

EXERCISE 17.1

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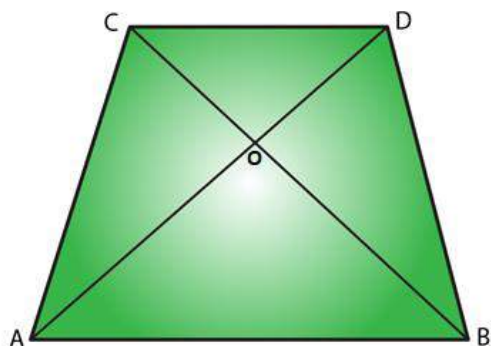
1. Given below is a parallelogram ABCD. Complete each statement along with the definition or property used.

(i) $AD =$

(ii) $\angle DCB =$

(iii) $OC =$

(iv) $\angle DAB + \angle CDA =$



Solution:

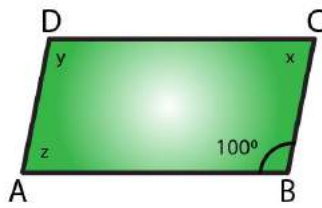
(i) $AD = BC$. Because, diagonals bisect each other in a parallelogram.

(ii) $\angle DCB = \angle BAD$. Because, alternate interior angles are equal.

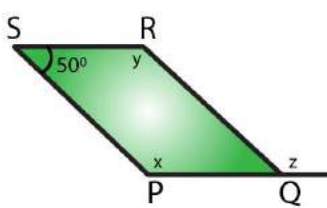
(iii) $OC = OA$. Because, diagonals bisect each other in a parallelogram.

(iv) $\angle DAB + \angle CDA = 180^\circ$. Because sum of adjacent angles in a parallelogram is 180° .

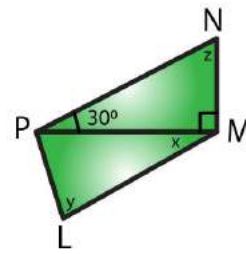
2. The following figures are parallelograms. Find the degree values of the unknowns x , y , z .



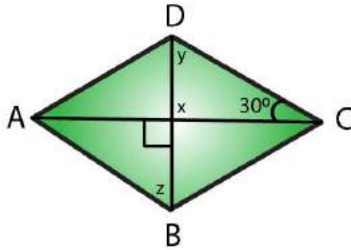
(i)



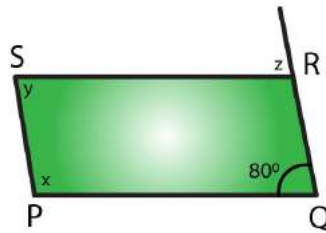
(ii)



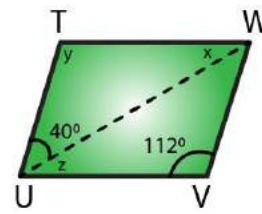
(iii)



(iv)



(v)



(vi)

Solution:

(i) $\angle ABC = \angle y = 100^\circ$ (opposite angles are equal in a parallelogram)
 $\angle x + \angle y = 180^\circ$ (sum of adjacent angles is $= 180^\circ$ in a parallelogram)
 $\angle x + 100^\circ = 180^\circ$
 $\angle x = 180^\circ - 100^\circ$
 $= 80^\circ$
 $\therefore \angle x = 80^\circ \angle y = 100^\circ \angle z = 80^\circ$ (opposite angles are equal in a parallelogram)

(ii) $\angle RSP + \angle y = 180^\circ$ (sum of adjacent angles is $= 180^\circ$ in a parallelogram)
 $\angle y + 50^\circ = 180^\circ$
 $\angle y = 180^\circ - 50^\circ$
 $= 130^\circ$
 $\therefore \angle x = \angle y = 130^\circ$ (opposite angles are equal in a parallelogram)
 $\angle RSP = \angle RQP = 50^\circ$ (opposite angles are equal in a parallelogram)
 $\angle RQP + \angle z = 180^\circ$ (linear pair)
 $50^\circ + \angle z = 180^\circ$
 $\angle z = 180^\circ - 50^\circ$
 $= 130^\circ$
 $\therefore \angle x = 130^\circ \angle y = 130^\circ \angle z = 130^\circ$

(iii) In $\triangle PMN$

$\angle NPM + \angle NMP + \angle MNP = 180^\circ$ [Sum of all the angles of a triangle is 180°]

$$30^\circ + 90^\circ + \angle z = 180^\circ$$

$$\angle z = 180^\circ - 120^\circ$$

$$= 60^\circ$$

$$\angle y = \angle z = 60^\circ \text{ [opposite angles are equal in a parallelogram]}$$

$$\angle z = 180^\circ - 120^\circ \text{ [sum of the adjacent angles is equal to } 180^\circ \text{ in a parallelogram]}$$

$$\angle z = 60^\circ$$

$$\angle z + \angle LMN = 180^\circ \text{ [sum of the adjacent angles is equal to } 180^\circ \text{ in a parallelogram]}$$

$$60^\circ + 90^\circ + \angle x = 180^\circ$$

$$\angle x = 180^\circ - 150^\circ$$

$$\angle x = 30^\circ$$

$$\therefore \angle x = 30^\circ \angle y = 60^\circ \angle z = 60^\circ$$

$$\text{(iv) } \angle x = 90^\circ \text{ [vertically opposite angles are equal]}$$

In $\triangle DOC$

$$\angle x + \angle y + 30^\circ = 180^\circ \text{ [Sum of all the angles of a triangle is } 180^\circ]$$

$$90^\circ + 30^\circ + \angle y = 180^\circ$$

$$\angle y = 180^\circ - 120^\circ$$

$$\angle y = 60^\circ$$

$$\angle y = \angle z = 60^\circ \text{ [alternate interior angles are equal]}$$

$$\therefore \angle x = 90^\circ \angle y = 60^\circ \angle z = 60^\circ$$

$$\text{(v) } \angle x + \angle POR = 180^\circ \text{ [sum of the adjacent angles is equal to } 180^\circ \text{ in a parallelogram]}$$

$$\angle x + 80^\circ = 180^\circ$$

$$\angle x = 180^\circ - 80^\circ$$

$$\angle x = 100^\circ$$

$$\angle y = 80^\circ \text{ [opposite angles are equal in a parallelogram]}$$

$$\angle SRQ = \angle x = 100^\circ$$

$$\angle SRQ + \angle z = 180^\circ \text{ [Linear pair]}$$

$$100^\circ + \angle z = 180^\circ$$

$$\angle z = 180^\circ - 100^\circ$$

$$\angle z = 80^\circ$$

$$\therefore \angle x = 100^\circ \angle y = 80^\circ \angle z = 80^\circ$$

$$\text{(vi) } \angle y = 112^\circ \text{ [In a parallelogram opposite angles are equal]}$$

$$\angle y + \angle VUT = 180^\circ \text{ [In a parallelogram sum of the adjacent angles is equal to } 180^\circ]$$

$$\angle z + 40^\circ + 112^\circ = 180^\circ$$

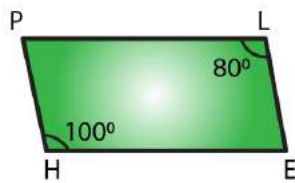
$$\angle z = 180^\circ - 152^\circ$$

$$\angle z = 28^\circ$$

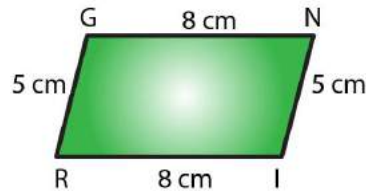
$$\angle z = \angle x = 28^\circ \text{ [alternate interior angles are equal]}$$

$$\therefore \angle x = 28^\circ \angle y = 112^\circ \angle z = 28^\circ$$

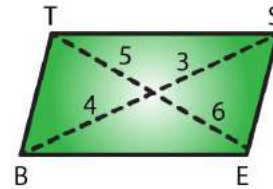
3. Can the following figures be parallelograms? Justify your answer.



(i)



(ii)



(iii)

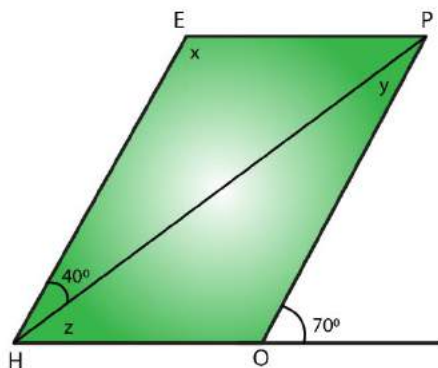
Solution:

(i) No, opposite angles are equal in a parallelogram.

(ii) Yes, opposite sides are equal and parallel in a parallelogram.

(iii) No, diagonals bisect each other in a parallelogram.

4. In the adjacent figure HOPE is a parallelogram. Find the angle measures x, y and z. State the geometrical truths you use to find them.



Solution:

We know that

$$\angle POH + 70^\circ = 180^\circ \text{ [Linear pair]}$$

$$\angle POH = 180^\circ - 70^\circ$$

$$\angle POH = 110^\circ$$

$\angle POH = \angle x = 110^\circ$ [opposite angles are equal in a parallelogram]

$\angle x + \angle z + 40^\circ = 180^\circ$ [sum of the adjacent angles is equal to 180° in a parallelogram]

$$110^\circ + \angle z + 40^\circ = 180^\circ$$

$$\angle z = 180^\circ - 150^\circ$$

$$\angle z = 30^\circ$$

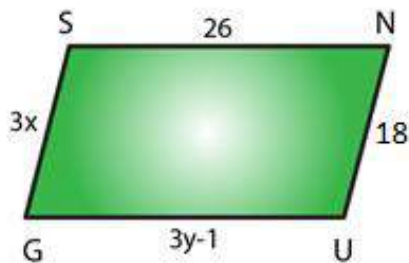
$$\angle z + \angle y = 70^\circ$$

$$\angle y + 30^\circ = 70^\circ$$

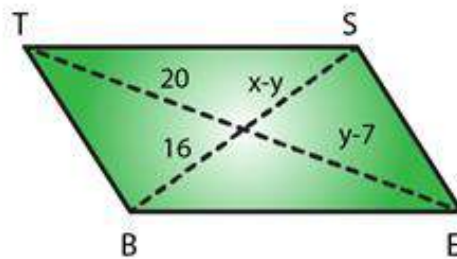
$$\angle y = 70^\circ - 30^\circ$$

$$\angle y = 40^\circ$$

5. In the following figures *GUNS* and *RUNS* are parallelograms. Find x and y .



(i)



(ii)

Solution:

(i) $3y - 1 = 26$ [opposite sides are of equal length in a parallelogram]

$$3y = 26 + 1$$

$$y = 27/3$$

$$y = 9$$

$3x = 18$ [opposite sides are of equal length in a parallelogram]

$$x = 18/3$$

$$x = 6$$

$$\therefore x = 6 \text{ and } y = 9$$

(ii) $y - 7 = 20$ [diagonals bisect each other in a parallelogram]

$$y = 20 + 7$$

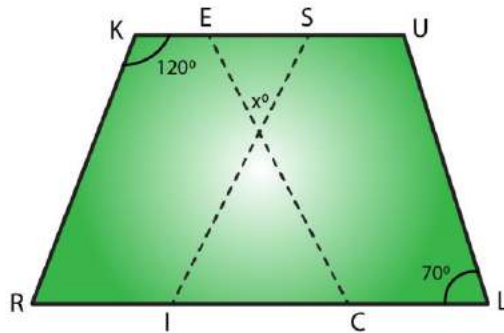
$$y = 27$$

$x - y = 16$ [diagonals bisect each other in a parallelogram]

$$x - 27 = 16$$

$$\begin{aligned}x &= 16 + 27 \\&= 43 \\ \therefore x &= 43 \text{ and } y = 27\end{aligned}$$

6. In the following figure RISK and CLUE are parallelograms. Find the measure of x.



Solution:

In parallelogram RISK

$\angle RKS + \angle KSI = 180^\circ$ [sum of the adjacent angles is equal to 180° in a parallelogram]

$$120^\circ + \angle KSI = 180^\circ$$

$$\angle KSI = 180^\circ - 120^\circ$$

$$\angle z = 60^\circ$$

In parallelogram CLUE

$$\angle CEU = \angle z = 70^\circ \text{ [opposite angles are equal in a parallelogram]}$$

In $\triangle EOS$

$$70^\circ + \angle x + 60^\circ = 180^\circ \text{ [Sum of angles of a triangles is } 180^\circ]$$

$$\angle x = 180^\circ - 130^\circ$$

$$\angle x = 50^\circ$$

$$\therefore x = 50^\circ$$

7. Two opposite angles of a parallelogram are $(3x - 2)^\circ$ and $(50 - x)^\circ$. Find the measure of each angle of the parallelogram.

Solution:

We know that opposite angles of a parallelogram are equal.

$$\text{So, } (3x - 2)^\circ = (50 - x)^\circ$$

$$3x^\circ - 2^\circ = 50^\circ - x^\circ$$

$$3x^\circ + x^\circ = 50^\circ + 2^\circ$$

$$4x^\circ = 52^\circ$$

$$x^\circ = 52^\circ/4$$

$$= 13^\circ$$

Measure of opposite angles are,

$$(3x - 2)^\circ = 3 \times 13 - 2 = 37^\circ$$

$$(50 - x)^\circ = 50 - 13 = 37^\circ$$

We know that Sum of adjacent angles = 180°

Other two angles are $180^\circ - 37^\circ = 143^\circ$

\therefore Measure of each angle is $37^\circ, 143^\circ, 37^\circ, 143^\circ$

8. If an angle of a parallelogram is two-third of its adjacent angle, find the angles of the parallelogram.

Solution:

Let us consider one of the adjacent angle as x°

Other adjacent angle is $= 2x^\circ/3$

We know that sum of adjacent angles = 180°

So,

$$x^\circ + 2x^\circ/3 = 180^\circ$$

$$(3x^\circ + 2x^\circ)/3 = 180^\circ$$

$$5x^\circ/3 = 180^\circ$$

$$x^\circ = 180^\circ \times 3/5$$

$$= 108^\circ$$

Other angle is $= 180^\circ - 108^\circ = 72^\circ$

\therefore Angles of a parallelogram are $72^\circ, 72^\circ, 108^\circ, 108^\circ$

9. The measure of one angle of a parallelogram is 70° . What are the measures of the remaining angles?

Solution:

Let us consider one of the adjacent angle as x°

Other adjacent angle = 70°

We know that sum of adjacent angles = 180°

So,

$$x^\circ + 70^\circ = 180^\circ$$

$$x^\circ = 180^\circ - 70^\circ$$

$$= 110^\circ$$

\therefore Measures of the remaining angles are $70^\circ, 70^\circ, 110^\circ$ and 110°

10. Two adjacent angles of a parallelogram are as 1 : 2. Find the measures of all the

angles of the parallelogram.

Solution:

Let us consider one of the adjacent angle as x°

Other adjacent angle = $2x^\circ$

We know that sum of adjacent angles = 180°

So,

$$x^\circ + 2x^\circ = 180^\circ$$

$$3x^\circ = 180^\circ$$

$$x^\circ = 180^\circ/3$$

$$= 60^\circ$$

So other angle is $2x = 2 \times 60 = 120^\circ$

\therefore Measures of the remaining angles are 60° , 60° , 120° and 120°

11. In a parallelogram ABCD, $\angle D = 135^\circ$, determine the measure of $\angle A$ and $\angle B$.

Solution:

Given, one of the adjacent angle $\angle D = 135^\circ$

Let other adjacent angle $\angle A$ be = x°

We know that sum of adjacent angles = 180°

$$x^\circ + 135^\circ = 180^\circ$$

$$x^\circ = 180^\circ - 135^\circ$$

$$= 45^\circ$$

$$\angle A = x^\circ = 45^\circ$$

We know that measure of opposite angles are equal in a parallelogram.

$$\text{So, } \angle A = \angle C = 45^\circ$$

$$\text{And } \angle D = \angle B = 135^\circ$$

12. ABCD is a parallelogram in which $\angle A = 70^\circ$. Compute $\angle B$, $\angle C$ and $\angle D$.

Solution:

Given, one of the adjacent angle $\angle A = 70^\circ$

Other adjacent angle $\angle B$ be = x°

We know that sum of adjacent angles = 180°

$$x^\circ + 70^\circ = 180^\circ$$

$$x^\circ = 180^\circ - 70^\circ$$

$$= 110^\circ$$

$$\angle B = x^\circ = 110^\circ$$

We know that measure of opposite angles are equal in a parallelogram.

$$\text{So, } \angle A = \angle C = 70^\circ$$

$$\text{And } \angle D = \angle B = 110^\circ$$

13. The sum of two opposite angles of a parallelogram is 130° . Find all the angles of the parallelogram.

Solution:

Given, one of the adjacent angle $\angle A = 130^\circ$

Other adjacent angle $\angle B$ be $= x^\circ$

We know that sum of adjacent angles $= 180^\circ$

$$x^\circ + 130^\circ = 180^\circ$$

$$x^\circ = 180^\circ - 130^\circ$$

$$= 50^\circ$$

$$\angle B = x^\circ = 110^\circ$$

We know that measure of opposite angles are equal in a parallelogram.

$$\text{So, } \angle A = \angle C = 130^\circ$$

$$\text{And } \angle D = \angle B = 50^\circ$$

14. All the angles of a quadrilateral are equal to each other. Find the measure of each. Is the quadrilateral a parallelogram? What special type of parallelogram is it?

Solution:

Let us consider each angle of a parallelogram as x°

We know that sum of angles $= 360^\circ$

$$x^\circ + x^\circ + x^\circ + x^\circ = 360^\circ$$

$$4x^\circ = 360^\circ$$

$$x^\circ = 360^\circ/4$$

$$= 90^\circ$$

\therefore Measure of each angle is 90°

Yes, this quadrilateral is a parallelogram.

Since each angle of a parallelogram is equal to 90° , so it is a rectangle.

15. Two adjacent sides of a parallelogram are 4 cm and 3 cm respectively. Find its perimeter.

Solution:

We know that opposite sides of a parallelogram are parallel and equal.

So, Perimeter = Sum of all sides (there are 4 sides)

$$\text{Perimeter} = 4 + 3 + 4 + 3$$

$$= 14 \text{ cm}$$

\therefore Perimeter is 14cm.

16. The perimeter of a parallelogram is 150 cm. One of its sides is greater than the other by 25 cm. Find the length of the sides of the parallelogram.

Solution:

Given, Perimeter of the parallelogram = 150 cm

Let us consider one of the sides as = 'x' cm

Other side as = (x + 25) cm

We know that opposite sides of a parallelogram are parallel and equal.

So, Perimeter = Sum of all sides

$$x + x + 25 + x + x + 25 = 150$$

$$4x + 50 = 150$$

$$4x = 150 - 50$$

$$x = 100/4$$

$$= 25$$

∴ Sides of the parallelogram are (x) = 25 cm and (x+25) = 50 cm.

17. The shorter side of a parallelogram is 4.8 cm and the longer side is half as much again as the shorter side. Find the perimeter of the parallelogram.

Solution:

Given, Shorter side of the parallelogram = 4.8 cm

Longer side of the parallelogram = $4.8 + 4.8/2$

$$= 4.8 + 2.4$$

$$= 7.2\text{cm}$$

We know that opposite sides of a parallelogram are parallel and equal.

So, Perimeter = Sum of all sides

$$\text{Perimeter of the parallelogram} = 4.8 + 7.2 + 4.8 + 7.2$$

$$= 24\text{cm}$$

∴ Perimeter of the parallelogram is 24 cm.

18. Two adjacent angles of a parallelogram are $(3x-4)^\circ$ and $(3x+10)^\circ$. Find the angles of the parallelogram.

Solution:

We know that adjacent angles of a parallelogram are equal.

$$\text{So, } (3x - 4)^\circ + (3x + 10)^\circ = 180^\circ$$

$$3x^\circ + 3x^\circ - 4 + 10 = 180^\circ$$

$$6x = 180^\circ - 6^\circ$$

$$x = 174^\circ/6$$

$$= 29^\circ$$

Measure of adjacent angles are,

$$(3x - 4)^\circ = 3 \times 29 - 4 = 83^\circ$$

$$(3x + 10)^\circ = 3 \times 29 + 10 = 97^\circ$$

We know that Sum of adjacent angles = 180°

∴ Measure of each angle is $83^\circ, 97^\circ, 83^\circ, 97^\circ$

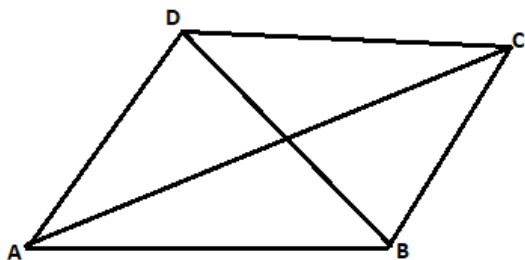
19. In a parallelogram ABCD, the diagonals bisect each other at O.

If $\angle ABC = 30^\circ$, $\angle BDC = 10^\circ$ and $\angle CAB = 70^\circ$. Find:

$\angle DAB$, $\angle ADC$, $\angle BCD$, $\angle AOD$, $\angle DOC$, $\angle BOC$, $\angle AOB$, $\angle ACD$, $\angle CAB$, $\angle ADB$, $\angle ACB$, $\angle DBC$, and $\angle DBA$.

Solution:

Firstly let us draw a parallelogram



Given, $\angle ABC = 30^\circ$,

$\angle ABC = \angle ADC = 30^\circ$ [We know that measure of opposite angles are equal in a parallelogram]

$\angle BDC = 10^\circ$

$\angle CAB = 70^\circ$

$\angle BDA = \angle ADB = \angle ADC - \angle BDC = 30^\circ - 10^\circ = 20^\circ$

$\angle DAB = 180^\circ - 30^\circ = 150^\circ$

$\angle ADB = \angle DBC = 20^\circ$ (alternate angles)

$\angle BCD = \angle DAB = 150^\circ$ [we know, opposite angles are equal in a parallelogram]

$\angle DBA = \angle BDC = 10^\circ$ [we know, Alternate interior angles are equal]

In $\triangle ABC$

$\angle CAB + \angle ABC + \angle BCA = 180^\circ$ [since, sum of all angles of a triangle is 180°]

$70^\circ + 30^\circ + \angle BCA = 180^\circ$

$\angle BCA = 180^\circ - 100^\circ$
 $= 80^\circ$

$\angle DAB = \angle DAC + \angle CAB = 70^\circ + 80^\circ = 150^\circ$

$\angle BCD = 150^\circ$ (opposite angle of the parallelogram)

$\angle DCA = \angle CAB = 70^\circ$

In $\triangle DOC$

$\angle BDC + \angle ACD + \angle DOC = 180^\circ$ [since, sum of all angles of a triangle is 180°]

$10^\circ + 70^\circ + \angle DOC = 180^\circ$

$\angle DOC = 180^\circ - 80^\circ$

$$\angle DOC = 100^\circ$$

So, $\angle DOC = \angle AOB = 100^\circ$ [Vertically opposite angles are equal]

$$\angle DOC + \angle AOD = 180^\circ \text{ [Linear pair]}$$

$$100^\circ + \angle AOD = 180^\circ$$

$$\angle AOD = 180^\circ - 100^\circ$$

$$\angle AOD = 80^\circ$$

So, $\angle AOD = \angle BOC = 80^\circ$ [Vertically opposite angles are equal]

$$\angle CAB = 70^\circ$$

$$\angle ABC + \angle BCD = 180^\circ \text{ [In a parallelogram sum of adjacent angles is } 180^\circ]$$

$$30^\circ + \angle ACB + \angle ACD = 180^\circ$$

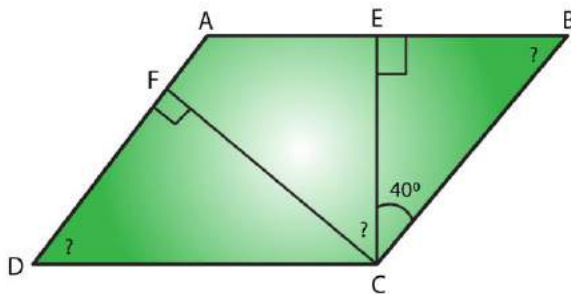
$$30^\circ + \angle ACB + 70^\circ = 180^\circ$$

$$\angle ACB = 180^\circ - 100^\circ$$

$$\angle ACB = 80^\circ$$

$\therefore \angle DAB = 150^\circ, \angle ADC = 30^\circ, \angle BCD = 150^\circ, \angle AOD = 80^\circ, \angle DOC = 100^\circ, \angle BOC = 80^\circ,$
 $\angle AOB = 100^\circ, \angle ACD = 70^\circ, \angle CAB = 70^\circ, \angle ADB = 20^\circ, \angle ACB = 80^\circ, \angle DBC = 20^\circ,$
 and $\angle DBA = 10^\circ$.

20. Find the angles marked with a question mark shown in Figure.



Solution:

In $\triangle BEC$

$$\angle BEC + \angle ECB + \angle CBE = 180^\circ \text{ [Sum of angles of a triangle is } 180^\circ]$$

$$90^\circ + 40^\circ + \angle CBE = 180^\circ$$

$$\angle CBE = 180^\circ - 130^\circ$$

$$\angle CBE = 50^\circ$$

$$\angle CBE = \angle ADC = 50^\circ \text{ (Opposite angles of a parallelogram are equal)}$$

$$\angle B = \angle D = 50^\circ \text{ [Opposite angles of a parallelogram are equal]}$$

$$\angle A + \angle B = 180^\circ \text{ [Sum of adjacent angles of a triangle is } 180^\circ]$$

$$\angle A + 50^\circ = 180^\circ$$

$$\angle A = 180^\circ - 50^\circ$$

$$\text{So, } \angle A = 130^\circ$$

In $\triangle DFC$

$$\angle DFC + \angle FCD + \angle CDF = 180^\circ \text{ [Sum of angles of a triangle is } 180^\circ]$$

$$90^\circ + \angle FCD + 50^\circ = 180^\circ$$

$$\angle FCD = 180^\circ - 140^\circ$$

$$\angle FCD = 40^\circ$$

$$\angle A = \angle C = 130^\circ \text{ [Opposite angles of a parallelogram are equal]}$$

$$\angle C = \angle FCE + \angle BCE + \angle FCD$$

$$\angle FCD + 40^\circ + 40^\circ = 130^\circ$$

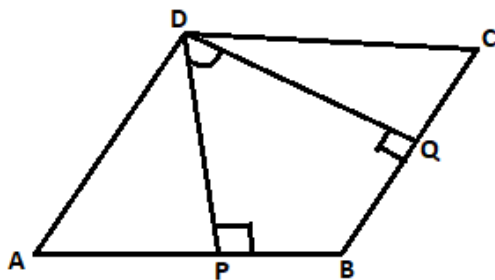
$$\angle FCD = 130^\circ - 80^\circ$$

$$\angle FCD = 50^\circ$$

$$\therefore \angle EBC = 50^\circ, \angle ADC = 50^\circ \text{ and } \angle FCD = 50^\circ$$

21. The angle between the altitudes of a parallelogram, through the same vertex of an obtuse angle of the parallelogram is 60° . Find the angles of the parallelogram.

Solution:



Let us consider a parallelogram, ABCD. Where, $DP \perp AB$ and $DQ \perp BC$.

$$\text{Given } \angle PDQ = 60^\circ$$

In quadrilateral DPBQ

$$\angle PDQ + \angle DPB + \angle B + \angle BQD = 360^\circ \text{ [Sum of all the angles of a Quadrilateral is } 360^\circ]$$

$$60^\circ + 90^\circ + \angle B + 90^\circ = 360^\circ$$

$$\angle B = 360^\circ - 240^\circ$$

$$\angle B = 120^\circ$$

$$\angle B = \angle D = 120^\circ \text{ [Opposite angles of parallelogram are equal]}$$

$$\angle B + \angle C = 180^\circ \text{ [Sum of adjacent interior angles in a parallelogram is } 180^\circ]$$

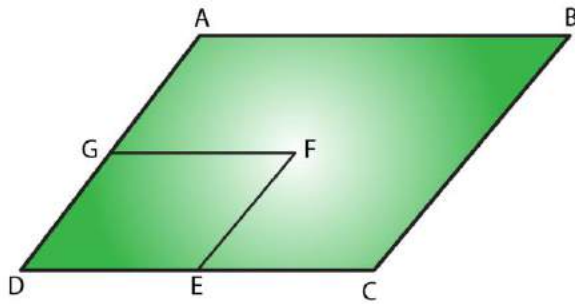
$$120^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 120^\circ = 60^\circ$$

$$\angle A = \angle C = 70^\circ \text{ (Opposite angles of parallelogram are equal)}$$

$$\therefore \text{Angles of a parallelogram are } 60^\circ, 120^\circ, 60^\circ, 120^\circ$$

22. In Figure, ABCD and AEFB are parallelograms. If $\angle C = 55^\circ$, what is the measure of $\angle F$?



Solution:

In parallelogram ABCD

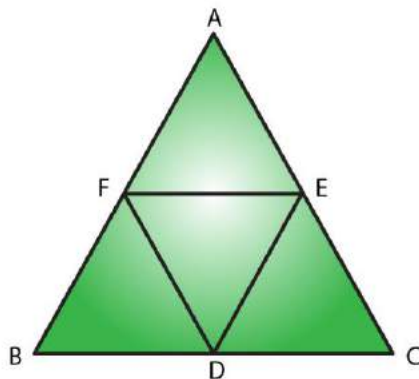
$$\angle C = \angle A = 55^\circ \text{ [In a parallelogram opposite angles are equal in a parallelogram]}$$

In parallelogram AEFB

$$\angle A = \angle F = 55^\circ \text{ [In a parallelogram opposite angles are equal in a parallelogram]}$$

$$\therefore \text{Measure of } \angle F = 55^\circ$$

23. In Figure, BDEF and DCEF are each a parallelogram. Is it true that $BD = DC$? Why or why not?



Solution:

In parallelogram BDEF

$BD = EF$ [In a parallelogram opposite sides are equal]

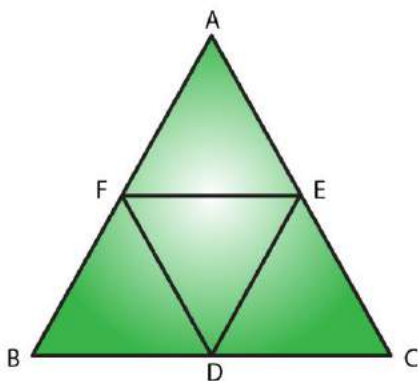
In parallelogram DCEF

$DC = EF$ [In a parallelogram opposite sides are equal]

Since, $BD = EF = DC$

So, $BD = DC$

24. In Figure, suppose it is known that $DE = DF$. Then, is $\triangle ABC$ isosceles? Why or why not?

**Solution:**

In parallelogram BDEF

$BD = EF$ and $BF = DE$ [opposite sides are equal in a parallelogram]

In parallelogram DCEF

$DC = EF$ and $DF = CE$ [opposite sides are equal in a parallelogram]

In parallelogram AFDE

$AF = DE$ and $DF = AE$ [opposite sides are equal in a parallelogram]

So, $DE = AF = BF$

Similarly: $DF = CE = AE$

Given, $DE = DF$

Since, $DF = DF$

$$AF + BF = CE + AE$$

$$AB = AC$$

$\therefore \triangle ABC$ is an isosceles triangle.

25. Diagonals of parallelogram ABCD intersect at O as shown in Figure. XY contains O, and X, Y are points on opposite sides of the parallelogram. Give reasons for each of the following:

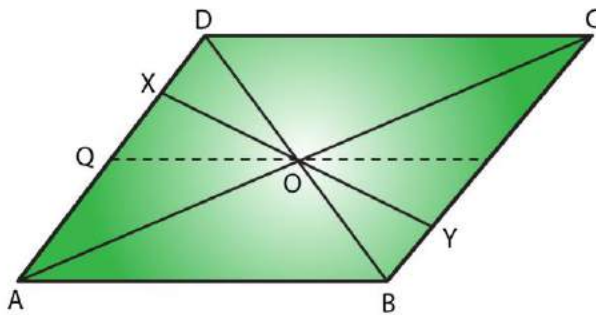
(i) $OB = OD$

(ii) $\angle OBY = \angle ODX$

(iii) $\angle BOY = \angle DOX$

(iv) $\triangle BOY \cong \triangle DOX$

Now, state if XY is bisected at O.



Solution:

(i) $OB = OD$

$OB = OD$. Since diagonals bisect each other in a parallelogram.

(ii) $\angle OBY = \angle ODX$

$\angle OBY = \angle ODX$. Since alternate interior angles are equal in a parallelogram.

(iii) $\angle BOY = \angle DOX$

$\angle BOY = \angle DOX$. Since vertical opposite angles are equal in a parallelogram.

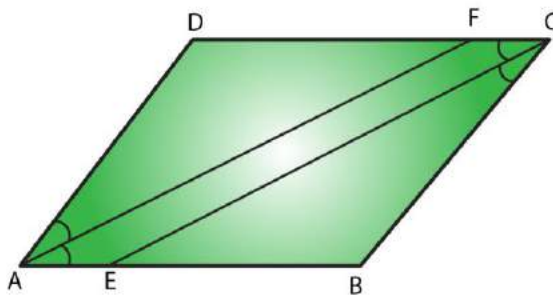
(iv) $\triangle BOY \cong \triangle DOX$

$\triangle BOY$ and $\triangle DOX$. Since $OB = OD$, where diagonals bisect each other in a parallelogram.

$\angle OBY = \angle ODX$ [Alternate interior angles are equal]
 $\angle BOY = \angle DOX$ [Vertically opposite angles are equal]
 $\triangle BOY \cong \triangle DOX$ [by ASA congruence rule]
 $OX = OY$ [Corresponding parts of congruent triangles]
 $\therefore XY$ is bisected at O.

26. In Fig. 17.31, ABCD is a parallelogram, CE bisects $\angle C$ and AF bisects $\angle A$. In each of the following, if the statement is true, give a reason for the same:

- (i) $\angle A = \angle C$
- (ii) $\angle FAB = \frac{1}{2} \angle A$
- (iii) $\angle DCE = \frac{1}{2} \angle C$
- (iv) $\angle CEB = \angle FAB$
- (v) $CE \parallel AF$



Solution:

- (i) $\angle A = \angle C$

True, Since $\angle A = \angle C = 55^\circ$ [opposite angles are equal in a parallelogram]

- (ii) $\angle FAB = \frac{1}{2} \angle A$

True, Since AF is the angle bisector of $\angle A$.

- (iii) $\angle DCE = \frac{1}{2} \angle C$

True, Since CE is the angle bisector of angle $\angle C$.

- (iv) $\angle CEB = \angle FAB$

True,

Since $\angle DCE = \angle FAB$ (opposite angles are equal in a parallelogram).

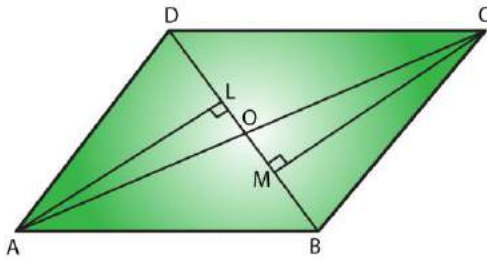
$\angle CEB = \angle DCE$ (alternate angles)

$\frac{1}{2} \angle C = \frac{1}{2} \angle A$ [AF and CE are angle bisectors]

(v) $CE \parallel AF$

True, since one pair of opposite angles are equal, therefore quad. AEFC is a parallelogram.

27. Diagonals of a parallelogram ABCD intersect at O. AL and CM are drawn perpendiculars to BD such that L and M lie on BD. Is $AL = CM$? Why or why not? Solution:



Given, AL and CM are perpendiculars on diagonal BD.

In $\triangle AOL$ and $\triangle COM$

$\angle AOL = \angle COM$ (vertically opposite angle) (i)

$\angle ALO = \angle CMO = 90^\circ$ (each right angle) (ii)

By using angle sum property

$\angle AOL + \angle ALO + \angle LAO = 180^\circ$ (iii)

$\angle COM + \angle CMO + \angle OCM = 180^\circ$ (iv)

From (iii) and (iv)

$\angle AOL + \angle ALO + \angle LAO = \angle COM + \angle CMO + \angle OCM$

$\angle LAO = \angle OCM$ (from (i) and (ii))

In $\triangle AOL$ and $\triangle COM$

$\angle ALO = \angle CMO$ (each right angle)

$AO = OC$ (diagonals of a parallelogram bisect each other)

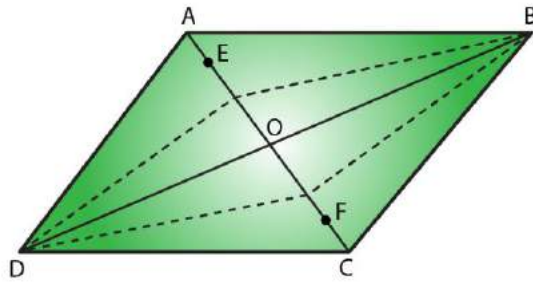
$\angle LAO = \angle OCM$ (proved)

So, $\triangle AOL$ is congruent to $\triangle COM$

$\therefore AL = CM$ (Corresponding parts of congruent triangles)

28. Points E and F lie on diagonals AC of a parallelogram ABCD such that $AE = CF$. what type of quadrilateral is BFDE?

Solution:



In parallelogram ABCD:

$AO = OC$ (i) (Diagonals of a parallelogram bisect each other)

$AE = CF$ (ii) Given

On subtracting (ii) from (i)

$AO - AE = OC - CF$

$EO = OF$ (iii)

In $\triangle DOE$ and $\triangle BOF$

$EO = OF$ (proved)

$DO = OB$ (Diagonals of a parallelogram bisect each other)

$\angle DOE = \angle BOF$ (vertically opposite angles are equal in a parallelogram)

By the rule of SAS congruence $\triangle DOE \cong \triangle BOF$

So, $DE = BF$ (Corresponding parts of congruent triangles)

In $\triangle BOE$ and $\triangle DOF$

$EO = OF$ (proved)

$DO = OB$ (diagonals of a parallelogram bisect each other)

$\angle DOF = \angle BOE$ (vertically opposite angles are equal in a parallelogram)

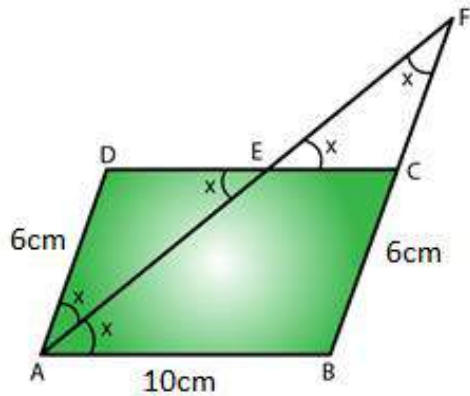
By the rule of SAS congruence $\triangle DOE \cong \triangle BOF$

$\therefore DF = BE$ (Corresponding parts of congruent triangles)

$\therefore BFDE$ is a parallelogram, since one pair of opposite sides are equal and parallel.

29. In a parallelogram ABCD, $AB = 10\text{cm}$, $AD = 6\text{ cm}$. The bisector of $\angle A$ meets DC in E , AE and BC produced meet at F . Find the length CF .

Solution:



In a parallelogram ABCD

Given, $AB = 10$ cm, $AD = 6$ cm

$\Rightarrow CD = AB = 10$ cm and $AD = BC = 6$ cm [In a parallelogram opposite sides are equal]

AE is the bisector of $\angle DAB$, $\angle DAE = \angle BAE = x$

$\angle BAE = \angle AED = x$ (alternate angles are equal)

$\triangle ADE$ is an isosceles triangle. Since opposite angles in $\triangle ADE$ are equal.

$AD = DE = 6$ cm (opposite sides are equal)

$CD = DE + EC$

$EC = CD - DE$

$$= 10 - 6$$

$$= 4 \text{ cm}$$

$\angle DEA = \angle CEF = x$ (vertically opposite angle are equal)

$\angle EAD = \angle EFC = x$ (alternate angles are equal)

$\triangle EFC$ is an isosceles triangle. Since opposite angles in $\triangle EFC$ are equal.

$CF = CE = 4$ cm (opposite side are equal to angles)

$\therefore CF = 4$ cm.

EXERCISE 17.2**PAGES NO: 17.16****1. Which of the following statements are true for a rhombus?**

- (i) It has two pairs of parallel sides.
- (ii) It has two pairs of equal sides.
- (iii) It has only two pairs of equal sides.
- (iv) Two of its angles are at right angles.
- (v) Its diagonals bisect each other at right angles.
- (vi) Its diagonals are equal and perpendicular.
- (vii) It has all its sides of equal lengths.
- (viii) It is a parallelogram.
- (ix) It is a quadrilateral.
- (x) It can be a square.
- (xi) It is a square.

Solution:

- (i) It has two pairs of parallel sides.

True, Rhombus is a parallelogram having two pairs of parallel sides.

- (ii) It has two pairs of equal sides.

True, Rhombus has two pairs of equal sides .

- (iii) It has only two pairs of equal sides.

False, Rhombus has all four sides equal.

- (iv) Two of its angles are at right angles.

False, rhombus has no angle of right angle.

- (v) Its diagonals bisect each other at right angles.

True, in rhombus diagonals bisect each other at right angles.

- (vi) Its diagonals are equal and perpendicular.

False, in rhombus diagonals are of unequal length.

- (vii) It has all its sides of equal lengths.

True, Rhombus has all four sides equal.

- (viii) It is a parallelogram.

True, Rhombus is a parallelogram since opposite sides equal and parallel.

(ix) It is a quadrilateral.

True, Rhombus is a quadrilateral since it has four sides.

(x) It can be a square.

True, Rhombus becomes square when any one angle is 90° .

(xi) It is a square.

False, Rhombus is never a square. Since in a square each angle is 90° .

2. Fill in the blanks, in each of the following, so as to make the statement true:

(i) A rhombus is a parallelogram in which _____.

(ii) A square is a rhombus in which _____.

(iii) A rhombus has all its sides of _____ length.

(iv) The diagonals of a rhombus _____ each other at _____ angles.

(v) If the diagonals of a parallelogram bisect each other at right angles, then it is a _____.

Solution:

(i) A rhombus is a parallelogram in which **adjacent sides are equal**.

(ii) A square is a rhombus in which **one angle is right angle**.

(iii) A rhombus has all its sides of **equal** length.

(iv) The diagonals of a rhombus **bisect** each other at **right** angles.

(v) If the diagonals of a parallelogram bisect each other at right angles, then it is a **rhombus**.

3. The diagonals of a parallelogram are not perpendicular. Is it a rhombus? Why or why not?

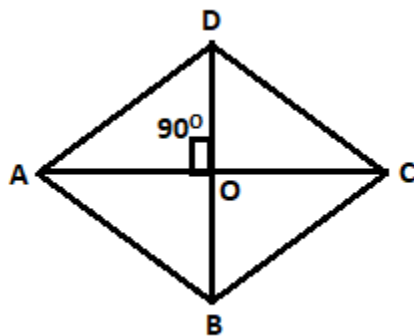
Solution:

No, Diagonals of a rhombus bisect each other at 90° . Where, diagonals must be perpendicular.

4. The diagonals of a quadrilateral are perpendicular to each other. Is such a quadrilateral always a rhombus? If your answer is 'No', draw a figure to justify your answer.

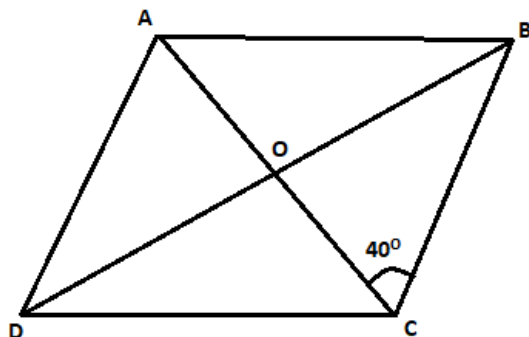
Solution:

No, because diagonals of a rhombus are perpendicular and bisect each other and its sides are equal. Below in the diagram we can see that diagonals are perpendicular to each other but they don't bisect each other.



5. ABCD is a rhombus. If $\angle ACB = 40^\circ$, find $\angle ADB$.

Solution:



ABCD is a rhombus. Diagonals are perpendicular.

So, $\angle BOC = 90^\circ$

From $\triangle BOC$, the sum of angles is 180°

$$\angle CBO + \angle BOC + \angle OBC = 180^\circ$$

$$\angle CBO = 180^\circ - \angle BOC - \angle OBC$$

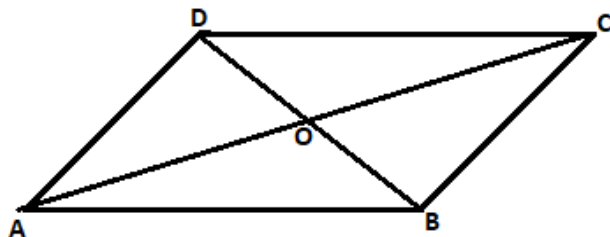
$$= 180^\circ - 40^\circ - 90^\circ$$

$$= 50^\circ$$

$\therefore \angle ADB = \angle CBO = 50^\circ$ (alternate angles are equal)

6. If the diagonals of a rhombus are 12 cm and 16 cm, find the length of each side.

Solution:



We know in rhombus diagonals bisect each other at right angle.

In $\triangle AOB$

$$AO = 12/2 = 6\text{cm}, BO = 16/2 = 8\text{cm}$$

Using Pythagoras theorem in $\triangle AOB$

$$AB^2 = AO^2 + BO^2$$

$$AB^2 = 6^2 + 8^2$$

$$AB^2 = 36 + 64$$

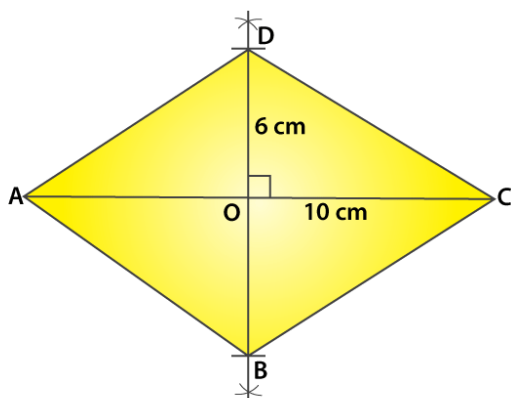
$$AB^2 = 100$$

$$AB = \sqrt{100} = 10\text{cm}$$

\therefore Each side of a rhombus is 10cm.

7. Construct a rhombus whose diagonals are of length 10 cm and 6 cm.

Solution:

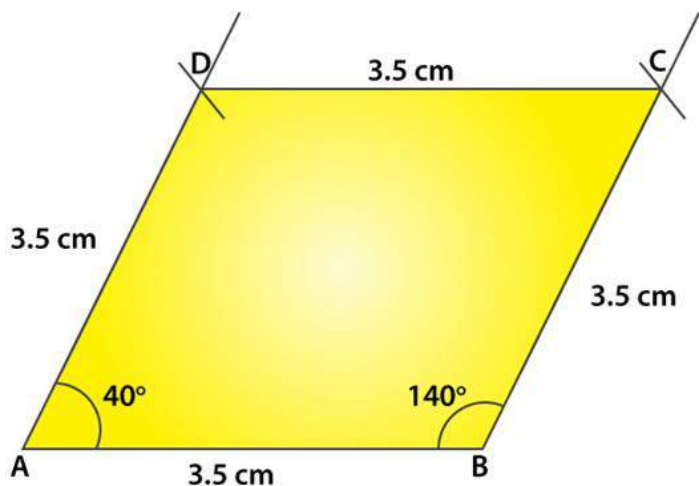


Steps to Construct a rhombus:

- (i) Draw diagonal AC of length 10 cm.
- (ii) With O as centre, draw perpendicular bisector of AC to point O.
- (iii) From point 'O' cut two arcs of length 3cm each to get points OB and OD.
- (iv) Now join AD and AB, BC and CD to get rhombus ABCD.

8. Draw a rhombus, having each side of length 3.5 cm and one of the angles as 40° .

Solution:

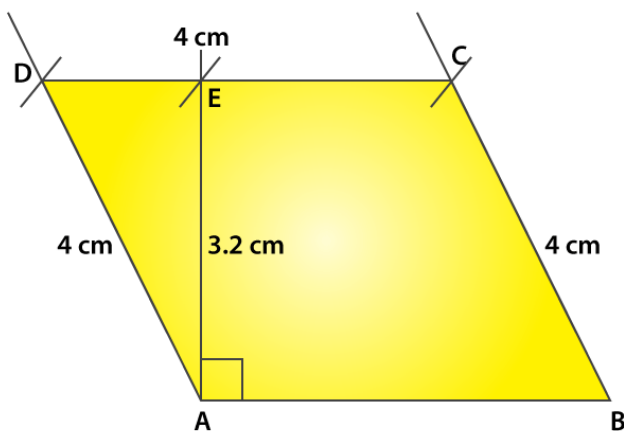


Steps to construct a rhombus:

- (i) Draw a line segment AB of length 3.5 cm.
- (ii) From point A and B draw angles of 40 and 140 respectively.
- (iii) From points A and B cut two arcs of length 3.5 cm each to get points D and C.
- (iv) Now, join AD, BC, CD to get rhombus ABCD.

9. One side of a rhombus is of length 4 cm and the length of an altitude is 3.2 cm. Draw the rhombus.

Solution:

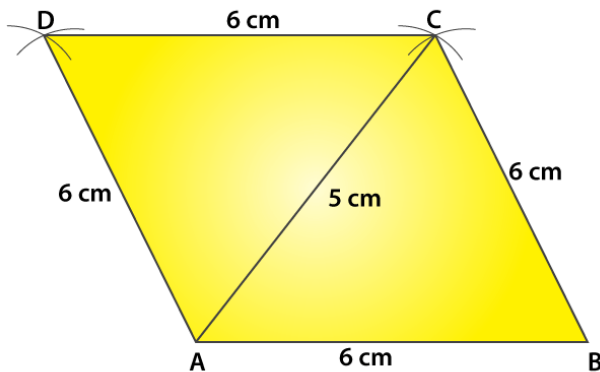


Steps to construct a rhombus:

- (i) Draw a line segment of 4 cm
- (ii) From point A draw a perpendicular line and cut a length of 3.2 cm to get point E.
- (iii) From point E draw a line parallel to AB.
- (iv) From points A and B cut two arcs of length 4 cm on the drawn parallel line to get points D and C.
- (v) Join AD, BC and CD to get rhombus ABCD.

10. Draw a rhombus ABCD, if AB = 6 cm and AC = 5 cm.

Solution:



Steps of construction:

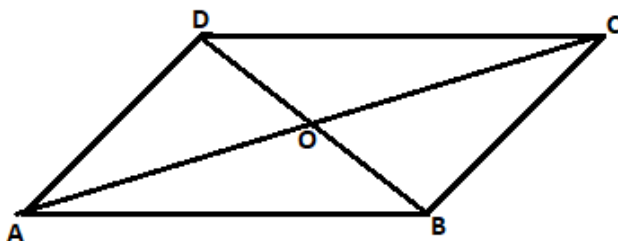
- (i) Draw a line segment AB of length 6 cm.
- (ii) From point 'A' cut an arc of length 5 cm and from point B cut an arc of length 6 cm. Such that both the arcs intersect at 'C'.
- (iii) Join AC and BC.
- (iv) From point A cut an arc of length 6 cm and from point C cut an arc of 6cm, so that both the arcs intersect at point D.
- (v) Join AD and DC to get rhombus ABCD.

11. ABCD is a rhombus and its diagonals intersect at O.

(i) is $\triangle BOC \cong \triangle DOC$? State the congruence condition used?

(ii) Also state, if $\angle BCO = \angle DCO$.

Solution:



(i) Yes,

In $\triangle BOC$ and $\triangle DOC$

$BO = DO$ [In a rhombus diagonals bisect each other]

$CO = CO$ Common

$BC = CD$ [All sides of a rhombus are equal]

By using SSS Congruency, $\triangle BOC \cong \triangle DOC$

(ii) Yes,
 $\angle BCO = \angle DCO$, by corresponding parts of congruent triangles.

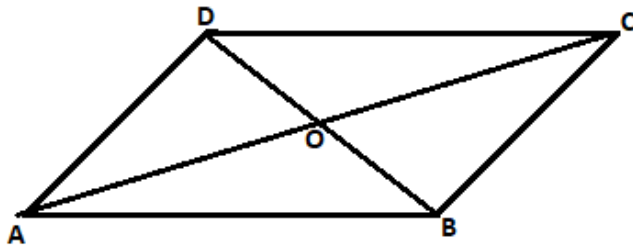
12. Show that each diagonal of a rhombus bisects the angle through which it passes.

Solution:

(i) In $\triangle BOC$ and $\triangle DOC$

$BO = DO$ [In a rhombus diagonals bisect each other]

$CO = CO$ Common



$BC = CD$ [All sides of a rhombus are equal]

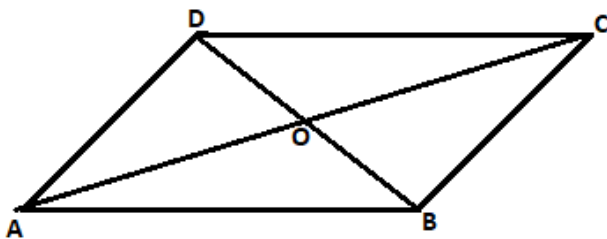
By using SSS Congruency, $\triangle BOC \cong \triangle DOC$

$\angle BCO = \angle DCO$, by corresponding parts of congruent triangles

\therefore Each diagonal of a rhombus bisect the angle through which it passes.

13. ABCD is a rhombus whose diagonals intersect at O. If $AB=10$ cm, diagonal $BD = 16$ cm, find the length of diagonal AC .

Solution:



We know in rhombus diagonals bisect each other at right angle.

In $\triangle AOB$

$$BO = BD/2 = 16/2 = 8\text{cm}$$

Using Pythagoras theorem in $\triangle AOB$

$$AB^2 = AO^2 + BO^2$$

$$10^2 = AO^2 + 8^2$$

$$100 - 64 = AO^2$$

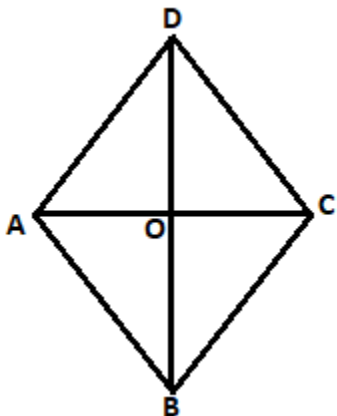
$$AO^2 = 36$$

$$AO = \sqrt{36} = 6\text{cm}$$

\therefore Length of diagonal AC is $6 \times 2 = 12\text{cm}$.

14. The diagonal of a quadrilateral are of lengths 6 cm and 8 cm. If the diagonals bisect each other at right angles, what is the length of each side of the quadrilateral?

Solution:



We know in rhombus diagonals bisect each other at right angle.

In $\triangle AOB$

$$BO = BD/2 = 6/2 = 3\text{cm}$$

$$AO = AC/2 = 8/2 = 4\text{cm}$$

Using Pythagoras theorem in $\triangle AOB$

$$AB^2 = AO^2 + BO^2$$

$$AB^2 = 4^2 + 3^2$$

$$AB^2 = 16 + 9$$

$$AB^2 = 25$$

$$AB = \sqrt{25} = 5\text{cm}$$

\therefore Length of each side of a Quadrilateral ABCD is 5cm.

EXERCISE 17.3**PAGES NO: 17.22****1. Which of the following statements are true for a rectangle?**

- (i) It has two pairs of equal sides.**
- (ii) It has all its sides of equal length.**
- (iii) Its diagonals are equal.**
- (iv) Its diagonals bisect each other.**
- (v) Its diagonals are perpendicular.**
- (vi) Its diagonals are perpendicular and bisect each other.**
- (vii) Its diagonals are equal and bisect each other.**
- (viii) Its diagonals are equal and perpendicular, and bisect each other.**
- (ix) All rectangles are squares.**
- (x) All rhombuses are parallelograms.**
- (xi) All squares are rhombuses and also rectangles.**
- (xii) All squares are not parallelograms.**

Solution:

- (i)** It has two pairs of equal sides.

True, in a rectangle two pairs of sides are equal.

- (ii)** It has all its sides of equal length.

False, in a rectangle only two pairs of sides are equal.

- (iii)** Its diagonals are equal.

True, in a rectangle diagonals are of equal length.

- (iv)** Its diagonals bisect each other.

True, in a rectangle diagonals bisect each other.

- (v)** Its diagonals are perpendicular.

False, Diagonals of a rectangle need not be perpendicular.

- (vi)** Its diagonals are perpendicular and bisect each other.

False, Diagonals of a rectangle need not be perpendicular. Diagonals only bisect each other.

- (vii)** Its diagonals are equal and bisect each other.

True, Diagonals are of equal length and bisect each other.

- (viii)** Its diagonals are equal and perpendicular, and bisect each other.

False, Diagonals are of equal length and bisect each other. Diagonals of a rectangle need not be perpendicular

(ix) All rectangles are squares.

False, in a square all sides are of equal length.

(x) All rhombuses are parallelograms.

True, all rhombuses are parallelograms, since opposite sides are equal and parallel.

(xi) All squares are rhombuses and also rectangles.

True, all squares are rhombuses, since all sides are equal in a square and rhombus. All squares are rectangles, since opposite sides are equal and parallel.

(xii) All squares are not parallelograms.

False, all squares are parallelograms, since opposite sides are parallel and equal.

2. Which of the following statements are true for a square?

(i) It is a rectangle.

(ii) It has all its sides of equal length.

(iii) Its diagonals bisect each other at right angle.

(v) Its diagonals are equal to its sides.

Solution:

(i) It is a rectangle.

True. Since, opposite sides are equal and parallel where, each angle is right angle.

(ii) It has all its sides of equal length.

True. Since, sides of a square are of equal length.

(iii) Its diagonals bisect each other at right angle.

True. Since, diagonals of a square bisect each other at right angle.

(v) Its diagonals are equal to its sides.

False. Since, diagonals of a square are of equal length. Length of diagonals is not equal to the length of sides

3. Fill in the blanks in each of the following, so as to make the statement true :

(i) A rectangle is a parallelogram in which _____.

(ii) A square is a rhombus in which _____.

(iii) A square is a rectangle in which _____.

Solution:

- (i) A rectangle is a parallelogram in which **one angle is a right angle**.
- (ii) A square is a rhombus in which **one angle is a right angle**.
- (iii) A square is a rectangle in which **adjacent sides are equal**.

4. A window frame has one diagonal longer than the other. Is the window frame a rectangle? Why or why not?

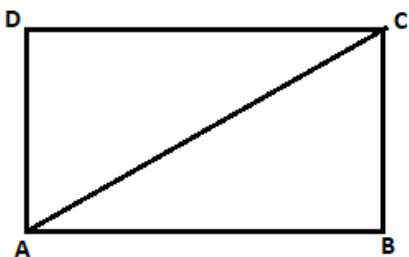
Solution:

No, diagonals of a rectangle are equal length.

5. In a rectangle $ABCD$, prove that $\triangle ACB \cong \triangle CAD$.

Solution:

Let us draw a rectangle,



In rectangle $ABCD$, AC is the diagonal.

In $\triangle ACB$ and $\triangle CAD$

$AB = CD$ [Opposite sides of a rectangle are equal]

$BC = DA$

$AC = CA$ [Common]

By using SSS congruency

$\triangle ACB \cong \triangle CAD$

6. The sides of a rectangle are in the ratio 2 : 3, and its perimeter is 20 cm. Draw the rectangle.

Solution:

In rectangle $ABCD$,

Given, perimeter of a rectangle = 20cm

Ratio = 2:3

So, let us consider the side as 'x'

Length of rectangle (l) = $3x$

Breadth of the rectangle (b) = $2x$

We know that,

Perimeter of the rectangle = $2(\text{length} + \text{breadth})$

$$20 = 2(3x + 2x)$$

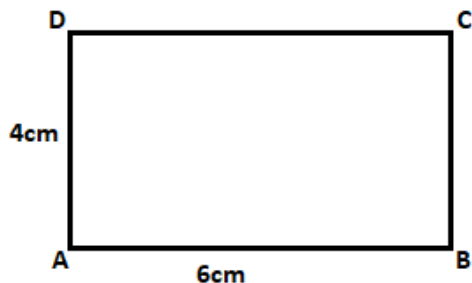
$$10x = 20$$

$$x = 20/10 = 2$$

Length of the rectangle = $3 \times 2 = 6\text{cm}$

Breadth of the rectangle = $2 \times 2 = 4\text{cm}$

Here, is the diagram of rectangle



7. The sides of a rectangle are in the ratio 4 : 5. Find its sides if the perimeter is 90 cm.

Solution:

In rectangle ABCD,

Given, perimeter of a rectangle = 90cm

Ratio = 4:5

So, let us consider the side as 'x'

Length of rectangle (l) = 5x

Breadth of the rectangle (b) = 4x

We know that,

Perimeter of the rectangle = $2(\text{length} + \text{breadth})$

$$90 = 2(5x + 4x)$$

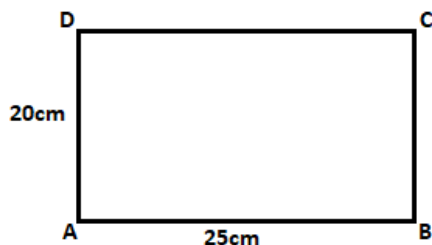
$$18x = 90$$

$$x = 90/18 = 5$$

Length of the rectangle = $5 \times 5 = 25\text{cm}$

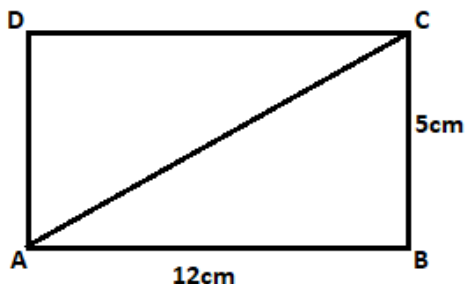
Breadth of the rectangle = $4 \times 5 = 20\text{cm}$

Here, is the diagram of rectangle



8. Find the length of the diagonal of a rectangle whose sides are 12 cm and 5 cm.

Solution:



In rectangle ABCD,

Given, sides of a rectangle ABCD are 5cm and 12cm

In $\triangle ABC$ using Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 12^2 + 5^2$$

$$AC^2 = 144 + 25$$

$$AC^2 = 169$$

$$AC = \sqrt{169}$$

$$AC = 13\text{cm}$$

\therefore Length of the diagonal AC is 13cm.

9. Draw a rectangle whose one side measures 8 cm and the length of each of whose diagonals is 10 cm.

Solution:

Given, one side of the rectangle is 8cm.

Length of the diagonal = 10cm

Now let us construct a rectangle,

Steps to construct a rectangle,

(i) Draw a line segment AB of length 8 cm

(ii) From point 'A' cut an arc of length 10 cm and mark that point as C.

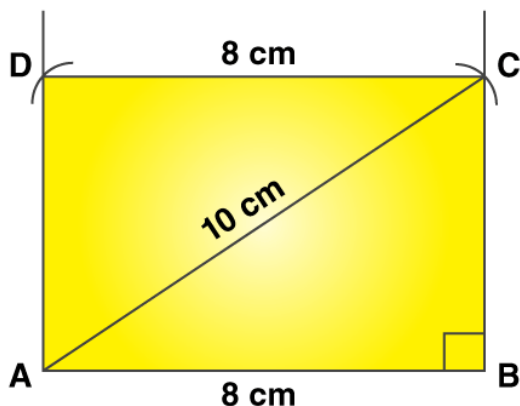
(iii) From point B draw an angle of 90° , and join the arc from point A which cuts at point C.

(iv) now join AC and BC

(v) From point A draw an angle of 90° and from point C cut an arc of length 8 cm to get point D.

(vi) Join CD and AD to form required rectangle.

Here, is the constructed diagram of rectangle



10. Draw a square who's each side measures 4.8 cm.

Solution:

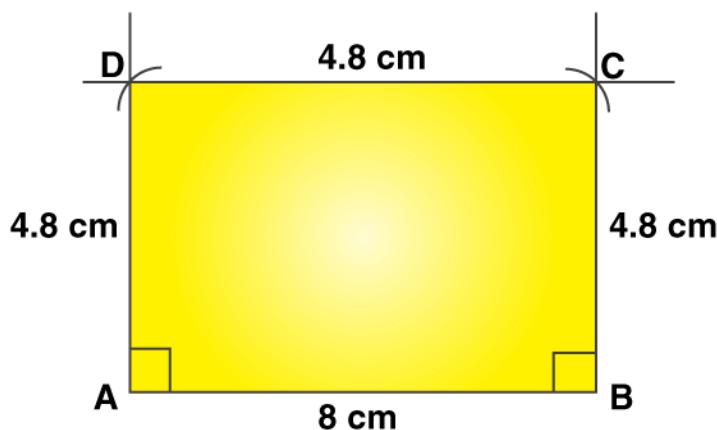
Given, side of a square is 4.8cm.

Now let us construct a square,

Steps to construct a square,

- (i) Draw a line segment AB of length 4.8 cm.
- (ii) From points A and B draw perpendiculars at 90° each.
- (iii) Cut an arc of 4.8 cm from point A and B on the perpendiculars to get point D and C.
- (iv) Join DC, AD and BC to form the required square.

Here, is the constructed diagram of square



11. Identify all the quadrilaterals that have:

- (i) Four sides of equal length
- (ii) Four right angles

Solution:

- (i) Four sides of equal length

The quadrilaterals which has all the four sides of equal length are Square and Rhombus.

(ii) Four right angles

The quadrilaterals which has four right angles are Square and Rectangle.

12. Explain how a square is

(i) A quadrilateral?

(ii) A parallelogram?

(iii) A rhombus?

(iv) A rectangle?

Solution:

(i) A square is a quadrilateral since it has all sides of equal length.

(ii) A square is a parallelogram since it's opposite sides are equal and parallel.

(iii) A square is a rhombus since it has all sides of equal length and opposite sides are parallel.

(iv) A square is a rectangle since it's opposite sides are equal and each angle is a 90° .

13. Name the quadrilaterals whose diagonals:

(i) bisect each other

(ii) are perpendicular bisector of each other

(iii) are equal.

Solution:

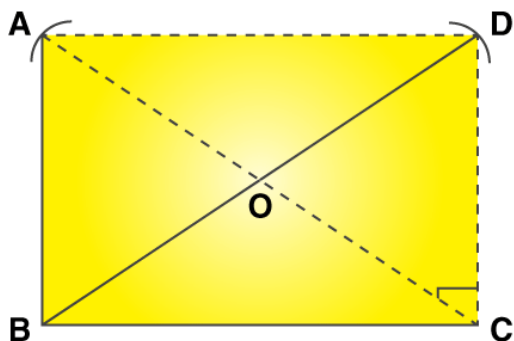
(i) Quadrilaterals whose diagonals bisect each other are: Parallelogram, Rectangle, Rhombus and Square.

(ii) Quadrilaterals whose diagonals are perpendicular bisector of each other are: Rhombus and Square.

(iii) Quadrilaterals whose diagonals are equal in Square and Rectangle.

14. ABC is a right angled triangle and O is the mid-point of the side opposite to the right angle. Explain why O is equidistant from A, B, and C.

Solution:



ABC is a right angled triangle. O is the midpoint of hypotenuse AC, such that $OA = OC$.
Now, draw $CD \parallel AB$ and join AD, such that $AB = CD$ and $AD = BC$.
Now, quadrilateral ABCD is a rectangle, since each angle is a right angle and opposite sides are equal and parallel.

We know in a rectangle diagonals are of equal length and they bisect each other.

So, $AC = BD$

And also, $AO = OC = BO = OD$

Hence, O is equidistant from A, B and C.

15. A mason has made a concrete slab. He needs it to be rectangular. In what different ways can he make sure that it is rectangular?

Solution:

For a concrete slab to be rectangular the mason has to check,

- (i) By measuring each angle and
- (ii) By measuring the lengths of diagonals.