Exercise 14.5

1. Find the area of a triangles whose vertices are
(i) (6, 3), (-3, 5) and (4, - 2)
(ii) [(at_1^2, at_1),(at_2^2, $2at_2$)(at_3^2, $2at_3$)]
(iii) (a, c + a), (a, c) and (-a, c - a)
Solution:

(i) Let A(6, 3), B(-3, 5) and C(4,-2) be the given points
We know that, area of a triangle is given by:
$1/2[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 + y_2)]$
Here,
$x_1 = 6$, $y_1 = 3$, $x_2 = -3$, $y_2 = 5$, $x_3 = 4$, $y_3 = -2$
So,
Area of ΔABC = 1/2 [6(5+2)+(-3)(- 2 -3)+ 4(3 - 5)]
=1/2 [6 × 7- 3 × (– 5) + 4(– 2)]
= 1/2[42 +15 - 8]
= 49/2 sq. units

(ii) Let A = $(x_1, y_1) = (at_1^2, 2at_1)$, B = $(x_2,y_2) = (at_2^2, 2at_2)$, C= $(x_3, y_3) = (at_3^2, 2at_2)$ be the given points.
Then,
The area of ΔABC is given by

$$= \frac{1}{2}[at_1^2(2at_2 - 2at_3) + at_2^2(2at_3 - 2at_1) + at_3^2(2at_1 - 2at_2)]$$

$$= \frac{1}{2}[at^2t_1^2t_2 - 2a^2t_1^2t_3 + 2a^2t_2^2t_3 - 2a^2t_2^2t_1 - 2a^2t_3^2t_2]$$

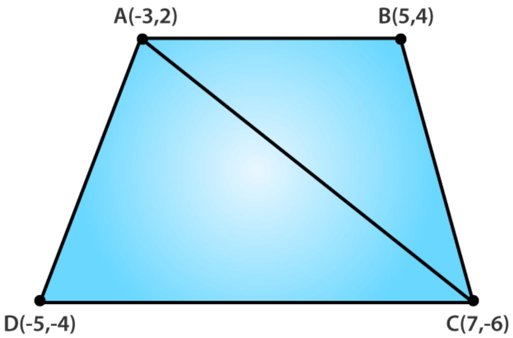
$$= \frac{1}{2} \times 2[a^2t_1^2(t_2 - t_3) + a^2t_2^2(t_3 - t_1) + a^2t_2^2(t_3 - t_1) + a^2 + t_3^2(t_1 - t_2)]$$

$$= a^2[t_1^2(t_2 - t_3) + t_2^2(t_3 - t_1) + t_3^2\ (t_1 - t_2)]$$

(iii) Let A = (x_1,y_1) = (a, c + a), B = (x_2, y_2) = (a, c) and C = (x_3, y_3) = (- a, c - a) be the given points
Then,
The area of ΔABC is given by
= 1/2[a (– {c - a}) + a(c – a - (c + a)) +(- a)(c + a - a)]
= 1/2 [a(c – c + a) + a(c – a – c - a) - a(c + a - c)]
= 1/2[a × a + ax(- 2a) – a × a]
= $1/2[a^2 - 2a^2 - a^2]$
= $1/2×(-2a)^2$
= $- a^2$

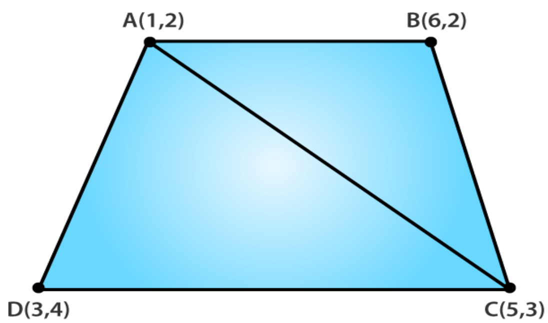
2. Find the area of the quadrilaterals, the coordinates of whose vertices are

(i) (-3, 2), (5, 4), (7, -6) and (-5, - 4)
(ii) (1, 2), (6, 2), (5, 3) and (3, 4)
(iii) (-4, -2), (-3, -5), (3, -2), (2, 3)
Solution:



(i)

Let A(-3, 2), B(5, 4), C(7,- 6) and D (-5, – 4) be the given points.
Area of ΔABC is given by
= 1/2[-3(4 + 6) + 5(- 6 - 2) + 7(2 - 4)]
= 1/2[-3×1 + 5×(-8) + 7(-2)]
= 1/2[- 30 – 40 -14]
= - 42
As the area cannot be negative,
The area of ΔADC = 42 square units
Now, area of ΔADC is given by
= 1/2[-3(– 6 + 4) + 7(- 4 - 2) + (- 5)(2 + 6)]
= 1/2[- 3(– 2) + 7(- 6) – 5 × 8]
= 1/2[6 – 42 - 40]
= 1/2 × – 76
= - 38
But, as the area cannot be negative,
The area of ΔADC = 38 square units
Thus, the area of quadrilateral ABCD = Ar. of ABC+ Ar. of ADC
= (42 + 38)
= 80 sq. units



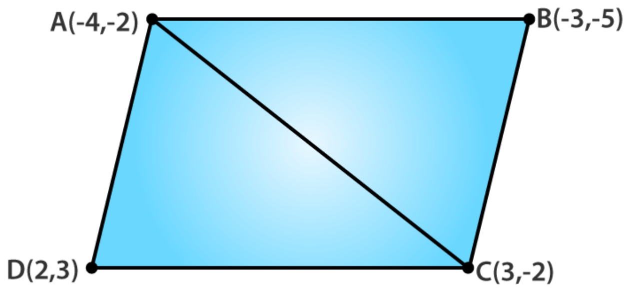
(ii)

Let A(1, 2) , B (6, 2) , C (5, 3) and (3, 4) be the given points

Firstly, area of ΔABC is given by
= 1/2[1(2 - 3) + 6(3 - 2) + 5(2 - 2)]
= 1/2[-1 + 6 × (1) + 0]
= 1/2[– 1 + 6]
= 5/2
Now, area of ΔADC is given by
= 1/2[1(3 - 4) + 5(4 - 2) + 3(2 - 3)]
= 1/2[-1 × 5 × 2 + 3(-1)]
= 1/2[-1 + 10 - 3]
= 1/2[6]
= 3
Thus, Area of quadrilateral ABCD = Area of ABC + Area of ADC

$$= \left(\frac{5}{2} + 3\right) \text{ sq. units}$$

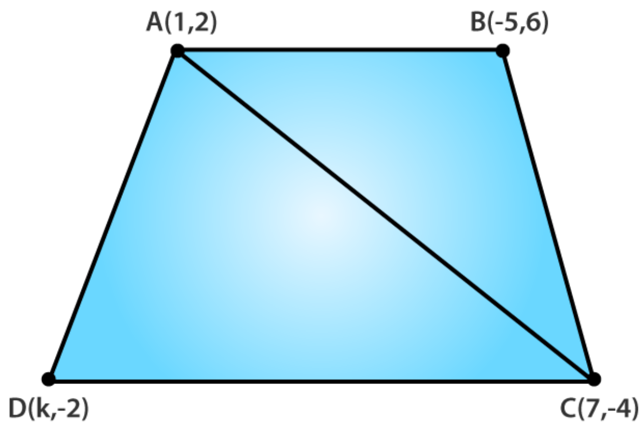
$$= \frac{11}{2} \text{ sq. units}$$



(iii)

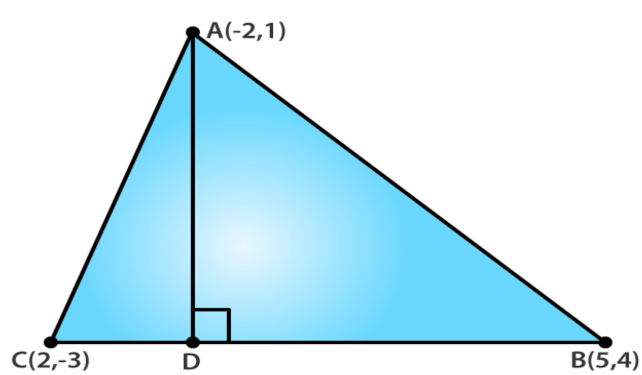
Let A (- 4, 2), B(– 3, – 5), C (3,- 2) and D(2, 3) be the given points
Firstly, area of ΔABC is given by
= 1/2|(- 4)(- 5 + 2) - 3(-2 + 2) + 3(- 2 + 5)|
= 1/2|(-4)(-3) – 3(0) + 3(3)|
= 21/2
Now, the area of ΔACD is given by
= 1/2|(– 4)(3 + 2) + 2(– 2 + 2) + 3(– 2 – 3)|
= 1/2|- 4(5) + 2(0) + 3(- 5)|= (- 35)/2
But, as the area can't negative,
The area of ΔADC = 35/2
Thus, the area of quadrilateral (ABCD) = ar(ΔABC) + ar(ΔADC)
= 21/2 + 35/2
= 56/2
= 28 sq. units

3. The four vertices of a quadrilateral are (1, 2), (-5, 6), (7, -4) and (k, -2) taken in order. If the area of the quadrilateral is zero, find the value of k.
Solution:



Let A(1, 2), B(-5, 6), C(7, -4) and D(k, -2) be the given points
Firstly, area of ΔABC is given by
= 1/2|(1)(6 + 4) - 5(-4 + 2) + 7(2 - 6)|
= 1/2|10 + 30 - 28|
= ½ x 12
= 6
Now, the area of ΔACD is given by
= 1/2|(1)(-4 + 2) + 7(– 2 - 2) + k(2 + 4)|
= 1/2|- 2 + 7x(-4) + k(6)|
= (- 30 + 6k)/2
= -15 + 3k
= 3k - 15
Thus, the area of quadrilateral (ABCD) = ar(ΔABC) + ar(ΔADC)
= 6 + 3k - 15
= 3k - 9
But, given area of quadrilateral is O.
So, 3k – 9 = 0
k = 9/3 = 3

4. The vertices of ΔABC are (-2, 1), (5, 4) and (2, -3) respectively. Find the area of the triangle and the length of the altitude through A.
Solution:



Let A(-2, 1), B(5, 4) and C(2, -3) be the vertices of ΔABC.
And let AD be the altitude through A.
Area of ΔABC is given by
= 1/2|(-2)(4 + 3) - 5(-3 - 1) + 2(1 - 4)|
= 1/2|-14 - 20 - 6|
= ½ x -40
= -20
But as the area cannot be negative,
The area of ΔABC = 20 sq. units
Now,

$$BC = \sqrt{(5-2)^2 + (4+3)^2}$$

$$BC = \sqrt{(3)^2 + (7)^2}$$

$$BC = \sqrt{58}$$

We know that, area of triangle
= ½ x Base x Altitude
20 = ½ x √58 x AD
AD = 40/ √58
Therefore, the altitude AD = 40/ √58

5. Show that the following sets of points are collinear.
(a) (2, 5), (4, 6) and (8, 8) (ii) (1, -1), (2, 1) and (4, 5)
Solution:

Condition: For the 3 points to be collinear the area of the triangle formed with the 3 points has to be zero.

(a) Let A(2, 5), B(4, 6) and C(8, 8) be the given points
Then, the area of ΔABC is given by

$$= \frac{1}{2}[2(6-8) + 4(8-5) + 8(5-6)]$$

$$= \frac{1}{2}[2 \times (-2) + 4 \times 3 + 8 \times (-1)]$$

$$= \frac{1}{2}[-4 + 12 - 8]$$

$$= \frac{1}{2} \times 0$$

$$= 0$$

Since, the area (ΔABC) = 0 the given points (2, 5), (4, 6) and (8, 8) are collinear.

(b) Let A(1, -1), B(2, 1) and C(4, 5) be the given points
Then, the area of ΔABC is given by

$= \frac{1}{2}[1(1-5)+2(5+1)+4(-1-1)]$

$= \frac{1}{2}[-4+12-8]$

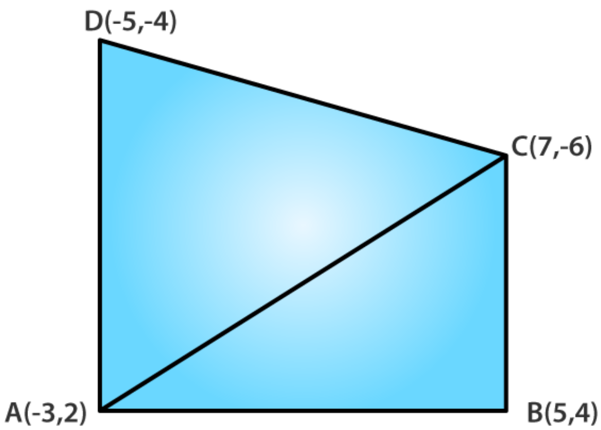
$= \frac{1}{2} \times 0$

$= 0$

Since, the area (ΔABC) = 0 the given points (1, -1), (2, 1) and (4, 5) are collinear.

6. Find the area of a quadrilateral ABCD, the coordinates of whose vertices are A (-3, 2), B (5, 4), C (7, 6) and D (-5, -4).

Solution:



Let's join AC. So, we have 2 triangles formed.

Now, the ar (ABCD) = Ar (ΔABC) + Ar (ΔACD)

Area of ΔABC is given by,

$\frac{1}{2}|-3(4+6)+5(-6-2)+7(2-4)|$

$= \frac{1}{2}|-30-40-14|$

$= \frac{1}{2} \times 84$

$= 42$ sq. units

Next, the area of ΔACD is given by,

$\frac{1}{2}|-3(-6+4)+7(-4-2)-5(2+6)|$

$= \frac{1}{2}|6-42-40|$

$= \frac{1}{2} \times 76$

$= 38$ sq. units

Thus, the area (ABCD) = 42 + 38 = 80 sq. units

7. In △ABC, the coordinates of vertex A are (0, -1) and D(1, 0) and E(0, 1) respectively the mid-points of the sides AB and AC. If F is the mid-point of side BC, find the area of △DEF.
Solution:

Let B(a, b) and C(p, q) be the other two vertices of the △ABC
Now, we know that D is the mid-point of AB
So, coordinates of D = (0+a/ 2, -1+b/ 2)
(1, 0) = (a/2, b-1/2)
1 = a/2 and 0 = (b-1)/ 2
a = 2 and b = 1
Hence, the coordinates of B = (2, 1)
And, now
E is the mid-point of AC.
So, coordinates of E = (0+p/ 2, -1+q/ 2)
(0, 1) = (p/2 , (q -1)/ 2)
p/2 = 0 and 1 = (q - 1)/2
p = 0 and 2 = q -1
p = 0 and q = 3
Hence, the coordinates of C = (0, 3)
Again, F is the mid-point of BC
Coordinates of F = (2+0/ 2, 1+3/ 2) = (1, 2)
Thus, the area of △DEF is given by

$$= \frac{1}{2}|1(1-2)+0(2-0)+1(0-1)|$$
$$= \frac{1}{2}|-1+0-1|$$
$$= \frac{1}{2}\times 2$$
$$= 1 \text{ sq. unit}$$

8. Find the area of the triangle PQR with Q (3, 2) and the mid-points of the sides through Q being (2, -1) and (1, 2).
Solution:

Let the coordinates of P and R be (x_1, y_1) and (x_2, y_2) respectively.
And, let the points E and F be the mid-points of PQ and QR respectively.

$$\frac{x_1+3}{2} = 1, \frac{y_1+2}{2} = 2 \text{ and } \frac{x_2+3}{2} = 2, \frac{y_2+2}{2} = -1$$

$x_1 + 3 = 2$, $y_1 + 2 = 4$ and $x_2 + 3 = 4$, $y_2 + 2 = -2$
$x_1 = -1$, $y_1 = 2$ and $x_2 = 1$, $y_2 = -4$
Hence, the coordinates of P and R are (-1, 2) and (1, 0) respectively.
Therefore, the area of △PQR is given by

$= \frac{1}{2}|(3 \times 2 + (-1) \times (-4) + 1 \times 2) - (-1 \times 2 + 1 \times 2 + 3 \times (-4))|$

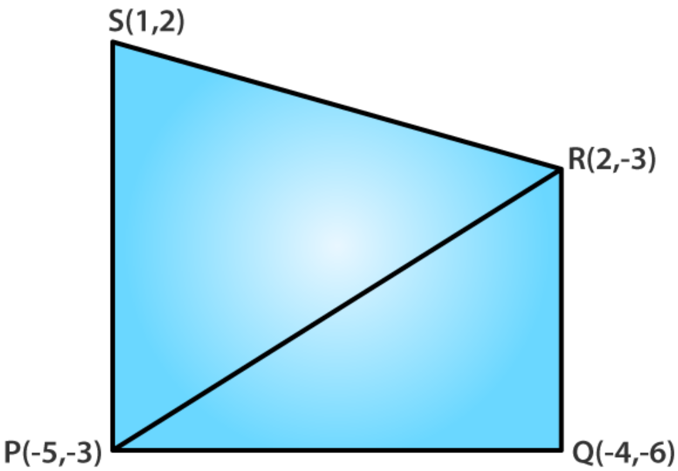
$= \frac{1}{2}|(6 + 4 + 2) - (-2 + 2 - 12)|$

$= \frac{1}{2} \times |24|$

$= \frac{1}{2} \times 24$

$= 12$ sq. units

9. If P(-5, -3), Q(-4, -6), R(2, -3) and S(1, 2) are the vertices of a quadrilateral PQRS, find its area.
Solution:



First, let's join P and R.
Then,
Area of △PSR is given by

$= \frac{1}{2}|-5(2+3) + 1(-3+3) + 2(-3-2)|$

$= \frac{1}{2}|-5 \times 5 + 1 \times 0 + 2 \times (-5)|$

$= \frac{1}{2}|-25 + 0 - 10|$

$= \frac{1}{2}|-35|$

$= \frac{35}{2}$

And, now
Area of △PQR is given by

$= \frac{1}{2}|-5(-6+3)-4(-3+3)+2(-3+6)|$

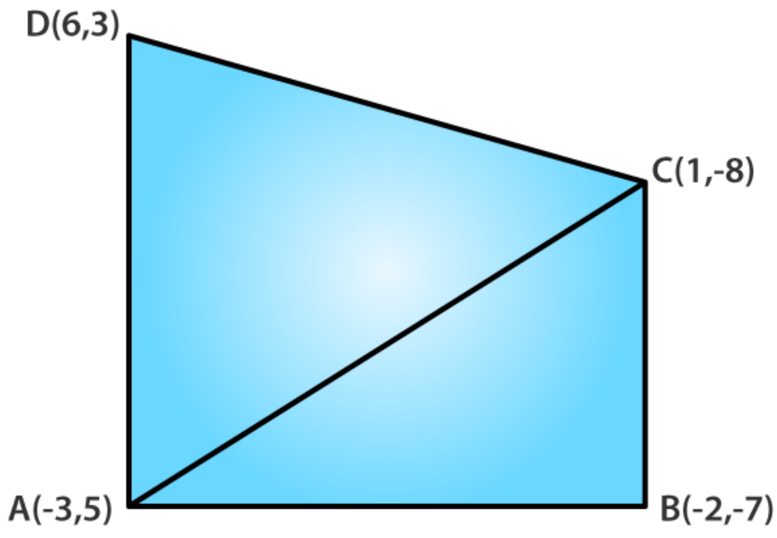
$= \frac{1}{2}|-5\times(-3)-4\times0+2\times3|$

$= \frac{1}{2}|15+0+6|$

$= \frac{21}{2}$

Thus,
Area of quad. PQRS = Area of △PSR + Area of △PQR
= 35/2 + 21/2
= 56/2
= 28 sq. units

10. If A (-3, 5), B(-2, -7), C(1, -8) and D(6, 3) are the vertices of a quadrilateral ABCD, find its area.
Solution:



Let's join A and C.
So, we get △ABC and △ADC
Hence,
The Area of quad. ABCD = Area of △ABC + Area of △ADC

$= \frac{1}{2}|-3(-7-(-8))+(-2)(-8-5)+1(5-(-7))| + \frac{1}{2}|-3(3+8)+6(-8-5)+1(5-3)|$

$= \frac{1}{2}|-3(1)+(-2)(-13)+1(12)| + \frac{1}{2}|-3(11)+6(-13)+1(2)|$

$= \frac{1}{2}|-3+26+12| + \frac{1}{2}|-33-78+2|$

$= \frac{1}{2}|35| + \frac{1}{2}|-109|$

$= \frac{1}{2} \times 35 + \frac{1}{2} \times 109$

$= \frac{35+109}{2}$

$= \frac{144}{2}$

$= 72$ sq. units

Therefore, the area of the quadrilateral ABCD is 72 sq. units

11. For what value of a the points (a, 1), (1, -1) and (11, 4) are collinear?
Solution:

Let A (a, 1), B (1, -1) and C (11, 4) be the given points
Then the area of △ABC is given by,

$= \frac{1}{2}\{a(-1-4)+1(4-1)+11(1+1)\}$

$= \frac{1}{2}\{-5a+3+22\}$

$= \frac{1}{2}\{-5a+25\}$

We know that for the points to be collinear the area of △ABC has to be zero.
½(-5a + 25) = 0
5a = 25
∴ a = 5

12. Prove that the points (a, b), (a_1, b_1) and (a-a_1, b-b_1) are collinear if $ab_1 = a_1b$
Solution:

Let A (a, b), B (a_1, b_1) and C (a-a_1, b-b_1) be the given points.
So, the area of △ABC is given by,

$= \frac{1}{2}\{a[b_1 - (b - b_1)] + a_1(b - b_1 - b) + (a - a_1)(b - b_1)\}$

$= \frac{1}{2}\{a(b_1 - b + b_1) + a_1(-b_1) + ab - ab_1 - a_1b + a_1b_1\}$

$= \frac{1}{2}\{ab_1 - ab + ab_1 - a_1b_1 + ab - ab_1 - a_1b + a_1b_1\}$

$= \frac{1}{2}\{ab_1 - a_1b\}$

So, only if $ab_1 = a_1b$ the area becomes zero
△ABC = ½ (0) = 0
Therefore, the given points are collinear if $ab_1 = a_1b$

13. If the vertices of a triangle are (1,-3), (4,p) and (-9, 7) and its area is 15 sq. units, find the value (s) of p.
Solution:

Let A(1,-3), B(4,p) and C(-9, 7) be the vertices of △ABC
Area of △ABC = 15 sq. units

$$15 = \frac{1}{2}|1(p-7) + 4(7+3) - 9(-3-p)|$$

$$15 = \frac{1}{2}|p - 7 + 40 + 27 + 9p|$$

$$15 = \frac{1}{2}|10p + 60|$$

When modulus is removed, two cases arise:

$30 = 10p + 60$ or $30 = -10p - 60$

$10p = -30$ or $10p = -90$

$p = -3$ or $p = -9$

14. If (x, y) be on the line joining the two points (1, -3) and (-4, 2). Prove that x + y + 2 = 0
Solution:

Let A (x, y), B (1, -3) and C (-4, 2) be the given points.
Area of △ABC is given by,

$= \frac{1}{2}\{x(-3-2) + 1(2-y) + (-4)(y+3)\}$

$= \frac{1}{2}\{-5x + 2 - y - 4y - 12\}$

$= \frac{1}{2}\{-5x - 5y - 10\}$

As, the three points lie on the same line (that means they are collinear).
Then, the area of △ABC = 0
½ (-5x – 5y - 10) = 0

-5x – 5y – 10 = 0
-5(x + y + 2) = 0
x + y + 2 = 0
- Hence proved

15. Find the value of k if points (k, 3), (6, -2) and (-3, 4) are collinear.
Solution:

Let A (k, 3), B (6, -2) and C (-3, 4) be the given points.
Then, the area of △ABC is given by,

$$= \frac{1}{2}\{k(-2-4)+6(4-3)+(-3)(3+2)\}$$

$$= \frac{1}{2}\{-6k+6-15\}$$

$$= \frac{1}{2}\{-6k-9\}$$

As, the points are collinear.
Area of △ABC has to be zero.
½ x (-6k – 9) = 0
-6k – 9 = 0
k = -9/6
∴ k = -3/2

16. Find the value of k, if points A(7, -2), B(5, 1) and C(3, 2k) are collinear.
Solution:

Given,
Points A(7, -2), B(5, 1) and C(3, 2k)
Then, the area of △ABC is given by,

$$= \frac{1}{2}\{7(1-2k)+5(2k+2)+3(-2-1)\}$$

$$= \frac{1}{2}\{7-14k+10k+10-9\}$$

$$= \frac{1}{2}\{-4k+8\}$$

As, the points are collinear.
Area of △ABC has to be zero.
½ (-4k + 8) = 0
-4k + 8 = 0
-4k = -8
∴ k = 2