EXERCISE 15.3 PAGE NO: 15.19

1. In Fig. 35, ∠CBX is an exterior angle of ΔABC at B. Name
(i) The interior adjacent angle
(ii) The interior opposite angles to exterior ∠CBX
Also, name the interior opposite angles to an exterior angle at A.

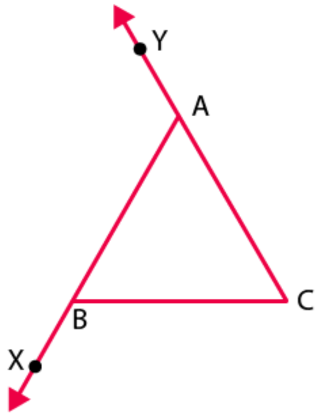


Fig. 35

Solution:
(i) The interior adjacent angle is ∠ABC
(ii) The interior opposite angles to exterior ∠CBX is ∠BAC and ∠ACB
Also the interior angles opposite to exterior are ∠ABC and ∠ACB

2. In the fig. 36, two of the angles are indicated. What are the measures of ∠ACX and ∠ACB?

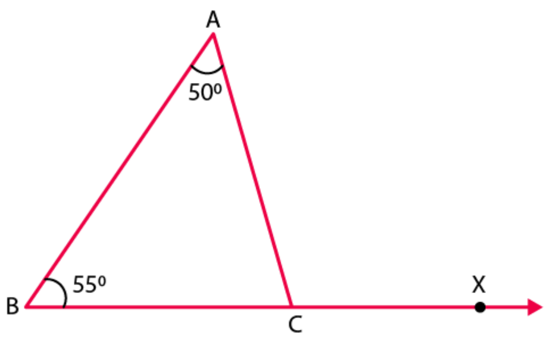


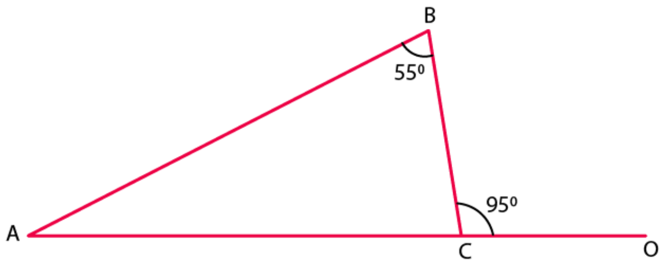
Fig. 36

Solution:
Given that in △ABC, ∠A = 50^o and ∠B = 55^o
We know that the sum of angles in a triangle is 180^o

Therefore we have
$\angle A + \angle B + \angle C = 180^o$
$50^o + 55^o + \angle C = 180^o$
$\angle C = 75^o$
$\angle ACB = 75^o$
$\angle ACX = 180^o - \angle ACB = 180^o - 75^o = 105^o$

3. In a triangle, an exterior angle at a vertex is 95° and its one of the interior opposite angles is 55°. Find all the angles of the triangle.

Solution:



We know that the sum of interior opposite angles is equal to the exterior angle.
Hence, for the given triangle, we can say that:
$\angle ABC + \angle BAC = \angle BCO$
$55^o + \angle BAC = 95^o$
$\angle BAC = 95^o - 95^o$
$\angle BAC = 40^o$
We also know that the sum of all angles of a triangle is 180°.
Hence, for the given △ABC, we can say that:
$\angle ABC + \angle BAC + \angle BCA = 180^o$
$55^o + 40^o + \angle BCA = 180^o$
$\angle BCA = 180^o - 95^o$
$\angle BCA = 85^o$

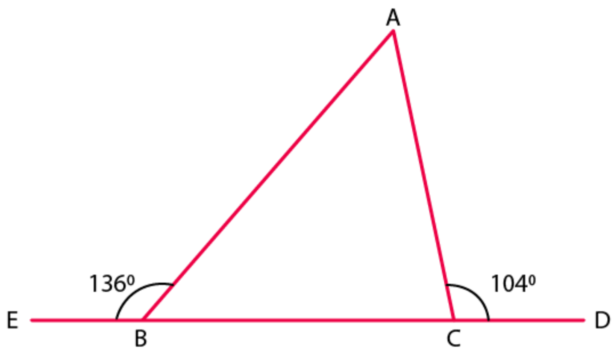
4. One of the exterior angles of a triangle is 80°, and the interior opposite angles are equal to each other. What is the measure of each of these two angles?

Solution:
Let us assume that A and B are the two interior opposite angles.
We know that ∠A is equal to ∠B.

We also know that the sum of interior opposite angles is equal to the exterior angle.
Therefore from the figure we have,
∠A + ∠B = 80°
∠A +∠A = 80° (because ∠A = ∠B)
2∠A = 80°
∠A = 40/2 =40°
∠A= ∠B = 40°
Thus, each of the required angles is of 40°.

5. The exterior angles, obtained on producing the base of a triangle both ways are 104° and 136°. Find all the angles of the triangle.

Solution:



In the given figure, ∠ABE and ∠ABC form a linear pair.
∠ABE + ∠ABC =180°
∠ABC = 180°– 136°
∠ABC = 44°
We can also see that ∠ACD and ∠ACB form a linear pair.
∠ACD + ∠ACB = 180°
∠AUB = 180°– 104°
∠ACB = 76°
We know that the sum of interior opposite angles is equal to the exterior angle.
Therefore, we can write as
∠BAC + ∠ABC = 104°
∠BAC = 104° – 44° = 60°
Thus,
∠ACE = 76° and ∠BAC = 60°

6. In Fig. 37, the sides BC, CA and BA of a △ABC have been produced to D, E and F respectively. If ∠ACD = 105° and ∠EAF = 45°; find all the angles of the △ABC.

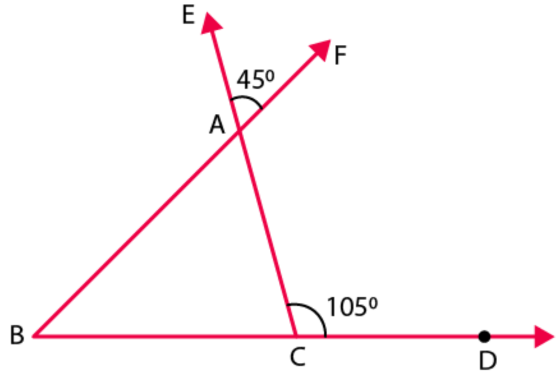


Fig. 37

Solution:

In a △ABC, ∠BAC and ∠EAF are vertically opposite angles.

Hence, we can write as

∠BAC = ∠EAF = 45°

Considering the exterior angle property, we have

∠BAC + ∠ABC = ∠ACD = 105°

On rearranging we get

∠ABC = 105°– 45° = 60°

We know that the sum of angles in a triangle is 180°

∠ABC + ∠ACS +∠BAC = 180°

∠ACB = 75°

Therefore, the angles are 45°, 65° and 75°.

7. In Fig. 38, AC perpendicular to CE and C ∠A: ∠B: ∠C= 3: 2: 1. Find the value of ∠ECD.

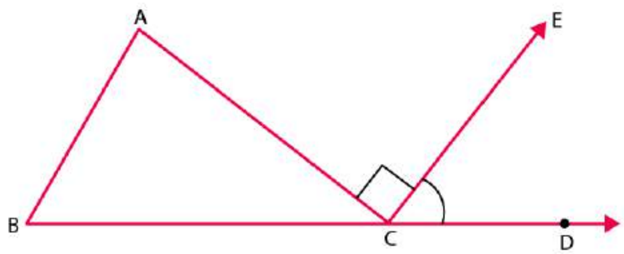


Fig. 38

Solution:

In the given triangle, the angles are in the ratio 3: 2: 1.

Let the angles of the triangle be 3x, 2x and x.

We know that sum of angles in a triangle is 180°

$3x + 2x + x = 180^o$
$6x = 180^o$
$x = 30^o$
Also, ∠ACB + ∠ACE + ∠ECD = 180^o
x + 90^o + ∠ECD = 180^o (∠ACE = 90^o)
We know that x = 30^o
Therefore
∠ECD = 60^o

8. A student when asked to measure two exterior angles of △ABC observed that the exterior angles at A and B are of 103^o and 74^o respectively. Is this possible? Why or why not?
Solution:
We know that sum of internal and external angle is equal to 180^o
Internal angle at A + External angle at A = 180^o
Internal angle at A + 103^o =180^o
Internal angle at A = 77^o
Internal angle at B + External angle at B = 180^o
Internal angle at B + 74^o = 180^o
Internal angle at B = 106^o
Sum of internal angles at A and B = 77^o + 106^o = 183^o
It means that the sum of internal angles at A and B is greater than 180^o, which cannot be possible.

9. In Fig.39, AD and CF are respectively perpendiculars to sides BC and AB of △ABC. If ∠FCD = 50^o, find ∠BAD

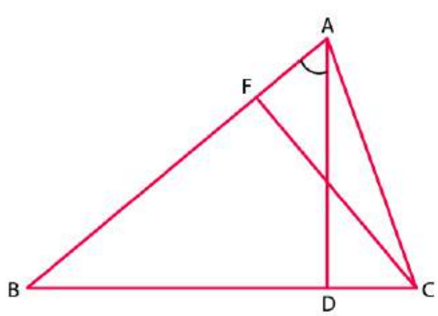


Fig. 39

Solution:
We know that the sum of all angles of a triangle is 180^o
Therefore, for the given △FCB, we have

∠FCB + ∠CBF + ∠BFC = 180°
50° + ∠CBF + 90° = 180°
∠CBF = 180° – 50° – 90° = 40°
Using the above steps for △ABD, we can say that:
∠ABD + ∠BDA + ∠BAD = 180°
∠BAD = 180° – 90° – 40° = 50°

10. In Fig.40, measures of some angles are indicated. Find the value of x.

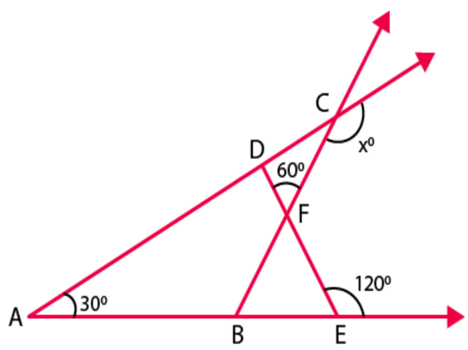


Fig. 40

Solution:
We know that the sum of the angles of a triangle is 180°
From the figure we have,
∠AED + 120° = 180° (Linear pair)
∠AED = 180° – 120° = 60°
We know that the sum of all angles of a triangle is 180°.
Therefore, for △ADE, we have
∠ADE + ∠AED + ∠DAE = 180°
60°+ ∠ADE + 30° =180°
∠ADE = 180° – 60° – 30° = 90°
From the given figure, we have
∠FDC + 90° = 180° (Linear pair)
∠FDC = 180° – 90° = 90°
Using the same steps for △CDF, we get
∠CDF + ∠DCF + ∠DFC = 180°
90° + ∠DCF + 60° = 180°
∠DCF = 180° – 60° – 90° = 30°
Again from the figure we have
∠DCF + x = 180° (Linear pair)
30° + x = 180°

x = 180° – 30° = 150°

11. In Fig. 41, ABC is a right triangle right angled at A. D lies on BA produced and DE perpendicular to BC intersecting AC at F. If ∠AFE = 130°, find
(i) ∠BDE
(ii) ∠BCA
(iii) ∠ABC

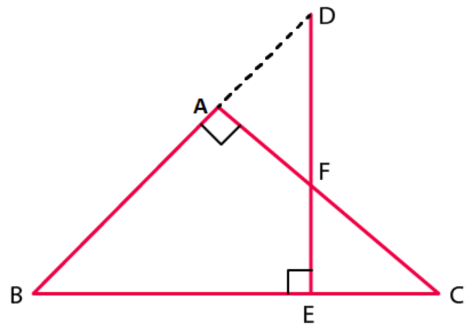


Fig. 41

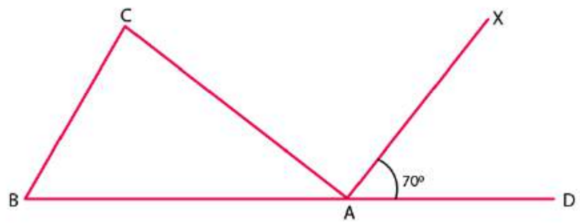
Solution:
(i) Here,
∠BAF + ∠FAD = 180° (Linear pair)
∠FAD = 180° - ∠BAF = 180° – 90° = 90°
Also from the figure,
∠AFE = ∠ADF + ∠FAD (Exterior angle property)
∠ADF + 90° = 130°
∠ADF = 130° – 90° = 40°

(ii) We know that the sum of all the angles of a triangle is 180°.
Therefore, for △BDE, we have
∠BDE + ∠BED + ∠DBE = 180°
∠DBE = 180° – ∠BDE
∠BED = 180° – 90° – 40° = 50° …. Equation (i)
Again from the figure we have,
∠FAD = ∠ABC + ∠ACB (Exterior angle property)
90° = 50° + ∠ACB
∠ACB = 90° – 50° = 40°

(iii) From equation we have
∠ABC = ∠DBE = 50°

12. ABC is a triangle in which ∠B = ∠C and ray AX bisects the exterior angle DAC. If ∠DAX = 70°. Find ∠ACB.

Solution:



Given that ABC is a triangle in which ∠B = ∠C

Also given that AX bisects the exterior angle DAC

∠CAX = ∠DAX (AX bisects ∠CAD)

∠CAX =70° [given]

∠CAX +∠DAX + ∠CAB =180°

70°+ 70° + ∠CAB =180°

∠CAB =180° –140°

∠CAB =40°

∠ACB + ∠CBA + ∠CAB = 180° (Sum of the angles of ΔABC)

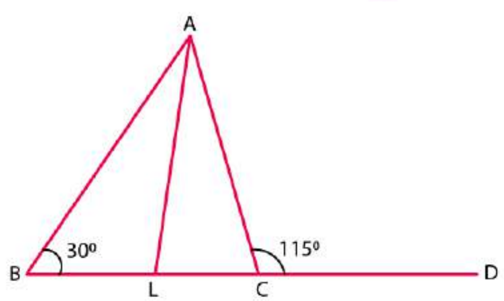
∠ACB + ∠ACB+ 40° = 180° (∠C = ∠B)

2∠ACB = 180° – 40°

∠ACB = 140/2

∠ACB = 70°

13. The side BC of △ABC is produced to a point D. The bisector of ∠A meets side BC in L. If ∠ABC= 30° and ∠ACD = 115°, find ∠ALC



Solution:

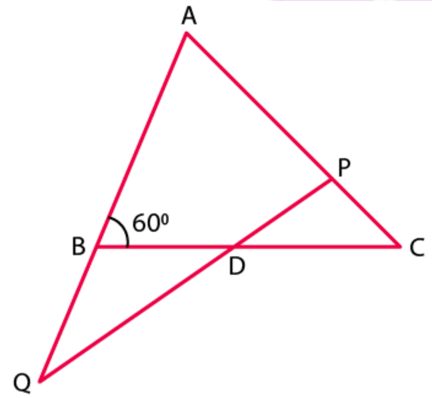
Given that ∠ABC= 30° and ∠ACD = 115°

From the figure, we have

∠ACD and ∠ACL make a linear pair.
∠ACD+ ∠ACB = 180°
115° + ∠ACB =180°
∠ACB = 180° – 115°
∠ACB = 65°
We know that the sum of all angles of a triangle is 180°.
Therefore, for△ ABC, we have
∠ABC + ∠BAC + ∠ACB = 180°
30° + ∠BAC + 65° = 180°
∠BAC = 85°
∠LAC = ∠BAC/2 = 85/2
Using the same steps for △ALC, we get
∠ALC + ∠LAC + ∠ACL = 180°
∠ALC + 82/2 + 65° = 180°
We know that ∠ALC = ∠ACB
∠ALC = 180° - 82/2 - 65°
∠ALC = 72 ½°

14. D is a point on the side BC of △ABC. A line PDQ through D, meets side AC in P and AB produced at Q. If ∠A = 80°, ∠ABC = 60° and ∠PDC = 15°, find
(i) ∠AQD
(ii) ∠APD

Solution:



From the figure we have
∠ABD and ∠QBD form a linear pair.
∠ABC + ∠QBC =180°
60° + ∠QBC = 180°

∠QBC = 120°
∠PDC = ∠BDQ (Vertically opposite angles)
∠BDQ = 75°
In △QBD:
∠QBD + ∠QDB + ∠BDQ = 180° (Sum of angles of △QBD)
120°+ 15° + ∠BQD = 180°
∠BQD = 180° – 135°
∠BQD = 45°
∠AQD = ∠BQD = 45°
In △AQP:
∠QAP + ∠AQP + ∠APQ = 180° (Sum of angles of △AQP)
80° + 45° + ∠APQ = 180°
∠APQ = 55°
∠APD = ∠APQ

15. Explain the concept of interior and exterior angles and in each of the figures given below. Find x and y (Fig. 42)

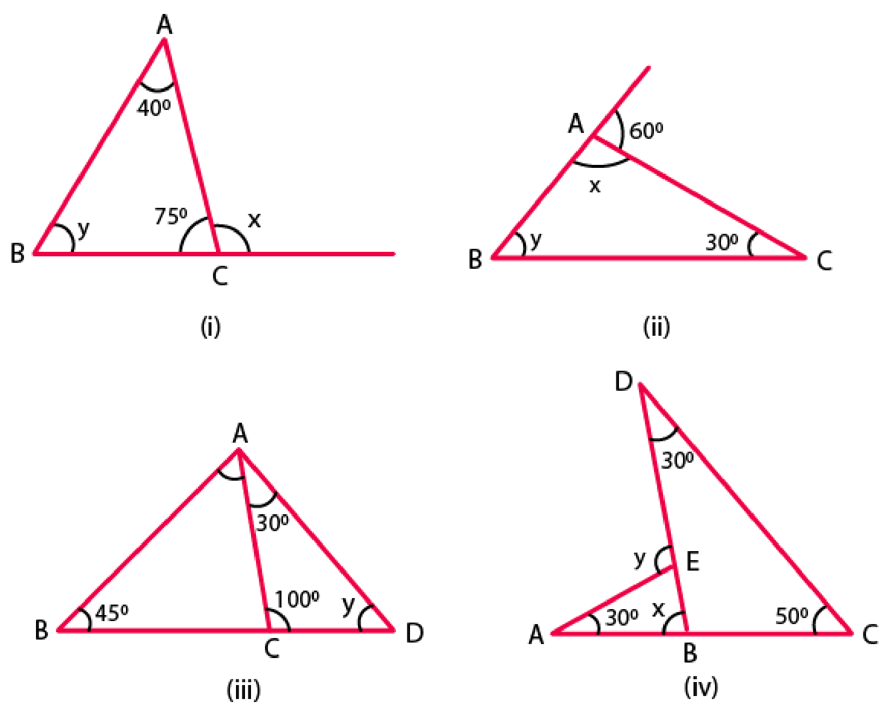


Fig. 42

Solution:
The interior angles of a triangle are the three angle elements inside the triangle.
The exterior angles are formed by extending the sides of a triangle, and if the side of a

triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.
Using these definitions, we will obtain the values of x and y.
(i) From the given figure, we have
$\angle ACB + x = 180^o$ (Linear pair)
$75^o + x = 180^o$
$x = 105^o$
We know that the sum of all angles of a triangle is 180^o
Therefore, for $\triangle ABC$, we can say that:
$\angle BAC + \angle ABC + \angle ACB = 180^o$
$40^o + y + 75^o = 180^o$
$y = 65^o$

(ii) From the figure, we have
$x + 80^o = 180^o$ (Linear pair)
$x = 100^o$
In $\triangle ABC$, we have
We also know that the sum of angles of a triangle is 180^o
$x + y + 30^o = 180^o$
$100^o + 30^o + y = 180^o$
$y = 50^o$

(iii) We know that the sum of all angles of a triangle is 180°.
Therefore, for $\triangle ACD$, we have
$30^o + 100^o + y = 180^o$
$y = 50^o$
Again from the figure we can write as
$\angle ACB + 100° = 180°$
$\angle ACB = 80^o$
Using the above rule for $\triangle ACD$, we can say that:
$x + 45^o + 80^o = 180^o$
$x = 55^o$

(iv) We know that the sum of all angles of a triangle is 180^o.
Therefore, for $\triangle DBC$, we have
$30^o + 50^o + \angle DBC = 180^o$
$\angle DBC = 100^o$

From the figure we can say that
x + ∠DBC = 180° is a Linear pair
x = 80°
From the exterior angle property we have
y = 30° + 80° = 110°

16. Compute the value of x in each of the following figures:

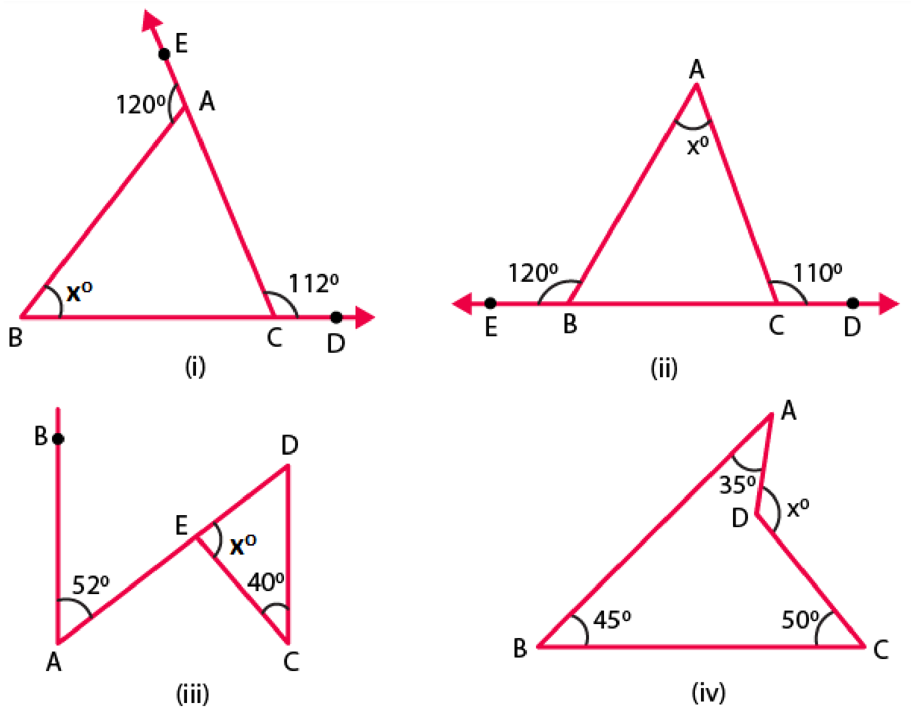


Fig. 43

Solution:
(i) From the given figure, we can write as
∠ACD + ∠ACB = 180° is a linear pair
On rearranging we get
∠ACB = 180° – 112° = 68°
Again from the figure we have,
∠BAE + ∠BAC = 180° is a linear pair
On rearranging we get,
∠BAC = 180°– 120° = 60°
We know that the sum of all angles of a triangle is 180°.
Therefore, for △ABC:
x + ∠BAC + ∠ACB = 180°

$x = 180^o - 60^o - 68^o = 52^o$
$x = 52^o$

(ii) From the given figure, we can write as
$\angle ABC + 120^o = 180^o$ is a linear pair
$\angle ABC = 60^o$
Again from the figure we can write as
$\angle ACB + 110^o = 180^o$ is a linear pair
$\angle ACB = 70^o$
We know that the sum of all angles of a triangle is 180^o.
Therefore, consider $\triangle ABC$, we get
$x + \angle ABC + \angle ACB = 180^o$
$x = 50^o$

(iii) From the given figure, we can write as
$\angle BAD = \angle ADC = 52^o$ are alternate angles
We know that the sum of all the angles of a triangle is 180^o.
Therefore, consider $\triangle DEC$, we have
$x + 40^o + 52^o = 180^o$
$x = 88^o$

(iv) In the given figure, we have a quadrilateral and also we know that sum of all angles is quadrilateral is 360^o.
Thus,
$35^o + 45^o + 50^o + \text{reflex } \angle ADC = 360^o$
On rearranging we get,
Reflex $\angle ADC = 230^o$
$230^o + x = 360^o$ (A complete angle)
$x = 130^o$