

(4)

Thousands	Hundreds	Tens	Units
	4	3	8
	8	6	7
	2	8	9

So, it is clear that 10 units = 1 tens
10 tens = 1 hundred
10 hundreds = 1 thousand
10 thousands = 1 ten thousand
10 ten thousands = 1 lakh.

Similarly, if we move in the opposite direction.

1 Lakh ÷ 10 → 10 Thousand ÷ 10 → 1 Thousand ÷ 10 → 1 Hundred ÷ 10

1 Ten ÷ 10 → 1 Unit ÷ 10 If we proceed on ...

Moving in the opposite direction :

100 Thousand 1 Lakh ← 10 Thousand 10 Thousand ← 10 hundred 1 Thousand ← 10 Ten 1 Hundred ← 10 Unit 1 Ten ← 1 Unit

On moving from right to left, the values increase in multiples of 10. In the opposite direction, the values get divided by 10. Now think if unit is divided by 10, what will happen? You would remember

$$1 \div 10 = \frac{1}{10} = 0.1$$

So, if this sequence is maintained:

1 Lakh $\xrightarrow{\div 10}$ 10 Thousand $\xrightarrow{\div 10}$ 1 Thousand $\xrightarrow{\div 10}$ 1 Hundred $\xrightarrow{\div 10}$ 1 Ten $\xrightarrow{\div 10}$

→ 1 Unit $\xrightarrow{\div 10}$ $\frac{1}{10}$ 10th part 0.1 $\xrightarrow{\div 10}$ $\frac{1}{100}$ 100th part 0.01 $\xrightarrow{\div 10}$ $\frac{1}{1000}$ 1000th part 0.001 $\xrightarrow{\div 10}$ $\frac{1}{10000}$ 10000th part 0.0001

Therefore, we can say that -

Just as while moving from right to left, the place value gets multiplied by 10; similarly, while moving from left to right, the place value gets multiplied by $\frac{1}{10}$ or becomes the tenth part of that value.

Let us observe the following examples.
Find out the place values in 0.325.

First place after the decimal or 0.1 = $\frac{1}{10}$	Second place after the decimal or 0.1 = $\frac{1}{100}$	Third place after the decimal or 0.1 = $\frac{1}{1000}$
3	2	5
Or 3 × .1 = .03	2 × .01 = .02	5 × .001 = .005

or .3 + .02 + .005 = .325

Similarly, .628 = .6 + .02 + .008

$$= \frac{6}{10} + \frac{2}{100} + \frac{8}{1000}$$

ACTIVITY 5

Complete the given table with the digits in the appropriate place values.

Number	100000	10000	1000	100	10	1	.1 =	.01 = $\frac{1}{100}$	.001 = $\frac{1}{1000}$	.0001 = $\frac{1}{10000}$
	One lakh	Ten thousand	Thousand	Hundred	Ten	Unit	First place after decimal	Second place after decimal	Third place after decimal	Fourth place after decimal
830000.3257										
63.0095										
30.8007										
968.038										
3235.0509										

We have studied about length in class 5th In which we have learned

1. 10 mm = 1 cm

$$1 \text{ mm} = \frac{1}{10} \text{ cm} = 0.1\text{cm}$$

2. 100 cm = 1 meter

$$1 \text{ cm} = \frac{1}{100} \text{ meter} = 0.01 \text{ meter}$$

3. 1000 meter = 1 Km

$$1 \text{ meter} = \frac{1}{1000} \text{ Km } = 0.001 \text{ Km}$$

Example 5. Ramesh covers the distance of 150.5 Km between two cities by train, 65.7 km by Bus and remaining distance of 900 meter by walk. Tell the total distance covered by Ramesh?

Solution :

Distance covered by Ramesh

By Train = 150.5 km

By Bus = 65.7 Km

By Walk = 900 Meter

We know that

$$1 \text{ meter} = \frac{1}{1000} \text{ Km}$$

$$900 \text{ meter} = \frac{1}{1000} \times 900 = 0.9 \text{ Km}$$

So,

$$\begin{array}{r} 150.5 \text{ Km} \\ 65.7 \text{ km} \\ +\ 0.9 \text{ km} \\ \hline 217.1 \text{ km} \\ \hline \end{array}$$

So, Ramesh covered the total distance of 217.1 Km.

You know that

Example 6. If costs of Pens are 72 Rs, So what is the cost of 1 Pen?

Solution : Rs. 1 = Paise 100

$$\text{Paise } 1 = \text{Rs. } \frac{1}{100} = \text{Rs.} 0.01$$

Cost of 6 Pens = Rs. 72

So cost of 1 pen = Rs. 72/6

= Rs. 12

So cost of 1 pen will be Rs. 12

Example 7 -

The temperature of a city at afternoon in a day 36ºC and temperature at night was 28.5ºC. So, catch late the temperature fall.

temperature at noon = 36.0ºC
temperature at night = 28.5ºC
change in temperature = 36.0ºC - 28.5ºC
= 7.5ºC

Practice 4

1. The cost of one meter cloth is Rs. 24.75, So find out the cost of 2.8 meter cloths.
2. Anju buys a book costing Rs .143.60 from a shopkeeper and he gives Rs. 500 notes to him tell. How much money the shopkeeper has returned to Anju.
3. Akshat travels a distance of 26 Km by car, distance of 105 Km 500m by bus and remaining distance of 1 Km 250m by walk up to village. Find out the total distance he travelled?
4. The temperatures of two cities are 20.50ºC and 24ºC respectively. Determine the temperature difference of these two cities.

EXERCISE 7

1. Write True/False against the given statements and correct the statements that are false :

(i) $\frac{13}{16}$ and $\frac{78}{119}$ are equivalent fractions. ____

(ii) $\frac{33}{17}$ is a proper fraction. ____

(iii) $\frac{15}{33}$ and $\frac{60}{88}$ are equivalent fractions. ____

(iv) $\frac{23}{103}$ is an improper fraction. ____

(v) $\frac{13}{3}$ can also be written as $4\frac{1}{3}$. ___

(vi) $\frac{3}{2} < \frac{2}{3}$. ____

(vii) $-1 < .01$ ____

(viii) .2 × .3 = .6 ____

(ix) $\frac{135}{10000} = .0135$ ____

(x) .056 × 1000 = 56 _____

2. (a) Write the given fractions in decreasing order.

(i) $\frac{5}{6}, \frac{7}{8}, \frac{8}{9}$ (ii) $\frac{1}{2}, \frac{3}{4}, \frac{1}{6}, \frac{7}{6}, \frac{8}{12}$

(b) Write the given numbers in decreasing order.

(i) .0008, .08, .008, .8, 8 (ii) .01, .0099, .00992, .0012

3. Write the given fractions in increasing order.

(i) $\frac{5}{6}, \frac{9}{24}, \frac{3}{2}, \frac{1}{3}, \frac{5}{8}$ (ii) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{2}{15}$

4. Find out the values of the following -

(i) $\frac{1}{3}+\frac{5}{8}+\frac{3}{5}+\frac{7}{4}+\frac{13}{6}$ (ii) 9 + .9 + .09 + .009 + .0009

(iii) $\frac{3}{5}\times\frac{7}{5}\times\frac{4}{3}\div\frac{28}{15}$ (iv) $\frac{13}{27}\times\frac{3}{26}\div\frac{1}{18}$

(v) $\frac{17}{6}+\frac{19}{4}+\frac{5}{2}+\frac{4}{3}$ (vi) $\frac{6}{7}+\frac{13}{14}-\frac{9}{21}$

5. Complete the following blanks -

(i) $\frac{4}{5}=\frac{.....}{30}$ (ii) $\frac{7}{5}=\frac{.....}{55}$ (iii) $\frac{6}{7}=\frac{.....}{42}$ (iv) $\frac{4}{9}=\frac{.....}{18}$

6. Choose the proper and improper fractions from the following :

$\frac{17}{4}, \frac{4}{5}, \frac{8}{9}, \frac{16}{13}, \frac{15}{16}, \frac{6}{5}, \frac{3}{7}, \frac{8}{5}$.

7. Find out the place value of the following :

(i) 843.23 (ii) 14.876 (iii) 8764.0314

What Have We Learnt ?

1. Any fraction can be converted into several equivalent fractions. For this, the numerator and denominator of the fraction should be multiplied or divided by the same number.
2. Comparision of fractions :
   (i) If the numerator of fractions are same, the fraction whose denominator is the smallest, would be the greatest fraction.
   (ii) If the denominator of fractions are equal, then the fraction whose numerator is the greatest would be greatest fraction.
   (iii) Fractions are compared by making the denominators of all fractions equal with the help of their L.C.M.
3. The fractions in which the numerator is bigger than the denominator are called improper fractions.
4. The fractions in which the numerator is smaller than the denominator are called proper fractions.
5. When two fractions are multiplied, numerator multiplied to numerator and the denominator is multiplied to a denominator.
6. In the division of a fraction, the divisor becomes inverse and sign of multiplication is used instead of the sign of division.

Chapter 8

THE ANGLE

When you open or close a door, then the door makes different angles with the wall or its frame in different situations. If the body be considered as a straight line and the hand as another straight line then while doing physical exercises, the hand makes different angles with respect to the body when we rotate the hand in various directions.

In our daily life we come across many examples when we observe angles being made like the angle between the two hands of a clock, the angle between the two blades in a pair of scissors etc.

Note down some other such examples of angles being made in your notebook.

Let us observe some examples of angles.

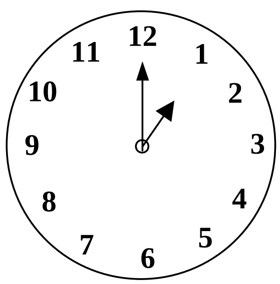


Fig 1

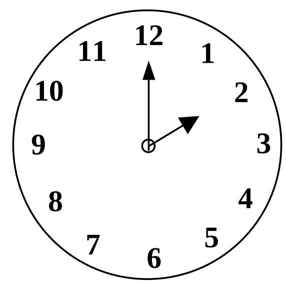


Fig 2

The anlges between two hands of a clock.

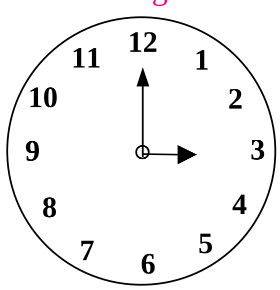


Fig 3

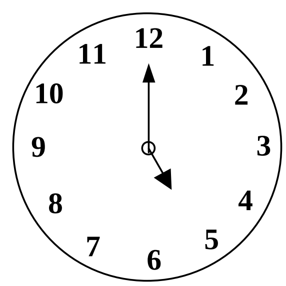


Fig 4

In all the above figures the bigger (minute) hand of the clock is at 12 which the smaller (hour) hand is at different situations. In figure 1, the inclination between the two hands of the clock is less but the rotation between the hands is seen to increase gradually in fig. 2, 3 and 4.

Similarly, while using a pair of scissors, the inclination between the two blades becomes lesser than the state of beginning to use it. While taking your food, you must have noticed that your elbow turns at different angles to make it possible for you to take food on to your mouth from the plate.

Thus,

*An inclination or turn between two arms or line segments at a point is known as an **angle.***

Similarly, when two line segments or rays cross or intersect each other, then the turn or inclination between those line segments are known as *angles*.

In which of figures given below. Do you find angles made?

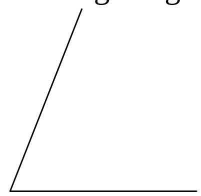


Fig 5

______

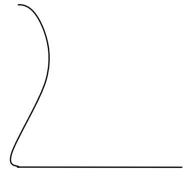


Fig 6

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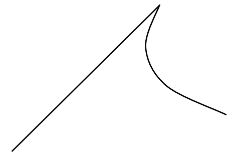

Fig 7

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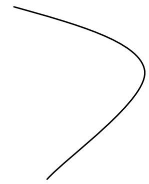

Fig 8

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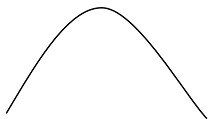

Fig 9

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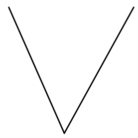

Fig 10

______

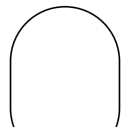

Fig 11

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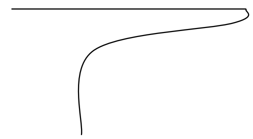


Fig 12

______

In the above diagrams you can see that fig5 makes an angle, but fig8 does not make an angle, though there is a turn. This is because, none of the two arms is a straight line.

This means an angle can occur only when there is a turn between two straight lines or line segments. Can you now tell which other figures do not represent angles?

ACTIVITY 1

Look at the anlges represented in the Hindi letter 'ञ' and the letter 'A' of the English alphabet, like those, shown in the figures below and try to find out whether angles are represented in other letters of the alphabet in Hindi and English as you write them down.

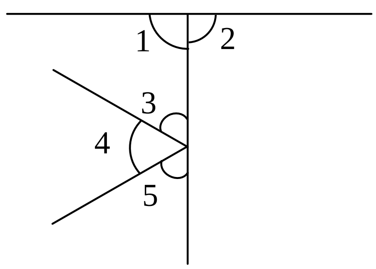


Fig 13

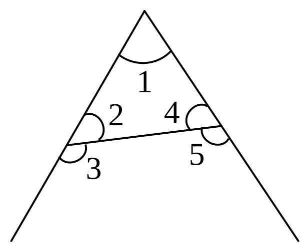


Fig 14

ACTIVITY 2

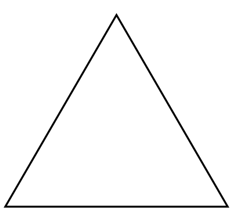


Fig 15

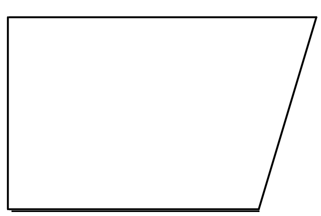


Fig 16

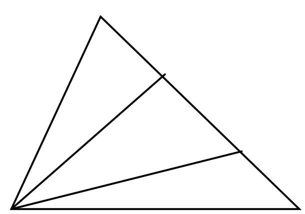


Fig 17

Observe the figures and say how many anlges do you see in each of the figures?

Figure	Number of angles
(15)	
(16)	
(17)	

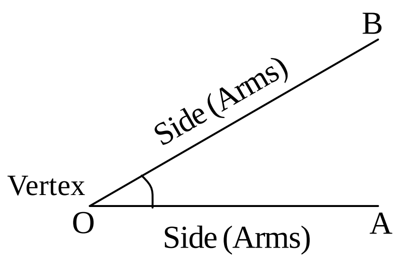


Fig 18

By this time you must have started recognizing angles and have come to understand how anlges are formed. Can you now say, what are the conditions necessary for the fomation of an angle?

Every angle has two arms that meet at a point. The point at which the two arms meet is known as the **vertex** of the angle.

For example, look at the angle AOB (fig 18). OA and OB are the two arms that meet at O. The turn that the arm OB has taken from the direction of OA is the way angle AOB has been formed. An angle is represented by this symbol Ď. Thus angle AOB would be written as $\angle$AOB.

Reading Angles by their Names

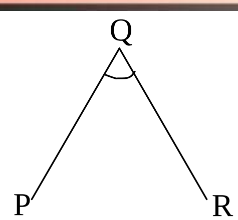


Fig 19

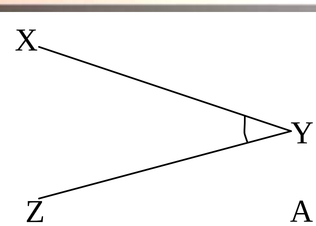


Fig 20

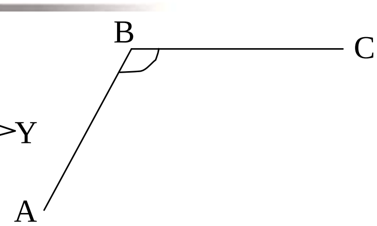


Fig 21

In figure 19, we can read or write the angle formed as $\angle$PQR or $\angle$RQP. In figure 20, the angle is $\angle$XYZ or $\angle$ZYX. What is the name of the angle in fig 21?

Remember that the point at B which angle is formed is known as its ***vertex***. While reading or writing the name of an angle, the position of the vertex is always kept in the middle. Thus, we read the angle in fig 21 as $\angle$ABC or $\angle$CBA.

ACTIVITY 3

In the figures given below, write the names of the angles in both ways in the space provided.

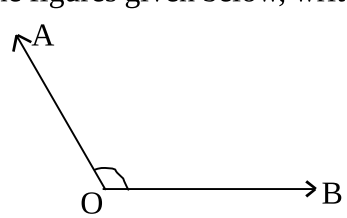


fig 22

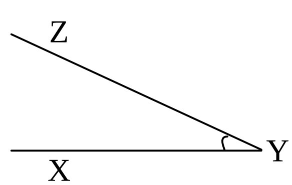


fig 23

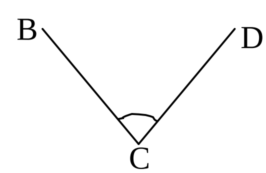


fig 24

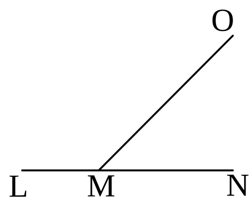


fig 25

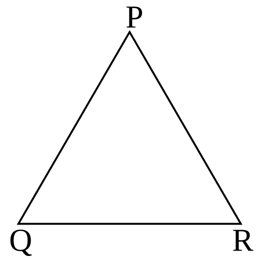


fig 26

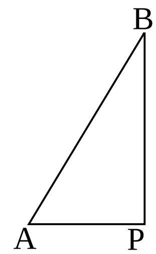


fig 27

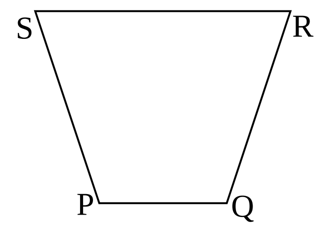


fig 28

Figure Number	**Name of the angle**
22	∠AOB or ∠BOA
23	
24	
25	
26	
27	
28	

Measuring the Angle

ACTIVITY 4

Let us do an activity. Take two sticks of broom or two bamboo sticks. Keep one end of one stick over the other and fix a pin at that point. Here's your angle-making *apparatus*. Now if you rotate one of the sticks, keeping the other fixed at one position, you will get angles of different values. Let us think about some such possibilities.

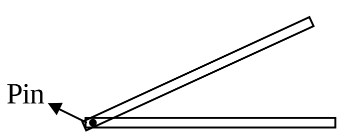


Fig 29

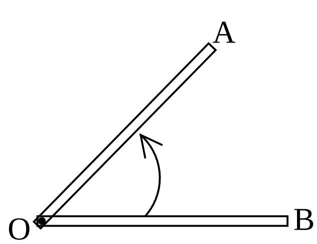


fig 30

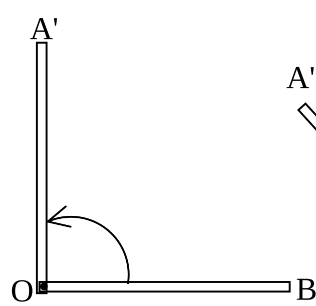


fig 31

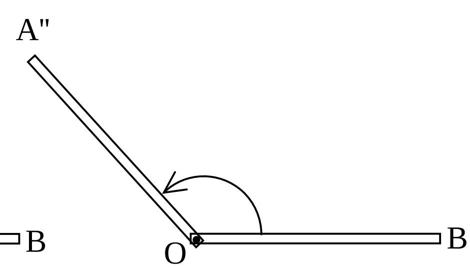


fig 32

Keeping OB fixed, if OA is rotated, the value of the angle would increase with increase in the turn of OA. Therefore, ∠AOB, ∠A'OB, ∠A"OB.

Show your teacher, the smallest angle or the biggest angle you can make with the help of your angle making apparatus.

Which is the largest angle among the pairs of figures given below?

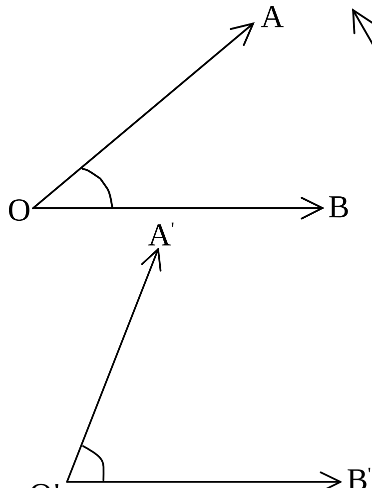


Fig 33

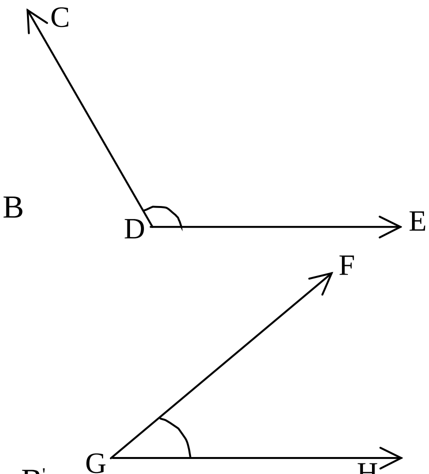


Fig 34

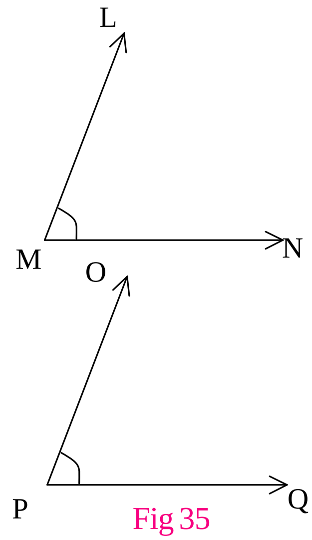


Fig 35

Is ∠LMN > ∠OPQ? Give reasons.

In such a state when we cannot say whether an angle is greater or smaller just by looking at it, we measure the angle with the help of a protractor and find out which of them is greater.

Now look at your protractor. On its curved side marks are put at equal distances. Count them and find out how many divisions are marked. Try to identify the divisions that show different values and notice their positions.

Now let us draw an angle of a particular measure. You must have drawn angles in your notebooks in Class V. To draw an angle, we need a scale and a protractor. Take out the protractor from your geometry box and look at the divisions marked on its curved surface. Write the total number of divisions in your notebook.

Just as we use units like metre and centimetre to measure lengths, similarly the angle between two line segments is measured with the help of an unit known as degree. We represent it like this :

45 degree = 45°

22.5 degree = 22.5°

ACTIVITY 5

Let us know something more about 'degree'. Draw a long straight line on a plane paper or on a plane page of your notebook. Put the base of the protractor on the line in such a way that the middle of the protractor comes in the centre of the line drawn. Draw the outer edge of the protractor with your pencil along the surface moving from 0° to 180° as in fig 36.

On removing your protractor, you will find a figure as shown in fig 37. Now, keep the protractor on the base line in a way that the curved surface is just opposite to the previous

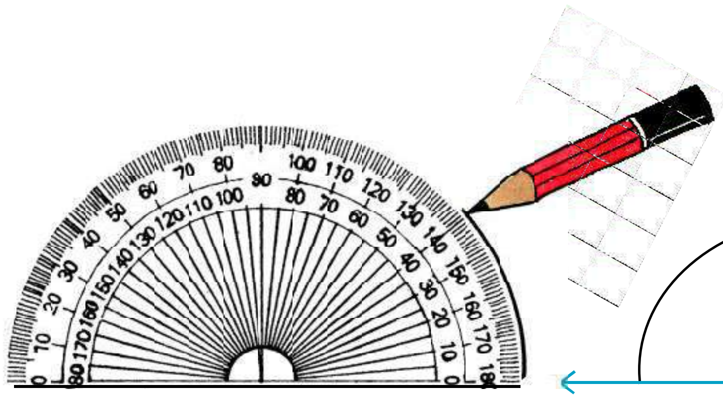


Fig 36

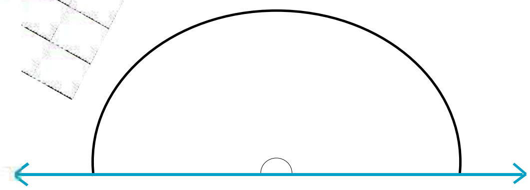


Fig 37

position. Draw the surface along the curved edge for 0° to 180° with the help of your pencil as shown in fig38. You will now get a figure as shown in fig39. When you remove your protractor.

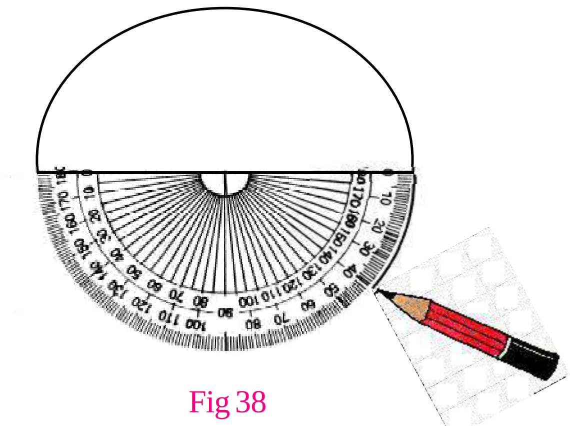


Fig 38

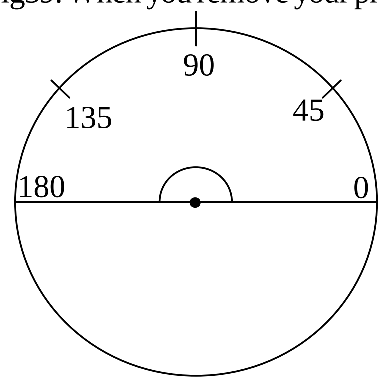


Fig 39

On the basis of this figure, answer the following questions, now.

(1) The angle formed at the centre of one semicircle (fig 37) = 180°

(2) The angle formed at the centre of the second semicircle (fig 38) = ?

(3) The complete angle formed at the centre of both the semicircles = ?

You will find that the first semicircle makes an angle of 180° and the second semicircle also make an angle of 180°, so both together make an angle of 360° at the centre. This point is also the centre of the circle that is made by both the semicircles.

If you fold a circular piece of paper in a way that the circle gets divided into two equal parts, you'll find that a straight line passes through the centre point O, which makes an angle of 180°. This is also known as the *straight angle*.

Can you tell, how many such straight angles can be formed at the centre of a circle?

You have got a straight angle by folding a circular piece of paper into half. Now fold it in such a way that the circle gets divided into four equal parts.

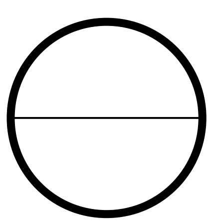
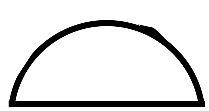
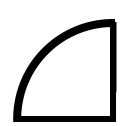


Fig 40



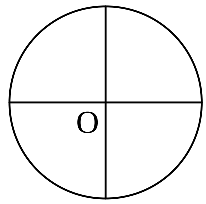
On folding the circular piece of paper once

Fig 41



On folding the paper twice

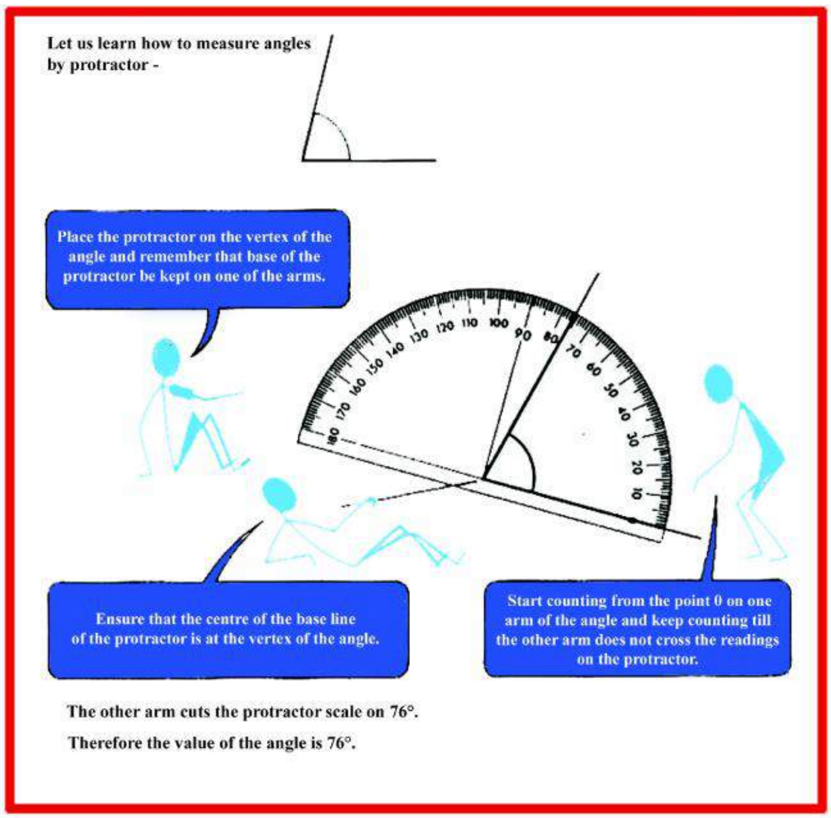
Fig 42



On opening the circular piece of paper

Fig 43

Now the circle has been divided into 4 equal parts at the point O. Now find out the measures of the four angles that you can see at point O separately. How much is it?



Practice 1

Make five different angles of five measures in your notebook. Measure them yourself and let your friends measure them too, so that the measures get checked.

In order to draw an angle with the help of the protractor, first a straight line AB in drawn. The point O on the straight line at which the angle is to be drawn, is taken as the centre and the protractor is kept on it in such a way that the base line of the protractor falls on the straight line drawn in your notebook. Now move ahead from the point on your protractor where 0° is written. Put a point at the mark/division for the measure that you want your angle to be drawn. Suppose, you are drawing an angle of 60°. Then you would move from mark 0° towards 60° and put a point at that mark along the protractor. Now remove the protractor and join that point to the point O on the straight line with you pencil. Now you have $\angle POB = 60^\circ$, i.e. an angle POB of 60°.

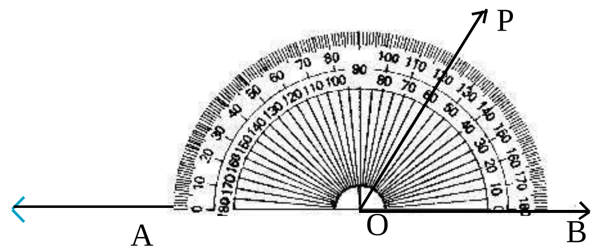
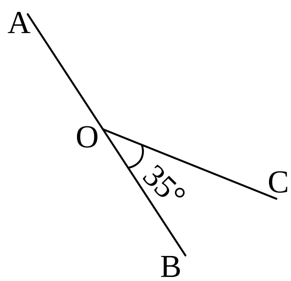


Fig 44

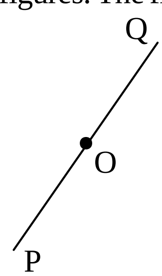
ACTIVITY 6

You have been given some lines below. At the point shown on each line, draw the angle that have been mentioned below the figures. The first one is done for you.



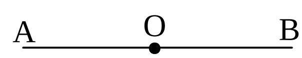
$\angle BOC = 35°$

Fig 45



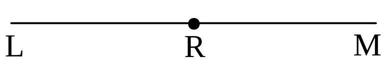
$\angle POR = 130°$

Fig 46



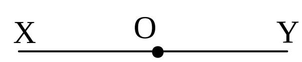
$\angle AOB = 180°$

Fig 47



$\angle MRQ = 170°$

Fig 48



$\angle YOZ = 70°$

Fig 49

On which points in the figures, did you draw the angle and in which direction to $0°$ did you move to draw them?

Figure No.	**Point at which the angle is formed**	**The point that shows on which side of 0° the measure began**
45	O	B
46		
47		
48		
49		

Note : Do you know that while doing physical exercise, when you stand at attention, the angle between your heels should be 30°?

Find out some more such information.

ACTIVITY 7

Write down the length of arms and the measure of the angle for the given names in the table according to the figures.

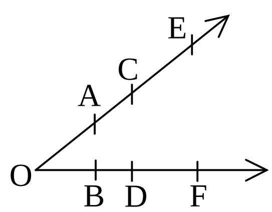


Fig 50

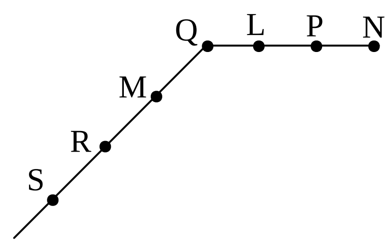


Fig 51

Figure No.	Name of the angle	Measure of the angle	Arms that make the angle		Measures of the arms	
50	$\angle$ A O B	43°	O A	O B		
50	$\angle$ C O D		O C	O D		
50	$\angle$ E O F		O E	O F		
50	$\angle$ A O F		O A	O F		
50	$\angle$ E O B		O E	O B		
51	$\angle$ L Q M		Q L	Q M		
51	$\angle$ P Q R		Q P	Q R		
51	$\angle$ P Q S		Q P	Q S		
51	$\angle$ L Q R		Q L	Q R		

(1) Is $\angle COD > \angle AOF$, Is $\angle EOF > \angle COD$, If no, why?

(2) Does the measure of the angle depend on the legth of its arms?

(3) On what does the greater or lesser degree of an angle between two arms depend?

ACTIVITY 8

In fig 52, measure the angle x° and y° with the help of your protractor. Is $x^\circ = y^\circ$? Note it down in your notebook.

It is clear from the activities that the measures of angles, do not depend on the measure of the arms that make the angle because they can be made by straight lines or rays which can be extended endlessly. Both the lines are made up of several points. Therefore, the distance between any two points between two straight lines does not make an angle.

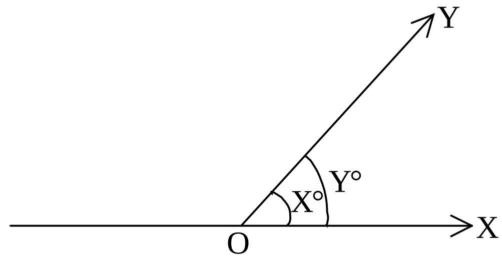
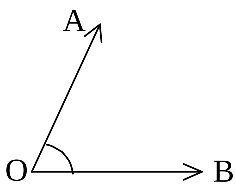


Fig 52

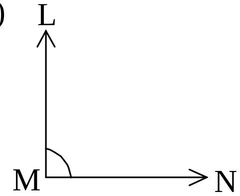
EXERCISE 8.1

(1) Name the angles

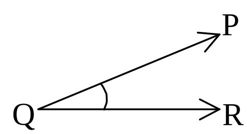
(i)



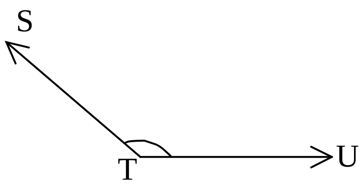
(ii)



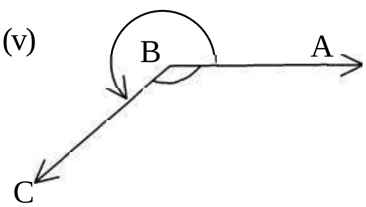
(iii)



(iv)

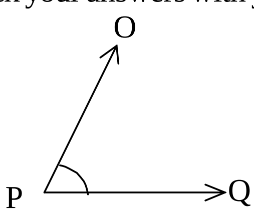


(v)



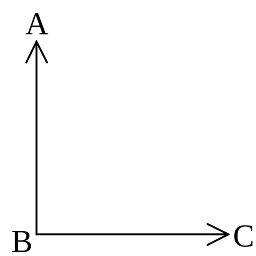
(2) With the help of your protractor measure the angles and write them in the space provided. Check your answers with your friends.

(i)



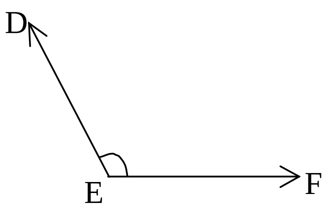
$\angle$OPQ = ...

(ii)



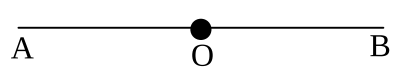
$\angle$ABC = ...

(iii)



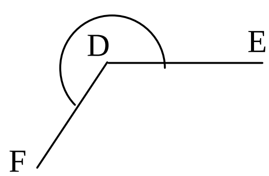
$\angle$DEF = ...

(iv)



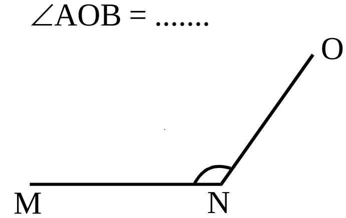
$\angle$AOB = ...

(v)



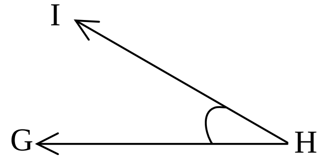
$\angle$ EDF = ...

(vi)



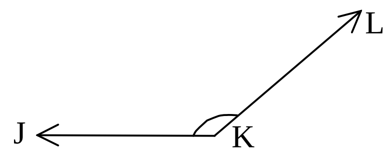
$\angle$MNO = ...

(vii)



$\angle GHI$ = ...

(viii)



$\angle JKL$ = ...

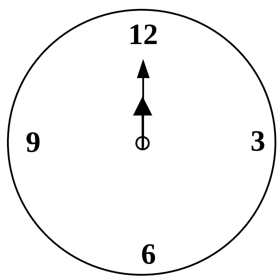
(3) Draw the following angles with the help of a protractor.

(i)	45°	(ii)	75°	(ii)	90°
(iv)	120°	(v)	155°	(vi)	210°

(4) What will be the measure of the angle between the two hands of the clock at 6 O'clock?

Types of Angles

Can you say when it is exactly 12 o'clock in your watch or on the clock, what is the measure of the angle between the smaller and the bigger hand?



Measurement of angle.........

Fig. 53

Can you tell, how many times will the hour hand overlap the minute hand of your clock in 12 O'clock in 12 hours?

Think about the fact, that when the two hands overlap ecah other, what will be the angle between the two?

Just like the clock hands, when one ray overlaps the other, the angle between both of them is zero degree. Now look at your watch at 2:45 and tell what would be the measure of the angle between the hour hand and the minute hand at this time?

In fig 54 $\angle AOB = 0^\circ$, that is line segment OA is just over line segment OB, so that the turn between them is 0°. What will be the value of $\angle BAO$ in fig 55.

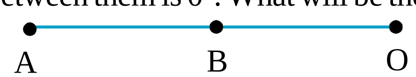


Fig 54

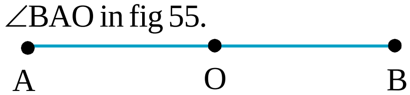
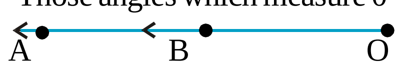


Fig 55

(1) Zero Angle: Those angles which measure 0° are called zero angles.

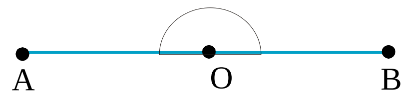


$\angle AOB = 0^\circ$

Fig 56

And in fig 55 OA and OB are line segments in opposite directions, so that they make a bigger line segment and then $\angle AOB = 180^\circ$, which is known as a *straight angle*.

(2) Straight Angle: The angle which measures 180° is known as a *straight angle*.



$\angle AOB = 180^\circ$

Fig 57

(3) Acute Angle: The angle which is more than 0° and less than 90°, is called an *acute angle*.

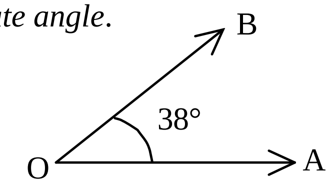


Fig 58

(4) Right Angle: The angle which measures 90° is known as a *right angle*. One arm of a right angle is perpendicular to the other arm.

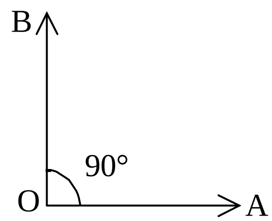


Fig 59

(5) Obtuse Angle: The angle which measures more than 90° but less than 180° is known as an *obtuse angle*.

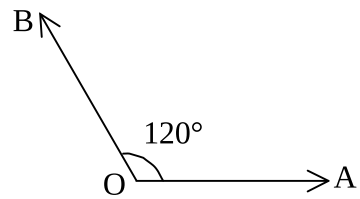


Fig 60

(6) Reflex Angle (wide angle): The angle which measures greater than a straight angle 180° but less than 360° is known as a *reflex angle*.

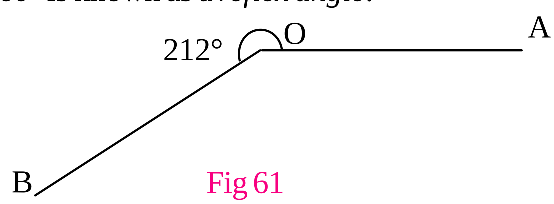


Fig 61

(8) Complete Angle: If a ray takes one complete turn or rotates round its end point to complete one full rotation and comes back to fall over its initial portion, then the angle it would make would be a full circle or a *complete angle*.

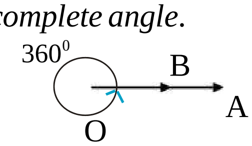
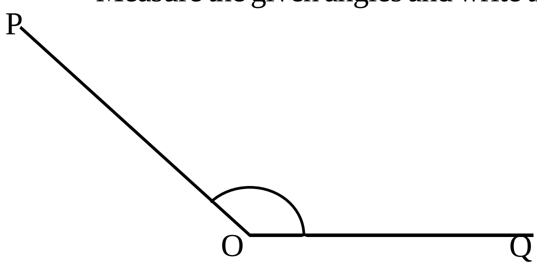


Fig 62

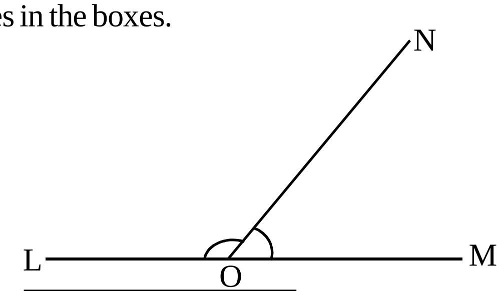
ACTIVITY 9

Measure the given angles and write their types in the boxes.



∠POQ = Obtuse angle

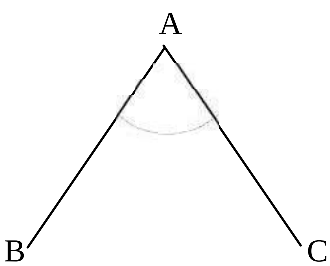
Fig 63



∠LON =

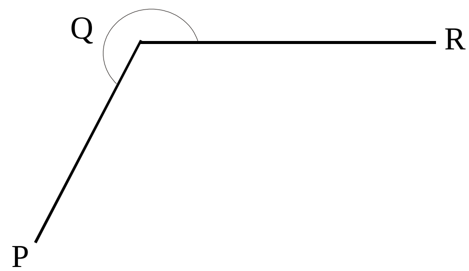
∠MON =

Fig 64



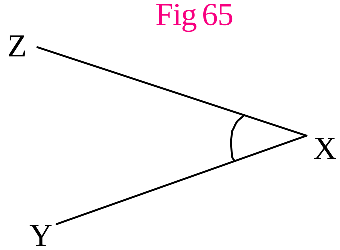
∠BAC =

Fig 65



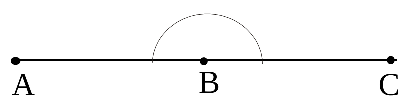
∠PQR =

Fig 66



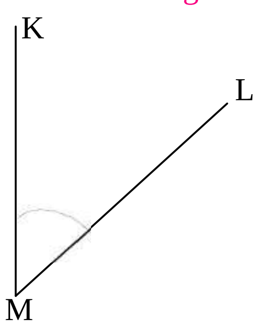
∠ZXY =

Fig 67



∠ABC =

Fig 68



∠KML =

Fig 69

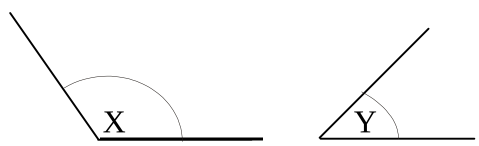
∠ABO =

Fig 70

EXERCISE 8.2

1. Find out the statements that are true correct the false statements.
   (i) A straight angle is 180°.
   (ii) An obtuse angle measures more than 180°.
   (iii) An acute angle is less than 90°.
   (iv) At three o’clock, the hour and minute hands of a clock makes a right angle.
2. Identify the acute angle, right angle, obtuse angle and straight angle from the following:

   (i) 120° (ii) 30° (iii) 90° (iv) 180°
   (v) 70° (vi) 105° (vii) 72° (viii) 36°
   (ix) 15° (x) 75°.
3. Compare the pairs of angles by measuring them:

(i) 

$\angle x$ $\square$ $\angle y$

(ii) 

$\angle x$ $\square$ $\angle y$

(iii) 

$\angle x$ $\square$ $\angle y$

(iv) 

$\angle x$ $\square$ $\angle y$

What Have We Learnt ?

1. The turn or inclination between two arms is known as the measure of an angles.

2. The unit to measure an angle is known an a degree. It is indicated by a small '°' at the top of a number e.g. 30°, 45°, 90°, 180°, 360°.

3. An angle whose measure is :

   0° is known as a zero angle.

   Between 0° - 90° is called an acute angle.

   Equal to 90° is a right angle.

   Between 90° - 180° is called obtuse angle.

   180 is called a straight angle.

   Between 180° and 360° is known as a reflex angle.

   360° is called a complete angle.

Chapter 9

TRIANGLE AND QUADRILATERAL

You have seen many objects around that are shaped like the flag on the temple, paper flags that you use to decorate your school on the Independence day and the paratha that you eat! Let us observe some more such shapes:

(1) The set-square in the compass box (fig1).

(2) A folded corner of your notebook (fig2).

(3) The figure obtained by joining the 3 points in fig 3 with the

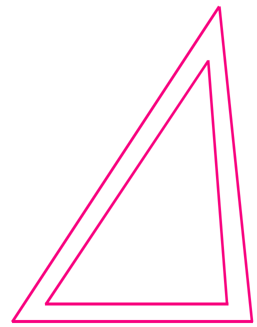


Fig 1

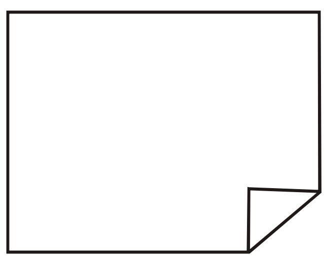


Fig 2

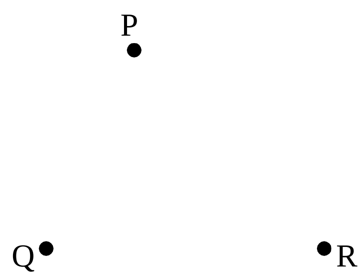


Fig 3

help of line segments.

What is the similarity in these figures?

Did you notice more shapes like these around yourself? Write down where have you seen them? Find out these kind of shapes in the figures given below.

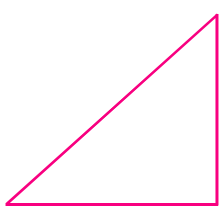


Fig 4

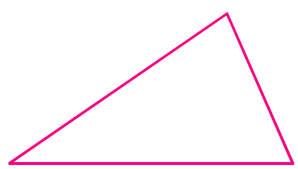


Fig 5

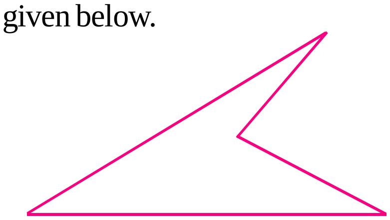


Fig 6

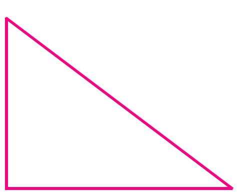


Fig 7

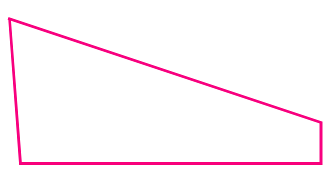


Fig 8

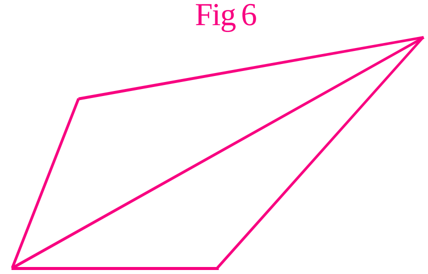


Fig 9

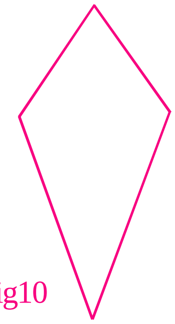


Fig10

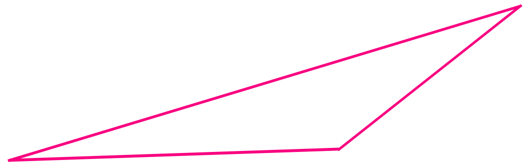


Fig11

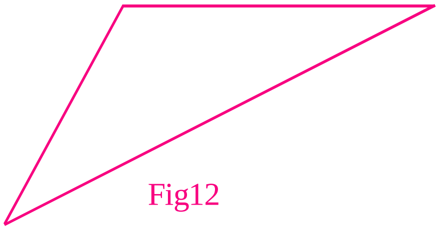


Fig12

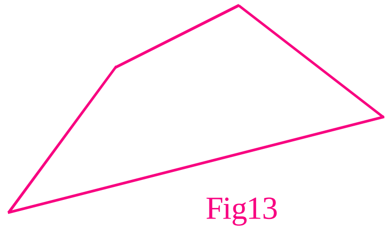


Fig13

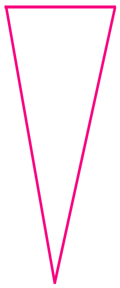


Fig14

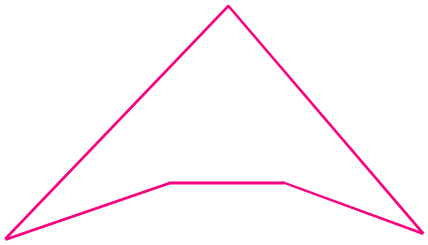


Fig15

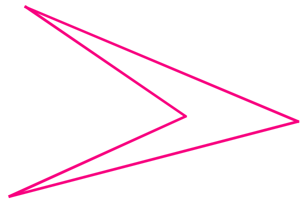


Fig16

On what basis have you selected/categorised the figures?

You must have noticed that the similarity in all these figures selected. There are 3 arms in all of them that meet at 3 points. So, they are called triangles.

All the figures from 17 to 23 given below are made up of 3 arms but all of them are not triangles. Why do you think some of them are not triangles?

Fig 17

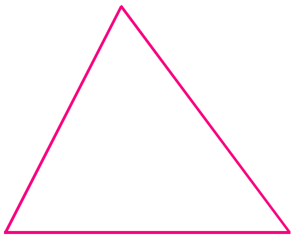


Fig 18

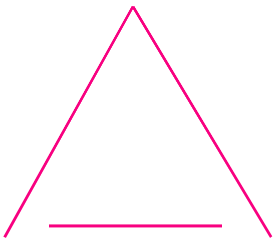


Fig 19

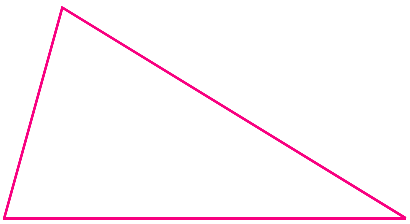


Fig 20

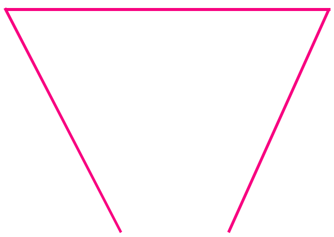


Fig 21

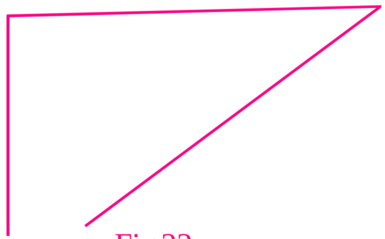


Fig 22

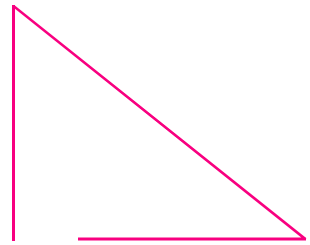


Fig 23

You have read about closed and open figures in the lesson on line segments. In the figures given above fig no. 18 & 20 are closed figures. Others are all open figures. The closed figures are made up of 3 line segments that make three angles also. In the open figures, also there are 3 arms but these arms are not making 3 angles. Therefore, all the figures that are made up of three line segments are not triangles. **The closed figures made up of three line segments are known as triangles.**

ACTIVITY 1

In each of the figures given below. There are 3 points. Can you make triangles by joining them at the three given points.

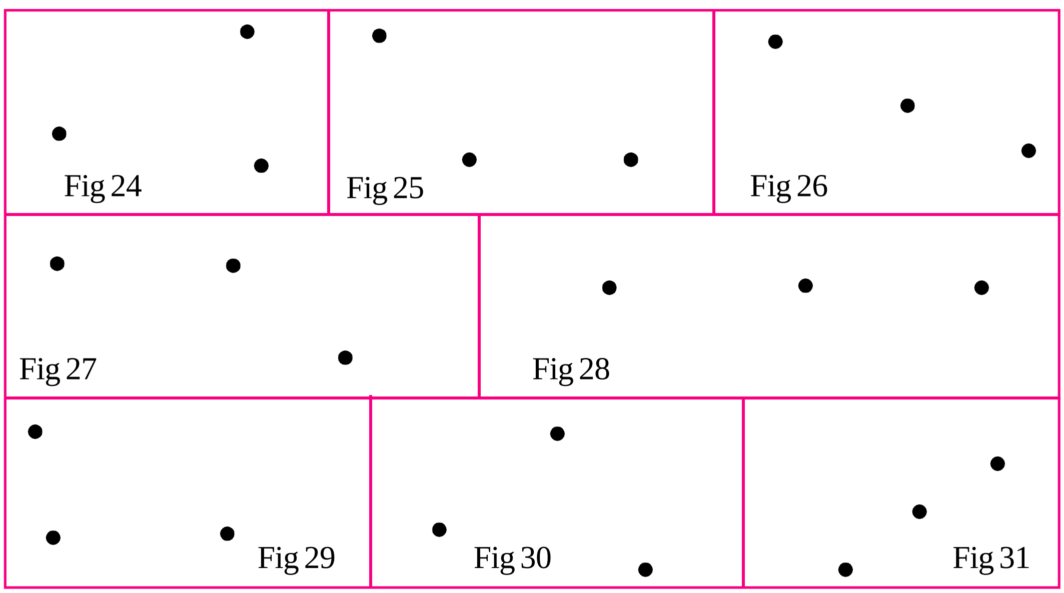


Fig 24 Fig 25 Fig 26 Fig 27 Fig 28 Fig 29 Fig 30 Fig 31

In the figures above you must have noticed that whenever the three given points are in a straight line, they cannot be joined by three line segments. Therefore, they are not making triangles. This means three points that are not in a straight line and can be joined by line segments can make a slope that is called triangle.

The Parts of A Triangle

The triangle ABC has three angle ∠ABC, ∠CAB and ∠BCA. A, B and C are the vertices AB, BC & CA are the arms. ∠ABC, ∠BCA and ∠CAB are the angles.

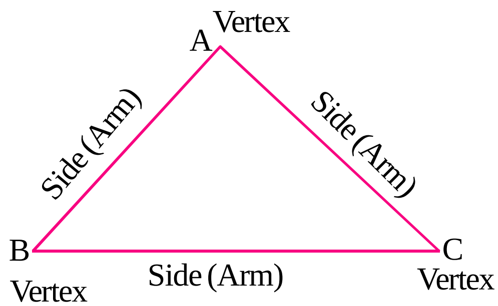
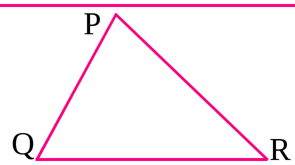
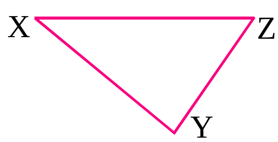
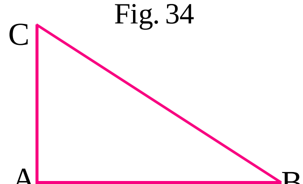
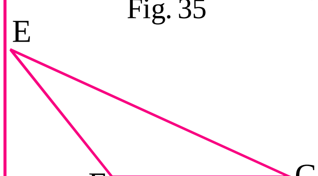


Fig 32

ACTIVITY 2

In the figure above, two arms meet at each vertex and make an angle. For the triangles given below. Write the names of the vertices, arms and the angles.

S.No.	Triangle	Vertex	Side	Angle
1.	 Fig. 33	______ ______ ______	______ ______ ______	______ ______ ______
2.	 Fig. 34	______ ______ ______	______ ______ ______	______ ______ ______
3.	 Fig. 35	______ ______ ______	______ ______ ______	______ ______ ______
4.	 Fig. 36	______ ______ ______	______ ______ ______	______ ______ ______

The Internal Angles of a Triangle

The angles made by the arms of a triangle are all in a closed area, and so are interior angles. All the names of angles that you have listed in figures 33, 34, 35 & 36 are all interior angles.

ACTIVITY 3

Now measure the internal angles of the triangles in figures 37, 38, 39 & 40 with the help of your protractor and write them in the given table. Find the sum of the angles that you have measured for each triangle.

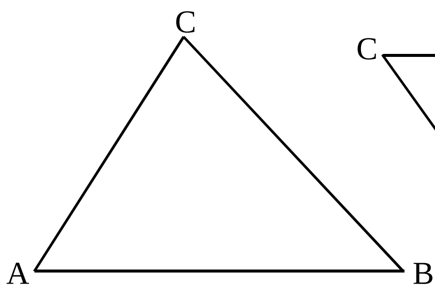


Fig 37

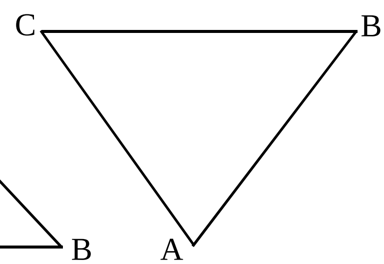


Fig 38

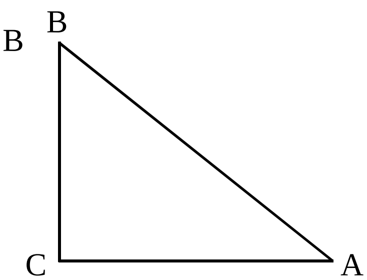


Fig 39

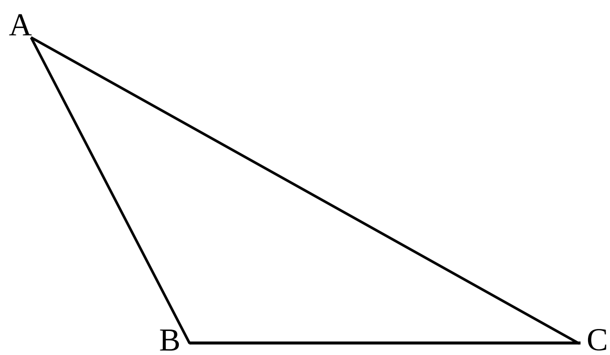


Fig 40

Fig No.	$\angle A$	$\angle B$	$\angle C$	$\angle A+\angle B+\angle C$ The sum of all the triangles
37				
38				
39				
40				

On measuring the interior angles of a triangles, we find that the sum of the three angles is approximately 180°. The sum of the three internal angles of a triangle is equal to 180°. We will learn how to prove this statement in our higher classes. Presently, let us try to find out the value of one angle of a triangle on the basis of values of two given angles.

S. No.	Measure of the 1^{st} angle	Measure of the 2^{nd} angle	Value of the 3^{rd} angle = 180° – (1^{st} angle + 2^{nd} angle)
01	40°	60°	$180^\circ - (40^\circ + 60^\circ) = 80^\circ$
02	40°	30°	....
03	45°	95°	....
04	70°	50°	....

The External Angles of a Triangle

If one of the arms of a triangle is extended out of the closed area, it makes an angle, which is called the exterior angle.

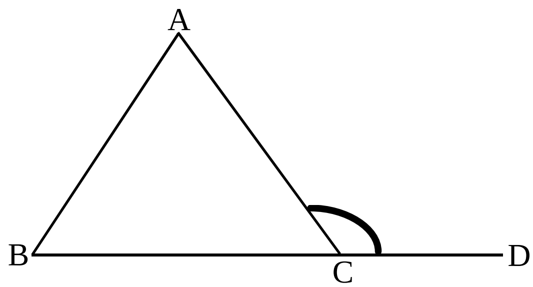


Fig 41

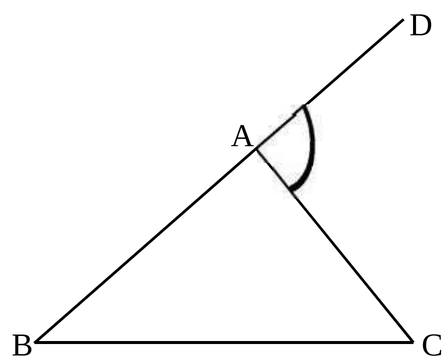


Fig 42

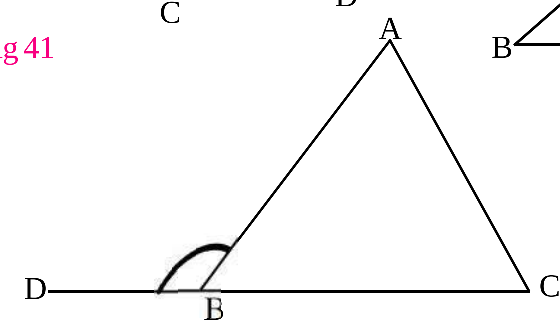


Fig 43

In all the above figures, one of the arms of the given triangles have been extended to point D which show ∠ACD, ∠CAD and ∠ABD respectively. These are all *exterior angles*. Each exterior angle is attached to an interior angle, which is complementary angle to the exterior angle. This interior angle which is joined to the exterior angle is kown as the *adjacent interior angle*. For example:

Fig no.	Exterior angle	Interior angle
41	∠ACD	∠ACB
42	∠CAD	∠CAB
43	∠ABD	∠ABC

The interior angle attached to the external angle is called the adjacent interior or adjoining interior angle and the other two interior angles are known as distant interior angles. Thus in fig 41 ∠BAC & ∠CBA, in fig42 ∠ABC & ∠BCA and in fig43 ∠BCA & ∠CAB are distant interior angles.

Point out the exterior angles. Then find out adjoining interior angles for the exterior angles in the given figures. If no exterior angles are being formed in any figure, what do you think is the reason behind this?

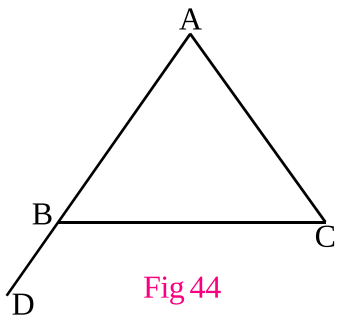


Fig 44

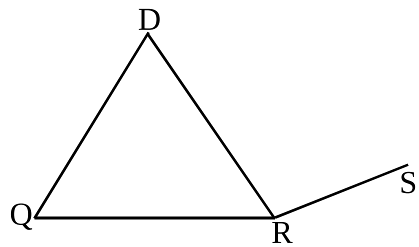


Fig 45

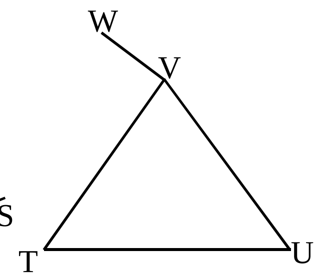


Fig 46

In fig45 & 46, no exterior angles are being formed because QRS & UVW are not straight lines.

ACTIVITY 4

In the figures given below, fill in the table with the names of adjoining interior angles, distant angles and the exterior angles of the triangles drawn.

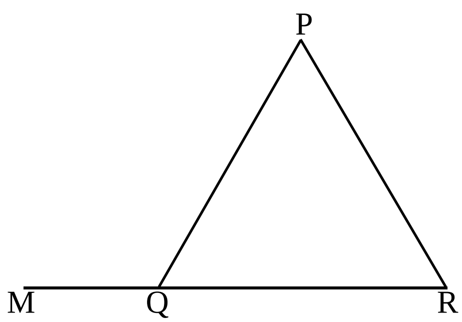


Fig 47

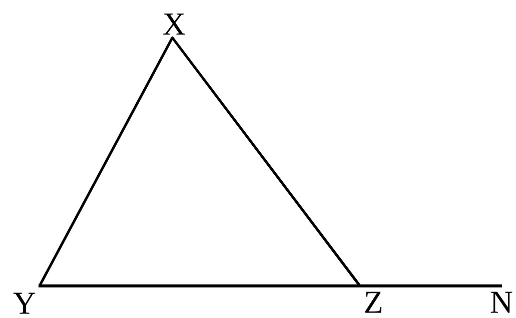


Fig 48

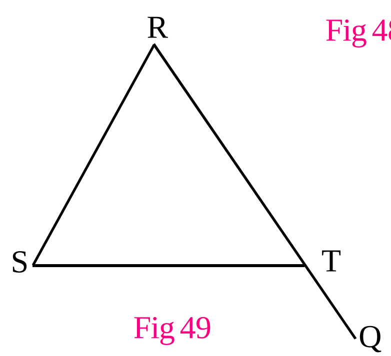


Fig 49

(See the table in next page.)

You have now learnt to identify the exterior, adjoining and distant interior angles of a triangle. Let us now take up another activity with angles.

Fig No.	Name of the triangle	Name of the exterior angle	Adjoining interior angle	Distant interior angle	
				I	II
47					
48					
49					

ACTIVITY 5

Find out the values of ∠X, ∠Y and ∠Z in the triangles given below and complete the table.

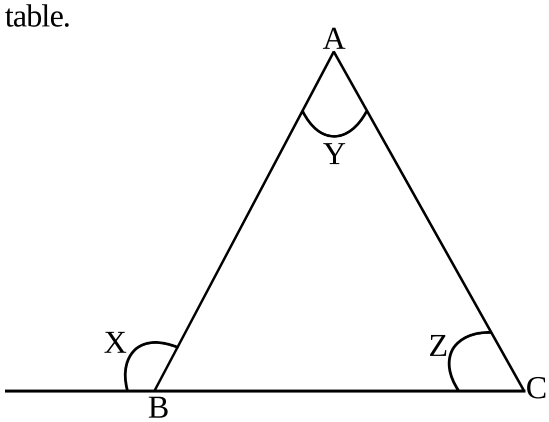


Fig 50

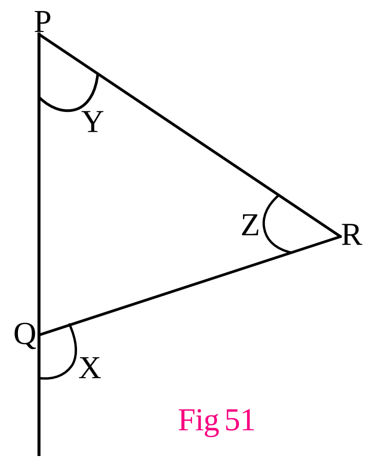


Fig 51

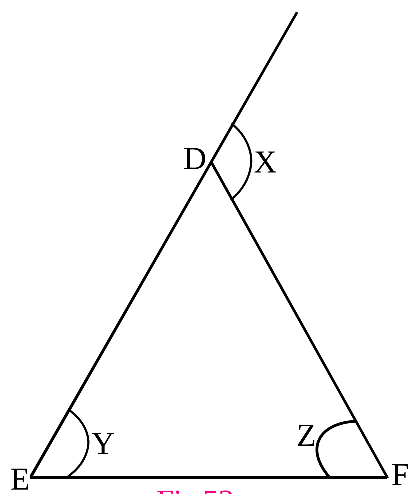


Fig 52

What is the relationship between ∠X and ∠Y +∠Z? It is clear from the table below that **the measure of an exterior angle of a triangle is equal to the sum of the distant interior angles of a triangle.**

Fig No.	$\angle X$	$\angle Y$	$\angle Z$	$\angle Y + \angle Z$
50				
51				
52				

Classification of Triangles

Till now you have seen different shapes of triangles. On the basis of arms & angles of triangles, they can be classified into the following types:

1. Classification of triangles according to measures of arm length

(a) Triangles in which all the three arms are of different lengths are called **scalene triangles**.

(b) Triangles in which two arms are of equal length and the third is of a different measure are known as **isoceles triangles**.

(c) Triangles in which all the three arms are of equal length are called **equilateral triangles.**

Practice 1

In the table given below, measures of the arms (sides) of triangles are shown. Classify the triangles on the basis of the given measures.

S. No.	Measures of arms	Types of triangles
1	4cm, 5cm, 6cm	
2	7cm, 7cm, 7cm	
3	6cm, 5cm, 6cm	
4	7.2cm, 7.2cm, 6cm	

ACTIVITY 6

Measure the sides or arms of the triangles in the figures given below and classify the triangles into three types.

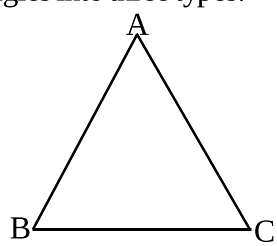


Fig 53

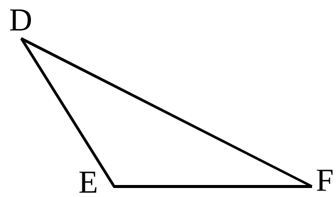


Fig 54

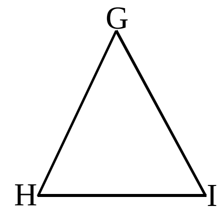


Fig 55

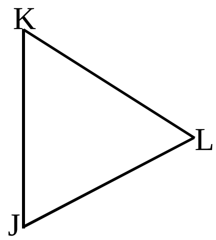


Fig 56

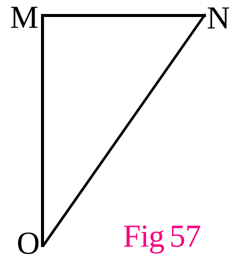


Fig 57

Fig No.	LENGTH OF THE ARMS			Types of triangle
	1	2	3	
53				
54				
55				
56				
57				

2. Classification of triangles on the basis of angles

(a) A triangle in which all the three angles are acute angles is called a **Acute angled triangle.**

(b) A triangle in which one of the angles is right angle is known as a **right angled triangle.**

(c) A triangle in which one of the angles is obtuse is an **obtuse angled triangle.**

ACTIVITY 7

Classify the triangles on the basis of the measures of angles given:

S. No.	Angles of a triangle	Types of triangles
1	$30^o, 30^o, 120^o$	
2	$60^o, 90^o, 30^o$	
3	$45^o, 40^o, 95^o$	
4	$30^o, 70^o, 80^o$	
5	$60^o, 60^o, 60^o$	

ACTIVITY 8

Measure the angles of the triangles in the given figures and write the type of triangle in the space given below each figure.

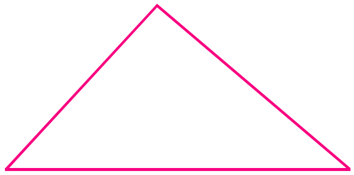


Fig 58 (................)

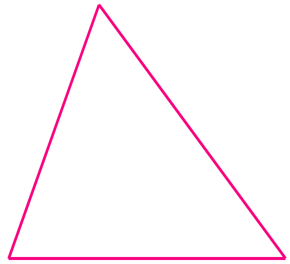


Fig 59 (................)

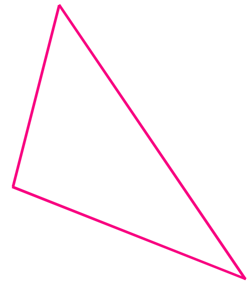


Fig 60 (................)

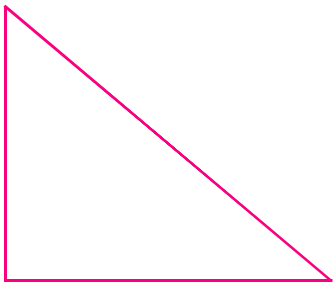


Fig 61 (................)

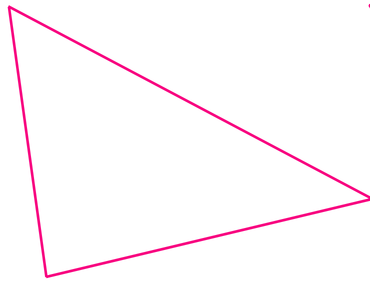


Fig 62 (................)

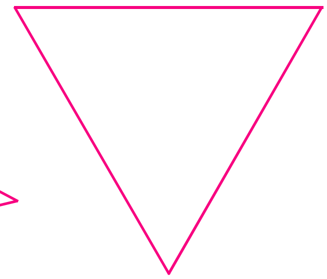


Fig 63 (................)

Now make some triangles and classify them according to their angles and arm lengths.

3. Classification of triangles on the basis of both arm as well as angles

In the given figures measure the lengths of arms and the angles of the triangles and write them separately in the table below. Classify the triangles according to the arm lengths and angles.

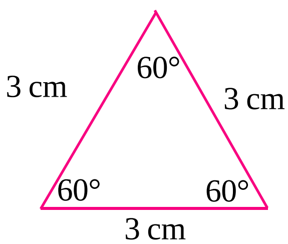


Fig 64

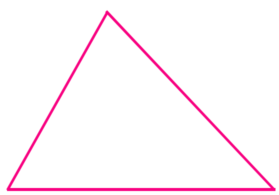


Fig 65

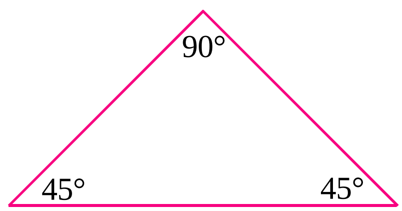


Fig 66

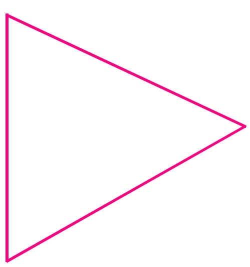


Fig 67

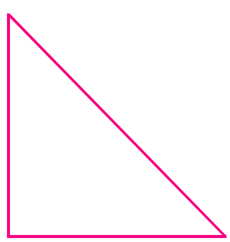


Fig 68

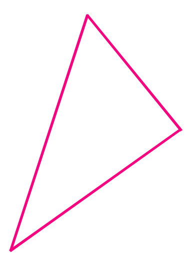


Fig 69

ACTIVITY 9

Fig. No.	Measures of the 3 arms			Measures of the 3 angles			Type of triangle	
	1	2	3	1	2	3	On the basis of angle	On the basis of arm length
64	3cm	3cm	3cm	60^o	60^o	60^o	Acute angled triangle	Equilateral triangle
65								
66								
67								
68								
69								

The following conclusions can be drawn from the observations made above:

(1) In a scalene triangle, the measures of all the three sides of the triangle are different and all the three angles also are of different measures.

(2) In an isoceles triangle, two arms and two angles are equal.

(3) In an equilateral triangle, all the three arms & all the three angles are of equal measures.

ACTIVITY 10

Take sticks of different lengths and make triangles.

for e.g. A triangle whose arm lengths are 8cm, 10cm & 12cm long.

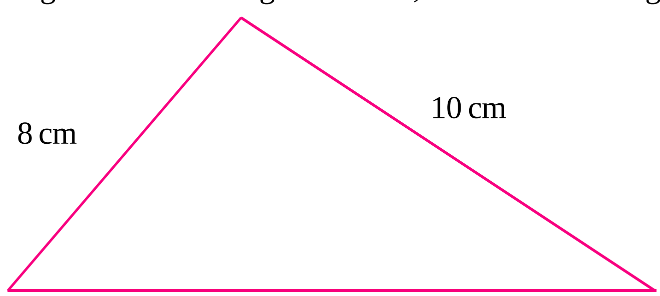


Fig 70

You find that triangles of some given measures can be made.

Make triangles in the same way with sticks of the measures given below. Note whether triangles can be made in all the situations. If not, find out reasons for it.

(1) 8cm, 10cm & 12cm

(2) 5cm, 9cm & 3cm

(3) 6cm, 8cm & 9cm

(4) 5cm, 7cm & 12cm

(5) 15cm, 5cm & 12cm

Now verify the conclusions that you have drawn

(1) If the sum of lengths of two sides (arms) of a triangle is greater than the length of the third side, only then can we make a triangle.

(2) If the sum of lengths of two arms of the triangle is less than the given length of the third arm, no triangle can be formed.

The situation in example 2, shows that when measures of arm lengths are as follows:

Sum of lengths of 2 arms of the triangle

5cm + 3cm = 8cm

which is less than the given measure of the third arm i.e. 9cm.

So, this triangle cannot be formed.

In example 4, sum of the two arm lengths of the triangle given is 5cm + 7cm = 12cm, which is equal to the given measure 12cm of the third arm.

In this situation also the triangle cannot be made.

Make triangles of similar measures yourself and help your friends to make the angles of different measures.

EXERCISE 9.1

1. Julie has made the following statements, Identify whether they are true or false. Select the false statements & correct them.
   (i) One arm or side of a triangle cannot be smaller than the sum of the other two arm lengths or sides.
   (ii) A triangle has 3 arms, 3 vertices and 3 internal angles.
   (iii) The length of one arm of a triangle is equal to the sum of the other two arm-lengths.
   (iv) When one angle of a triangle is obtuse, then the triangle is known as an obtuse angled triangle.
   (v) A triangle can have two angles of 90°.
   (vi) All the 3 angles of an acute angled triangle need not be acute.
   (vii) In the measure of two angles of a triangle are given, the measure of the third angle can be determined.
   (viii) All the three sides (arms) of an equilateral triangle are equal but all its three angles are not equal.
   (ix) The angles infront of the two equal arms of an isoceles triangle are also equal.
   (x) An equilateral triangle is always an acute angled triangle.
2. If the two angles of a triangle are 65° and 75°, find out the measure of the third angle ?
3. One angle of right angled triangle is 45°, find the other angle ?
4. What is the measure of each angle in an equilateral triangle?
5. If one angle of a triangle is equal to the measure of the other two angles, will the triangle be a right angled triangle?
6. Can the following situations lead to construction of triangles? Say yes or no.
   (i) If two angles are right angles
   (ii) When two angles are obtuse angles
   (iii) Sum of all the 3 angles measure 60°.

(iv) All the three angles are acute angles

(v) All the angles are less than 60°.

(vi) All the angles are greater than 60°.

Quadrilateral

You know about triangles. Everyday you see shapes like a black board, football playground, Kabaddi playground and pages of your copies, books etc. How many sides do each of these have? Where else have you seen shapes like these? Write down more names.

Choose figures like these from among the following: -

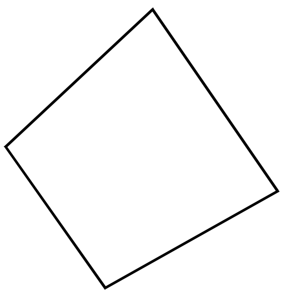


Fig 71

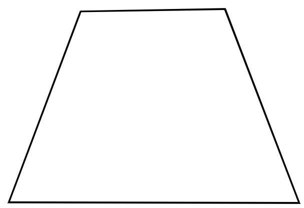


Fig 72

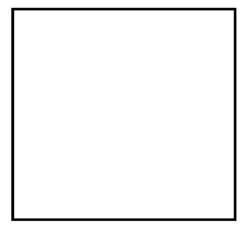


Fig 73

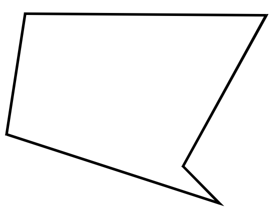


Fig 74

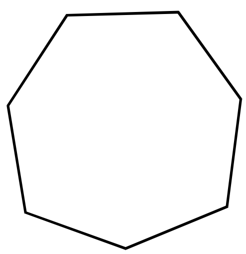


Fig 75

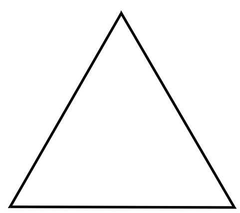


Fig 76

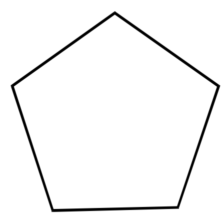


Fig 77

You have selected the four cornered shapes from the figures above. These shapes have 4 sides and are therefore called Quadrilaterals.

Some more shapes each of which, is formed by joining four sides are given below. Are all these quadrilaterals? If not, then think of the reason for each of your answers?

Fig 78

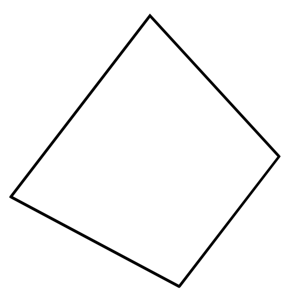


Fig 79

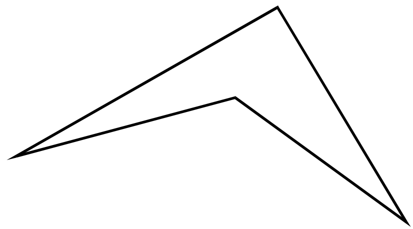


Fig 80

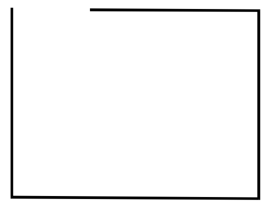


Fig 81

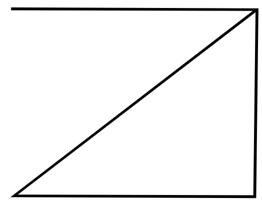


Fig 82

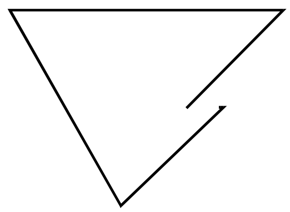


Fig 83

ou can observe that figures 78, 89 and 80 are closed shapes enclosed by four sides and the enclosed region has four angles. All these are therefore Quadrilaterals.

Figures 81, 82 and 83 are not closed shapes and, therefore are not quadrilaterals. **In this way, we say "Closed shapes having four sides where four angles are formed are called quadrilaterals".**

Parts of A Quadrilateral

In the quadrilateral ABCD, AB, BC, CD and DA are the four sides and A, B, C, D are the four vertices. Every vertex is formed by the joining of two sides and at every vertex the two sides form one interior angle. In this way four interior angles are formed. These are $\angle BAD$, $\angle ADC$, $\angle DCB$ and $\angle CBA$ respectively.

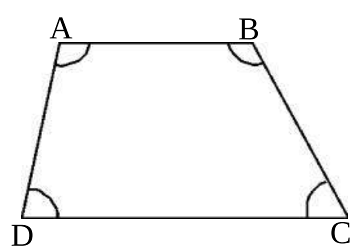
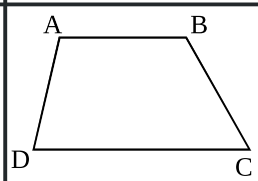
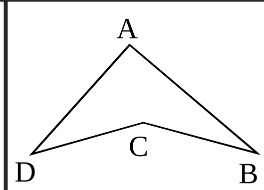
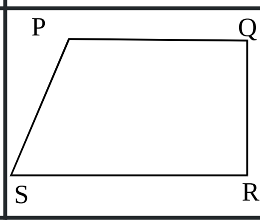


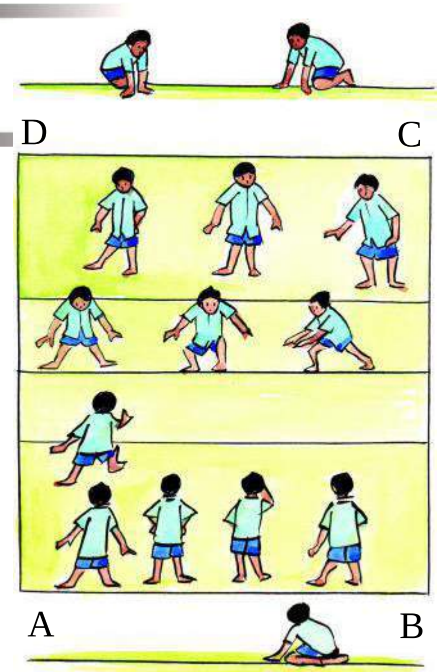
Fig 84

ACTIVITY 11

In the figures below, identify the sides, angles & vertices and write them at the appropriate places.

Figure No.	**Figure**	**Vertices**	**Sides**	**Angles**
85		(i) A (ii) B (iii) C (iv) D	(i) AB (ii) BC (iii) CD (iv) DA	(i) ∠ADC or ∠CDA (ii) ∠DCB or ∠BCD (iii) ∠CBA or ∠ABC (iv) ∠BAD or ∠DAB
86		(i)________ (ii)______ (iii)______ (iv)______	(i)________ (ii)______ (iii)______ (iv)______	(i)________ (ii)______ (iii)______ (iv)______
87		(i)________ (ii)______ (iii)______ (iv)______	(i)________ (ii)______ (iii)______ (iv)______	(i)________ (ii)______ (iii)______ (iv)______

Interior Region and Exterior egions of a quadrilateral



Kabbadi Ground

We are familiar with Kabaddi grounds. The adjacent figure shows players playing Kabaddi. Can you tell the number of players in the ground?

We can see in the picture that some players are outside the ground. They are 3 in number.

Is the Kabaddi ground ABCD a quadrilateral?

In the adjacent figure, the region inside the boundary of the quadrilateral is called the interior region of the quadrilateral. In figure 88, points P and Q are shown in the interior region of the quadrilateral.

The part of the plane (ground), outside the quadrilateral, is called exterior region of the The part of the plane

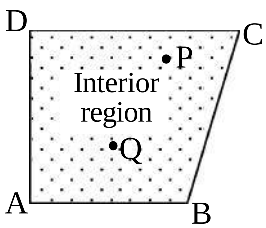


Fig 88

(ground), outside the quadrilateral, is called exterior region of the quadrilateral. In figure 89 points R and S are in the exterior region of the quadrilateral.

Numbers, letters etc written on any page of your book, are located in which region of the page?

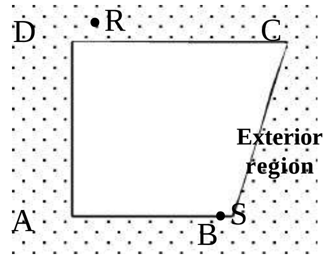


Fig 89

Adjacent Sides and Opposite Sides

In figure 90, you can see that the sides SP and QP are meeting at the vertex P. Similarly, the sides PQ and RQ meet at the vertex Q.

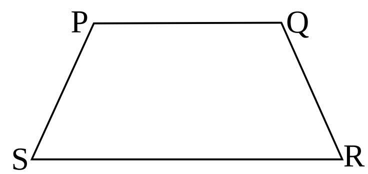


Fig 90

The sides of a quadrilateral, that meet each other at a point (vertex), are called adjacent sides. Here RS and PS are adjacent sides, which meet at the vertex S. Write the name of the adjacent sides, which meet at the vertices Q and R.

In figure 90, sides PQ and RS do not meet; therefore these sides are known as opposite sides. In figure 90 write the second pair of opposite sides.

ACTIVITY 12

Identify the pairs of adjacent sides in the following figures and write them along with their vertices in the following table-

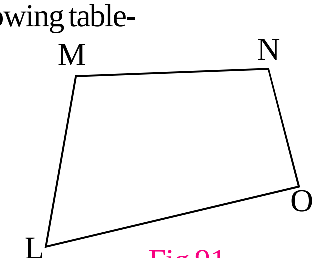


Fig 91

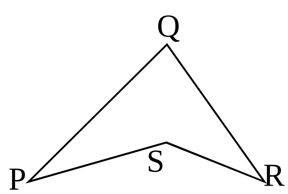


Fig 92

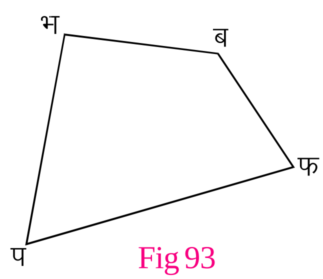


Fig 93

Figure No.	Adjacent sides	Vertices	Opposite side
91	(i) .... (ii) .... (iii) .... (iv)	(i) .... (ii) .... (iii) .... (iv)	(i) .... (ii) .... (iii) .... (iv) ....
92	(i) .... (ii) .... (iii) .... (iv)	(i) .... (ii) .... (iii) .... (iv)	(i) .... (ii) .... (iii) .... (iv) ....
93	(i) .... (ii) .... (iii) .... (iv)	(i) .... (ii) .... (iii) .... (iv)	(i) .... (ii) .... (iii) .... (iv) ....

Adjacent Angles and Opposite Angles

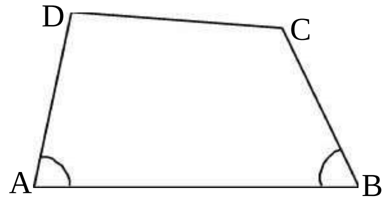


Fig 94

We have studied that a quadrilateral has four interior angles. Out of these, two such angles, which are formed by one common side, are called adjacent angles.

In figure 94, ∠A is formed by sides DA and AB, and ∠B is formed by sides AB and BC. Here AB is the common side therefore ∠A and ∠B are adjacent angles.

Is there any other angle adjacent to ∠A?

In the same manner, write the adjacent angles of ∠B, ∠C & ∠D.

In the above figure 94, ∠B has 2 adjacent angles ∠A & ∠C but ∠B & ∠D are not adjacent angles.

Therefore any two angles of a quadrilateral, which are not adjacent are called opposite angles.

In figure 94, ∠D is the opposite angle of ∠B, and ∠A is the opposite angle of ∠C. Opposite angles face each other.

Diagonal of a Quadrilateral and the Sum of the Interior Angles

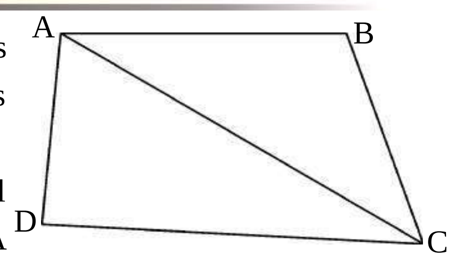


Fig 95

ABCD is a quadrilateral. If two opposite vertices of this quadrilateral are joined by a line segment then it gets divided into two triangles.

Line segment AC is the diagonal of the quadrilateral ABCD. This is formed by joining the opposite vertices A and C.

Similarly, the line segment BD will also be a diagonal.

The sum of the interior angles of a quadrilateral is eqal to 360^o .

Types of Quadrilaterals

Use a scale to take broom sticks of lengths mentioned below. Join them head to head and form quadrilaterals of different shapes.

(i) 8cm, 4cm, 8cm and 4cm.

Following are some of the quadrilaterals so formed by these: -

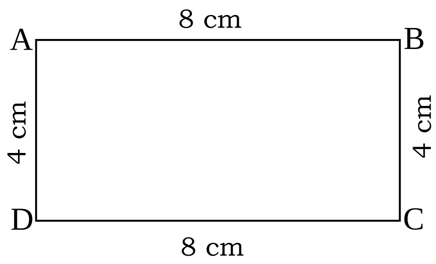


Fig 96

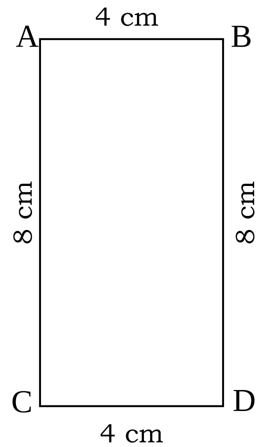


Fig 97

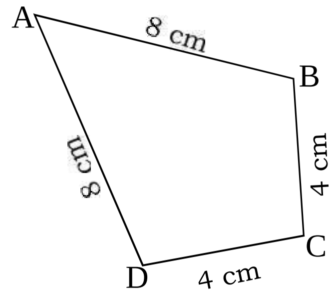


Fig 98

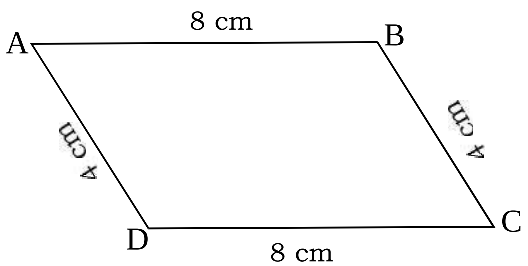


Fig 99

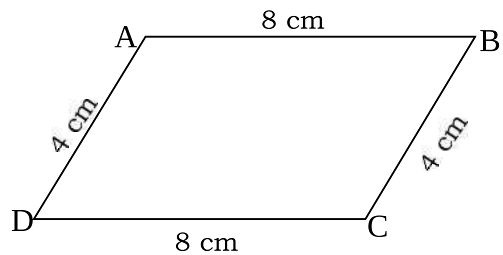


Fig 100

Among these figures, 96, 97, 99 and 100 have both pairs of opposite sides parallel to each other and equal in length. These are known as parallelograms.

Therefore, those quadrilaterals in which opposite sides are parallel and equal to each other are called parallelograms.

Figures 96, 97 are parallelograms with all angles of 90° each. These are called rectangles. **Thus, those parallelograms, which have all angles as right angles, are called rectangles.**

In figure 98, the opposite sides are neither parallel nor equal. Therefore it is not a parallelogram.

(ii) Take 4 sticks of 4 cm length each and make quadrilaterals: -

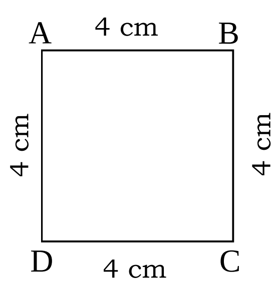


Fig 101

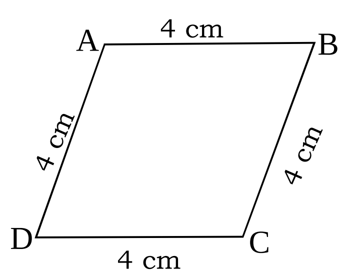


Fig 102

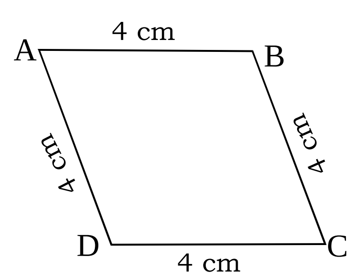


Fig 103

Some of the quadrilaterals made by you would be similar to the figures drawn above. Are these quadrilaterals parallelograms?

You will observe that, all the pairs of opposite sides in these figures are parallel and equal. Thus, all these are parallelograms. Since all sides of these quadrilaterals are equal, so they are a special type of parallelogram.

Those parallelograms, which have all sides equal are known as Rhombus.

Figure 101 is also a Rhombus. Apart from having all equal sides, this parallelogram has another specialty too. Each angle of this quadrilateral is of 90°.

Such a quadrilateral, which has all equal sides and all angles as right angles, is known as a square. Thus, square is a special type of Rhombus.

(iii) Now take sticks of lengths 3 cm, 4cm, 5 cm and 6 cm respectively, join them head to head and form many - different quadrilaterals. Some of the quadrilaterals formed by you may be of the following types: -

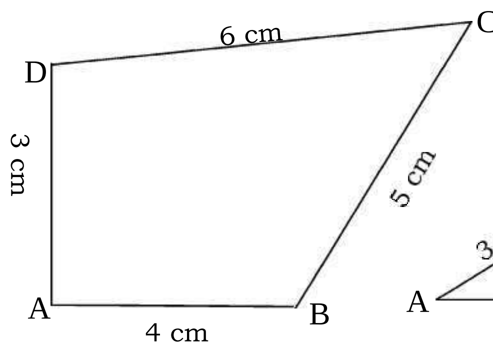


Fig 104

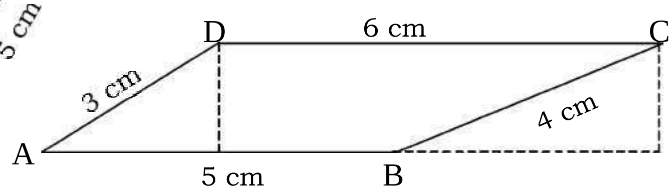


Fig 105

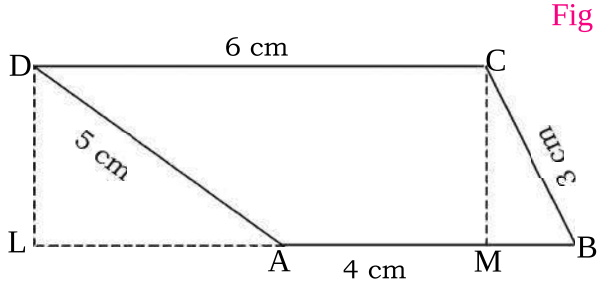


Fig 106

With the help of the sticks of specified lengths try and form some more quadrilaterals.

Each side of figure 104 is of a different length and opposite sides are not parallel. This is a quadrilateral having all sides of different lengths.

Quadrilaterals shown in figures 105 and 106 have two of their opposite sides (AB and DC) parallel but of different lengths. These are called Trapeziums. In a trapezium, perpendiculars drawn from the vertex on the opposite parallel side are of equal lengths.

Thus those quadrilaterals, in which one pair of opposite sides is parallel, are called Trapeziums.

ACTIVITY 15

Classify the following figures as rectangles, squares, rhombuses, trapezium and quadrilaterals with all sides of different lengths and fill the table given below:

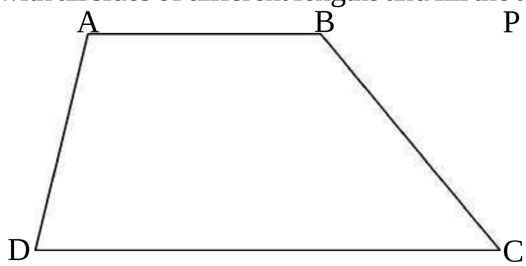


Fig 107

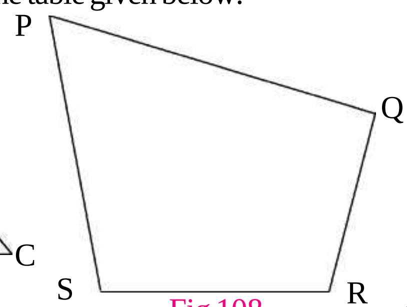


Fig 108

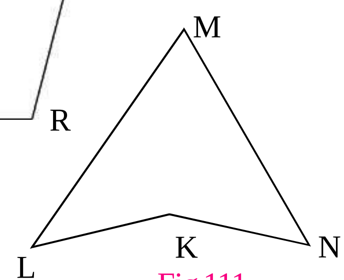


Fig 111

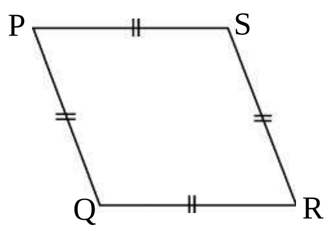


Fig 109

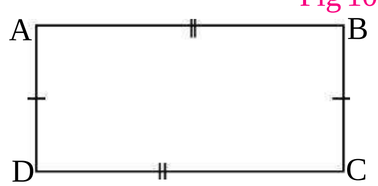


Fig 110

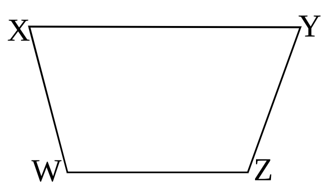


Fig 112

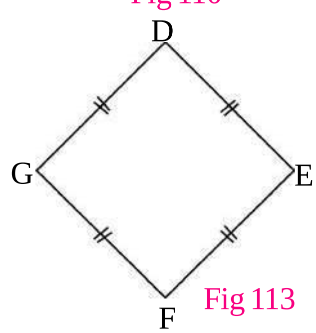


Fig 113

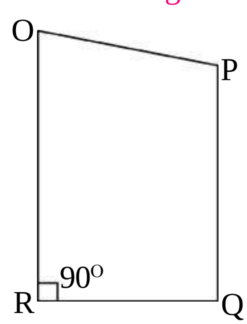


Fig 114

Fig. No.	Names of Parallel sides	Names of Equal sides	Type of the quadrilateral
107	AB | | DC	None	Trapezium
108	…………………	…………………	…………………
109	…………………	…………………	…………………
110	…………………	…………………	…………………
111	…………………	…………………	…………………
112	…………………	…………………	…………………
113	…………………	…………………	…………………
114	…………………	…………………	…………………

EXERCISE 9.2

Q1. Fill in the blanks-

(i) A quadrilateral has _____ diagonals.

(ii) The diagonal of a quadrilateral divides the quadrilateral into two _____.

(iii) The sum of all the interior angles of a quadrilateral is _______ degrees.

(iv) _______ pair(s) of opposite angle(s) are/is formed in a quadrilateral.

(v) Every quadrilateral has __________ vertices, among which more than __________ vertices cannot lie on a straight line.

Q2. (i) If in a quadrilateral, only one pair of opposite sides is parallel, then such a quadrilateral is called ____________________

(ii) Each angle of a rectangle is of __________degrees

(iii) In a rhombus, opposite sides are _____________ and all four sides are ________________to each other.

(iv) A parallelogram in which each angle is of 90°and all sides are of equal lengths is called ___________________.

(v) A quadrilateral whose all sides are equal is called _________________.

Q3. State true or false and correct the false statements: -

(i) A Rectangle is a parallelogram.

(ii) Every parallelogram is a rectangle.

(iii) Every Rhombus is a square

(iv) Opposite sides of a trapezium are parallel.

What Have We Learnt ?

1. Triangle is an area circumscribed by 3 arms.
2. A triangle is a closed figure. If the 3 arms together do not form a closed figure, it doesn’t make a triangle.
3. The vertex, arms and angles are parts of a triangle.

4. Triangles have 3 angles.

5. The measures of the sum of three internal angles of a triangle is equal to two right angles.

6. The exterior angle formed on extending the length of one of the arms of a triangle is equal to the sum of the two distant interior angles of the triangle.

7. Triangles can be classified into equilateral, isoceles & scalene triangles on the basis of the measures of arm length of these triangles.

8. On the basis of angles, triangles can be classified into acute angled, right angled and obtuse angled triangle.

9. The measures of the three arm lengths and the three angles of a scalene triangle are different from each other.

10. In an isoceles triangle, two arms and two angles are equal.

11. In an euqilateral triangle, the three arm lengths and three angles are equal.

12. A triangle can be constructed only when the sum of two arm lengths is greater than the length of the third arm.

13. A closed shape formed by four sides having four interior angles is known as a quadrilateral.

14. There are four vertices, four sides and four angles in a quadrilateral.

15. The line, joining opposite vertices of a quadrilateral is called a diagonal. There are two diagonals in a quadrilateral.

16. Sides of a quadrilateral having one common vertex are called adjacent sides.

17. Sides of a quadrilateral, which do not have any common vertex, are called opposite sides.

18. Interior of the quadrilateral ABCD together with the boundary of the quadrilateral forms the region of quadrilateral ABCD.

19. Sum of all the angles of a quadrilateral is 360°.

20. Opposite sides of a parallelogram are equal and parallel to each other.

21. A parallelogram each of whose angles is of 90° is called a rectangle.

22. A parallelogram in which all the sides are equal is called a rhombus.

23. A quadrilateral in which one pair of opposite sides is parallel is called a trapezium.

24. A parallelogram each of whose angle is of 90° and all sides are equal, is called a square.

Chapter 10

RATIO

Every morning Mohan and Rama take milk in cups. Mohan uses three spoons of sugar for two cups of milk while Rama uses two spoons of sugar for one cup of milk. How do we compare the quantity of sugar in the milk both the children take?

In our everyday life, when we have to buy things, play game or choose the more appropriate option out of two choices, we need to compare the situations. We often need to decide, which vegetable is better and how different their rates or prices are. Let us take an example. Shyam goes to the market to buy potatoes. One shopkeeper prices potatoes at Rs. 20 for 3 kg. The other shopkeeper prices them at Rs. 30 for 5 kg. Shyam is confused. Which is a better option? In such situations we require to think about ratios. Have you ever faced any such situation when you needed to decide about a better choice between almost similar options. Think of some such situations & write them down.

Ratios are indicated in different ways. We indicate them by the "**:**" symbol.

For example, a shopkeeper says, "This year the sale was twice compared to last year." By this he means that the sale this year is two times that of last year that is the sale ratio between this year and last year is 2:1.

Take another example, a school has one teacher for 45 students. This means the ratio of students and teacher for that school is 45:1. Now if we say that the school has 90 students, then it means that the school has 2 teachers. This is because the pupil teacher ratio is 45:1 or 90:2.

Reeta says, "The ratio of the number of teachers to the number of students in the school is 1:45. Is she right? In talking about ratios we need to remember which quantity is being compared to which quantity? For example, if we compare the number of teachers with the number of pupils/ students, then the ratio would be 1:45, whereas if we compare the number of pupils to that of teachers the ratio would be 45:1. Now, if there are five teachers in that school in all, what would be the number of students in that school?

ACTIVITY 1

(1) Write the following statements in ratios :

(i) The number of men sitting in a hall are 150 and the number of women are 100. Write down the ratio of the number of men & women. **150:100**

(ii) Mr.Sharma is 40 years old & his wife is 35 years old. Write the ratio between their ages. ______

(2) A class has 20 boys & 25 girls. State the ratios between:

(i) Girls and boys: ______________

(ii) Boys and girls: ______________

(iii) Girls and total no. of students : ______________

(iv) Boys & total no. of students: ______________

(3) Ramesh walks 6km in one hour. Tara walks 4 km in one hour. What is the ratio between the speeds of Ramesh and Tara? $\boxed{}$

(4) Ram is 30 years old and Shyam is 20 years old. What would be the ratio between the age of Ram & Shyam? $\boxed{}$

What would be the ratio between the age of Shyam & Ram? $\boxed{}$

Think

One year has 20 weeks of rains and 120 days of raining weather in the next year. What would be the ratio of rainy days in both the years?

Can you show it as 20:120? If not, why?

Let us take another example

Rani takes 50 minutes to reach school from her house. Uma takes 1 hour to cover the same distance. What would be the ratio of time taken by Rani and Uma to travel to school? Can we indicate this at 50:___ ? Think about your response.

Now frame such questions and ask your friends to solve them.

In your every day life, we face many such problems when we compare different measures with one another. Such situations where direct comparisons are not possible, we need to change the comparable quantities into similar units. Look at the example about rainy days we talked about-

In one year the rains are indicated by 20 weeks and in the following year by 120 days. To change this into similar units, we will change 20 weeks into days.

We know 1 week = 7 days

$\therefore$ 20 weeks = 7×20

= 140 days

Now it is easy to compare the ratio of rain in the two years because we have the same units of days i.e. 140 days and 120 days. So the ratio of rainy days in the first and second year would be 140:120 and it can be simplified as 7:6. Can you now find out the ratio between the time taken by Rani and Uma to reach school from their house?

ACTIVITY 2

Write the ratios for the given statements:

(i) The lenght of a tree in the picture is 25 cm and the tree is 13 m high (long).

(ii) Ram takes 40 minutes to complete his homework and Shyam takes 1 hour to do his homework.

(iii) Anand came to Raipur after 15 months and Amina came after 2 years.

Can you now find out some problems of unlike or dissimilar units and write them in ratios?

The two quantities that compare in a ratio are known as terms. First term and second term. For example if we are comparing quantity ‘a’ with quantity ‘b’, then we will have the first term as ‘a’ and the second term as ‘b’ when written as ratio a:b. Whereas when ‘b’ is compared to ‘a’, the first term would be ‘b’ and the second term would be ‘a’. In another example given below, the lengths of two rectangles have been shown b:a. Let us compare them.

S. No.	Figures	Sides	Area
1.	1  2	Length = 2 units, breadth = 1 unit, ratio of length & breadth = 2:1	$A = 2 \times 1$ = 2 square units
2.	2  4	Length = 4 units, breadth = 2 units, ratio of length & breadth = 4:2	$A = 4 \times 2$ = 8 square units

In the figure above, rectangle 1, shows a ratio of 2:1 for their length and breadth.
In fig 2, the ratio of length and breadth = 4:2
The ratio of area in the two cases would be 2:8 or 1:4.

You must have seen the map of India. How is it possible to depict a big country like India on a small map? Think about this.

Below every map, we can see the scale for the map indicated e.g. 1cm = 100km; which means the distance 100 km has been depicted in the map by 1cm. Thus, a map is a figure of proportion.

Before you use a ratio, remember to check that the quantities are in the same units.

ACTIVITY 3

1. All the students of the class find out their heights one by one.
2. Students stretch their arms and find the distance between the ends of both the hands.
3. What relationship do you observe between the height of each student and the distance between the ends of their arms.

Complete the table.

S. No.	Name of the student	Height	Distance between the ends of arms stretched	Ratio	Simplified form
1.					
2.					
3.					
4.					

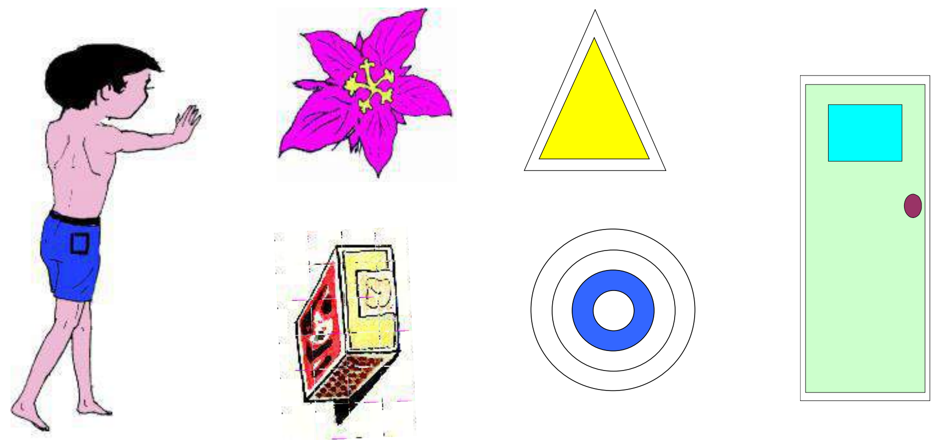
Draw the conclusions.

Aesthetics of Proportionate Figures

Ratio does not take place only in number but in many other examples around us. For example, if the legs of a man shown in a picture are very long or the head is larger than the body.

Then how does the man in the picture look? Naturally, it looks awkward. Similarly, suppose your class has a wall which has a short length & breadth and on that small wall a very big picture is put up or the wall is very big and a very small picture is put up at one end or in the middle of that wall, or a big frame has a very small picture in it, then most of the time, it would seem very awkward or unusual.

This is because all the above descriptions lack proportion. Our eyes are accustomed to see things in a definite proportion. Given below are a few proportionate & unproportionate figures. Identify the pictures in which you find suitable proportion or ratio and try to think about the reasons that made you leave the pictures that you didn't select.



Comparing two Quantities

Together a picture is proportionate or not is decided by comparing the shapes of different parts of the pictures. For example, the plank of a door can be very long, a match box wouldn't look like a match box if one of its sides become longer in length than the other sides. We can compare such situations in several ways and base the comparisons on different aspects.

The easiest way is to find out how big or small is the first quantity is from the second one. This information can be obtained in many different ways.

1. Bhawna get 40 marks & Renu get 20 marks in a test. Which means Renu get 20 marks less than Bhawna.
2. Out of 600 students, 200 students did not come to school on a day, it means only 400 students came to the school that day.
3. If two line segments measure 8 cm and 4 cm respectively, then the first line segment is 4 cm longer than the second line segment.

4. The height of a door is 8 feet and its width is 2 feet means that its length is 6 feet more than its width. Saying that the difference between the length & breadth of the door is 6 feet does not indicate whether the measurement is proportionate because if a door is 18 feet high & 12 feet wide, the difference is yet 6 feet but the two size are very different from each other. There can be another basis of comparing the quantities. We can also see how many times the second quantity is the first or how many times is the first quantity of the second. Thus the height of the door in the first case is 4 times the width, whereas in the case of the second door, the height of the door is $1\frac{1}{2}$ times its width.

We can find many such examples to see how many times is one quantity of the other.

1^{st} quantity	2^{nd} quantity	How many times of the first quantity is the second quantity	How many times of the second quantity is the first quantity
2 cm	6 cm	3 times	$\frac{1}{3}$ times
500 g	1000 g	2 times	$\frac{1}{2}$ times
200 rupees	1000 rupees	5 times	$\frac{1}{5}$ times
5 litre	20 litre	4 times	$\frac{1}{4}$ times
4 metre	32 metre	8 times	$\frac{1}{8}$ times
3 metre	5 metre	5/3 times or 5:3	$\frac{3}{5}$ times or 3:5

Now tell about some examples from your everyday experience where we need to know how many times of one quantity is the other quantity. The length of your room is 30 feet and its width is 15 feet. This means the length of the room is twice its width. We can write this as :

$$\frac{\text{Length of the room}}{\text{Width of the room}} = \frac{30 \text{ feet}}{15 \text{ feet}} = \frac{2}{1} = 2 \text{ times}$$

This means length is twice the width and the ratio of length & breadth is 2:1. We can also say that the ratio of breadth & length is 1:2.

Therefore, ratio is a relationship based on quantity.

1. The ratio between 50 books and 10 books = 50:10 = 5:1.
2. Ram is 20 yrs old and Shyam is 30 yrs old. The ratio between their age is 20:30 = 2:3.
3. The ratio between 400 kg wheat & 100 kg wheat = 400:100 = 4:1.
4. The ratio between quantity a & quantity b = a:b.

Practice

S. No.	Name of the school	No. of teachers	No. of pupils	Ratio	Simplified form
1.	Govt. Model Middle School	06	150		
2.	Bhagat Singh M.S.	10	350		
3.	Paramount M. S.	15	600		
4.	Lakshmibai Girls School	08	264		

Some Points to Remember About Ratios

1. If the ratio between two quantities ‘a’ and ‘b’ is shown by a:b, then ‘a’ is the first term and ‘b’ the second term (where ‘a’ & ‘b’ are whole numbers).
2. Like quantities in a ratio are indicated in the same units.

ACTIVITY 4

Shally and her family members were standing near a tree in an open pasture. Shally saw the shadows on the ground & began to measure them. The heights of the family members and the lengths of their shadows have been given in the table below.

S. No.	Members	Length of the shadow	Length of the Member
1.	Father	92 cm	184 cm
2.	Mother	80 cm	160 cm
3.	Brother	45 cm	90 cm
4.	Self	75 cm	150 cm
5.	Tree	215 cm	------

Now Shally had a problem. She knew the heights of her family members & could measure the lengths of their shadow, but she could measure only the length of the shadow of the tree because it was difficult for her to measure its height.

Can she fiind out the height of the tree with the help of the length of its shadow? Let us see how Shally solved her problem?

She noticed a relationship between the numbers in the table. She found that the height of every person is twice the length of the shadow. So, she thought that if the ratio of the height of every person & the length of his/her shadow is 2:1, then height of the tree also should be twice the length of its shadow. Thus she could find out that the height of the tree was 430cm.

Example 1.

A person gave 25 rupees to his son and 36 rupees to his daughter. Find out the ratio of money given by the person ?

Solution : Son's share = 25 rupees

Daughter's share = 36 rupees

Son's share : Daughter's share

$\Rightarrow$ 25:36.

Example 2.

A stick is 90 cm long and a bamboo is 4 m and 50cm long. Find out the ratio between the stick and the bamboo ?

Solution: Length of the stick = 90cm

Length of the bamboo = 4m 50cm

= 400cm + 50cm ($\because$ 1 meter = 100cm)

= 450cm

[We have converted the length of the stick and the bamboo into the same units (cm)]

Therefore, the length of the stick : length of the bamboo.

= 90 : 450

= 1 : 5 (simplified form)

Example 3.

Rajesh earns Rs. 12500/- per month. Out of this, he saves Rs. 2500. Find out (i) the ratio of Rajesh's earning & expenditure. (ii) Rajesh's earning and savings.

Solution: Rajesh's monthly income = 12500 Rs.

Rajesh's monthly savings = 2500 Rs.

Rajesh's monthly expenditure = 12500 - 2500 rupees

= 10,000 rupees

Therefore, the ratio of Rajesh's income and his expenditure = 12500 : 10000

The ratio of Rajesh's income and his savings = 12500 : 2500

= 5 : 1

Example 4.

Find the ratio of the cost of a pen and pencil. When the price of pens are Rs. 144 per dozen and Rs. 90 for 10 pencil ?

Solution: Here, we will have to calculate the cost of one pen and one pencil first.

One dozen or 12 pens cost = Rs. 144

$\therefore$ 1 pen will cost $\frac{144}{12}$ = Rs. 12

10 pencil cost = Rs. 90

$\therefore$ 1 pencil will cost $\frac{90}{10}$ = Rs. 9

Thus the ratio -

cost of 1 pen : cost of pencil

$= \frac{12}{9} = \frac{4}{3}$

= 4 : 3

Example 5.

40 toffees have to distribute between Chhotu and Mintoo in the ratio 4:1.

Solution: Total number of divisions = 4 + 1 = 5 or 40 toffees are divided into 5 parts, Chhotu will get 4 parts and Mintoo will get 1 part of the total number of toffees.

Therefore, Chhotu's share $= \frac{40}{5} \times 4$

= 32 toffees

Mintoo's share $= \frac{40}{5} \times 1$

= 8 toffees.

Example 6.

Find out ratio of the following measures:

(a) Rs. 5 and 50 paise.

(b) 500 cm and 10 meter.

(c) 8 kilogram and 640 gram.

Solution:

(a) The ratio of Rs. 5 and 50 paise.

5 Rs. $= 5 \times 100$ paise

$= 500$ paise

$\therefore$ Ratio $= \dfrac{500}{50} = \dfrac{10}{1} = 10:1$

(b) The ratio of 500 cm and 10 meter.

10 meter $= 10 \times 100$ cm $= 1000$ cm

500 cm : 1000 cm

1 : 2

(c) The ratio of 8 kg and 600 g

8 kilogram $= 8 \times 1000$ gram

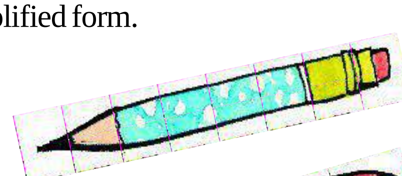
$= 8000$ gram

$\therefore$ Ratio $= \dfrac{8000}{640} = \dfrac{25}{2} = 25:2$

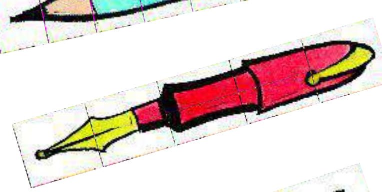
EXERCISE 10.1

1. Collect things around you. Find out the length of the objects and find the ratio of lengths between the different objects. Write them in simplified form.

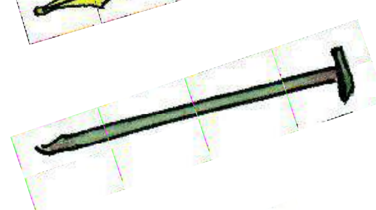
New pencil ______________



Pen ______________



Nail ______________



Alpin ______________

Find the ratio of lengths of :

(i) alpin and nail (ii) nail and pencil
(iii) pen and nail (iv) nail and alpin
(v) alpin and pen (vi) pen and pencil
(vii) pen and alpin (viii) pencil and alpin

2. Find out the ratio of the following:

(i) 15 minutes and 1 hour (ii) 250 gms and 1 kilogram
(iii) 15 paise and 1 rupee (iv) 12 paise and rupees 5
(v) 2½ cms and 1 meter (vi) 10 meters and 25 cms
(vii) 40 cm and 2.5 meters.

3. Write each of the ratios in simplified forms:

(i) 50 : 400 (ii) 85 : 255 (iii) 1 dozen : 1 score
(iv) 27 : 57 (v) 24 : 68 (vi) 250 : 375
(vii) 65 : 91 (viii) 2.5 : 7.5 (ix) 50 : 255
(x) 500 : 10000

4. Vishakha's annual income is Rs. 80,000; out of which she gave Rs. 5000 as income tax. Find the ratios of :

(i) income tax : income (ii) income : income tax

5. Munnu and Bunnu participated in race. In the given time Munnu covered 210 meter and Bunnu covered 180 meter in the same time. What will be the ratio of the distance covered by Munnu and Bunnu in the race?

6. Satish is a scientist and earns Rs. 20000 per month. His wife Anita is a doctor and earns rupees Rs. 15000 per month. Find put the ratios of :

(i) Satish's income : Anita's income
(ii) Satish's income : Their total income

7. The number of students in a school is 1500. Out of this 600 are girls. Find out the ratio of the number of boys and girls in the school.

8. Divide 20 balloons in the ratio of 2:3 between Bhanu and Bangaroo. How many balloons will Bhanu and Bangaroo get?

9. Rajesh and Javed together opened a shop. In this shop Rajesh's share was Rs. 45000 while Javed gave Rs. 36000. Find out the investment ratio of Rajesh and Javed.

10. In an examination out of 117 candidates 65 failed. Find out the ratio of successful and unsuccessful candidates.

11. Ratna and Sheela picked 18 mangoes. Both now wish to divide the mangoes between each other. Ratna wants the mangoes to be distributed in the ratio of their ages. Find out how many mangoes would Ratna & Sheela get if it is divided in this manner, when Ratna is 15 years old and Sheela is 12 years old?

12. The present age of a father is 50 years & that of a son is 20 years. Find the ratio of :

(i) present age of the father and the son.
(ii) both their ages when the son's age was 10 years.
(iii) both their ages when the father's age was 35 years.
(iv) both their ages when the son's age is 40 years.
(v) both their ages when the father's age was 75 years.

13. The ratio of income of Ram & Shyam is 3:4. If their total income is Rs. 21000. Find out the earnings of Ram & Shyam individually.

14. Point B is placed in between A & C in such a way that AB:BC is equal to 7:3. If AC = 40 km. Find out the values of AB and BC.

15. I have 6 samosas which I wish to divide with my friends. If

    i) I divide it in the ratio 1:1 between me and my friend, how many samosas will each one of us get?

    ii) I divide it in the ratio 2:1:3 with my two friends, how many samosas will each one of us get?

UNITARY METHOD

Look at some situations given below :-

1. You go to the market and buy two copies for Rs. 20. Now, if you need 5 more copies, how much money should you have?

2. Shyam has 2 litre of petrol in his scooter. He thinks that he can easily travel 50 kilometre with that amount of petrol. He has to travel 100 kilometre, what is the minimum amount of petrol that he needs to have in his vehicle to travel that distance?

3. On your birthday, you wish to give a small token gift to your friends. You choose a small car in the toy shop for a gift. If 3 toys cost Rs.75 and you need to buy 15 such toy cars how much money should you have?

Let us now think about the solutions to the above situations :

If two copies costs Rs. 20

Then 1 copy costs $\frac{20}{2}$ = Rs.10

Now if 1 copy costs Rs. 10

5 copy would cost = 10×5 = Rs. 50

Now are these methods to solve the other two situations also.

You can try several such problems with your friends.

In such conditions where the cost of many objects is known and the cost of one object is found in order to get the cost of the number of objects asked for, unitary method or unitary law is used.

ACTIVITY 5

Observe the given table & fill in the blanks.

Time taken	Distance covered on foot (km)	Distance covered by bicycle (km)	Distance covered by car (km)	Distance covered by train (km)
2 hours	8	20	70	120
1 hours	4	___	___	___
5 hours	20	___	___	___

We find that :

Distance travelled in 2 hours = 8 km

Distance travelled in 1 hour = $\frac{8}{2}$ = 4 km

Distance travelled in 5 hours = 4 × 5 = 20 km

Here to find out the distance covered in 5 hours bycycle, car, train or on foot, first we will have to find out the distance travelled in 1 hour.

Example 7

In a hostel the consumption of rice for 8 students is 4 kilogram, how much rice would be consumed by 30 students ?

Solution :

We shall solve this problem in 2 steps :

Step 1: First, we shall find out how much rice is consumed by 1 student .
When 8 students consume 4 kg of rice

$$1 \text{ student consumes } \frac{4}{8} = \frac{1}{2} \text{kg of rice.}$$

Step 2: Now from this, we shall find out the consumption of rice by 30 students.

$$\therefore \quad 30 \text{ students would consume } \frac{1}{2} \times 30 = 15 \text{ kilogram}$$

Thus, 30 students would consume 15 kg of rice.

Example 8

An aeroplane flies 4000 km in 5 hours. How much distance does it fly in 3 hours?

Solution :

We shall find out the distance covered by the aeroplane is one hour and in 2nd step, the distance covered in the asked period of time (3 hours) can be determined.

Step 1: Distance covered in 5 hours = 4000 km

Distance covered in 1 hour $= \frac{4000}{5}$ km

= 800 km

Step 2: Distance covered in 1 hours = 800 km
Distance covered in 3 hours = 800 × 3 km
= 2400 km

Therefore, the aeroplane would fly 2400 km in 3 hours.

Example 9

A woman saves Rs. 18000 in 15 months.

(i) What would be her savings in 7 months?

(ii) In how many months will she save Rs. 30,000?

Solution :

Step 1: Savings in 15 months = Rs18,000

$\therefore$ Savings in 1 month $= \frac{18000}{15}$

= Rs1200

Step 2: Savings in One month = Rs1200

$\therefore$ Savings in 7 months = 1200 × 7 = Rs.8400

Again Rs.1200 are saved in 1 month.

Rs.30,000 are saved in $\frac{1}{1200} \times 30{,}000 = 25$ months.

EXERCISE 10.2

1. Three copies cost Rs.16.5 What will be the cost of 7 copies?
2. A car moves 165 kms in 3 hours. Then find out
   (i) How much time would it need to move 440 kms?
   (ii) How much distance will be covered by the car in 6½ hours?
3. 72 books weight 9 kilograms ?
   (i) Find the weigh of 80 books.
   (ii) How many books would weight 6 kgs?
4. A worker earns Rs1500 in 25 days. Find out his income in 30 days ?
5. If 22 metres of cloth cost Rs704 , what would be the cost of 20 metres of cloth?
6. Complete the given table :

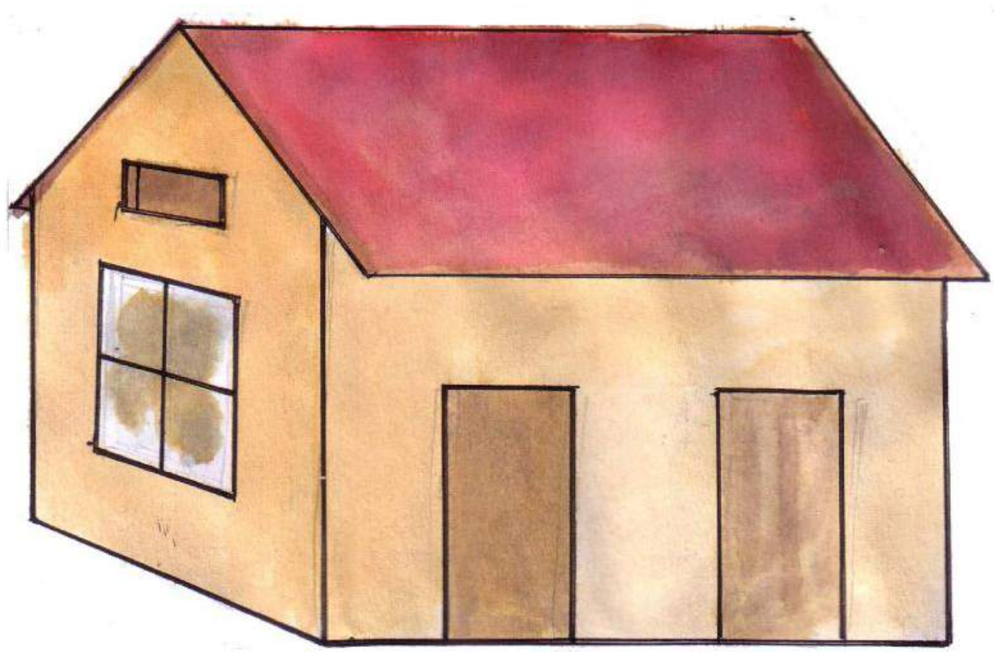
Number of books	Price (in Rupees
50	2500
75	____
____	100
____	3000

What Have We Learnt ?

(1) Ratio of two similar quantities shows how many times is one quantity of the other.

(2) The ratio of two quantities is generally written in its simplified form. For example na : nb is written as a : b.

(3) When the price or value of a unit quantity is found out from the given number of the quantity and then the value/ price for the asked number of quantity is determined, the method is known as the unitary method.



Chapter 11

VARIABLES

We often see around us several things whose values are fixed or constant and many other things whose values keep on changing. For example, the weight of a chair tomorrow and day after and many days later remains the same what it is today but if we notice a germinating seed, the length of the growing plant changes very slowly everyday. Similarly, the length & width of a class room remain constant, but we cannot say anything about the change in the level of water in a well after a month and in the rainy season.

Write down five examples in your notebook about values that remain constant and five examples whose values keep changing.

While writing the examples in the notebooks in the class, Anu said to Rohan, “ I shall write that my father’s age is changing but his height is constant.” Rohan said, “I’m going to write that the area of my fields measure 4 acres, but the harvest is sometimes less & sometimes more.”

These are interesting examples and you notice that some values are constant while others are changing. Below are provided some situations, discuss amongst yourself and write in the blanks whether the concerning values are constant or keep changing.

ACTIVITY 1

S. No.	Situations	Values constant / changing
1.	The number of days in a week.	
2.	The temperature of the day in the month of May.	
3.	The no. of student coming to your class everyday.	
4.	No. of players in hockey team.	
5.	No. of potatoes in one kg of potatoes.	

While solving the last problem, Hamida said to her friends, “If the potatoes are big, then lesser number of potatoes will make 1 kilogram, but if they are smaller in size, their number in one kilogram would be more. Similarly, we cannot say how many potatoes can be put into a bag. Something like this happened at home yesterday. My father put all of us into a maze. He had some toffees tied up in his handkerchief. He asked, “How many toffees are there in this handkerchief?” Now none of us knew the number, so how could we tell him that. We kept thinking, is there not any way to tell the number of toffees in the handkerchief? Raju was listening carefully & said, “Let us go to our mathematics teacher and ask her about this.”

The mathematics teacher listened to their problem and put another problem before the students. She asked, “How many pieces of chalk are there in this box? The students told different numbers as per their assumtions. Hamida said ‘12’ and so on, Raju said ‘18’, Anu said ‘16’, Rohan said ‘20’ and so on. The teacher said, “If I take out 5 pieces of chalk from the box, then according to you, how many chalk sticks would be left.” Hamida, Raju, Anu and Rohan calculated on the basis of their own numbers respectively to get 12-5 = 7, 18-5 = 13, 16-5 = 11 & 20-5 =15.

Since the number of chalk sticks in the box were unknown, so the answers were different. But if instead of the original numbers thought of, we write the no. of chalk sticks as 5, then every one would have a common answer, for this we will have to write the number of chalk sticks everytime. Can’t we write it in brief? Do we have a method for this?

If the number of chalk in the box is considered as ‘C’ and take out 5 sticks of chalk from it, then the number of pieces of chalk in the box would be C-5. Similarly if we add 3 pieces of chalk to the box, the number of chalk sticks in the box would be C+3. Let us take another example.

In a packet of toffees, there are 20 toffees but the cost of the packet is not known to us.

If 1 toffee costs 50 paise, then the price of the packet would be

$$= 20 \times 0.50 \text{ rupee} = 10 \text{ rupee}$$

If 1 toffee costs 1 rupee, then the price of the packet

$$= 20 \times 1 \text{ rupee} = 20 \text{ rupee}$$

If 1 toffee costs 2 rupees, then the price of the packet

$$= 20 \times 2 \text{ rupees} = 40 \text{ rupees}$$

Thus, the price of a packet here = 20 × (price of 1 toffee)

Then if instead of the price of one toffee, we write x rupees, y rupees and z rupees or any letter of the alphabet, then the price of the packet will be 20x rupees, 20y rupees and 20z rupees.

Let us consider another such example.

In a square, the length of each side is equal to 2 units, the perimeter of the square would be 4 × 2 units. If the length of each side of the square is 3 units, then the perimeter is = 4 × 3 units. If the length of the side is 7 units, then the perimeter would be 4 × 7 units and similarly, if the length of a side of the square be ‘a’ unit, then the perimeter of the square would be $4 \times a$ units.

In all the examples taken above, you have seen that some questions are constant like when the price of packet of toffee is 20x rupees, 20 here is constant, but x rupees means the cost of the packet changes according to the price of 1 toffee.

Similarly, in the perimeter of a square 4a, 4 is constant (sides), but as the value (length) of the side changes (a unit), the perimeter of square also changes.

After observing all the examples, Hamida came to a conclusion that all changing values are indicated by some letter. So, she could have said that ‘my father had z toffees in his handkerchief.’ The value of z can be found out only we had some extra information regarding the toffees, otherwise not!

These quantities which keep changing are known as variables. These can have any value they can be denoted by any letter of Hindi or English alphabet like अ,उब,उस,उद or a, b, c, d or x, y, z etc. These numbers (denotions) are called variable numbers or algebraic numbers.

Whenever we have a number whose value is not known, instead of the number, we use an algebriac number or valuable number. It is easy to solve such problems by using variable numbers.

Similarly, mathematical examples are also done with the help of variables for example:

1. What is the relationship between a number and its succeeding number? What number comes after 4? This number is 5 which means 4 + 1. Similarly, what comes after 1000? It is 1001, which means 1000 + 1. So, to any given number if one (1) is added, we get the next number. If any number is x, then the next number will be $x + 1$.
2. Can you make a similar rule about the number preceeding any given number?
3. Can you write even numbers as variables? 2, 4, 6, 8 etc. are even numbers. All these numbers have a common multiple factor 2. This means any integer multiplied by 2 will give us an even number. Suppose, 'n' is a natural / countable number, then 2n will be a even number or an even number can be denoted by '2n'.
4. Can you write odd numbers in the form of variables? If you look at numbers, you will find that even and odd numbers come alternately even, odd, even, odd, even, ... like 1, 2, 3, 4, 5, 6, 7, 8, ... Here every number before an even number is an odd number and the number after the even number is also an odd number. We have denoted even numbers by '2n'. $x - 1$, and the number that comes after it is $x + 1$. So, the number before an even number would be 2n - 1 and the number after the even number will be 2n + 1. Thus (2n - 1) or (2n + 1) can be used to denote an odd number.

Below are given some numbers that are related to some rules. Write them in the n^{th} term.

S. No.	Numbers that are related to some rules	First term	Second term	Third term	Seventh term	Ninth term	n^{th} term
1.	3, 6, 9, 12, ... etc	3	6	12	21	27	3n
2.	5, 8, 11, 14, ... etc	5	8	14	----	----	----
3.	3, 7, 11, 15, ... etc	3	7	15	----	----	----

EXERCISE 11

1. Indicate the following by numbers, literal numbers and basic operation signs. Also say what each letter denotes?
   (i) The diameter of a circle is twice its radius
   (ii) The area of a rectangle is the product its length & width.
   (iii) Selling price is equal to the sum of cost price and profit.
   (iv) A number is added to another number.

(v) 7 is subtracted from any number.

(vi) Composite amount is equal to the sum of principle and the Interest.

2. Identify the true and false statements and rewrite the wrong statements correctly.

(i) The number succeeding 'a' will be $a + 1$.

(ii) The value of 'x' changes according to situations.

(iii) 2n would be an odd number.

(iv) $m \times n$ would be the product of any two numbers.

(v) Variables are generally indicated by small letters of the English alphabet.

What Have We Learnt ?

1. Any letter that is used to denote numbers, is known as a numerical letter.
2. These letters show numbers as variables, hence follow all rules followed by or applicable to numbers.
3. The quantity that has a definite numerical value is not a variable but a constant.
4. The quantity that can have many numerical values is known as a variable.
5. In arithmatic we use numbers with definite numerical values, where in algebra, we use letters that can have more than one numerical values.
6. The algebraic part of a number is a variable quantity. If $p = 4a$, 4 is a constant, but a is variable.

Chapter 12

ALGEBRAIC EXPRESSIONS

In the last lesson you have learnt about variables and you know how to write them in the form of variables or algebraic numerals. You can now easily indicate the changing or unknown quantities by variables and the constant values by constant quantities or numbers. These statements or expressions that are formed with the help of constants and variables are kown as Algebraic Expressions. In the last lesson wherever we have used variables, we can call them algebraic expressions also.

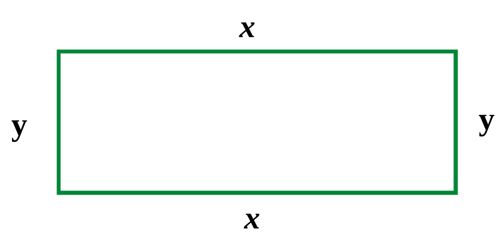
Let us observe how algebraic expressions are used in our daily life.

Razia does not know her age. Now if she needs to know what her age 5 years ago was or what will her age be 3 years hence, we would better answer the questions with the help of algebraic expressions.

Let us denote Razia's age by 'y' years then 5 years before, her age must have been y-5 years and after 3 years, her age will be y + 3 years. Here y - 5 and y + 3 are algebraic expressins.

Now let us put some more mathematical problems into algebraic forms.

1. Radha's marks are 3 less than Neha's marks. If Neha's marks are x, then Radha's marks would be x - 3.
2. The length of a rectangle is 4 units more than its width. If the width of the rectangle is 'y' units, then the length of the rectangle will be y + 4 units.
3. The length of a rectangular ground is x and y is its width, then to take complete round, what will be the distance to be covered? A complete round will cover p = x + y + x + y distance.



ACTIVITY 1

Write down the following in algebraic expressions:

1. Three times a number.
2. 6 more than a number.
3. 17 less than a number.
4. Fifth part of a number.
5. 12 more than twice of a number.
6. 3 less than seven times a number.
7. $\frac{1}{3}$ of 4 times a number.
8. 7 times of a number added to itself.

9. Subtracting 6 times of a number from 5 times the same number.

10. 5 litres of milk is taken out from a can full of milk. How much milk remains in the can?

In all the above examples, you must have noticed that algebraic expressions are made up of constants and variables. In the table given below, sort out the constant and variables. Write them in the proper columns.

ACTIVITY 2

S. No.	Algebraic expression	Variable	Constant
1.	z	z	1
2.	$x + 5$	x	1, 5
3.	$y - 8$	___	___
4.	$3x + 2y$	-----	-----
5.	$2xy - 3$	x, y	2, -3
6.	$3x^2 - 7$	-----	-----
7.	$33x$	-----	-----
8.	$y - x$	-----	-----

Are all the algebraic expressions similar?

Here the number of terms in z and 33x is 1, whereas in the rest of the statements there are 2 terms. This means that in an algebraic expression, the number of terms or items in the algebraic expression is always 1 more than the number of times + or - sign is used in the expression.

ACTIVITY 3

Write the number of terms or items in the given algebraic expressions.

1. $3x + 8$ two
2. $3 + y - 7p$ ___
3. $2x + 3y + z$ ___
4. $8x$ ___
5. $9 + 3a - 3x + b$ ___
6. $5xyz$ ___

You know about variables and constants now for examples in $3x + 8y + 7$, x and y are variables but 3, 8 and 7 are constants. In the lesson on variables, you have seen that $3x$ is actually 3 times x; similarly 8y is 8 times y. This means the constant that is with the variable is multiplied to the variable and hence we call it a multiple. Therefore in $3x + 8y + 7$; 3 is a multiple of x, 8 is a multiple of y and 7 is a constant.

ACTIVITY 4

S. No.	Algebraic expression	Multiple	Variable
1.	$8x$	8	x
2.	9py		
3.	xyz		
4.	18ab		
5.	yz		
6.	$-\frac{1}{2}$yz		
7.	$3xyz$		
8.	$32x$		
9.	- 3py		
10.	$-\frac{3}{5}$yz		

In the above activity, you have found that in $8x$ and $32x$ the value of the variable is x; similarly in 9py and -3py, the value of the variable is py and in $3xyz$, the value of the variable is xyz. In yz, $-\frac{1}{2}$yz and $-\frac{3}{5}$yz, the value of the variable is yz. Such quantities with the same variables are kown as like variables/

Like Terms

All terms in which the variable or algebraic part are same are called like terms. Their multiples can be different.

Unlike Terms

When the variable or algebraic part of the expressions are not same, they are called unlike terms.

ACTIVITY 5

Encircle the like variables in the given list of algebraic expressions.

Like x	$6xy,\ 5y,\ \frac{2}{3}x,\ 5xz,\ 7z,\ 2x$
Like yz	$2y,\ 7xz,\ 5z,\ 2yz,\ \frac{1}{2}yz,\ 6xy$
Like a	$2a,\ \frac{6}{7}ab,\ \frac{7}{6}a,\ -3b,\ 6a,\ 2c$
Like lmn	$6l,\ 5mn,\ \frac{2}{3}lm,\ lmn,\ 2l,\ -6ln$
Like $2pq$	$6r,\ pqr,\ -5pq,\ 7qr,\ 2q,\ 2p$
Like st	$4rs,\ 7st,\ -14rt,\ 2rst,\ 6r,\ 4t$

EXERCISE 12

1. Recognise the single term (univariant) and two termed (bivariant) algebraic expressions and write term separately.

(i)	$3x + 4y$	(ii)	$9z + 3y$	(iii)	$4a - 7b$
(iv)	$5x + 1$	(v)	$a - 30$	(vi)	$4ab$
(vii)	$abc - 1$	(viii)	$3xy$	(ix)	$ab + bc$
(x)	$a + abc$				

2. Select the like terms from the following algebraic expressions.

$$5xy,\ 7c,\ -\frac{4}{5}yz,\ -7bc,\ -\frac{9}{4}xy,\ \frac{2}{7}z,\ -2c,\ bc,\ -37pqr,\ \frac{11}{13}yz,\ 7z,\ 9pqr.$$

What Have We Learnt ?

1. The letters used in place of numbers are called variables. They are also called algebraic quantities.
2. The terms with same letter numerals and exponents are called like terms.
3. The terms with dissimilar letter numerals and exponents are called unlike terms.
4. When a two variables or a variable and a constant are combined using +, -, ×, or ÷ signs, then we get an algebraic expression.
5. Expressions with one term are known as single termed or univariate algebraic expressions.
6. Expressions with two terms are known as two termed or bivariate algebraic expressions.

Chapter 13

PERCENTAGE

Some students in the class were discussing about some problem. Many said, "When I was coming to school today, I saw a big banner on the cloth shop. It said, "Discount, Discount, Discount, 10 percent!" What does this mean?"

Arun said, "If you buy cloth for 100 rupees, you will get a relaxation of 10 rupees. That means you'll get the cloth for Rs. 90, that is the discount you get."

Mary enquired, "If I buy cloth for Rs. 40; how much discount shall I get?"

Salma replied, "You will get a discount of Rs. 4, which means you'll have to pay 36 Rs."

Mary said, "But in the examination marksheet also. The marks obtained are written in percent."

Ramesh said slowly, "Where else do we use this?"

Salma was excited and said, "Let us make a list of situations, where we use percentage."

All the students thought about it and wrote some examples:

(i) In the exams Anil got 93 percent marks.

(ii) In the last annual examination, 87 percent girls and 76 percent boys passed.

(iii) Bank gives an interest of 5 percent on savings or deposited amount.

(iv) In business loss and gain is expressed in percent.

(v) 70 percent people in the country live in villages.

(vi) 15 percent of discount on fans sold during writer.

Why do we Need Percentage?

In an exam Uma got 8 marks out of 10, Vinay got 15 out of 20. Can you say, who got better marks?

To compare the marks obtained by both Uma & Vinay. The denominators of the fractions will have to be equated

Therefore Uma's marks will be : $\frac{8}{10}=\frac{8\times 2}{10\times 2}=\frac{16}{20}$

And Vinay's marks are : $\frac{15}{20}$

Now since both the deniminators are same, we can compare them and say that Uma's marks are better than Vinay. This means wher ever we need to compare marks obtained out of maximum marks, we would require to equate the fractions.

If these marks are converted in a way that the denominator is 100, then

Uma's marks would be $\frac{8}{10}=\frac{8\times 10}{10\times 10}=\frac{80}{100}$.

Since 8 marks out of 10 is equivalent to 80 marks out of 100, therefore we can say that Uma scored 80 percent marks.

Vinay's marks would be $\frac{15}{20} = \frac{15 \times 5}{20 \times 5} = \frac{75}{100}$.

Since 15 marks out of 20 is equivalent to 75 marks out of 100, we can say that Vinay scored 75 percent marks. And thereby, the percentage of Uma's marks is higher.

You find in the above examples that percentage means per 100. It is helpful in comparison of data. 100 is taken as a common base that can assist comparison in every situation when the marks obtained are compared on the basis of 100, then the marks/score obtained out of 100 would be known as per hundred. Hence in percentage, 'per' means 'every' and 'cent' means 'hundred'. Percent is denoted by "%".

ACTIVITY 1

Fill in the given table & compare the marks. Who scored the highest? For this we shall find out whose percentage of marks is the highest?

S. No.	Name	Maximum Marks	Marks obtained	$\frac{\text{M.O.}}{\text{M.M.}}$	Percentage of Marks obtained
1.	Golu	80	60	$\frac{60}{80}$	$\frac{60}{80}$ ò 100 Z 79%
2.	Salma	100	90	$\frac{90}{100}$	$\frac{90}{100}$ ò 100 Z 90%
3.	George	150	120	$\frac{120}{150}$	$\frac{120}{150}$ ò 100 Z 80%

The percentage of marks obtained by Salma is the highest. Think that if we compare the scores of students in any other way, will it be so easy?

Fill in the table below and say which school got a better results? For this again, you'll find out percentages.

Name of the school	Students enrolled in class VI *a*	No. of students passed *b*	No. of students out of 100 who passed $\frac{b}{a} \times 10$	Percentage result
Govt. High School, Jagdalpur	500	450	$\frac{450}{500} \times 100$	90%
Govt. High School, Raipur	300	195	?	?
Govt. High School, Sarguja	200	140	?	?

The above activities indicate that two or more situations can easily be compared with the help of percentage.

If percentage means comparison on the basis of 100, question arises, can the value in percentage exceed 100?

Let us find out!

Example 1.

On a particular day 300 kg of potatoes were bought in the market, 750 kg of potatoes were purchased on the second day, what percentage increase took place in the purchase of potatoes?

Potatoes bought on the first day = 300 kg

Potatoes bought on the second day = 750 kg

The increase on the purchase of potatoes = 750 -300 kg = 450 kilograms

Now, on 300kg of potatoes, the increase in purchase was 450kg

$\therefore$ on 1kg of potatoes $\frac{450}{300}$ kg increase in purchase took place.

So, the percentage of increase in the purchase would be $\frac{450}{300} \times 100 = 150\%$

Can you think about situations from your everyday life where percentage (i) less (ii) equal or (iii) more than 100% are used.

Think about 5 examples of each and tell your friends.

ACTIVITY 2

What percentage of the given picture is shaded?

What is the fractional value for the shaded parts?

For example:

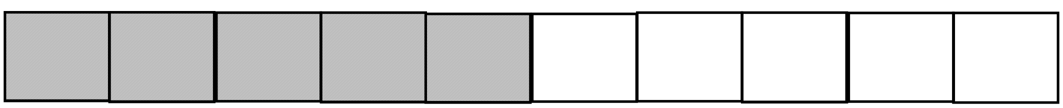


Fig 1

$\frac{5}{10} = \frac{1}{2} = 50\%$

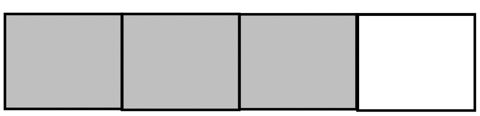


Fig 2

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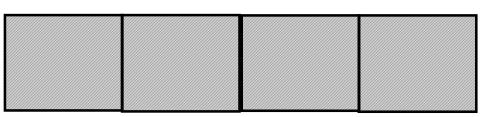

Fig 3

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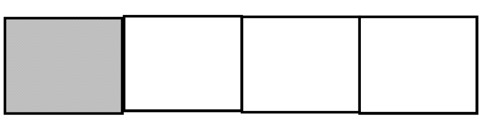

Fig 4

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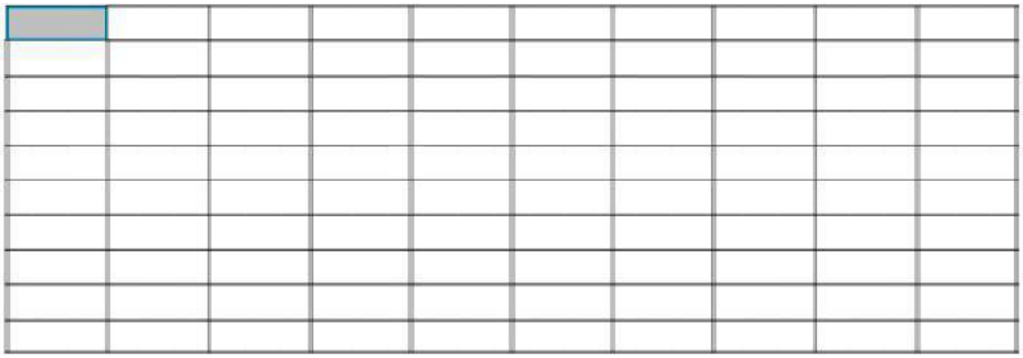

Fig 5

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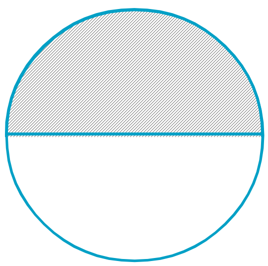

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Fig 6

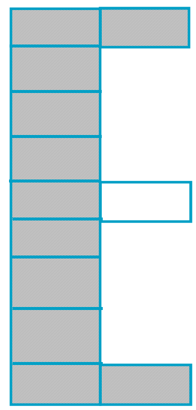

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Fig 7

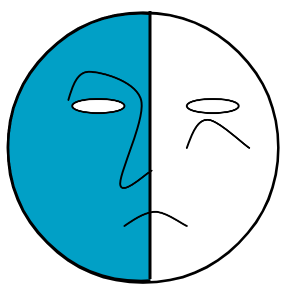

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Fig 8

You have just changed the shaded parts of the figures into percentage & fractions.
Is percentage a different form of fraction?
Let us solve some examples.

ACTIVITY 3

You have learnt that in a fraction, if the denominator is 100, the numerator of that fraction is equal to its percentage. Now change the following fractions into percentages.

S. No.	Fraction	Fraction $\times \frac{100}{100}$	Multiple of $\frac{1}{100}$	percentage
1.	$\frac{1}{1}$	$\frac{1}{1} \times \frac{100}{100}$	$\frac{1}{100} \times 100$	100%
2.	$\frac{1}{2}$	$\frac{1}{2} \times \frac{100}{100}$	$\frac{1}{100} \times 50$	50%
3.	$\frac{1}{4}$			
4.	$\frac{3}{4}$			
5.	$\frac{1}{10}$			
6.	$\frac{1}{100}$			

ACTIVITY 4

Below are given some percentage changed into fractions, complete the rest.

S. No.	percentage	Multiple of $\frac{1}{100}$	fraction
(i)	100%	$100 \times \frac{1}{100}$	$\frac{1}{1}$
(ii)	50%	$50 \times \frac{1}{100}$	$\frac{1}{2}$
(iii)	25%		
(iv)	75%		
(v)	10%		
(vi)	1%		

Note for the teacher : (If a student multiplies $\frac{1}{2}$ by 100 directly, to convert it into percent, then let him do so by explaining the basic rule.)

In the above activity, you've seen that the fractional form of 100% is $\frac{1}{1}$.

Similarly, fractional form of 50% is $\frac{1}{2}$.

Therefore, we can say that percentage is a form of fraction.

Example 2.

On 26th January, 200 laddoos were brought to a school. If 90% of the laddoos were distributed to students, find out the number of laddoos that remained ?

Solution. Total no. of laddoos = 200

No.of laddoos distributed = 90 percent of 200

No. of laddoos distributed $= 200 \times \frac{90}{100}$

$= 180$

Remaining laddoos = 200 - 180 = 20 laddoos.

Example 3.

The population of a village is 10,000. Out of which 60% are women, 25% are men & the rest are children. Find the number of men, women & children ?

Solution: Population of the village = 10,000

Since 60% are women = 60% of 10,000

So, the number of women $= \frac{10,000 \times 60}{100}$

$\therefore$ Number of women = 6000

Given 25% of population are men

= 25% of 10,000

$\therefore$ The number of men $= \frac{10,000 \times 25}{100}$

$\therefore$ The number of men = 2500

The number of children = 10000 - (6000 + 2500)

= 10,000 - 8500

= 1500 children.

Example 4.

Shyamu bought a book for Rs. 50 from a shop. The shopkeeper gave him a discount of 20%. How much did Shyamu pay the shopkeeper?

Solution:

20% discount means

Out of Rs. 100 discount got is of Rs. 20.

$\therefore$ Out of Rs. 1 discount got is of Rs. $\frac{20}{100}$.

Out of Rs. 50 discount got is of Rs. $\frac{20}{100} \times 50$ = Rs. 10

$= 10.$

Therefore, the amount Shyamu had to pay the shopkeeper was 50 - 10 = Rs. 40.

Example 5.

(i) Find out 50% of rupees 650 ?

(ii) Find 5% of 750 kilograms ?

Solution:

(i) 50% of Rs. 650 $= \dfrac{650 \times 50}{100}$

$=$ Rs. 325

(ii) 5% of 750 kilograms $= \dfrac{750 \times 5}{100}$

$= 37.5$ kilograms.

Example 6.

Dhawal got 450 marks out of 500 and Yash got 675 out of 900. Whose result is better?

Solution:

According to the question

Dhawal gets 450 out of 500

$\because$ Marks out of 500 $= 450$

Marks out of 1 $= \dfrac{450}{500}$

Marks out of 100 $= \dfrac{450}{500} \times \dfrac{100}{1} = 90 = 90\%$

Yash gets 675 out of 900

$\because$ Marks out of 900 $= 675$

Marks out of 1 $= \dfrac{675}{900}$

Marks out of 100 $= \dfrac{675}{900} \times 100 = 75\%$

The solution shows that Dhawal's result is better than that of Yash.

Try to solve the problems by some other methods or a new method. Discuss the method you have used with your teacher.

EXERCISE 13

1. Change the following into percentage:

(i) $\dfrac{3}{2}$ (ii) $\dfrac{5}{2}$ (iii) $\dfrac{1}{5}$ (iv) $\dfrac{3}{20}$

2. Change the percentages into fractions:

(i) 50% (ii) 15% (iii) 2% (iv) 10%

3. Find out 60% of Rs. 360.
4. How much is 15% of 480 kilograms?
5. Seeta got 250 marks out of 500. Convert her marks into percentage.
6. If Ram gets a maths book priced Rs. 10 at 10% discount. How much will he pay for it?
7. On Independence Day, 300 toffees were brought to a school. 99% toffees were distributed to students. What number of toffees remained?
8. If Rupa gets 390 out of 600 marks in her annual exams, what percentage of marks did she get?
9. If a rubber is stretched to twice its length, find out the percentage increase in length.
10. 40% of the total population of a city are men and 35% are women while the rest are children. If the children number 18,000, find the number of men & women in the city.
11. The population of a village is 3000. It increased by 10% in the first year and after one year the population decreased by 10%. Find the percentage of increase or decrease in the population.
12. A person buys things of Rs. 630. The shopkeeper takes only 567 rupees from him. What is the percentage of discount that the person enjoyed?
13. 75% of a number is 600. Find the number ?
14. A person deposited Rs. 5000 in the bank . After a few years he got Rs. 6000. What the percentage increase his deposit?
15. Out of 40 students enrolled in a class, 36 boys have passed in the examination. Find out the percentage of successful and unsuccessful students ?
16. 90% of the people in a village are literate. If the population of that village is 1600, find out the number of literate & illiterate people ?

What Have We Learnt ?

1. Percentage means per hundred.
2. We can compare quantities through percentage.
3. Percentage can be expressed as fraction, in decimals and as ratio and fraction, decimal and ratio can be converted into or expressed as percentage.

Chapter 14

EQUATIONS

When you compare two numbers, you say that one number is greater or smaller or equal to the other number. Below are given some statements of comparison. The statements are incopmlete. Put the symbols =, > or < in the boxes to complete the statements.

ACTIVITY 1

Example: (1) $3 + 5 \boxed{>} 7$

(2) $8 + 7 \boxed{} 15$

(3) $4 + 6 \boxed{} 11$

(4) $13 + 8 \boxed{} 18$

(5) $23 + 7 \boxed{} 30$

How did you find out the appropriate symbol for the statements. Think about the reasons that you gave yourself.

These are two sides of the statements. Those towards the left of the box and those towards the right of the box. In the statements 3 + 5 is the left side & because it is greater than 7, so we wrote 3 + 5 > 7.

Select these statements for which you have used the = symbol and note them in your copy. Those statements which you have not noted down are statements of inequality. Let us, look at some more statements in which variables have been used for e.g. in $x + 5 = 13$, if $x = 5$, then substituting 5 in the place of x, we would get 5 + 5 = 10, whereas the right side = 13. Therefore, the statement that left hand side is equal to right hand side is not true

So, L.H.S. $\neq$ R.H.S., in the statement if $x = 8$, then both sides would become equal and the statement L.H.S. = R.H.S. would become true.

ACTIVITY 2

Below are given some statements. The value of x is given with them. Write down whether the statements are true or false according to the given values of x.

1) $x + 3 = 8$, if $x = 5$, then the statement is $\boxed{\text{true.}}$

2) $x - 2 = 4$, if $x = 7$, then the statement is ________.

3) $x + 2 = 10$, if $x = 8$, then the statement is _______.

4) $7 = 12 - x$, if $x = 3$, then the statement is _______.

5) $3 = x - 9$, if $x = 5$, then the statement is _______.

Find out whether these statements which are false, would become true for some value of x. If yes, then write down these values of x for the false statements in your notebooks. In that case, the above statements would stand true only when both the sides are equal to each other. These statements in which variables are included and both the sides are equal are known as equations.

These equivalent statements which have one or more than one algebraic numbers are called equations. Thus in all the variable and nonvariable elements of the equivalents symbol (=) represent. The Left Hand Side and all those variables and nonvariable elements on the right of the sign represent the Right Hand Side of the equation.

Why Equation ?

One day, Naresh asked his friends a question.

"Guavas have been kept in two baskets. The second basket contains two times of the guavas in the first basket. If 8 guavas are added to the first basket, the number of guavas in the second basket become equal to the number of guavas in the first basket. Can anyone tell the number of guavas in both the baskets?"

All the friends of Naresh began to think about the problem but they couldn't make anything out of it. Just then Anu said that the first basket contains 8 and the second basket contains 16 guavas. Naresh said, "The answer is correct, but how did you solve it?" Anu said, "I have read that if a number is added to the same number, then we get twice that number since the number of guavas in the first basket is 8, then 8 more guavas added to the first basket would make 16, which is the number of guavas kept in the second basket, so I got the answer."

Naresh said, "We can solve this by another method.

Guavas in the first basket.	Guavas kept in the second basket, twice the number of guavas in the 1^{st} basket.

Guavas (1^{st} basket) + 8 = 2 × guavas (2^{nd} basket)

8 added & 8 alone will make it two times (twice). Therefore, the first basket has 8 and the second basket has 16 guavas.

Farida remembered, "We have read about it in the lesson on 'Variables' that when we do not know any number, we can consider it as a variable."

Suppose, the first basket has $\boldsymbol{x}$ guavas. Then in the second basket we will have $2\boldsymbol{x}$ number of guavas.

Now, 8 guavas added to the first basket, will make $\boldsymbol{x} + 8$ guavas in that basket which is equal to the number of guavas in the second basket. This means,

$$\boldsymbol{x + 8 = 2x}$$

Naresh exclaimed, "Wow! that makes it an equation! Here if we put 8 as the value of $\boldsymbol{x}$, the statement will become true. This also means that all equations with unknown values or variables can easily be solved!" So, you have seen that equations are quite useful to help us find out values of unknown quantities. Now, let us understand how variables are formed:

How to Make Equations

Let us play a game. How old are you? Think of your own age. Add 5 to it. Multiply the sum by 2 and subtract 10 from the product. Now from this difference subtract your age. The answer will be your age!

Naresh's solution		Teacher's Instructions		Anu's solution
12 years	←	Think of your age.	→	11 years
$12 + 5 = 17$ years	←	Add 5	→	$11 + 5 = 16$ years
$17 \times 2 = 34$ years	←	Multiply by 2	→	$2 \times 16 = 32$ years
$34 - 10 = 24$ years	←	Subtract 10	→	$32 - 10 = 22$ years
$24 - 12 = 12$ years	←	Subtract your age	→	$22 - 11 = 11$ years

Thus all students find that the age that they thought of in the beginning comes as the answer in the end. How did this happen? Let's find out.

Suppose, the age thought of is x years.

5 added to age	=	$x + 5$
Sum multiplied by 2	=	$2(x + 5) = 2x + 10$
10 subtracted	=	$2x + 10 - 10 = 2x$
Age thought of subtracted	=	$2x - x = x$

This means you are getting the number that you thought of in the beginning as your age.

As soon as Raju looked at the steps of the equation, he was excited and said, "Now I can also ask questions about making equations!" He asked, "If a number is multiplied by 2 and 5 is subtracted from its product, the result is 3, what will the equation be?"

Anu at once formed the equation.

Suppose, the number is x; multiplied by 2, we get $2x$, 5 subtracted from the product given $2x - 5$ which is equal to 3. This means the equation is

$$2x - 5 = 3$$

Anu said, "Now, I'm giving you an equation you'll have to write it in words?"

Equation : $7y - 5 = 9$

Hamida could at once think about it that 7 multiplied to any number and 5 subtracted from it gives 9.

Now, all children in the class started taking interest in framing & solving equations. It was fun.

EXERCISE 14.1

1. Identify which of these are equations:

(i)	$x - 4 = 10$	(vi)	$7 = 2x - 5$
(ii)	$x - 4 - 10$	(vii)	$3x - 2x = 2x$

(iii) $2y - 3 + 9$ (viii) $\frac{5}{x} = 3$

(iv) $5(2y - 3) = 15$ (ix) $4.5 + 3.2\boldsymbol{x} = z$

(v) $3\boldsymbol{x} + 4$ (x) $ly + lx = px$

2. Identify the L.H.S. and R.H.S. in the given equations
   (i) $x - 5 = 9$
   (ii) $2x - 3 = 7$
   (iii) $2y = 9 - y$
   (iv) $2y = 6$
   (v) $15 = 2a + 5$
3. In the following statements use **"y"** for the unknown numbers & change them into equations.
   (i) 3 subtracted from twice of the number gives 17.
   (ii) The sixth part of the number is 7.
   (iii) The difference of the number and 5 is 8.
   (iv) 7 multiplied by the number and 5 subtracted gives 9.
4. Write the following equations as statements:
   (i) $x - 6 = 9$
   (ii) $7y - 14 = 0$
   (iii) $\frac{2x}{3} = 6$
   (iv) $\frac{x}{2} + 5 = 10$
   (v) $38 - 2x = 4$

Solving Equations

In Activity 2, you have seen that each statement is true for only one value of x, e.g. In $x + 2 = 4$, if $x = 7$, the statement would become false because on putting the value of x as 7, L.H.S. will not be equal to R.H.S. Hence, the statement becomes true only when $x = 2$. This means that these kind of equations have only one solution.

ACTIVITY 3

In the table below are given some equations with x as the variable, decide whether the L.H.S. and R.H.S. of the equations are equivalent or not for the different values of x shown in the table.

S. No.	Equation	L. H. S. of the equation	R. H. S. of the equation	Whether L. H. S. is <, = or > R. H. S. for the different value of x					
				$x = 0$	$x = 1$	$x = 2$	$x = 3$	$x = 4$	$x = 5$
1.	$x + 3 = 5$	$x + 3$	5	$<$	$<$	$=$	$>$	$>$	$>$
2.	$x - 2 = 3$								
3.	$x + 3 = 2x$								
4.	$x + 4 = 4$								
5.	$3x + 1 = 3 + x$								
6.	$x + 5 = 10$								

In the above example, the value of x for which both L.H.S. and R.H.S. are equal, is the only correct solution for the equation. This method is known as the Trial and Error method.

Let us compare some of the characters of equations with the help of a physical balance.

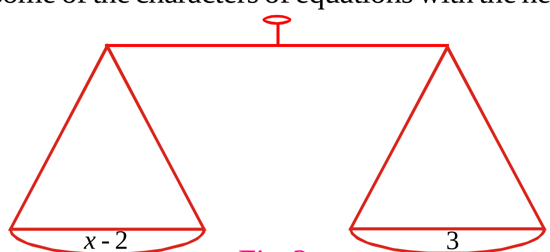


Fig 2

In figure 2, the equation $x - 2 = 3$ is $x - 2$ and the R.H.S. is 3. The balance is in a state of equilibrium. Now if we put some weight on the left pan of the balance, then to bring the balance into a state of equilibrium, we shall have to put the same amount of weight from the R.H.S. too. Similarly, if we take out some weight from the R.H.S., we will have to take out the same amount of weight from L.H.S. also to maintain the state of balance. This means if any operation is carried out on one side of an equation, the same operation has to take place on the other side of the equation too, then only we can maintain equilibrium in the equation and that is what an equation by its name means.

Therefore, in $x - 2 = 3$

knowing that $(-2) + (2) = 0$, if we add 2 to the L.H.S. of the equation, only x will remain. Since 2 is added to the L.H.S., 2 is added to R.H.S. also.

Therefore, $x - 2 = 3$ is equivalent to

$x - 2 + 2 = 3 + 2$

or $x + (-2)(+2) = 5$

or $x = 5$

Similarly, if $7x = 21$.

We know that if 7 is divided by 7, we get 1, so if $7x$ is divided by 7, we shall have x. Since L.H.S. is being divided by 7, the R.H.S. also will have to be divided by 7. This means

$7x = 21$ is the same as

$$\frac{7x}{7} = \frac{21}{7}$$

or $x = 3$

From the above examples, you have learnt that if a constant is added to both sides of an equation or is subtracted from both sides of equation or gets multiplied to or divided by a constant on both sides, these is no change in the state of equilibrium of the equation.

ACTICITY 4

In the table given below, which of the operations - addition, subtraction, multiplication or division will be carried out on both sides of the equations, so that the value of x is obtained. Fill in the table as shown in the example.

S.No	Equation	Which operation would remove the constant from the variable's side	Equation after theoperation	Value of x after the equation is solved
1.	x +3 = 5	3 subtracted	x + 3 - 3 = 5-3	x = 2
2.	x -5 = 7			
3.	2x = 6			
4.	x/3 = 5			
5.	x + 7 = 2			
6.	7 = z- 4			
7.	5 + x= 9			
8.	4 + x = 2			
9.	-7 = 3 + y			
10.	4 = 8y			

Solve the given equations:

(i) $x + 3 = 10$

(ii) 6 = y + 4

(iii) S + 6 = 15

(iv) 7 + t = 25

ACTIVITY 5

You have learnt how to solve simple equations, in which any operation has to be carried out once. Now let us solve some equations where two operations need to be carried out, in order to find a solution.

S.No	Equation	The operation that would remove the constant from the variable's side	Equation after the 1st operation	Operation for both sides that would remove	Equation after the 2nd operation	Value of x
1.	$2x + 3 = 9$	Subtracted 3	$2x + 3 - 3 = 9 - 3$	$2x = 6$; or dividing both side by 2	$\frac{2x}{2}=\frac{6}{2}=3$	$x = 3$
2.	$18x - 11 = 61$					
3.	$\frac{x}{7} - 13 = 1$					
4.	$1 + \frac{x}{5} = 3$					
5.	$\frac{x}{4} - 5 = -6$					
6.	$0 = \frac{x}{14} - \frac{1}{7}$					

Practice

1. Solve the following equations -

i) $3x + 8 = 20$ ii) $4x + 10 = 30$

iii) $5x - 7 = 8$ iv) $6x - 7 = 11$

v) $3x + \frac{21}{7} = 0$ vi) $29 = 7x + 1$

vii) $60 - 8x = -4$ viii) $19x + 7 = 45$

You have learnt solving and making equations by now. Here are some problems about numbers. Try to solve them using equations.

Example 1.

If 5 is added to a number, the number becomes 20. What is the number?

Solution:

Suppose the number is x

According to the given problem:

$x + 5$

$x + 5 = 20$

5 subtracted on both sides

$x + 5 - 5 = 20 - 5$

$x = 15$

Verification:

L.H.S.= $x + 5$

$= 15 + 5$ (putting the value of x) $= 20 =$ R.H.S.

Example 2.

6 subtracted from a number makes it 10, what is the number ?

Solution :

Suppose the number is x

From the above statement, 6 subtracted from the number makes it $x - 6$, that is equal to 10.

This makes the equation : $x - 6 = 10$

6 added to both sides means

$$x - 6 + 6 = 10 + 6$$

here ($-6 + 6 = 0$ and $10 + 6 = 16$)

or $x = 16$

Verification:

L.H.S. $= x - 6$ $= 16 - 6$ (putting the value of x)

$= 10$

$=$ R.H.S.

Example 3.

7 added to twice of a number makes it 37. What will the number be?

Solution:

Suppose the number is x.

Twice the number would be $2x$.

According to the problem, 7 added to twice the number makes 37.

Step 1: Twice the number

$= 2x$

Step 2: 7 added to $2x$

$= 2x + 7$

Step 3: As per the problem

$2x + 7 = 37.$

(7 subtracted from both sides) $2x + 7 - 7 = 37 - 7$

$2x = 30$

(On dividing both side by 2) $\frac{2x}{2} = \frac{30}{2}$

$x = 15$

Verification:

L.H.S. $= 2x + 7$

$= 2 \times 15 + 7$ (on putting the value of x)

$= 30 + 7$

$= 37$

$=$ R.H.S.

Example 4.

One third of a number gives 11. Find the number.

Solution:

Let the number be x.

$\frac{1}{3}$ the number would be $\frac{x}{3}$ which is equal to 11..

$\therefore \frac{x}{3} = 11$

To find out the value of x. 3 has to be removed from the denominator of the L.H.S. of the equation. For this 3 is multiplied to both sides of the eqation $\frac{x}{3} \times 3 = 11 \times 3$

$x = 33.$

Example 5.

The sum of the ages of Malati and her father is 49 years. If Malati is 12 years old. Find out how old is her father ?

Solution:

Suppose the age of Malati's father is x

The sum of both their ages would be

$x + 12.$

Given the sum of age of Malati & her father = 49 years.

Therefore, $x + 12 = 49$

$x + 12 - 12 = 49 - 12$ (On subtracting 12 from both sides of the eq^n.)

or $x + 0 = 37$

or $x = 37$

Thus Malati's father is 37 years old.

Verification:

Sum of Malati's age & her father's age

$12 + 37 = 49$ years.

Example 6.

Shivani has only 50 paise coins in her purse. If she has 25 rupees in her purse, how many coins are there in her purse?

Solution:

Suppose the number of coins in Shivani's purse is x.

The value of each coin = 50 paise.

or Value of each coin $= \frac{1}{2}$ rupee

$\therefore$ Value of all the coins $= \frac{1}{2}\ x$ rupees

According to the condition given on the equation

$$\frac{1}{2}\ x = 25$$

$$\frac{1}{2}\ x \times 2 = 25 \times 2 \text{ (On multiplying both sides by 2)}$$

$$x = 50$$

Therefore, Shivani's purse has 50 coins.

Verification:

The value of 50 coins $= 50 \times 50$

$= 2500$ paise

$= 25$ rupees.

EXERCISE 14.2

1. Solve the following equations.

   i) $x - 3 = -4$

   ii) $z - 8 = 0$

   iii) $3y = 9$

   iv) $16 = 3y + 7$

   v) $5 + \frac{x}{3} = 7$

   vi) $9z - 7 = 14$

2. Solve the given equations and verify your answer.

   i) $3(2 + x) = 12$

   ii) $10 - z = 6$

iii) $\frac{x}{5} = 15$

iv) $7 - 4y = 3$

3. Twice a number makes it 10, what is the number?
4. If 35 is added to twice a number, 85 is obtained. Find the number ?
5. How many 25 paise coins would make 10 rupees?
6. If 4 is subtracted from the half of a number, we get 6. What will that number be?
7. Uma has few meters of cloth. If she makes 4 curtains of 2 metres each, she still has 5 metres of cloth left. Find out how much cloth did she have in the beginning?

What Have We Learnt ?

To solve any problem with the help of equations, we will have to keep the following things in mind:

(i) Read the problem well and identify the known variables and the unknown variables.

(ii) Denote the unknown numbers/ variables by x, y, z etc.

(iii) Change every word of the problem (as far as possible) into a mathematical statement.

(iv) Identify the variables that are equal and make a proper equation.

(v) Solve the equation to find out the unknown variable.

(vi) Varify whether the solution satisfies the conditions and the equation that has been made.

Chapter 15

GEOMETRICAL FIGURES

Using the Scale

When you go to the market to buy clothes, the shopkeeper generally uses an iron rod or scale to measure the cloth. You have also used the scale in your compass box several times to measure lengths. Look at the scale of your compass box carefully and try to find out the answers to the following questions:

There are two kinds of measuring units on a scale. Find out into how many small parts/divisions are the units of both kinds divided? What is the measure of the smallest division?

We use the scale on many occassions. Can you draw three line segments in your notebooks that measure 3.5 cm, 4.2 cm and 8.9 cm respectively.

Draw more line segments of different measures. List the situations in your daily life when you need to use a scale.

Drawing A Circle

You must have used the compass to draw circles. You also know what a circle is and what kind of figures or objects are circular. Let us make a list of objects around us that are circular. You have already made such a list before, so this time your list ought to be longer.

Knowing About the Compass

1) How many arms does a compass have?
2) Are the arms of the same length?
3) What is the pointed arm used for? Should the point be inclined when you use the compass?
4) While drawing a circle, if the pointed arm shifts from its position, will you be able to draw a proper circle?

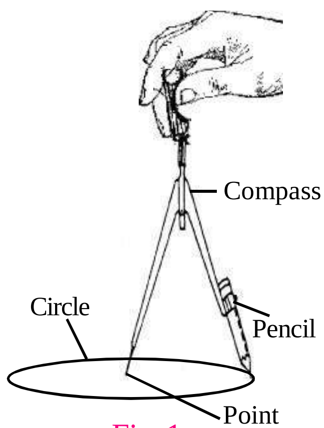


Fig. 1

You now understand how the compass works and how it is used. Draw in your notebook circles of radii 3.2 cm, 4.7 cm and 5.1 cm respectively. Think of more measures of radius and draw a few more circles yourself.

You have also learnt to use the divider in the lesson on line segments . Can you tell the uses of the compass and the divider?

The Protractor

Your geometry box has a protractor too. Look at it carefully and answer the questions given below:

i) What is the shape of the protractor?

ii) Into, how many divisions is the semicircular part of the protractor divided?

iii) Can you draw angles of 47°, 95° and 170° in your notebooks?

The Setsquare

By now, you know about the scale, the compass, the divider and the protractor in your geometry box. Is there any other instrument in your geometry box?

Take out the two triangular instrument that remain in your geometry box and keep them on your notebook. Now trace the outline of these instruments with the help of your pencil that you get the shape of the instrument on the paper.

Measure the angles of the two triangular shapes. You will find that one angle of each of the instruments is of 90°. The remaining two angles are of 45° each in one instrument and in another instrument the angles are of 30° and 60°.

These two instruments are known as **setsquares**.

Now with the help of the setsquares draw an angle of 90° on any line on your notebook and verify the measure with the protractor.

If the angle made by the setsquare is not exactly of 90°, how different is it? Think of the reason for the difference.

You know that the setsquare is useful for making angles of 90°. Let us construct some more figures using the setsquare.

Drawing a perpenicular on a given line segment from a point that is not situated on the line.

Suppose PQ is a line segment and M is a point outside the line segment.

Steps of Construction

1) Put the scale on the paper in such a way that it is aligned to the line segment PQ (Fig 2).
2) Keep one perpendicular side of the setsquare along the scale. Be careful that the scale doesn't slide or move. The other side of the setsquare is now perpendicular to the scale.
3) Hold the scale tightly on the paper and slide the setsquare along the scale in such a way that the perpendicular side of the setsquare touches the point M (Fig 4).
4) Draw a line segment from point M along the perpendicular side of the setsquare.
5) This line segment would be perpendicular or PQ.

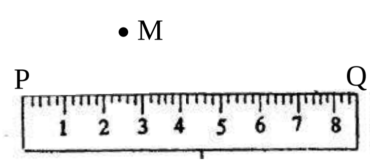


Fig 2

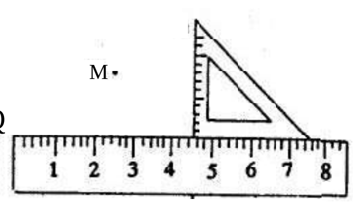


Fig 3

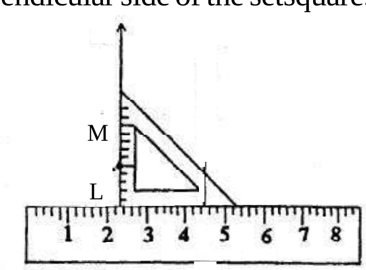


Fig 4

Drawing Parallel Lines with the Help of Setsquare and Scale

You have learnt that the perpendicular distance between two parallel lines is always the same. You have also learnt to draw perpendicular on a line segment with the help of setsquare.

Can you draw a parallel line with respect to the line in your notebook with the help of setsquare and scale? Try to do so, Write how you draw the parallel line.

Draw A Line Parallel to The Given Line Segment From A Point Outside the Line Segment.

Steps of Construction

P is a point outside line AB. We need to draw a line parallel to AB from point P.

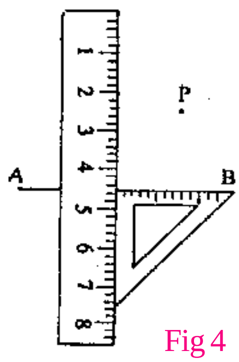


Fig 4

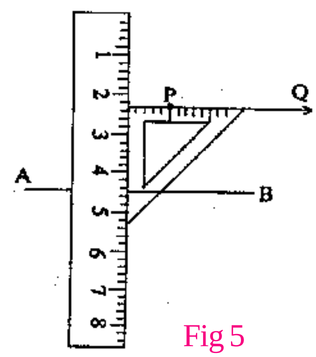


Fig 5

1) Place one perpendicular side of the setsquare along the line AB.
2) Keep the setsquare fixed and place the scale along the other perpendicular arm of the setsquare (fig 4).
3) Hold the scale in a way that it doesn't move.
4) Slide the setsquare along the scale till the perpendicular arm of the setsquare touches the point P.
5) Finally, keeping the setsquare at that point draw a straight line along the side of the setsquare through P. This line PQ will be parallel to AB. You can verify it by measuring the distance between PQ and AB at different points.

The Theory of Construction

Drawing a line parallel to another line means drawing a perpendicular on the given line and then drawing another perpendicular to the drawn perpendicular. Do you agree with this? As it is shown in fig 6, RQ is perpendicular to line AB, and RS is again perpendicular to RQ. So now AB and RS are parallel.

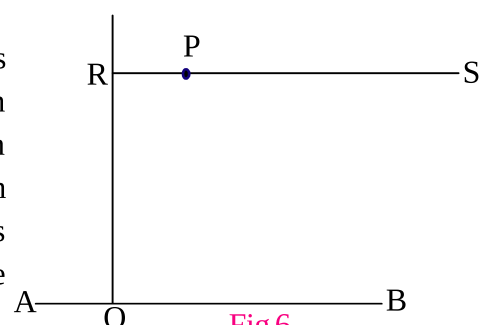


Fig 6

Drawing a Parallel Line at a Particular Distance from the given Line

Suppose we need to draw a line parallel at a distance of 6cms.

1) Draw line AB.
2) Use setsquare and scale to draw a perpendicular on AB (fig 7).
3) Take a point R on PQ in such a way that the distance between Q and R be 6cm.
4) Draw a perpendicular RS on R with the help of setsquare. RS will be parallel to AB and the distance of RS from AB will be 6 cm.

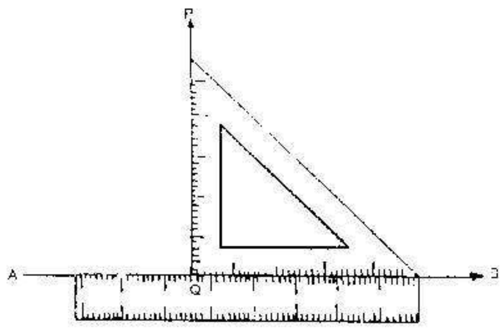


Fig 7

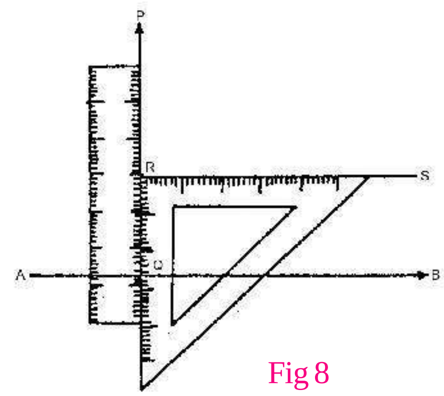


Fig 8

Practice 1

1. Draw a line segment of 3cm and construct parallel lines at the given distances.

   (i) 1.5cm (ii) 2.0cm (iii) 2.2cm (iv) 3.1cm

Using the Scale and Compass to Bisect a Line Segment

Let us do an activity

Activity 1

Draw a line segment AB of any measure. Keeping the point of the compass on A, stretch the other arm upto B and draw a circle taking A as the centre. Now draw another circle of the same measure taking B as the centre. Mark the points and name them.

Now reduce the stretch of the compass, place the point of the compass on A and draw a circle, then draw another circle of same measure by placing point of the compass on B.

Mark the points at which the two circles intersect and name them as R and S.

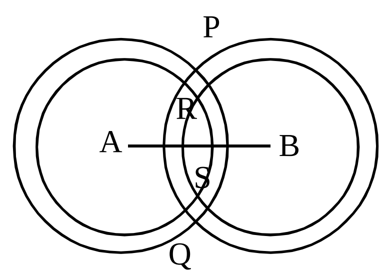


Fig 9

Similarly, go on reducing the stretch of the compass and go on drawing intersecting circles from point A and B. Keep marking the intersecting points. Now can you answer the questions below?

1) You are reducing the radii of the circles gradually. Will two circles of same radius that are being drawn from point A and B always intersect each other? If not, till what measure will the circles drawn at point A and B intersect each other?

2) Are the point P, Q, R, S, T, U etc. that you have got colinear? Can you tell why they are so?

3) In what ratio does line PQ bisect line AB?

4) What angle does line PQ make with line AB?

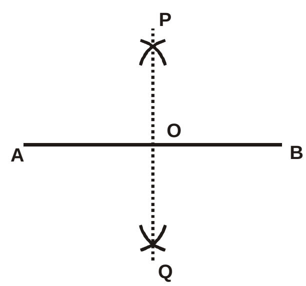
While doing the activity you must have found that the length of the radii get reduced to less than half. Consequently, the radii or circle drawn from point A and B go on reducing and they do not intersect or cut across each other.

Thus, you can say that if you want to draw a bisecting line for a line segment of a given measure, we need to stretch the compass at more than half of the line's distance and draw circles or arcs with the end points of the given line segments as the two centre. Now, if the intersecting pointsbisecting a line of the circles or the arcs are joined, we get a line bisecting the original line which is also perpendicular on it?

ACTIVITY 2

1) Given are two points A and B in the picture. Join them.

A B



2) Keeping the point of the compass at A measure more than half of the length of AB and draw arcs on both sides of the line.

3) Repeat the process with placing the compass on point B and draw arcs on both sides of the line in such a way that the arcs intersect each other. Name them as P and Q.

4) Now join PQ.

5) Name the point at which line segment PQ cuts AB on O. Now measure AO and OB and see if AO = OB?

6) Measure $\angle POB$. Is $\angle POB = 90°$? The line PQ obtained like this would be the bisector of line AB.

Making Angles of Different Measures Using the Compass

ACTIVITY 3

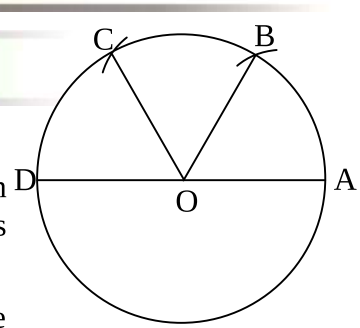


Fig 10

1) Draw a circle of any radius and name its centre.

2) Take a point A on the circle. Keep the compass on point A and draw an arc on the circle with the same measure as the radius.

3) Place the compass on the point where you have drawn the arc and draw another arc with the same radius.

4) Repeat this process of drawing arcs on the circle.

Find the Answer to the Questions Below :

1. In how many arcs the circle can be divided equal to the measure of the radius?
2. Join the intersecting points of all the arcs to the centre of the circle. What is the measure of the angle that is being made by joining two consecutive arcs?
3. Are all the angles of the same measures?
4. If all the angles are of equal measures, what is the measure of one angle?

While finding the answers to the above questions, you have found that on a circle, six arcs equal to the measure of its radius can be cut. The consecutive intersect points on the circumference of the circle make 60° angles with the centre. Each angle made by consecutive intersecting points is of 60°.

Now you make 60° angles with the help of compass and scale. You must have drawn an angle of 60° with the help of compass and scale, let us repeat the process.

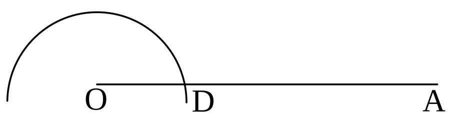


Fig 11

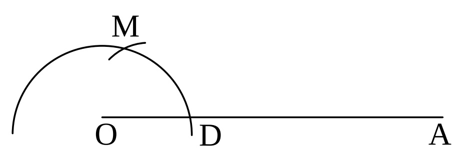


Fig 12

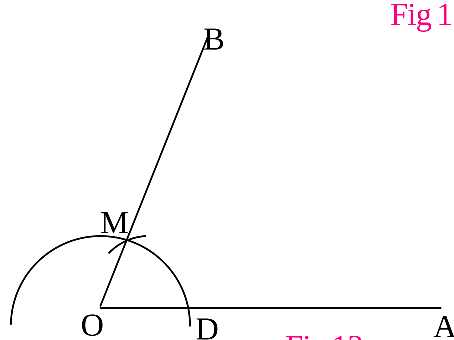
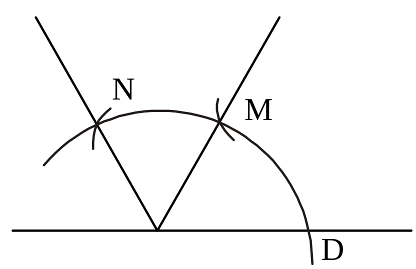


Fig 13

1. Draw a line segment and make an arc from point O, that intersects OA at D (fig 11).
2. Keeping the compass at D, cut another arc of the measure of the radius at M (fig 12).
3. Join OM and extend it (fig 13).
4. $\angle AOB = 60°$



You have already divided the circle into six equal parts with the measure of the radius of the circle.

Every part makes an angle of 60° with the centre. In the above example you're got an angle of 60° by cutting an arc. Taking the same arc if you make another arc ahead from point M, then you'll get an angle 120° and if you cut arcs thrice, you'll get an angle of 180°.

Practice 2

1. Make an angle of 120° with the help of compass and scale.

Bisecting an Angle

Steps of construction:

1. Take point B on $\angle ABC$.

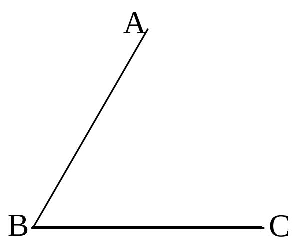


Fig 14

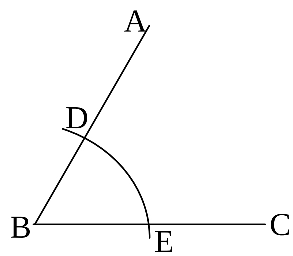


Fig 15

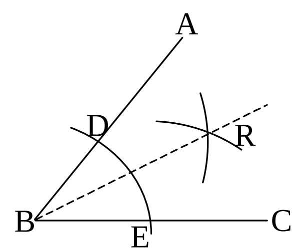


Fig 16

2. Place your compass on point B and draw an arc of any radius to cut AB at D and BC at E.
3. Make another arc taking D as the centre and make another arc E of the same radius, so that both arcs intersect each other at point R (fig 16).
4. Join BR and extend it.
5. The line BR is the bisector of $\angle ABC$.

Practice 3

1. Make an angle of 52° and draw its bisector.
2. Make an angle of 170° and draw its bisector.
3. Bisect an angle of 60°. Now tell what is the measure of each new angles you got?

Constructing an Angle Equivalent to the Measure of A Given Angle

Suppose $\angle AOB$ is given, we need to construct another angle equal to $\angle AOB$.

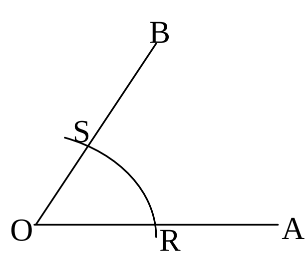


Fig 17

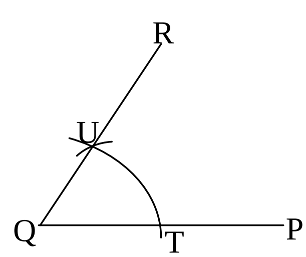


Fig18

1. Draw line QP, we have to make an angle equal to $\angle AOB$ at point Q.
2. Stretch the compass and keep its pointed end at O. Cut an arc in a way that both arms OA and OB get cut at R and S respectively (fig 17).
3. Cut an arc of the same measure with the compass at Q which cuts QP at T.
4. Keeping the compass on R, stretch it to S and use the measure to draw an arc at T that cuts TU (fig 18).
5. Join QU and extend it to R.

Now $\angle PQR = \angle AOB$

Practice 4

1. Make an angle of 55° with the help of the protractor and also make an equal angle with the help of scale and compass.
2. Draw an angle of 120° with the help of the protractor and draw an equivalent angle with the help of a scale and compass.
3. Given a point P outside the line segment AB. Draw a perpendicualr from P on AB.

Steps of Construction

1. Draw a line AB and take a point P outside AB.
2. Taking P as the centre draw an arc of radius that is convinient which cuts AB on D and E (fig 20).
3. Considering D and E as the centre draw two arcs which cut each other on R.

   Join PR (fig 21) and extend it.

   So, $PR \perp AB$
4. PR and AB meet at Q (fig 22).

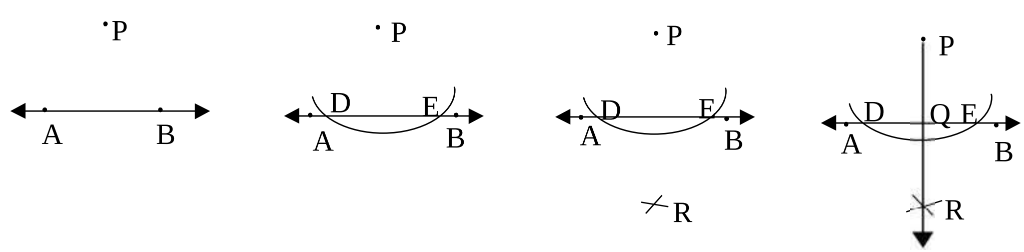


Fig 19 Fig 20 Fig 21 Fig 22

Drawing a perpendicular on AB from point P situated on line segment AB.

Steps of Construction

1. First, draw a line segment AB and make a point P on it.
2. Keep the pointed end of the compass on point P and with any radius draw an arc on line segment AB that cuts AB on the two points Q and R.

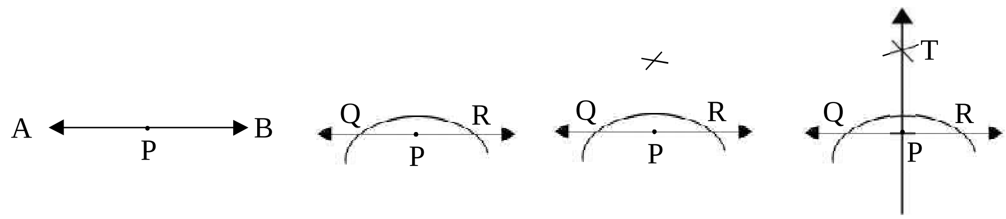


Fig 23 Fig 24 Fig 25 Fig 26

3. Now keep the compass on R and draw an arc with any radius.

4. Repeat the process by keeping the compass at point Q and draw an arc of the same radius, it should be drawn in such a way that it intersects previously at point T.

5. Join point T to P.

The obtained line segment PT, is the perpendicular. Therefore, $PT \perp AB$.

EXERCISE 15

1. Draw a line segment of 5cm and draw a line parallel to it at a distance of 3cm.
2. Construct the following angles with the help of setsquare.

   (i) 45° (ii) 60° (iii) 30° (iv) 90° (v) 120°
3. Draw line segments of the given measures and bisect them.

   (i) 5cm (ii) 4.5cm (iii) 3.6cm (iv) 5.4cm
4. Draw the following angles with the help of compass and scale.

   (i) 60° (ii) 90° (iii) 120° (iv) 150°
5. Bisect the above angles using the compass and scale.
6. Draw the following angles with the help of protractor and draw equal angles with the help of compass and scale.

   (i) 65° (ii) 92° (iii) 108° (iv) 126° (v) 153°

♦ ♦ ♦

Chapter 16

MENSURATION-1
AREA

In lesson 2, you have learnt about close and open figures. In our daily life, you see many figures. Look at the given figures carefully.

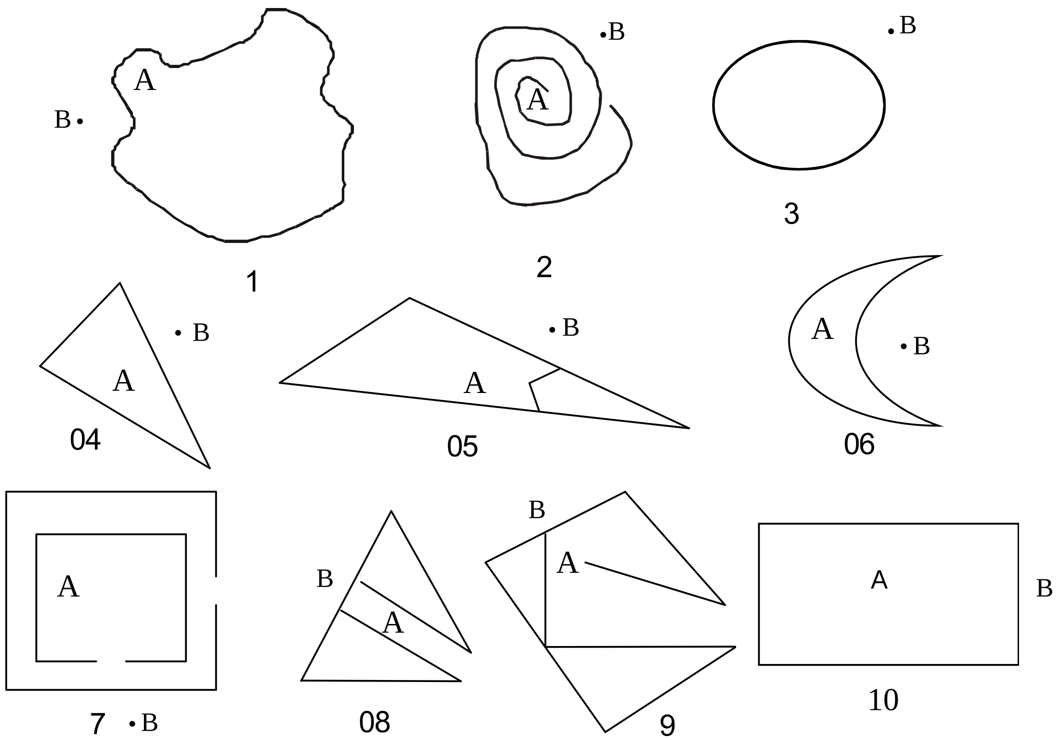


Fig. 1

In the figures given above can you start at point B and move around the figure to reach point A without crossing the outlines? If yes, then the figure in which you can do so, is an open figure and if not, it would be a closed figure.

So, now you must have recognised an open and a close figure. Can you say whether the given examples are open or closed in nature?

1. a kho-kho playground.
2. a football playground.
3. the ground on which we play 'gilli-danda'.
4. the kabaddi playground.
5. the place where we play 'billas'.

Area

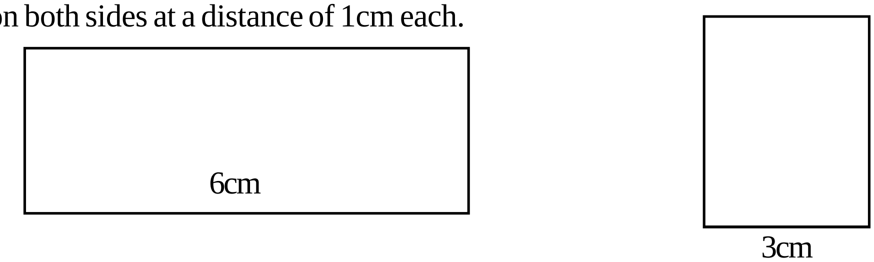
Every close figure has some space within itself. Some points are outside it and we cannot reach the point inside without crossing the outline figure. The space inside a closed figure is its area. Some figures have more space inside. Those which have more space are bigger.

Area of a Rectangle

You have learnt about rectangles in Class V. It is a quadrilateral whose opposite sides are equal and every angle is a right angle.

ACTIVITY 2

1. A rectangle has a length of 6cm and width of 3cm. Draw vertical and horizontal lines on both sides at a distance of 1cm each.



horizontal state of the rectangle   vertical state of the rectangle

Fig 4

2. Divide the rectangle into 1cm × 1cm parts, like this

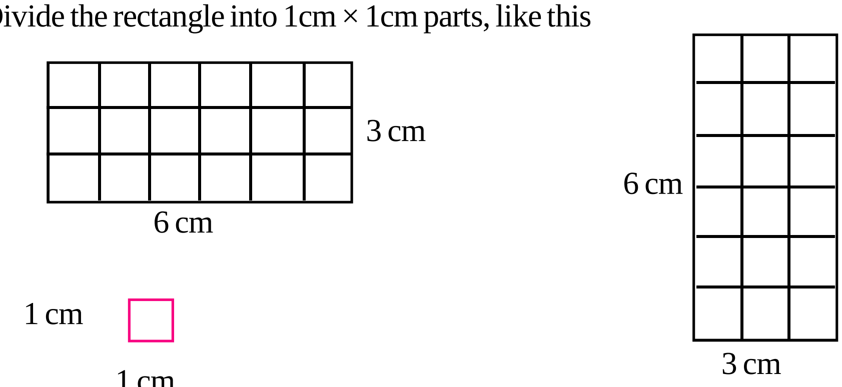


Fig 5

In the figure above each small square is 1cm × 1cm. Count there 1cm × 1cm small squares.

No. of squares	= 18
Area of 1 square	= 1 square cm.
18 square area	= 18 square cm.

Conclusion:

The larger the rectangle, the more will be the number of 1square cm squares.

Area = 18 square cm

= 6cm × 3cm

or = 3cm × 6cm

Area of a Rectangle = Length × breadth

Since the operation of multiplication follows the commutative law. Therefore we could also write Area of the Rectangle = Breadth × Length.

Practice

1. The place that your mathematics book covers on a surface.
2. The spaces covered by the blackboard.
3. You must be thinking that these covers larger spaces. How shall we measure them?

Area of a Square

A square is a special kind of rectangle whose sides are equal, that is the length and breadth of a square are equal.

If we divide a square of 4cm × 4cm into unit squares of 1cm × 1cm.

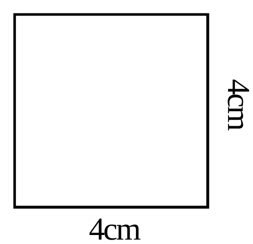


Fig 6

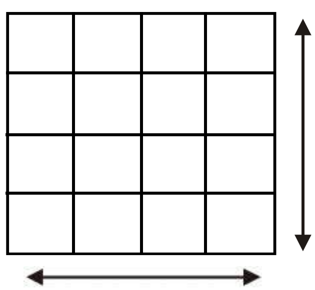


Fig 7

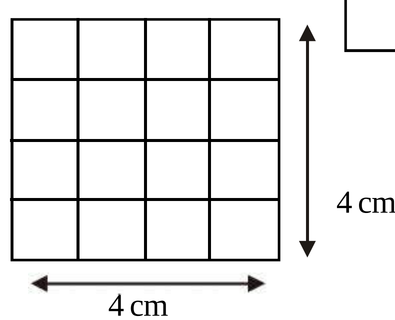


Fig 8

1 square cm = 1cm × 1cm

Area of the square = the number of unit squares

= 16

Area of one unit square = 1 square cm

Area of 16 unit square = 16 square cm

Area of a square = 16 square cm

The area represented = 4cm × 4cm

Thus, **Area of a square = Side × side**

or **Area of a square = $(side)^2$**

Example 1.

If the side of a square is 5cm. What would be its area?

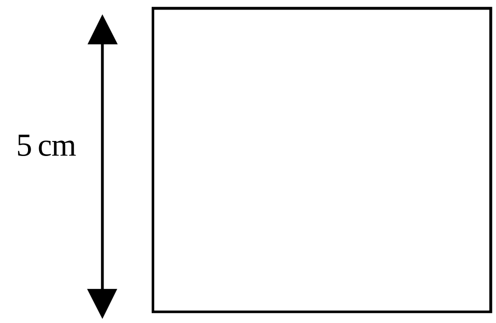


Fig 9

In the figure, a square of 5cm has been shown. On each arm mark point at gaps of 1cm.

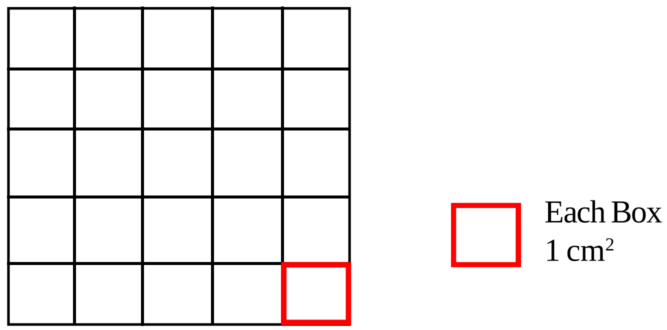


Fig 10

Now join the two marks with the help of horizontal and vertical lines. This will divide the bigger square into smaller squares. Now count the 1cm long and 1cm wide boxes inside the square.

Area of the square = No. of 1cm long and 1cm wide boxes inside the square.

= 25

= 25 × Area of 1 box

= 25 × 1square cm

= 25 square cm

∴ The area of the square = side × side

= square of the side

Example 2.

A rectangle is 7cm long and 3cm wide. Find its area.

Solution:

Here length of the rectangle	=	7cm
breadth of the rectangle	=	3cm
Area of the rectangle	=	length × breadth
	=	7cm × 3cm
	=	$21cm^2$
	or	21 square cm.

Example 3.

Find the area of a square whose side is 8cm. long.

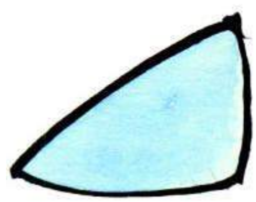
Solution:

Area of a square	=	side × side
	=	8cm × 8cm
	=	$64\ cm^2$
or		64 square cm.

EXERCISE 16

1. Recognise the closed figures.

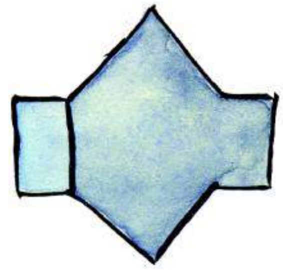
(i) 

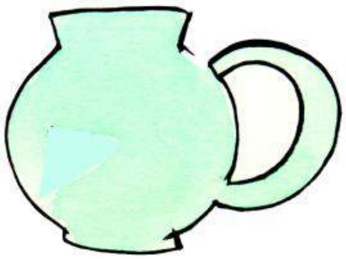
(ii) 

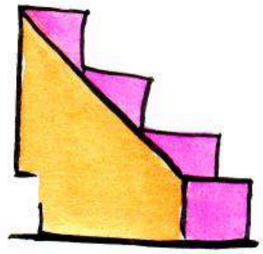
(iii) 

(iv) 

(v) 

(vi) 

(vii) 

(viii) 

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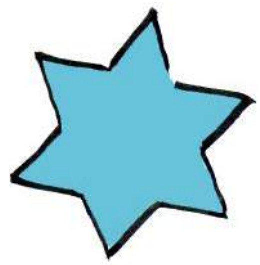
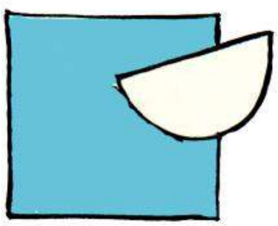
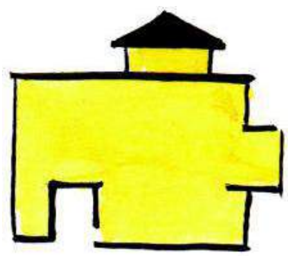
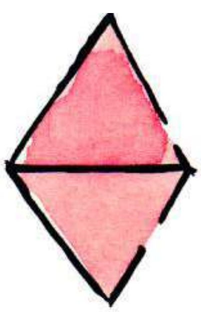
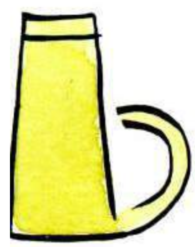
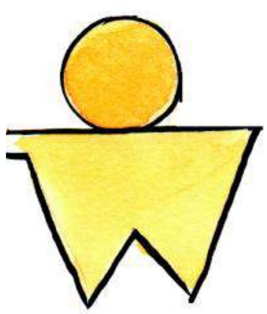
(x) 

(xi) 

(xii) 

(xiii) 

(xiv) 

(xv) 

2. Find out the area of the rectangles whose length and breadth are as follows-

   (i) length = 6 cm; width = 2 cm

   (ii) length = 10 cm; width = 1cm

   (iii) length = 12 cm; width = 6 cm

   (iv) length = 13.5 cm; width = 10 cm

3. Find the areas of the squares whose sides are :

   (i) 6 cm (ii) 12 cm (iii) 13 cm (iv) 3.5 cm

4. Draw lines at 1 cm distance horizontally and vertically inside the squares and find out the area of the square. Also veryify your answer with the formula.

   (i) length = 5 cm; width = 4 cm

   (ii) length = 12 cm; width = 2 cm

5. The side of a square is 6cm. Draw lines at 1cm distance horizontally and vertically inside the square to find the area and verify your answer with Q3. (i).

What Have We Learnt ?

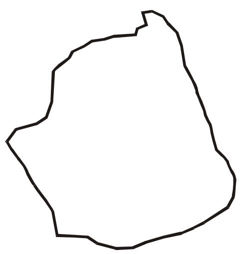
1. The area of an object is the space covered by it on a plane surface.
2. The area of a rectangle = length × width
3. The area of a square = side × side = $(\text{side})^2$
4. The unit of area is square unit.

Chapter 17

MENSURATION - 2

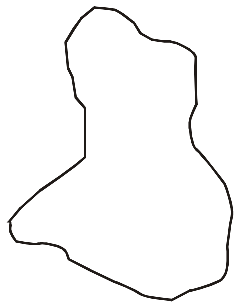
PERIMETER

Look at the given figures. They have been made by thread.



A

Fig 1



A

Fig 2

A

Fig 3

In the above figures if we start moving from a point A and complete a full circle and return to A again, it is definite that the distance covered would be equal to the length of the thread required to make this figure. This is therefore, the length of the circumference of the figure.

Make some more such figures with the help of thread and find the length of the circumference of the figures you have made. This is known as the perimeter of the area.

You must have noticed that only one round of the circumference of an area is the perimeter, therefore, when we are trying to make a boundary of wire or bricks for a particular area, we need to measure its perimeter.

Perimeter

Many objects are used in our daily life which are circular, triangular and rectangular in shape. You have already seen objects of these shapes. The page of your notebook, chess board, carrom board etc. are rectangular in shape.

The notebook, chess board, book, posters, blackboard are all rectangular. Some of these are square in shape too. Find out some more objects around you and calegolise the rectangular and square shaped objects.

List them out in the space given below:

Only rectangular object	Objects that are in square shaped also
1. Page of your copy.	1.
2.	2.
3.	3.
4.	4.
5.	5.

Now look at edges of the following figures and say how many edges do they have?

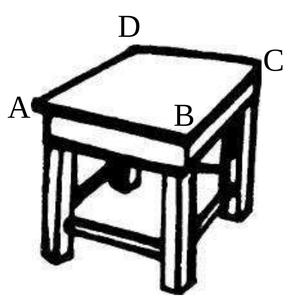


Fig. 4

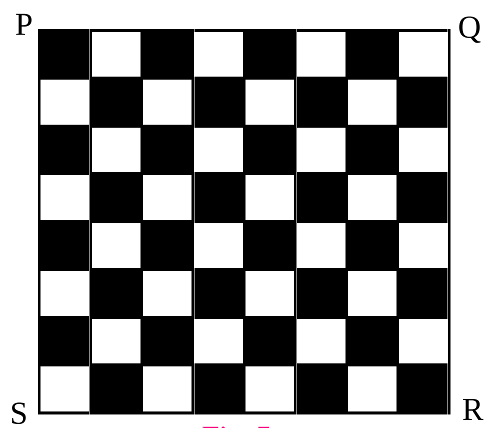


Fig. 5

Here, you can see that the table has 4 sides. Similarly, the chessboard also has 4 sides.

Let us find the perimeter of some rectangular objects.

ACTIVITY 1

Measure the edges of the upper surface of the table (fig. 4) and write them down:

AB =…………….. cm BC =……………………cm

CD = …..……….. cm DA =……………………cm

Now add all the lengths and write them down

The sum of all the 4 edges =

= AB + BC + C D + DA = ... + ... + ... + ...+

= .... cm

ACTIVITY 2

Similarly, measure the outer edges of the chessboard and add them

PQ + QR + RS + SP = ...+................+................+................ = ... cm

ACTIVITY 3

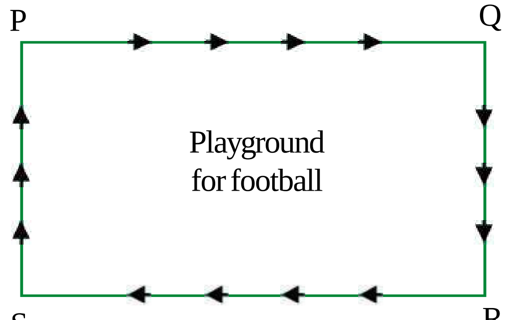


Fig. 6

Figure 6 is a football playground PQRS. A student of class VI, Golu, wakes up at 5 O'clock and runs around the playground once everyday. Can you say, how much distance does Raju walks everyday?

Total distance covered by Golu in one round = length of PQ + length of QR + length of RS + length of SP.

In all these activities 1, 2, and 3, you have seen that the sum of the lengths of all the edges of the figure is the perimeter of the figure.

Find out perimeter of the given figures and fill in the blanks :

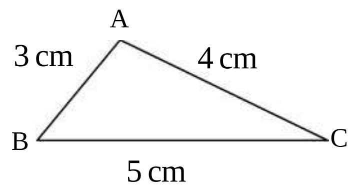


Fig. 7

Perimeter = ...

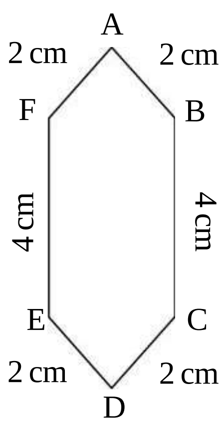


Fig. 8

AB + BC + CD + DE + EF + FA

= __ + __ + __ + __ + __ + __

Perimeter = ...

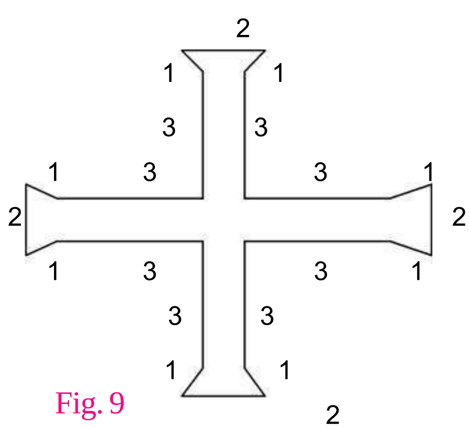
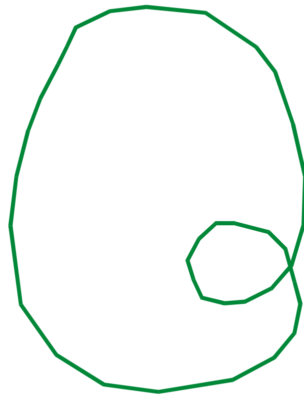


Fig. 9

Perimeter = ...

Keep a thread on the picture and measure its length. That would be the perimeter of the figure.



Perimeter =......

Fig. 10

Perimeter =......

Fig. 11

Perimeter =......

Fig. 12

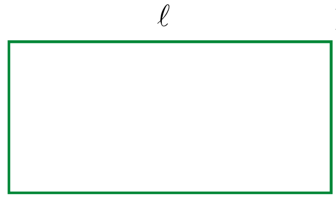
Practice 1

1. The length of a rectangular garden is 6cm and its width is 3cm. You need to make a boundary of wire around it, find the length of the wire you'll require.
2. In a rectangular playground that is 100m long and 50m wide, how much distance will be covered while taking 2 rounds of the playground.

How will you find perimeter? You must have understood that

Perimeter of the rectangle = the length of its four sides

Now if the length of the rectangle is 'ℓ' and 'b' is its breadth then,



Perimeter of rectangle = the sum of the 4 sides of the rectangle

= length of AB + length of BC + length of CD + length of DA

= ℓ unit + b unit + ℓ unit + b unit

$= \ell$ unit $+ \ell$ unit + b unit + b unit

$= (\ell + \ell)$ units + (b + b) units

$= 2\ \ell$ units + 2 b units

$= 2\ (\ell + b)$ units

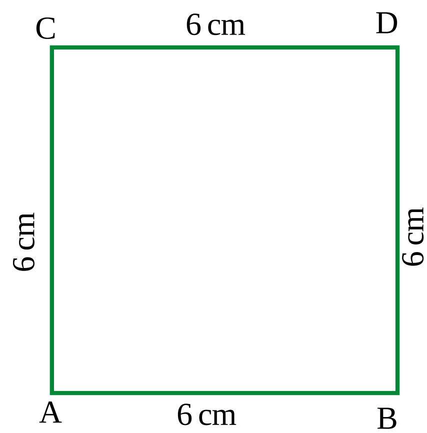
Therefore, the perimeter of a ℓ unit long and b unit wide rectangle

$= 2\ (\ell + b)$ units

The perimeter of the rectangle = 2 (length + width)

Perimeter of Square

The length of the side of a square is 6cm. What is its perimeter?



Perimeter of a square = Total length of the 4 sides of square

= 6 cm + 6 cm + 6 cm + 6 cm

= 4 × 6 cm (one side of 6 cm)

∴ Perimeter of square = 4 × length of one side

Perimeter of square = 4 × side

Unit of Perimeter

Perimeter is the total length of the circumference of the closed figure. So, what should be its unit? Since perimeter is actually the length and its unit would be the unit of length.

Fill in the blanks in the given table:

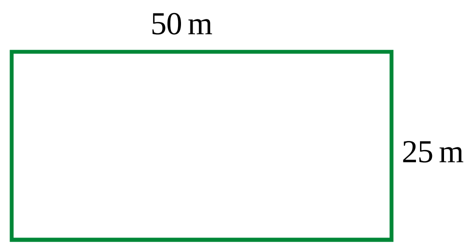
S. no.	Length of the rectangle (ℓ)	Breadth of the rectangle (*b*)	The sum of the sides of the rectangle	The perimeter of the rectangl	Perimeter of rectangl with the help of formula
1.	10 cm	5 cm	10cm + 5cm + 10cm + 5cm = 30cm	30cm	2(10+5)cm = 2×15cm = 30cm
2.	5 cm	5 cm	5cm + 5cm + 5cm + 5cm = 20cm	20cm	4×5cm = 20cm
3.	6 cm	4 cm			
4.	7 cm	7 cm			

Let us see some more examples.

Example 1.

The length and width of a rectangular field are 50 meter and 25 meter respectively. An athelete runs around the field 10 times. Find out how much distance does he run?

Solution:



Here, the length of the rectangle (ℓ) $= 50m$

breadth of the rectangle (b) $= 25m$

The perimeter of the rectangle $= 2\ (\ell + b)$

$= 2\ (50m + 25m)$

$= 150m$

Now the athlete cover 150m in one round.

$\therefore$ In 10 rounds the athelete runs $= 10 \times 150$ m $= 1500$m distance covered.

Example 2.

If a square has a perimeter of 200m, find its area.

Solution:

Here, perimeter $= 200m$

$4 \times$ length of one side $= 200m$

length of a side of the square $= \frac{200}{4}$ meters

$= 50$ meters

Now, the area of the square $=$ side $\times$ side

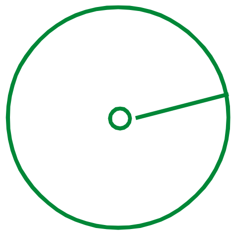
$= 50m \times 50m$

$= 2500m^2$

or 2500 square meter.

Finding the Perimeter of a Circle

You have already done the activity of finding out the circumference of a circle. You have also seen that the length of the circumference of the circle and the diameter of a circle are in a ratio that is equal to π, where π is a constant. The relationship can be written as follows-



$$\frac{\text{The length of the circumference of the circle}}{\text{diameter of the circle}} = \pi$$

The boundary around the circle is the perimeter, which is also known as its circumference.

If the radius of the circle = r

then the diameter of the circle = $2r$

$$\therefore \frac{\text{The circumference of the circle}}{2r} = \pi,$$

or the circumference of the circle = $2\pi r$ (where $\pi = \frac{22}{7}$)

So, **Circumference of the circle (C) = 2®*r*.**

Example 3.

If the radius of a circle is 7cm, find the circumference of the circle.

$C = 2\pi r$

$$C = \frac{2 \times 22 \times 7}{7} = 44\text{cm.}$$

Example 4.

If one round of the circle is of distance 1km. What would be the radius of the circle?

$C = 2\pi r$

$$1000 = \frac{2 \times 22 \times r}{7}$$

$\therefore$ $7 \times 1000 = 2 \times 22 \times r$

$$\frac{7 \times 1000}{2 \times 22} = r$$

$$r = \frac{1750}{11} \text{ m.}$$

$r = 150.9$ m.

Practice 2

(1) The following are the radius of different circle. Find out the perimeter.

1. 3.5cm 2. 10.5cm 3. 17.5cm

(2) The following are perimeters of different circles. Find out the radius -

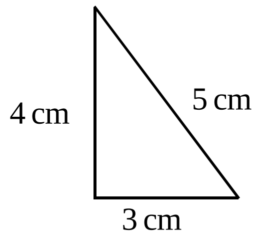
1. 500 metre 2. 100 metre 3. 22 cm. 4. 11 cm.

(3) The radius of a wheel is 1/2m. How many rounds do you need to take to cover 11km. of distance?

EXERCISE 17

(1) Select the close figures and find their perimeter.

(i)



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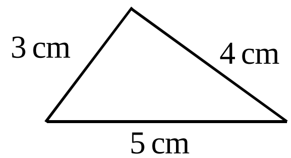

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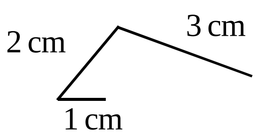

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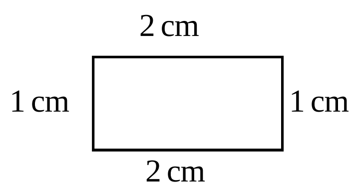

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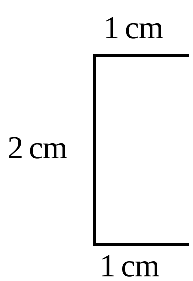

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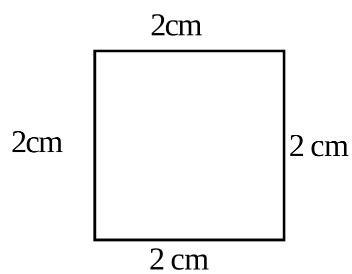

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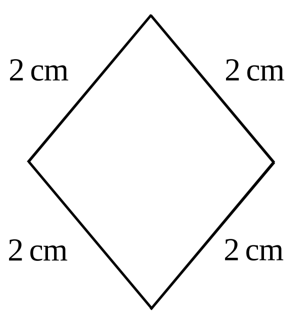

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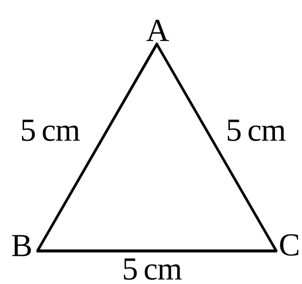


(2) Find out the perimeter of the given rectangles. The length and breadth are as follows:

(i) length = 15 cm	breadth = 6 cm
(ii) length = 12 cm	breadth = 6 cm
(iii) length = 3.5 cm	breadth = 2.5 cm
(iv) length = 100 cm	breadth = 50 cm

(3) Write in the brackets, whether the statements are true or false. Correct those that are false:

(i) Every square is a rectangle. ( )

(ii) Every rectangle is a square. ( )

(iii) If all the sides of a rectangle are measured in cm, the perimeter would be in meter. ( )

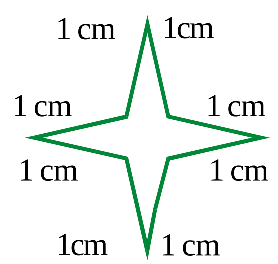
(iv) The perimeter of the rectangle is the sum of its 4 sides. ( )

(4) If the side of a square is 15cm, find its perimeter.

(5) The length of a rectangle is 20 cm and breadth is 0.5 m, find out the perimeter in cm and meter.

(6) In the given figure, the measures are in cms. Find its perimeter.

(i)



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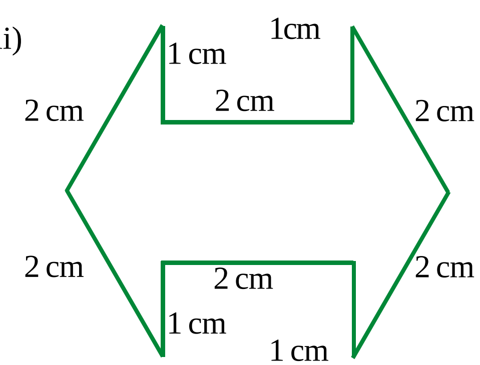

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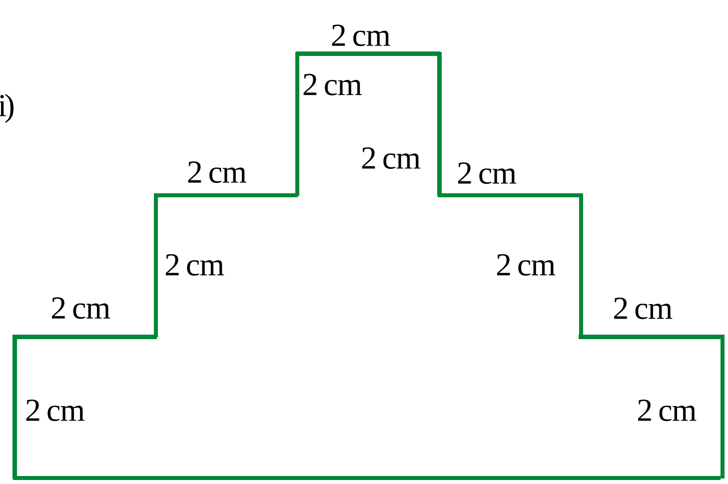

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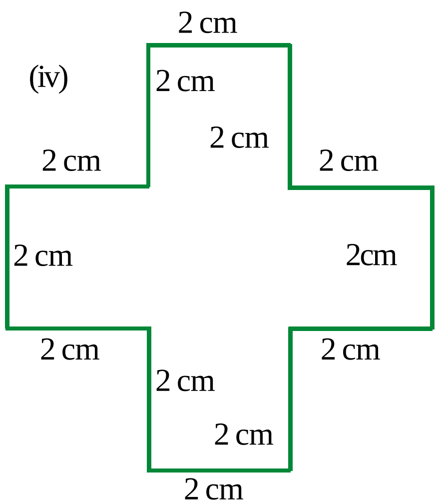

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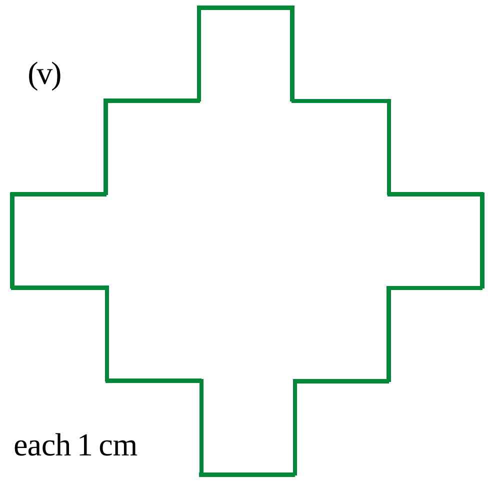
(v)



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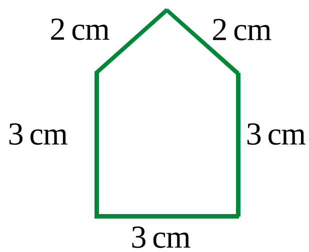
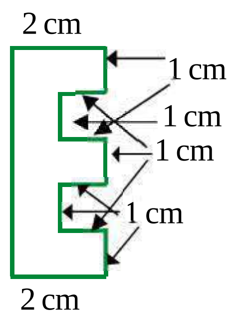


(vii)



(7) The length of a rectangular ground is 25m and the width is 10meter. An athlete completes 4 rounds, how much distance does he cover?

What Have We Learnt ?

1) It is possible to find out perimeter and area only in closed figures.
2) Close figures are those that end at the starting point without passing through any point 2 times.
3) Area of the rectangle = the area of the inner space of a rectangle.
4) Area of the rectangle = length × width.
5) Every square is a rectangle, but not all rectangles are squares.
6) Area of a square = $(\text{side})^2$.
7) Perimeter of a rectangle = 2 (length + breadth).
8) Perimeter of the square = 4 × side.
9) 1 meter^2 = 1m × 1m = 100cm × 100cm = 10,000 square cm = 10,000 cm^2.

Project work

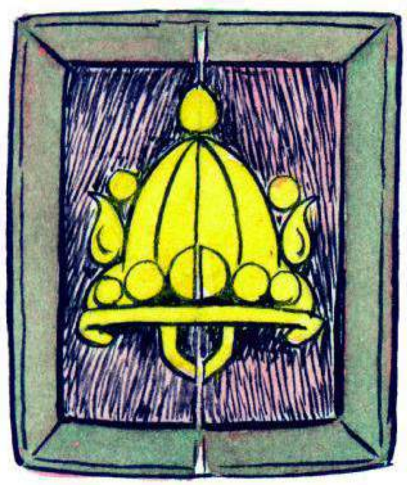
Measure the length and breadth of your class room and calculate its perimeter.

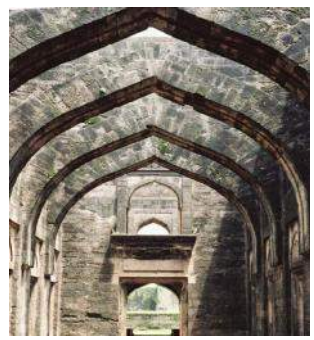
Chapter 18

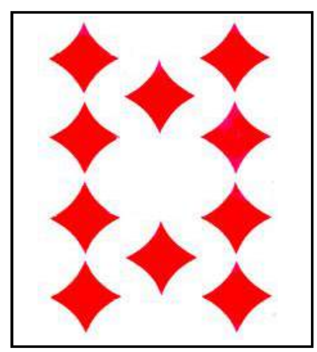
SYMMETRY

Many types of shapes exist around us. We look at the flowers, beautiful paintings, buildings and other things. In all these, we see symmetry and some kind of harmony.

Many of these shapes are in balanced proportions. Some of these look the same at different positions. Some of these look as if they are made up of two similar figures.







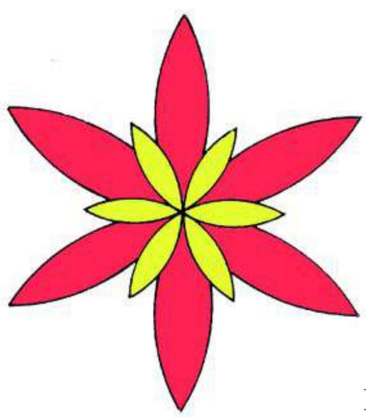


Fig 1

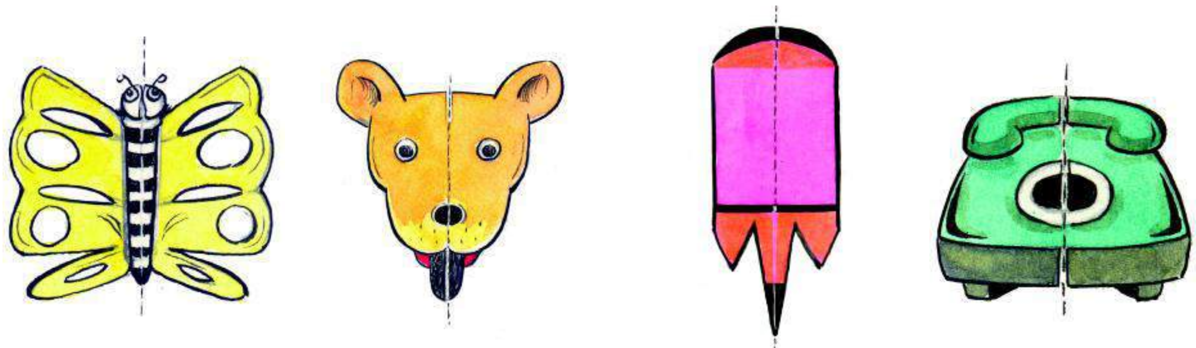
All these are symmetric shapes. When we see such shapes around us everyday then we say these are symmetric shapes.

ACTIVITY 1

Axis of symmetry

Observe the given figures. If we can fold any one of these figures in such a way that it's left half portion coincides with the other right half portion OR the upper half part coincides completely with the lower half then we say that these figures have a axis of symmetry. In such a case, both the halves are mirror images of each other.

See figures A. If it is folded along the broken line, then one part will completely hide the other part. Observe similar lines in the remaining figures.



(a) (b) (c) (d)

Fig 2

If we place a plane mirror at the line of fold then in symmetrical figures, the mirror image of one part of the figure will completely cover the other part. In these figures make a fold (using a real or imaginary line) and after placing a plane mirror on the broken line, observe the figures.

Is the image seen in the mirror same as the remaining part? This mirror line, is called symmetrical line or the axis of symmetry of the figure. According to Rohan all the figures drawn above are symmetric. Do you agree with him? Why?

Try to draw 5 symmetrical figures and draw their axis of symmetry.

ACTIVITY 2

Recognize the symmetrical figures:-

Which one of the figures given below are symmetric?

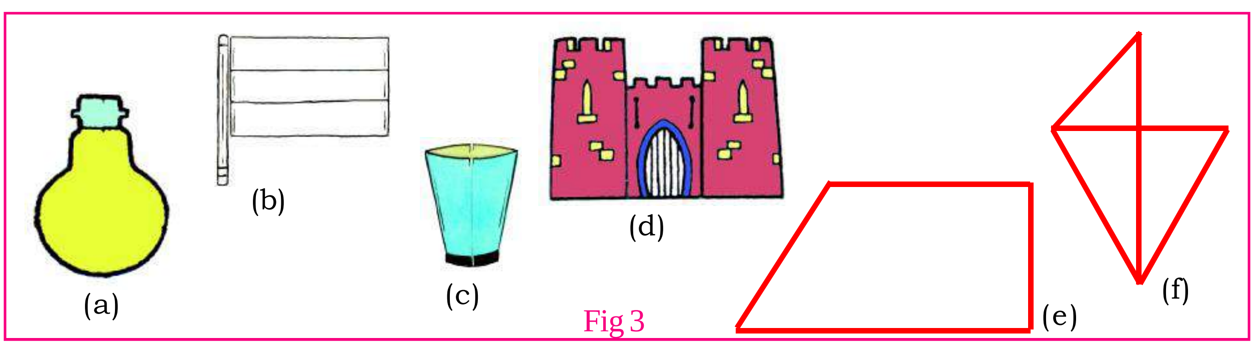


Fig 3

How did you recognize the symmetrical figures?

Now, draw their axis of symmetry. Can you change non-symmetrical figures into symmetric figures by adding something? Choose one figure and think this over.

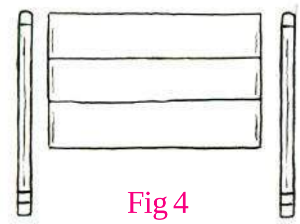


Fig 4

In symmetric figures, one half of the figure completely covers the other half on the axis of symmetry.

Figure (B) is not symmetric but if we add one more pole to it then the new structure will be symmetrical. In this figure, where is the axis of symmetry. Do the same with the remaining non-symmetric figures to make them symmetric.

ACTIVITY 3

Which of these letters are symmetrical?

Cut out the shapes of letters A,B,..., Y,Z from a thick paper. Take two boxes and paste the slip marked "Symmetrical" on one box and paste the other slip marked "Non-Symmetrical" on the second box.

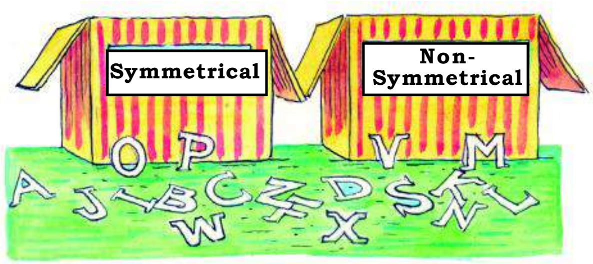


Fig 5

Now observe A,B,C,D....... one by one and check whether half part of the letter coincides completely with the remaining half part on the axis of symmetry
In which box, will you put the letter whose parts coincide with each other?
Which letters did you put in the symmetrical box? Which box has more letters?
Do the same practice for the letters क ख ग ....ह
Which of these letters are symmetric?

ACTIVITY 4

Another type of Symmetry

Take a paper and fold it into two equal parts. Put some ink or colour drops on one half. Fold the second part over the first and press it. What do you see?

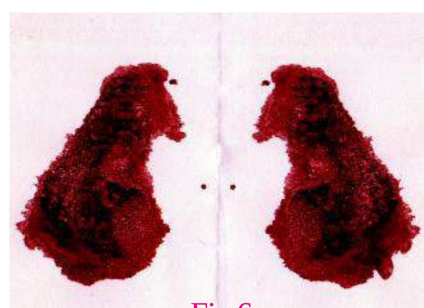


Fig 6

Is the obtained figure symmetric? If yes, then where is the line of it's symmetry? Is there any other line along which if the paper is folded, two similar parts would be obtained? Try to make some more symmetrical patterns of this type.

Observe the different objects available in your classroom. List the objects that have symmetrical shapes example the black board, top surface of a table, your notebook etc. Is the shape of the wing of the fan also symmetrical? After discussion, show your list to your teacher. After drawing all the symmetrical figures also draw a line of symmetry for each.

ACTIVITY 5

Now, look at the following figures:-

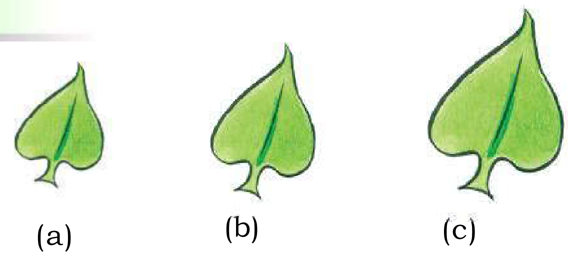


Fig 7

Are they symmetric?

You have seen the figures drawn on the walls of your home or on the walls of other houses located in your village. Draw similar figures in your notebook. Are these figures symmetric? Draw the axis of symmetry for each.

ACTIVITY 6

Recognize the symmetry:

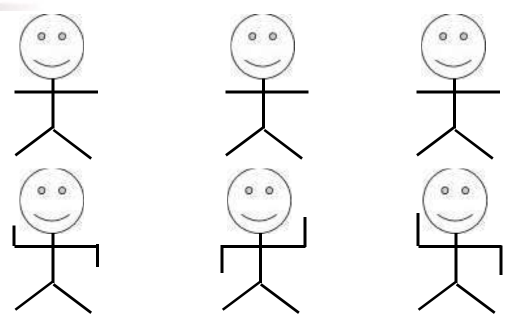


Fig 8

Which of the above figures are symmetrical? Convert the non-symmetrical figures into symmetrical ones.

Have you ever made a 'Rangoli'? It involves kind of figures shown below.

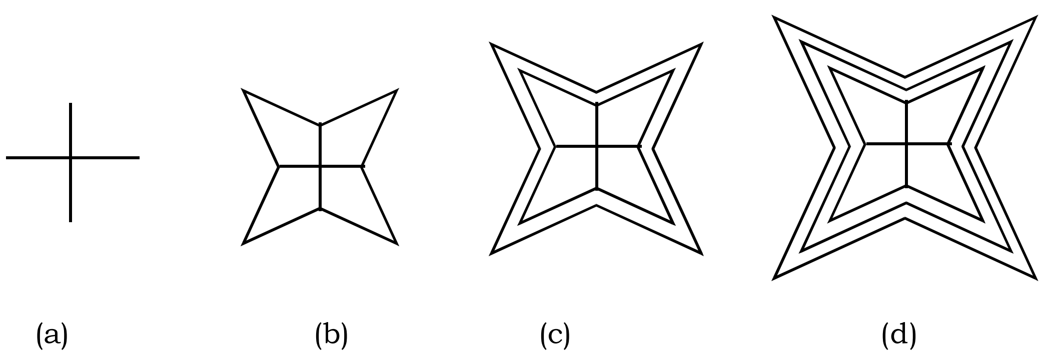


Fig 9

In different parts of these figures, different-different colours can be filled.

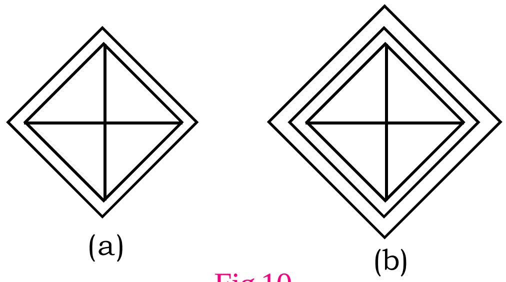


Fig 10

These figures look beautiful after being coloured. Do they have a line of symmetry? Observe every shape. Does any of the shapes have more than one line of symmetry?

ACTIVITY 7

There are two set squares in your geometry box, one of them has angles 90°, 60° and 30°. Take two such set squares.

Join these two to make the shape of a kite as shown in the figure. How many lines of symmetry does this figure have?

In the same way take two set-squares of the other kind (involving 90°, 45° & 45°) and keep them along side each other as earlier.

What shape do you obtain?

How many lines of symmetry in this?

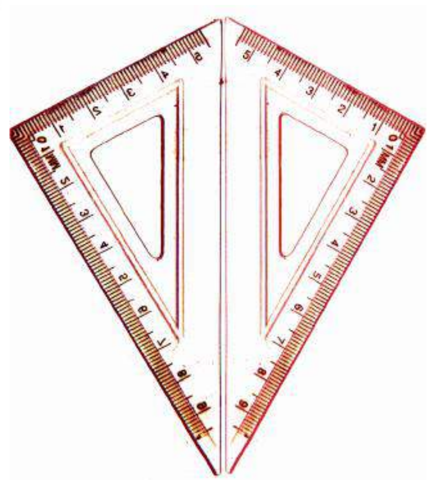


Fig 11

Think of more shapes having more than one line of symmetry.

ACTIVITY 8

Rectangle and Symmetry

Take a postcard. Fold it along its length (Figure 12a) so that one part completely covers the other part. Is this fold line, a line of symmetry?

Give reasons for your answer.

Open this postcard and fold it again in the same way along its breadth (Figure 12b)

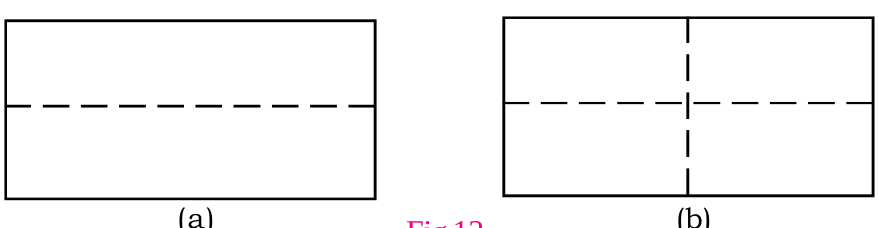


Fig 12

Is the line of the second fold also, a line of symmetry?

Do you think there are only two lines of symmetry in this figure?

Now again think of the square formed above by using set-squares.

How many lines of symmetry does it have?

ACTIVITY 9

Mirror and symmetry:

A picture of an umbrella (Figure a) is shown below. In figure b, one half of the umbrella is shown placed in front of a plane mirror. Carefully observe the front half of the umbrella and its reflection in the mirror. Does, the figure of the umbrella look complete?

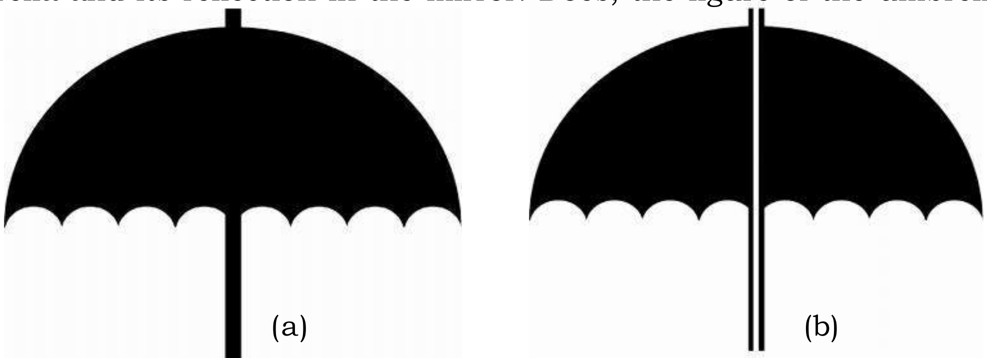


Fig 13

ACTIVITY 10

One half of a face is shown in the figure. Would you see the full face if you place a mirror along the line AB?

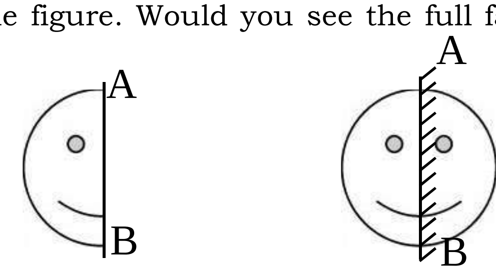


Fig 14

AB is the line of symmetry of the complete figure.

ACTIVITY 11

In which figures are both parts reflected when a plane mirror is placed on its line of symmetry.

Look at these figures and determine such position for the plane mirror from where the image and the object look the same.

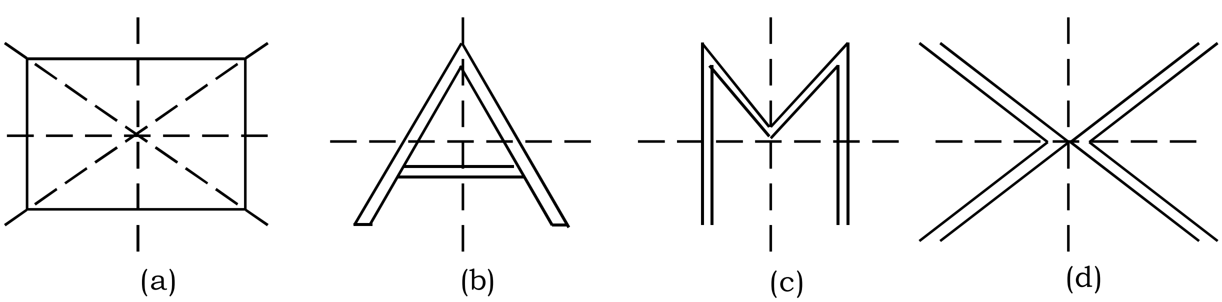
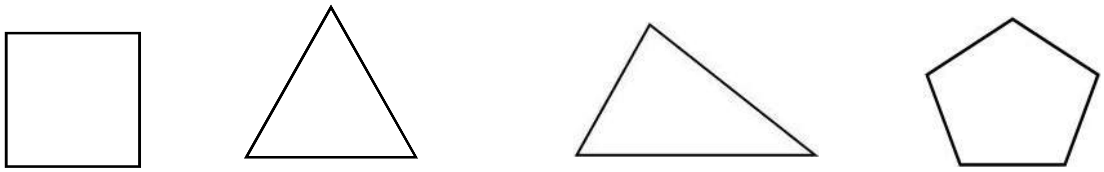


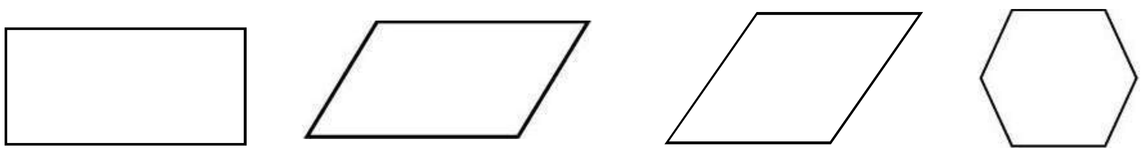
Fig 15

EXERCISE 18.1

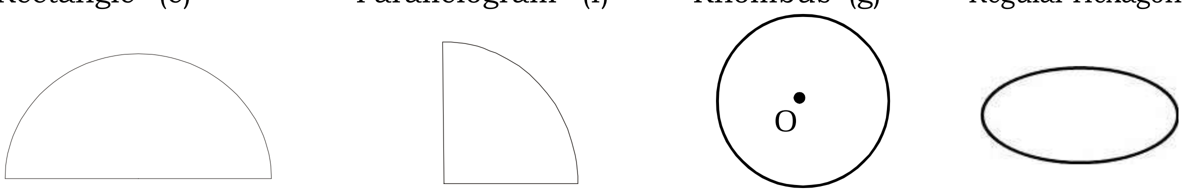
Identify, which of the following figures are symmetric? Find the lines of symmetry for these. For the symmetric figures draw the lines of symmetry and for each figure write the number of the lines of symmetry.



Square(a) Isoceless Triangle (b) Scalen Triangle (c) Regular Pentagon (d)



Rectangle (e) Parallelogram (f) Rhombus (g) Regular Hexagon(h)



Semi-circle (i) Quarter circle (j) Circle (k) Ellipse (l)

Fig 16

How many lines of symmetry are there in a circle?

A circle is symmetrical about each of its diameters. This means, on cutting the circle along any diameter, two equal parts are obtained.

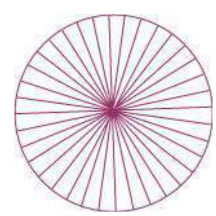


Fig 17

ACTIVITY 12

Take a square shaped paper. Fold it first from top to bottom and then from left to right. Now make a design on it, according to the given figure. Cut along the border of the figure made, open the paper out.

How many lines of symmetry are there?

Fig 18

ACTIVITY 13

Many lines of symmetry.

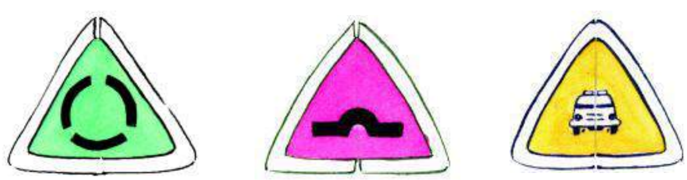
Take 3 boxes. Now paste a paper strip on each and write '1 line of symmetry on first box '2 lines of symmetry on the second box, and '3 or more lines of symmetry' on the third box.

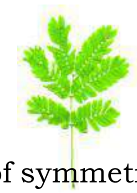
Observe the cut out shapes of letters A, B, C, ... Y, Z and determine the number of lines of symmetry for each. Put the letters having one line of symmetry in 1st box, 2 lines of symmetry in 2nd box and those with 3 or more lines of symmetry in the 3rd box. Discuss what you do with your classmates.

Fig 19

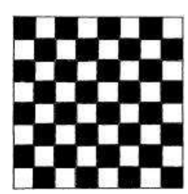
Can you say which English letter has the maximum lines of symmetry? Classify more figures and shapes on the basis of the number of lines of symmetry.

1. We observe different road signs and signals while traveling in a bus. Identify those shapes that have lines of symmetry and draw them in your notebook.
2. Look at plants/leaves/petiole of leaves, do they have lines of symmetry?
3. Are there any lines of symmetry in the given figure?


(i) (ii) (iii)



4. Are there lines of symmetry in the playing Cards? Classify them into groups of 'no line of symmetry', '1 line of symmetry', 2 lines of symmetry, 3 lines of symmetry and more lines of symmetry.
5. There are lines of symmetry in playgrounds and in play boards. List such play grounds and play boards and discuss them with your teacher.



6. There is symmetry in every kind of a vehicle. Like Buses, trucks etc.

Take a rectangular coloured paper. Fold it many times and cut it as shown in the figure below. Now unfold the paper.

Place the shape on your notebook and fill different colours in it. Do you observe any symmetry?

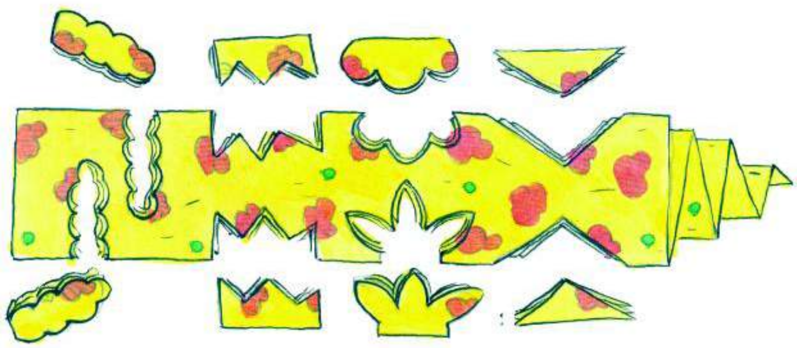


Fig 20

Rangoli

Have you ever made 'Rangoli' during the festivals? Have you noticed the symmetry in making these figures? Copy these rangoli patterns on paper and make an album of different patterns.

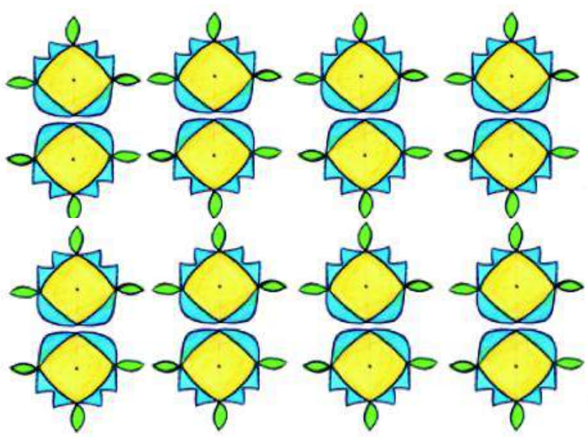


Fig 21

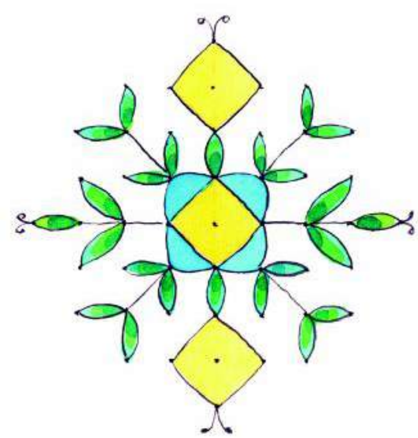


Fig 22

Mehandi

You would have seen ladies applying 'mehandi' on their palms. Is there any symmetry in mehandi designs? Discuss it with the girls of your class.

EXERCISE 18.2

Q1. Classify the following figures as symmetric and non symmetric:-

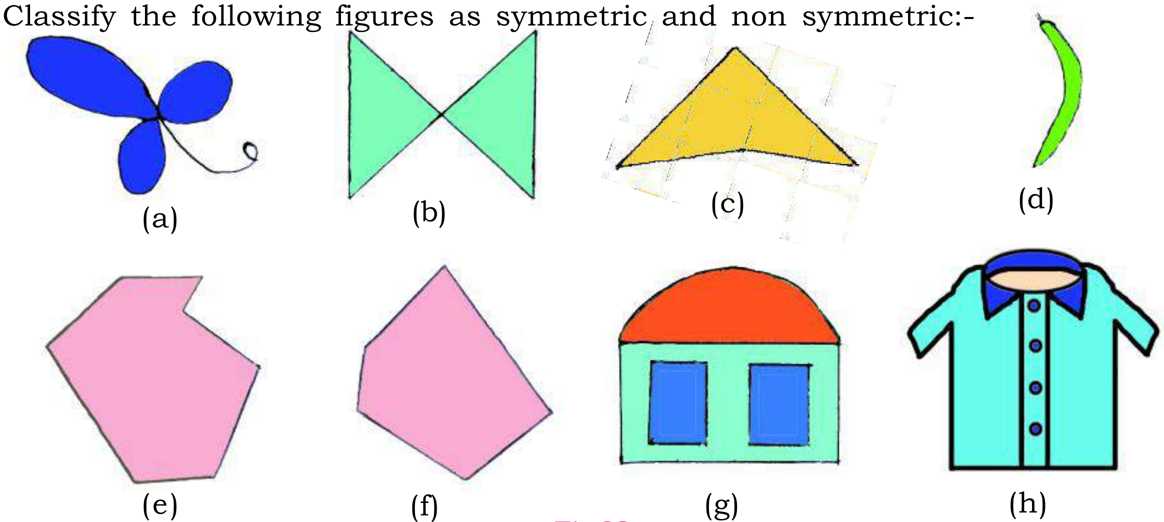


Fig 23

Q2. List any 5 non symmetric figures around you, which have not appeared in this book.

Q3. Draw a line segment of 6 cm and mark a line of symmetry on it.

Q4. Complete the following figures. They have PQ as the line of symmetry.

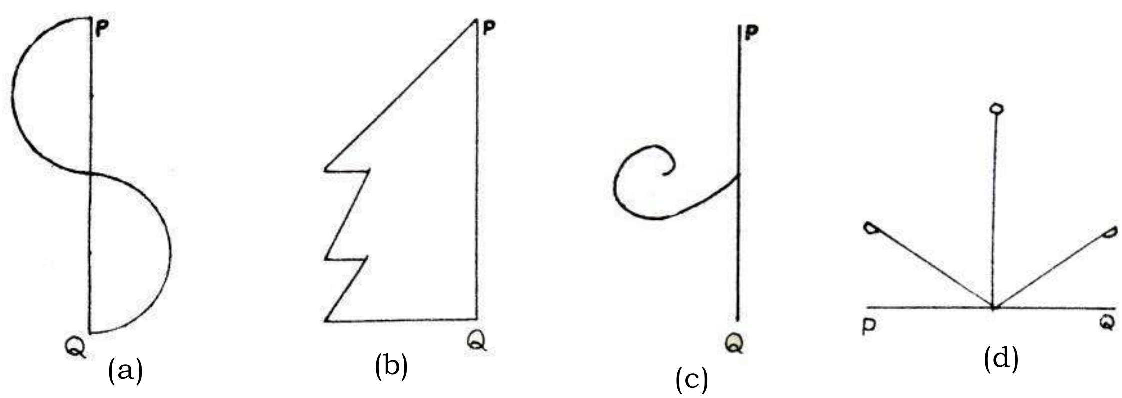


Fig 24

Q5. The shapes of some folded papers are drawn below. On their folds shapes are drawn. In each draw the complete shape that would be obtained when we cut along the design for each of the figures:-

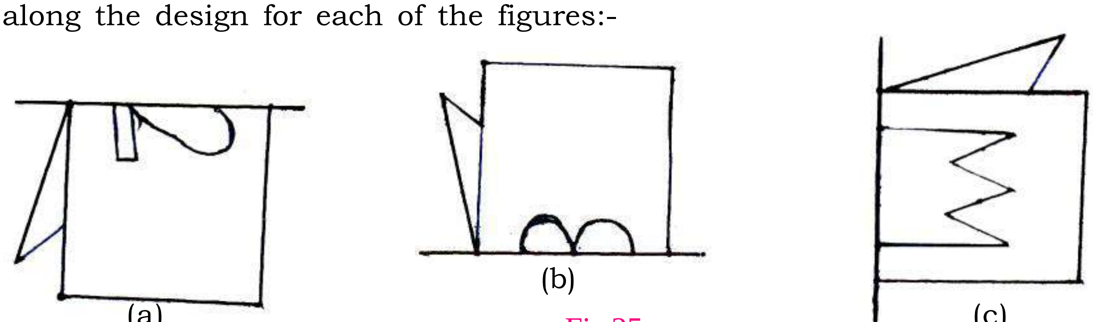


Fig 25

Q6. If the following figure were drawn on one section of a 4 folded paper, then how would the full figure be? Think of the shapes and draw them in your notebook. If you cannot find the shapes by thinking then find them by paper cutting.

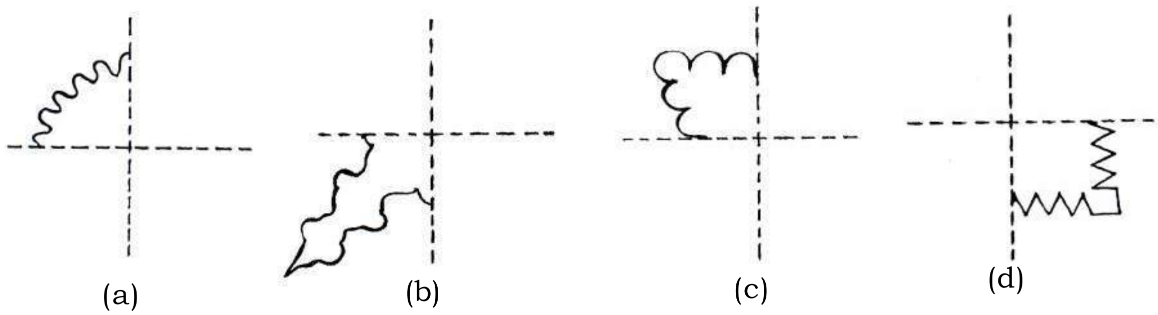
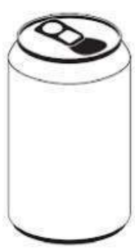


Fig 26

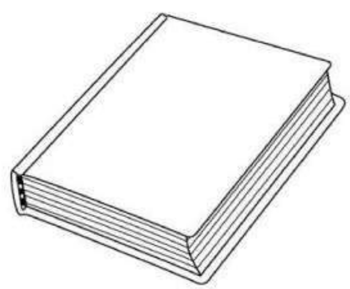
Q8. Write down in your notebook the numbers upto 100 which are symmetrical?

Three Dimensional Shapes

In our daily life we see some solid object which are not plane.



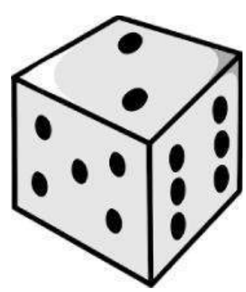
Canl : cylinder



Book : Shape of cuboid

Icecream - Shape of cone

Ball : Sphere

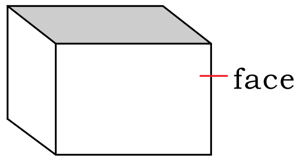


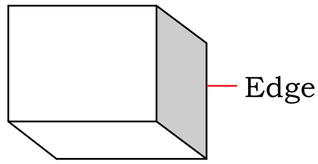
Dice : Shape of cube

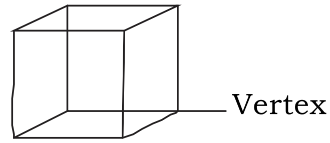
Fig 27

Faces, Edges and Vertices

We can easily indentify the faces, edges and their vertices in the three diamensional figures.





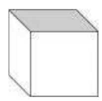
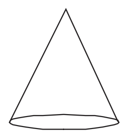
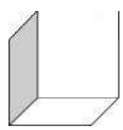
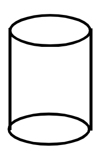


For example, take a cuboid. The each upper phase of cuboid is a rectangle it's self. The two faces of the cuboid at a line segment which is known as edge of cuboid. The three adjacent edges of cuboidmeets at a point, which is called as vertex.

In this manner a cub oil have 6 rectangular faces, 12 edges and 8 vertex.

ACTIVITY - 13

1. Match the following -

(i) Cone	(i) 
(ii) Sphere	(ii) 
(iii) Cylinder	(iii) 
(iv) Cube	(iv) 
(v)Cuboid	(v) 

2. Identify the shape of object-

(i) Chalk box

(ii) Tennis Ball

(iii) Pipe

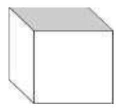
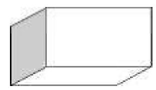
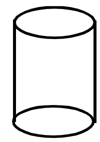
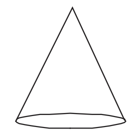
(iv) Cap of Juicer

(v) Dice

3. Name four objects whose shape is similar to cuboid.

4. Name three objects whose shape is similar to cylinder.

5. In the table given below, write down the number of faces, edges and vertices.

Shape						
face	Plane					
	Curve					
Edge	straight					
	Curve					
Vertex						

What Have We Learnt ?

1. We see flowers, beautiful paintings, buildings and other objects, many of things are symmetric.
2. Objects look beautiful if they have symmetry.
3. There are many kinds of three diamensional figures around us some of them are cube cuboid sphere cylinder and pyramid.

Chapter 19

STATISTICS

Introduction

A programme to decorate the classroom was to be organized in the school. The students of class 7 could not decide the colour to be used to paint the walls of the class room. Only 4 colours viz light yellow, pink, light green and sky blue were available in their school. The class monitor asked all the students to write their names and their favorite colour on a paper. This is represented in the following table.

No.	Name of the student	Colour
1	Rajesh	Light yellow
2	Ruchi	Pink
3	Meena	Light Yellow
4	Raheem	Sky blue
5	Hameeda	Light yellow
6	Julie	Light green
7	Anita	Light green
8	Francis	Sky blue

No.	Name of the student	Colour
9	Keshav	Light Yellow
10	Basant	Sky blue
11	Shekhar	Light green
12	Reeta	Pink
13	Sunil	Light yellow
14	Anamika	Light yellow
15	Balwant	Pink
16	Raghu	Light yellow

On the basis of this data, can you decide the colour to use on the walls of the classroom? Rita got an idea, she wrote all the colours on the board and asked each student to write his or her name in front of his or her favorite colour.

Now, the following list was formed:-

Fig 1

Colour	Students Name
Pink	Ruchi, Reeta, Balwant
Light Yellow	Anamika, Rajesh, Meena, Hameda, Keshav, Sunil, Raghu
Light Green	Julie, Anita, Shekhar
Sky blue	Raheem, Basant, Francis

Since light yellow was the favorite colour for more students, it was decided to paint the walls with this colour.

Have you ever adopted this method to take a decision in your daily life?

Now, you construct a list classifying students scoring above and below 34% marks in each subject. On the basis of this data, can you say in which subject is the result the best and in which subject the result is the worst?

Data

We always require some information to take a decision. This necessary quantitative information is called data.

Suppose you have to buy a newspaper for the students of your class. Which newspaper will you buy so that the largest number of students read it? How will you take this decision?

All the students of the class made a table in which they wrote their name in front of their favorite newspaper. The newspaper that the largest number of students liked was selected.

While looking at the tables again and again Julie kept thinking that there was no point writing their names in the table. They only needed the number of students in favour of a particular newspaper. So instead of writing names in the table a symbol could be used to indicate the choice.

Do you agree with Julie? Can you think of a way to count the data using only a symbol instead of having to use names in the table?

Basant suggested that in place of each name we can use a small vertical line to represent the student and then these lines could be counted. Everyone agreed with Basant's suggestion.

Anita said, "Let us find out the order of popularity of some games". She wrote the names of 4 games on the board and asked each student to draw a small vertical line in front of their favourite game. The following table was generated:-

Name of the game	**Tally sign (Vertical line)**	**No. of students**
Football	| | |	3
Cricket	| | | | | | |	7
Vollyball	|	1
Kabaddi	| | | | |	5

But in such tables, it is inconvenient to count a very large number of vertical lines. So, as in earlier classes while learning counting, we made bundles of 10 units, in the same way if we make bundles of 5 vertical lines here then it will become much easier for us to count the lines. We draw 4 vertical lines and represent the 5th line by a slanted line which cuts these 4 lines (as shown below). E.g. for 5:-

For 5 : ~~||||~~

For 19 : ~~||||~~ ~~||||~~ ~~||||~~ | | | | | | | |

This makes counting easier.

According to the data in the table above, the number of students liking Cricket is ~~||||~~ | | i.e. 7 This is called **frequency**. The procedure of representing each data by a vertical line is called **marking a Tally** and the method is known as collection of data using **Tally method**. The table constructed by this, is called the **Frequency Table**.

Use this method to collect data for quantities around you.

Example 1: The number of children in 20 houses of a village are represented by the following table:-

House No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
No. of children	2	3	2	1	3	2	0	1	3	4	2	2	1	1
House No.	15	16	17	18	19	20								
No. of children	2	4	3	2	0	3								

Construct an appropriate frequency table for the above data using Tally method.

Solution: Let us make columns for the number of children in the house, for tally marks and for the frequency. Mark the tally sign for each house. For convenience we represent the 5th sign by a slanted line cutting the 4 previously drawn vertical lines.

No. of Children	Tally sign	Frequency
0	II	2
1	IIII	4
2	~~IIII~~ II	7
3	IIII	5
4	II	2

In the above table why have we chosen the no. of children to be between 0 to 4 only?

What would happen if we start with 1 ?

What happens, if we were to write the number of children in the table to be from 0,1,2,3 —— upto 7?

Pictograph

Rajesh was reading the newspaper. The newspaper said that -"Girls score over boys"

In the class 8th board examinations of this year, girls are ahead of boys in all areas. While looking at the figures, Rajesh thought that– "this is a good method of data display. By looking at these figures, it is very easy to see that the girls have scored over boys in all aspects of the result". A similar picture can be seen when we stand in queues during prayer, the number of students in a class can be compared by the length of the queues. Rajesh asked his friends, "Why do we not represent the popularity of games in the same form using the data collected in table 3?"

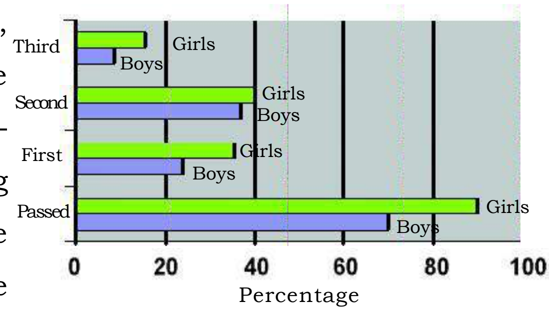


Fig 2

The total No. of students in table 3 was 16. In this, 3 students liked Football, 7 liked Cricket, 1 liked Volleyball and 5 liked Kabaddi. How can this be represented in the form of a figure?

Julie said, "If we make a picture for each student, then 3 pictures in front of football, 7 in front of cricket, 1 in front of volleyball and 5 in front of Kabaddi will have to be made.

Football

Cricket

Volleyball

Kabaddi

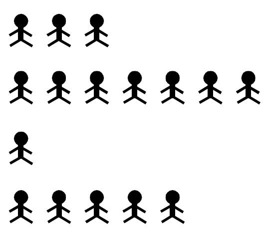
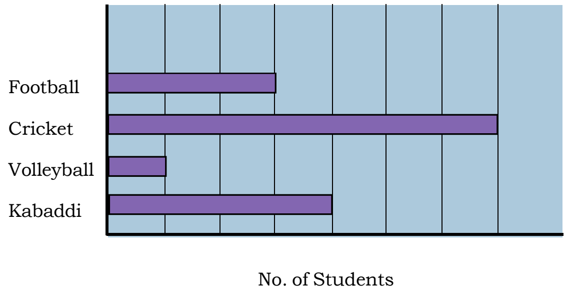


Fig 3

The representation of data with the help of pictures in this form is called a Pictograph.

Bar Graph

Pictograph is easy to understand and conclusions can be drawn by looking at the pictures. But this method requires a lot of pictures to be drawn which sometime becomes impractical. If we take a bar of length 1 cm for each student, then the representation of data becomes even more easy. These bars can be drawn in both horizontal and vertical form.



(Horizontal Bar Graph)

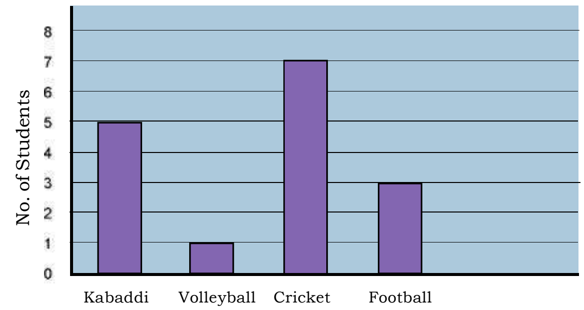


Fig 4

(Vertical Bar Graph)

Note that width of the bars is kept equal in the above graphs. It is easy to estimate the extent of the popularity of these games by looking at these bar graphs. Since the number of students in the above example is small, the data can be easily represented on a notebook using a bar of 1 cm. length for each student. But in case the number of students is large, how can we depict it on the notebook? In such a situation, the main problem is to choose the height of the bars. Let us think this over-

There are 750 men, 660 women and 140 children in the locality where Rajesh resides. We are required to represent this data in a graph.

What should be the height of the bars in order to represent the above data? If we take 1cm for each person, then we need to draw 750 cm high bar for men, 660 cm bar for women and 140 cm bar for children. But it is impossible to draw such bars in our notebooks.

If we take 1cm bar for every 10 people, then we need to draw bars of 75 cm, 66 cm and 14 cm for men, women and children respectively. Even these heights cannot be represented in our notebooks. But, if we take 1cm bar for every 100 people, then we need to draw bars 7.5 cm, 6.6 cm and 1.4 cm long for men, women and children respectively. These bars can be easily represented on our notebooks. So, let us see how we will represent this data using a Bar graph.

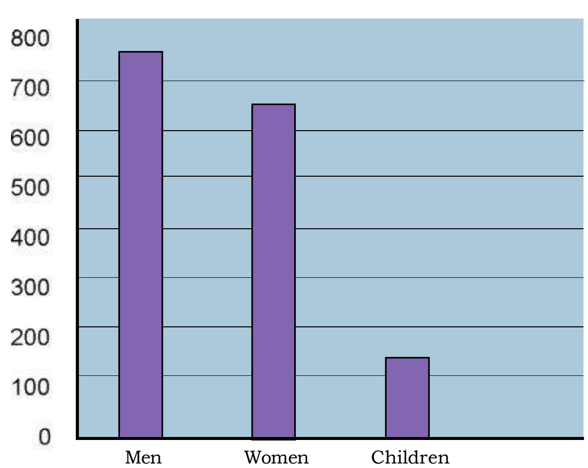


Fig 5

This data is represented by vertical bars. This is called a **vertical bar graph**. Bars can also be drawn horizontally.

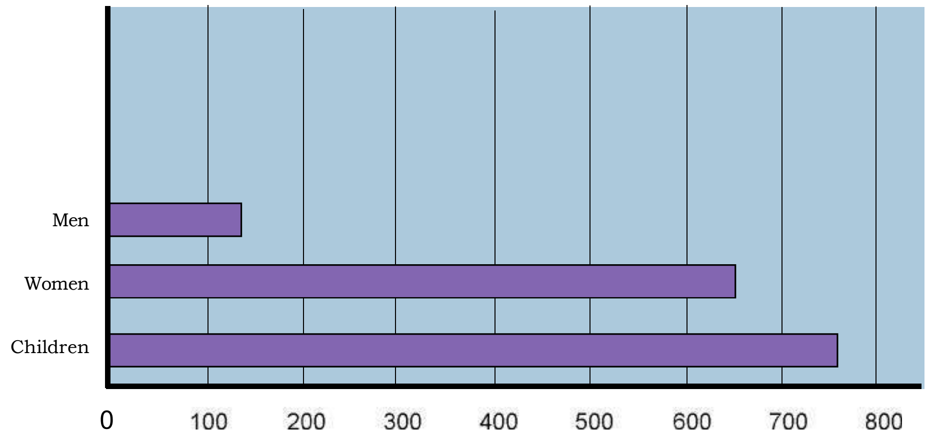


Fig 6

If the bars are drawn horizontally, then the graph obtained is called a **horizontal bar graph** (Fig. 6). Anita was wondering about the use of these bar graphs. She thought we get the same information from the graphs as we get from the frequency tables.

Let us find a solution to Anita's question.

Following table gives the production of wheat from year the 1991 to 2000:-

Year	Wheat Production (in lakh tones)
1991	72
1992	90
1993	82
1994	103
1995	110
1996	94
1997	99
1998	88
1999	90
2000	78

The above data can be represented in the form of a bar graph in the following way:-

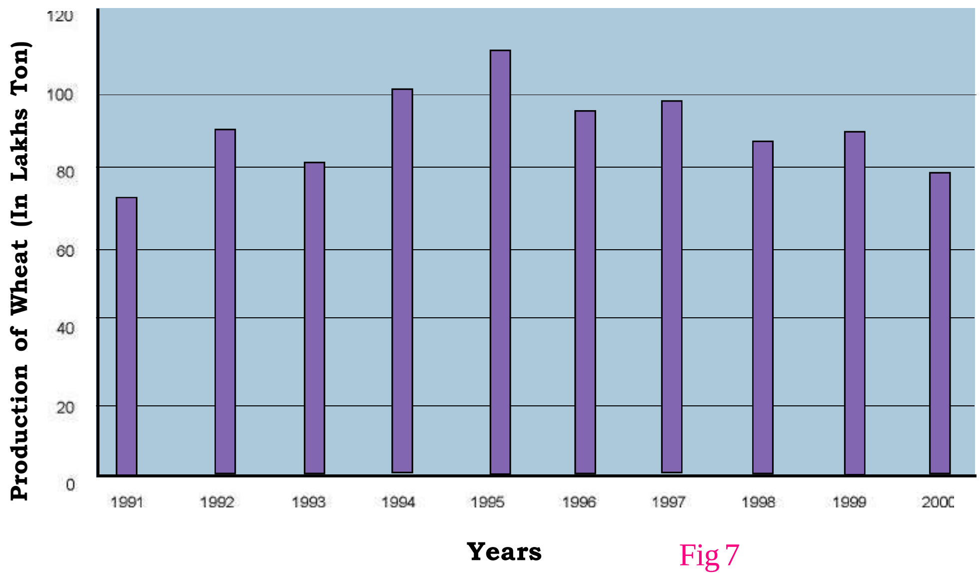


Fig 7

By looking at this bar graph, can you tell which year had the minimum wheat production and which year had the maximum production? What other information can you obtain from this graph? Write down.

You will observe that the maximum wheat production was in the year 1995 and the minimum was in the year 1991. We can also observe that the years 1992 and 1999 had equal wheat production; can you make the same observations using a frequency table?

Clearly, it is difficult to draw conclusions just by looking at the data in the table. For this, one needs to examine the data minutely, whereas with just a glance at the bar graphs we can see which year had the maximum and which year the minimum production. Thus, the major advantage of a bar graph is that it can be easily understood just by looking at it and it can be easily compared with other data.

EXERCISE 19.1

Q1. In a class 20 students obtained the following marks out of 5, in their mathematics test:-

3 2 5 4 0 1 2 3 5 2 2 3 5
4 1 0 3 2 3 4

Construct a table for the above using the Tally method.

Q2. The maximum daily temperature of a city in degree Celsius between 1st April 2005 to 15th April 2005 was recorded as follows:

37.8, 37.8, 37.9, 38.0, 37.9, 37.9, 38.0, 38.1, 38.1,
38.2, 38.3, 38.3, 38.2, 38.1, 38.2

Construct a table for the daily temperature from the above data using the Tally Method.

Q3. The following table represents the results of students of class VI according to divisions obtained. Observe the table and answer the following questions:

Division	No. of students
Ist Division	12
IInd Division	14
IIIrd Division	10
Failed	04

(a) In which division do the maximum number of students fall?
(b) How many students appeared for the exams?
(c) How many students passed the examination?

Q4. The following table represents the yearly income of a company for 5 years. Represent the data by a bar graph.

Year	1996	1997	1998	1999	2000
Yearly income (in Lakhs)	10	20	15	12	22

Q5. The following table represents the percentage of people buying different TV sets. Represent the data in a bar graph.

Brand	% purchased
p	25
q	30
r	15
S	10
T	10
Others	10

Q6. The following table represents the percentage of average marks obtained by the students of a school, in their annual examinations. Represent the data in a bar graph.

Subject	Average Marks obtained by students (%)
English	55
Maths	60
Science	65
Social Science	90
Hindi	70

What Have We Learnt ?

1. Depiction of quantitative data in the form of pictures is called a pictograph.
2. A bar graph, is a representation of quantitative data using bars of equal width taken at equal distances either horizontally or vertically.
3. It is easy to infer many things by observing a bar graph.

Answers

EXERCISE 1

1. 1

2. 41600,

3. (i) > (ii) > (iii) = (iv) <
(v) > (vi) >

4. 20 5. 9899

EXERCISE 2.1

1. 1 2. 0 3. 4 4. 46,47,48 5. 41806

6. (i) False (ii) True (iii) True (iv) True (v) True (vi) True
(vii) False (viii) False

7. (i) 24 (ii) 78 (iii) 519 (iv) 1099 (v) 52331

8. (i) 26 (ii) 521 (iii) 1101 (iv) 52333

9. 100000 10. 99999 11. 1

12. 18, 252,421,497, 557, 731

13. 617, 458, 225, 69, 59

14. (ii)

15. 6387

EXERCISE 2.2

1. (i) 391 (ii) 40 (iii) 40 (iv) 39 (v) C

2. (i) (23589+411) + 1248=25248 (ii) (32+68)+(2546+544)= 3190
(iii) (247+153)+376 = 776 (iv) (143+857)+456= 1456
(v) (32958+12042)+5000= 50000

3. 0 4. Sum of two whole numbers is always a whole number.

5. (i) 4559 (ii) 0 (iii)

$$\begin{array}{r} 8\,7\,6 \\ \boxed{2}\,3\,\boxed{9} \\ \hline 6\,\boxed{3}\,7 \end{array}$$

6. 1 7. 0 8. 1216 ÷ 76 = 16 9. 32 10. 20310

11. (i) Quotient= 215, Remainder = 32 (ii) Quotient = 14, Remainder = 735
(iii) Quotient = 309, Remainder = 145

12. (i)
$$\begin{array}{r} 7\,3\,5 \\ -\,4\,2\,\boxed{9} \\ \hline \boxed{3}\boxed{0}\,6 \end{array}$$
(ii)
$$\begin{array}{r} 4\,9\,3\,1 \\ -\;\boxed{3}\boxed{0}\,7\,8 \\ \hline 1\,8\,\boxed{5}\boxed{3} \end{array}$$

13. Rs. 390 14. Rs. 3414 15. 31825 16. 9

17. 780

EXERCISE 3

1. (i) True (ii) False (iii) True (iv) True (v) False
2. (i)
3. (i) AB and CD (ii) RS and PQ (iii) SR and TU
4. C and E

EXERCISE 4

1.	(i)	-2	(ii)	2	(iii)	-9
	(iv)	8	(v)	-4	(vi)	3
	(vii)	-5	(viii)	8		
2.	(i)	1028	(ii)	-266	(iii)	36
	(iv)	154				
3.	(i)	=	(ii)	<	(iii)	<
	(iv)	=	(v)	=	(vi)	>
4.	(i)	30	(ii)	90	(iii)	24
	(iv)	24	(v)	0	(vi)	42
5.	(i)	>	(ii)	=	(iii)	<
	(iv)	=	(v)	=	(vi)	<
	(vii)	=				
6.	13				7.	100
8.	(i)	15	(ii)	-10	(iii)	-4
	(iv)	Undefined	(v)	-14	(vi)	-19
9.	(i)	4	(ii)	-2	(iii)	-1
	(iv)	-12				
10.	(i)	-17	(ii)	23	(iii)	-68,
	(iv)	75				
11.	(i)	18	(ii)	-26	(iii)	-161
	(iv)	79				

EXERCISE 5

5.	44 cm					
6.	(i)	Radius	(ii)	Centre	(iii)	Diameter
	(iv)	Centre	(v)	Equal	(vi)	Radius
	(vii)	Chord				

EXERCISE 6.1

1. (i) 12 (ii) 18 (iii) 10 (iv) 27
2. (i) 36 (ii) 150 (iii) 24 (iv) 15
3. 1
4. (i) 361/64, (ii) 201/275 5. 17
6. 11 7. 44 8. 17, 113 heaps 9. 30

EXERCISE 6.2

Oral :

1. 4 2. 338 3. 24 4. Greater than 7 5. Not

Written :

1. i. 28 ii. 324 iii. 180 iv. 2520
2. i. 56 ii. 1904 iii. 360 iv. 16560
3. 10 4. 15 times 5. 23 January
6. 72 7. 221 8. 20 days
9. No, because H.C.F is always a factor of L.C.M.
10. 4.15 Evening

EXERCISE 7

1. (i) False (ii) False (iii) True
(iv) False (v) True (vi) False
(vii) True (viii) False (ix) True (x) True

2. (a) (i) $\frac{8}{9} \rangle \frac{7}{8} \rangle \frac{5}{6}$ (ii) $\frac{7}{6} > \frac{3}{4} > \frac{8}{12} > \frac{1}{2} > \frac{1}{6}$
(b) (i) 8 > .8 > .08 > .008 > .0008 (ii) .01 > .00992 > .0099 > .0012

3. (i) $\frac{1}{3} < \frac{9}{24} < \frac{5}{8} < \frac{5}{6} < \frac{3}{2}$ (ii) $\frac{1}{8} < \frac{2}{15} < \frac{1}{6} < \frac{1}{4} < \frac{1}{2}$

4. (i) $5\frac{19}{40}$ (ii) 8.0001 (iii) $\frac{3}{4}$ (iv) 1 (v) $11\frac{5}{12}$ (vi) $1\frac{5}{14}$

5. (i) 24 (ii) 77 (iii) 36 (iv) 8

6. Proper $\frac{4}{5}, \frac{8}{9}, \frac{15}{16}, \frac{3}{7}$ Improper $\frac{17}{4}, \frac{16}{13}, \frac{6}{5}, \frac{8}{5}$

EXERCISE 8.1

1. (i) $\angle$AOB (ii) $\angle$LMN (iii) $\angle$PQR (iv) $\angle$STU (v) $\angle$ABC
4. 180°

EXERCISE 8.2

1.	(i)	True	(ii)	False	(iii)	True	(iv)	True
2.	(i)	Obtuse angle	(ii)	Acute angle	(iii)	Right angle	(iv)	Straight angle
	(v)	Acute angle	(vi)	Obtuse angle	(vii)	Acute angle	(viii)	Acute angle
	(ix)	Acute angle	(x)	Acute angle				

EXERCISE 9.1

1.	(i)	False	(ii)	True	(iii)	False	(iv)	True	(v)	False
	(vi)	False	(vii)	True	(viii)	False	(ix)	True	(x)	True
2.	40°		3. 45°		4. 60°		5. Yes			
6.	(i)	No	(ii)	No	(iii)	Yes	(iv) No	(v) Yes	(vi) No	

EXERCISE 9.2

(1)	(i)	Two	(ii)	Triangles	(iii)	360°	(iv)	Two
	(v)	Four, Three						
(2)	(i)	Trapizium	(ii)	90°	(iii)	Parallel, eqaul		
	(iv)	Square	(v)	Rhombus				
(3)	(i)	True	(ii)	False	(iii)	False	(iv)	False

EXERCISE 10.1

2.	(i)	1 : 4	(ii)	1 : 4	(iii)	3 : 20	(iv)	3 : 100
	(v)	21 : 200	(vi)	40 : 1	(vii)	4 : 25		
3.	(i)	3 : 8	(ii)	1 : 3	(iii)	3 : 5	(iv)	9 : 19
	(v)	6 : 17	(vi)	2 : 3	(vii)	5 : 7	(viii)	1 : 3
	(ix)	2 : 9	(x)	1 : 20				
4.	(i)	1 : 16	(ii)	16 : 1				
5.	7 : 6							
6.	(i)	4 : 3	(ii)	4 : 7	7-	3 : 2		
8.	8 to Bhanu and 12 to Bangaru			9.	5 : 4	10.	4 : 5	
11.	10 Mangoes to Ratna and 8 Mangoes to Sheela							

12. (i) 5 : 2 (ii) 4 : 1 (iii) 7 : 1 (iv) 7 : 4 (v) 5 : 3

13. Rs. 9000 to Ram and Rs.12000 to Shyam.

14 AB = 28 Km BC = 12 Km 16. (i) 3 to each (ii) 2, 1, 3

EXERCISE 10.2

1. Rs. 38.50 2. (i) In 8 hours (ii) 357.50 Kilometre

3. (i) 10 Kilograms (ii) 48 Kilograms 4. Rs.1800 5. Rs. 640

6.

No. of Books	Price (in rupees)
50	2500
75	3750
2	100
60	3000

EXERCISE 11

1. (i) a=2r (ii) A = 1 x b (iii) s = c + p (iv) a + b (v) x - 7
(vi) A = p + c

2. (i) True (ii) True (iii) False (iv) True (v) True

EXERCISE 12

1. vi and viii - Monomial, i, ii,iii,iv,v,vi,viii, ix and x—binomial.

2. 5xy and $\frac{9}{4}$xy, 7c and 2c, $\frac{4}{5}$yz and $\frac{11}{13}$yz

7bc and bc, $\frac{2}{7}$z and 7z, 37pqr and 9pqr

EXERCISE 13

1. (i) 150% (ii) 250% (iii) 20% (iv) 15%

2. (i) $\frac{1}{2}$ (ii) $\frac{3}{20}$ (iii) $\frac{1}{50}$ (iv) $\frac{1}{10}$

3. Rs. 216 4. 72 kilograms 5. 50% 6. Rs. 9

7. 3 toffees 8. 65% 9. 100%

10. 28,800 gents, 25,200 ladies 11. Defeciency of 1%

12. 10% 13. 800 14. 20% 15. 90% pass, 10% fail

16. 1440 literate, 160 illiterate

EXERCISE 14.1

1. i and iv, vi, vii, viii,ix and x are equations of x.

2.

i.	left = x – 5	right = 9	ii.	left = 2 x - 3	right = 7
iii.	left = 2y	right = 9 – y	iv.	left = 2y	right = 6
v.	left = 15	right = 2a + 5			

3.

i.	$2y – 3 = 17$	ii.	$\frac{y}{6} = 7$	iii.	$y – 5 = 8$
iv.	$3y + 11 = 44$	v.	$7y - 5 = 9$		

4. (i) 9 obtained by subtracting 6 from any number.
   (ii) 0 obtained by subtracting 14 from 7 times of any number.
   (iii) Two third of a number is 6.
   (iv) 10 obtained by adding 5 to the half of any number.
   (v) 4 obtained by subtracting half of any number from 38.

EXERCISE 14.2

1. (i) $x = -1$ (ii) $z = 8$ (iii) $y = 3$ (iv) $y = 3$ (v) $x = 6$
   (vi) $z = \frac{7}{3}$

2. (i) $x= 2$ (ii) $z = 4$ (iii) $x = 75$ (iv) $y = 1$

3. 5 4. 25 5. 40 6. 20 7. 13 metre

EXERCISE 16

(1) Closed figures:
(i) (ii) (iii) (iv) (v)
(vii) (x) (xii) (xiii)

(2) (i) 12 sq.cm (ii) 10 sq.cm
(iii) 72 sq.m (iv) 135 sq.m

(3) (i) 36 sq.cm (ii) 144 sq.cm
(iii) 169 sq.cm (iv) 12.25 sq.cm

(4) (i) 20 sq.cm (ii) 24 sq.cm

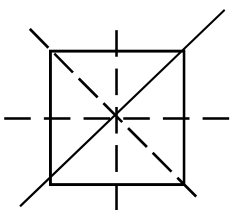
(5) 36 sq.cm

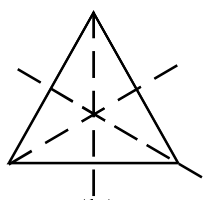
EXERCISE 17

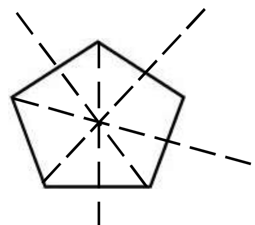
1)	(i) 12 cm	(ii) 12 cm	(iv) 6 cm	
	(vi) 8 cm	(vii) 8 cm	(viii) 15 cm	
2)	(i) 42 cm	(ii) 36 cm		
	(iii) 12 cm	(iv) 3 metre or 300 cm		
3)	(i) True	(ii) False	(iii) False	(iv) True
4)	60 cm			
5)	140 cm or 1.4 metre			
6)	(i) 8cm	(ii) 16cm	(iii) 32 cm	(iv) 24 cm
	(v) 20 cm	(vi) 13 cm	(vii) 18 cm	
7)	280 metre			

EXERCISE 18.1

1.


(a)


(b)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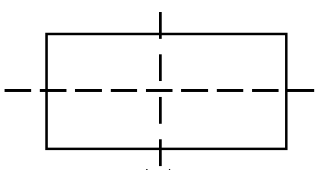
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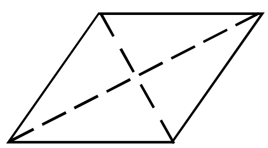
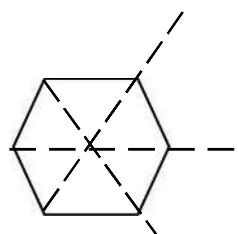
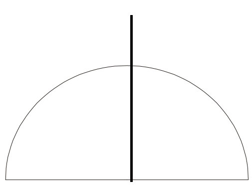
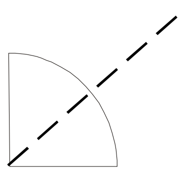
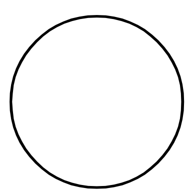
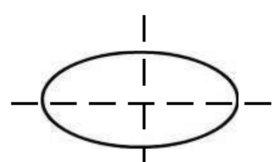

(g)


(h)


(i)


(k)


(l)


(m)

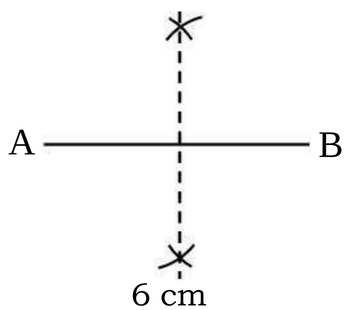
EXERCISE 18.2

Q.1.

Symmetry	Non-symmetry
b	a
c	d
g	e
h	f

Q.2. Do yourself.

Q.3.



EXERCISE 19

Q. 1

No.	Tally sign	Frequency
0	$\mid\mid$	2
1	$\mid\mid$	2
2	$\cancel{\mid\mid\mid\mid}$	5
3	$\cancel{\mid\mid\mid\mid}$	5
4	$\mid\mid\mid$	3
5	$\mid\mid\mid$	3

Q. 2

Temperature	Tally sign	Frequency
37.8	$\mid\mid$	2
37.9	$\mid\mid\mid$	3
38.0	$\mid\mid$	2
38.1	$\mid\mid\mid$	3
38.2	$\mid\mid\mid$	3
38.3	$\mid\mid$	2

Q. 3. (a) Second class (b)40 (c) 36

Methods of Vedic Maths

You have already learnt addition, subtraction, multiplication and division. There are a few simple and interesting methods for this processes in Vedic Maths also. Here we will introduce them to you. Before knowing about these methods let us get acquainted with digits.

Digits (Ank)- 0,1,2,3,4,5,6,7,8,9. These are the ten digits. All the numbers are written using these digits.

Bijank- In Vedic Maths digits from 1 to 9 are called Bijank. To find out the Bijank of any number, the digit of the number are added till a single digit number is obtained.

For example –

To find out the Bijank of 35, we will add its digits.

$3 + 5 = 8$

So the Bijank of 35 is 8

Similarly -

Bijank of 97

$9 + 7 = 16$ but 16 has 2 digits So we will add these digits also

$1 + 6 = 7$

So the Bijank of 97 is 7

Param Mitra Ank –

Any 2 digits whose total is 10 are called Param Mitra of each other.

For example –

$1 + 9 = 10$

So 1 is Param Mitra of 9

and 9 is Param Mitra of 1

Now let's practice it a bit

Practice

Q. 1 - What are the digits that are used for writing numbers?

Q. 2 - Write the Bijank of following numbers.

(i) 12	(ii) 15	(iii) 17	(iv) 19	(v) 37
(vi) 44	(vii) 56	(viii) 67	(ix) 96	(x) 183

Q. 3 - Write the Param Mitra number of the following numbers.

(i) 2 (ii) 3 (iii) 4 (iv) 5

Ekadhiken Poorven

The meaning of **Ekadhiken Poorven** is take one more than the previous number.

For example - 3 is the ekadhik of 2

Similarly - 4 is the ekadhik of 3

Can you tell the ekadhik of each digit from 1 to 9 ?

Eknyunen Poorven

The meaning of **Eknyunen Poorven** is take one less than the previous number.

For example - 7 is eknyune of 8, Similarly 4 is eknyune of 5

Now you tell the eknyune of all the digit from 1 to 9.

In the methods of Vedic Maths, Ekadhiken Poorven and Eknyunen Poorven are used of many places.

Now tell –

What numbers will you get from the following numbers by doing Ekadhik?

(i) 22 (ii) 43 (iii) 30 (iv) 58

Sometimes it is necessary to do Ekadhik or Eknyun more than once.

For example –

We get 13 by doing Ekadhik of 12 and 14 when we again do Ekadhik of 13 that is get 14 when we do Ekadhik of 12 twice.

Now lets do Eknyune of 12 twice.

We get 11 by doing Eknyune and 10 when we again do Eknyun of 11 that is we get 10 when we do Eknyune of 12 twice.

What numbers we will get when we do Ekadhik of these numbers thrice?

(i) 23 (ii) 15 (iii) 36 (iv) 42

Choose some numbers on your own and practice Ekadhik of these numbers.

Now tell –

What numbers will you get by doing Eknyunen twice?

(i) 16 (ii) 30 (iii) 67 (iv) 75

What numbers will you get from these numbers by doing Eknyunen thrice?

Choose some numbers on your own and practice doing Eknyune twice or thrice.

Addition with the help of Param Mitra.

If we have to add 1,2 or 3 to any digit, we can do it by doing Ekadhik. But if both the digits are greater than 5, it is easy to add with the help of Param Mitra.

Lets, look at an example.

$$\begin{array}{r} 9 \\ +\ 7 \\ \hline \end{array}$$

Here we have to add 9 and 7. Param Mitra Ank of 9 is 1.

So we taken 1from 7 and add it to 9.

Now 9 + 1 = 10

And taking out 1 from 7 makes it 6. By adding 6 to 10, we get 16

i.e.

$$\begin{array}{r} 9 \\ +\ 7 \\ \hline 16 \end{array}$$

Similarly practice addition with the help of Param Mitra.

(i) 7 + 8 (ii) 8 + 6 (iii) 9 + 8 (iv) 6 + 9

In a similar way, take two digits greater than 5 and try adding them with the help of Param Mitra.

Ekadhik sign {One more} Addition by (.)

You know about addition with carry. Let us from here. Take an example

Solve these

$$\begin{array}{r} 5\ 4 \\ +1\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} (1) \\ 5\ 4 \\ +\ 1\ 8 \\ \hline 2 \end{array}$$

12 is obtain by addition of unit digits (4+8)

Unit digit 2 of this addition is written as its sum and carry 1 is written upon the 5 in ten's column.

$$\begin{array}{r} (1) \\ 5\ 4 \\ +\ 1\ 8 \\ \hline 7\ 2 \end{array}$$

1+5+1=7 Is written as sum of ten's digit.

Sum 72 is obtained

If carry obtaind from addition of unit place digit 1 is written in the from of point in ten's column then also sum is as usual. See this addition again.

$$\begin{array}{r} 5\ 4 \\ +1\ 8 \\ \hline \end{array}$$

Addition of 4 and 8 gives 12

$$\begin{array}{r} 5\ 4 \\ +\dot{1}\ 8 \\ \hline 2 \end{array}$$

Write 2 of 12 as addition of unit digits and mark carry 1 as one point above 1 of ten's place digit. This point is known as Ekadhik sign(.).

$$\begin{array}{r} 5\ 4 \\ +\dot{1}\ 8 \\ \hline 7\ 2 \end{array}$$

Now add ten's place digits 5+(.)+1=7{count (.) as1}

Total 72 is obtained

Let us one more example

Example2 **Solve these**

$$\begin{array}{r} 4\ 6 \\ +2\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 4\ 6 \\ +\dot{2}\ 4 \\ \hline 0 \end{array}$$

Add 6 and 4 of unit place. Will get 6+4=10

Write 0 of 10 of addition in unit column.

$$\begin{array}{rr} 4 & 6 \\ +\dot{2} & 4 \\ \hline 7 & 0 \end{array}$$

Make carry 1 as(.) above 2.

Now add ten's digits. 4+(.)+2=7 Count (.)as1. Total addition 70 obtained.

This method is easier for addition of more than two numbers

Example 3 Solve these

$$\begin{array}{rr} 2 & 7 \\ 4 & 8 \\ +1 & 9 \\ \hline \end{array}$$

$$\begin{array}{rrl} 2 & 7 & \\ \dot{4} & 8 & 7+8=15 \\ \dot{1} & 9 & 5+9=14 \\ \hline 9 & 4 & \end{array}$$

Add 7 and 8 of unit 15 obtained. Mark 1 in the form of ekadhik sign above 4 and add 5 and 9. 14 obtained. Mark 1 0f 14 as ekadhik sign above 1 0f ten's column. Write 4 as result of addition. Now add digit of ten's 2+(.)+4+(.)+1=9

Example 4 Solve these

$$\begin{array}{rr} 1 & 8 \\ 2 & 5 \\ +1 & 9 \\ \hline \end{array}$$

Solution :

$$\begin{array}{rrl} 1 & 8 & 8+5=13 \\ \dot{2} & 5 & \\ +1 & 9+ & \\ \hline \end{array}$$

$$\begin{array}{rrl} 1 & 8 & \\ \dot{2} & 5 & \\ +\dot{1} & 9 & 9+3=12 \\ \hline & 2 & \end{array}$$

$$\begin{array}{rr} 1 & 8 \\ \dot{2} & 5 \\ +\dot{1} & 9 \\ \hline 6 & 2 \end{array}$$

$$1 + (\cdot) + 2 + (\cdot) + 1 = 6$$

Exercise

Add with Ekadhik sign

1. $\begin{array}{r} 2\ 3 \\ +3\ 6 \\ \hline \end{array}$ **2.** $\begin{array}{r} 3\ 8 \\ +4\ 5 \\ \hline \end{array}$ **3.** $\begin{array}{r} 1\ 7 \\ +2\ 4 \\ \hline \end{array}$ **4.** $\begin{array}{r} 1\ 5 \\ 1\ 7 \\ +2\ 8 \\ \hline \end{array}$ **5.** $\begin{array}{r} 3\ 7 \\ 2\ 8 \\ +1\ 9 \\ \hline \end{array}$ **6.** $\begin{array}{r} 2\ 8 \\ 1\ 7 \\ +3\ 6 \\ \hline \end{array}$

Subtraction with Ekadhik sign {one more sign}

Problems of subtraction where borrowing of number is required. we use Ekadhik sign for subtraction. Here we have to use one more concept Parammitra of vedic maths. (Any two number whose sum is 10, are callwd as parammitra. Like 3 is Parammitra of 7 and 7 is Parammitra of 3 since 3+7=10. In this manner 6 and 4 are Parammitra. 5 is Parammitra of itself)

Let us understand this process with an example.

Example 1. Solve these $\begin{array}{r} 3\ 6 \\ -1\ 7 \end{array}$

$\begin{array}{r} 3\quad 6^{+3} \\ -\dot{1}\quad 7 \\ \hline 9 \end{array}$

$\begin{array}{r} 3\ 6 \\ -\dot{1}\ 7 \\ \hline 1\ 9 \end{array}$

7 can not be subtraction from6. Add Parammitra of 7, 3 to 6. It gives 9. Write it below as result and mark ekadhik sign above1. Now subtract (.)+1 means 2 from 3. 1 obtained. Write it below as result. Solution 19 obtained.

Example 2 Solve these $\begin{array}{r} 7\ 5 \\ -2\ 8 \end{array}$

$\begin{array}{r} 7\quad 5^{+2} \\ -\dot{2}\quad 8 \\ \hline 7 \end{array}$

$\begin{array}{r} 7\ 5 \\ -\dot{2}\ 8 \\ \hline 4\ 7 \end{array}$

8 cannot be subtracted from 5. (add 2 Parammitra of 8 to 5, gives 7) Write it below as result. Mark Ekadhik sign(.) above 2. Subtract (.)+2 means 3 from7. Obtained 4. Write it below as result.

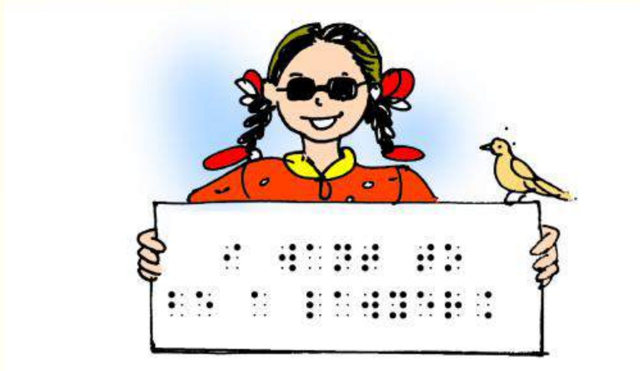
Solution 47 obtained.

Practice

Subtract with Ekadhik sign.

1. $\begin{array}{r} 7\ 2 \\ -1\ 8 \end{array}$ **2.** $\begin{array}{r} 3\ 7 \\ -1\ 9 \end{array}$ **3.** $\begin{array}{r} 4\ 0 \\ -2\ 8 \end{array}$ **4.** $\begin{array}{r} 3\ 5 \\ -2\ 6 \end{array}$ **5.** $\begin{array}{r} 4\ 6 \\ -2\ 8 \end{array}$ **6.** $\begin{array}{r} 6\ 8 \\ -3\ 9 \end{array}$

Braille An Introduction



Do you know what is written here?

It is: I want to be a lawyer.

Like devnaagri and Gurumukhi etc. Braille is also a script. Braille script is used by Blind persons to read and write. Braille was invented by Louis Braille in 1829. Braille script is based on six dots. These six dots are referred as the Braille cell. Each cell comprises of one Braille character. To write Braille script Blind person uses Stylus and Braille slate. Braille slate consist essentially of two metal or plastic plates hinged together to permit a sheet of paper to be inserted between the two plates. While writing on a Braille sheet (drawing sheet) it is to be written from right to left and then reverse the normal numbering of the Braille cell. Blind person reads these raised (embossed) dots with the help of their finger tip.

1	4
2	5
3	6

Braille cell

Total 63 combinations are possible using these 6 dots.
Some combinatios given below:

Braille Chart

a	b	c	d	e	f	g	h	i	j
k	l	m	n	o	p	q	r	s	t
u	v	w	x	y	z				

A Number sign (.:) is used before the alphabets 'a' to 'j' to convert them to numbers.

1	2	3	4	5	6	7	8	9	0