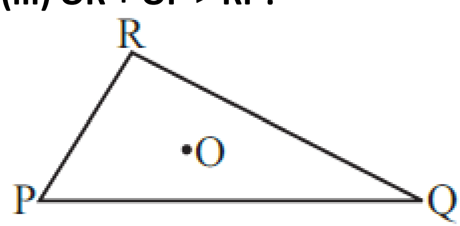
EXERCISE 6.4 PAGE: 126

1. Is it possible to have a triangle with the following sides?
(i) 2 cm, 3 cm, 5 cm
Solution:-
Clearly, we have:
(2 + 3) = 5
5 = 5
Thus, the sum of any two of these numbers is not greater than the third.
Hence, it is not possible to draw a triangle whose sides are 2 cm, 3 cm and 5 cm.

(ii) 3 cm, 6 cm, 7 cm
Solution:-
Clearly, we have:
(3 + 6) = 9 > 7
(6 + 7) = 13 > 3
(7 + 3) = 10 > 6
Thus, the sum of any two of these numbers is greater than the third.
Hence, it is possible to draw a triangle whose sides are 3 cm, 6 cm and 7 cm.

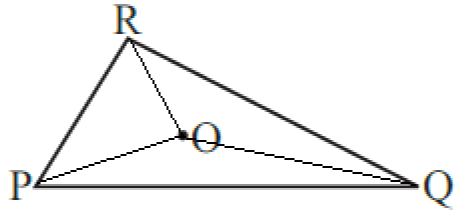
(iii) 6 cm, 3 cm, 2 cm
Solution:-
Clearly, we have:
(3 + 2) = 5 < 6
Thus, the sum of any two of these numbers is less than the third.
Hence, it is not possible to draw a triangle whose sides are 6 cm, 3 cm and 2 cm.

2. Take any point O in the interior of a triangle PQR. Is
(i) OP + OQ > PQ?
(ii) OQ + OR > QR?
(iii) OR + OP > RP?



Solution:-

If we take any point O in the interior of a triangle PQR and join OR, OP, OQ.
Then, we get three triangles ΔOPQ, ΔOQR and ΔORP is shown in the figure below.



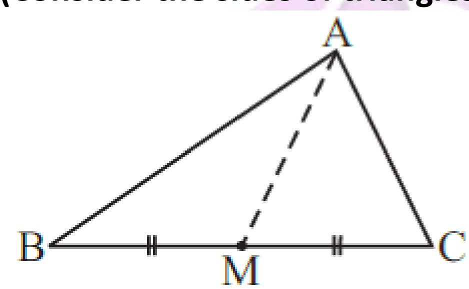
We know that,
The sum of the length of any two sides is always greater than the third side.

(i) Yes, ΔOPQ has sides OP, OQ and PQ.
So, OP + OQ > PQ

(ii) Yes, ΔOQR has sides OR, OQ and QR.
So, OQ + OR > QR

(iii) Yes, ΔORP has sides OR, OP and PR.
So, OR + OP > RP

3. AM is a median of a triangle ABC.
Is AB + BC + CA > 2 AM?
(Consider the sides of triangles ΔABM and ΔAMC.)



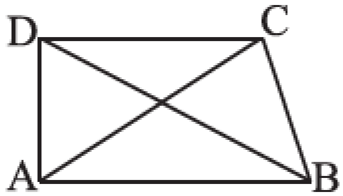
Solution:-
We know that,
The sum of the length of any two sides is always greater than the third side.
Now consider the ΔABM,
Here, AB + BM > AM … [equation i]
Then, consider the ΔACM
Here, AC + CM > AM … [equation ii]
By adding equation [i] and [ii] we get,
AB + BM + AC + CM > AM + AM

From the figure we have, BC = BM + CM

AB + BC + AC > 2 AM

Hence, the given expression is true.

4. ABCD is a quadrilateral.
Is AB + BC + CD + DA > AC + BD?



Solution:-

We know that,

The sum of the length of any two sides is always greater than the third side.

Now consider the ΔABC,

Here, AB + BC > CA … [equation i]

Then, consider the ΔBCD

Here, BC + CD > DB … [equation ii]

Consider the ΔCDA

Here, CD + DA > AC … [equation iii]

Consider the ΔDAB

Here, DA + AB > DB … [equation iv]

By adding equation [i], [ii], [iii] and [iv] we get,

AB + BC + BC + CD + CD + DA + DA + AB > CA + DB + AC + DB

2AB + 2BC + 2CD + 2DA > 2CA + 2DB

Take out 2 on both the side,

2(AB + BC + CA + DA) > 2(CA + DB)

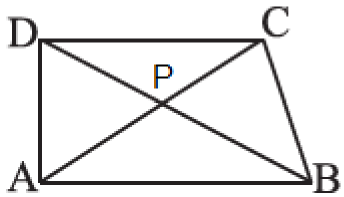
AB + BC + CA + DA > CA + DB

Hence, the given expression is true.

5. ABCD is quadrilateral. Is AB + BC + CD + DA < 2 (AC + BD)

Solution:-

Let us consider ABCD is quadrilateral and P is the point where the diagonals are intersect. As shown in the figure below.



We know that,
The sum of the length of any two sides is always greater than the third side.
Now consider the ΔPAB,

Here, PA + PB < AB ... [equation i]

Then, consider the ΔPBC

Here, PB + PC < BC ... [equation ii]

Consider the ΔPCD

Here, PC + PD < CD ... [equation iii]

Consider the ΔPDA

Here, PD + PA < DA ... [equation iv]

By adding equation [i], [ii], [iii] and [iv] we get,
PA + PB + PB + PC + PC + PD + PD + PA < AB + BC + CD + DA
2PA + 2PB + 2PC + 2PD < AB + BC + CD + DA
2PA + 2PC + 2PB + 2PD < AB + BC + CD + DA
2(PA + PC) + 2(PB + PD) < AB + BC + CD + DA
From the figure we have, AC = PA + PC and BD = PB + PD
Then,
2AC + 2BD < AB + BC + CD + DA
2(AC + BD) < AB + BC + CD + DA
Hence, the given expression is true.

6. The lengths of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?

Solution:-

We know that,
The sum of the length of any two sides is always greater than the third side.
From the question, it is given that two sides of triangle are 12 cm and 15 cm.
So, the third side length should be less than the sum of other two sides,

12 + 15 = 27 cm.

Then, it is given that the third side is cannot not be less than the difference of the two sides, 15 – 12 = 3 cm
So, the length of the third side falls between 3 cm and 27 cm.