Exercise 14.4

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1. Find the centroid of the triangle whose vertices are:
(i) (1, 4), (-1, -1) and (3, -2) **(ii) (-2, 3), (2, -1) and (4, 0)**
Solution:

We know that the coordinates of the centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2), (x_3, y_3) are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y + y_2 + y_3}{3}\right)$$

(i) So, the coordinates of the centroid of a triangle whose vertices are (1, 4), (-1, -1) and (3, -2) are

$$\left(\frac{1 - 1 + 3}{3}, \frac{4 - 1 - 2}{3}\right)$$

(1, 1/3)
Thus, centroid of the triangle is (1, 1/3)

(ii) So, the coordinates of the centroid of a triangle whose vertices are (-2, 3), (2, -1) and (4, 0) are

$$\left(\frac{2 - 2 + 4}{3}, \frac{3 - 1 + 0}{3}\right)$$

(4/3, 2/3)
Thus, centroid of the triangle is (4/3, 2/3)

2. Two vertices of a triangle are (1, 2), (3, 5) and its centroid is at the origin. Find the coordinates of the third vertex.
Solution:

Let the coordinates of the third vertex be (x, y)
Then, we know that the coordinates of centroid of the triangle are

$$\left(\frac{x + 1 + 3}{3}, \frac{y + 2 + 5}{3}\right)$$

Given that the centroid for the triangle is at the origin (0, 0)

$$\therefore \frac{x + 1 + 3}{3} = 0 \text{ and } \frac{y + 2 + 5}{3} = 0$$

$\Rightarrow x + 4 = 0$ $\Rightarrow y + 7 = 0$
$\Rightarrow x = -4$ $\Rightarrow y = -7$
Therefore, the coordinates of the third vertex is (-4, -7)

3. Find the third vertex of a triangle, if two of its vertices are at (-3, 1) and (0, -2) and the centroid is at the origin.

Solution:

Let the coordinates of the third vertex be (x, y)
Then, we know that the coordinates of centroid of the triangle are

$$\left(\frac{x - 3 + 0}{3}, \frac{y + 1 - 2}{3}\right)$$

Given that the centroid for the triangle is at the origin (0, 0)

$$\therefore \frac{x - 3 + 0}{3} = 0 \text{ and } \frac{y + 1 - 2}{3} = 0$$

$\Rightarrow$ x - 3 = 0 $\quad \Rightarrow$ y - 1 = 0
$\Rightarrow$ x = 3 $\quad \Rightarrow$ y = 1
Therefore, the coordinates of the third vertex is (3, 1)

4. A(3, 2) and B(-2, 1) are two vertices of a triangle ABC whose centroid G has the coordinates (5/3, -1/3). Find the coordinates of the third vertex C of the triangle.
Solution:

Let the coordinates of the third vertex C be (x, y)
Given, A(3, 2) and B(-2, 1) are two vertices of a triangle ABC
Then, we know that the coordinates of centroid of the triangle are

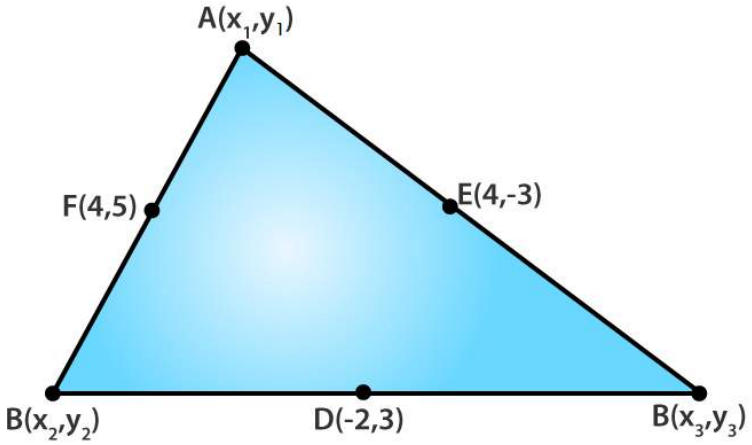
$$\left(\frac{x + 3 - 2}{3}, \frac{y + 2 + 1}{3}\right)$$

Given that the centroid for the triangle is **(**5/3, -1/3).

$$\therefore \frac{x + 3 - 2}{3} = 5/3 \text{ and } \frac{y + 2 + 1}{3} = -1/3$$

$\Rightarrow$ x + 1 = 5 $\quad \Rightarrow$ y + 3 = -1
$\Rightarrow$ x = 4 $\quad \Rightarrow$ y = -4
Therefore, the coordinates of the third vertex C is (4, -4)

5. If (-2, 3), (4, -3) and (4, 5) are the mid-points of the sides of a triangle, find the coordinates of its centroid.
Solution:



Let A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) be the vertices of triangle ABC.
Let D (-2, 3), E (4, -3) and F (4, 5) be the mid-points of sides BC, CA and AB respectively.
As D is the mid-point of BC

$$\frac{x_2+x_3}{2}=-2 \text{ and } \frac{y_2+y_3}{2}=3$$

$$x_2+x_3=-4 \text{ and } y_2+y_3=6 \quad \ldots\ldots. (1)$$

Similarly E and F are the mid-points of AC and AB

$$\frac{x_1+x_3}{2}=4 \text{ and } \frac{y_1+y_3}{2}=-3$$

$$x_1+x_3=8 \text{ and } y_1+y_3=-6 \quad \ldots\ldots.. (2)$$

And,

$$\frac{x_1+x_2}{2}=4 \text{ and } \frac{y_1+y_2}{2}=5$$

$$x_1+x_2=8 \text{ and } y_1+y_2=10 \quad \ldots\ldots (3)$$

From (1), (2) and (3), we have

$$x_2+x_3+x_1+x_3+x_1+x_2=-4+8+8 \text{ and}$$

$$y_2+y_3+y_1+y_3+y_1+y_2=6-6+10$$

$$2(x_1+x_2+x_3)=12 \text{ and } 2(y_1+y_2+y_3)=10$$

$$x_1+x_2+x_3=6 \text{ and } y_1+y_2+y_3=5 \quad \ldots\ldots.. (4)$$

Form (1) and (4), we get

$$x_1-4=6 \text{ and } y_1+6=5$$

$$x_1=10 \Rightarrow y_1=-1$$

Thus, the coordinates of A are (10, -1)
From (2) and (4), we get

$$x_2+8=6 \text{ and } y_2-6=5$$

$$x_2=-2 \Rightarrow y_2=11$$

Thus, the coordinates of B are (-2, 11)
From (3) and (4), we get

$$x_3+8=6 \text{ and } y_3+10=5$$

$$x_3=-2 \Rightarrow y_3=-5$$

Thus, the coordinates of C are (-2, -5)
Hence, the vertices of triangle ABC are A (10, -1), B (-2, 11) and C (-2, -5).
Therefore, the coordinates of the centroid of triangle ABC are

$$\left(\frac{10-2-2}{3}, \frac{-1+11-5}{3}\right)=\left(2, \frac{5}{3}\right)$$