EXERCISE 8.1 PAGE NO: 8.2

1. Write the degree of each of the following polynomials:
(i) $2x^3 + 5x^2 - 7$
(ii) $5x^2 – 3x + 2$
(iii) $2x + x^2 - 8$
(iv) $1/2y^7 – 12y^6 + 48y^5 - 10$
(v) $3x^3 + 1$
(vi) 5
(vii) $20x^3 + 12x^2y^2 – 10y^2 + 20$
Solution:
(i) $2x^3 + 5x^2 - 7$
We know that in a polynomial, degree is the highest power of the variable.
The degree of the polynomial, $2x^3 + 5x^2 - 7$ is 3.

(ii) $5x^2 – 3x + 2$
The degree of the polynomial, $5x^2 – 3x + 2$ is 2.

(iii) $2x + x^2 – 8$
The degree of the polynomial, $2x + x^2 – 8$ is 2.

(iv) $1/2y^7 – 12y^6 + 48y^5 – 10$
The degree of the polynomial, $1/2y^7 – 12y^6 + 48y^5 – 10$ is 7.

(v) $3x^3 + 1$
The degree of the polynomial, $3x^3 + 1$ is 3

(vi) 5
The degree of the polynomial, 5 is 0 (since 5 is a constant number).

(vii) $20x^3 + 12x^2y^2 – 10y^2 + 20$
The degree of the polynomial, $20x^3 + 12x^2y^2 – 10y^2 + 20$ is 4.

2. Which of the following expressions are not polynomials?
(i) $x^2 + 2x^{-2}$
(ii) $\sqrt{(ax)} + x^2 – x^3$
(iii) $3y^3 - \sqrt{5}y + 9$
(iv) $ax^{1/2} + ax + 9x^2 + 4$
(v) $3x^{-3} + 2x^{-1} + 4x + 5$

Solution:
(i) $x^2 + 2x^{-2}$
The given expression is not a polynomial.
Because a polynomial does not contain any negative powers or fractions.

(ii) $\sqrt{(ax)} + x^2 – x^3$
The given expression is a polynomial.
Because the polynomial has positive powers.

(iii) $3y^3 - \sqrt{5}y + 9$
The given expression is a polynomial.
Because the polynomial has positive powers.

(iv) $ax^{1/2} + ax + 9x^2 + 4$
The given expression is not a polynomial.
Because a polynomial does not contain any negative powers or fractions.

(v) $3x^{-3} + 2x^{-1} + 4x + 5$
The given expression is not a polynomial.
Because a polynomial does not contain any negative powers or fractions.

3. Write each of the following polynomials in the standard from. Also, write their degree:
(i) $x^2 + 3 + 6x + 5x^4$
(ii) $a^2 + 4 + 5a^6$
(iii) $(x^3 - 1) (x^3 - 4)$
(iv) $(y^3 - 2) (y^3 + 11)$
(v) $(a^3 – 3/8) (a^3 + 16/17)$
(vi) $(a + 3/4) (a + 4/3)$
Solution:
(i) $x^2 + 3 + 6x + 5x^4$
The standard form of the polynomial is written in either increasing or decreasing order of their powers.
$3 + 6x + x^2 + 5x^4$ or $5x^4 + x^2 + 6x + 3$
The degree of the given polynomial is 4.

(ii) $a^2 + 4 + 5a^6$
The standard form of the polynomial is written in either increasing or decreasing order of their powers.

$4 + a^2 + 5a^6$ or $5a^6 + a^2 + 4$
The degree of the given polynomial is 6.

(iii) $(x^3 - 1)(x^3 - 4)$
$x^6 – 4x^3 – x^3 + 4$
$x^6 – 5x^3 + 4$
The standard form of the polynomial is written in either increasing or decreasing order of their powers.
$x^6 – 5x^3 + 4$ or $4 – 5x^3 + x^6$
The degree of the given polynomial is 6.

(iv) $(y^3 - 2)(y^3 + 11)$
$y^6 + 11y^3 – 2y^3 – 22$
$y^6 + 9y^3 – 22$
The standard form of the polynomial is written in either increasing or decreasing order of their powers.
$y^6 + 9y^3 – 22$ or $-22 + 9y^3 + y^6$
The degree of the given polynomial is 6.

(v) $(a^3 – 3/8)(a^3 + 16/17)$
$a^6 + 16a^3/17 – 3a^3/8 – 6/17$
$a^6 + 27/136a^3 – 48/136$
The standard form of the polynomial is written in either increasing or decreasing order of their powers.
$a^6 + 27/136a^3 – 48/136$ or $-48/136 + 27/136a^3 + a^6$
The degree of the given polynomial is 6.

(vi) $(a + 3/4)(a + 4/3)$
$a^2 + 4a/3 + 3a/4 + 1$
$a^2 + 25a/12 + 1$
The standard form of the polynomial is written in either increasing or decreasing order of their powers.
$a^2 + 25a/12 + 1$ or $1 + 25a/12 + a^2$
The degree of the given polynomial is 2.

EXERCISE 8.2

PAGE NO: 8.4

Divide:

1. $6x^3y^2z^2$ by $3x^2yz$

Solution:

We have,

$6x^3y^2z^2 / 3x^2yz$

By using the formula $a^n / a^m = a^{n-m}$

$6/3\ x^{3-2}\ y^{2-1}\ z^{2-1}$

$2xyz$

2. $15m^2n^3$ by $5m^2n^2$

Solution:

We have,

$15m^2n^3 / 5m^2n^2$

By using the formula $a^n / a^m = a^{n-m}$

$15/5\ m^{2-2}\ n^{3-2}$

$3n$

3. $24a^3b^3$ by $-8ab$

Solution:

We have,

$24a^3b^3 / -8ab$

By using the formula $a^n / a^m = a^{n-m}$

$24/-8\ a^{3-1}\ b^{3-1}$

$-3a^2b^2$

4. $-21abc^2$ by $7abc$

Solution:

We have,

$-21abc^2 / 7abc$

By using the formula $a^n / a^m = a^{n-m}$

$-21/7\ a^{1-1}\ b^{1-1}\ c^{2-1}$

$-3c$

5. $72xyz^2$ by $-9xz$

Solution:

We have,

$72xyz^2 / -9xz$

By using the formula $a^n / a^m = a^{n-m}$
$72/-9\ x^{1-1}\ y\ z^{2-1}$
$-8yz$

6. $-72a^4b^5c^8$ by $-9a^2b^2c^3$
Solution:
We have,
$-72a^4b^5c^8 / -9a^2b^2c^3$
By using the formula $a^n / a^m = a^{n-m}$
$-72/-9\ a^{4-2}\ b^{5-2}\ c^{8-3}$
$8a^2b^3c^5$

Simplify:

7. $16m^3y^2 / 4m^2y$
Solution:
We have,
$16m^3y^2 / 4m^2y$
By using the formula $a^n / a^m = a^{n-m}$
$16/4\ m^{3-2}\ y^{2-1}$
$4my$

8. $32m^2n^3p^2 / 4mnp$
Solution:
We have,
$32m^2n^3p^2 / 4mnp$
By using the formula $a^n / a^m = a^{n-m}$
$32/4\ m^{2-1}\ n^{3-1}\ p^{2-1}$
$8m^2n^2p$

EXERCISE 8.3 PAGE NO: 8.6

Divide:

1. $x + 2x^2 + 3x^4 – x^5$ by 2x

Solution:

We have,

$(x + 2x^2 + 3x^4 – x^5) / 2x$

$x/2x + 2x^2/2x + 3x^4/2x – x^5/2x$

By using the formula $a^n / a^m = a^{n-m}$

$1/2\ x^{1-1} + x^{2-1} + 3/2\ x^{4-1} – 1/2\ x^{5-1}$

$1/2 + x + 3/2\ x^3 – 1/2\ x^4$

2. $y^4 – 3y^3 + 1/2y^2$ by 3y

Solution:

We have,

$(y^4 – 3y^3 + 1/2y^2)/ 3y$

$y^4/3y – 3y^3/3y + (½)y^2/3y$

By using the formula $a^n / a^m = a^{n-m}$

$1/3\ y^{4-1} – y^{3-1} + 1/6\ y^{2-1}$

$1/3y^3 – y^2 + 1/6y$

3. $-4a^3 + 4a^2 + a$ by 2a

Solution:

We have,

$(-4a^3 + 4a^2 + a) / 2a$

$-4a^3/2a + 4a^2/2a + a/2a$

By using the formula $a^n / a^m = a^{n-m}$

$-2a^{3-1} + 2a^{2-1} + 1/2\ a^{1-1}$

$-2a^2 + 2a + ½$

4. $–x^6 + 2x^4 + 4x^3 + 2x^2$ by $\sqrt{2}x^2$

Solution:

We have,

$(–x^6 + 2x^4 + 4x^3 + 2x^2) / \sqrt{2}x^2$

$-x^6/\sqrt{2}x^2 + 2x^4/\sqrt{2}x^2 + 4x^3/\sqrt{2}x^2 + 2x^2/\sqrt{2}x^2$

By using the formula $a^n / a^m = a^{n-m}$

$-1/\sqrt{2}\ x^{6-2} + 2/\sqrt{2}\ x^{4-2} + 4/\sqrt{2}\ x^{3-2} + 2/\sqrt{2}\ x^{2-2}$

$-1/\sqrt{2}\ x^4 + \sqrt{2}x^2 + 2\sqrt{2}x + \sqrt{2}$

5. $-4a^3 + 4a^2 + a$ by $2a$

Solution:

We have,

$(-4a^3 + 4a^2 + a) / 2a$

$-4a^3/2a + 4a^2/2a + a/2a$

By using the formula $a^n / a^m = a^{n-m}$

$-2a^{3-1} + 2a^{2-1} + 1/2a^{1-1}$

$-2a^2 + 2a + ½$

6. $\sqrt{3}a^4 + 2\sqrt{3}a^3 + 3a^2 – 6a$ by $3a$

Solution:

We have,

$(\sqrt{3}a^4 + 2\sqrt{3}a^3 + 3a^2 – 6a) / 3a$

$\sqrt{3}a^4/3a + 2\sqrt{3}a^3/3a + 3a^2/3a – 6a/3a$

By using the formula $a^n / a^m = a^{n-m}$

$\sqrt{3}/3\ a^{4-1} + 2\sqrt{3}/3\ a^{3-1} + a^{2-1} – 2a^{1-1}$

$1/\sqrt{3}\ a^3 + 2/\sqrt{3}\ a^2 + a – 2$

EXERCISE 8.4 PAGE NO: 8.11

Divide:

1. $5x^3 – 15x^2 + 25x$ by $5x$
Solution:
We have,
$(5x^3 – 15x^2 + 25x) / 5x$
$5x^3/5x – 15x^2/5x + 25x/5x$
By using the formula $a^n / a^m = a^{n-m}$
$5/5\ x^{3-1} – 15/5\ x^{2-1} + 25/5\ x^{1-1}$
$x^2 – 3x + 5$

2. $4z^3 + 6z^2 – z$ by $-1/2z$
Solution:
We have,
$(4z^3 + 6z^2 – z) / -1/2z$
$4z^3/(-1/2z) + 6z^2/(-1/2z) – z/(-1/2z)$
By using the formula $a^n / a^m = a^{n-m}$
$-8\ z^{3-1} – 12z^{2-1} + 2\ z^{1-1}$
$-8z^2 – 12z + 2$

3. $9x^2y – 6xy + 12xy^2$ by $-3/2xy$
Solution:
We have,
$(9x^2y – 6xy + 12xy^2) / -3/2xy$
$9x^2y/(-3/2xy) – 6xy/(-3/2xy) + 12xy^2/(-3/2xy)$
By using the formula $a^n / a^m = a^{n-m}$
$(-9×2)/3\ x^{2-1}y^{1-1} – (-6×2)/3\ x^{1-1}y^{1-1} + (-12×2)/3\ x^{1-1}y^{2-1}$
$-6x + 4 – 8y$

4. $3x^3y^2 + 2x^2y + 15xy$ by $3xy$
Solution:
We have,
$(3x^3y^2 + 2x^2y + 15xy) / 3xy$
$3x^3y^2/3xy + 2x^2y/3xy + 15xy/3xy$
By using the formula $a^n / a^m = a^{n-m}$
$3/3\ x^{3-1}y^{2-1} + 2/3\ x^{2-1}y^{1-1} + 15/3\ x^{1-1}y^{1-1}$
$x^2y + 2/3x + 5$

5. $x^3 + 7x + 12$ by $x + 4$

Solution:

We have,

$(x^3 + 7x + 12) / (x + 4)$

By using long division method

$$
\begin{array}{r|rrr}
 & x & +3 & \\
\hline
x+4 & x^2 & +7x & +12 \\
 & - & & \\
 & x^2 & +4x & \\
\hline
 & & 3x & +12 \\
 & & - & \\
 & & 3x & +12 \\
\hline
 & & & 0
\end{array}
$$

$\therefore (x^3 + 7x + 12) / (x + 4) = x + 3$

6. $4y^2 + 3y + 1/2$ by $2y + 1$

Solution:

We have,

$4y^2 + 3y + 1/2$ by $(2y + 1)$

By using long division method

$$
\begin{array}{r|rrr}
 & 2y & +\frac{1}{2} & \\
\hline
2y+1 & 4y^2 & +3y & +\frac{1}{2} \\
 & - & & \\
 & 4y^2 & +2y & \\
\hline
 & & y & +\frac{1}{2} \\
 & & - & \\
 & & y & +\frac{1}{2} \\
\hline
 & & & 0
\end{array}
$$

$\therefore (4y^2 + 3y + 1/2) / (2y + 1) = 2y + 1/2$

7. $3x^3 + 4x^2 + 5x + 18$ by $x + 2$

Solution:

We have,

$(3x^3 + 4x^2 + 5x + 18) / (x + 2)$

By using long division method

$$
\begin{array}{r|rrrr}
 & 3x^2 & -2x & +9 & \\
\hline
x+2 & 3x^3 & +4x^2 & +5x & +18 \\
 - & 3x^3 & +6x^2 & & \\
\hline
 & & -2x^2 & +5x & +18 \\
 & & - & & \\
 & & -2x^2 & -4x & \\
\hline
 & & & 9x & +18 \\
 & & & - & \\
 & & & 9x & +18 \\
\hline
 & & & & 0
\end{array}
$$

∴ $(3x^3 + 4x^2 + 5x + 18) / (x + 2) = 3x^2 – 2x + 9$

8. $14x^2 – 53x + 45$ by $7x - 9$

Solution:

We have,

$(14x^2 – 53x + 45) / (7x – 9)$

By using long division method

$$
\begin{array}{r|rrr}
 & 2x & -5 & \\
\hline
7x-9 & 14x^2 & -53x & +45 \\
 - & 14x^2 & -18x & \\
\hline
 & & -35x & +45 \\
 & & - & \\
 & & -35x & +45 \\
\hline
 & & & 0
\end{array}
$$

∴ $(14x^2 – 53x + 45) / (7x – 9) = 2x – 5$

9. $-21 + 71x – 31x^2 – 24x^3$ by $3 – 8x$

Solution:

We have,

$-21 + 71x – 31x^2 – 24x^3$ by $3 – 8x$

$(-24x^3 – 31x^2 + 71x – 21) / (3 – 8x)$

By using long division method

$$
\begin{array}{r|rrrr}
 & 3x^2 & +5x & -7 & \\
\hline
-8x+3 & -24x^3 & -31x^2 & +71x & -21 \\
 & - & & & \\
 & -24x^3 & +9x^2 & & \\
\hline
 & & -40x^2 & +71x & -21 \\
 & & - & & \\
 & & -40x^2 & +15x & \\
\hline
 & & & 56x & -21 \\
 & & & - & \\
 & & & 56x & -21 \\
\hline
 & & & & 0
\end{array}
$$

∴ $(-24x^3 – 31x^2 + 71x – 21) / (3 – 8x) = 3x^2 + 5x – 7$

10. $3y^4 – 3y^3 – 4y^2 – 4y$ by $y^2 - 2y$

Solution:

We have,

$(3y^4 – 3y^3 – 4y^2 – 4y) / (y^2 - 2y)$

By using long division method

$$
\begin{array}{r|rrrrr}
 & 3y^2 & +3y & +2 & & \\
\hline
y^2-2y & 3y^4 & -3y^3 & -4y^2 & -4y & +0 \\
 & - & & & & \\
 & 3y^4 & -6y^3 & & & \\
\hline
 & & 3y^3 & -4y^2 & -4y & +0 \\
 & & - & & & \\
 & & 3y^3 & -6y^2 & & \\
\hline
 & & & 2y^2 & -4y & +0 \\
 & & & - & & \\
 & & & 2y^2 & -4y & \\
\hline
 & & & & & 0
\end{array}
$$

∴ $(3y^4 – 3y^3 – 4y^2 – 4y) / (y^2 - 2y) = 3y^2 + 3y + 2$

11. $2y^5 + 10y^4 + 6y^3 + y^2 + 5y + 3$ by $2y^3 + 1$

Solution:

We have,

$(2y^5 + 10y^4 + 6y^3 + y^2 + 5y + 3) / (2y^3 + 1)$

By using long division method

$$
\begin{array}{rrrrrrr}
 & y^2 & +5y & +3 & & & \\
2y^3+1 & \big) 2y^5 & +10y^4 & +6y^3 & +y^2 & +5y & +3 \\
 & - & & & & & \\
 & 2y^5 & +0y^4 & +0y^3 & +y^2 & & \\
\hline
 & & 10y^4 & +6y^3 & +0y^2 & +5y & +3 \\
 & & - & & & & \\
 & & 10y^4 & +0y^3 & +0y^2 & +5y & \\
\hline
 & & & 6y^3 & +0y^2 & +0y & +3 \\
 & & & - & & & \\
 & & & 6y^3 & +0y^2 & +0y & +3 \\
\hline
 & & & & & & 0
\end{array}
$$

∴ $(2y^5 + 10y^4 + 6y^3 + y^2 + 5y + 3) / (2y^3 + 1) = y^2 + 5y + 3$

12. $x^4 – 2x^3 + 2x^2 + x + 4$ by $x^2 + x + 1$
Solution:
We have,
$(x^4 – 2x^3 + 2x^2 + x + 4) / (x^2 + x + 1)$
By using long division method

$$
\begin{array}{rrrrrr}
 & x^2 & -3x & +4 & & \\
x^2+x+1 & \big) x^4 & -2x^3 & +2x^2 & +x & +4 \\
 & - & & & & \\
 & x^4 & +x^3 & +x^2 & & \\
\hline
 & & -3x^3 & +x^2 & +x & +4 \\
 & & - & & & \\
 & & -3x^3 & -3x^2 & -3x & \\
\hline
 & & & 4x^2 & +4x & +4 \\
 & & & - & & \\
 & & & 4x^2 & +4x & +4 \\
\hline
 & & & & & 0
\end{array}
$$

∴ $(x^4 – 2x^3 + 2x^2 + x + 4) / (x^2 + x + 1) = x^2 – 3x + 4$

13. $m^3 – 14m^2 + 37m – 26$ by $m^2 – 12m + 13$
Solution:
We have,
$(m^3 – 14m^2 + 37m – 26) / (m^2 – 12m + 13)$
By using long division method

$$
\begin{array}{r|rrrr}
 & m & -2 & & \\
\hline
m^2 - 12m + 13 & m^3 & -14m^2 & +37m & -26 \\
- & & & & \\
 & m^3 & -12m^2 & +13m & \\
\hline
 & & -2m^2 & +24m & -26 \\
 & - & & & \\
 & & -2m^2 & +24m & -26 \\
\hline
 & & & & 0
\end{array}
$$

$\therefore (m^3 - 14m^2 + 37m - 26) / (m^2 - 12m + 13) = m - 2$

14. $x^4 + x^2 + 1$ by $x^2 + x + 1$

Solution:

We have,

$(x^4 + x^2 + 1) / (x^2 + x + 1)$

By using long division method

$$
\begin{array}{r|rrrrr}
 & x^2 & -x & +1 & & \\
\hline
x^2 + x + 1 & x^4 & +0x^3 & +x^2 & +0x & +1 \\
- & & & & & \\
 & x^4 & +x^3 & +x^2 & & \\
\hline
 & & -x^3 & +0x^2 & +0x & +1 \\
 & - & & & & \\
 & & -x^3 & -x^2 & -x & \\
\hline
 & & & x^2 & +x & +1 \\
 & & - & & & \\
 & & & x^2 & +x & +1 \\
\hline
 & & & & & 0
\end{array}
$$

$\therefore (x^4 + x^2 + 1) / (x^2 + x + 1) = x^2 - x + 1$

15. $x^5 + x^4 + x^3 + x^2 + x + 1$ by $x^3 + 1$

Solution:

We have,

$(x^5 + x^4 + x^3 + x^2 + x + 1) / (x^3 + 1)$

By using long division method

$$
\begin{array}{r|rrrrrr}
 & x^2 & & +1 & & & \\
\hline
x^3+1 & x^5 & +x^4 & +x^3 & +x^2 & +x & +1 \\
 & - & & & & & \\
 & x^5 & +0x^4 & +0x^3 & +x^2 & & \\
\hline
 & & & +x^3 & +0x^2 & +x & +1 \\
 & & - & & & & \\
 & & x^4 & +0x^3 & +0x^2 & +x & \\
\hline
 & & & x^3 & +0x^2 & +0x & +1 \\
 & & & - & & & \\
 & & & x^3 & +0x^2 & +0x & +1 \\
\hline
 & & & & & & 0
\end{array}
$$

∴ $(x^5 + x^4 + x^3 + x^2 + x + 1) / (x^3 + 1) = x^2 + x + 1$

Divide each of the following and find the quotient and remainder:

16. $14x^3 - 5x^2 + 9x - 1$ by $2x - 1$
Solution:
We have,
$(14x^3 - 5x^2 + 9x - 1) / (2x - 1)$
By using long division method

$$
\begin{array}{r|rrrr}
 & 7x^2 & +x & +5 & \\
\hline
2x-1 & 14x^3 & -5x^2 & +9x & -1 \\
 & - & & & \\
 & 14x^3 & -7x^2 & & \\
\hline
 & & 2x^2 & +9x & -1 \\
 & & - & & \\
 & & 2x^2 & -x & \\
\hline
 & & & 10x & -1 \\
 & & & - & \\
 & & & 10x & -5 \\
\hline
 & & & & 4
\end{array}
$$

∴ Quotient is $7x^2 + x + 5$ and the Remainder is 4.

17. $6x^3 - x^2 - 10x - 3$ by $2x - 3$

Solution:

We have,

$(6x^3 - x^2 - 10x - 3) / (2x - 3)$

By using long division method

$$
\begin{array}{r|rrrr}
 & 3x^2 & +4x & +1 & \\
\cline{2-5}
2x-3 & 6x^3 & -x^2 & -10x & -3 \\
 - & 6x^3 & -9x^2 & & \\
\hline
 & & 8x^2 & -10x & -3 \\
 & - & 8x^2 & -12x & \\
\hline
 & & & 2x & -3 \\
 & & - & 2x & -3 \\
\hline
 & & & & 0
\end{array}
$$

∴ Quotient is $3x^2 + 4x + 1$ and the Remainder is 0.

18. $6x^3 + 11x^2 - 39x - 65$ by $3x^2 + 13x + 13$

Solution:

We have,

$(6x^3 + 11x^2 - 39x - 65) / (3x^2 + 13x + 13)$

By using long division method

$$
\begin{array}{r|rrrr}
 & 2x & -5 & & \\
\cline{2-5}
3x^2+13x+13 & 6x^3 & +11x^2 & -39x & -65 \\
 - & 6x^3 & +26x^2 & +26x & \\
\hline
 & & -15x^2 & -65x & -65 \\
 & - & -15x^2 & -65x & -65 \\
\hline
 & & & & 0
\end{array}
$$

∴ Quotient is $2x - 5$ and the Remainder is 0.

19. $30x^4 + 11x^3 - 82x^2 - 12x + 48$ by $3x^2 + 2x - 4$

Solution:

We have,

$(30x^4 + 11x^3 - 82x^2 - 12x + 48) / (3x^2 + 2x - 4)$
By using long division method

$$
\begin{array}{r|rrrrr}
 & 10x^2 & -3x & -12 & & \\
\hline
3x^2+2x-4 & 30x^4 & +11x^3 & -82x^2 & -12x & +48 \\
- & & & & & \\
 & 30x^4 & +20x^3 & -40x^2 & & \\
\hline
 & & -9x^3 & -42x^2 & -12x & +48 \\
 & & - & & & \\
 & & -9x^3 & -6x^2 & +12x & \\
\hline
 & & & -36x^2 & -24x & +48 \\
 & & & - & & \\
 & & & -36x^2 & -24x & +48 \\
\hline
 & & & & & 0
\end{array}
$$

∴ Quotient is $10x^2 - 3x - 12$ and the Remainder is 0.

20. $9x^4 - 4x^2 + 4$ by $3x^2 - 4x + 2$
Solution:
We have,
$(9x^4 - 4x^2 + 4) / (3x^2 - 4x + 2)$
By using long division method

$$
\begin{array}{r|rrrrr}
 & 3x^2 & +4x & +2 & & \\
\hline
3x^2-4x+2 & 9x^4 & +0x^3 & -4x^2 & +0x & +4 \\
- & & & & & \\
 & 9x^4 & -12x^3 & +6x^2 & & \\
\hline
 & & 12x^3 & -10x^2 & +0x & +4 \\
 & & - & & & \\
 & & 12x^3 & -16x^2 & +8x & \\
\hline
 & & & 6x^2 & -8x & +4 \\
 & & & - & & \\
 & & & 6x^2 & -8x & +4 \\
\hline
 & & & & & 0
\end{array}
$$

∴ Quotient is $3x^2 + 4x + 2$ and the Remainder is 0.

21. Verify division algorithm i.e. Dividend = Divisor × Quotient + Remainder, in each of the following. Also, write the quotient and remainder:

Dividend	**divisor**
(i) $14x^2 + 13x - 15$	$7x - 4$
(ii) $15z^3 - 20z^2 + 13z - 12$	$3z - 6$
(iii) $6y^5 - 28y^3 + 3y^2 + 30y - 9$	$2x^2 - 6$
(iv) $34x - 22x^3 - 12x^4 - 10x^2 - 75$	$3x + 7$
(v) $15y^4 - 16y^3 + 9y^2 - 10/3y + 6$	$3y - 2$
(vi) $4y^3 + 8y + 8y^2 + 7$	$2y^2 - y + 1$
(vii) $6y^4 + 4y^4 + 4y^3 + 7y^2 + 27y + 6$	$2y^3 + 1$

Solution:

(i) Dividend — divisor

$14x^2 + 13x - 15$ — $7x - 4$

By using long division method

$$
\begin{array}{r|rrr}
 & 2x & +3 & \\
\hline
7x-4 & 14x^2 & +13x & -15 \\
- & & & \\
 & 14x^2 & -8x & \\
\hline
 & & 21x & -15 \\
 & & - & \\
 & & 21x & -12 \\
\hline
 & & & -3
\end{array}
$$

Let us verify, Dividend = Divisor × Quotient + Remainder

$14x^2 + 13x - 15 = (7x - 4) \times (2x + 3) + (-3)$

$= 14x^2 + 21x - 8x -12 -3$

$= 14x^2 + 13x - 15$

Hence, verified.

∴ Quotient is $2x + 3$ and the Remainder is -3.

(ii) Dividend — divisor

$15z^3 - 20z^2 + 13z - 12$ — $3z - 6$

By using long division method

$$\begin{array}{r|llll}
 & 5z^2 & +\frac{10z}{3} & +11 & \\ \hline
3z-6 & 15z^3 & -20z^2 & +13z & -12 \\
 & - & & & \\
 & 15z^3 & -30z^2 & & \\ \hline
 & & 10z^2 & +13z & -12 \\
 & & - & & \\
 & & 10z^2 & -20z & \\ \hline
 & & & 33z & -12 \\
 & & & - & \\
 & & & 33z & -66 \\ \hline
 & & & & 54
\end{array}$$

Let us verify, Dividend = Divisor × Quotient + Remainder

$15z^3 – 20z^2 + 13z – 12 = (3z - 6) × (5z^2 + 10z/3 + 11) + 54$

$= 15z^3 + 10z^2 + 33z – 30z^2 – 20z + 54$

$= 15z^2 – 20z^2 + 13z – 12$

Hence, verified.

∴ Quotient is $5z^2 + 10z/3 + 11$ and the Remainder is 54.

(iii) Dividend divisor

$6y^5 – 28y^3 + 3y^2 + 30y – 9$ $2x^2 – 6$

By using long division method

$$\begin{array}{r|llllll}
 & 3y^3 & -5y & +\frac{3}{2} & & & \\ \hline
2y^2-6 & 6y^5 & +0y^4 & -28y^3 & +3y^2 & +30y & -9 \\
 & - & & & & & \\
 & 6y^5 & +0y^4 & -18y^3 & & & \\ \hline
 & & & -10y^3 & +3y^2 & +30y & -9 \\
 & & & - & & & \\
 & & & -10y^3 & +0y^2 & +30y & \\ \hline
 & & & & 3y^2 & +0y & -9 \\
 & & & & - & & \\
 & & & & 3y^2 & +0y & -9 \\ \hline
 & & & & & & 0
\end{array}$$

Let us verify, Dividend = Divisor × Quotient + Remainder

$6y^5 – 28y^3 + 3y^2 + 30y – 9 = (2x^2 – 6) × (3y^3 – 5y + 3/2) + 0$

$= 6y^5 – 10y^3 + 3y^2 – 18y^3 + 30y – 9$

$= 6y^5 – 28y^3 + 3y^2 + 30y – 9$

Hence, verified.

∴ Quotient is $3y^3 – 5y + 3/2$ and the Remainder is 0.

(iv) Dividend $\quad$ divisor

$34x – 22x^3 – 12x^4 – 10x^2 – 75 \quad 3x + 7$

$-12x^4 – 22x^3 – 10x^2 + 34x - 75$

By using long division method

$$
\begin{array}{r|rrrrr}
 & -4x^3 & +2x^2 & -8x & +30 & \\
\hline
3x+7 & -12x^4 & -22x^3 & -10x^2 & +34x & -75 \\
 & - & & & & \\
 & -12x^4 & -28x^3 & & & \\
\hline
 & & 6x^3 & -10x^2 & +34x & -75 \\
 & & - & & & \\
 & & 6x^3 & +14x^2 & & \\
\hline
 & & & -24x^2 & +34x & -75 \\
 & & & - & & \\
 & & & -24x^2 & -56x & \\
\hline
 & & & & 90x & -75 \\
 & & & & - & \\
 & & & & 90x & +210 \\
\hline
 & & & & & -285
\end{array}
$$

Let us verify, Dividend = Divisor × Quotient + Remainder

$-12x^4 – 22x^3 – 10x^2 + 34x – 75 = (3x + 7) \times (-4x^3 + 2x^2 – 8x + 30) – 285$

$= -12x^4 + 6x^3 – 24x^2 – 28x^3 + 14x^2 + 90x – 56x + 210 -285$

$= -12x^4 – 22x^3 – 10x^2 + 34x – 75$

Hence, verified.

∴ Quotient is $-4x^3 + 2x^2 - 8x + 30$ and the Remainder is -285.

(v) Dividend $\quad$ divisor

$15y^4 – 16y^3 + 9y^2 – 10/3y + 6 \quad 3y – 2$

By using long division method

$$
\begin{array}{r|rrrrr}
 & 5y^3 & -2y^2 & +\frac{5y}{3} & & \\
\hline
3y-2 & 15y^4 & -16y^3 & +9y^2 & -\frac{10y}{3} & +6 \\
- & 15y^4 & -10y^3 & & & \\
\hline
 & & -6y^3 & +9y^2 & -\frac{10y}{3} & +6 \\
 & & - & & & \\
 & & -6y^3 & +4y^2 & & \\
\hline
 & & & 5y^2 & -\frac{10y}{3} & +6 \\
 & & & - & & \\
 & & & 5y^2 & -\frac{10y}{3} & \\
\hline
 & & & & 0 & 6
\end{array}
$$

Let us verify, Dividend = Divisor × Quotient + Remainder

$15y^4 – 16y^3 + 9y^2 – 10/3y + 6 = (3y – 2) \times (5y^3 – 2y^2 + 5y/3) + 6$

$= 15y^4 – 6y^3 + 5y^2 – 10y^3 + 4y^2 – 10y/3 + 6$

$= 15y^4 – 16y^3 + 9y^2 – 10/3y + 6$

Hence, verified.

∴ Quotient is $5y^3 – 2y^2 + 5y/3$ and the Remainder is 6.

(vi) Dividend divisor

$4y^3 + 8y + 8y^2 + 7$ $2y^2 – y + 1$

$4y^3 + 8y^2 + 8y + 7$

By using long division method

$$
\begin{array}{r|rrrr}
 & 2y & +5 & & \\
\hline
2y^2-y+1 & 4y^3 & +8y^2 & +8y & +7 \\
- & 4y^3 & -2y^2 & +2y & \\
\hline
 & & 10y^2 & +6y & +7 \\
 & & - & & \\
 & & 10y^2 & -5y & +5 \\
\hline
 & & & 11y & +2
\end{array}
$$

Let us verify, Dividend = Divisor × Quotient + Remainder

$4y^3 + 8y^2 + 8y + 7 = (2y^2 – y + 1) \times (2y + 5) + 11y + 2$

$= 4y^3 + 10y^2 – 2y^2 – 5y + 2y + 5 + 11y + 2$
$= 4y^3 + 8y^2 + 8y + 7$

Hence, verified.

∴ Quotient is $2y + 5$ and the Remainder is $11y + 2$.

(vii) Dividend divisor

$6y^5 + 4y^4 + 4y^3 + 7y^2 + 27y + 6$ $2y^3 + 1$

By using long division method

$$
\begin{array}{r|rrrrrr}
 & 3y^2 & +2y & +2 & & & \\
\hline
2y^3+1 & 6y^5 & +4y^4 & +4y^3 & +7y^2 & +27y & +6 \\
- & 6y^5 & +0y^4 & +0y^3 & +3y^2 & & \\
\hline
 & & 4y^4 & +4y^3 & +4y^2 & +27y & +6 \\
 & - & 4y^4 & +0y^3 & +0y^2 & +2y & \\
\hline
 & & & 4y^3 & +4y^2 & +25y & +6 \\
 & & - & 4y^3 & +0y^2 & +0y & +2 \\
\hline
 & & & & 4y^2 & +25y & +4
\end{array}
$$

Let us verify, Dividend = Divisor × Quotient + Remainder

$6y^5 + 4y^4 + 4y^3 + 7y^2 + 27y + 6 = (2y^3 + 1) \times (3y^2 + 2y + 2) + 4y^2 + 25y + 4$
$= 6y^5 + 4y^4 + 4y^3 + 3y^2 + 2y + 2 + 4y^2 + 25y + 4$
$= 6y^5 + 4y^4 + 4y^3 + 7y^2 + 27y + 6$

Hence, verified.

∴ Quotient is $3y^2 + 2y + 2$ and the Remainder is $4y^2 + 25y + 4$.

22. Divide $15y^4+ 16y^3 + 10/3y – 9y^2 – 6$ by $3y - 2$ Write down the coefficients of the terms in the quotient.

Solution:

We have,

$(15y^4+ 16y^3 + 10/3y – 9y^2 – 6) / (3y – 2)$

By using long division method

$$
\begin{array}{r|llllll}
 & 5y^3 & +\frac{26y^2}{3} & +\frac{25y}{9} & +\frac{80}{27} & & \\
\hline
3y-2 & 15y^4 & +16y^3 & -9y^2 & +\frac{10y}{3} & -6 & \\
 & - & & & & & \\
 & 15y^4 & -10y^3 & & & & \\
\hline
 & & 26y^3 & -9y^2 & +\frac{10y}{3} & -6 & \\
 & & - & & & & \\
 & & 26y^3 & -\frac{52y^2}{3} & & & \\
\hline
 & & & \frac{25y^2}{3} & +\frac{10y}{3} & -6 & \\
 & & & - & & & \\
 & & & \frac{25y^2}{3} & -\frac{50y}{9} & & \\
\hline
 & & & & \frac{80y}{9} & -6 & \\
 & & & & - & & \\
 & & & & \frac{80y}{9} & -\frac{160}{27} & \\
\hline
 & & & & & -\frac{2}{27} &
\end{array}
$$

∴ Quotient is $5y^3 + 26y^2/3 + 25y/9 + 80/27$
So the coefficients of the terms in the quotient are:
Coefficient of $y^3 = 5$
Coefficient of $y^2 = 26/3$
Coefficient of $y = 25/9$
Constant term = $80/27$

23. Using division of polynomials state whether
(i) $x + 6$ is a factor of $x^2 – x - 42$
(ii) $4x - 1$ is a factor of $4x^2 – 13x - 12$
(iii) $2y - 5$ is a factor of $4y^4 – 10y^3 – 10y^2 + 30y – 15$
(iv) $3y^2 + 5$ is a factor of $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$
(v) $z^2 + 3$ is a factor of $z^5 – 9z$
(vi) $2x^2 – x + 3$ is a factor of $6x^5 – x^4 + 4x^3 – 5x^2 – x – 15$
Solution:
(i) $x + 6$ is a factor of $x^2 – x - 42$
Firstly let us perform long division method

$$\begin{array}{r|lll}
 & x & -7 & \\
\hline
x+6 & x^2 & -x & -42 \\
 & - & & \\
 & x^2 & +6x & \\
\hline
 & & -7x & -42 \\
 & & - & \\
 & & -7x & -42 \\
\hline
 & & & 0
\end{array}$$

Since the remainder is 0, we can say that x + 6 is a factor of $x^2 – x – 42$

(ii) 4x - 1 is a factor of $4x^2 – 13x - 12$
Firstly let us perform long division method

$$\begin{array}{r|lll}
 & x & -3 & \\
\hline
4x-1 & 4x^2 & -13x & -12 \\
 & - & & \\
 & 4x^2 & -x & \\
\hline
 & & -12x & -12 \\
 & & - & \\
 & & -12x & +3 \\
\hline
 & & & -15
\end{array}$$

Since the remainder is -15, 4x - 1 is not a factor of $4x^2 – 13x - 12$

(iii) 2y - 5 is a factor of $4y^4 – 10y^3 – 10y^2 + 30y – 15$
Firstly let us perform long division method

$$\begin{array}{r|lllll}
 & 2y^3 & -5y & +\frac{5}{2} & & \\
\hline
2y-5 & 4y^4 & -10y^3 & -10y^2 & +30y & -15 \\
 & - & & & & \\
 & 4y^4 & -10y^3 & & & \\
\hline
 & & 0 & -10y^2 & +30y & -15 \\
 & & - & & & \\
 & & -10y^3 & +25y^2 & & \\
\hline
 & & 10y^3 & -35y^2 & +30y & -15 \\
 & & - & & & \\
 & & 5y^3 & -\frac{25y^2}{2} & & \\
\hline
 & & 5y^3 & -\frac{45y^2}{2} & +30y & -15
\end{array}$$

Since the remainder is $5y^3 - 45y^2/2 + 30y - 15$, $2y - 5$ is not a factor of $4y^4 - 10y^3 - 10y^2 + 30y - 15$

(iv) $3y^2 + 5$ is a factor of $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$
Firstly let us perform long division method

$$
\begin{array}{r|rrrrrr}
 & 2y^3 & +5y^2 & +2y & -7 & & \\
\hline
3y^2+5 & 6y^5 & +15y^4 & +16y^3 & +4y^2 & +10y & -35 \\
 - & 6y^5 & +0y^4 & +10y^3 & & & \\
\hline
 & & 15y^4 & +6y^3 & +4y^2 & +10y & -35 \\
 - & & 15y^4 & +0y^3 & +25y^2 & & \\
\hline
 & & & 6y^3 & -21y^2 & +10y & -35 \\
 - & & & 6y^3 & +0y^2 & +10y & \\
\hline
 & & & & -21y^2 & +0y & -35 \\
 - & & & & -21y^2 & +0y & -35 \\
\hline
 & & & & & & 0
\end{array}
$$

Since the remainder is 0, $3y^2 + 5$ is a factor of $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$

(v) $z^2 + 3$ is a factor of $z^5 - 9z$
Firstly let us perform long division method

$$
\begin{array}{r|rrrrrr}
 & z^3 & -3z & & & & \\
\hline
z^2+3 & z^5 & +0z^4 & +0z^3 & +0z^2 & -9z & +0 \\
 - & z^5 & +0z^4 & +3z^3 & & & \\
\hline
 & & & -3z^3 & +0z^2 & -9z & +0 \\
 - & & & -3z^3 & +0z^2 & -9z & \\
\hline
 & & & & & 0 & 0
\end{array}
$$

Since the remainder is 0, $z^2 + 3$ is a factor of $z^5 - 9z$

(vi) $2x^2 - x + 3$ is a factor of $6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$
Firstly let us perform long division method

$$
\begin{array}{r|rrrrrr}
 & 3x^3 & +x^2 & -2x & -5 & & \\
\hline
2x^2 - x + 3 & 6x^5 & -x^4 & +4x^3 & -5x^2 & -x & -15 \\
 & - & & & & & \\
 & 6x^5 & -3x^4 & +9x^3 & & & \\
\hline
 & & 2x^4 & -5x^3 & -5x^2 & -x & -15 \\
 & & - & & & & \\
 & & 2x^4 & -x^3 & +3x^2 & & \\
\hline
 & & & -4x^3 & -8x^2 & -x & -15 \\
 & & & - & & & \\
 & & & -4x^3 & +2x^2 & -6x & \\
\hline
 & & & & -10x^2 & +5x & -15 \\
 & & & & - & & \\
 & & & & -10x^2 & +5x & -15 \\
\hline
 & & & & & & 0
\end{array}
$$

Since the remainder is 0, $2x^2 - x + 3$ is a factor of $6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

24. Find the value of a, if x + 2 is a factor of $4x^4 + 2x^3 - 3x^2 + 8x + 5a$
Solution:
We know that x + 2 is a factor of $4x^4 + 2x^3 - 3x^2 + 8x + 5a$
Let us equate x + 2 = 0
x = -2
Now let us substitute x = -2 in the equation $4x^4 + 2x^3 - 3x^2 + 8x + 5a$
$4(-2)^4 + 2(-2)^3 - 3(-2)^2 + 8(-2) + 5a = 0$
64 – 16 – 12 – 16 + 5a = 0
20 + 5a = 0
5a = -20
a = -20/5
= -4

25. What must be added to $x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$.
Solution:
Firstly let us perform long division method

$$\begin{array}{r|rrrrr} & x^2 & & +1 & & \\ \cline{2-6} x^2+2x-3 & x^4 & +2x^3 & -2x^2 & +x & -1 \\ & - & & & & \\ & x^4 & +2x^3 & -3x^2 & & \\ \cline{2-6} & & & x^2 & +x & -1 \\ & & & - & & \\ & & & x^2 & +2x & -3 \\ \cline{2-6} & & & & -x & +2 \end{array}$$

By long division method we got remainder as $-x + 2$,

$\therefore$ $x - 2$ has to be added to $x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$.

EXERCISE 8.5 PAGE NO: 8.15

1. Divide the first polynomial by the second polynomial in each of the following. Also, write the quotient and remainder:
(i) $3x^2 + 4x + 5$, $x - 2$
(ii) $10x^2 – 7x + 8$, $5x - 3$
(iii) $5y^3 – 6y^2 + 6y – 1$, $5y – 1$
(iv) $x^4 – x^3 + 5x$, $x - 1$
(v) $y^4 + y^2$, $y^2 – 2$

Solution:

(i) $3x^2 + 4x + 5$, $x - 2$

By using long division method

$$
\begin{array}{r|rrr}
 & 3x & +10 & \\
\hline
x-2 & 3x^2 & +4x & +5 \\
- & 3x^2 & -6x & \\
\hline
 & & 10x & +5 \\
 & - & & \\
 & & 10x & -20 \\
\hline
 & & & 25
\end{array}
$$

∴ the Quotient is $3x + 10$ and the Remainder is 25.

(ii) $10x^2 – 7x + 8$, $5x - 3$

By using long division method

$$
\begin{array}{r|rrr}
 & 2x & -\frac{1}{5} & \\
\hline
5x-3 & 10x^2 & -7x & +8 \\
- & 10x^2 & -6x & \\
\hline
 & & -x & +8 \\
 & - & & \\
 & & -x & +\frac{3}{5} \\
\hline
 & & & \frac{37}{5}
\end{array}
$$

∴ the Quotient is $2x – 1/5$ and the Remainder is $37/5$.

(iii) $5y^3 - 6y^2 + 6y - 1,\ 5y - 1$
By using long division method

$$
\begin{array}{l|lllll}
 & & y^2 & -y & +1 & \\
\hline
5y-1 & & 5y^3 & -6y^2 & +6y & -1 \\
 & - & & & & \\
 & & 5y^3 & -y^2 & & \\
\hline
 & & & -5y^2 & +6y & -1 \\
 & & - & & & \\
 & & & -5y^2 & +y & \\
\hline
 & & & & 5y & -1 \\
 & & & - & & \\
 & & & & 5y & -1 \\
\hline
 & & & & & 0
\end{array}
$$

∴ the Quotient is $y^2 - y + 1$ and the Remainder is 0.

(iv) $x^4 - x^3 + 5x,\ x - 1$
By using long division method

$$
\begin{array}{l|llllll}
 & & x^3 & +5 & & & \\
\hline
x-1 & & x^4 & -x^3 & +0x^2 & +5x & +0 \\
 & - & & & & & \\
 & & x^4 & -x^3 & & & \\
\hline
 & & & 0 & +0x^2 & +5x & +0 \\
 & & - & & & & \\
 & & & 5x^3 & -5x^2 & & \\
\hline
 & & & -5x^3 & +5x^2 & +5x & +0
\end{array}
$$

∴ the Quotient is $x^3 + 5$ and the Remainder is 5.

(v) $y^4 + y^2,\ y^2 - 2$
By using long division method

$$
\begin{array}{r|l}
 & y^2 \quad +3 \\
\cline{2-2}
y^2-2 & y^4 \quad +0y^3 \quad +y^2 \quad +0y \quad +0 \\
 & - \\
 & y^4 \quad +0y^3 \quad -2y^2 \\
\cline{2-2}
 & \qquad\qquad\quad 3y^2 \quad +0y \quad +0 \\
 & \qquad\qquad\quad - \\
 & \qquad\qquad\quad 3y^2 \quad +0y \quad -6 \\
\cline{2-2}
 & \qquad\qquad\qquad\qquad\qquad\quad 6
\end{array}
$$

∴ the Quotient is $y^2 + 3$ and the Remainder is 6.

2. Find Whether or not the first polynomial is a factor of the second:
(i) $x + 1$, $2x^2 + 5x + 4$
(ii) $y – 2$, $3y^3 + 5y^2 + 5y + 2$
(iii) $4x^2 – 5$, $4x^4 + 7x^2 + 15$
(iv) $4 – z$, $3z^2 – 13z + 4$
(v) $2a – 3$, $10a^2 – 9a - 5$
(vi) $4y + 1$, $8y^2 – 2y + 1$
Solution:
(i) $x + 1$, $2x^2 + 5x + 4$
Let us perform long division method,

$$
\begin{array}{r|l}
 & 2x \quad +3 \\
\cline{2-2}
x+1 & 2x^2 \quad +5x \quad +4 \\
 & - \\
 & 2x^2 \quad +2x \\
\cline{2-2}
 & \qquad\quad 3x \quad +4 \\
 & \qquad\quad - \\
 & \qquad\quad 3x \quad +3 \\
\cline{2-2}
 & \qquad\qquad\quad 1
\end{array}
$$

Since remainder is 1 therefore the first polynomial is not a factor of the second polynomial.

(ii) $y – 2$, $3y^3 + 5y^2 + 5y + 2$
Let us perform long division method,

$$\begin{array}{rrrrr}
 & 3y^2 & +11y & +27 & \\
\cline{2-5}
y-2 \;\big) & 3y^3 & +5y^2 & +5y & +2 \\
- & 3y^3 & -6y^2 & & \\
\hline
 & & 11y^2 & +5y & +2 \\
 & - & 11y^2 & -22y & \\
\hline
 & & & 27y & +2 \\
 & & - & 27y & -54 \\
\hline
 & & & & 56
\end{array}$$

Since remainder is 56 therefore the first polynomial is not a factor of the second polynomial.

(iii) $4x^2 - 5$, $4x^4 + 7x^2 + 15$
Let us perform long division method,

$$\begin{array}{rrrrrr}
 & x^2 & +3 & & & \\
\cline{2-6}
4x^2-5 \;\big) & 4x^4 & +0x^3 & +7x^2 & +0x & +15 \\
- & 4x^4 & +0x^3 & -5x^2 & & \\
\hline
 & & & 12x^2 & +0x & +15 \\
 & & - & 12x^2 & +0x & -15 \\
\hline
 & & & & & 30
\end{array}$$

Since remainder is 30 therefore the first polynomial is not a factor of the second polynomial.

(iv) $4 - z$, $3z^2 - 13z + 4$
Let us perform long division method,

$$\begin{array}{r|rrr} & -3z & +1 & \\ \hline -z+4 & 3z^2 & -13z & +4 \\ & - & & \\ & 3z^2 & -12z & \\ \hline & & -z & +4 \\ & & - & \\ & & -z & +4 \\ \hline & & & 0 \end{array}$$

Since remainder is 0 therefore the first polynomial is a factor of the second polynomial.

(v) $2a – 3$, $10a^2 – 9a - 5$

Let us perform long division method,

$$\begin{array}{r|rrr} & 5a & +3 & \\ \hline 2a-3 & 10a^2 & -9a & -5 \\ & - & & \\ & 10a^2 & -15a & \\ \hline & & 6a & -5 \\ & & - & \\ & & 6a & -9 \\ \hline & & & 4 \end{array}$$

Since remainder is 4 therefore the first polynomial is not a factor of the second polynomial.

(vi) $4y + 1$, $8y^2 – 2y + 1$

Let us perform long division method,

$$\begin{array}{r|l} & 2y \quad -1 \\ \hline 4y+1 & 8y^2 \quad -2y \quad +1 \\ & -\\ & 8y^2 \quad +2y \\ \hline & \qquad -4y \quad +1 \\ & \qquad - \\ & \qquad -4y \quad -1 \\ \hline & \qquad\qquad 2 \end{array}$$

Since remainder is 2 therefore the first polynomial is not a factor of the second polynomial.

EXERCISE 8.6 PAGE NO: 8.17

Divide:

1. $x^2 - 5x + 6$ by $x - 3$
Solution:
We have,
$(x^2 - 5x + 6) / (x - 3)$
Let us perform long division method,

$$\begin{array}{r|llll} & x & -2 & & \\ \hline x-3 & x^2 & -5x & +6 & \\ & - & & & \\ & x^2 & -3x & & \\ \hline & & -2x & +6 & \\ & & - & & \\ & & -2x & +6 & \\ \hline & & & 0 & \end{array}$$

∴ the Quotient is $x - 2$

2. $ax^2 - ay^2$ by $ax+ay$
Solution:
We have,
$(ax^2 - ay^2)/ (ax+ay)$
$(ax^2 - ay^2)/ (ax+ay) = (x - y) + 0/(ax+ay)$
$= (x - y)$
∴ the answer is $(x - y)$

3. $x^4 - y^4$ by $x^2 - y^2$
Solution:
We have,
$(x^4 - y^4)/ (x^2 - y^2)$
$(x^4 - y^4)/ (x^2 - y^2) = x^2 + y^2 + 0/(x^2 - y^2)$
$= x^2 + y^2$
∴ the answer is $(x^2 + y^2)$

4. $acx^2 + (bc + ad)x + bd$ by $(ax + b)$

Solution:
We have,
$(acx^2 + (bc + ad)\ x + bd) / (ax + b)$
$(acx^2 + (bc + ad)\ x + bd) / (ax + b) = cx + d + 0/ (ax + b)$
$= cx + d$
∴ the answer is (cx + d)

5. $(a^2 + 2ab + b^2) – (a^2 + 2ac + c^2)$ by 2a + b + c
Solution:
We have,
$[(a^2 + 2ab + b^2) – (a^2 + 2ac + c^2)] / (2a + b + c)$
$[(a^2 + 2ab + b^2) – (a^2 + 2ac + c^2)] / (2a + b + c) = b – c + 0/(2a + b + c)$
$= b – c$
∴ the answer is (b – c)

6. $1/4x^2 – 1/2x – 12$ by $1/2x – 4$
Solution:
We have,
$(1/4x^2 – 1/2x – 12) / (1/2x – 4)$
Let us perform long division method,

$$\begin{array}{r|lll} & \frac{x}{2} & +3 & \\ \hline \frac{x}{2}-4 & \frac{x^2}{4} & -\frac{x}{2} & +0 \\ & - & & \\ & \frac{x^2}{4} & -2x & \\ \hline & & \frac{3x}{2} & +0 \\ & & - & \\ & & \frac{3x}{2} & -12 \\ \hline & & & 12 \end{array}$$

∴ the Quotient is x/2 + 3