EXERCISE 8.5 PAGE NO: 8.15

1. Divide the first polynomial by the second polynomial in each of the following. Also, write the quotient and remainder:
(i) $3x^2 + 4x + 5$, $x - 2$
(ii) $10x^2 – 7x + 8$, $5x - 3$
(iii) $5y^3 – 6y^2 + 6y – 1$, $5y – 1$
(iv) $x^4 – x^3 + 5x$, $x - 1$
(v) $y^4 + y^2$, $y^2 – 2$

Solution:

(i) $3x^2 + 4x + 5$, $x - 2$

By using long division method

$$\begin{array}{r|lll} & 3x & +10 & \\ \hline x-2 & 3x^2 & +4x & +5 \\ - & 3x^2 & -6x & \\ \hline & & 10x & +5 \\ & - & & \\ & & 10x & -20 \\ \hline & & & 25 \end{array}$$

∴ the Quotient is $3x + 10$ and the Remainder is 25.

(ii) $10x^2 – 7x + 8$, $5x - 3$

By using long division method

$$\begin{array}{r|lll} & 2x & -\frac{1}{5} & \\ \hline 5x-3 & 10x^2 & -7x & +8 \\ - & 10x^2 & -6x & \\ \hline & & -x & +8 \\ & - & & \\ & & -x & +\frac{3}{5} \\ \hline & & & \frac{37}{5} \end{array}$$

∴ the Quotient is $2x – 1/5$ and the Remainder is $37/5$.

(iii) $5y^3 - 6y^2 + 6y - 1, 5y - 1$
By using long division method

$$\begin{array}{r|llll}
 & y^2 & -y & +1 & \\
\hline
5y-1 & 5y^3 & -6y^2 & +6y & -1 \\
- & 5y^3 & -y^2 & & \\
\hline
 & & -5y^2 & +6y & -1 \\
 & - & -5y^2 & +y & \\
\hline
 & & & 5y & -1 \\
 & & - & 5y & -1 \\
\hline
 & & & & 0
\end{array}$$

∴ the Quotient is $y^2 - y + 1$ and the Remainder is 0.

(iv) $x^4 - x^3 + 5x, x - 1$
By using long division method

$$\begin{array}{r|lllll}
 & x^3 & +5 & & & \\
\hline
x-1 & x^4 & -x^3 & +0x^2 & +5x & +0 \\
- & x^4 & -x^3 & & & \\
\hline
 & & 0 & +0x^2 & +5x & +0 \\
 & - & & & & \\
 & & 5x^3 & -5x^2 & & \\
\hline
 & & -5x^3 & +5x^2 & +5x & +0
\end{array}$$

∴ the Quotient is $x^3 + 5$ and the Remainder is 5.

(v) $y^4 + y^2, y^2 - 2$
By using long division method

$$\begin{array}{r|lllll} & y^2 & +3 & & & \\ \hline y^2-2 & y^4 & +0y^3 & +y^2 & +0y & +0 \\ - & & & & & \\ & y^4 & +0y^3 & -2y^2 & & \\ \hline & & & 3y^2 & +0y & +0 \\ & & - & & & \\ & & & 3y^2 & +0y & -6 \\ \hline & & & & & 6 \end{array}$$

∴ the Quotient is $y^2 + 3$ and the Remainder is 6.

2. Find Whether or not the first polynomial is a factor of the second:
(i) $x + 1$, $2x^2 + 5x + 4$
(ii) $y – 2$, $3y^3 + 5y^2 + 5y + 2$
(iii) $4x^2 – 5$, $4x^4 + 7x^2 + 15$
(iv) $4 – z$, $3z^2 – 13z + 4$
(v) $2a – 3$, $10a^2 – 9a - 5$
(vi) $4y + 1$, $8y^2 – 2y + 1$
Solution:
(i) $x + 1$, $2x^2 + 5x + 4$
Let us perform long division method,

$$\begin{array}{r|lll} & 2x & +3 & \\ \hline x+1 & 2x^2 & +5x & +4 \\ - & & & \\ & 2x^2 & +2x & \\ \hline & & 3x & +4 \\ & - & & \\ & & 3x & +3 \\ \hline & & & 1 \end{array}$$

Since remainder is 1 therefore the first polynomial is not a factor of the second polynomial.

(ii) $y – 2$, $3y^3 + 5y^2 + 5y + 2$
Let us perform long division method,

$$\begin{array}{r|l} & 3y^2 \quad +11y \quad +27 \\ \hline y-2 & 3y^3 \quad +5y^2 \quad +5y \quad +2 \\ & -\;(3y^3 \quad -6y^2) \\ \hline & 11y^2 \quad +5y \quad +2 \\ & -\;(11y^2 \quad -22y) \\ \hline & 27y \quad +2 \\ & -\;(27y \quad -54) \\ \hline & 56 \end{array}$$

Since remainder is 56 therefore the first polynomial is not a factor of the second polynomial.

(iii) $4x^2 - 5$, $4x^4 + 7x^2 + 15$

Let us perform long division method,

$$\begin{array}{r|l} & x^2 \quad +3 \\ \hline 4x^2-5 & 4x^4 \quad +0x^3 \quad +7x^2 \quad +0x \quad +15 \\ & -\;(4x^4 \quad +0x^3 \quad -5x^2) \\ \hline & 12x^2 \quad +0x \quad +15 \\ & -\;(12x^2 \quad +0x \quad -15) \\ \hline & 30 \end{array}$$

Since remainder is 30 therefore the first polynomial is not a factor of the second polynomial.

(iv) $4 - z$, $3z^2 - 13z + 4$

Let us perform long division method,

$$
\begin{array}{r|rrr}
 & -3z & +1 & \\
\hline
-z+4 & 3z^2 & -13z & +4 \\
- & & & \\
 & 3z^2 & -12z & \\
\hline
 & & -z & +4 \\
 & - & & \\
 & & -z & +4 \\
\hline
 & & & 0
\end{array}
$$

Since remainder is 0 therefore the first polynomial is a factor of the second polynomial.

(v) $2a – 3$, $10a^2 – 9a - 5$

Let us perform long division method,

$$
\begin{array}{r|rrr}
 & 5a & +3 & \\
\hline
2a-3 & 10a^2 & -9a & -5 \\
- & & & \\
 & 10a^2 & -15a & \\
\hline
 & & 6a & -5 \\
 & - & & \\
 & & 6a & -9 \\
\hline
 & & & 4
\end{array}
$$

Since remainder is 4 therefore the first polynomial is not a factor of the second polynomial.

(vi) $4y + 1$, $8y^2 – 2y + 1$

Let us perform long division method,

$$\begin{array}{r|l} & 2y \quad -1 \\ \hline 4y+1 & 8y^2 \quad -2y \quad +1 \\ & -\left(8y^2 \quad +2y\right) \\ \hline & \qquad -4y \quad +1 \\ & \qquad -\left(-4y \quad -1\right) \\ \hline & \qquad\qquad 2 \end{array}$$

Since remainder is 2 therefore the first polynomial is not a factor of the second polynomial.