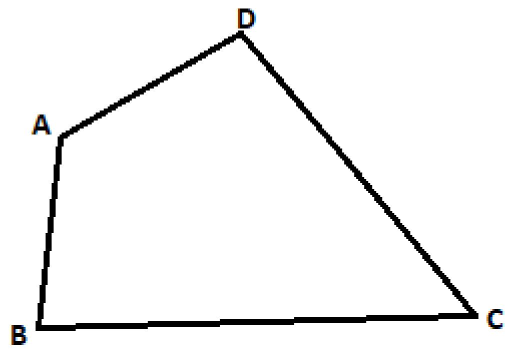
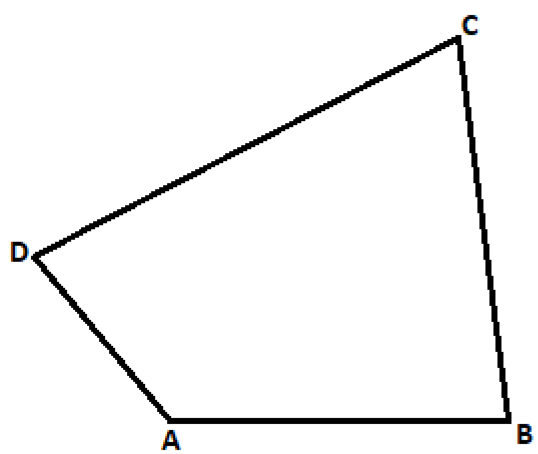
EXERCISE 16.1 PAGE NO: 16.15

1. Define the following terms:
(i) Quadrilateral
(ii) Convex Quadrilateral
Solution:
(i) Quadrilateral
Definition: Let A, B, C and D be four points in a plane such that: (a) no three of them are collinear. (b) The line segments AB, BC, CD and DA do not intersect except at their end points. Then an Enclosed figure with four sides is termed as Quadrilateral.

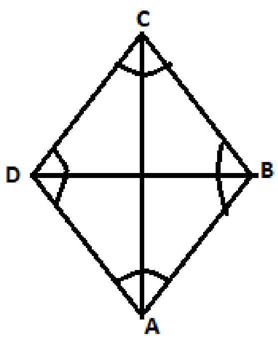


(ii) Convex Quadrilateral
Definition: If the line containing any side of the quadrilateral has the remaining vertices on the same side of it is termed as Convex Quadrilateral.



Vertices A, B lie on the same side of line CD, vertices B, C lie on the same side of line DA, vertices C, D lie on the same side of line AB, vertices D, A lie on the same side of line BC.

2. In a quadrilateral, define each of the following:
(i) Sides
(ii) Vertices
(iii) Angles
(iv) Diagonals
(v) Adjacent angles
(vi) Adjacent sides
(vii) Opposite sides
(viii) Opposite angles
(ix) Interior
(x) Exterior
Solution:



(i) Sides: In a quadrilateral. All the sides may have same of different length.
The four line segments AB, BC, CD and DA are called its sides.

(ii) Vertices
Vertices are the angular points where two sides or edges meet.
A, B, C and D are the four vertices in a quadrilateral.

(iii) Angles
Angle is the inclination between two sides of a quadrilateral. i.e. meeting point of two sides is an angle. ABC, BCA, CDA and DAB are the four angles in a quadrilateral.

(iv) Diagonals
The lines joining two opposite vertices is called the diagonals in a quadrilateral.
BD and AC are the two diagonals.

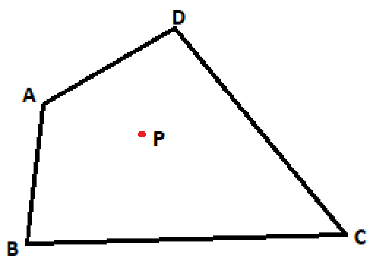
(v) Adjacent angles
Angles having one common arm onto the sides is called the adjacent angles.
ABC, BCD are adjacent angles in a quadrilateral.

(vi) Adjacent sides
When two sides have common endpoint is termed as adjacent sides.
AB BC, BC CA, CD DA, DA AB are pairs of adjacent sides in a quadrilateral.

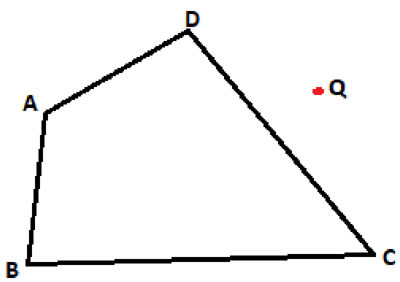
(vii) Opposite sides: Opposite sides when they don't meet at any point is termed as opposite sides.
AB CD, BC DA are the pairs of opposite sides in a quadrilateral.

(viii) Opposite angles
Two angles, which are not adjacent angles are termed as opposite angles.
A and C, angles B and D are opposite angles in a quadrilateral.

(ix) Interior
The part of plane when points are enclosed within the quadrilateral is called as interior.



(x) Exterior
The part of plane when points are not enclosed within the quadrilateral is called as exterior.



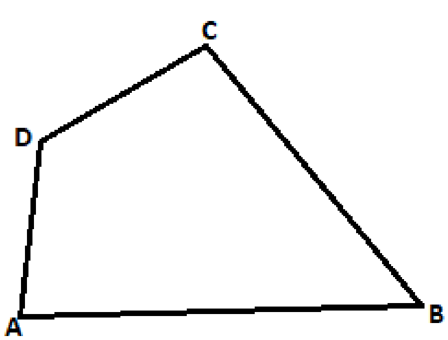
3. Complete each of the following, so as to make a true statement:
(i) A quadrilateral has ________ sides.
(ii) A quadrilateral has ________angles.
(iii) A quadrilateral has ________, no three of which are ________.
(iv) A quadrilateral has ________diagonals.
(v) The number of pairs of adjacent angles of a quadrilateral is ________.
(vi) The number of pairs of opposite angles of a quadrilateral is ________.

(vii) The sum of the angles of a quadrilateral is ________.
(viii) A diagonal of a quadrilateral is a line segment that joins two ________ vertices of the quadrilateral.
(ix) The sum of the angles of a quadrilateral is ________ right angles.
(x) The measure of each angle of a convex quadrilateral is ________ 180°.
(xi) In a quadrilateral the point of intersection of the diagonals lies in ________ of the quadrilateral.
(xii) A point is in the interior of a convex quadrilateral, if it is in the ________ of its two opposite angles.
(xiii) A quadrilateral is convex if for each side, the remaining ________ lie on the same side of the line containing the side.

Solution:

(i) A quadrilateral has **four** sides.
(ii) A quadrilateral has **four** angles.
(iii) A quadrilateral has **four**, no three of which are **collinear**.
(iv) A quadrilateral has **two** diagonals.
(v) The number of pairs of adjacent angles of a quadrilateral is **four**.
(vi) The number of pairs of opposite angles of a quadrilateral is **two**.
(vii) The sum of the angles of a quadrilateral is **360^0**.
(viii) A diagonal of a quadrilateral is a line segment that joins two **opposite** vertices of the quadrilateral.
(ix) The sum of the angles of a quadrilateral is **four** right angles.
(x) The measure of each angle of a convex quadrilateral is **less than** 180°.
(xi) In a quadrilateral the point of intersection of the diagonals lies in **interior** of the quadrilateral.
(xii) A point is in the interior of a convex quadrilateral, if it is in the **interiors** of its two opposite angles.
(xiii) A quadrilateral is convex if for each side, the remaining **vertices** lie on the same side of the line containing the side.

4. In Fig. 16.19, ABCD is a quadrilateral.



(i) Name a pair of adjacent sides.
(ii) Name a pair of opposite sides.
(iii) How many pairs of adjacent sides are there?
(iv) How many pairs of opposite sides are there?
(v) Name a pair of adjacent angles.
(vi) Name a pair of opposite angles.
(vii) How many pairs of adjacent angles are there?
(viii) How many pairs of opposite angles are there?
Solution:
(i) Name a pair of adjacent sides.
Adjacent sides are: AB, BC or BC, CD or CD, DA or AD, AB

(ii) Name a pair of opposite sides.
opposite sides are: AB, CD or BC, DA

(iii) How many pairs of adjacent sides are there?
Four pairs of adjacent sides i.e. AB BC, BC CD, CD DA and DA AB

(iv) How many pairs of opposite sides are there?
Two pairs of opposite sides. AB, DC and DA, BC

(v) Name a pair of adjacent angles.
Four pairs of Adjacent angles are: D∠AB A∠BC, A∠BC B∠CA, B∠CA C∠DA or C∠DA D∠AB

(vi) Name a pair of opposite angles.
Four pair of opposite angles are: D∠AB B∠CA and A∠BC C∠DA

(vii) How many pairs of adjacent angles are there?
Four pairs of adjacent angles. D∠AB A∠BC, A∠BC B∠CA, B∠CA C∠DA and C∠DA D∠AB

(viii) How many pairs of opposite angles are there?
Two pairs of opposite angles. D∠AB B∠CA and A∠BC C∠DA

5. The angles of a quadrilateral are 110°, 72°, 55° and *x*°. Find the value of *x*.
Solution:
We know that Sum of angles of a quadrilateral is = 360°

So,
$110° + 72° + 55° + x° = 360°$
$x° = 360° - 237°$
$x° = 123^o$
∴ Value of x is 123^o

6. The three angles of a quadrilateral are respectively equal to 110°, 50° and 40°. Find its fourth angle.
Solution:
We know that Sum of angles of a quadrilateral is = 360°
So,
$110° + 50° + 40° + x° = 360°$
$x° = 360° - 200°$
$x° = 160^o$
∴ Value of fourth angle is 160^o

7. A quadrilateral has three acute angles each measures 80°. What is the measure of the fourth angle?
Solution:
We know that Sum of angles of a quadrilateral is = 360°
So,
$80° + 80° + 80° + x° = 360°$
$x° = 360° - 240°$
$x° = 120^o$
∴ Value of fourth angle is 120^o

8. A quadrilateral has all its four angles of the same measure. What is the measure of each?
Solution:
We know that Sum of angles of a quadrilateral is = 360°
Let each angle be x^o
So,
$x^o + x^o + x^o + x^o = 360^o$
$x^o = 360^o/4$
$= 90^o$
∴ Value of angle is 90^o each.

9. Two angles of a quadrilateral are of measure 65° and the other two angles are equal. What is the measure of each of these two angles?

Solution:
We know that Sum of angles of a quadrilateral is = 360°
Let each angle be x^o
So,
$65^o + 65^o + x^o + x^o = 360^o$
$2x^o = 360^o – 130^o$
$x^o = 230^o/2$
$= 115^o$
∴ Value of two angles is 115^o each.

10. Three angles of a quadrilateral are equal. Fourth angle is of measure 150°. What is the measure of equal angles?
Solution:
We know that Sum of angles of a quadrilateral is = 360°
Let each angle be x^o
So,
$150^o + x^o + x^o + x^o = 360^o$
$3x^o = 360^o – 150^o$
$x^o = 210^o/3$
$= 70^o$
∴ Value of equal angles is 70^o each.

11. The four angles of a quadrilateral are as 3 : 5 : 7 : 9. Find the angles.
Solution:
We know that Sum of angles of a quadrilateral is = 360°
Let each angle be x^o
So,
$3x^o + 5x^o + 7x^o + 9x^o = 360^o$
$24x^o = 360^o$
$x^o = 360^o/24$
$= 15^o$
Value of angles are
$3x = 3 \times 15 = 45^o$
$5x = 5 \times 15 = 75^o$
$7x = 7 \times 15 = 105^o$
$9x = 9 \times 15 = 135^o$
∴ Value of angles are 45^o, 75^o, 105^o, 135^o

12. If the sum of the two angles of a quadrilateral is 180°. What is the sum of the

remaining two angles?

Solution:

We know that Sum of angles of a quadrilateral is = 360°

Let the sum of two angles be 180°

Let angle be x°

So,

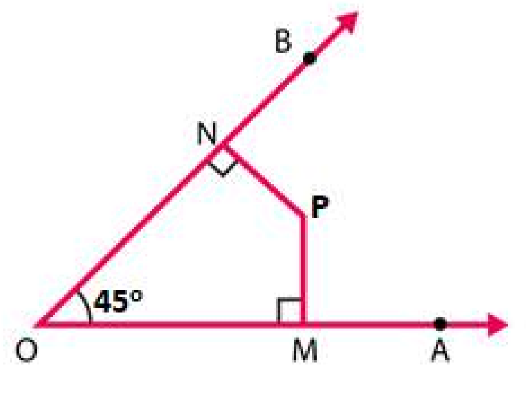
$180^o + x^o = 360^o$

$x^o = 360^o – 180^o$

$x^o = 180^o$

∴ Sum of remaining two angles is 180°

13. In Figure, find the measure of ∠*MPN*.



Solution:

We know that Sum of angles of a quadrilateral is = 360°

In the quadrilateral MPNO

∠NOP = 45°, ∠OMP = ∠PNO = 90°

Let angle ∠MPN is x°

∠NOP + ∠OMP + ∠PNO + ∠MPN = 360°

$45° + 90° + 90° + x° = 360°$

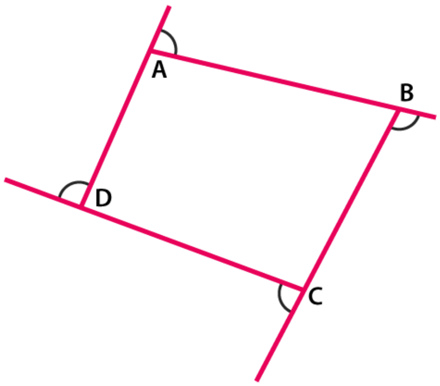
$x° = 360° - 225°$

$x° = 135°$

∴ Measure of ∠MPN is 135°

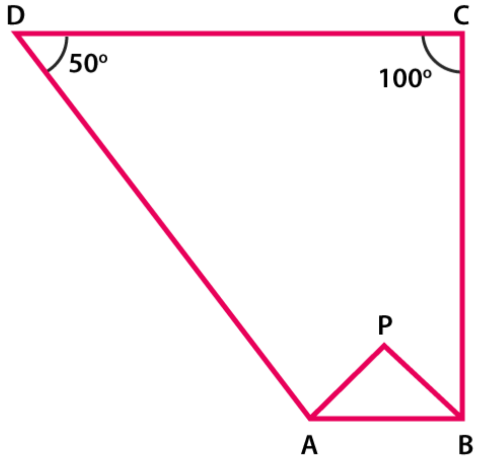
14. The sides of a quadrilateral are produced in order. What is the sum of the four exterior angles?

Solution:



We know that, exterior angle + interior adjacent angle = 180° [Linear pair]
Applying relation for polygon having n sides
Sum of all exterior angles + Sum of all interior angles = n × 180°
Sum of all exterior angles = n × 180° - Sum of all interior angles
= n × 180° - (n -2) × 180° [Sum of interior angles is = (n - 2) x 180°]
= n × 180° - n × 180° + 2 × 180°
= 180°n - 180°n + 360°
= 360°
∴ Sum of four exterior angles is 360°

15. In Figure, the bisectors of ∠A and ∠B meet at a point P. If ∠C =100° and ∠D = 50°, find the measure of ∠APB.



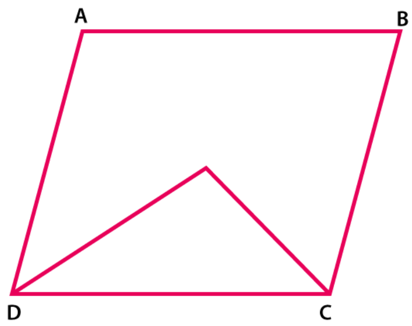
Solution:
We know that Sum of angles of a quadrilateral is = 360°
In the quadrilateral ABCD
Given, ∠*C* =100° and ∠*D* = 50°
∠A + ∠B + ∠C + ∠D = 360°

$\angle A + \angle B + 100^o + 50^o = 360^o$
$\angle A + \angle B = 360^o – 150^o$
$\angle A + \angle B = 210^o$ ……. (Equation 1)

Now in Δ APB
½ ∠A + ½ ∠B + ∠APB = 180º (since, sum of triangle is 180º)
∠APB = 180º – ½ (∠A + ∠B)………. (Equation 2)
On substituting value of ∠A + ∠B = 210 from equation (1) in equation (2)
∠APB = 180º – ½ (210º)
= 180º – 105º
= 75º
∴ The measure of ∠APB is 75º

16. In a quadrilateral *ABCD,* the angles *A, B, C* and *D* are in the ratio 1 : 2 : 4 : 5. Find the measure of each angle of the quadrilateral.
Solution:
We know that Sum of angles of a quadrilateral is = 360°
Let each angle be xº
So,
$x^o + 2x^o + 4x^o + 5x^o = 360^o$
$12x^o = 360^o$
$x^o = 360^o/12$
$= 30^o$
Value of angles are
x = 30º
2x = 2 × 30 = 60º
4x = 4 × 30 = 120º
5x = 5 × 30 = 150º
∴ Value of angles are 30º, 60º, 120º, 150º

17. In a quadrilateral ABCD, CO and DO are the bisectors of ∠C and ∠D respectively. Prove that ∠COD = 1/2 (∠A +∠B).
Solution:
We know that sum of angles of a quadrilateral is 360°
In the quadrilateral ABCD



∠A + ∠B + ∠C + ∠D = 360°

∠A + ∠B = 360° – (∠C + ∠D)

½ (∠A + ∠B) = ½ [360° – (∠C + ∠D)]

= 180° – ½ (∠C + ∠D)]……… (Equation 1)

Now in Δ DOC

½ ∠D + ½ ∠C + ∠COD = 180° (since sum of triangle = 180°)

½ (∠C + ∠D) + ∠COD = 180°

∠COD = 180° - ½ (∠C + ∠D)………. (Equation 2)

From above equations (1) and (2) RHS is equal, then LHS will also be equal.

∴ ∠COD = ½ (∠A + ∠B) is proved.

18. Find the number of sides of a regular polygon, when each of its angles has a measure of

(i) 160°

(ii) 135°

(iii) 175°

(iv) 162°

(v) 150°

Solution:

The measure of interior angle A of a polygon of n sides is given by A = [(n-2) ×180°]/n

(i) 160°

Angle of quadrilateral is 160°

160° = [(n-2) ×180°]/n

160°n = (n-2) ×180°

160°n = 180°n – 360°

180°n – 160° = 360°

20°n = 360°

n = 360°/20

= 18

∴ Number of sides are 18

(ii) 135^o
Angle of quadrilateral is 135°
$135^o = [(n-2) \times 180^o]/n$
$135^o n = (n-2) \times 180^o$
$135^o n = 180^o n – 360^o$
$180^o n – 135^o = 360^o$
$45^o n = 360^o$
$n = 360^o/45$
$= 8$
∴ Number of sides are 8

(iii) 175^o
Angle of quadrilateral is 175°
$175^o = [(n-2) \times 180^o]/n$
$175^o n = (n-2) \times 180^o$
$175^o n = 180^o n – 360^o$
$180^o n – 175^o = 360^o$
$5^o n = 360^o$
$n = 360^o/5$
$= 72$
∴ Number of sides are 72

(iv) 162^o
Angle of quadrilateral is 162°
$162^o = [(n-2) \times 180^o]/n$
$162^o n = (n-2) \times 180^o$
$162^o n = 180^o n – 360^o$
$180^o n – 162^o = 360^o$
$18^o n = 360^o$
$n = 360^o/18$
$= 20$
∴ Number of sides are 20

(v) 150^o
Angle of quadrilateral is 160°
$150^o = [(n-2) \times 180^o]/n$
$150^o n = (n-2) \times 180^o$

$150^o n = 180^o n - 360^o$
$180^o n - 150^o = 360^o$
$30^o n = 360^o$
$n = 360^o/30$
$= 12$
∴ Number of sides are 12

19. Find the numbers of degrees in each exterior angle of a regular pentagon.
Solution:
We know that the sum of exterior angles of a polygon is 360°
Measure of each exterior angle of a polygon is = $360^o/n$, where n is the number of sides
We know that number of sides in a pentagon is 5
Measure of each exterior angle of a pentagon is = $360^o/5 = 72^o$
∴ Measure of each exterior angle of a pentagon is 72^o

20. The measure of angles of a hexagon are $x°$, $(x-5)°$, $(x-5)°$, $(2x-5)°$, $(2x-5)°$, $(2x+20)°$. Find value of x.
Solution:
By using the formula,
The sum of interior angles of a polygon = (n – 2) × 180°, (where n = number of sides of polygon.)
We know, a hexagon has 6 sides. So,
The sum of interior angles of a hexagon = (6 – 2) × 180° = 4 × 180° = 720°

$x° + (x-5)° + (x-5)° + (2x-5)° + (2x-5)° + (2x+20)° = 720°$
$x° + x° - 5° + x° - 5° + 2x° - 5° + 2x° - 5° + 2x° + 20° = 720°$
$9x° = 720°$
$x = 720^o/9$
$= 80^o$
∴ Value of x is 80^o

21. In a convex hexagon, prove that the sum of all interior angle is equal to twice the sum of its exterior angles formed by producing the sides in the same order.
Solution:
By using the formulas,
The sum of interior angles of a polygon = (n – 2) × 180°
The sum of interior angles of a hexagon = (6 – 2) × 180° = 4 × 180° = 720°
The Sum of exterior angle of a polygon is 360°
∴ Sum of interior angles of a hexagon = twice the sum of interior angles.

Hence proved.

22. The sum of the interior angles of a polygon is three times the sum of its exterior angles. Determine the number of sides of the polygon.

Solution:

By using the formulas,

The sum of interior angles of a polygon = $(n - 2) \times 180°$ …..(i)

The Sum of exterior angle of a polygon is 360°

So,

Sum of interior angles = 3 × sum of exterior angles

= $3 \times 360° = 1080°$…..(ii)

Now by equating (i) and (ii) we get,

$(n - 2) \times 180° = 1080°$

$n - 2 = 1080°/180°$

$n - 2 = 6$

$n = 6 + 2$

$= 8$

∴ Number of sides of a polygon is 8

23. Determine the number of sides of a polygon whose exterior and interior angles are in the ratio 1 : 5.

Solution:

By using the formulas,

The sum of interior angles of a polygon = $(n - 2) \times 180°$ ……..(i)

The Sum of exterior angle of a polygon is 360°

We know that Sum of exterior angles/Sum of interior angles = 1/5…..(ii)

So, equating (i) and (ii) we get

$360°/(n - 2) \times 180° = 1/5$

On cross multiplication,

$(n - 2) \times 180° = 360° \times 5$

$(n - 2) \times 180° = 1800°$

$(n - 2) = 1800°/180°$

$(n - 2) = 10$

$n = 10 + 2$

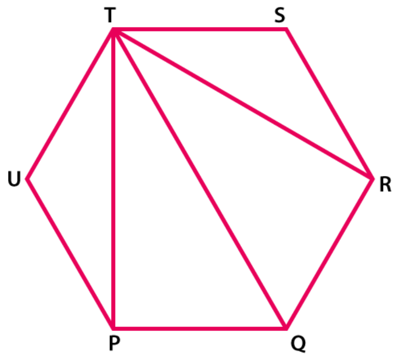
$= 12$

∴ Numbers of sides of a polygon is 12

24. PQRSTU is a regular hexagon, determine each angle of ΔPQT.

Solution:

We know that the sum of interior angles of a polygon = (n – 2) × 180°
The sum of interior angles of a hexagon = (6 – 2) × 180° = 4 × 180° = 720°
Measure of each angle of hexagon = 720°/6 = 120°
∠PUT = 120° Proved.



In Δ PUT
∠PUT + ∠UTP + ∠TPU = 180° (sum of triangles)
120° + 2∠UTP = 180° (since Δ PUT is an isosceles triangle)
2∠UTP = 180° – 120°
2∠UTP = 60°
∠UTP = 60°/2
= 30°

∠UTP = ∠TPU = 30° similarly ∠RTS = 30°
∴ ∠PTR = ∠UTS - ∠UTP - ∠RTS
= 120° – 30° – 30°
= 60°

∠TPQ = ∠UPQ - ∠UPT
= 120° – 30°
= 90°

∠TQP = 180° – 150°
= 30° (by using angle sum property of triangle in ΔPQT)
∴ ∠P = 90°, ∠Q = 60°, ∠T = 30°