EXERCISE

SHORT ANSWER TYPE

1. Let A = {–1, 2, 3} and B = {1, 3}. Determine
(i) A × B
(ii) B × A
(iii) B × B
(iv) A × A
Solution:

According to the question,
A = {–1, 2, 3} and B = {1, 3}
(i) A × B
{–1, 2, 3} × {1, 3}
So, A × B = {(–1, 1), (–1, 3), (2, 1), (2, 3), (3, 1), (3, 3)}
Hence, the Cartesian product = {(–1, 1), (–1, 3), (2, 1), (2, 3), (3, 1), (3, 3)}
(ii) B × A.
{1, 3} × {–1, 2, 3}
So, B × A = {(1, –1), (1, 2), (1, 3), (3, –1), (3, 2), (3, 3)}
Hence, the Cartesian product = {(1, –1), (1, 2), (1, 3), (3, –1), (3, 2), (3, 3)}
(iii) B × B
{1, 3} ×{1, 3}
So, B × B = {(1, 1), (1, 3), (3, 1), (3, 3)}
Hence, the Cartesian product = {(1, 1), (1, 3), (3, 1), (3, 3)}
(iv) A × A
{–1, 2, 3} × {–1, 2, 3}
So, A × A = {(–1, –1), (–1, 2), (–1, 3), (2, –1), (2, 2), (2, 3), (3, –1), (3, 2), (3, 3)}
Hence,
the Cartesian product ={(–1, –1), (–1, 2), (–1, 3), (2, –1), (2, 2), (2, 3), (3, –1), (3, 2), (3, 3)}

2. If P = {x : x < 3, x ∈ N}, Q = {x : x ≤ 2, x ∈ W}. Find (P ∪ Q) × (P ∩ Q), where W is the set of whole numbers.
Solution:

According to the question,
P = {x: x < 3, x ∈N}, Q = {x : x ≤ 2, x ∈W} where W is the set of whole numbers
P = {1, 2}
Q = {0, 1, 2}
Now
(P∪Q) = {1, 2}∪{0, 1, 2} = {0, 1, 2}
And,
(P∩Q) = {1, 2}∩{0, 1, 2} = {1, 2}
We need to find the Cartesian product of (P∪Q) = {0, 1, 2} and (P∩Q) = {1, 2}
So,
(P∪Q) × (P∩Q) = {0, 1, 2} × {1, 2}
= {(0, 1), (0, 2), (1, 1), (1, 2), (2, 1), (2, 2)}
Hence, the Cartesian product = {(0, 1), (0, 2), (1, 1), (1, 2), (2, 1), (2, 2)}

3. If A = {x : x ∈ W, x < 2}, B = {x : x ∈ N, 1 < x < 5}, C = {3, 5} find
(i) A × (B ∩ C)
(ii) A × (B ∪ C)
Solution:

According to the question,
A = {x: x ∈ **W**, x < 2}, B = {x : x ∈**N**, 1 < x < 5} C = {3, 5}; **W** is the set of whole numbers
A = {x: x ∈ **W**, x < 2} = {0, 1}
B = {x : x ∈**N**, 1 < x < 5} = {2, 3, 4}
(i)
(B∩C) = {2, 3, 4} ∩ {3, 5}
(B∩C) = {3}
A × (B∩C) = {0, 1} × {3} = {(0, 3), (1, 3)}
Hence, the Cartesian product = {(0, 3), (1, 3)}

(ii)
(B∪C) = {2, 3, 4} ∪ {3, 5}
(B∪C) = {2, 3, 4, 5}
A × (B∪C) = {0, 1} × {2, 3, 4, 5} = {(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)}
Hence, the Cartesian product = {(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)}

4. In each of the following cases, find a and b.
(i) (2a + b, a – b) = (8, 3)
(ii) (a/4 , a – 2b) = (0, 6 + b)
Solution:

(i)
According to the question,
(2a + b, a – b) = (8, 3)
Given the ordered pairs are equal, so corresponding elements will be equal.
Hence,
2a + b = 8 and a–b = 3
Now a–b = 3
⇒a = 3 + b
Substituting the value of a in the equation 2a + b = 8,
We get,
2(3 + b) + b = 8
⇒ 6 + 2b + b = 8
⇒ 3b = 8–6 = 2
⇒ b = 2/3
Substituting the value of b in equation (a–b = 3),
We get,
⇒ a – 2/3 = 3
⇒ a = 3 + 2/3
⇒ a = (9 + 2)/3
⇒ a = 11/3
Hence the value of a = 11/3 and b = 2/3 respectively.

(ii)
According to the question,

$$\left(\frac{a}{4}, a - 2b\right) = (0, 6 + b)$$

Given the ordered pairs are equal, so corresponding elements will be equal.
a/4 = 0 and a – 2b = 6 + b
Now a/4 = 0
⇒a = 0
Substituting the value of a in the equation (a–2b = 6 + b),
We get,
0 – 2b = 6 + b
⇒ – 2b – b = 6
⇒ – 3b = 6
⇒ b = – 6/3
⇒ b = – 2
Hence, the value of a = 0 and b = – 2 respectively

5. Given A = {1, 2, 3, 4, 5}, S = {(x, y) : x ∈ A, y ∈ A}. Find the ordered pairs which satisfy the conditions given below:
(i) x + y = 5
(ii) x + y < 5
(iii) x + y > 8
Solution:

According to the question, A = {1, 2, 3, 4, 5}, S = {(x, y) : x ∈A, y ∈A}
(i) x + y = 5
So, we find the ordered pair such that x + y = 5, where x and y belongs to set A = {1, 2, 3, 4, 5},
1 + 1 = 2≠5
1 + 2 = 3≠5
1 + 3 = 4≠5
1 + 4 = 5⇒ the ordered pair is (1, 4)
1 + 5 = 6≠5
2 + 1 = 3≠5
2 + 2 = 4≠5
2 + 3 = 5⇒ the ordered pair is (2, 3)
2 + 4 = 6≠5
2 + 5 = 7≠5
3 + 1 = 4≠5
3 + 2 = 5⇒ the ordered pair is (3, 2)
3 + 3 = 6≠5
3 + 4 = 7≠5
3 + 5 = 8≠5
4 + 1 = 5⇒ the ordered pair is (4, 1)
4 + 2 = 6≠5
4 + 3 = 7≠5
4 + 4 = 8≠5
4 + 5 = 9≠5

$5 + 1 = 6 \neq 5$
$5 + 2 = 7 \neq 5$
$5 + 3 = 8 \neq 5$
$5 + 4 = 9 \neq 5$
$5 + 5 = 10 \neq 5$
Therefore, the set of ordered pairs satisfying x + y = 5 = {(1,4), (2,3), (3,2), (4,1)}.

(ii) $x + y < 5$
So, we find the ordered pair such that $x + y<5$, where x and y belongs to set A = {1, 2, 3, 4, 5}
$1 + 1 = 2<5 \Rightarrow$ the ordered pairs is (1, 1)
$1 + 2 = 3<5 \Rightarrow$ the ordered pairs is (1, 2)
$1 + 3 = 4<5 \Rightarrow$ the ordered pairs is (1, 3)
$1 + 4 = 5$
$1 + 5 = 6>5$
$2 + 1 = 3<5 \Rightarrow$ the ordered pairs is (2, 1)
$2 + 2 = 4<5 \Rightarrow$ the ordered pairs is (2, 2)
$2 + 3 = 5$
$2 + 4 = 6>5$
$2 + 5 = 7>5$
$3 + 1 = 4<5 \Rightarrow$ the ordered pairs is (3, 1)
$3 + 2 = 5$
$3 + 3 = 6>5$
$3 + 4 = 7>5$
$3 + 5 = 8>5$
$4 + 1 = 5$
$4 + 2 = 6>5$
$4 + 3 = 7>5$
$4 + 4 = 8>5$
$4 + 5 = 9>5$
$5 + 1 = 6>5$
$5 + 2 = 7>5$
$5 + 3 = 8>5$
$5 + 4 = 9>5$
$5 + 5 = 10>5$
Therefore, the set of ordered pairs satisfying x + y< 5 = {(1,1), (1,2), (1,3), (2, 1), (2,2), (3,1)}.

(iii) $x + y > 8$
So, we find the ordered pair such that $x + y>8$, where x and y belongs to set A = {1, 2, 3, 4, 5}
$1 + 1 = 2<8$
$1 + 2 = 3<8$
$1 + 3 = 4<8$
$1 + 4 = 5<8$
$1 + 5 = 6<8$
$2 + 1 = 3<8$
$2 + 2 = 4<8$
$2 + 3 = 5<8$

2 + 4 = 6<8
2 + 5 = 7<8
3 + 1 = 4<8
3 + 2 = 5<8
3 + 3 = 6<8
3 + 4 = 7<8
3 + 5 = 8
4 + 1 = <8
4 + 2 = 6<8
4 + 3 = 7<8
4 + 4 = 8
4 + 5 = 9>8, so one of the ordered pairs is (4, 5)
5 + 1 = 6<8
5 + 2 = 7<8
5 + 3 = 8
5 + 4 = 9>8, so one of the ordered pairs is (5, 4)
5 + 5 = 10>8, so one of the ordered pairs is (5, 5)
Therefore, the set of ordered pairs satisfying x + y > 8 = {(4, 5), (5, 4), (5,5)}.

6. Given R = {(x, y) : x, y ∈ W, $x^2 + y^2 = 25$}. Find the domain and Range of R.
Solution:

According to the question,
R = {(x, y) : x, y ∈W, $x^2 + y^2 = 25$}
R = {(0,5), (3,4), (4, 3), (5,0)}
The domain of R consists of all the first elements of all the ordered pairs of R.
Domain of R = {0, 3, 4, 5}
The range of R contains all the second elements of all the ordered pairs of R.
Range of R = {5, 4, 3, 0}

7. If R_1 = {(x, y) | y = 2x + 7, where x ∈ R and – 5 ≤ x ≤ 5} is a relation. Then find the domain and Range of R_1.
Solution:

According to the question,
R_1 = {(x, y) | y = 2x + 7, where x ∈R and – 5 ≤ x ≤ 5} is a relation
The domain of R_1 consists of all the first elements of all the ordered pairs of R_1, i.e., x,
It is also given – 5 ≤ x ≤ 5.
Therefore,
Domain of R_1 = [–5, 5]
The range of R contains all the second elements of all the ordered pairs of R_1, i.e., y
It is also given y = 2x + 7
Now x ∈ [–5,5]
Multiply LHS and RHS by 2,
We get,
2x ∈ [–10, 10]
Adding LHS and RHS with 7,
We get,

$2x + 7 \in [-3, 17]$
Or, $y \in [-3, 17]$
So,
Range of $R_1 = [-3, 17]$

8. If $R_2 = \{(x, y) \mid x$ and y are integers and $x^2 + y^2 = 64\}$ is a relation. Then find R_2.
Solution:
We have,
$R_2 = \{(x, y) \mid x$ and y are integers and $x^2 + y^2 - 64\}$
So, we get,
$x^2 = 0$ and $y^2 = 64$ or $x^2 = 64$ and $y^2 = 0$
$x = 0$ and $y = \pm 8$ or $x = \pm 8$ and $y = 0$
Therefore, $R_2 = \{(0, 8), (0, -8), (8,0), (-8,0)\}$

9. If $R_3 = \{(x, |x|) \mid x$ is a real number$\}$ is a relation. Then find domain and range of R_3.
Solution:
According to the question,
$R_3 = \{(x, |x|) \mid x$ is a real number$\}$ is a relation
Domain of R_3 consists of all the first elements of all the ordered pairs of R_3, i.e., x,
It is also given that x is a real number,
So, Domain of $R_3 = R$
Range of R contains all the second elements of all the ordered pairs of R_3, i.e., $|x|$
It is also given that x is a real number,
So, $|x| = |R|$
$\Rightarrow |x| \geq 0$,
i.e., $|x|$ has all positive real numbers including 0
Hence,
Range of $R_3 = [0, \infty)$

10. Is the given relation a function? Give reasons for your answer.
(i) $h = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$
(ii) $f = \{(x, x) \mid x$ is a real number$\}$
(iii) $g = n, (1/n) \mid n$ is a positive integer
(iv) $s = \{(n, n^2) \mid n$ is a positive integer$\}$
(v) $t = \{(x, 3) \mid x$ is a real number.
Solution:
(i) According to the question,
$h = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$
Therefore, element 3 has two images, namely, 9 and 11.
A relation is said to be function if every element of one set has one and only one image in other set.
Hence, h is not a function.

(ii) According to the question,
$f = \{(x, x) \mid x$ is a real number$\}$
This means the relation f has elements which are real number.

Therefore, for every $x \in R$ there will be unique image.
A relation is said to be function if every element of one set has one and only one image in other set.
Hence, f is a function.

(iii) According to the question,
g = n, (1/n) |n is a positive integer
Therefore, the element n is a positive integer and the corresponding 1/n will be a unique and distinct number.
Therefore, every element in the domain has unique image.
A relation is said to be function if every element of one set has one and only one image in other set.
Hence, g is a function.

(iv) According to the question,
$s = \{(n, n^2) \mid n \text{ is a positive integer}\}$
Therefore, element n is a positive integer and the corresponding n^2 will be a unique and distinct number, as square of any positive integer is unique.
Therefore, every element in the domain has unique image.
A relation is said to be function if every element of one set has one and only one image in other set.
Hence, s is a function.

(v) According to the question,
t = {(x, 3) | x is a real number.
Therefore, the domain element x is a real number.
Also, range has one number i.e., 3 in it.
Therefore, for every element in the domain has the image 3, it is a constant function.
A relation is said to be function if every element of one set has one and only one image in other set.
Hence, t is a function.

11. **If f and g are real functions defined by $f(x) = x^2 + 7$ and $g(x) = 3x + 5$, find each of the following**
(a) f (3) + g (– 5)
(b) f(½) × g(14)
(c) f (– 2) + g (– 1)
(d) f (t) – f (– 2)
(e) (f(t) – f(5))/ (t – 5), if t ≠ 5
Solution:
According to the question,
f and g are real functions such that $f(x) = x^2 + 7$ and $g(x) = 3x + 5$

(a) f (3) + g (– 5)
$f(x) = x^2 + 7$
Substituting x = 3 in f(x), we get

$f(3) = 3^2 + 7 = 9 + 7 = 16$ …(i)
And,
$g(x) = 3x + 5$
Substituting $x = –5$ in $g(x)$, we get
$g(–5) = 3(–5) + 5 = –15 + 5 = –10$…………(ii)
Adding equations (i) and (ii),
We get,
$f(3) + g(–5) = 16–10 = 6$

(b) $f(½) \times g(14)$
$f(x) = x^2 + 7$
Substituting $x = ½$ in $f(x)$, we get
$f(½) = (½)^2 + 7 = ¼ + 7 = 29/4$ …(i)
And,
$g(x) = 3x + 5$
Substituting $x = 14$ in $g(x)$, we get
$g(14) = 3(14) + 5 = 42 + 5 = 47$…………(ii)
Multiplying equation (i) and (ii),
We get,
$f(½) \times g(14) = (29/4) \times 47 = 1363/4$

(c) $f(–2) + g(–1)$
$f(x) = x^2 + 7$
Substituting $x = –2$ in $f(x)$, we get
$f(–2) = (–2)^2 + 7 = 4 + 7 = 11$……..(i)
And,
$g(x) = 3x + 5$
Substituting $x = –1$ in $g(x)$, we get
$g(–1) = 3(–1) + 5$
$= –3 + 5 = 2$…………(ii)
Adding equation (i) and (ii),
We get,
$f(–2) + g(–1) = 11 + 2 = 13$

(d) $f(t) – f(–2)$
$f(x) = x^2 + 7$
Substituting $x = t$ in $f(x)$, we get
$f(t) = t^2 + 7$……..(i)
Considering the same function,
$f(x) = x^2 + 7$
Substituting $x = –2$ in $f(x)$, we get
$f(–2) = (–2)^2 + 7 = 4 + 7 = 11$…….(ii)
Subtracting equation (i) with (ii),
We get,
$f(t) – f(–2) = t^2 + 7 – 11 = t^2 – 4$

(e) (f(t) – f(5))/ (t – 5), if $t \neq 5$
$f(x) = x^2 + 7$
Substituting x = t in f(x), we get
$f(t) = t^2 + 7$……..(i)
Considering the same function,
$f(x) = x^2 + 7$
Substituting x = 5 in f(x), we get
$f(5) = (5)^2 + 7 = 25 + 7 = 32$……..(ii)
From equation (i) and (ii), we get

$$\frac{f(t) - f(5)}{t-5} = \frac{t^2 + 7 - 32}{t-5}$$

$$= \frac{t^2 - 25}{t-5}$$

$$= \frac{t^2 - 5^2}{t-5}$$

But we know $a^2-b^2 = (a + b)(a-b)$, so above equation becomes,

$$\frac{f(t) - f(5)}{t-5} = \frac{(t + 5)(t-5)}{t-5}$$

Cancelling the like terms, we get

$$\frac{f(t) - f(5)}{t-5} = t + 5$$

12. Let f and g be real functions defined by f (x) = 2x + 1 and g (x) = 4x – 7.
(a) For what real numbers x, f (x) = g (x)?
(b) For what real numbers x, f (x) < g (x)?
Solution:

According to the question,
f and g be real functions defined by f(x) = 2x + 1 and g(x) = 4x – 7
(a) For what real numbers x, f (x) = g (x)
To satisfy the condition f(x) = g(x),
Should also satisfy,
2x + 1 = 4x–7
⇒ 7 + 1 = 4x–2x
⇒ 8 = 2x
Or, 2x = 8
⇒ x = 4
Hence, we get,
For x = 4, f (x) = g (x)

(b) For what real numbers x, f (x) < g (x)
To satisfy the condition f(x) < g(x),
Should also satisfy,
2x + 1 < 4x–7

$\Rightarrow 7 + 1 < 4x–2x$
$\Rightarrow 8 < 2x$
Or, $2x > 8$
$\Rightarrow x > 4$
Hence, we get,
For $x > 4$, $f(x) > g(x)$

13. If f and g are two real valued functions defined as f (x) = 2x + 1, g (x) = x^2 + 1, then find.
(i) f + g (ii) f – g (iii) fg (iv)f/g
Solution:

According to the question,
f and g be real valued functions defined as $f(x) = 2x + 1$, $g(x) = x^2 + 1$,
(i) f + g
$\Rightarrow f + g = f(x) + g(x)$
$= 2x + 1 + x^2 + 1$
$= x^2 + 2x + 2$

(ii) f – g
$\Rightarrow f – g = f(x) – g(x)$
$= 2x + 1 – (x^2 + 1)$
$= 2x – x^2$

(iii) fg
$\Rightarrow fg = f(x)\, g(x)$
$= (2x + 1)(x^2 + 1)$
$= 2x(x^2) + 2x(1) + 1(x^2) + 1(1)$
$= 2x^3 + 2x + x^2 + 1$
$= 2x^3 + x^2 + 2x + 1$

(iv) f/g
$f/g = f(x)/g(x)$
$$\Rightarrow \frac{f}{g} = \frac{(2x + 1)}{x^2 + 1}$$

14. Express the following functions as set of ordered pairs and determine their range.
f: X → R, f (x) = x^3 + 1, where X = {–1, 0, 3, 9, 7}
Solution:

According to the question,
A function f: X →R, $f(x) = x^3 + 1$, where X = {–1, 0, 3, 9, 7}
Domain = f is a function such that the first elements of all the ordered pair belong to the set X = {–1, 0, 3, 9, 7}.
The second element of all the ordered pair are such that they satisfy the condition $f(x) = x^3 + 1$
When x = – 1,
$f(x) = x^3 + 1$
$f(– 1) = (– 1)^3 + 1 = – 1 + 1 = 0 \Rightarrow$ ordered pair = (–1, 0)

When $x = 0$,
$f(x) = x^3 + 1$
$f(0) = (0)^3 + 1 = 0 + 1 = 1 \Rightarrow$ ordered pair $= (0, 1)$
When $x = 3$,
$f(x) = x^3 + 1$
$f(3) = (3)^3 + 1 = 27 + 1 = 28 \Rightarrow$ ordered pair $= (3, 28)$
When $x = 9$,
$f(x) = x^3 + 1$
$f(9) = (9)^3 + 1 = 729 + 1 = 730 \Rightarrow$ ordered pair $= (9, 730)$
When $x = 7$,
$f(x) = x^3 + 1$
$f(7) = (7)^3 + 1 = 343 + 1 = 344 \Rightarrow$ ordered pair $= (7, 344)$
Therefore, the given function as a set of ordered pairs is
$f = \{(-1, 0), (0, 1), (3, 28), (7, 344), (9, 730)\}$
And,
Range of $f = \{0, 1, 28, 730, 344\}$

15. Find the values of x for which the functions
$f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal
Solution:
According to the question,
f and g functions defined by $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$
For what real numbers x, $f(x) = g(x)$
To satisfy the condition $f(x) = g(x)$,
Should also satisfy,
$3x^2 - 1 = 3 + x$
$\Rightarrow 3x^2 - x - 3 - 1 = 0$
$\Rightarrow 3x^2 - x - 4 = 0$
Splitting the middle term,
We get,
$\Rightarrow 3x^2 + 3x - 4x - 4 = 0$
$\Rightarrow 3x(x + 1) - 4(x + 1) = 0$
$\Rightarrow (3x - 4)(x + 1) = 0$
$\Rightarrow 3x - 4 = 0$ or $x + 1 = 0$
$\Rightarrow 3x = 4$ or $x = -1$
$\Rightarrow x = 4/3, -1$
Hence, for $x = 4/3, -1$, $f(x) = g(x)$,
i.e., given functions are equal.

LONG ANSWER TYPE

16. Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ a function? Justify. If this is described by the relation, $g(x) = \alpha x + \beta$, then what values should be assigned to α and β?
Solution:
According to the question,
$g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$, and is described by relation $g(x) = \alpha x + \beta$

Now, given the relation,
g = {(1, 1), (2, 3), (3, 5), (4, 7)}
g (x) = αx + β
For ordered pair (1,1), g (x) = αx + β, becomes
g (1) = α(1) + β = 1
⇒ α + β = 1
⇒ α = 1 – β …(i)
Considering ordered pair (2, 3), g (x) = αx + β, becomes
g (2) = α(2) + β = 3
⇒ 2α + β = 3
Substituting value of α from equation (i), we get
⇒ 2(2) + β = 3
⇒ β = 3 – 4 = – 1
Substituting value of β in equation (i), we get
α = 1–β = 1–(–1) = 2
Now, the given equation becomes,
i.e., g (x) = 2x–1

17. Find the domain of each of the following functions given by

(i) $f(x) = \dfrac{1}{\sqrt{1-\cos x}}$

(ii) $f(x) = \dfrac{1}{\sqrt{x+|x|}}$

(iii) $f(x) = x\,|x|$

(iv) $f(x) = \dfrac{x^3 - x + 3}{x^2 - 1}$

(v) $f(x) = \dfrac{3x}{2x-8}$

Solution:

(i)
$f(x) = \dfrac{1}{\sqrt{1-\cos x}}$
According to the question,
We know the value of cos x lies between –1, 1,
–1 ≤ cos x ≤ 1
Multiplying by negative sign, we get
Or 1 ≥ – cos x ≥ –1
Adding 1, we get
2 ≥ 1– cos x ≥ 0 …(i)
Now,

$$f(x) = \frac{1}{\sqrt{1-\cos x}},$$

$1 - \cos x \neq 0$
$\Rightarrow \cos x \neq 1$
Or, $x \neq 2n\pi \ \forall \ n \in Z$
Therefore, the domain of $f = R - \{2n\pi : n \in Z\}$

(ii)

$$f(x) = \frac{1}{\sqrt{x + |x|}}$$

According to the question,
For real value of f,
$x + |x| > 0$
When $x > 0$,
$x + |x| > 0 \Rightarrow x + x > 0 \Rightarrow 2x > 0 \Rightarrow x > 0$
When $x < 0$,
$x + |x| > 0 \Rightarrow x - x > 0 \Rightarrow 2x > 0 \Rightarrow x > 0$
So, $x > 0$, to satisfy the given equation.
Therefore, the domain of $f = R^+$

(iii)
$f(x) = x|x|$
According to the question,
We know x and |x| are defined for all real values.
Therefore, the domain of f = R

(iv)

$$f(x) = \frac{(x^3 - x + 3)}{x^2 - 1}$$

According to the question,
For real value of
$x^2 - 1 \neq 0$
$\Rightarrow (x-1)(x + 1) \neq 0$
$\Rightarrow x-1 \neq 0$ or $x + 1 \neq 0$
$\Rightarrow x \neq 1$ or $x \neq -1$
Therefore, the domain of $f = R - \{-1, 1\}$

(v)

$$f(x) = \frac{3x}{2x - 8}$$

According to the question,
For real value of
$28 - x \neq 0$
$\Rightarrow x \neq 28$

Therefore, the domain of f = R–{28}

18. Find the range of the following functions given by

(i) $f(x) = \dfrac{3}{2-x^2}$

(ii) $f(x) = 1 - |x-2|$

(iii) $f(x) = |x-3|$

(iv) $f(x) = 1 + 3\cos 2x$

Solution:

(i)

$$f(x) = \frac{3}{2-x^2}$$

According to the question,

Let f(x) = y,

$$y = \frac{3}{2-x^2}$$

$$\Rightarrow 2 - x^2 = \frac{3}{y}$$

$$\Rightarrow x^2 = 2 - \frac{3}{y}$$

But, we know that, $x^2 \geq 0$

$$2 - \frac{3}{y} \geq 0$$

$$\Rightarrow \frac{2y-3}{y} \geq 0$$

⇒ y>0 and 2y–3≥0

⇒ y>0 and 2y≥3

⇒ y>0 and y ≥ 3/2

Or f(x)>0 and f(x) ≥ 3/2

f(x) ∈ (– ∞, 0) ∪ [3/2 , ∞)

$$\Rightarrow f(x) \in (-\infty, 0) \cup \left[\frac{3}{2}, \infty\right)$$

Therefore, the range of f = (– ∞, 0) ∪ [3/2 , ∞)

(ii) f(x) = 1–|x–2|

According to the question,

For real value of f,

|x–2|≥ 0

Adding negative sign, we get

Or –|x–2|≤ 0

Adding 1 we get

⇒ 1–|x–2|≤ 1

Or f(x)≤1

$\Rightarrow$ f(x)$\in$ $(-\infty, 1]$
Therefore, the range of f = $(-\infty, 1]$

(iii) f(x) = |x–3|
According to the question,
We know |x| are defined for all real values.
And |x–3| will always be greater than or equal to 0.
i.e., f(x) $\geq$ 0
Therefore, the range of f = $[0, \infty)$

(iv) f (x) = 1 + 3 cos2x
According to the question,
We know the value of cos 2x lies between –1, 1, so
$-1 \leq \cos 2x \leq 1$
Multiplying by 3, we get
$-3 \leq 3\cos 2x \leq 3$
Adding 1, we get
$-2 \leq 1 + 3\cos 2x \leq 4$
Or, $-2 \leq f(x) \leq 4$
Hence f(x)$\in$ [–2, 4]
Therefore, the range of f = [–2, 4]

19. Redefine the function f(x) = |x – 2| + |2 + x|, – 3 ≤ x ≤ 3
Solution:

According to the question,
function f(x) = |x–2| + |2 + x|, $-3 \leq x \leq 3$
We know that,
when x>0,
|x – 2| is (x–2), x≥2
|2 + x| is (2 + x), x≥–2
when x>0
|x – 2| is –(x–2), x<2
|2 + x| is –(2 + x), x<–2
Given that, f(x) = |x–2| + |2 + x|, $-3 \leq x \leq 3$
It can be rewritten as,

$$f(x) = \begin{cases} -(x-2) - (2+x), -3 \leq x < -2 \\ -(x-2) + (2+x), -2 \leq x < 2 \\ (x-2) + (2+x), 2 \leq x \leq 3 \end{cases}$$

Or

$$f(x) = \begin{cases} -x + 2 - 2 - x, -3 \leq x < -2 \\ -x + 2 + 2 + x, -2 \leq x < 2 \\ x - 2 + 2 + x, 2 \leq x \leq 3 \end{cases}$$

Or,

$$f(x) = \begin{cases} -2x, -3 \le x < -2 \\ 4, -2 \le x < 2 \\ 2x, 2 \le x \le 3 \end{cases}$$

20. If

$f(x) = \dfrac{x-1}{x+1}$**, then show that:**

(i) $f\left(\dfrac{1}{x}\right) = -f(x)$

(ii) $f\left(-\dfrac{1}{x}\right) = \dfrac{-1}{f(x)}$

Solution:

(i)

$$f(x) = \frac{x-1}{x+1}$$

Substituting x by 1/x, we get

$$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x} - 1}{\frac{1}{x} + 1}$$

$$= \frac{\frac{1-x}{x}}{\frac{1+x}{x}}$$

$$= \frac{1-x}{1+x}$$

$$= \frac{-(x-1)}{1+x}$$

$$= -\frac{x-1}{x+1}$$

Therefore,

We get,

$$f\left(\frac{1}{x}\right) = -f(x)$$

Hence proved

(ii)

$$f(x) = \frac{x-1}{x+1}$$

Substituting x by – 1/x, we get

$$f\left(-\frac{1}{x}\right) = \frac{\left(-\frac{1}{x}\right) - 1}{\left(-\frac{1}{x}\right) + 1}$$

$$= \frac{\frac{-1 - x}{x}}{\frac{-1 + x}{x}}$$

$$= \frac{-1 - x}{-1 + x}$$

$$= \frac{-(x + 1)}{x - 1}$$

$$= \frac{-1}{\frac{x - 1}{x + 1}}$$

Therefore,

$$f\left(-\frac{1}{x}\right) = \frac{-1}{f(x)}$$

Hence proved

21. Let f(x) = √x and g (x) = x be two functions defined in the domain $R^+ \cup \{0\}$. Find
(i) (f + g) (x)
(ii) (f – g) (x)
(iii) (fg) (x)
(iv) (f/g) (x)
Solution:

(i)
(f + g)(x)
⇒ (f + g)(x) = f(x) + g(x)
⇒ f(x) + g(x) = √x + x

(ii)
(f – g)(x)
⇒ (f – g)(x) = f(x) – g(x)
⇒ f(x) – g(x) = √x–x

(iii)
(fg)(x)
⇒ (fg)(x) = f(x) g(x)
⇒ (fg)(x) = (√x)(x)
⇒ f(x)g(x)= x√x

(iv)
(f/q)(x) = f(x)/g(x)

$$\Rightarrow \left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{x}$$

Multiplying and dividing by √x,

We get

$$= \frac{\sqrt{x}}{x} \times \frac{\sqrt{x}}{\sqrt{x}}$$

$$= \frac{x}{x\sqrt{x}}$$

$$\Rightarrow \left(\frac{f}{g}\right)(x) = \frac{1}{\sqrt{x}}$$