EXERCISE 8.4

Divide:

1. $5x^3 – 15x^2 + 25x$ by $5x$

Solution:

We have,

$(5x^3 – 15x^2 + 25x) / 5x$

$5x^3/5x – 15x^2/5x + 25x/5x$

By using the formula $a^n / a^m = a^{n-m}$

$5/5\ x^{3-1} – 15/5\ x^{2-1} + 25/5\ x^{1-1}$

$x^2 – 3x + 5$

2. $4z^3 + 6z^2 – z$ by $-1/2z$

Solution:

We have,

$(4z^3 + 6z^2 – z) / -1/2z$

$4z^3/(-1/2z) + 6z^2/(-1/2z) – z/(-1/2z)$

By using the formula $a^n / a^m = a^{n-m}$

$-8\ z^{3-1} – 12z^{2-1} + 2\ z^{1-1}$

$-8z^2 – 12z + 2$

3. $9x^2y – 6xy + 12xy^2$ by $-3/2xy$

Solution:

We have,

$(9x^2y – 6xy + 12xy^2) / -3/2xy$

$9x^2y/(-3/2xy) – 6xy/(-3/2xy) + 12xy^2/(-3/2xy)$

By using the formula $a^n / a^m = a^{n-m}$

$(-9×2)/3\ x^{2-1}y^{1-1} – (-6×2)/3\ x^{1-1}y^{1-1} + (-12×2)/3\ x^{1-1}y^{2-1}$

$-6x + 4 – 8y$

4. $3x^3y^2 + 2x^2y + 15xy$ by $3xy$

Solution:

We have,

$(3x^3y^2 + 2x^2y + 15xy) / 3xy$

$3x^3y^2/3xy + 2x^2y/3xy + 15xy/3xy$

By using the formula $a^n / a^m = a^{n-m}$

$3/3\ x^{3-1}y^{2-1} + 2/3\ x^{2-1}y^{1-1} + 15/3\ x^{1-1}y^{1-1}$

$x^2y + 2/3x + 5$

5. $x^3 + 7x + 12$ by $x + 4$

Solution:

We have,

$(x^3 + 7x + 12) / (x + 4)$

By using long division method

$$\begin{array}{r|rrr} & x & +3 & \\ \hline x+4 & x^2 & +7x & +12 \\ - & x^2 & +4x & \\ \hline & & 3x & +12 \\ & - & 3x & +12 \\ \hline & & & 0 \end{array}$$

$\therefore (x^3 + 7x + 12) / (x + 4) = x + 3$

6. $4y^2 + 3y + 1/2$ by $2y + 1$

Solution:

We have,

$4y^2 + 3y + 1/2$ by $(2y + 1)$

By using long division method

$$\begin{array}{r|rrr} & 2y & +\frac{1}{2} & \\ \hline 2y+1 & 4y^2 & +3y & +\frac{1}{2} \\ - & 4y^2 & +2y & \\ \hline & & y & +\frac{1}{2} \\ & - & y & +\frac{1}{2} \\ \hline & & & 0 \end{array}$$

$\therefore (4y^2 + 3y + 1/2) / (2y + 1) = 2y + 1/2$

7. $3x^3 + 4x^2 + 5x + 18$ by $x + 2$

Solution:

We have,

$(3x^3 + 4x^2 + 5x + 18) / (x + 2)$

By using long division method

$$
\begin{array}{r|l}
 & 3x^2 \quad -2x \quad +9 \\
x+2 & 3x^3 \quad +4x^2 \quad +5x \quad +18 \\
 & - \\
 & 3x^3 \quad +6x^2 \\
\hline
 & -2x^2 \quad +5x \quad +18 \\
 & - \\
 & -2x^2 \quad -4x \\
\hline
 & 9x \quad +18 \\
 & - \\
 & 9x \quad +18 \\
\hline
 & 0
\end{array}
$$

∴ $(3x^3 + 4x^2 + 5x + 18) / (x + 2) = 3x^2 – 2x + 9$

8. $14x^2 – 53x + 45$ by $7x - 9$
Solution:
We have,
$(14x^2 – 53x + 45) / (7x – 9)$
By using long division method

$$
\begin{array}{r|l}
 & 2x \quad -5 \\
7x-9 & 14x^2 \quad -53x \quad +45 \\
 & - \\
 & 14x^2 \quad -18x \\
\hline
 & -35x \quad +45 \\
 & - \\
 & -35x \quad +45 \\
\hline
 & 0
\end{array}
$$

∴ $(14x^2 – 53x + 45) / (7x – 9) = 2x – 5$

9. $-21 + 71x – 31x^2 – 24x^3$ by $3 – 8x$
Solution:
We have,
$-21 + 71x – 31x^2 – 24x^3$ by $3 – 8x$
$(-24x^3 – 31x^2 + 71x – 21) / (3 – 8x)$
By using long division method

$$3x^2 \quad +5x \quad -7$$
$$-8x+3 \;\overline{)\,-24x^3 \quad -31x^2 \quad +71x \quad -21}$$
$$-$$
$$-24x^3 \quad +9x^2$$
$$\overline{\qquad -40x^2 \quad +71x \quad -21}$$
$$-$$
$$-40x^2 \quad +15x$$
$$\overline{\qquad 56x \quad -21}$$
$$-$$
$$56x \quad -21$$
$$\overline{\qquad 0}$$

∴ $(-24x^3 – 31x^2 + 71x – 21) / (3 – 8x) = 3x^2 + 5x – 7$

10. $3y^4 – 3y^3 – 4y^2 – 4y$ by $y^2 - 2y$
Solution:
We have,
$(3y^4 – 3y^3 – 4y^2 – 4y) / (y^2 - 2y)$
By using long division method

$$3y^2 \quad +3y \quad +2$$
$$y^2-2y \;\overline{)\,3y^4 \quad -3y^3 \quad -4y^2 \quad -4y \quad +0}$$
$$-$$
$$3y^4 \quad -6y^3$$
$$\overline{\qquad 3y^3 \quad -4y^2 \quad -4y \quad +0}$$
$$-$$
$$3y^3 \quad -6y^2$$
$$\overline{\qquad 2y^2 \quad -4y \quad +0}$$
$$-$$
$$2y^2 \quad -4y$$
$$\overline{\qquad 0}$$

∴ $(3y^4 – 3y^3 – 4y^2 – 4y) / (y^2 - 2y) = 3y^2 + 3y + 2$

11. $2y^5 + 10y^4 + 6y^3 + y^2 + 5y + 3$ by $2y^3 + 1$
Solution:
We have,
$(2y^5 + 10y^4 + 6y^3 + y^2 + 5y + 3) / (2y^3 + 1)$
By using long division method

$$
\begin{array}{r|llllll}
 & y^2 & +5y & +3 & & & \\
2y^3+1 & 2y^5 & +10y^4 & +6y^3 & +y^2 & +5y & +3 \\
 & - & & & & & \\
 & 2y^5 & +0y^4 & +0y^3 & +y^2 & & \\
\hline
 & & 10y^4 & +6y^3 & +0y^2 & +5y & +3 \\
 & & - & & & & \\
 & & 10y^4 & +0y^3 & +0y^2 & +5y & \\
\hline
 & & & 6y^3 & +0y^2 & +0y & +3 \\
 & & & - & & & \\
 & & & 6y^3 & +0y^2 & +0y & +3 \\
\hline
 & & & & & & 0
\end{array}
$$

∴ $(2y^5 + 10y^4 + 6y^3 + y^2 + 5y + 3) / (2y^3 + 1) = y^2 + 5y + 3$

12. $x^4 – 2x^3 + 2x^2 + x + 4$ by $x^2 + x + 1$

Solution:

We have,

$(x^4 – 2x^3 + 2x^2 + x + 4) / (x^2 + x + 1)$

By using long division method

$$
\begin{array}{r|lllll}
 & x^2 & -3x & +4 & & \\
x^2+x+1 & x^4 & -2x^3 & +2x^2 & +x & +4 \\
 & - & & & & \\
 & x^4 & +x^3 & +x^2 & & \\
\hline
 & & -3x^3 & +x^2 & +x & +4 \\
 & & - & & & \\
 & & -3x^3 & -3x^2 & -3x & \\
\hline
 & & & 4x^2 & +4x & +4 \\
 & & & - & & \\
 & & & 4x^2 & +4x & +4 \\
\hline
 & & & & & 0
\end{array}
$$

∴ $(x^4 – 2x^3 + 2x^2 + x + 4) / (x^2 + x + 1) = x^2 – 3x + 4$

13. $m^3 – 14m^2 + 37m – 26$ by $m^2 – 12m + 13$

Solution:

We have,

$(m^3 – 14m^2 + 37m – 26) / (m^2 – 12m + 13)$

By using long division method

$$
\begin{array}{r|rrrr}
 & m & -2 & & \\
\hline
m^2 - 12m + 13 & m^3 & -14m^2 & +37m & -26 \\
- & m^3 & -12m^2 & +13m & \\
\hline
 & & -2m^2 & +24m & -26 \\
- & & -2m^2 & +24m & -26 \\
\hline
 & & & & 0
\end{array}
$$

∴ $(m^3 - 14m^2 + 37m - 26) / (m^2 - 12m + 13) = m - 2$

14. $x^4 + x^2 + 1$ by $x^2 + x + 1$

Solution:

We have,

$(x^4 + x^2 + 1) / (x^2 + x + 1)$

By using long division method

$$
\begin{array}{r|rrrrr}
 & x^2 & -x & +1 & & \\
\hline
x^2 + x + 1 & x^4 & +0x^3 & +x^2 & +0x & +1 \\
- & x^4 & +x^3 & +x^2 & & \\
\hline
 & & -x^3 & +0x^2 & +0x & +1 \\
- & & -x^3 & -x^2 & -x & \\
\hline
 & & & x^2 & +x & +1 \\
- & & & x^2 & +x & +1 \\
\hline
 & & & & & 0
\end{array}
$$

∴ $(x^4 + x^2 + 1) / (x^2 + x + 1) = x^2 - x + 1$

15. $x^5 + x^4 + x^3 + x^2 + x + 1$ by $x^3 + 1$

Solution:

We have,

$(x^5 + x^4 + x^3 + x^2 + x + 1) / (x^3 + 1)$

By using long division method

$$
\begin{array}{r|rrrrrr}
 & x^2 & & +1 & & & \\
\hline
x^3+1 & x^5 & +x^4 & +x^3 & +x^2 & +x & +1 \\
- & x^5 & +0x^4 & +0x^3 & +x^2 & & \\
\hline
 & & & +x^3 & +0x^2 & +x & +1 \\
 & - & & & & & \\
 & & x^4 & +0x^3 & +0x^2 & +x & \\
\hline
 & & & x^3 & +0x^2 & +0x & +1 \\
 & & & - & & & \\
 & & & x^3 & +0x^2 & +0x & +1 \\
\hline
 & & & & & & 0
\end{array}
$$

∴ $(x^5 + x^4 + x^3 + x^2 + x + 1) / (x^3 + 1) = x^2 + x + 1$

Divide each of the following and find the quotient and remainder:

16. $14x^3 – 5x^2 + 9x – 1$ by $2x – 1$
Solution:
We have,
$(14x^3 – 5x^2 + 9x – 1) / (2x – 1)$
By using long division method

$$
\begin{array}{r|rrrr}
 & 7x^2 & +x & +5 & \\
\hline
2x-1 & 14x^3 & -5x^2 & +9x & -1 \\
- & 14x^3 & -7x^2 & & \\
\hline
 & & 2x^2 & +9x & -1 \\
 & & - & & \\
 & & 2x^2 & -x & \\
\hline
 & & & 10x & -1 \\
 & & & - & \\
 & & & 10x & -5 \\
\hline
 & & & & 4
\end{array}
$$

∴ Quotient is $7x^2 + x + 5$ and the Remainder is 4.

17. $6x^3 – x^2 – 10x – 3$ by $2x – 3$

Solution:

We have,

$(6x^3 – x^2 – 10x – 3) / (2x – 3)$

By using long division method

$$
\begin{array}{r|rrrr}
 & 3x^2 & +4x & +1 & \\
\hline
2x-3 & 6x^3 & -x^2 & -10x & -3 \\
- & 6x^3 & -9x^2 & & \\
\hline
 & & 8x^2 & -10x & -3 \\
 & - & 8x^2 & -12x & \\
\hline
 & & & 2x & -3 \\
 & & - & 2x & -3 \\
\hline
 & & & & 0
\end{array}
$$

∴ Quotient is $3x^2 + 4x + 1$ and the Remainder is 0.

18. $6x^3 + 11x^2 – 39x – 65$ by $3x^2 + 13x + 13$

Solution:

We have,

$(6x^3 + 11x^2 – 39x – 65) / (3x^2 + 13x + 13)$

By using long division method

$$
\begin{array}{r|rrrr}
 & 2x & -5 & & \\
\hline
3x^2+13x+13 & 6x^3 & +11x^2 & -39x & -65 \\
- & 6x^3 & +26x^2 & +26x & \\
\hline
 & & -15x^2 & -65x & -65 \\
 & - & -15x^2 & -65x & -65 \\
\hline
 & & & & 0
\end{array}
$$

∴ Quotient is 2x - 5 and the Remainder is 0.

19. $30x^4 + 11x^3 – 82x^2 – 12x + 48$ by $3x^2 + 2x – 4$

Solution:

We have,

$(30x^4 + 11x^3 – 82x^2 – 12x + 48) / (3x^2 + 2x – 4)$
By using long division method

$$
\begin{array}{r|rrrrr}
 & 10x^2 & -3x & -12 & & \\
\hline
3x^2+2x-4 & 30x^4 & +11x^3 & -82x^2 & -12x & +48 \\
- & 30x^4 & +20x^3 & -40x^2 & & \\
\hline
 & & -9x^3 & -42x^2 & -12x & +48 \\
 & & - & & & \\
 & & -9x^3 & -6x^2 & +12x & \\
\hline
 & & & -36x^2 & -24x & +48 \\
 & & & - & & \\
 & & & -36x^2 & -24x & +48 \\
\hline
 & & & & & 0
\end{array}
$$

∴ Quotient is $10x^2 – 3x - 12$ and the Remainder is 0.

20. $9x^4 – 4x^2 + 4$ by $3x^2 – 4x + 2$

Solution:

We have,
$(9x^4 – 4x^2 + 4) / (3x^2 – 4x + 2)$
By using long division method

$$
\begin{array}{r|rrrrr}
 & 3x^2 & +4x & +2 & & \\
\hline
3x^2-4x+2 & 9x^4 & +0x^3 & -4x^2 & +0x & +4 \\
- & 9x^4 & -12x^3 & +6x^2 & & \\
\hline
 & & 12x^3 & -10x^2 & +0x & +4 \\
 & & - & & & \\
 & & 12x^3 & -16x^2 & +8x & \\
\hline
 & & & 6x^2 & -8x & +4 \\
 & & & - & & \\
 & & & 6x^2 & -8x & +4 \\
\hline
 & & & & & 0
\end{array}
$$

∴ Quotient is $3x^2 + 4x + 2$ and the Remainder is 0.

21. Verify division algorithm i.e. Dividend = Divisor × Quotient + Remainder, in each of the following. Also, write the quotient and remainder:

Dividend	**divisor**
(i) $14x^2 + 13x - 15$	$7x - 4$
(ii) $15z^3 - 20z^2 + 13z - 12$	$3z - 6$
(iii) $6y^5 - 28y^3 + 3y^2 + 30y - 9$	$2x^2 - 6$
(iv) $34x - 22x^3 - 12x^4 - 10x^2 - 75$	$3x + 7$
(v) $15y^4 - 16y^3 + 9y^2 - 10/3y + 6$	$3y - 2$
(vi) $4y^3 + 8y + 8y^2 + 7$	$2y^2 - y + 1$
(vii) $6y^4 + 4y^4 + 4y^3 + 7y^2 + 27y + 6$	$2y^3 + 1$

Solution:

(i) Dividend   divisor

$14x^2 + 13x - 15$ $7x - 4$

By using long division method

$$
\begin{array}{r|rrr}
 & 2x & +3 & \\
\hline
7x-4 & 14x^2 & +13x & -15 \\
- & 14x^2 & -8x & \\
\hline
 & & 21x & -15 \\
 & - & 21x & -12 \\
\hline
 & & & -3
\end{array}
$$

Let us verify, Dividend = Divisor × Quotient + Remainder

$14x^2 + 13x - 15 = (7x - 4) \times (2x + 3) + (-3)$

$= 14x^2 + 21x - 8x - 12 - 3$

$= 14x^2 + 13x - 15$

Hence, verified.

∴ Quotient is $2x + 3$ and the Remainder is -3.

(ii) Dividend   divisor

$15z^3 - 20z^2 + 13z - 12$ $3z - 6$

By using long division method

$$
\begin{array}{rl}
 & \quad 5z^2 \quad +\frac{10z}{3} \quad +11 \\
3z-6 & \overline{)\,15z^3 \quad -20z^2 \quad +13z \quad -12} \\
 & - \\
 & \underline{15z^3 \quad -30z^2 \qquad\qquad\qquad} \\
 & \qquad\qquad 10z^2 \quad +13z \quad -12 \\
 & \qquad\qquad - \\
 & \underline{\qquad\qquad 10z^2 \quad -20z \qquad\quad} \\
 & \qquad\qquad\qquad\quad 33z \quad -12 \\
 & \qquad\qquad\qquad\quad - \\
 & \underline{\qquad\qquad\qquad\quad 33z \quad -66} \\
 & \qquad\qquad\qquad\qquad\qquad 54
\end{array}
$$

Let us verify, Dividend = Divisor × Quotient + Remainder

$15z^3 – 20z^2 + 13z – 12 = (3z - 6) × (5z^2 + 10z/3 + 11) + 54$

$= 15z^3 + 10z^2 + 33z – 30z^2 – 20z + 54$

$= 15z^2 – 20z^2 + 13z – 12$

Hence, verified.

∴ Quotient is $5z^2 + 10z/3 + 11$ and the Remainder is 54.

(iii) Dividend divisor

$6y^5 – 28y^3 + 3y^2 + 30y – 9$ $2x^2 – 6$

By using long division method

$$
\begin{array}{rl}
 & \quad 3y^3 \quad -5y \quad +\frac{3}{2} \\
2y^2-6 & \overline{)\,6y^5 \quad +0y^4 \quad -28y^3 \quad +3y^2 \quad +30y \quad -9} \\
 & - \\
 & \underline{6y^5 \quad +0y^4 \quad -18y^3 \qquad\qquad\qquad\qquad\quad} \\
 & \qquad\qquad\qquad -10y^3 \quad +3y^2 \quad +30y \quad -9 \\
 & \qquad\qquad\qquad - \\
 & \underline{\qquad\qquad\qquad -10y^3 \quad +0y^2 \quad +30y \qquad\quad} \\
 & \qquad\qquad\qquad\qquad\qquad 3y^2 \quad +0y \quad -9 \\
 & \qquad\qquad\qquad\qquad\qquad - \\
 & \underline{\qquad\qquad\qquad\qquad\qquad 3y^2 \quad +0y \quad -9} \\
 & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 0
\end{array}
$$

Let us verify, Dividend = Divisor × Quotient + Remainder

$6y^5 – 28y^3 + 3y^2 + 30y – 9 = (2x^2 – 6) × (3y^3 – 5y + 3/2) + 0$

$= 6y^5 – 10y^3 + 3y^2 – 18y^3 + 30y – 9$

$= 6y^5 – 28y^3 + 3y^2 + 30y – 9$

Hence, verified.
∴ Quotient is $3y^3 – 5y + 3/2$ and the Remainder is 0.

(iv) Dividend divisor
$34x – 22x^3 – 12x^4 – 10x^2 – 75$ $3x + 7$
$-12x^4 – 22x^3 – 10x^2 + 34x - 75$
By using long division method

$$
\begin{array}{r|rrrrr}
 & -4x^3 & +2x^2 & -8x & +30 & \\
\hline
3x+7 & -12x^4 & -22x^3 & -10x^2 & +34x & -75 \\
- & -12x^4 & -28x^3 & & & \\
\hline
 & & 6x^3 & -10x^2 & +34x & -75 \\
 & - & 6x^3 & +14x^2 & & \\
\hline
 & & & -24x^2 & +34x & -75 \\
 & & - & -24x^2 & -56x & \\
\hline
 & & & & 90x & -75 \\
 & & & - & 90x & +210 \\
\hline
 & & & & & -285
\end{array}
$$

Let us verify, Dividend = Divisor × Quotient + Remainder
$-12x^4 – 22x^3 – 10x^2 + 34x – 75 = (3x + 7) \times (-4x^3 + 2x^2 – 8x + 30) – 285$
$= -12x^4 + 6x^3 – 24x^2 – 28x^3 + 14x^2 + 90x – 56x + 210 -285$
$= -12x^4 – 22x^3 – 10x^2 + 34x – 75$
Hence, verified.
∴ Quotient is $-4x^3 + 2x^2 - 8x + 30$ and the Remainder is -285.

(v) Dividend divisor
$15y^4 – 16y^3 + 9y^2 – 10/3y + 6$ $3y – 2$
By using long division method

$$
\begin{array}{r|l}
 & 5y^3 \quad -2y^2 \quad +\frac{5y}{3} \\
\hline
3y-2 & 15y^4 \quad -16y^3 \quad +9y^2 \quad -\frac{10y}{3} \quad +6 \\
 & - \\
 & 15y^4 \quad -10y^3 \\
\hline
 & -6y^3 \quad +9y^2 \quad -\frac{10y}{3} \quad +6 \\
 & - \\
 & -6y^3 \quad +4y^2 \\
\hline
 & 5y^2 \quad -\frac{10y}{3} \quad +6 \\
 & - \\
 & 5y^2 \quad -\frac{10y}{3} \\
\hline
 & 0 \quad 6
\end{array}
$$

Let us verify, Dividend = Divisor × Quotient + Remainder

$15y^4 – 16y^3 + 9y^2 – 10/3y + 6 = (3y – 2) × (5y^3 – 2y^2 + 5y/3) + 6$

$= 15y^4 – 6y^3 + 5y^2 – 10y^3 + 4y^2 – 10y/3 + 6$

$= 15y^4 – 16y^3 + 9y^2 – 10/3y + 6$

Hence, verified.

∴ Quotient is $5y^3 – 2y^2 + 5y/3$ and the Remainder is 6.

(vi) Dividend divisor

$4y^3 + 8y + 8y^2 + 7$ $2y^2 – y + 1$

$4y^3 + 8y^2 + 8y + 7$

By using long division method

$$
\begin{array}{r|l}
 & 2y \quad +5 \\
\hline
2y^2-y+1 & 4y^3 \quad +8y^2 \quad +8y \quad +7 \\
 & - \\
 & 4y^3 \quad -2y^2 \quad +2y \\
\hline
 & 10y^2 \quad +6y \quad +7 \\
 & - \\
 & 10y^2 \quad -5y \quad +5 \\
\hline
 & 11y \quad +2
\end{array}
$$

Let us verify, Dividend = Divisor × Quotient + Remainder

$4y^3 + 8y^2 + 8y + 7 = (2y^2 – y + 1) × (2y + 5) + 11y + 2$

$= 4y^3 + 10y^2 – 2y^2 – 5y + 2y + 5 + 11y + 2$
$= 4y^3 + 8y^2 + 8y + 7$
Hence, verified.
∴ Quotient is $2y + 5$ and the Remainder is $11y + 2$.

(vii) Dividend divisor
$6y^5 + 4y^4 + 4y^3 + 7y^2 + 27y + 6$ $2y^3 + 1$
By using long division method

$$
\begin{array}{r|rrrrrr}
 & 3y^2 & +2y & +2 & & & \\
2y^3+1 & 6y^5 & +4y^4 & +4y^3 & +7y^2 & +27y & +6 \\
 & - & & & & & \\
 & 6y^5 & +0y^4 & +0y^3 & +3y^2 & & \\
\hline
 & & 4y^4 & +4y^3 & +4y^2 & +27y & +6 \\
 & & - & & & & \\
 & & 4y^4 & +0y^3 & +0y^2 & +2y & \\
\hline
 & & & 4y^3 & +4y^2 & +25y & +6 \\
 & & & - & & & \\
 & & & 4y^3 & +0y^2 & +0y & +2 \\
\hline
 & & & & 4y^2 & +25y & +4
\end{array}
$$

Let us verify, Dividend = Divisor × Quotient + Remainder
$6y^5 + 4y^4 + 4y^3 + 7y^2 + 27y + 6 = (2y^3 + 1) × (3y^2 + 2y + 2) + 4y^2 + 25y + 4$
$= 6y^5 + 4y^4 + 4y^3 + 3y^2 + 2y + 2 + 4y^2 + 25y + 4$
$= 6y^5 + 4y^4 + 4y^3 + 7y^2 + 27y + 6$
Hence, verified.
∴ Quotient is $3y^2 + 2y + 2$ and the Remainder is $4y^2 + 25y + 4$.

22. Divide $15y^4 + 16y^3 + 10/3y – 9y^2 – 6$ by $3y - 2$ Write down the coefficients of the terms in the quotient.
Solution:
We have,
$(15y^4 + 16y^3 + 10/3y – 9y^2 – 6) / (3y – 2)$
By using long division method

$$
\begin{array}{r|l}
 & 5y^3 + \frac{26y^2}{3} + \frac{25y}{9} + \frac{80}{27} \\
\hline
3y - 2 & 15y^4 + 16y^3 - 9y^2 + \frac{10y}{3} - 6 \\
 & - \\
 & 15y^4 - 10y^3 \\
\hline
 & 26y^3 - 9y^2 + \frac{10y}{3} - 6 \\
 & - \\
 & 26y^3 - \frac{52y^2}{3} \\
\hline
 & \frac{25y^2}{3} + \frac{10y}{3} - 6 \\
 & - \\
 & \frac{25y^2}{3} - \frac{50y}{9} \\
\hline
 & \frac{80y}{9} - 6 \\
 & - \\
 & \frac{80y}{9} - \frac{160}{27} \\
\hline
 & -\frac{2}{27}
\end{array}
$$

∴ Quotient is $5y^3 + 26y^2/3 + 25y/9 + 80/27$
So the coefficients of the terms in the quotient are:
Coefficient of $y^3 = 5$
Coefficient of $y^2 = 26/3$
Coefficient of $y = 25/9$
Constant term = 80/27

23. Using division of polynomials state whether
(i) $x + 6$ is a factor of $x^2 – x - 42$
(ii) $4x - 1$ is a factor of $4x^2 – 13x - 12$
(iii) $2y - 5$ is a factor of $4y^4 – 10y^3 – 10y^2 + 30y – 15$
(iv) $3y^2 + 5$ is a factor of $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$
(v) $z^2 + 3$ is a factor of $z^5 – 9z$
(vi) $2x^2 – x + 3$ is a factor of $6x^5 – x^4 + 4x^3 – 5x^2 – x – 15$
Solution:
(i) $x + 6$ is a factor of $x^2 – x - 42$
Firstly let us perform long division method

$$\begin{array}{r|l} & x \quad -7 \\ \hline x+6 & x^2 \quad -x \quad -42 \\ & -\ (x^2 \quad +6x) \\ \hline & \quad -7x \quad -42 \\ & -\ (-7x \quad -42) \\ \hline & \qquad 0 \end{array}$$

Since the remainder is 0, we can say that x + 6 is a factor of $x^2 – x – 42$

(ii) 4x - 1 is a factor of $4x^2 – 13x - 12$
Firstly let us perform long division method

$$\begin{array}{r|l} & x \quad -3 \\ \hline 4x-1 & 4x^2 \quad -13x \quad -12 \\ & -\ (4x^2 \quad -x) \\ \hline & \quad -12x \quad -12 \\ & -\ (-12x \quad +3) \\ \hline & \qquad -15 \end{array}$$

Since the remainder is -15, 4x - 1 is not a factor of $4x^2 – 13x - 12$

(iii) 2y - 5 is a factor of $4y^4 – 10y^3 – 10y^2 + 30y – 15$
Firstly let us perform long division method

$$\begin{array}{r|l} & 2y^3 \quad -5y \quad +\frac{5}{2} \\ \hline 2y-5 & 4y^4 \quad -10y^3 \quad -10y^2 \quad +30y \quad -15 \\ & -\ (4y^4 \quad -10y^3) \\ \hline & \quad 0 \quad -10y^2 \quad +30y \quad -15 \\ & -\ (-10y^3 \quad +25y^2) \\ \hline & \quad 10y^3 \quad -35y^2 \quad +30y \quad -15 \\ & -\ (5y^3 \quad -\frac{25y^2}{2}) \\ \hline & \quad 5y^3 \quad -\frac{45y^2}{2} \quad +30y \quad -15 \end{array}$$

Since the remainder is $5y^3 – 45y^2/2 + 30y – 15$, $2y - 5$ is not a factor of $4y^4 – 10y^3 – 10y^2 + 30y – 15$

(iv) $3y^2 + 5$ is a factor of $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$
Firstly let us perform long division method

$$
\begin{array}{r|rrrrrr}
 & 2y^3 & +5y^2 & +2y & -7 & & \\
\hline
3y^2+5 & 6y^5 & +15y^4 & +16y^3 & +4y^2 & +10y & -35 \\
- & 6y^5 & +0y^4 & +10y^3 & & & \\
\hline
 & & 15y^4 & +6y^3 & +4y^2 & +10y & -35 \\
- & & 15y^4 & +0y^3 & +25y^2 & & \\
\hline
 & & & 6y^3 & -21y^2 & +10y & -35 \\
- & & & 6y^3 & +0y^2 & +10y & \\
\hline
 & & & & -21y^2 & +0y & -35 \\
- & & & & -21y^2 & +0y & -35 \\
\hline
 & & & & & & 0
\end{array}
$$

Since the remainder is 0, $3y^2 + 5$ is a factor of $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$

(v) $z^2 + 3$ is a factor of $z^5 – 9z$
Firstly let us perform long division method

$$
\begin{array}{r|rrrrrr}
 & z^3 & -3z & & & & \\
\hline
z^2+3 & z^5 & +0z^4 & +0z^3 & +0z^2 & -9z & +0 \\
- & z^5 & +0z^4 & +3z^3 & & & \\
\hline
 & & & -3z^3 & +0z^2 & -9z & +0 \\
- & & & -3z^3 & +0z^2 & -9z & \\
\hline
 & & & & & 0 & 0
\end{array}
$$

Since the remainder is 0, $z^2 + 3$ is a factor of $z^5 – 9z$

(vi) $2x^2 - x + 3$ is a factor of $6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

Firstly let us perform long division method

$$\begin{array}{r|rrrrrr}
 & 3x^3 & +x^2 & -2x & -5 & & \\
\hline
2x^2 - x + 3 & 6x^5 & -x^4 & +4x^3 & -5x^2 & -x & -15 \\
- & 6x^5 & -3x^4 & +9x^3 & & & \\
\hline
 & & 2x^4 & -5x^3 & -5x^2 & -x & -15 \\
 & - & 2x^4 & -x^3 & +3x^2 & & \\
\hline
 & & & -4x^3 & -8x^2 & -x & -15 \\
 & & - & -4x^3 & +2x^2 & -6x & \\
\hline
 & & & & -10x^2 & +5x & -15 \\
 & & & - & -10x^2 & +5x & -15 \\
\hline
 & & & & & & 0
\end{array}$$

Since the remainder is 0, $2x^2 - x + 3$ is a factor of $6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

24. Find the value of a, if x + 2 is a factor of $4x^4 + 2x^3 - 3x^2 + 8x + 5a$

Solution:

We know that x + 2 is a factor of $4x^4 + 2x^3 - 3x^2 + 8x + 5a$

Let us equate x + 2 = 0

x = -2

Now let us substitute x = -2 in the equation $4x^4 + 2x^3 - 3x^2 + 8x + 5a$

$4(-2)^4 + 2(-2)^3 - 3(-2)^2 + 8(-2) + 5a = 0$

64 – 16 – 12 – 16 + 5a = 0

20 + 5a = 0

5a = -20

a = -20/5

= -4

25. What must be added to $x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$.

Solution:

Firstly let us perform long division method

$$
\begin{array}{r|llllll}
 & x^2 & +1 & & & \\
x^2+2x-3 & x^4 & +2x^3 & -2x^2 & +x & -1 \\
 & - & & & & \\
 & x^4 & +2x^3 & -3x^2 & & \\
\hline
 & & & x^2 & +x & -1 \\
 & & & - & & \\
 & & & x^2 & +2x & -3 \\
\hline
 & & & & -x & +2
\end{array}
$$

By long division method we got remainder as $-x + 2$,

$\therefore$ $x - 2$ has to be added to $x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$.