EXERCISE

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SHORT ANSWER TYPE

1. Prove that

$$\frac{\tan A+\sec A-1}{\tan A-\sec A+1}=\frac{1+\sin A}{\cos A}$$

Solution:

According to the question,

$$LHS = \frac{\tan A+\sec A-1}{\tan A-\sec A+1}$$

$$= \frac{\frac{\sin A}{\cos A}+\frac{1}{\cos A}-1}{\frac{\sin A}{\cos A}-\frac{1}{\cos A}+1}$$

$$= \frac{\sin A + 1 - \cos A}{\sin A - 1 + \cos A}$$

$$= \frac{\sin A + (1-\cos A)}{\sin A - (1-\cos A)}$$

Using the identity,

$\sin^2 A + \cos^2 A = 1$, we get,

$\sin A + (1 - \cos A)$.

$$\therefore LHS = \frac{\sin A+(1-\cos A)}{\sin A-(1-\cos A)} \times \frac{\sin A+(1-\cos A)}{\sin A+(1-\cos A)}$$

$$= \frac{\{\sin A + (1-\cos A)\}^2}{\sin^2 A - (1-\cos A)^2}$$

$$= \frac{\sin^2 A + (1-\cos A)^2 + 2\sin A(1-\cos A)}{\sin^2 A - (1-\cos A)^2}$$

$$= \frac{(\sin^2 A + \cos^2 A) + 1 - 2\cos A + 2\sin A(1-\cos A)}{\sin^2 A - \{1+\cos^2 A - 2\cos A\}}$$

$$= \frac{(1) + 1 - 2\cos A + 2\sin A(1-\cos A)}{(\sin^2 A - 1) - \cos^2 A + 2\cos A}$$

$$= \frac{2(1-\cos A) + 2\sin A(1-\cos A)}{(-\cos^2 A) - \cos^2 A + 2\cos A}$$

$$= \frac{2(1+\sin A)(1-\cos A)}{-2\cos^2 A + 2\cos A}$$

$$= \frac{2(1+\sin A)(1-\cos A)}{2\cos A(1-\cos A)}$$

$$= \frac{(1+\sin A)}{\cos A} = RHS$$

Hence, L.H.S = R.H.S

2. If [2sinα / (1+cosα+sinα)] = y, then prove that [(1– cosα+sinα) / (1+sinα)] is also equal to y.

$$\left[\text{Hint : Express } \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} = \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}.\frac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}\right]$$

Solution:

According to the question,

y =2sinα /(1+cosα+sinα)

Multiplying numerator and denominator by (1 – cos α + sin α),

We get,

$$\Rightarrow y = \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} \times \frac{1-\cos\alpha+\sin\alpha}{1-\cos\alpha+\sin\alpha}$$
$$= \frac{2\sin\alpha}{(1+\sin\alpha)+\cos\alpha} \times \frac{(1+\sin\alpha)-\cos\alpha}{(1+\sin\alpha)-\cos\alpha}$$

Using $(a + b)(a-b) = a^2 - b^2$, we get:

$$= \frac{2\sin\alpha\{(1+\sin\alpha)-\cos\alpha\}}{(1+\sin\alpha)^2-\cos^2\alpha}$$
$$= \frac{2\sin\alpha(1+\sin\alpha)-2\sin\alpha\cos\alpha}{1+\sin^2\alpha+2\sin\alpha-\cos^2\alpha}$$

Since, $1 - \cos^2\alpha = \sin^2\alpha$

$$\therefore y = \frac{2\sin\alpha(1+\sin\alpha-\cos\alpha)}{\sin^2\alpha+2\sin\alpha+\sin^2\alpha}$$
$$= \frac{2\sin\alpha(1+\sin\alpha-\cos\alpha)}{2\sin\alpha(1+\sin\alpha)}$$
$$\Rightarrow y = \frac{(1+\sin\alpha-\cos\alpha)}{(1+\sin\alpha)}$$
$$\Rightarrow y = \frac{(1-\cos\alpha+\sin\alpha)}{(1+\sin\alpha)}$$

Hence Proved

3. If m sin θ = n sin (θ + 2α), then prove that
tan (θ + α) cot α = (m + n)/(m – n)
[Hints: Express sin(θ + 2α) / sinθ = m/n and apply componendo and dividend]
Solution:

According to the question,

m sin θ = n sin (θ + 2α)

To prove:

tan (θ + α)cot α =(m + n)/(m – n)

Proof:

m sin θ = n sin (θ + 2α)

⇒ sin(θ + 2α) / sinθ = m/n

Applying componendo-dividendo rule, we have,

$$\Rightarrow \frac{\sin(\theta+2\alpha)+\sin\theta}{\sin(\theta+2\alpha)-\sin\theta} = \frac{m+n}{m-n}$$

By transformation formula of T-ratios,

We know that,

sin A + sin B = 2 sin ((A+B)/2) cos ((A – B)/2)

And,

sin A – sin B = 2 cos ((A+B)/2) sin ((A – B)/2)

On applying the formula, we get,

$$\frac{2\sin\left(\frac{2\theta+2\alpha}{2}\right)\cos\left(\frac{\theta+2\alpha-\theta}{2}\right)}{2\cos\left(\frac{2\theta+2\alpha}{2}\right)\sin\left(\frac{\theta+2\alpha-\theta}{2}\right)} = \frac{m+n}{m-n}$$

$$\Rightarrow \frac{\sin(\theta+\alpha)\cos(\alpha)}{\cos(\theta+\alpha)\sin(\alpha)} = \frac{m+n}{m-n}$$

{∵ tan x = (sin x)/(cos x)}

$$\Rightarrow \tan(\theta+\alpha)\cot\alpha = \frac{m+n}{m-n}$$

Therefore, tan (θ + α) cot α = (m + n)/(m – n)

Hence Proved

4. If $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$, **where α lie between 0 and π/4, find value of tan 2α**

[Hint: Express tan 2α as tan (α + β + α – β)]

Solution:

According to the question,

cos(α + β) = 4/5 …(i)

We know that,

sin x = √(1 – $\cos^2$x)

Therefore,

sin (α + β) = √(1 – $\cos^2$(α + β))

⇒ sin (α + β) = √(1 – $(4/5)^2$) = 3/5 …(ii)

Also,

sin(α - β) = 5/13 {given} …(iii)

we know that,

cos x = √(1 – $\sin^2$x)

Therefore,

cos (α - β) = √(1 – $\sin^2$(α - β))

⇒ cos (α - β) = √(1 – $(5/13)^2$) = 12/13 …(iv)

Therefore,

tan 2α = tan (α + β + α – β)

We know that,

$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$\therefore \tan 2\alpha = \frac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha-\beta)}$

$\Rightarrow \tan 2\alpha = \frac{\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} + \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}}{1 - \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} \times \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}}$

From equation i, ii, iii and iv we have,

$$\Rightarrow \tan 2\alpha = \frac{\frac{\frac{3}{5}}{\frac{4}{5}} + \frac{\frac{5}{13}}{\frac{12}{13}}}{1 - \frac{\frac{3}{5}}{\frac{4}{5}} \times \frac{\frac{5}{13}}{\frac{12}{13}}}$$

$$= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \times \frac{5}{12}}$$

$$= \frac{\frac{9+5}{12}}{1 - \frac{15}{48}}$$

$$\Rightarrow \tan 2\alpha = \frac{14}{12\left(\frac{33}{48}\right)}$$

$$= \frac{56}{33}$$

Hence, tan 2α = 56/33

5. If tanx = b/a then find the value of

$\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}.$

Solution:

According to the question,

tan x = b/a

Let,

$$y = \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$$

$$\therefore y = \sqrt{\frac{a\left(1+\frac{b}{a}\right)}{a\left(1-\frac{b}{a}\right)}} + \sqrt{\frac{a\left(1-\frac{b}{a}\right)}{a\left(1+\frac{b}{a}\right)}}$$

$$= \sqrt{\frac{(1+\tan x)}{(1-\tan x)}} + \sqrt{\frac{(1-\tan x)}{(1+\tan x)}}$$

$$= \frac{\sqrt{1+\tan x}}{\sqrt{1-\tan x}} + \frac{\sqrt{1-\tan x}}{\sqrt{1+\tan x}}$$

$$= \frac{\left(\sqrt{1+\tan x}\right)^2 + \left(\sqrt{1-\tan x}\right)^2}{\left(\sqrt{1-\tan x}\right)\left(\sqrt{1+\tan x}\right)}$$

$$= \frac{1+\tan x + 1 - \tan x}{\sqrt{1-\tan^2 x}} = \frac{2}{\sqrt{1-\tan^2 x}}$$

$$\therefore y = \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \frac{2}{\sqrt{1-\tan^2 x}}$$

$$= \frac{2}{\sqrt{1-\frac{sin^2\theta}{cos^2\theta}}}$$

$$= \frac{2}{\frac{\sqrt{cos^2\theta - sin^2\theta}}{cos\theta}}$$

$$\because cos^2\theta - sin^2\theta = cos2\theta$$

$$= \frac{2cos\theta}{cos2\theta}$$

6. Prove that cos θ cos θ/2 – cos 3θ cos 9θ/2 = sin7θ sin4θ
[Hint: Express L.H.S. = ½ [2cos θcos θ/2 – 2cos 3θ cos 9θ / 2]
Solution:

Using transformation formula, we get,
2 cos A cos B = cos(A + B) + cos (A – B)
-2 sin A sin B = cos(A + B) - cos (A – B)
Multiplying and dividing the expression by 2.

$\therefore$ LHS $= \frac{1}{2}\left(2\cos\theta\cos\frac{\theta}{2} - 2\cos 3\theta\cos\frac{9\theta}{2}\right)$

Applying transformation formula, we get,

LHS $= \frac{1}{2}\left(\cos\left(\theta + \frac{\theta}{2}\right) + \cos\left(\theta - \frac{\theta}{2}\right) - \left\{\cos\left(3\theta + \frac{9\theta}{2}\right) + \cos\left(3\theta - \frac{9\theta}{2}\right)\right\}\right)$

$\Rightarrow$ LHS $= \frac{1}{2}\left(\cos\frac{3\theta}{2} + \cos\frac{\theta}{2} - \cos\left(\frac{15\theta}{2}\right) - \cos\left(-\frac{3\theta}{2}\right)\right)$

$\Rightarrow$ LHS $= \frac{1}{2}\left(\cos\frac{3\theta}{2} + \cos\frac{\theta}{2} - \cos\frac{15\theta}{2} - \cos\frac{3\theta}{2}\right)$ {∵ cos (-x) = cos x}

$\Rightarrow$ LHS $= \frac{1}{2}\left(\cos\frac{\theta}{2} - \cos\frac{15\theta}{2}\right)$

$\Rightarrow$ LHS $= \frac{1}{2}\left(2\sin\left(\frac{\frac{\theta}{2}+\frac{15\theta}{2}}{2}\right)\sin\left(\frac{\frac{15\theta}{2}-\frac{\theta}{2}}{2}\right)\right)$

$\Rightarrow$ LHS $= \frac{1}{2}\left(2\sin\left(\frac{8\theta}{2}\right)\sin\left(\frac{7\theta}{2}\right)\right)$

$\therefore$ LHS $= \sin 4\theta \sin\left(\frac{7\theta}{2}\right)$ = RHS

Hence,

$\cos\theta\cos\frac{\theta}{2} - \cos 3\theta\cos\frac{9\theta}{2} = \sin 4\theta\sin\left(\frac{7\theta}{2}\right)$

7. If a cos θ + b sin θ = m and a sin θ – b cos θ = n, then show that $a^2 + b^2 = m^2 + n^2$.

Solution:

According to the question,

a cos θ + b sin θ = m …(i)

a sin θ – b cos θ = n …(ii)

Squaring and adding equation 1 and 2, we get,

$(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2 = m^2 + n^2$

$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta = m^2 + n^2$

$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta = m^2 + n^2$

$\Rightarrow a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = m^2 + n^2$

Using, $\sin^2\theta + \cos^2\theta = 1$,

We get,

$\Rightarrow a^2 + b^2 = m^2 + n^2$

8. Find the value of tan 22°30’.

[Hint: Let θ = 45°, use $\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \frac{\sin\theta}{1+\cos\theta}$]

Solution:

Let, θ = 45°

As we need to find: tan 22°30’ = tan (θ/2)

We know that,

sin θ = cos θ = 1/√2 (for θ = 45°)

Since,

$\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$

Multiplying 2cos θ/2 in numerator and denominator, we get,

$\Rightarrow \tan\frac{\theta}{2} = \frac{2\cos\frac{\theta}{2}\sin\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$

By applying formula of T-ratios of multiple angles-

sin 2x = 2sin x cos x

cos 2x = $2\cos^2 x - 1$ or $1 + \cos 2x = 2\cos^2 x$

$\therefore \tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$

$\Rightarrow \tan 22°30' = \frac{\sin 45°}{1+\cos 45°}$

$= \frac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$

$= \frac{1}{\sqrt{2}+1}$

Therefore, tan 22°30' = √2 – 1

9. Prove that sin 4A = 4sinA $\cos^3$A – 4 cosA $\sin^3$A.

Solution:

sin 4A = sin (2A + 2A)

We know that,

sin(A + B) = sin A cos B + cos A sin B

Therefore, sin 4A = sin 2A cos 2A + cos 2A sin 2A

⇒ sin 4A = 2 sin 2A cos 2A

From T-ratios of multiple angle,

We get,

sin 2A = 2 sin A cos A and cos 2A = $\cos^2$A – $\sin^2$A

⇒ sin 4A = 2(2 sin A cos A)($\cos^2$A – $\sin^2$A)

⇒ sin 4A = 4 sin A $\cos^3$A – 4 cos A $\sin^3$A

Hence, sin 4A = 4 sin A $\cos^3$A – 4 cos A $\sin^3$A

10. If tan θ + sin θ = m and tan θ – sin θ = n, then prove that $m^2 - n^2$ = 4 sin θ tan θ
[Hint: m + n = 2tanθ, m – n = 2 sin θ, then use $m^2 - n^2$ = (m + n)(m – n)]

Solution:

According to the question,

tan θ + sin θ = m …(i)

tan θ – sin θ = n …(ii)

Adding equation i and ii,

2 tan θ = m + n …(iii)

Subtracting equation ii from i,

We get,

2sin θ = m – n …(iv)
Multiplying equations (iii) and (iv),
2sin θ (2tan θ) = (m + n)(m – n)
⇒ 4 sin θ tan θ = $m^2 - n^2$
Hence,
$m^2 - n^2$ = 4 sin θ tan θ

11. If tan (A + B) = p, tan (A – B) = q, then show that tan 2A = (p + q) / (1 – pq).
[Hint: Use 2A = (A + B) + (A – B)]
Solution:

We know that,
tan 2A = tan (A + B + A – B)
And also,

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\therefore \tan 2A = \frac{\tan(A+B) + \tan(A-B)}{1 - \tan(A+B)\tan(A-B)}$$

Substituting the values given in question,

$$\Rightarrow \tan 2A = \frac{p+q}{1-pq}$$

$$\text{Hence, } \tan 2A = \frac{p+q}{1-pq}$$

12. If cosα + cosβ = 0 = sinα + sinβ, then prove that cos 2α + cos 2β = –2cos (α + β).
[Hint: cosα + cosβ)^2 – (sinα + sinβ)^2 = 0]
Solution:

According to the question,
cosα + cosβ = 0 = sinα + sinβ …(i)
Since, LHS = cos 2α + cos 2β
We know that,
cos 2x = $\cos^2 x - \sin^2 x$
Therefore,
LHS = $\cos^2\alpha - \sin^2\alpha + (\cos^2\beta - \sin^2\beta)$
⇒ LHS = $\cos^2\alpha + \cos^2\beta - (\sin^2\alpha + \sin^2\beta)$
Also, since,
$a^2 + b^2 = (a+b)^2 - 2ab$
⇒ LHS = $(\cos\alpha + \cos\beta)^2 - 2\cos\alpha\cos\beta - (\sin\alpha + \sin\beta)^2 + 2\sin\alpha\sin\beta$
From equation (i),
⇒ LHS = 0 - 2cosα cosβ -0 + 2sinα sinβ
⇒ LHS = -2(cosα cosβ – sinα sinβ)
∵ cos (α + β) = cosα cosβ – sinα sinβ
Therefore, LHS = -2 cos (α + β) = RHS
Hence, cos 2α + cos 2β = –2cos (α + β)

13.

If $\frac{\sin(x+y)}{\sin(x-y)}=\frac{a+b}{a-b}$, **then show that** $\frac{\tan x}{\tan y}=\frac{a}{b}$.

[Hint: Use componendo and Dividendo]

Solution:

According to the question,

$\frac{\sin(x+y)}{\sin(x-y)}=\frac{a+b}{a-b}$

Since, sin(A+B) = sin A cos B + cos A sin B

$\therefore \frac{\sin(x+y)}{\sin(x-y)}=\frac{a+b}{a-b}$

$\Rightarrow \frac{\sin x\cos y+\cos x\sin y}{\sin x\cos y-\cos x\sin y}=\frac{a+b}{a-b}$

Applying componendo-dividendo rule,

We get,

$\Rightarrow \frac{(\sin x\cos y+\cos x\sin y)+(\sin x\cos y-\cos x\sin y)}{(\sin x\cos y+\cos x\sin y)-(\sin x\cos y-\cos x\sin y)}=\frac{(a+b)+(a-b)}{(a+b)-(a-b)}$

$\Rightarrow \frac{2\sin x\cos y}{2\cos x\sin y}=\frac{2a}{2b}$

$\Rightarrow \left(\frac{\sin x}{\cos x}\right)\left(\frac{\cos y}{\sin y}\right)=\frac{a}{b}$

Since, tan A = (sinA)/cos(A)

$\Rightarrow \tan x\left(\frac{1}{\tan y}\right)=\frac{a}{b}$

$\Rightarrow \frac{\tan x}{\tan y}=\frac{a}{b}$

14.

If $\tan\theta=\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}$, **then show that sinα + cosα = √2 cos θ.**

[Hint: Express tanθ = tan(α – π/2) θ = α – π/4]

Solution:

We know that,

$\tan\theta=\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}$

$\Rightarrow \tan\theta=\frac{\cos\alpha\left(\frac{\sin\alpha}{\cos\alpha}-1\right)}{\cos\alpha\left(\frac{\sin\alpha}{\cos\alpha}+1\right)}$

Since, tan A = (sin A)/(cos A)

$\Rightarrow \tan\theta = (\tan\alpha - 1) / (\tan\alpha + 1)$

Since, tan π/4 = 1

$\Rightarrow \tan\theta=\frac{\left(\tan\alpha-\tan\frac{\pi}{4}\right)}{\left(1+\tan\frac{\pi}{4}.\tan\alpha\right)}$

We know that,

tan(x-y) = (tan x – tan y) / (1 + tan x . tan y)
Therefore, tan θ = tan (α – π/4)
⇒ θ = α - π/4
⇒ α = θ + π/4 …(i)
To prove,
sinα + cosα = √2 cos θ
∵ LHS = sinα + cosα
From equation (i)
⇒ LHS = sin(θ + π/4) + cos(θ + π/4)
∵ sin(x + y) = sin x cos y + cos x sin y
And, cos(x + y) = cos x cos y – sin x sin y
Therefore, LHS = sin θ cos(π/4) + sin(π/4)cos θ + cos θ cos(π/4) - sin(π/4)sin θ
∵ sin(π/4)=cos(π/4) = 1/√2
⇒ LHS = sin θ (1/√2) + (1/√2) cos θ + cos θ (1/√2) – sin θ (1/√2)
⇒ LHS = 2 cos θ (1/√2)
⇒ LHS = √2 cos θ = RHS
Therefore, sinα + cosα = √2 cos θ

15. If sin θ + cos θ = 1, then find the general value of θ.
Solution:

According to the question,
sin θ + cos θ = 1
As, sin θ + cos θ = 1
⇒ $\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta\right) = 1$
We know that,
sin (π/4) = cos(π/4) = 1/√2
⇒ $\sqrt{2}\left(\sin\theta\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\theta\right) = 1$
We know that,
sin (A+B) = sinAcosB + cosAsinB
⇒ $\sin\left(\frac{\pi}{4} + \theta\right) = \frac{1}{\sqrt{2}}$
⇒ $\sin\left(\frac{\pi}{4} + \theta\right) = \sin\frac{\pi}{4}$
Since we know,
If sin θ = sinα ⇒ θ = nπ + $(-1)^n$α
We get,
θ + π/4 = nπ + $(-1)^n$(π/4)
⇒ θ = nπ + (π/4)($(-1)^n$ – 1)

16. Find the most general value of θ satisfying the equation tan θ = –1 and cos θ = 1/√2
Solution:

According to the question,
We have,

$\tan\theta = -1$
And $\cos\theta = 1/\sqrt{2}$.
$\Rightarrow \theta = -\pi/4$
So, we know that,
θ lies in IV quadrant.
$\theta = 2\pi - \pi/4 = 7\pi/4$
So, general solution is $\theta = 7\pi/4 + 2n\pi$, $n \in Z$

17. If cot θ + tan θ = 2 cosec θ, then find the general value of θ.
Solution:

According to the question,
$\Rightarrow \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = 2\,\text{cosec}\,\theta$
Since,
$\sin^2\theta + \cos^2\theta = 1$
$\Rightarrow \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} = 2\,\text{cosec}\,\theta$
$\Rightarrow 1 = 2\,\text{cosec}\,\theta\sin\theta\cos\theta$
We know that,
$\sin\theta\,\text{cosec}\,\theta = 1$
$\Rightarrow 1 = 2\cos\theta$
$\Rightarrow \cos\theta = 1/2 = \cos(\pi/3)$
Hence,
The solution of $\cos x = \cos\alpha$ can be given by,
$x = 2m\pi \pm \alpha\ \forall\ m \in Z$
$\Rightarrow \theta = 2n\pi \pm \pi/3$, $n \in Z$

18. If $2\sin^2\theta = 3\cos\theta$, where $0 \le \theta \le 2\pi$, then find the value of θ.
Solution:

According to the question,
$2\sin^2\theta = 3\cos\theta$
We know that,
$\sin^2\theta = 1 - \cos^2\theta$
Given that,
$2\sin^2\theta = 3\cos\theta$
$2 - 2\cos^2\theta = 3\cos\theta$
$2\cos^2\theta + 3\cos\theta - 2 = 0$
$(\cos\theta + 2)(2\cos\theta - 1) = 0$
Therefore,

$\cos\theta = ½ = \cos\pi/3$
$\theta = \pi/3$ or $2\pi - \pi/3$
$\theta = \pi/3, 5\pi/3$

Therefore, $2(1 - \cos^2\theta) = 3\cos\theta$
$\Rightarrow 2 - 2\cos^2\theta = 3\cos\theta$
$\Rightarrow 2\cos^2\theta + 3\cos\theta - 2 = 0$
$\Rightarrow 2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$

$\Rightarrow$ 2cos θ (cos θ+ 2) +1 (cos θ + 2) = 0
$\Rightarrow$ (2cos θ + 1)(cos θ + 2) = 0
Since, cos θ ∈ [-1,1] , for any value θ.
So, cos θ ≠ - 2
Therefore,
2 cos θ - 1 = 0
$\Rightarrow$ cos θ = ½
= π/3 or 2π – π/3
θ = π/3, 5π/3

19. If sec x cos 5x + 1 = 0, where 0 < x ≤ π/2, then find the value of x.
Solution:

According to the question,
sec x cos 5x = -1
$\Rightarrow$ cos 5x = -1/sec x
We know that,
sec x = 1/cos x
$\Rightarrow$ cos 5x + cos x = 0
By transformation formula of T-ratios,
We know that,

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\Rightarrow 2\cos\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right) = 0$$

$\Rightarrow$ 2 cos 3x cos 2x = 0
$\Rightarrow$ cos 3x = 0 or cos 2x = 0
∵ 0 < x ≤ π/2
Therefore, 0< 2x ≤ π or 0< 3x ≤ 3π/2
Therefore, 2x = π/2
$\Rightarrow$ x = π/4
3x = π/2
$\Rightarrow$ x = π/6
Or 3x = 3π/2
$\Rightarrow$ x = π/2
Hence, x = π/6, π/4, π/2.

20. If sin (θ + α) = a and sin (θ + β) = b, then prove that cos 2(α – β) – 4ab cos (α – β) = 1 – $2a^2 - 2b^2$
Solution:

According to the question,
sin (θ + α) = a and sin(θ + β) = b
LHS = cos 2(α – β) – 4ab cos (α – β)
Using cos 2x = $2\cos^2 x - 1$,
Let us solve,
$\Rightarrow$ LHS = $2\cos^2(\alpha - \beta) - 1 - 4ab\cos(\alpha - \beta)$

⇒ LHS = 2cos (α – β) {cos (α – β) – 2ab} – 1
Since,
cos (α – β) = cos {(θ + α) – (θ + β)}
cos (A – B) = cos A cos B + sin A sin B
⇒ cos (α – β) = cos(θ + α)cos(θ + β) + sin(θ + α)sin(θ + β)
Since,
sin(θ + α) = a
⇒ $\cos(\theta + \alpha) = \sqrt{(1 - \sin^2(\theta + \alpha)} = \sqrt{(1 - a^2)}$
Similarly,
$\cos(\theta + \beta) = \sqrt{(1 - b^2)}$
Therefore,
$\cos(\alpha - \beta) = \sqrt{(1-a^2)}\sqrt{(1-b^2)} + ab$
Therefore,
$LHS = 2\{ab + \sqrt{(1 - a^2)(1 - b^2)}\}\{ab + \sqrt{(1 - a^2)(1 - b^2)} -2ab\} - 1$
⇒ $LHS = 2\{\sqrt{(1 - a^2)(1 - b^2)} + ab\}\{\sqrt{(1 - a^2)(1 - b^2)} - ab\}-1$
Using $(x + y)(x - y) = x^2 - y^2$
⇒ $LHS = 2\{(1-a^2)(1-b^2) - a^2b^2\} - 1$
⇒ $LHS = 2\{1 - a^2 - b^2 + a^2b^2\} - 1$
⇒ $LHS = 2 - 2a^2 - 2b^2 - 1$
⇒ $LHS = 1 - 2a^2 - 2b^2 = RHS$
Therefore,
We get,
$\cos 2(\alpha - \beta) - 4ab \cos (\alpha - \beta) = 1 - 2a^2 - 2b^2$

21. If cos (θ + ϕ) = m cos (θ – ϕ), then prove that tan θ = ((1 – m)/(1 + m)) cot ϕ
[Hint: Express cos (θ + ϕ)/ cos (θ – ϕ) = m/l and apply Componendo and Dividendo]
Solution:

According to the question,

$\cos(\theta + \phi) = m \cos(\theta - \phi)$

$\because \cos(\theta + \phi) = m \cos(\theta - \phi)$

$\Rightarrow \frac{\cos(\theta-\phi)}{\cos(\theta+\phi)} = \frac{1}{m}$

Applying componendo – dividend, we get,

$\Rightarrow \frac{\cos(\theta-\phi)+\cos(\theta+\phi)}{\cos(\theta-\phi)-\cos(\theta+\phi)} = \frac{1+m}{1-m}$

From transformation formula, we know that,

$\cos(A+B) + \cos(A - B) = 2\cos A\cos B$

$\cos(A - B) - \cos(A + B) = 2\sin A\sin B$

$\Rightarrow \frac{2\cos\theta\cos\phi}{2\sin\theta\sin\phi} = \frac{1+m}{1-m}$

Since, $(\cos\theta)/(\sin\theta) = \cot\theta$

$\Rightarrow \cot\theta\cot\phi = \frac{1+m}{1-m}$

$\Rightarrow \left(\frac{1-m}{1+m}\right)\cot\phi = \frac{1}{\cot\theta}$

$\Rightarrow \tan\theta = \left(\frac{1-m}{1+m}\right)\cot\phi$

22. Find the value of the expression

$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$$

Solution:

According to the question,

Let, $y = 3[\sin^4(3\pi/2 - \alpha) + \sin^4(3\pi + \alpha)] - 2[\sin^6(\pi/2 + \alpha) + \sin^6(5\pi - \alpha)]$

We know that,

$\sin(3\pi/2 - \alpha) = -\cos\alpha$

$\sin(3\pi + \alpha) = -\sin\alpha$

$\sin(\pi/2 + \alpha) = \cos\alpha$

$\sin(5\pi - \alpha) = \sin\alpha$

Therefore,

$y = 3[(-\cos\alpha)^4 + (-\sin\alpha)^4] - 2[\cos^6\alpha + \sin^6\alpha]$

$\Rightarrow y = 3[\cos^4\alpha + \sin^4\alpha] - 2[\sin^6\alpha + \cos^6\alpha]$

$\Rightarrow y = 3[(\sin^2\alpha + \cos^2\alpha)^2 - 2\sin^2\alpha\cos^2\alpha] - 2[(\sin^2\alpha)^3 + (\cos^2\alpha)^3]$

Since, we know that,

$\sin^2\alpha + \cos^2\alpha = 1$

Also, we know that,

$a^3+b^3 = (a+b)(a^2 - ab + b^2)$

$\Rightarrow y = 3[1 - 2\sin^2\alpha\cos^2\alpha] - 2[(\sin^2\alpha + \cos^2\alpha)(\cos^4\alpha + \sin^4\alpha - \sin^2\alpha\cos^2\alpha)]$

$\Rightarrow y = 3[1 - 2\sin^2\alpha\cos^2\alpha] - 2[\cos^4\alpha + \sin^4\alpha - \sin^2\alpha\cos^2\alpha]$

$\Rightarrow y = 3[1 - 2\sin^2\alpha\cos^2\alpha] - 2[(\sin^2\alpha + \cos^2\alpha)^2 - 2\sin^2\alpha\cos^2\alpha - \sin^2\alpha\cos^2\alpha]$

$\Rightarrow y = 3[1 - 2\sin^2\alpha\cos^2\alpha] - 2[1 - 3\sin^2\alpha\cos^2\alpha]$

$\Rightarrow y = 3 - 6\sin^2\alpha\cos^2\alpha - 2 + 6\sin^2\alpha\cos^2\alpha$

$\Rightarrow y = 1$

23. If a cos2θ + b sin 2θ = c has α and β as its roots, then prove that tan α + tan β = 2b/(a + c)
[Hint: Use the identities cos 2θ = ((1 – $\tan^2$ θ)/(1 + $\tan^2$ θ) and sin 2θ = 2tan θ/(1 + $\tan^2$ θ)]
Solution:

According to the question,
a cos2θ + b sin 2θ = c
α and β are the roots of the equation.
Using the formula of multiple angles,
We know that,

$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$ and $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

$\therefore a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + b\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) - c = 0$

$\Rightarrow a(1 - \tan^2\theta) + 2b\tan\theta - c(1 + \tan^2\theta) = 0$

$\Rightarrow (-c - a)\tan^2\theta + 2b\tan\theta - c + a = 0$ …(i)

We know that,
The sum of roots of a quadratic equation, $ax^2 + bx + c = 0$ is given by (-b/a)
Therefore,
tan α + tan β = –2b/–(c + a) = 2b/(c + a)
Hence, tan α + tan β = 2b/(c + a)

24. If x = sec ϕ – tan ϕ and y = cosec ϕ + cot ϕ, then show that xy + x – y + 1 = 0.
[Hint: Find xy + 1 and then show tan x – y = –(xy + 1)]
Solution:

According to the question,
x = sec ϕ – tan ϕ and y = cosec ϕ + cot ϕ
Given that, LHS = xy + x – y + 1

$= (\sec\phi - \tan\phi)(\mathrm{cosec}\,\phi + \cot\phi) + (\sec\phi - \tan\phi) - (\mathrm{cosec}\,\phi + \cot\phi) + 1$

$= \sec\phi\,\mathrm{cosec}\,\phi + \cot\phi\sec\phi - \tan\phi\cot\phi - \tan\phi\,\mathrm{cosec}\,\phi + \sec\phi - \tan\phi - (\mathrm{cosec}\,\phi + \cot\phi) + 1$

$= \frac{1}{\sin\phi\cos\phi} + \frac{1}{\sin\phi} - 1 - \sec\phi + \sec\phi - \tan\phi - \left(\frac{1}{\sin\phi} + \frac{\cos\phi}{\sin\phi}\right) + 1$

$= \frac{1}{\sin\phi\cos\phi} + \frac{1}{\sin\phi} - \tan\phi - \left(\frac{1}{\sin\phi} + \frac{\cos\phi}{\sin\phi}\right)$

$= \frac{1}{\sin\phi\cos\phi} - \frac{\cos\phi}{\sin\phi} - \frac{\sin\phi}{\cos\phi}$

$= \frac{1}{\sin\phi\cos\phi} - \left(\frac{\cos\phi}{\sin\phi} + \frac{\sin\phi}{\cos\phi}\right)$

$= \frac{1}{\sin\phi\cos\phi} - \left(\frac{\cos^2\phi + \sin^2\phi}{\sin\phi\cos\phi}\right)$

Since, $\sin^2\theta + \cos^2\theta = 1$

$= \frac{1}{\sin\phi\cos\phi} - \left(\frac{1}{\sin\phi\cos\phi}\right) = 0$

Thus, LHS = xy + x – y + 1 = 0

25. If θ lies in the first quadrant and cos θ = 8/17, then find the value of cos(30° + θ) + cos (45° – θ) + cos (120° – θ)
Solution:

According to the question,
$\cos\theta = 8/17$
$\sin\theta = \pm\sqrt{(1 - \cos^2\theta)}$
Since, θ lies in first quadrant, only positive sign can be considered.
$\Rightarrow \sin\theta = \sqrt{(1 - 64/289)} = 15/17$
Let, $y = \cos(30° + \theta) + \cos(45° - \theta) + \cos(120° - \theta)$
We know that,
$\cos(x + y) = \cos x \cos y - \sin x \sin y$
Therefore,
$y = \cos30° \cos\theta - \sin30° \sin\theta + \cos45° \cos\theta + \sin45° \sin\theta + \cos120° \cos\theta + \sin120° \sin\theta$
Substituting values of cos30°, sin30°, cos 120°, sin120° and cos 45°

$$\Rightarrow y = \frac{\sqrt{3}}{2}.\frac{8}{17} - \frac{1}{2}.\frac{15}{17} + \frac{1}{\sqrt{2}}.\frac{8}{17} + \frac{1}{\sqrt{2}}.\frac{15}{17} + \left(-\frac{1}{2}\right)\left(\frac{8}{17}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{15}{17}\right)$$

$$= \frac{8\sqrt{3}}{34} - \frac{15}{34} + \frac{8+15}{17\sqrt{2}} - \frac{8}{34} + \frac{15\sqrt{3}}{34}$$

$$= \frac{23\sqrt{3}}{34} + \frac{23}{17\sqrt{2}} - \frac{23}{34}$$

$$\Rightarrow y = \frac{23}{34}(\sqrt{3} + \sqrt{2} - 1)$$

26. Find the value of the expression $\cos^4(\pi/8) + \cos^4(3\pi/8) + \cos^4(5\pi/8) + \cos^4(7\pi/8)$.
[Hint: Simplify the expression to

$$2\left(\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8}\right) = 2\left[\left(\cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8}\right)^2 - 2\cos^2\frac{\pi}{8}\cos^2\frac{3\pi}{8}\right]$$

Solution:

According to the question,
Let $y = \cos^4(\pi/8) + \cos^4(3\pi/8) + \cos^4(5\pi/8) + \cos^4(7\pi/8)$.
$\Rightarrow y = \cos^4(\pi/8) + \cos^4(3\pi/8) + \cos^4(\pi - 3\pi/8) + \cos^4(\pi - \pi/8)$.
Since we know that, $\cos(\pi - x) = -\cos x$, we get,

$$= \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{\pi}{8}\right)$$
$$= 2\left(\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8}\right)$$
$$= 2\left(\cos^4\frac{\pi}{8} + \cos^4\left(\frac{\pi}{2} - \frac{\pi}{8}\right)\right)$$
$$= 2\left(\cos^4\frac{\pi}{8} + \sin^4\frac{\pi}{8}\right)$$
$$= 2\left[\left(\cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8}\right)^2 - 2\cos^2\frac{\pi}{8}.\sin^2\frac{\pi}{8}\right]$$
$$= 2\left[1 - 2\cos^2\frac{\pi}{8}.\sin^2\frac{\pi}{8}\right]$$
$$= 2 - \left(2\cos\frac{\pi}{8}.\sin\frac{\pi}{8}\right)^2$$
$$= 2 - \left(\sin\frac{2\pi}{8}\right)^2$$
$= 2 – (1/\sqrt{2})^2$
$= 2 – ½$
$= 3/2$

27. Find the general solution of the equation
$5\cos^2\theta + 7\sin^2\theta – 6 = 0$
Solution:

According to the question,
$5\cos^2\theta + 7\sin^2\theta – 6 = 0$
We know that,
$\sin^2\theta = 1 – \cos^2\theta$
Therefore, $5\cos^2\theta + 7(1 – \cos^2\theta) – 6 = 0$
$\Rightarrow 5\cos^2\theta + 7 – 7\cos^2\theta – 6 = 0$
$\Rightarrow -2\cos^2\theta + 1 = 0$
$\Rightarrow \cos^2\theta = ½$
Therefore, $\cos \theta = \pm 1/\sqrt{2}$
Therefore, $\cos \theta = \cos \pi/4$ or $\cos \theta = \cos 3\pi/4$
Since, solution of $\cos x = \cos \alpha$ is given by
$x = 2m\pi \pm \alpha \ \forall\ m \in Z$
$\theta = n\pi \pm \pi/4, n \in Z$

28. Find the general solution of the equation sin x – 3sin2x + sin3x = cos x – 3cos2x + cos3x
Solution:

According to the question,
sin x – 3sin2x + sin3x = cos x – 3cos2x + cos3x
Grouping sin x and sin 3x in LHS and, cos x and cos 3x in RHS,
We get,
sin x + sin3x – 3sin2x = cos x + cos3x – 3cos2x
Applying transformation formula,
cos A + cos B = 2cos ((A + B)/2) cos((A – B)/2)

sin A + sin B = 2sin ((A + B)/2) cos((A – B)/2)

$\Rightarrow 2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - 3\sin 2x = 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - 3\cos 2x$

⇒ 2sin 2x cos x – 3sin 2x = 2cos 2x cos x – 3cos 2x

⇒ 2sin 2x cos x – 3sin 2x – 2cos 2x cos x + 3cos 2x = 0

⇒ 2cos x (sin 2x – cos 2x) – 3(sin 2x – cos 2x) = 0

⇒ (sin 2x – cos 2x)(2cos x – 3) = 0

⇒ cos x = 3/2 or sin 2x = cos 2x

As cos x ∈ [-1,1]

Hence, no value of x exists for which cos x = 3/2

Therefore, sin 2x = cos 2x

⇒ tan 2x = 1 = tan π/4

We know solution of tan x = tan α is given by,

x= nπ ± α , n ∈ Z

Therefore, 2x = nπ ± (π/4)

⇒ x = nπ/2 ± (π/8), n ∈ Z

29. Find the general solution of the equation (√3 – 1) cos θ + (√3 + 1) sin θ = 2
[Hint: Put √3 – 1 = r sin α, √3 + 1 = r cos α which gives tan α = tan((π/4) – (π/6)) α = π/12]
Solution:

Let, r sinα = √3 – 1 and r cosα = √3 + 1

Therefore, r = $\sqrt{\{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2\}}$ = √8 = 2√2

And, tan α = (√3 – 1) / (√3 + 1)

Therefore, r(sinα cos θ + cosα sin θ) = 2

⇒ r sin (θ+α) = 2

⇒ sin (θ+α) = 1/√2

⇒ sin (θ+α) = sin (π/4)

⇒ $\theta+\alpha = n\pi + (-1)^n (\pi/4)$, n ∈ Z

⇒ $\theta = n\pi + (-1)^n (\pi/4) - (\pi/12)$, n ∈ Z

OBJECTIVE TYPE QUESTIONS

30. If sin θ + cosec θ = 2, then $\sin^2\theta + \mathrm{cosec}^2\theta$ is equal to
A. 1
B.4
C. 2
D. None of these
Solution:

C. 2

Explanation:

According to the question,

sin θ + cosec θ = 2

Squaring LHS and RHS,

We get,

⇒ $(\sin\theta + \mathrm{cosec}\,\theta)^2 = 2^2$

⇒ $\sin^2\theta + \mathrm{cosec}^2\theta + 2\sin\theta\,\mathrm{cosec}\,\theta = 4$

⇒ $\sin^2\theta + \text{cosec}^2\theta + 2 \sin\theta (1/\sin)\theta = 4$
⇒ $\sin^2\theta + \text{cosec}^2\theta + 2 = 4$
⇒ $\sin^2\theta + \text{cosec}^2\theta = 2$
Thus, option (C) 2 is the correct answer.

31. If $f(x) = \cos^2 x + \sec^2 x$, then
A. $f(x) < 1$
B. $f(x) = 1$
C. $2 < f(x) < 1$
D. $f(x) \geq 2$
[Hint: A.M ≥ G.M.]
Solution:

D. $f(x) \geq 2$

Explanation:

According to the question,
We have, $f(x) = \cos^2 x + \sec^2 x$
We know that, A.M ≥ G.M.

$$\Rightarrow \frac{\cos^2 x + \sec^2 x}{2} \geq \sqrt{\cos^2 x \sec^2 x}$$

$$\Rightarrow \frac{\cos^2 x + \sec^2 x}{2} \geq \sqrt{\cos^2 x \frac{1}{\cos^2 x}}$$

$$\Rightarrow \frac{\cos^2 x + \sec^2 x}{2} \geq 1$$

⇒ $\cos 0^2 x + \sec^2 x \geq 2$
⇒ $f(x) \geq 2$
Thus, option (D) $f(x) \geq 2$ is the correct answer.

32. If $\tan\theta = 1/2$ and $\tan\phi = 1/3$, then the value of $\theta + \phi$ is
A. $\pi/6$
B. π
C. 0
D. $\pi/4$
Solution:

D. $\pi/4$

Explanation:

According to the question,

$\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$

We know that,

$$\tan(\theta+\phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$$

$$= \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$$\Rightarrow \tan(\theta+\phi) = \tan\frac{\pi}{4}$$

Thus, option (D) π/4 is the correct answer.

33. Which of the following is not correct?

A. sin θ = – 1/5

B. cos θ = 1

C. sec θ = ½

D. tan θ = 20

Solution:

C. sec θ = ½

Explanation:

According to the question,

We know that,

a) sin θ = – 1/5 is correct since Sin θ ∈ [-1,1]

b) cos θ = 1 is correct since Cos θ ∈ [-1,1]

c) sec θ = ½

⇒ (1/cos θ) = ½

⇒cos θ=2 is incorrect since Cos θ ∈ [-1,1]

d) tan θ = 20 is correct since tan θ ∈ R.

Thus, option (C) sec θ = ½ is the correct answer.

34. The value of tan 1° tan 2° tan 3°… tan 89° is

A. 0

B. 1

C. ½

D. Not defined

Solution:

B. 1

Explanation:

According to the question,

tan 1° tan 2° tan 3°… tan 89°

= tan 1° tan 2° … tan 45° tan (90-44°) tan(90-43°)…tan (90-1°)

= tan 1°tan 2° … tan 45°cot 44°cot 43°…cot 1° [∵tan (90-θ)=cot θ]

= tan 1° cot 1° tan 2° cot 2°…tan45°… tan 89° cot 89°

=1.1….1 = 1

Thus, option (B) 1 is the correct answer.

35. The value of $(1 - \tan^2 15^o)/(1 + \tan^2 15^o)$ is
A. 1
B. √3
C. √3/2
D. 2
Solution:

C. √3/2

Explanation:

According to the question,

Let θ = 15° ⇒ 2θ = 30°

Now, since we know that,

$$\text{Cos}2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$$

$$\Rightarrow \cos 30° = \frac{1 - \tan^2 15°}{1 + \tan^2 15°}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{1 - \tan^2 15°}{1 + \tan^2 15°}$$

Thus, option (C) √3/2 is the correct answer.

36. The value of cos 1° cos 2° cos 3°… cos 179° is
A. 1/√2
B. 0
C. 1
D. –1
Solution:

B. 0

Explanation:

According to the question,

Since cos90° =0

We get,

⇒ cos 1° cos 2° cos 3°… cos90°… cos 179° = 0

Thus, option (B) 0 is the correct answer.

37. If tan θ = 3 and θ lies in third quadrant, then the value of sin θ is
A. 1/√10
B. – 1/√10
C. – 3/√10
D. 3/√10
Solution:

C. – 3/√10

Explanation:

According to the question,

Given that, tan θ = 3 and θ lies in third quadrant

⇒ cot θ = 1/3

We know that,

$\text{Cosec}^2\theta = 1+\cot^2\theta$

$$= 1 + \left(\frac{1}{3}\right)^2 = 1 + \frac{1}{9} = \frac{10}{9}$$

$$\Rightarrow \sin^2\theta = \frac{9}{10}$$

$$\Rightarrow \sin\theta = \pm\frac{3}{\sqrt{10}}$$

$$\Rightarrow \sin\theta = -\frac{3}{\sqrt{10}}\text{ , since } \theta \text{ lies in third quadrant.}$$

Thus, option (C) – 3/√10 is the correct answer.

38. The value of tan 75°– cot 75° is equal to

A. 2√3

B. 2 + √3

C. 2 – √3

D. 1

Solution:

A. 2√3

Explanation:

According to the question,

We have,

tan 75°– cot 75°

$$= \frac{\sin 75°}{\cos 75°} - \frac{\cos 75°}{\sin 75°}$$

$$= \frac{\sin^2 75° - \cos^2 75°}{\cos 75° \sin 75°}$$

$$= \frac{2(\sin^2 75° - \cos^2 75°)}{2\cos 75° \sin 75°}$$

$$= \frac{-2\cos 150°}{\sin 150°}$$

= -2cot150°

= -2 cot (180°-30°)

= 2cot30°

=2√3

Thus, option (A) 2√3 is the correct answer.