

Exercise 14.3

Page No: 14.28

1. Find the coordinates of the point which divides the line segment joining (-1, 3) and (4, -7) internally in the ratio 3: 4.

Solution:

Let P(x, y) be the required point.

By section formula, we know that the coordinates are

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$y = \frac{my_2 + ny_1}{m + n}$$

Here,

$$x_1 = -1, y_1 = 3$$

$$x_2 = 4, y_2 = -7$$

$$m : n = 3 : 4$$

Then,

$$x = \frac{3 \times 4 + 4 \times (-1)}{3 + 4} \times 3$$

$$x = \frac{3 \times 4 + 4 \times (-1)}{3 + 4}$$

$$x = \frac{12 - 4}{7}$$

$$x = \frac{8}{7}$$

$$y = \frac{3 \times (-7) + 4 \times 3}{3 + 4}$$

$$y = \frac{-21 + 12}{7}$$

$$y = \frac{-9}{7}$$

Therefore, the coordinates of P are $(\frac{8}{7}, -\frac{9}{7})$

2. Find the points of trisection of the line segment joining the points:

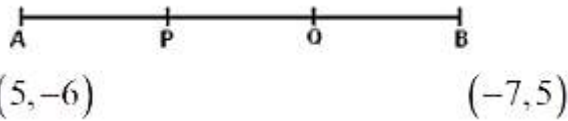
(i) (5, -6) and (-7, 5)

(ii) (3, -2) and (-3, -4)

(iii) (2, -2) and (-7, 4)

Solution:

(i) Let P and Q be the point of trisection of AB such that $AP = PQ = QB$



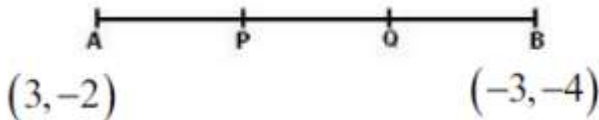
So, P divides AB internally in the ratio of 1: 2, thereby applying section formula, the coordinates of P will be

$$\left(\frac{2(-7) + 1(5)}{2 + 1}, \frac{2(5) + 1(-6)}{2 + 1} \right) \text{ i. e., } \left(1, -\frac{7}{3} \right)$$

Now, Q also divides AB internally in the ratio of 2:1 so their coordinates will be

$$\left(\frac{2(-7) + 1(5)}{2 + 1}, \frac{2(5) + 1(-6)}{2 + 1} \right) \text{ i. e., } \left(-3, \frac{4}{3} \right)$$

(ii) Let P and Q be the points of trisection of AB such that $AP = PQ = QB$



As, P divides AB internally in the ratio of 1: 2. Hence by applying section formula, the coordinates of P are

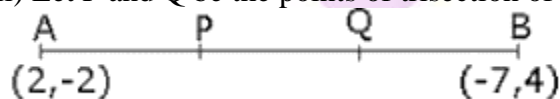
$$\left(\frac{1(-3) + 2(3)}{1 + 2}, \frac{1(-4) + 2(-2)}{1 + 2} \right) \text{ i. e., } \left(1, -\frac{8}{3} \right)$$

Now, Q also divides as internally in the ratio of 2: 1

So, the coordinates of Q are given by

$$\left(\frac{2(-3) + 1(3)}{2 + 1}, \frac{2(-4) + 1(-2)}{2 + 1} \right) \text{ i. e., } \left(-1, -\frac{10}{3} \right)$$

(iii) Let P and Q be the points of trisection of AB such that $AP = PQ = OQ$



As, P divides AB internally in the ratio 1:2. So, the coordinates of P, by applying the section formula, are given by

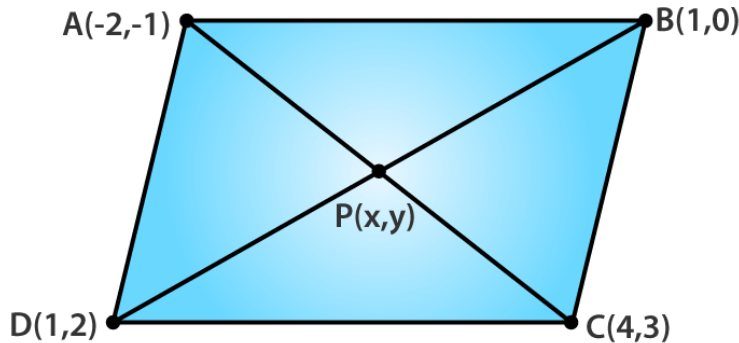
$$\left(\frac{1(-7) + 2(2)}{1 + 2}, \frac{1(4) + 2(-2)}{1 + 2} \right), \text{ i. e., } (-1, 0)$$

Now, Q also divides AB internally in the ration 2: 1. And the coordinates of Q are given by

$$\left(\frac{2(-7) + 1(2)}{2 + 1}, \frac{2(4) + 1(2)}{2 + 1} \right), \text{ i. e., } (-4, 2)$$

3. Find the coordinates of the point where the diagonals of the parallelogram formed by joining the points $(-2, -1)$, $(1, 0)$, $(4, 3)$ and $(1, 2)$ meet.

Solution:



Let $A(-2, -1)$, $B(1, 0)$, $C(4, 3)$ and $D(1, 2)$ be the given points.

Let $P(x, y)$ be the point of intersection of the diagonals of the parallelogram formed by the given points.

We know that, diagonals of a parallelogram bisect each other.

$$x = \frac{-2 + 4}{2}$$

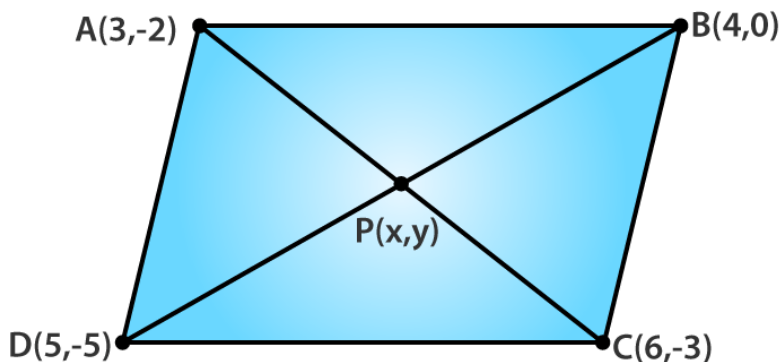
$$\Rightarrow x = \frac{2}{2} = 1$$

$$y = \frac{-1 + 3}{2} = \frac{2}{2} = 1$$

Therefore, the coordinates of P are $(1, 1)$

4. Prove that the points $(3, 2)$, $(4, 0)$, $(6, -3)$ and $(5, -5)$ are the vertices of a parallelogram.

Solution:



Let $A(3, -2)$, $B(4, 0)$, $C(6, -3)$ and $D(5, -5)$

Let $P(x, y)$ be the point of intersection of diagonals AC and BD of ABCD.

The mid-point of AC is given by,

$$x = \frac{3 + 6}{2} = \frac{9}{2}$$

$$y = \frac{-2 - 3}{2} = \frac{-5}{2}$$

Mid - point of AC $\left(\frac{9}{2}, \frac{-5}{2}\right)$

Again, the mid-point of BD is given by,

$$x = \frac{5 + 4}{2} = \frac{9}{2}$$

$$y = \frac{-5 + 0}{2} = \frac{-5}{2}$$

Thus, we can conclude that diagonals AC and BD bisect each other.

And, we know that diagonals of a parallelogram bisect each other

Therefore, ABCD is a parallelogram.

5. If P(9a - 2, -b) divides the line segment joining A(3a + 1, -3) and B(8a, 5) in the ratio 3 : 1, find the values of a and b.

Solution:

Given that, P(9a - 2, -b) divides the line segment joining A(3a + 1, -3) and B(8a, 5) in the ratio 3:1

Then, by section formula

Coordinates of P are

$$9a - 2 = \frac{3(8a) + 1(3a + 1)}{3 + 1}$$

And,

$$-b = \frac{3(5) + 1(-3)}{3 + 1}$$

Solving for a, we have

$$(9a - 2) \times 4 = 24a + 3a + 1$$

$$36a - 8 = 27a + 1$$

$$9a = 9$$

$$a = 1$$

Now, solving for b, we have

$$4 \times -b = 15 - 3$$

$$-4b = 12$$

$$b = -3$$

Therefore, the values of a and b are 1 and -3 respectively.

6. If (a, b) is the mid-point of the line segment joining the points A (10, -6), B(k, 4) and a - 2b = 18, find the value of k and the distance AB.

Solution:

As (a, b) is the mid-point of the line segment A(10, -6) and B(k, 4)

So,

$$(a, b) = (10 + k / 2, -6 + 4 / 2)$$

$$a = (10 + k) / 2 \quad \text{and} \quad b = -1$$

$$2a = 10 + k$$

$$k = 2a - 10$$

Given, $a - 2b = 18$

Using $b = -1$ in the above relation we get,

$$a - 2(-1) = 18$$

$$a = 18 - 2 = 16$$

So,

$$k = 2(16) - 10 = 32 - 10 = 22$$

Thus,

$$AB = \sqrt{[(22 - 10)^2 + (4 + 6)^2]} = \sqrt{[(12)^2 + (10)^2]} = \sqrt{[144 + 100]} = 2\sqrt{61} \text{ units}$$

7. Find the ratio in which the point (2, y) divides the line segment joining the points A(-2, 2) and B(3, 7). Also find the value of y.

Solution:

Let the point P(2, y) divide the line segment joining the points A(-2, 2) and B(3, 7) in the ratio k: 1

Then, the coordinates of P are given by

$$\left[\frac{3k + (-2) \times 1}{k + 1}, \frac{7k + 2 \times 1}{k + 1} \right]$$

$$= \left[\frac{3k - 2}{k + 1}, \frac{7k + 2}{k + 1} \right]$$

And, given the coordinates of P are (2, y)

So,

$$2 = (3k - 2) / (k + 1) \quad \text{and} \quad y = (7k + 2) / (k + 1)$$

Solving for k, we get

$$2(k + 1) = (3k - 2)$$

$$2k + 2 = 3k - 2$$

$$k = 4$$

Using k to find y, we have

$$y = (7(4) + 2) / (4 + 1)$$

$$= (28 + 2) / 5$$

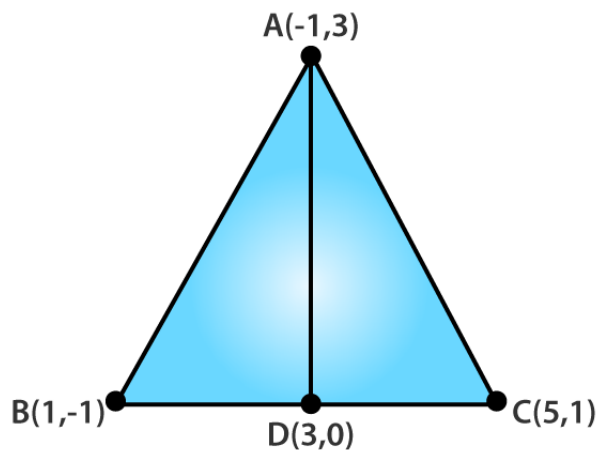
$$= 30 / 5$$

$$y = 6$$

Therefore, the ratio is 4: 1 and $y = 6$

8. If A(-1, 3), B(1, -1) and C(5, 1) are the vertices of a triangle ABC, find the length of median through A.

Solution:



Let AD be the median through A.

As, AD is the median, D is the mid-point of BC

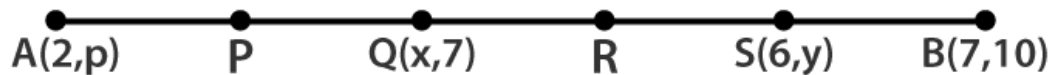
So, the coordinates of D are $(\frac{1+5}{2}, \frac{-1+1}{2}) = (3, 0)$

Therefore,

Length of median AD = $\sqrt{[(3+1)^2 + (0-3)^2]} = \sqrt{[(4)^2 + (-3)^2]} = \sqrt{[16+9]} = \sqrt{25} = 5$ units

9. If the points P, Q(x, 7), R, S(6, y) in this order divide the line segment joining A(2, p) and B (7, 10) in 5 equal parts, find x, y and p.

Solution:



From question, we have

$AP = PQ = QR = RS = SB$

So, Q is the mid-point of A and S

Then,

$$x = \frac{2+6}{2} = \frac{8}{2} = 4$$

$$7 = \frac{y+p}{2}$$

$$y+p = 14 \dots\dots (1)$$

Now, since S divides QB in the ratio 2: 1

$$y = \frac{2 \times 10 + 1 \times 7}{2+1} \Rightarrow \frac{20+7}{3} = \frac{27}{3} = 9$$

From (i), $y+p = 14 \Rightarrow 9+p = 14$

So, $p = 14 - 9 = 5$

Therefore, $x = 4$, $y = 9$ and $p = 5$

10. If a vertex of a triangle be (1, 1) and the middle points of the sides through it be (-2, 3) and (5, 2) find the other vertices.

Solution:

Let A(1, 1) be the given vertex and D(-2, 3), E(5, 2) be the mid-points of AB and AC
Now, as D and E are the mid-points of AB and AC

$$\frac{x_1 + 1}{2} = -2, \quad \frac{y_1 + 1}{2} = 3$$

$$x_1 + 1 = -4 \quad \Rightarrow y_1 + 1 = 6$$

$$x_1 = -5 \quad \Rightarrow y_1 = 5$$

So, the coordinates of B are (-5, 5)

Again,

$$\frac{x_2 + 1}{2} = 5, \quad \frac{y_2 + 1}{2} = 2$$

$$x_2 + 1 = 10 \quad \Rightarrow y_2 + 1 = 4$$

$$x_2 = 9 \quad \Rightarrow y_2 = 3$$

So, the coordinates of C are (9, 3)

Therefore, the other vertices of the triangle are (-5, 5) and (9, 3).

11. (i) In what ratio is the segment joining the points (-2, -3) and (3, 7) divides by the y-axis? Also, find the coordinates of the point of division.

Solution:

Let P(-2, -3) and Q(9, 3) be the given points.

Suppose y-axis divides PQ in the ratio k: 1 at R(0, y)

So, the coordinates of R are given by

$$\left[\frac{3k + (-2) \times 1}{k + 1}, \frac{7k + (-3) \times 1}{k + 1} \right]$$

Now, equating

$$\frac{3k + (-2) \times 1}{k + 1} = 0$$

$$3k - 2 = 0$$

$$k = 2/3$$

Therefore, the ratio is 2: 3

Putting k = 2/3 in the coordinates of R, we get

R (0, 1)

(ii) In what ratio is the line segment joining (-3, -1) and (-8, -9) divided at the point (-5, -21/5)?

Solution:

Let A(-3, -1) and B(-8, -9) be the given points.

And, let P be the point that divides AB in the ratio k: 1

So, the coordinates of P are given by

$$\left[\frac{-8k - 3}{k + 1}, \frac{-9k - 1}{k + 1} \right]$$

But, given the coordinates of P

On equating, we get

$$(-8k - 3) / (k + 1) = -5$$

$$-8k - 3 = -5k - 5$$

$$3k = 2$$

$$k = 2/3$$

Thus, the point P divides AB in the ratio 2: 3

12. If the mid-point of the line joining (3, 4) and (k, 7) is (x, y) and $2x + 2y + 1 = 0$ find the value of k.

Solution:

As (x, y) is the mid-point

$$x = (3 + k) / 2 \text{ and } y = (4 + 7) / 2 = 11/2$$

Also,

Given that the mid-point lies on the line $2x + 2y + 1 = 0$

$$2[(3 + k) / 2] + 2(11/2) + 1 = 0$$

$$3 + k + 11 + 1 = 0$$

$$\text{Thus, } k = -15$$

13. Find the ratio in which the point P(3/4, 5/12) divides the line segments joining the point A(1/2, 3/2) and B(2, -5).

Solution:

Given,

Points A(1/2, 3/2) and B(2, -5)

Let the point P(3/4, 5/12) divide the line segment AB in the ratio k: 1

Then, we know that

$$P(3/4, 5/12) = (2k + 1/2) / (k + 1), (2k + 3/2) / (k + 1)$$

Now, equating the abscissa we get

$$3/4 = (2k + 1/2) / (k + 1)$$

$$3(k + 1) = 4(2k + 1/2)$$

$$3k + 3 = 8k + 2$$

$$5k = 1$$

$$k = 1/5$$

Therefore, the ratio in which the point P(3/4, 5/12) divides is 1: 5

14. Find the ratio in which the line joining (-2, -3) and (5, 6) is divided by (i) x-axis (ii) y-axis. Also, find the coordinates of the point of division in each case.

Solution:

Let A(-2, -3) and B(5, 6) be the given points.

(i) Suppose x-axis divides AB in the ratio k: 1 at the point P

Then, the coordinates of the point of division are

$$\left[\frac{5k - 2}{k + 1}, \frac{6k - 3}{k + 1} \right]$$

As, P lies in the x-axis, the y – coordinate is zero.

So,

$$6k - 3/k + 1 = 0$$

$$6k - 3 = 0$$

$$k = \frac{1}{2}$$

Thus, the required ratio is 1: 2

Using k in the coordinates of P

We get, P (1/3, 0)

(ii) Suppose y-axis divides AB in the ratio k: 1 at point Q

The, the coordinates of the point of division is given by

$$\left[\frac{5k - 2}{k + 1}, \frac{6k - 3}{k + 1} \right]$$

As, Q lies on the y-axis, the x – ordinate is zero.

So,

$$5k - 2/k + 1 = 0$$

$$5k - 2 = 0$$

$$k = \frac{2}{5}$$

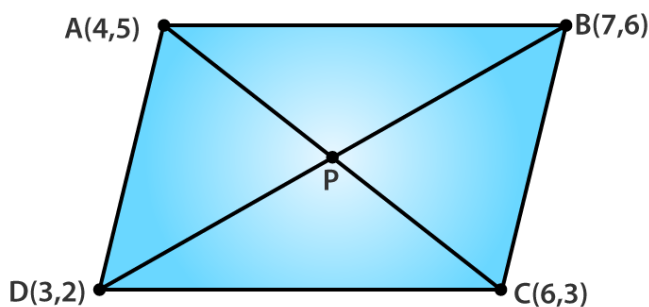
Thus, the required ratio is 2: 5

Using k in the coordinates of Q

We get, Q (0, -3/7)

15. Prove that the points (4, 5), (7, 6), (6, 3), (3, 2) are the vertices of a parallelogram. Is it a rectangle?

Solution:



Let A (4, 5), B(7, 6), C(6, 3) and D(3, 2) be the given points.

And, P be the point of intersection of AC and BD.

Coordinates of the mid-point of AC are $(\frac{4+6}{2}, \frac{5+3}{2}) = (5, 4)$

Coordinates of the mid-point of BD are $(\frac{7+3}{2}, \frac{6+2}{2}) = (5, 4)$

Thus, it's clearly seen that the mid-point of AC and BD are same.

So, ABCD is a parallelogram.

Now,

$$AC = \sqrt{[(6 - 4)^2 + (3 - 5)^2]} = \sqrt{[(2)^2 + (-2)^2]} = \sqrt{[4 + 4]} = \sqrt{8} \text{ units}$$

And,

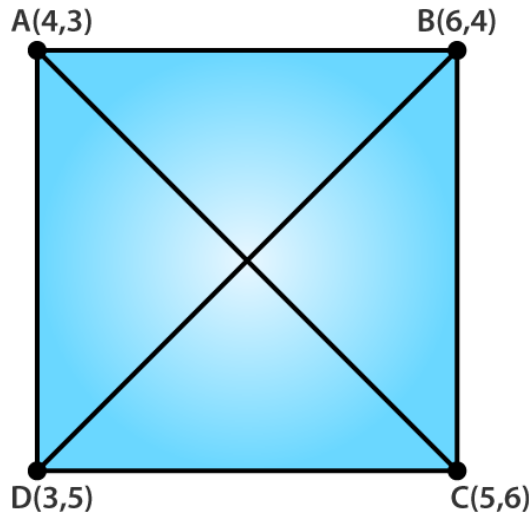
$$BD = \sqrt{[(7 - 3)^2 + (6 - 2)^2]} = \sqrt{[(4)^2 + (4)^2]} = \sqrt{[16 + 16]} = \sqrt{32} \text{ units}$$

Since, $AC \neq BD$

Therefore, ABCD is not a rectangle.

16. Prove that (4, 3), (6, 4), (5, 6) and (3, 5) are the angular points of a square.

Solution:



Let A(4,3), B(6,4), C(5,6) and D(3,5) be the given points.

The distance formula is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$AB = \sqrt{(4 - 6)^2 + (3 - 4)^2} = \sqrt{5}$$

$$BC = \sqrt{(6 - 5)^2 + (4 - 6)^2} = \sqrt{5}$$

$$CD = \sqrt{(5 - 3)^2 + (6 - 5)^2} = \sqrt{5}$$

$$AD = \sqrt{(4 - 3)^2 + (3 - 5)^2} = \sqrt{5}$$

It's seen that the length of all the sides are same.

Now, the length of diagonals are

$$AC = \sqrt{(4 - 5)^2 + (3 - 6)^2} = \sqrt{10}$$

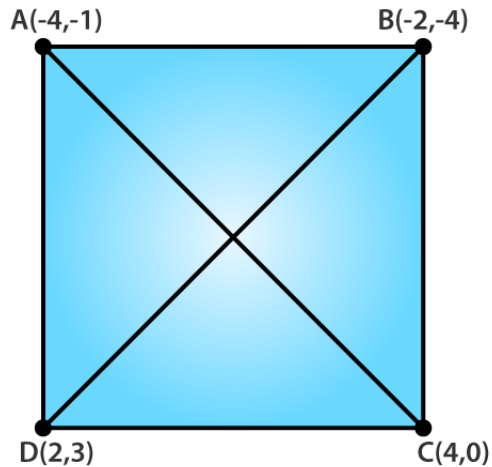
$$BD = \sqrt{(6 - 3)^2 + (4 - 5)^2} = \sqrt{10}$$

Also, the length of both the diagonals are same.

Therefore, we can conclude that the given points are the angular points of a square.

17. Prove that the points (-4, -1), (-2, -4), (4, 0) and (2, 3) are the vertices of a rectangle.

Solution:



Let A(-4, -1), B(-2, -4), C(4, 0) and D(2, 3) be the given points.

Now,

Coordinates of the mid-point of AC are $(-4 + 4/2, -1 + 0/2) = (0, -1/2)$

Coordinates of the mid-point of BD are $(-2 + 2/2, -4 + 3/2) = (0, -1/2)$

Thus, it's seen that AC and BD have the same point.

And, we have diagonals

$$AC = \sqrt{(4 + 4)^2 + (0 + 1)^2} = \sqrt{65}$$

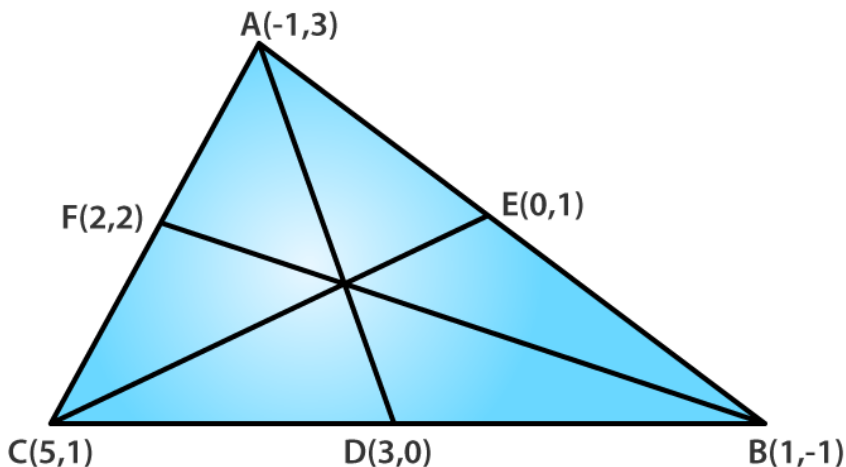
$$BD = \sqrt{(-2 - 2)^2 + (-4 - 3)^2} = \sqrt{65}$$

The length of diagonals are also same.

Therefore, the given points are the vertices of a rectangle.

18. Find the length of the medians of a triangle whose vertices are A(-1, 3), B(1, -1) and C(5, 1).

Solution:



Let AD, BF and CE be the medians of ΔABC

Coordinates of D are $(5 + 1/2, 1 - 1/2) = (3, 0)$

Coordinates of E are $(-1 + 1/2, 3 - 1/2) = (0, 1)$

Coordinates of F are $(5 - 1/2, 1 + 3/2) = (2, 2)$

Now,

Finding the length of the respectively medians:

$$\text{Length of } AD = \sqrt{(-1-3)^2 + (3-0)^2} = 5 \text{ units}$$

$$\text{Length of } BF = \sqrt{(2-1)^2 + (2+1)^2} = \sqrt{10} \text{ units}$$

$$\text{Length of } CE = \sqrt{(5-0)^2 + (1-1)^2} = 5 \text{ units}$$

19. Find the ratio in which the line segment joining the points A (3, -3) and B (-2, 7) is divided by x-axis. Also, find the coordinates of the point of division.

Solution:

Let the point on the x-axis be (x, 0). [y – coordinate is zero]

And, let this point divides the line segment AB in the ratio of k : 1.

Now using the section formula for the y-coordinate, we have

$$0 = (7k - 3)/(k + 1)$$

$$7k - 3 = 0$$

$$k = 3/7$$

Therefore, the line segment AB is divided by x-axis in the ratio 3: 7

20. Find the ratio in which the point P(x, 2) divides the line segment joining the points A (12, 5) and B (4, -3). Also, find the value of x.

Solution:

Let P divide the line joining A and B and let it divide the segment in the ratio k: 1

Now, using the section formula for the y – coordinate we have

$$2 = (-3k + 5)/(k + 1)$$

$$2(k + 1) = -3k + 5$$

$$2k + 2 = -3k + 5$$

$$5k = 3$$

$$k = 3/5$$

Thus, P divides the line segment AB in the ratio of 3: 5

Using value of k, we get the x – coordinate as

$$x = 12 + 60/8 = 72/8 = 9$$

Therefore, the coordinates of point P is (9, 2)

21. Find the ratio in which the point P(-1, y) lying on the line segment joining A(-3, 10) and B(6, -8) divides it. Also find the value of y.

Solution:

Let P divide A(-3, 10) and B(6, -8) in the ratio of k: 1

Given coordinates of P as (-1, y)

Now, using the section formula for x – coordinate we have

$$-1 = 6k - 3/k + 1$$

$$-(k + 1) = 6k - 3$$

$$7k = 2$$

$$k = 2/7$$

Thus, the point P divides AB in the ratio of 2: 7

Using value of k, to find the y-coordinate we have

$$y = (-8k + 10)/(k + 1)$$

$$y = (-8(2/7) + 10)/(2/7 + 1)$$

$$y = -16 + 70/2 + 7 = 54/9$$

$$y = 6$$

Therefore, the y-coordinate of P is 6

22. Find the coordinates of a point A, where AB is the diameter of circle whose center is (2, -3) and B is (1, 4).

Solution:

Let the coordinates of point A be (x, y)

If AB is the diameter, then the center is the mid-point of the diameter

So,

$$(2, -3) = (x + 1/2, y + 4/2)$$

$$2 = x + 1/2 \quad \text{and} \quad -3 = y + 4/2$$

$$4 = x + 1 \quad \text{and} \quad -6 = y + 4$$

$$x = 3 \quad \text{and} \quad y = -10$$

Therefore, the coordinates of A are (3, -10)

23. If the points (-2, 1), (1, 0), (x, 3) and (1, y) form a parallelogram, find the values of x and y.

Solution:

Let A(-2, 1), B(1, 0), C(x, 3) and D(1, y) be the given points of the parallelogram.

We know that the diagonals of a parallelogram bisect each other.

So, the coordinates of mid-point of AC = Coordinates of mid-point of BD

$$\left(\frac{x-2}{2}, \frac{3-1}{2}\right) = \left(\frac{1+1}{2}, \frac{y+0}{2}\right)$$

$$\left(\frac{x-2}{2}, 1\right) = \left(1, \frac{y}{2}\right)$$

$$\frac{x-2}{2} = 1 \quad \text{and} \quad \frac{y}{2} = 1$$

$$x - 2 = 2 \quad \Rightarrow y = 2$$

$$x = 4 \quad \Rightarrow y = 2$$

Therefore, the value of x is 4 and the value of y is 2.

24. The points A(2, 0), B(9, 1), C(11, 6) and D(4, 4) are the vertices of a quadrilateral ABCD. Determine whether ABCD is a rhombus or not.

Solution:

Given points are A(2, 0), B(9, 1), C(11, 6) and D(4, 4).

Coordinates of mid-point of AC are $(\frac{11+2}{2}, \frac{6+0}{2}) = (\frac{13}{2}, 3)$

Coordinates of mid-point of BD are $(\frac{9+4}{2}, \frac{1+4}{2}) = (\frac{13}{2}, \frac{5}{2})$

As the coordinates of the mid-point of AC \neq coordinates of mid-point of BD, ABCD is not even a parallelogram.

Therefore, ABCD cannot be a rhombus too.

25. In what ratio does the point (-4,6) divide the line segment joining the points A(-6,10) and B(3,-8)?

Solution:

Let the point (-4, 6) divide the line segment AB in the ratio k: 1.

So, using the section formula, we have

$$(-4, 6) = \left(\frac{3k - 6}{k + 1}, \frac{-8k + 10}{k + 1} \right)$$

$$-4 = \frac{3k - 6}{k + 1}$$

$$-4k - 4 = 3k - 6$$

$$7k = 2$$

$$k : 1 = 2 : 7$$

The same can be checked for the y-coordinate also.

Therefore, the ratio in which the point (-4, 6) divides the line segment AB is 2: 7

26. Find the ratio in which the y-axis divides the line segment joining the points (5, -6) and (-1, -4). Also find the coordinates of the point of division.

Solution:

Let P(5, -6) and Q(-1, -4) be the given points.

Let the y-axis divide the line segment PQ in the ratio k: 1

Then, by using section formula for the x-coordinate (as it's zero) we have

$$\frac{-k + 5}{k + 1} = 0$$

$$-k + 5 = 0$$

$$k = 5$$

Thus, the ratio in which the y-axis divides the given 2 points is 5: 1

Now, for finding the coordinates of the point of division

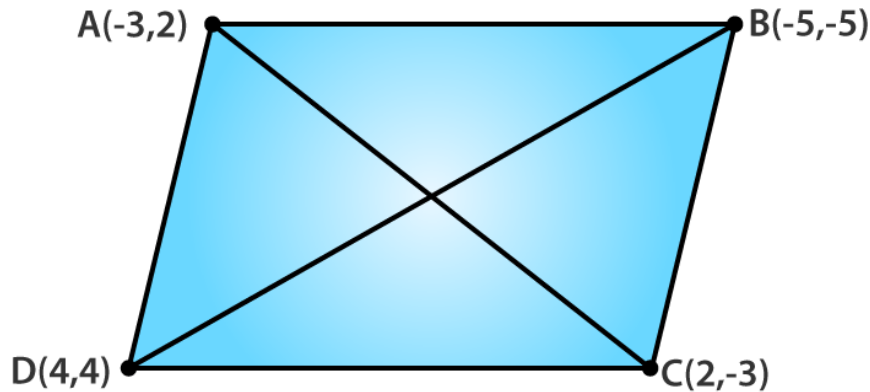
Putting k = 5, we get

$$\left(\frac{-5 + 5}{5 + 1}, \frac{-4 \times 5 - 6}{5 + 1} \right) = \left(0, \frac{-13}{3} \right)$$

Hence, the coordinates of the point of division are (0, -13/3)

27. Show that A(-3, 2), B(-5, 5), C(2, -3) and D(4, 4) are the vertices of a rhombus.

Solution:



Given points are A(-3, 2), B(-5, 5), C(2, -3) and D(4, 4)

Now,

Coordinates of the mid-point of AC are $(-3+2/2, 2-3/2) = (-1/2, -1/2)$

And,

Coordinates of mid-point of BD are $(-5+4/2, -5+4/2) = (-1/2, -1/2)$

Thus, the mid-point for both the diagonals are the same. So, ABCD is a parallelogram.

Next, the sides

$$AB = \sqrt{(-5+3)^2 + (-5-2)^2}$$

$$AB = \sqrt{4+49}$$

$$AB = \sqrt{53}$$

$$BC = \sqrt{(-5-2)^2 + (-5+3)^2}$$

$$BC = \sqrt{49+4}$$

$$BC = \sqrt{53}$$

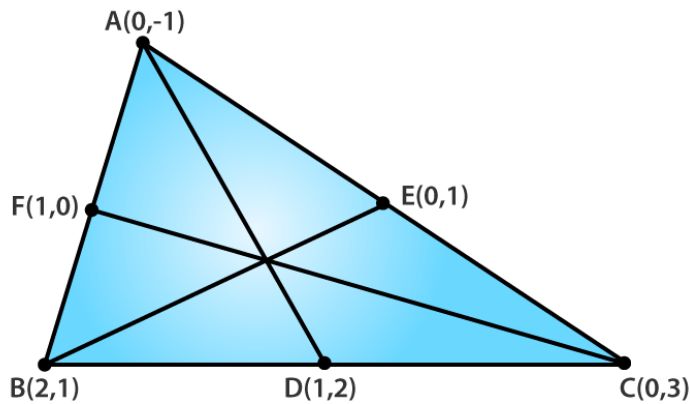
$$AB = BC$$

It's seen that ABCD is a parallelogram with adjacent sides equal.

Therefore, ABCD is a rhombus.

28. Find the lengths of the medians of a ΔABC having vertices at A(0, -1), B(2, 1) and C(0, 3).

Solution:



Let AD, BE and CF be the medians of ΔABC

Then,

Coordinates of D are $(\frac{2+0}{2}, \frac{1+3}{2}) = (1, 2)$

Coordinates of E are $(\frac{0+0}{2}, \frac{3-1}{2}) = (0, 1)$

Coordinates of F are $(\frac{2+0}{2}, \frac{1-1}{2}) = (1, 0)$

Now, the length of the medians

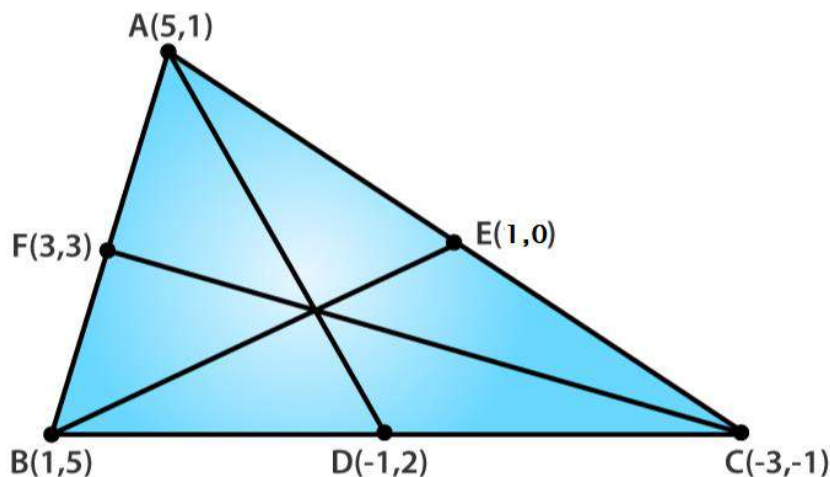
$$\text{Length of median } AD = \sqrt{(1-0)^2 + (2+1)^2} = \sqrt{10} \text{ units}$$

$$\text{Length of median } BE = \sqrt{(2-0)^2 + (1-1)^2} = 2 \text{ units}$$

$$\text{Length of median } CF = \sqrt{(1-0)^2 + (0-3)^2} = \sqrt{10} \text{ units}$$

29. Find the lengths of the median of a ΔABC having vertices at A(5, 1), B(1, 5) and C(-3, -1).

Solution:



Given vertices of ΔABC as A(5, 1), B(1, 5) and C(-3, -1).

Let AD, BE and CF be the medians

Coordinates of D are $(\frac{1-3}{2}, \frac{5-1}{2}) = (-1, 2)$

Coordinates of E are $(\frac{5-3}{2}, \frac{1-1}{2}) = (1, 0)$

Coordinates of F are $(5+1/2, 1+5/2) = (3, 3)$

Now, the length of the medians

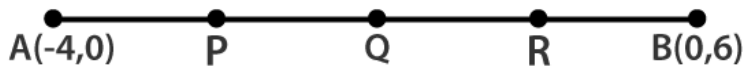
$$\text{Length of median } AD = \sqrt{(5+1)^2 + (1-2)^2} = \sqrt{37} \text{ units}$$

$$\text{Length of median } BE = \sqrt{(1-1)^2 + (5-0)^2} = 5 \text{ units}$$

$$\text{Length of median } CF = \sqrt{(3+3)^2 + (3+1)^2} = 2\sqrt{13} = \sqrt{52} \text{ units}$$

30. Find the coordinates of the point which divide the line segment joining the points $(-4, 0)$ and $(0, 6)$ in four equal parts.

Solution:



Let $A(-4, 0)$ and $B(0, 6)$ be the given points

And, let P, Q and R be the points which divide AB in four equal parts.

Now, we know that $AP: PB = 1: 3$

Using section formula the coordinates of P are

$$\left(\frac{1 \times 0 + 3(-4)}{1+3}, \frac{1 \times 6 + 3 \times 0}{1+3} \right) = \left(-3, \frac{3}{2} \right)$$

And, it's seen that Q is the mid-point of AB

So, the coordinates of Q are

$$\left(\frac{-4+0}{2}, \frac{0+6}{2} \right) = (-2, 3)$$

Finally, the ratio of $AR: BR$ is $3: 1$

Then by using section formula the coordinates of R are

$$\left(\frac{3 \times 0 + 1 \times (-4)}{3+1}, \frac{3 \times 6 + 1 \times 0}{3+1} \right) = \left(-1, \frac{9}{2} \right)$$